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STRUCTURES NOTE 456

ANALYSIS OF SERVICE CRACKING IN F-27 AIRCRAFT WINGS

by

G. S. JOST

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SUMMARY

Weibull extreme value distributions have been fitted to times to first cracking in the wing access openings of Australian and Papua-New Guinea F-27 aircraft. A method is given for estimating the distribution parameters from the service data, which include very often more runouts than cracks.

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16.   *ABSTRACT*

Weibull extreme value distribution have been fitted to times to first cracking in the wing access openings of Australian and Papua-New Guinea F-27 aircraft. A method is given for estimating the distribution parameters from the service data, which include very often more runouts than cracks.

CONTENTS

1. BACKGROUND

The Department of Transport is concerned about service cracking in the holes of bolts securing the tank access doors to the lower surfaces of the wings of Australian and Papua-New Guinea F-27 aircraft. Each wing contains five* such openings (ten per aircraft) and each door is secured by 62 bolts, Figure 1. Information concerning the detection times of cracks at these bolt holes is available, together with total flying time for each aircraft.

This note presents an appropriate and well based method for fitting an extreme value distribution to such data along with examples of its application.

2. THE WEIBULL DISTRIBUTION

It is presumed at the outset that that crack which is of prime importance in a structure, in which many cracks may occur, is the first one to appear. This is an extreme value situation for which appropriate distributions are available. That chosen, as much for its manipulative ease as for its demonstrated suitability, is the two parameter Weibull distribution. It takes the following forms:

$$f(t) = (\alpha/\beta)(t/\beta)^{\alpha-1} \exp\left[-(t/\beta)^\alpha\right], 0 \leqslant t \leqslant \infty \tag{1}$$

and

$$F(t) = 1 - \exp\left[-(t/\beta)^\alpha\right] \tag{2}$$

Here, $f(t)$ is the probability density function of the random variable t (time to crack initiation), α and β are the dispersion and location parameters respectively and $F(t)$ is the cumulative distribution function

$$F(t) = \int_0^t f(t)\, dt.$$

2.1 Parameter Estimation

There are several methods for fitting α and β to given data[1]. Another approach, which has received publicity in the aeronautical fatigue field, deals with the remarkable situation in which no cracks have yet been detected[2]. In this method, a value of α is assumed a priori and, given the current lives of all aircraft in the fleet, and assuming that the first crack is just about to occur, β may be calculated. The necessity of assuming α is seen as one of the less satisfactory aspects of this approach, although no sensible alternative presents itself from so little data. For the F-27 fleet there is no shortage of information on cracking in the wings, and the standard method of maximum likelihood[1] may be applied to estimate the distribution parameters.

2.2 Maximum Likelihood Estimators

The situation here is one of random censoring. At any particular inspection time each item under investigation is either cracked, or it is not. As this is the case for all aircraft in the fleet, every cracked item has associated with it a time to initiate (and have detected) that crack, and every uncracked item has associated with it a known time less than that to initiate a crack,

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* No cracking has yet been detected in the outermost access door opening of each wing.

i.e. a runout time. Thus, if there exist n cracked items at independent times t_i ($1 \leqslant i \leqslant n$) and m uncracked items at times t_j ($1 \leqslant j \leqslant m$), the likelihood function of these data is given by the joint probabilities of the times to cracking and the runout times:

$$\prod_{i=1}^{n} f(t_i) \prod_{j=1}^{m} [1 - F(t_j)] \quad e^L \text{ say.}$$

Taking logarithms,

$$L = \sum^{n} \ln f(t_i) + \sum^{m} \ln [1 - F(t_j)] \tag{3}$$

Substituting from (1) for $f(t)$ and (2) for $F(t)$ into (3) and simplifying yields

$$L = n(\ln \alpha - \ln \beta) + (\alpha - 1)(\sum^{n} \ln t - n \ln \beta) - \sum^{m+n} (t_i/\beta)^\alpha \tag{4}$$

Maximum likelihood estimates for the parameters α and β are obtained from (4) by differentiating with respect to each parameter and equating to zero. Putting

$$\partial L / \partial \alpha = 0$$

yields

$$\hat{\alpha} = \sum^{m+n} t^\alpha \left[\sum^{m+n} t^\alpha \ln t - \sum^{n} \ln t \sum^{m+n} t^\alpha / n \right] \tag{5}$$

and putting

$$\partial L / \partial \beta = 0$$

yields

$$n\beta^\alpha = \sum^{m+n} t^\alpha \tag{6}$$

Equation (5) requires solution by iterative substitution for $\hat{\alpha}$, after which $\hat{\beta}$ may be found from (6).

The maximum likelihood method may also be applied to provide a pooled α estimate from k data groups. In this case each group contains n_k cracks and m_k runouts, and the pooled α is given by

$$\hat{\alpha}_p = \sum_k n_k \sum_k \left[(m_k \sum_{i=1}^{m_k} t_{ik}^\alpha \ln t_{ik})(\sum_{i=1}^{m_k} t_{ik}^\alpha) - \sum_{i=1}^{n_k} \ln t_{ik} \right] \tag{7}$$

3. THE DATA

Comprehensive data[4] have been made available giving the utilisation and time to detection of cracking in the F-27 fleet. Utilisation data are listed and graphed in Table 1 and Figure 2, and condensed versions of the data on cracking are given in Tables 2 and 3. Although there are several models of the F-27 in the Australian and Papua-New Guinea fleets, all wings are assumed here to be nominally identical, or at least not significantly different, in the regions of interest.

3.1 Treatment of the Data

There are several levels at which the data may be considered, not all of which are necessarily meaningful. For example, a coarse level would be that in which only the time to detection of the first crack in the wing was noted, irrespective of its location. This corresponds to treating the data on a wing by wing basis. Similarly, time to first crack detection may be considered for each access door opening regardless of which bolt hole is cracked, for a given bolt hole (but not which position in the hole) and so on, on finer, and probably progressively more pointless, bases, were the appropriate data available. The data of Tables 2 and 3 have been processed

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here on three levels: by wings (two per aircraft), by openings (five pairs per aircraft) and for two pairs of bolt holes in the innermost wing openings (Numbers 94 and 155).

A check on the comparative times to first cracking in port and starboard wings, Figure 3, reveals no significant difference. Thus the times to detection of the first crack in each wing may legitimately be pooled. A similar check on data at differing spanwise openings shows no significant difference port to starboard. However, as expected, an increasing incidence of cracking occurs from outboard to inboard, Figure 5. Pooling of openings data is therefore legitimate only for corresponding port and starboard openings. Finally, data from four bolt holes, two from corresponding positions in each of the innermost port and starboard openings have been analysed.

By way of example, the pooled data for wings are given in Table 4 after every thousand flight hours. Corresponding listings are required for data considered by opening and by bolt hole for Weibull parameter estimation.

Figures 4 and 5 show the average number of cracked wings per aircraft and cracked openings per aircraft as functions of flying time. In addition, maxima and minima are shown, these being the data for the extreme aircraft in the fleet. The data of Figure 4 show an approximately linear relationship beyond about 14 000 hours, indicating a constant crack incidence rate for wings. Considered in terms of openings, however, particularly those inboard, the data of Figure 5 show some trends towards an increasing incidence rate with time: in these cases power laws might better relate the number of cracked openings per aircraft as a function of time.

3.2 Comment on the Data

There is an aspect of these data which should be noted. The time at which each crack is detected, and its actual initiation time (however that be defined) will rarely be the same. Similarly, known runout times are actually those of the most recent inspection, not current aircraft hours (as used here). Both sources of error act in the same sense, i.e. cracking times and runout times as used here will be slightly greater than they should be. Thus means established from these data will be slightly high, but variability should be little affected.

4. FITTING THE DISTRIBUTION TO THE DATA

4.1 Calculation of Parameters

The data of Tables 1, 2 and 3 have been used, in the form of Table 4, with equations (5) and (6) to estimate the dispersion and location parameters, α and β, of the Weibull distribution for the data considered by wings, openings and the selected bolt holes. The values are listed in Table 5 along with the corresponding numbers of data points for each estimate and pooled α for openings and bolt holes.

The relative magnitudes of the estimates may be understood from the nature of the data, which is the time to first cracking of the wing, opening or bolt hole. As the chosen basis becomes more selective, progressively more of the total available data are being truncated, and the percentage of cracked data points is being reduced. Thus the range of the α may be expected to increase as the estimates of β increase: these trends can be seen from the Table.

The incidence of cracking is largest in the two selected pairs of bolt holes. Even so, only one of these holes is cracked for every nine which are uncracked. Legitimate general pooling of bolt hole data to improve confidence in their parameter estimates may be justified only on the basis of local stress analysis. As the data stand, the dispersion parameter values are generally less than that of four, assumed in Reference 2, corresponding to a little more scatter in the F-27 data considered here.

4.2 The Fit of the Distributions

Fitted Weibull density functions, normalised histograms of corresponding cracked items and those for uncracked items for wings, openings and bolt holes are shown in Figures 6, 7 and 8 respectively. These give a useful visual indication of comparative parameter changes, and the

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relative confidence which may be associated with their estimation as the basis for treating the data becomes more selective. Survivorship functions and frequency of survival data points for wings and the inboard openings are shown on extreme value probability plots in Figures 9 and 10.

A better idea of the change in survivorship function with time is, perhaps, gained by replotting the functional relationships of Figures 9 and 10, along with those of the other data, on linear scales, Figure 11. After an initial plateau region early in life, a virtually linear reduction of survivorship with time takes place down to the 10% to 15% level. The relative positions of the sets of curves demonstrate that the more particular and restricted the inspection detail, the greater is the probability of that detail remaining uncracked to a given time.

4.3 Risk Rates

The hourly risk rate, or probability that cracking will occur in the next time increment, given that the item has not yet cracked, is given by

$$r(t) = f(t)/[1 - F(t)] \tag{7}$$

For the Weibull distribution this becomes

$$r(t) = (\alpha/\beta) \, (t/\beta)^{\alpha - 1} \tag{8}$$

Using the values of α and β estimated here for wings, openings and bolt holes yields the plots of Figure 12. Irrespective of the data base, the risk rate increases at an ever increasing rate with time.

5. CONCLUSIONS

The availability of unusually detailed data on times to crack detection in a fleet of aircraft has permitted the application of a soundly based procedure for fitting the appropriate two parameter Weibull distribution. The results presented here for the details considered may be used with confidence in assessing airworthiness actions for the F-27 fleet, the method itself being applicable to any like situation for which inspection data are available.

ACKNOWLEDGMENTS

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2. Thompson, P. A. A Procedure for Estimating the Demonstrated Fatigue Life of Airplane Structure from Fleet Service Experience. The Boeing Company, Document No. D6-41246 TN, July 1973.

3. — Data from the Department of Transport, Airworthiness Branch, Melbourne, February 1978.

TABLE 1

Papua-New Guinea and Australian F-27 Aircraft Utilisation
[December 1977]

Aircraft No.	Current Aircraft Hours	Aircraft No.	Current Aircraft Hours
1	34 393	31	38 500
2	35 769	32	47 500
3	35 843	33	45 500
4	34 878	34	37 172
5	29 257	35	33 896
6	22 968	36	24 282
7	27 752	37	27 528
8	40 178	38	28 933
9	43 594	39	26 855
10	26 389	40	26 220
11	24 158	41	26 590
12	25 475	42	16 892
13	32 975	43	27 369
14	31 220	44	27 852
15	29 500	45	12 828
16	14 332	46	13 171
17	17 516	47	30 500
18	1 610	48	37 841
19	1 696	49	20 965
20	36 825	50	37 706
21	44 183	51	38 096
22	33 727	52	24 439
23	52 123	53	24 039
24	24 005	54	26 974
25	5 488	55	23 265
26	5 065	56	23 005
27	3 896	57	23 323
28	3 801	58	23 072
29	1 111	59	18 651
30	593	60	15 697
		61	13 624

TABLE 2

Wing Opening Crack Incidence Data, Papua-New Guinea and Australia F-27 Fleets

Aircraft No.	Cracked Opening		Aircraft No.	Cracked Opening	
	No.	Hours		No.	Hours
1	94	26 625	21	155	33 700*
	93	27 775		93	36 337
	155	27 775		94	36 337
2	94	34 629		153	36 337
3	94	32 456	22	94	20 200*
	155	32 456		155	20 200*
4	92	24 817		92	23 392
	154	24 817		93	23 392
	155	24 817	23	93	41 800
	153	33 532		94	41 800
5	151	27 615		154	41 800
	154	27 615		155	41 800
	155	27 615	24	90	17 197
7	94	17 038		91	17 197
	153	22 583		151	17 197
	90	25 677		152	17 197
8	93	29 900*		153	17 197
	94	35 330		92	22 798
	154	35 330	31	155	34 038
	155	35 330		93	36 517
9	94	34 836		151	36 517
	154	34 836		154	36 517
	91	38 402	32	94	36 236
	92	38 402		155	36 236
	151	38 402		90	45 568
	152	38 402		91	45 568
11	92	17 602		93	45 568
	94	19 955		151	45 568
	155	19 955		152	45 568
12	91	22 828	33	154	44 236
14	155	17 450	34	92	27 012
15	91	28 295		93	27 012
	92	28 295		94	27 012
	93	28 295		90	32 267
	94	28 295		155	35 021
	153	28 295	35	90	23 200*
	154	28 295		94	23 200*
				155	23 200*

* Estimated.

