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A SIMPLIFIED METHOD FOR PLOTTING WAVE REFRACTION.(U)  
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ROSENSTIEL SCHOOL OF MARINE AND ATMOSPHERIC SCIENCE

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TECHNICAL REPORT  
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A SIMPLIFIED METHOD FOR PLOTTING WAVE REFRACTION

KEY WORDS: Beaches; Coastal engineering; Graphs, Harbors; Refraction; Waves; Wave height

ABSTRACT: A simplified method for plotting the refraction of waves as they traverse water of varying depths is presented. Traditional methods base the refraction diagram on a series of bathymetric contours spaced at equal intervals. This necessitates a relatively large number of computations of wave celerity and complicated plotting methods. The simplified method uses a variable contour interval so that the ratio between the celerity of the wave on the shallower side of the contour and that on the deeper side is a constant. Dimensionless values for the depth of the contours related to the wave period are computed for the celerity ratio desired. The method gives better resolution with fewer steps in plotting than the traditional method. Templates for plotting the refraction according to Snell's Law are greatly simplified because of the constant ratio of the celerities. Tables and a refraction template are provided as well as a general discussion of wave refraction.

REFERENCE: Michel, John F., "A Simplified Method for Plotting Wave Refraction," Journal of the Waterways, Harbours and Coastal Engineering Division, ASCE, Vol. \_\_\_\_, No. \_\_\_\_, Proc. Paper \_\_\_\_

P-1

Letter on file

A

## A SIMPLIFIED METHOD FOR PLOTTING WAVE REFRACTION<sup>a</sup>

By John F. Michel,<sup>1</sup> F. ASCE

### INTRODUCTION

Gravity waves in water crossing bottom contours at an angle to the normal are bent or refracted toward the shallower contours. This is caused by the fact that the portion of a wave in deeper water moves with a greater velocity than the portion in shallow water. Assuming that no energy is transmitted parallel to the wave crest, the amount of energy between any pair of orthogonals or lines drawn perpendicular to the crest will remain constant. Therefore, the energy per unit length of wave crest will vary inversely as the distance between the orthogonals. Since the energy of a unit length of crest is proportional to the square of the wave height, the wave height as modified by refraction will vary inversely as the square root of the distance between orthogonals. In addition to the effect of refraction it should be realized that shoaling also causes modification of the wave height. This will be discussed in more detail later.

Refraction diagrams such as the one shown on Figure 1 are extremely useful in predicting wave height and angle of approach in the nearshore region. The construction of such a diagram is based on the relationship between the phase velocity or celerity  $C$  and the depth  $h$  for a wave having a length  $L$ , which is (2)

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<sup>a</sup>The manuscript of this paper was submitted to ASCE on 10 December 1973 for publication in the Journal of Waterways, Harbours and Coastal Engineering Division.