TABLE 2—[Continued]

Aircraft No.	Cracked Opening		Aircraft No.	Cracked Opening	
	No.	Hours		No.	Hours
36	91	18 401	44	92	21 374
	154	18 401		94	24 623
37	93	16 400*	46	155	10 298
	94	16 400*	47	92	29 060
	155	16 400*		94	29 060
	90	23 935		153	29 060
	92	23 935		154	29 060
	151	23 935		155	29 060
	153	24 287	48	92	28 329
	154	24 287		93	28 329
38	94	16 900*		94	28 329
	153	16 900*		154	28 329
	154	19 900*		155	28 329
	155	17 502	49	93	17 228
	90	23 695		94	17 228
39	93	17 100*		153	17 228
	94	17 100*		155	17 228
	154	17 100*	50	92	28 723
	155	17 100*		93	28 723
	92	25 642		94	28 723
	151	25 642		153	28 723
40	92	15 200*		154	28 723
	93	15 200*		155	28 723
	94	15 200*		152	34 280
	154	15 200*	51	93	29 052
	151	21 784		94	29 052
	153	21 784		153	29 052
41	93	16 727		154	29 052
	153	16 727		155	29 052
	154	16 727	52	153	19 865
	90	23 360		155	19 865
	91	23 360	54	155	21 376
	151	23 360	55	154	19 351
42	92	7 648	56	155	15 792
	93	7 648	57	154	16 330
	154	7 648		155	18 584
	155	7 648	58	154	19 054
	94	11 557	59	154	14 933
				155	14 933

* Estimated.

TABLE 3

**Cracking at Bolt Holes 1 and 9, Opening 94 and 155
Papua-New Guinea and Australian F-27 Fleets**

Aircraft No.	Cracked Opening and Hole			Current Aircraft Hours
	Opening	Hole	Hours	
1	94	1	33 829	34 393
	155	9	33 829	
3	155	9	32 456	35 843
5	155	9	27 615	29 257
8	94	9	35 330	40 178
	155	1	35 330	
11	94	9	19 955	24 158
21	155	1	33 686	44 183
22	94	9	23 392	33 727
23	155	1	41 800*	52 123
		9	41 800*	
31	155	1	34 038	38 500*
34	94	1	27 012	37 172
35	94	1	23 200*	33 896
38	94	1	23 695	28 933
40	94	9	15 200*	26 220
49	94	9	17 228	20 965
	155	1	17 228	
50	94	1	32 445	37 706
	155	9	32 445	
51	94	9	29 052	38 096
	155	1	29 052	
59	155	9	14 933	18 651

* Estimated.

TABLE 4

Wing Cracking Incidence Data-Statistics

Flying Time (Khr)	No. of Aircraft in Service	No. of Cracked Wings*	Cracked Wings per Aircraft	No. of Uncracked Wings	Frequency of Survival
0	61	0	0	122	1
1	60	0	0	120	1
2	57	0	0	114	1
3	57	0	0	114	1
4	55	0	0	110	1
5	55	0	0	110	1
6	53	0	0	106	1
7	53	0	0	106	1
8	53	2	0·038	104	0·981
9	53	2	0·038	104	0·981
10	53	2	0·038	104	0·981
11	53	3	0·057	103	0·972
12	53	3	0·057	103	0·972
13	52	3	0·058	101	0·971
14	50	3	0·060	97	0·970
15	49	3	0·061	95	0·969
16	48	6	0·125	88	0·917
17	47	11	0·234	83	0·883
18	46	19	0·413	73	0·793
19	45	20	0·444	70	0·778
20	45	25	0·556	65	0·722
21	44	25	0·568	63	0·716
22	44	27	0·614	61	0·693
23	43	29	0·674	57	0·663
24	39	27	0·692	51	0·654
25	34	22	0·647	46	0·676
26	33	21	0·636	45	0·682
27	28	15	0·536	41	0·732
28	24	13	0·542	35	0·729
29	23	17	0·739	29	0·630
30	21	19	0·905	23	0·548
31	20	17	0·85	23	0·575
32	19	16	0·842	22	0·579
33	18	18	1	18	0·5
34	16	15	0·938	17	0·531
35	14	15	1·071	13	0·464
36	12	14	1·167	10	0·417
37	11	18	1·636	4	0·182
38	8	12	1·5	4	0·25
39	6	8	1·333	4	0·33
40	6	8	1·333	4	0·33

* In remaining aircraft.

TABLE 4—[Continued]

Flying Time (Khr)	No. of Aircraft in Service	No. of Cracked Wings*	Cracked Wings per Aircraft	No. of Uncracked Wings	Frequency of Survival
41	5	6	1·2	4	0·4
42	5	8	1·6	2	0·2
43	5	8	1·6	2	0·4
44	4	6	1·5	2	0·5
45	3	5	1·667	1	0·333
46	2	4	2	0	0
47	2	4	2	0	0
48	1	2	2	0	0
49	1	2	2	0	0
50	1	2	2	0	0
51	1	2	2	0	0
52	1	2	2	0	0
53	0				

* In remaining aircraft.

TABLE 5

Weibull Distribution Parameters F-27 Fleet of 61 Aircraft

Time to First Cracking in (Component)		No. of Data Points		Parameter	
		Cracked	Uncracked	α	β (hours)
Wings		70	52	3·24	30 800
Openings (Port Stbd)	90 151	17	105	3·27	54 300
	91 152	11	111	3·74	59 000
	92 153	29	93	2·67	48 600
	93 154	41	81	3·03	39 700
	94 155	54	68	3·29	34 400
	Pooled			3·12	
Bolt Holes (Hole Opening)	1 94	5	56	3·59	61 500
	9 155	6	55	4·21	53 800
	9 94	6	55	2·50	72 600
	1 155	6	55	5·34	48 900
	1 94 & 9 155	11	111	3·87	57 400
	9 94 & 1 155	12	110	3·46	58 700
	Pooled			3·66	
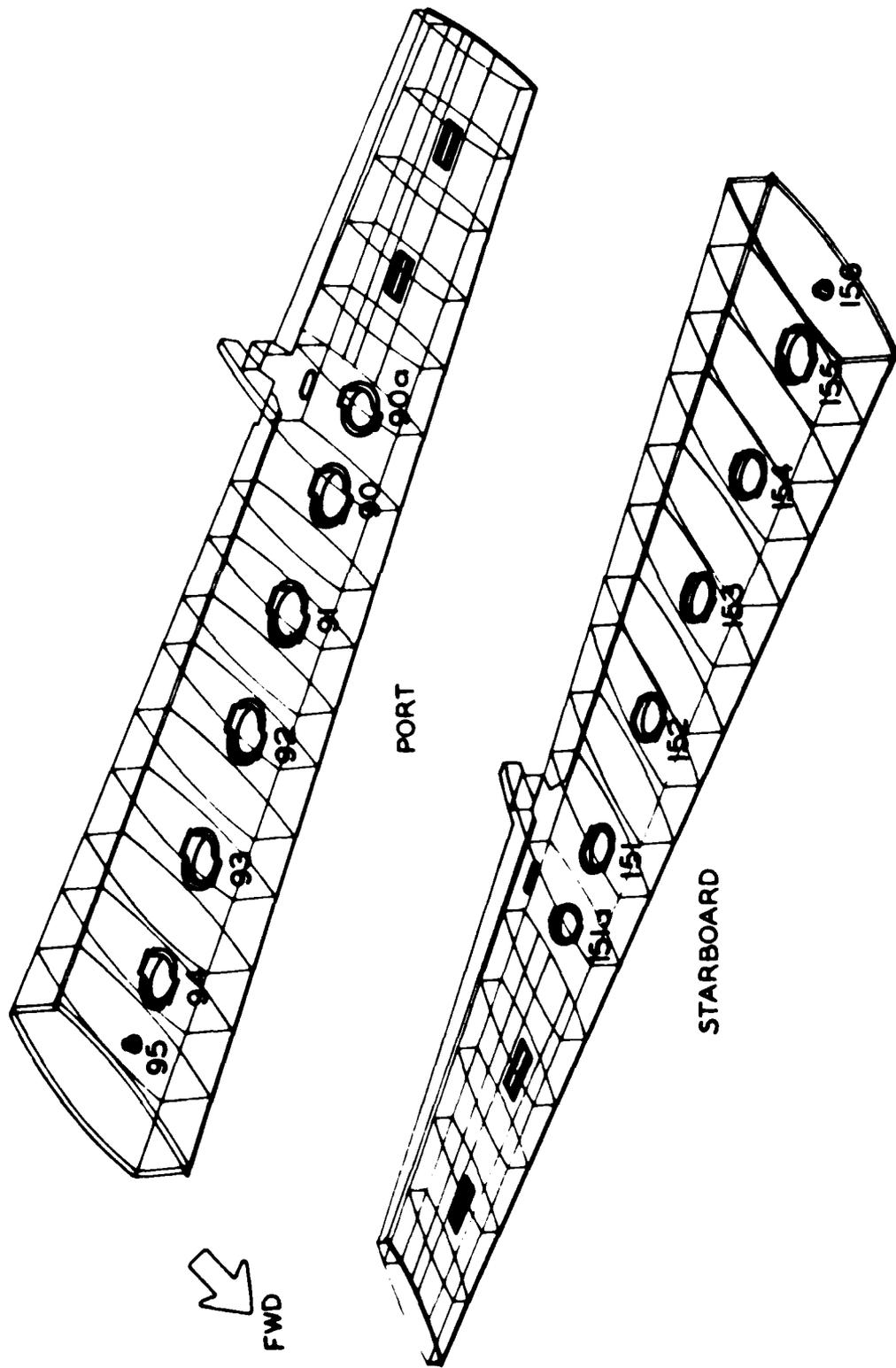
FIG. 1(a) F-27 WING ACCESS OPENING DETAIL
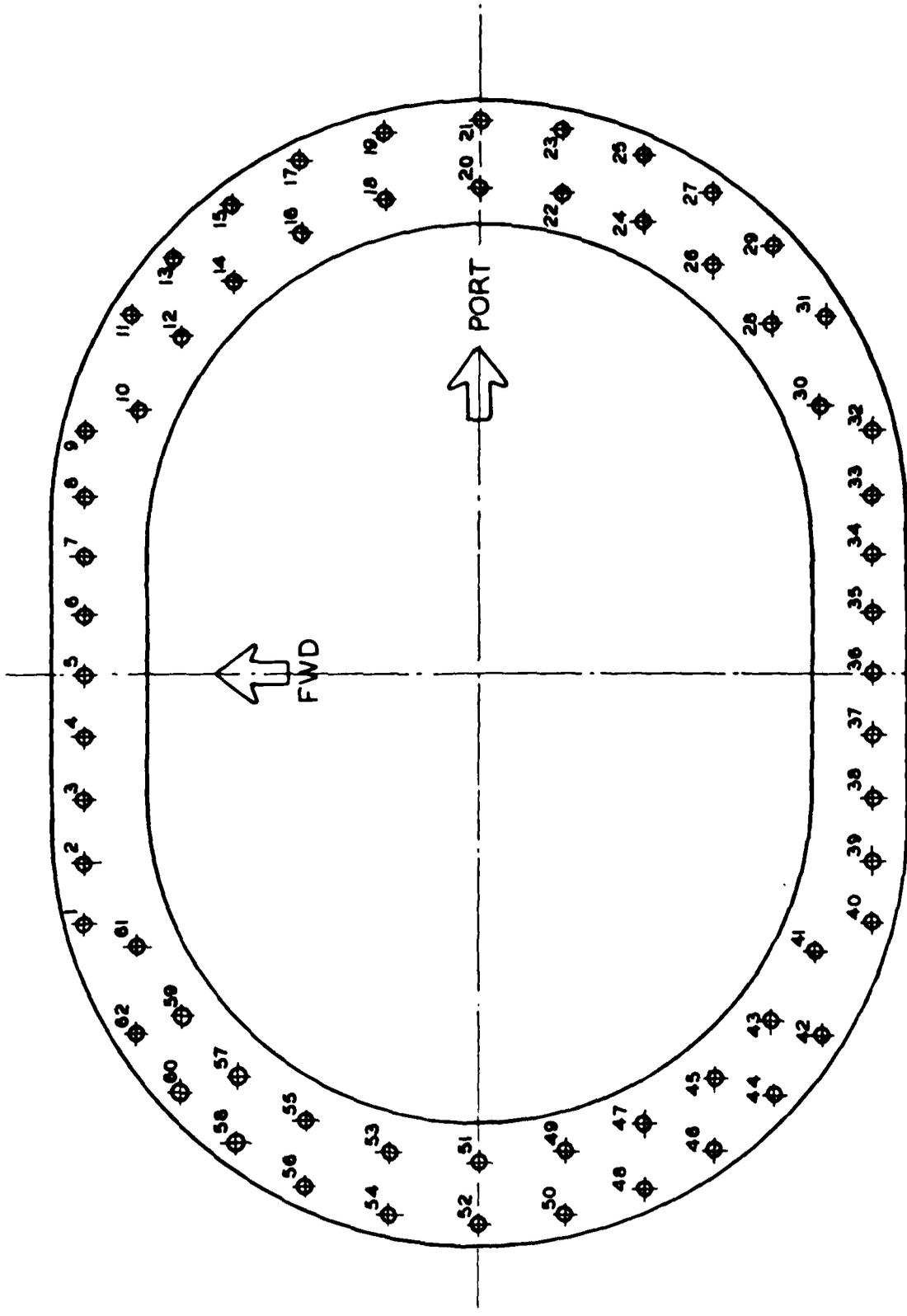
FIG. 1(b) F-27 TANK ACCESS DOOR HOLE NUMBERS – VIEW LOOKING UP
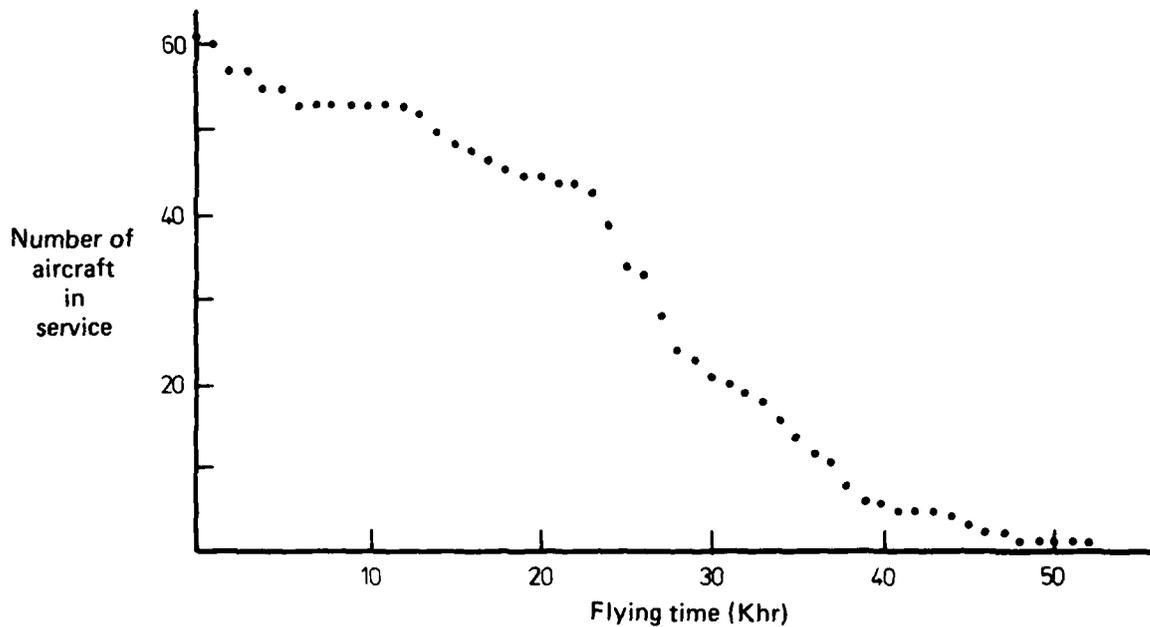
FIG. 2   F-27 FLEET UTILISATION (DEC 1977)
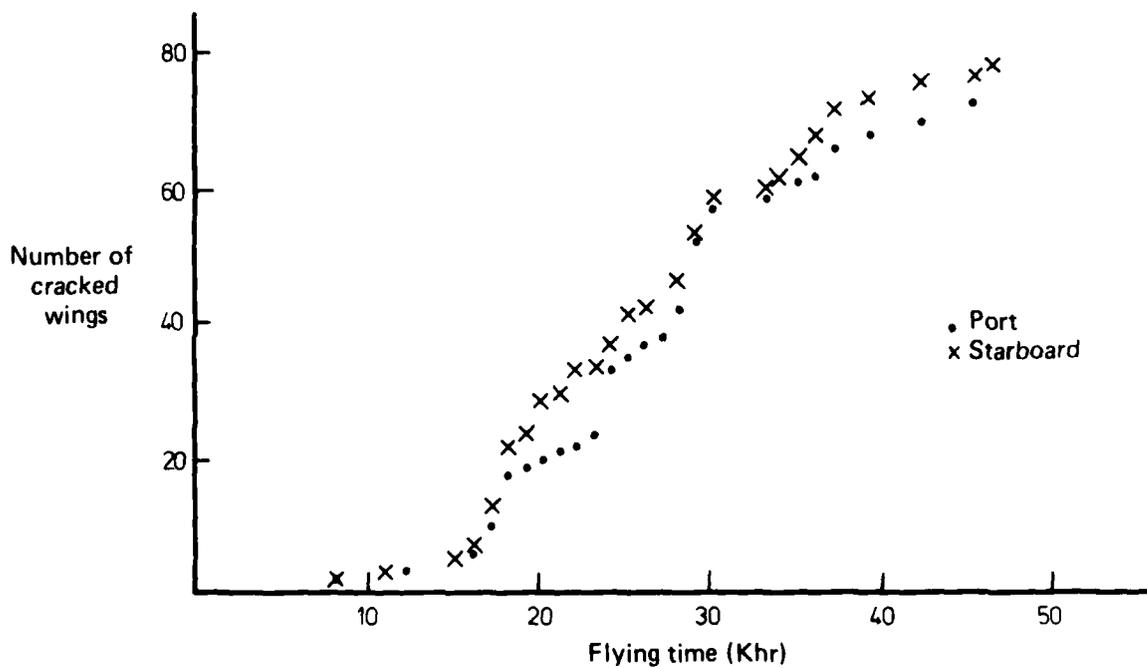
FIG. 3.   COMPARISON OF PORT AND STARBOARD WING CRACKING
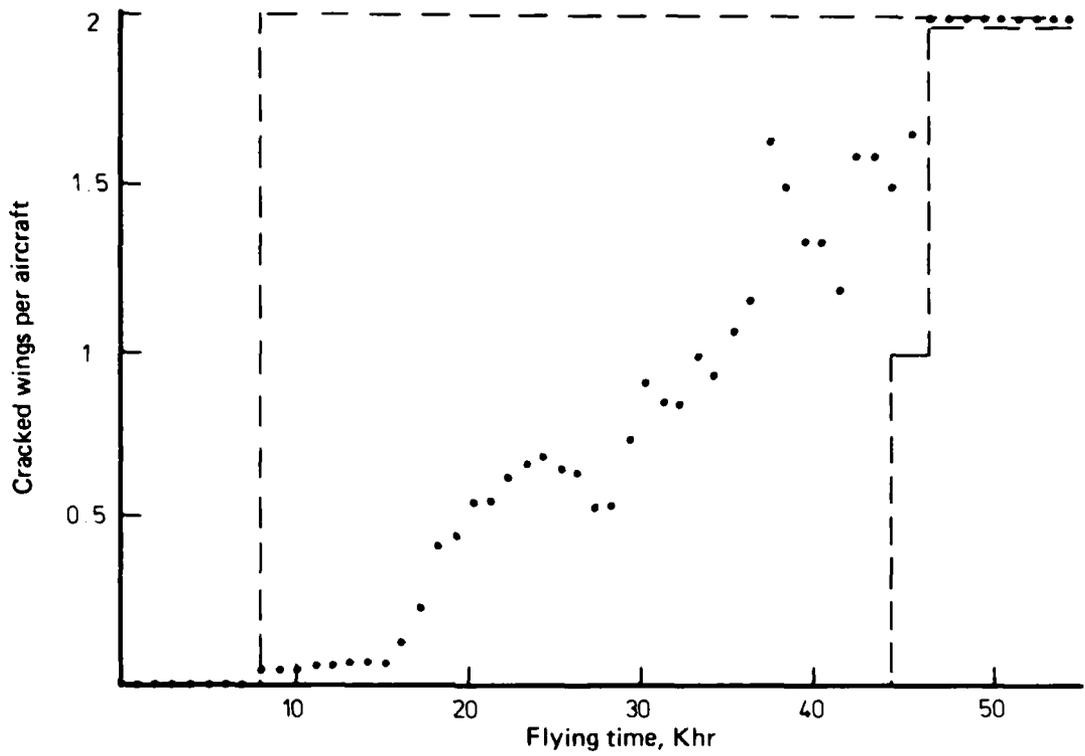
FIG. 4.   INCIDENCE OF WING CRACKING — AVERAGES AND INDIVIDUAL
EXTREMES

Fig 5 (cont)

Openings 93 and 154

Openings 94 and 155
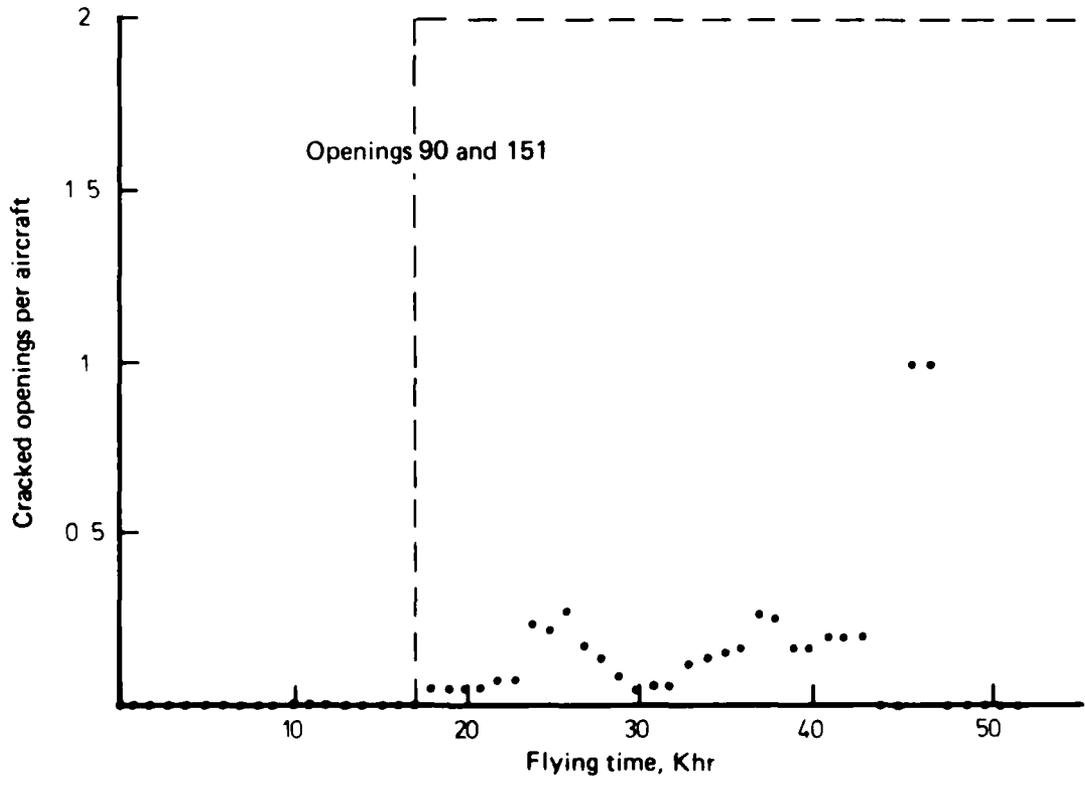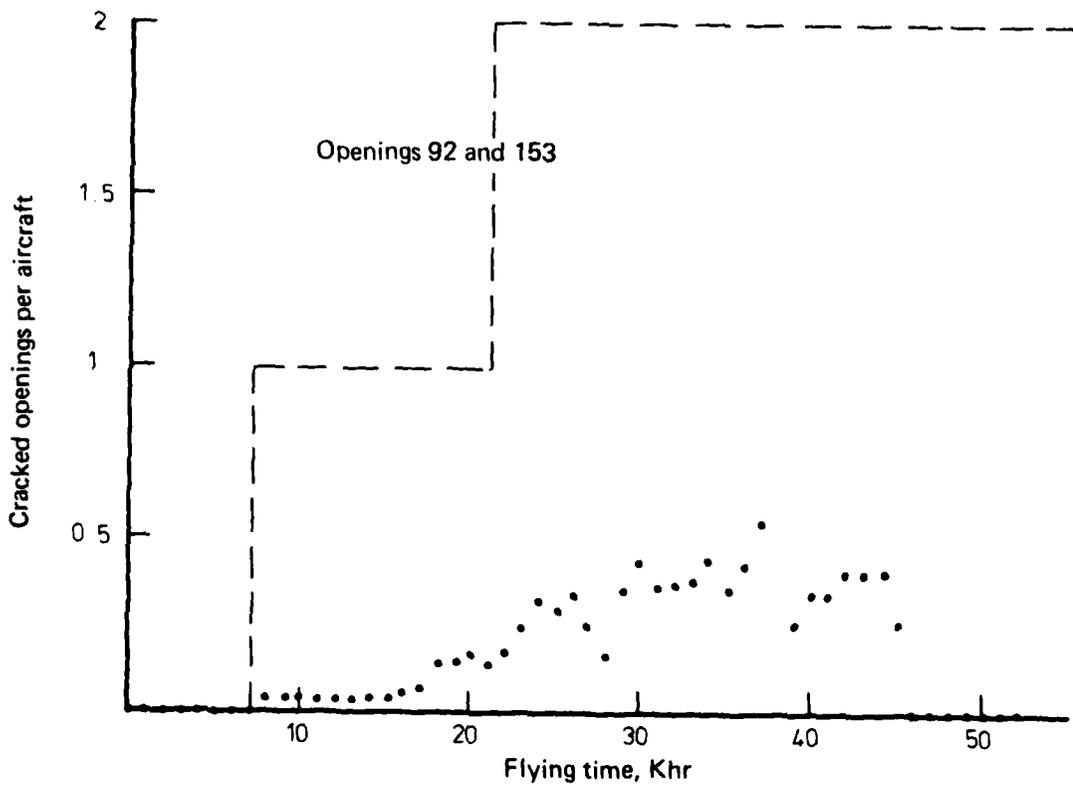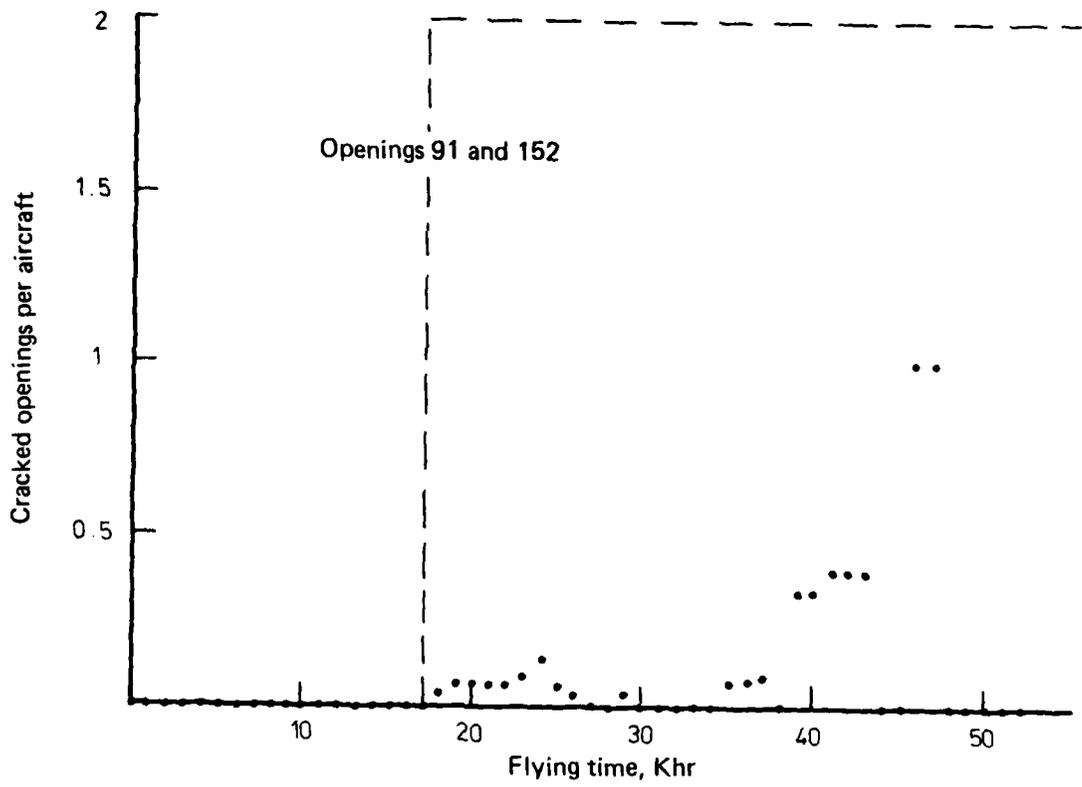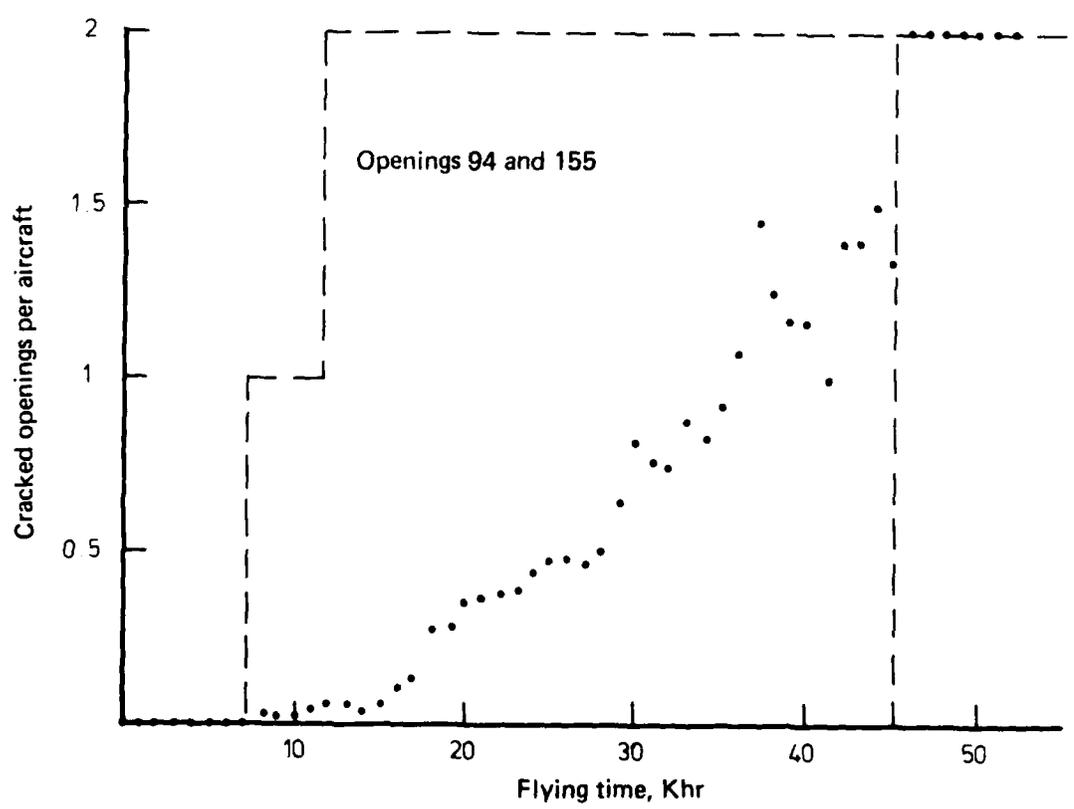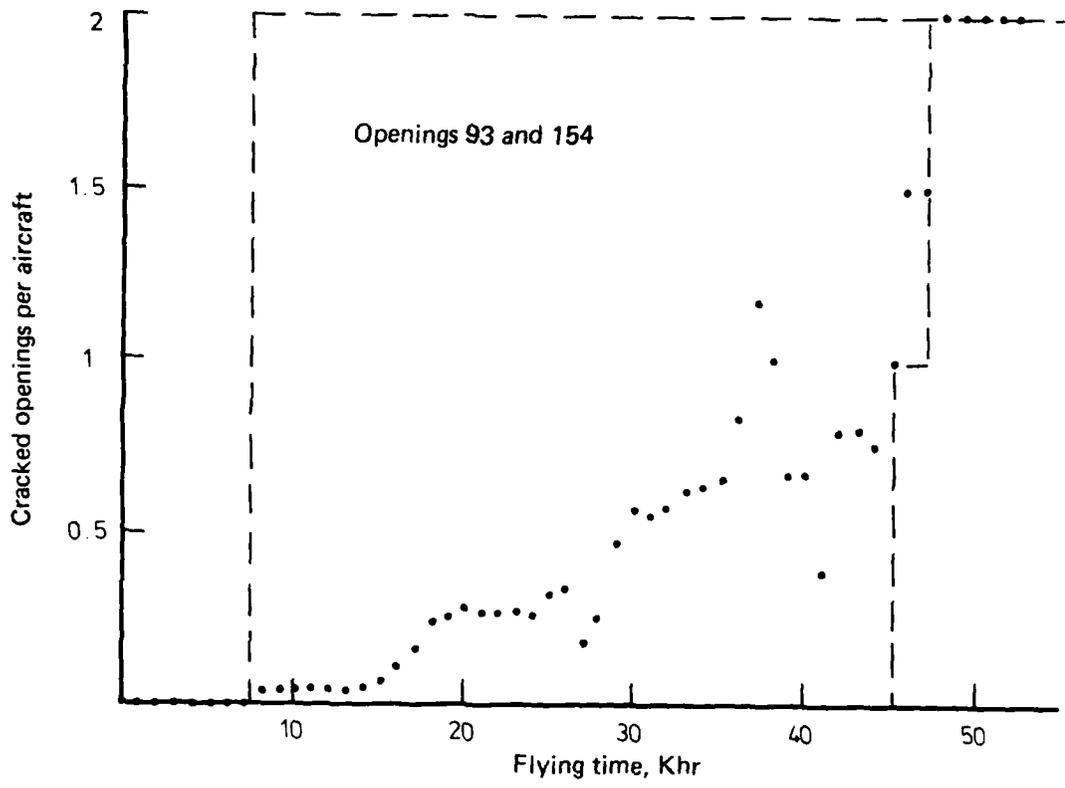
Fig 5 (cont)
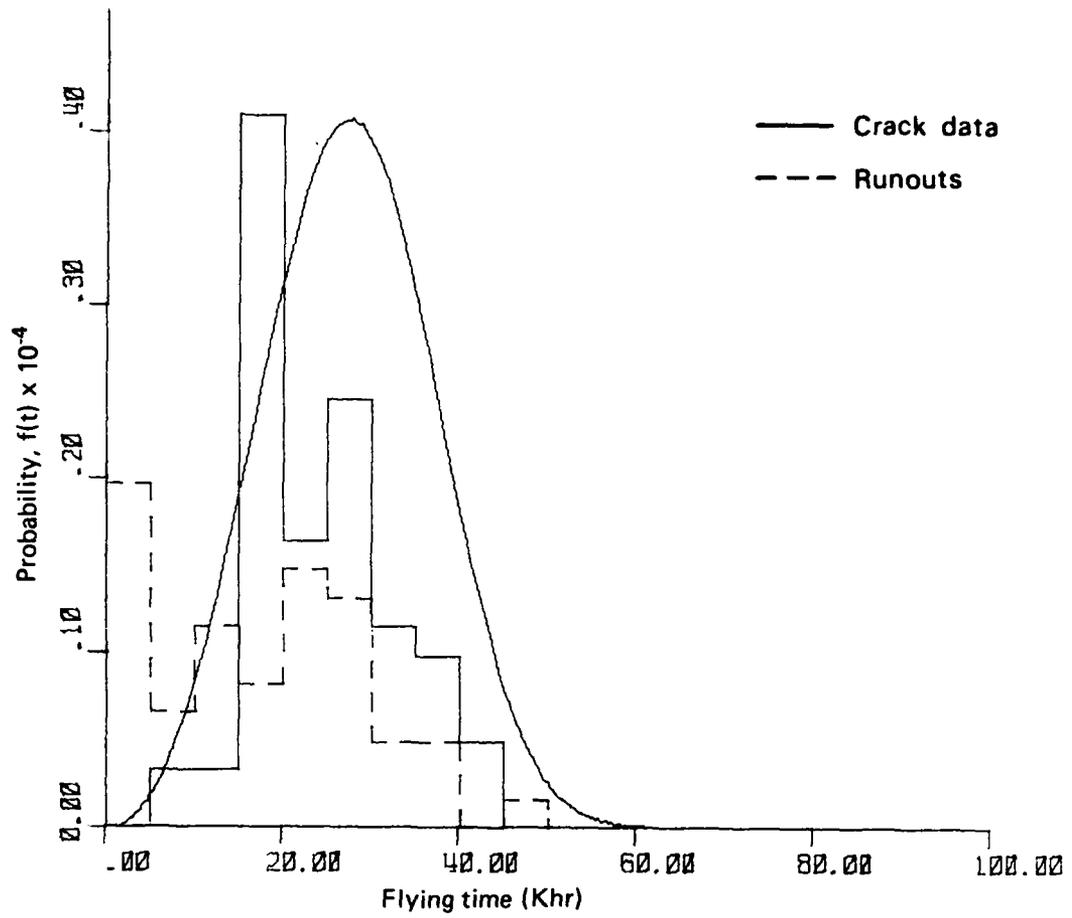
FIG. 6   PROBABILITY OF CRACKING BASED ON RELATIVE FREQUENCIES
OF CRACKING AND RUNOUTS — WINGS DATA
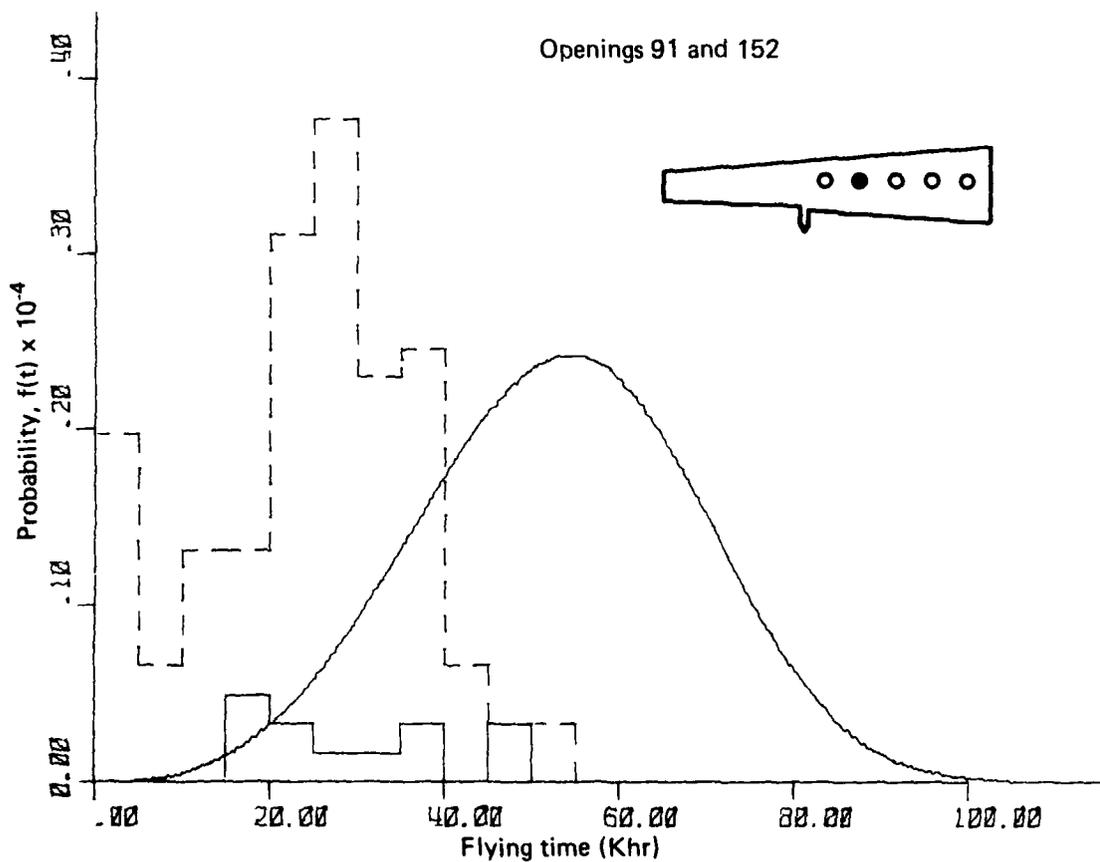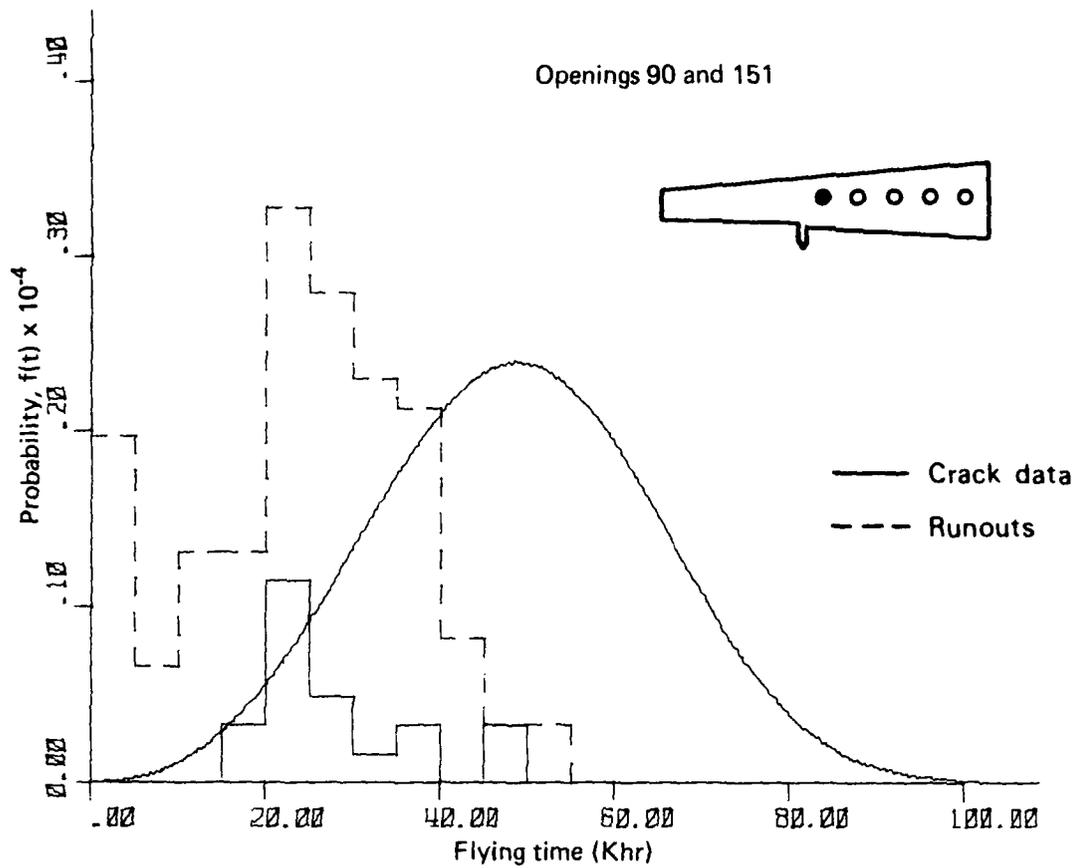
FIG 7   PROBABILITY OF CRACKING BASED ON RELATIVE FREQUENCIES
OF CRACKING AND RUNOUTS — OPENINGS DATA

Openings 92 and 153

Openings 93 and 154

Fig 7 (cont)

Openings 94 and 155

Fig 7 (cont)

Bolt hole 1, opening 94

Bolt holt 9, opening 155
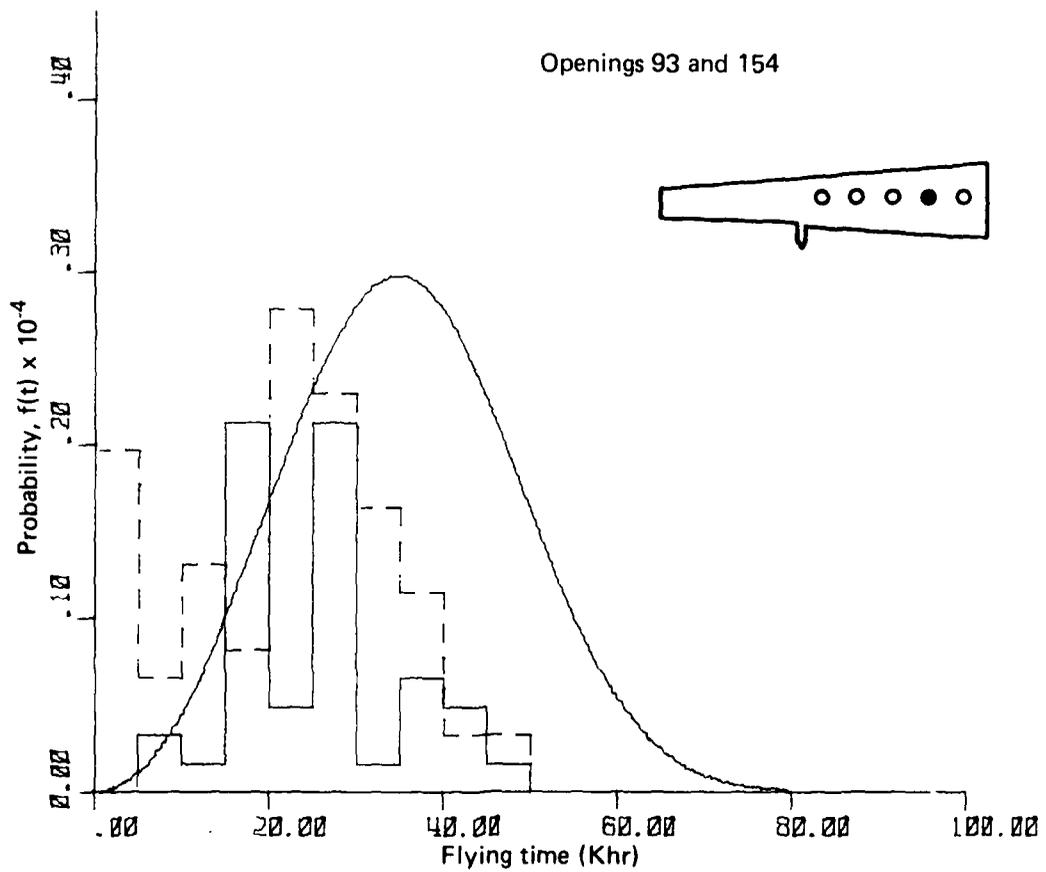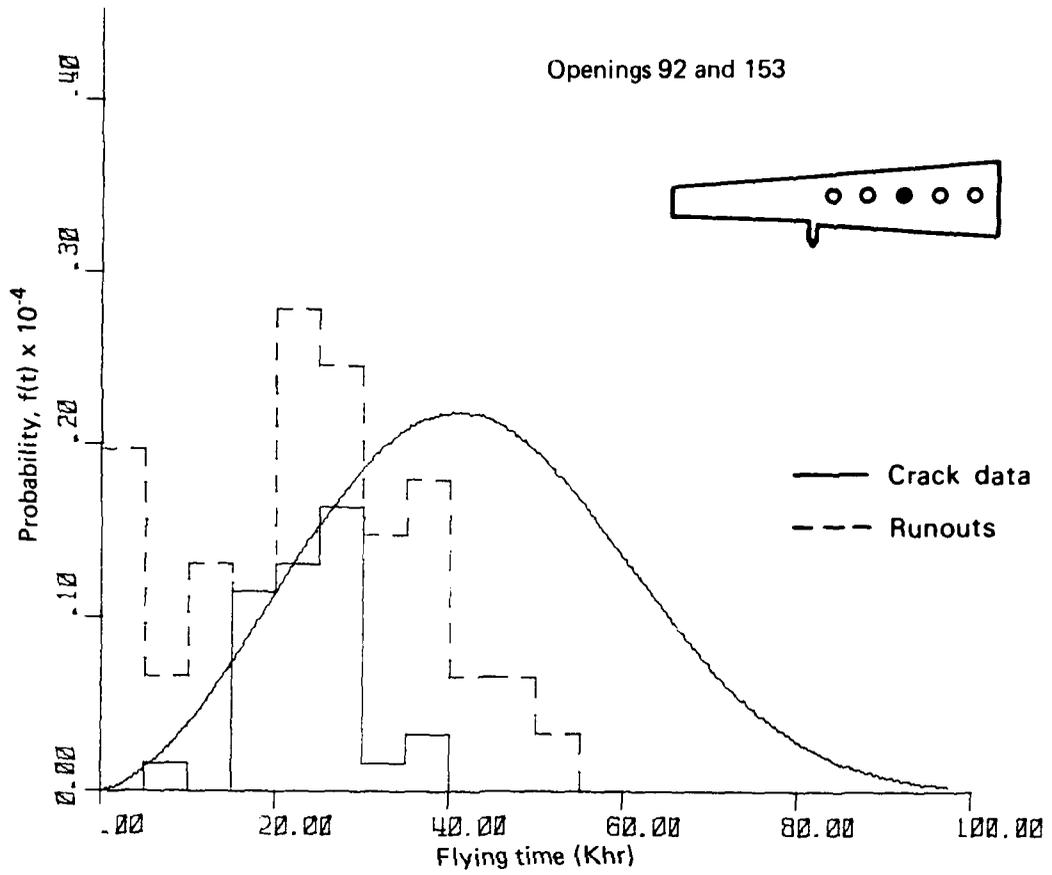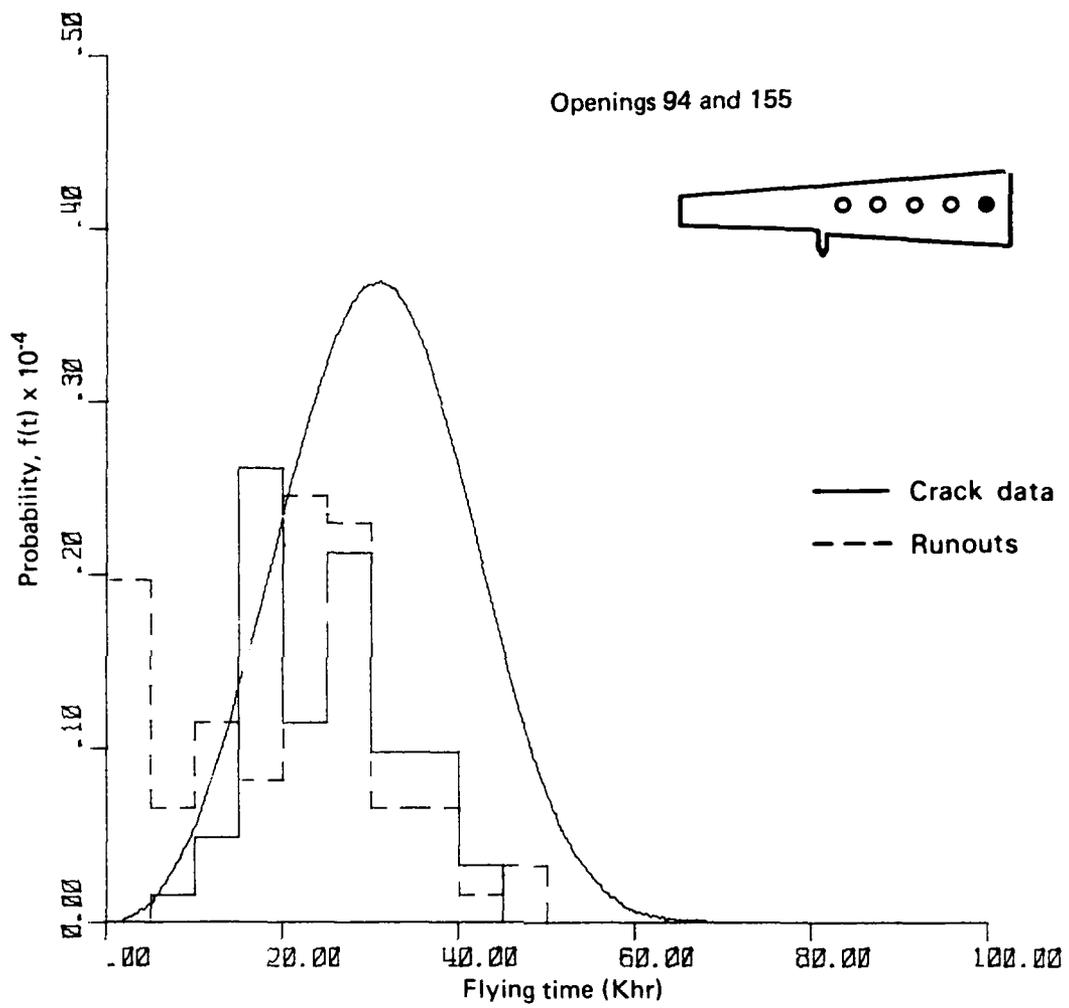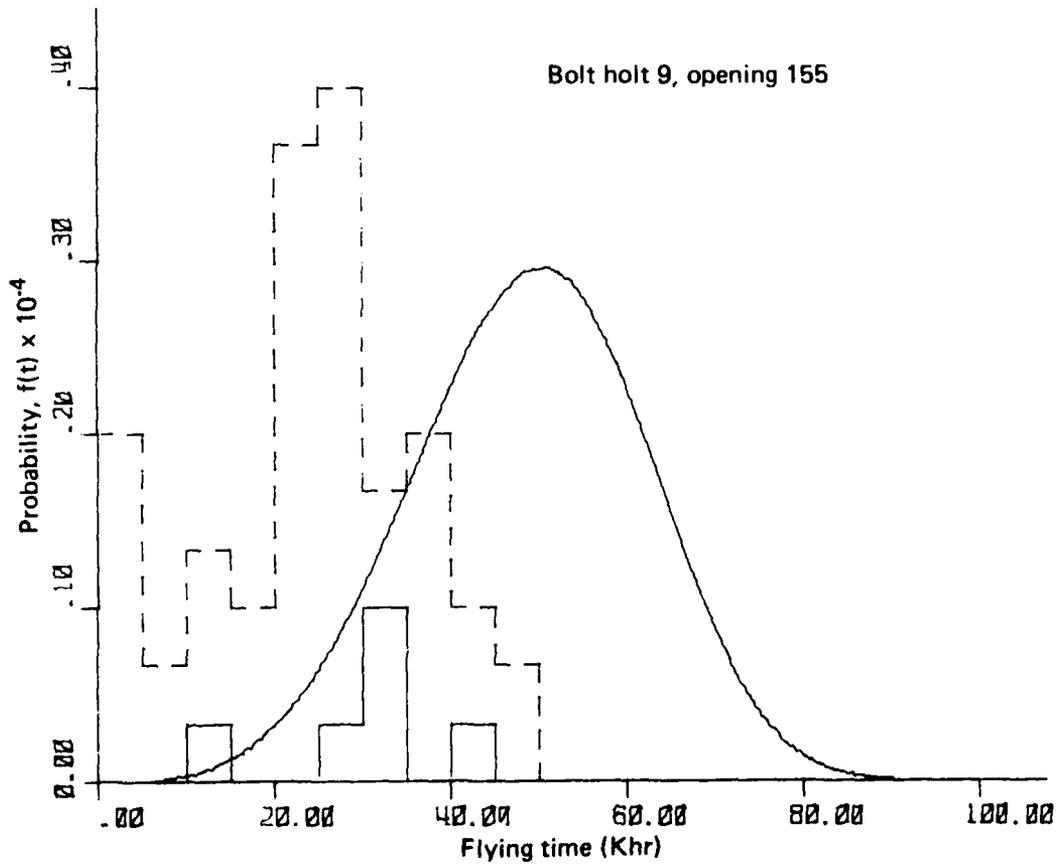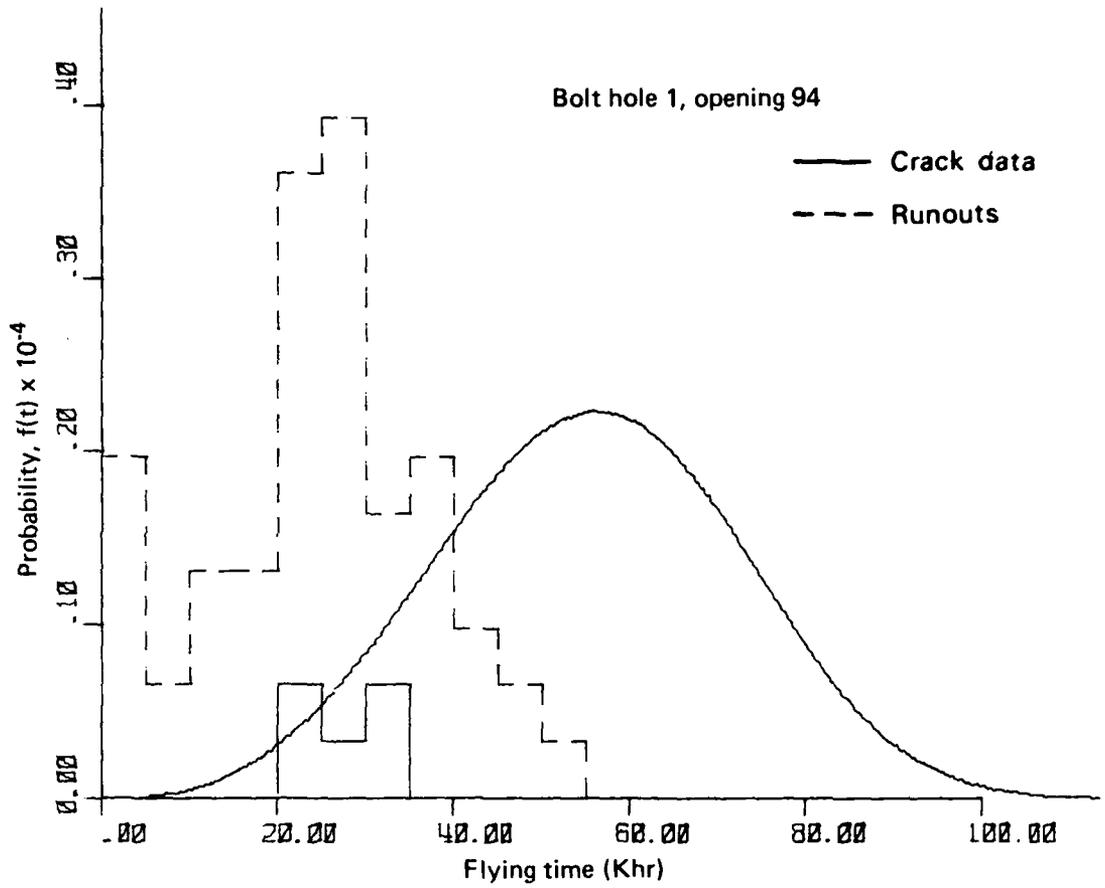
FIG. 8   PROBABILITY OF CRACKING BASED ON RELATIVE FREQUENCIES
OF CRACKING AND RUNOUTS — BOLT HOLE DATA

Bolt hole 9, opening 94

Bolt hole 1, opening 155
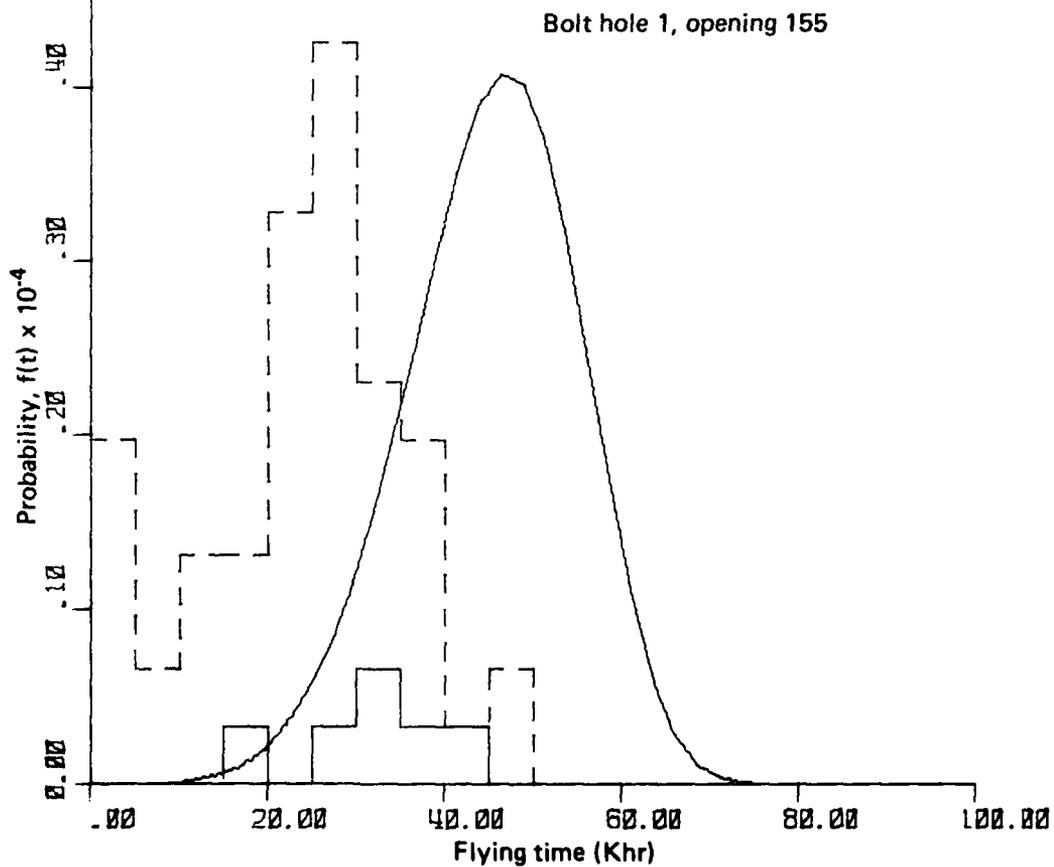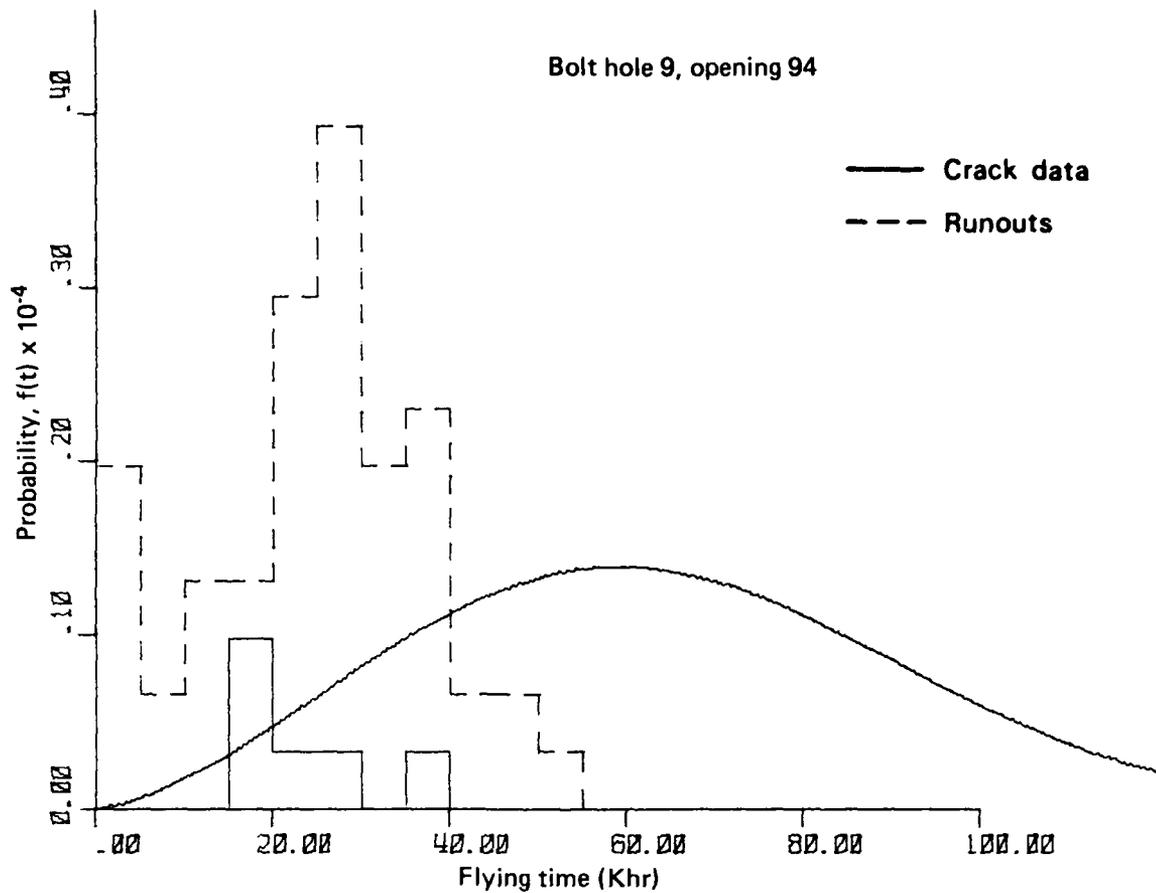
Fig 8 (cont)

Bolt hole/opening 1/94 and 9/155
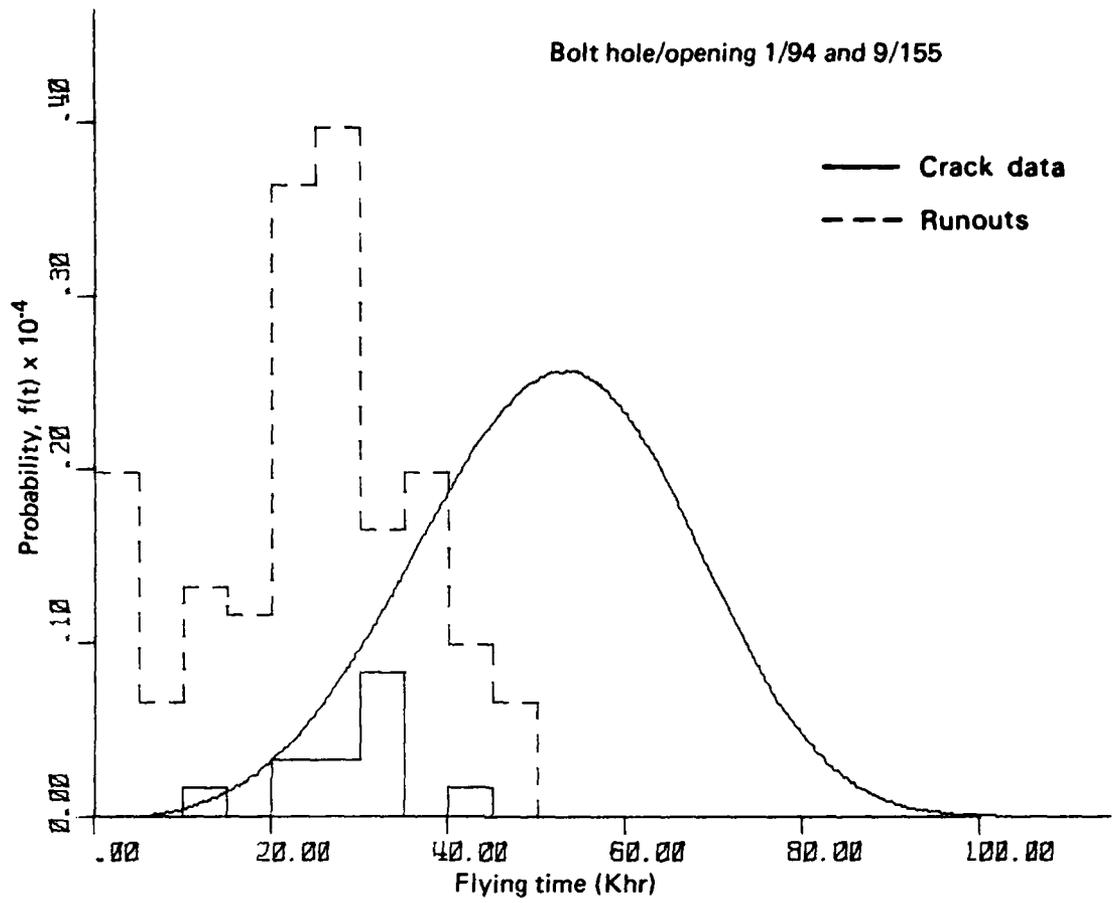
Bolt hole/opening 9/94 and 1/155
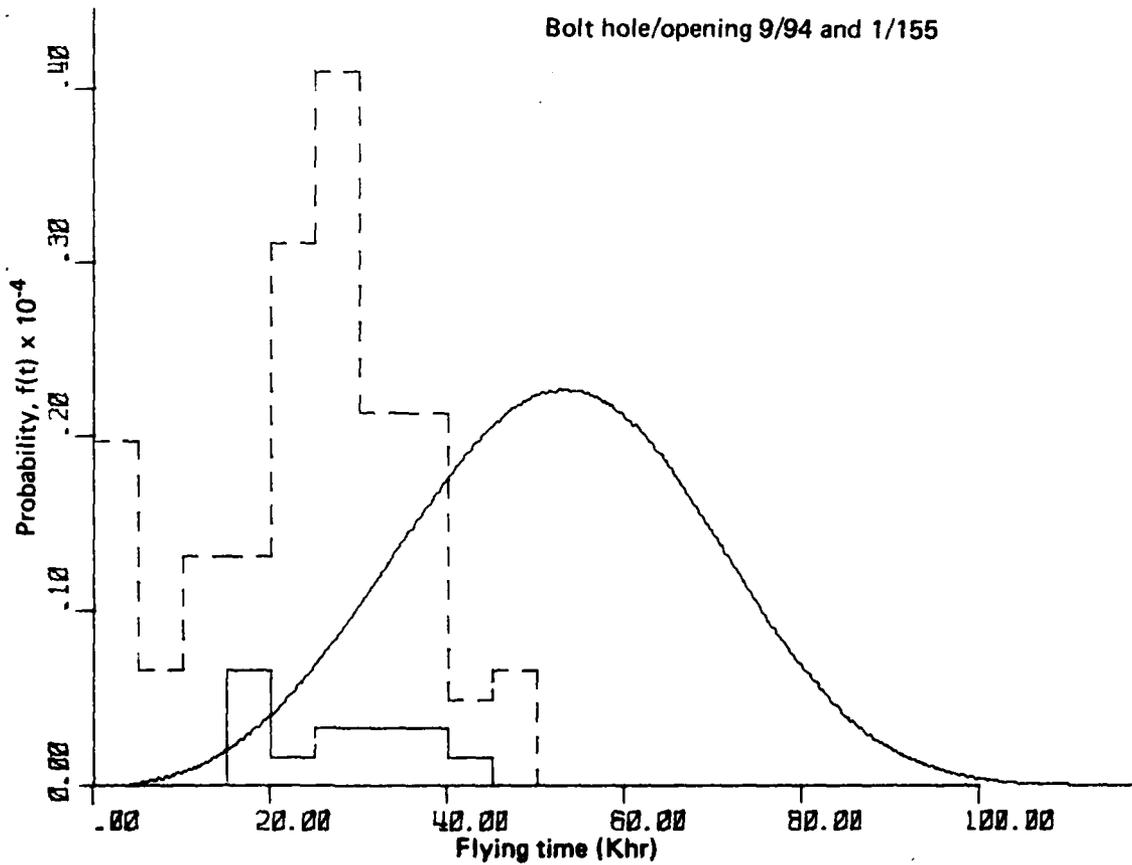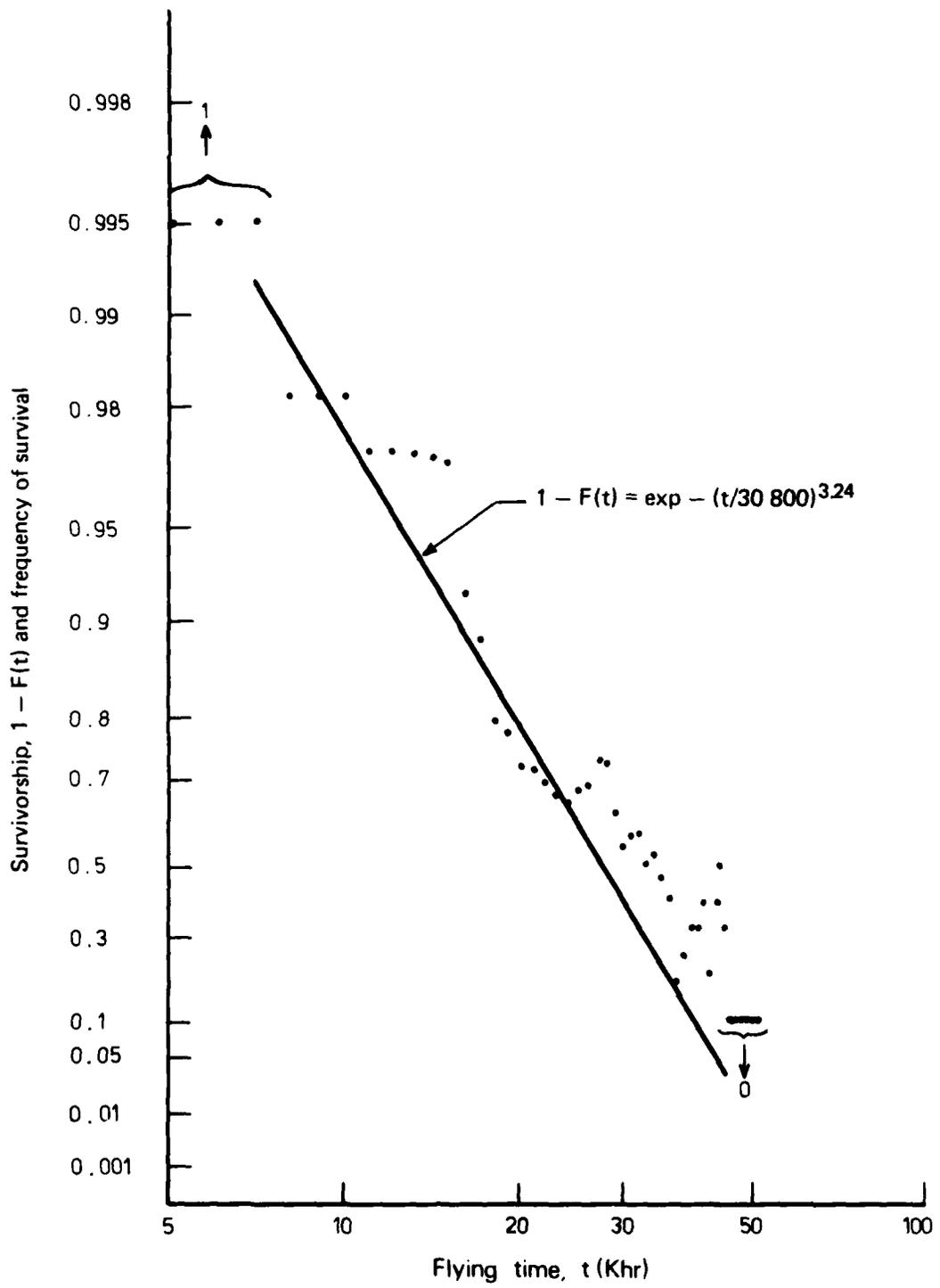
FIG. 9. SURVIVORSHIP DATA: WING CRACKING
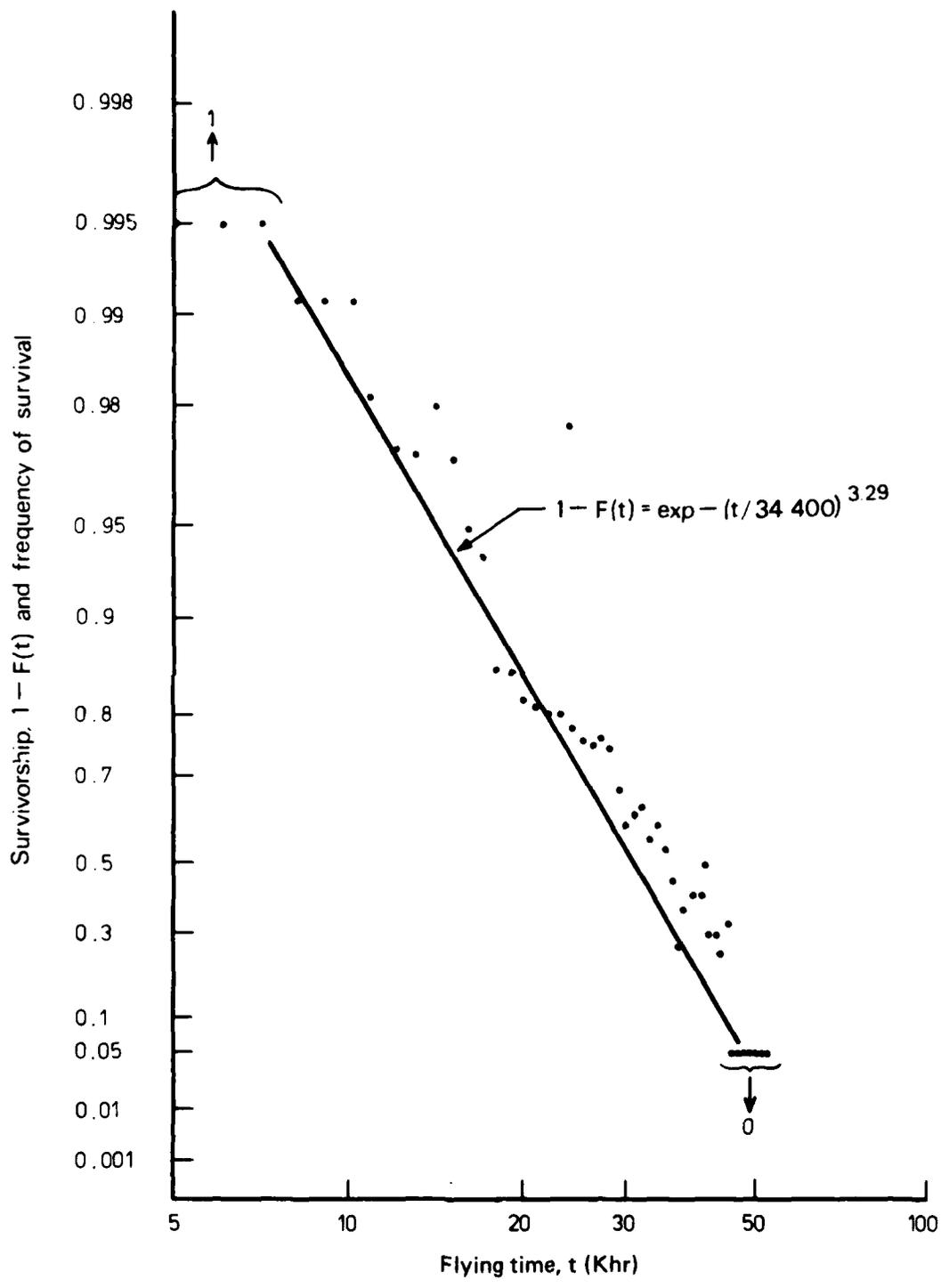
FIG. 10. SURVIVORSHIP DATA CRACKING IN OPENINGS 94 and 155
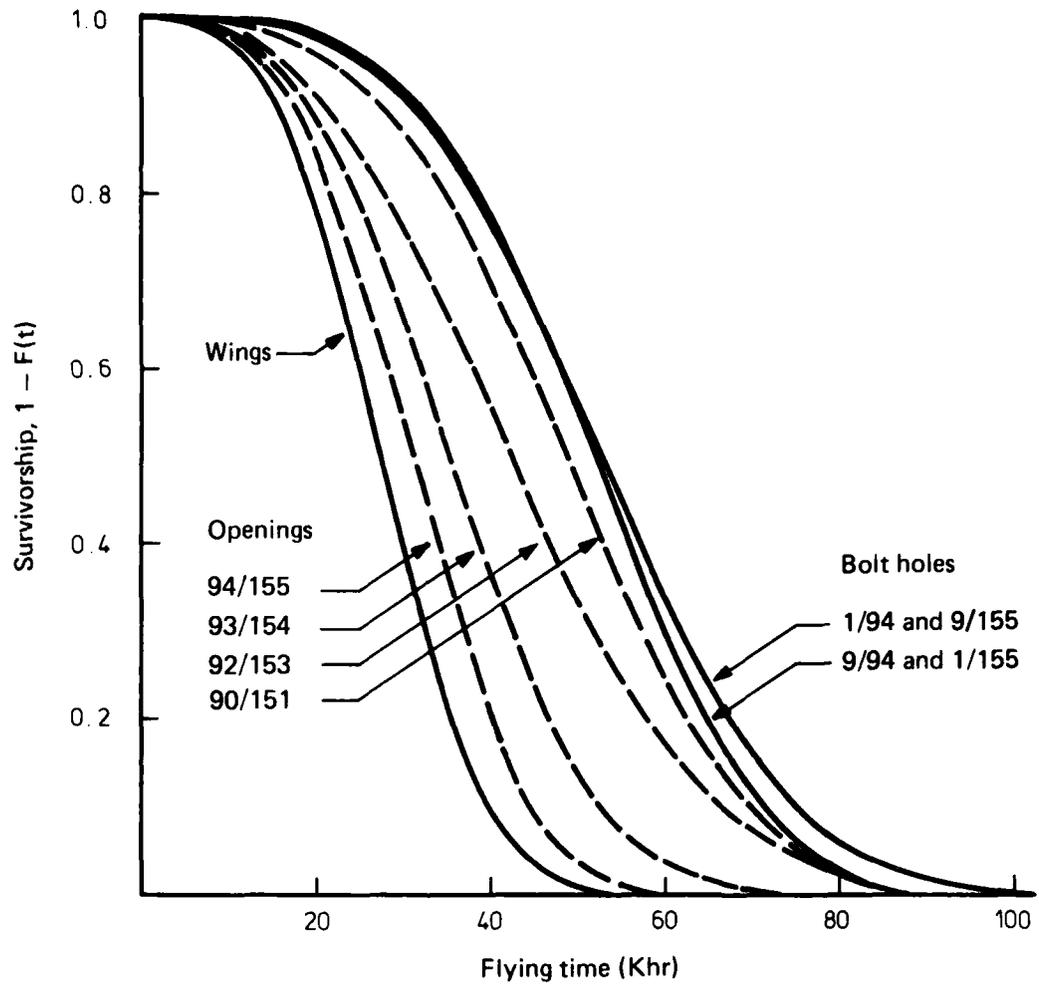
FIG. 11. SURVIVORSHIP: WINGS, OPENINGS AND BOLT HOLES
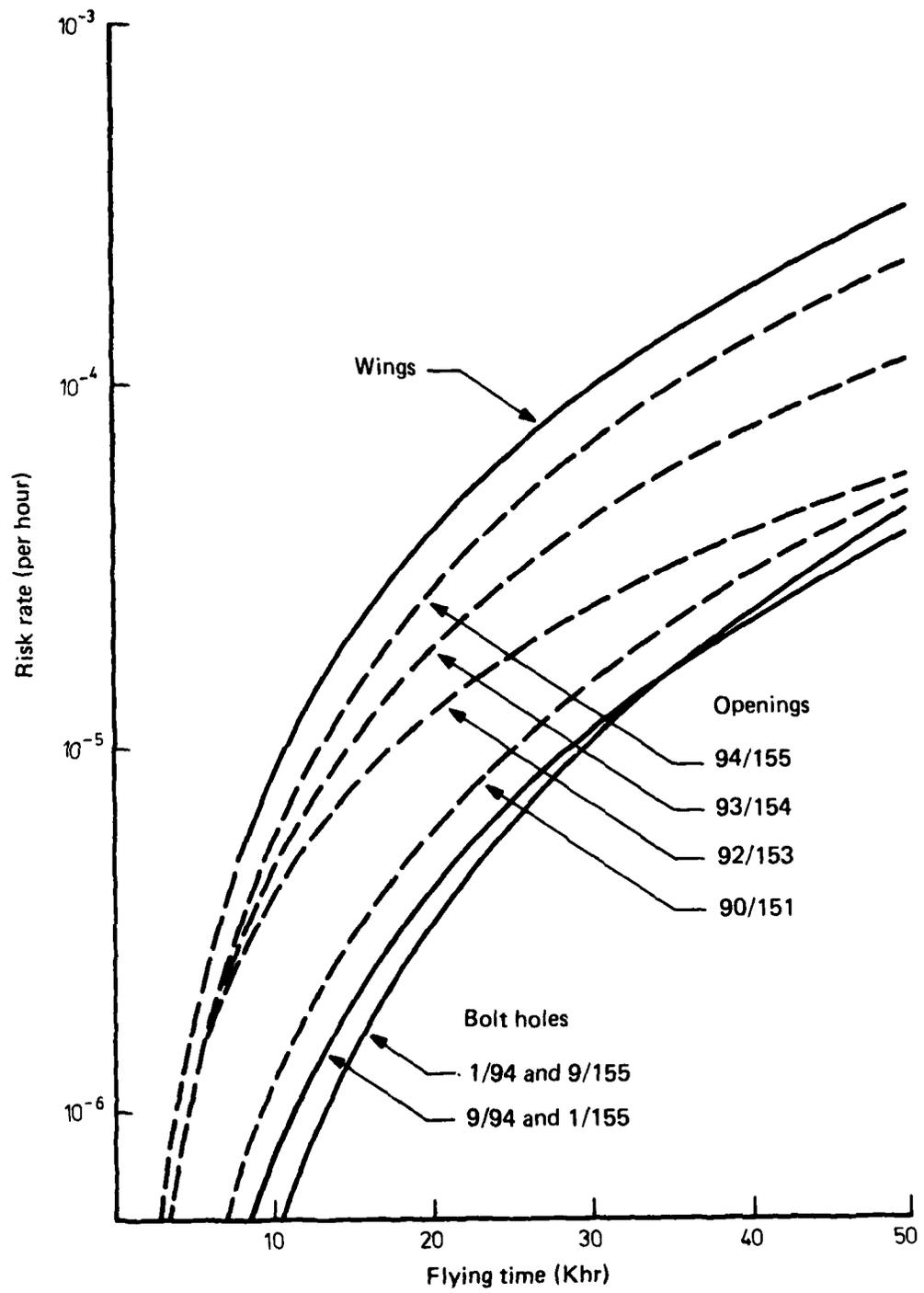
FIG. 12.   RISK RATES FOR WING, OPENINGS AND BOLT HOLES

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