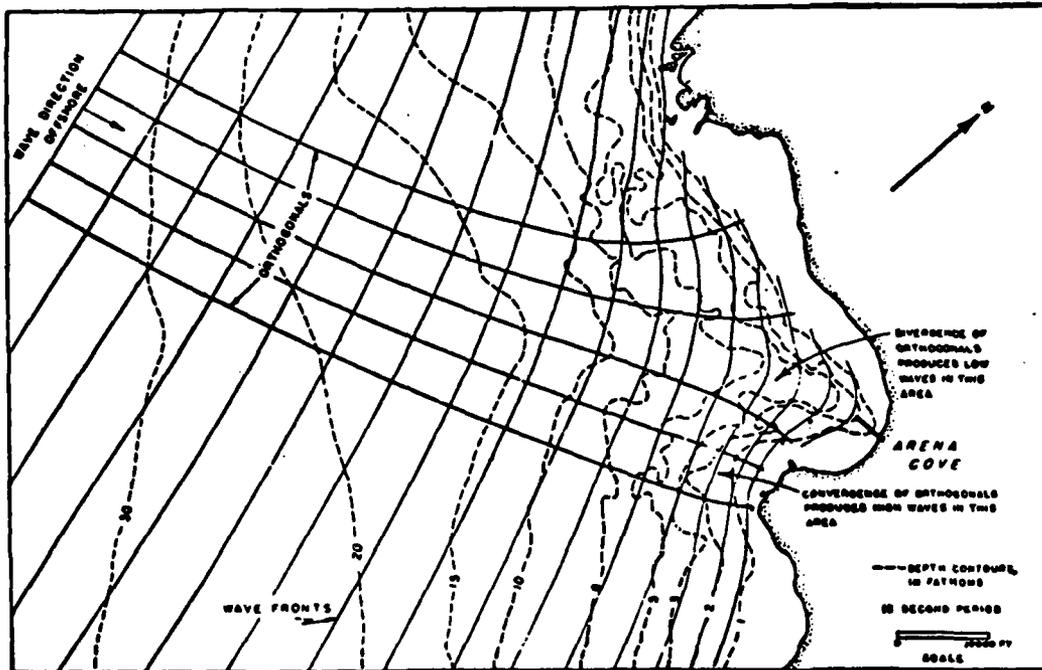


FIGURE A-6 REFRACTION DIAGRAM

(Wiegol, 238)

A-47

FIG. 1.--Refraction Diagram (Ref. 1, p. A-47)

$$C = \left( \frac{gL}{2\pi} \tanh \frac{2\pi h}{L} \right)^{1/2} \quad (1)$$

and the relationship of length to celerity and period T which can be shown to be constant for any train of waves where

$$L = CT \text{ or } \frac{L}{C} = \text{Constant} \quad (2)$$

Thus the ratio between the length and celerity of a wave in depths  $h_1$  and  $h_2$  will be:

$$\frac{L_1}{L_2} = \frac{C_1}{C_2} = \frac{\tanh \frac{2\pi h_1}{L_1}}{\tanh \frac{2\pi h_2}{L_2}} \quad (3)$$

For ratios of  $\frac{h}{L} > .5$ ,  $\tanh \frac{2\pi h}{L}$  is nearly unity so that for deep water conditions the relationships denoted by a subscript "o" become:

$$C_o = \left( \frac{gL_o}{2\pi} \right)^{1/2} = \frac{gT}{2\pi} \quad (4)$$

and

$$L_o = \frac{gT^2}{2\pi} \quad (5)$$

From this it is seen that

$$\frac{C}{C_o} = \tanh \frac{2\pi h}{L} = \frac{L}{L_o} \quad (6)$$

Tables (5) are available which give values of  $\tanh \frac{2\pi h}{L}$  for specified values of  $\frac{h}{L}$  and  $\frac{h}{L_o}$  which is easier to compute since  $L_o$  is independent of  $h$ .

It is shown (1) that a wave crest tracking in a depth  $h_1$  at an angle  $\alpha_2$  to a bathymetric contour will conform to Snell's law or:

$$\sin \alpha_2 = \frac{C_2}{C_1} \sin \alpha_1 \quad (7)$$

substituting Equation 3 in Equation 7 gives:

$$\sin \alpha_2 = \frac{\tanh \frac{2\pi h_2}{L_2}}{\tanh \frac{2\pi h_1}{L_1}} \sin \alpha_1 \quad (8)$$

On the basis of these relationships, orthogonals or wave fronts can be plotted and the wave heights related to the deep water wave height by:

$$H = H_o \left( \frac{b_o}{b} \right)^{1/2} D_d \quad (9)$$

where  $H$  is the wave height at a given location,  $H_o$  is the wave height in deep water,  $b_o$  is the distance between the orthogonals in deep water,  $b$  is the distance between orthogonals at the given location and  $D_d$  or  $\frac{H}{H_o}$  is the shoaling coefficient which expresses the effect of depth on the unrefracted waves.

$$D_d = \sqrt{\left(\frac{1}{2}\right) \left(\frac{1}{n}\right) \left(\frac{C}{C_o}\right)} \quad (10)$$

where

$$n = \frac{1}{2} \left( 1 + \frac{\frac{4\pi h}{L}}{\sinh \frac{4\pi h}{L}} \right) \quad (11)$$

Values of  $n$  or  $D_d$  can be computed or obtained from the tables (5) mentioned above.

### THE TRADITIONAL APPROACH

In the traditional approach (1) to plotting refraction diagrams a series of bathymetric contours with more or less uniform spacing such as 6 feet are plotted and smoothed and the ratios of celerities are computed. Figure 2 typifies this approach. Note that for a wave having a period of 10 seconds as specified,  $L_0 = 512$  feet so that the table would be carried out to a depth of 255 feet requiring 43 separate depths for the 6-foot interval shown. In actual practice as the ratio between the celerities of deeper and shallower water approaches unity the depth interval is increased by leaving out some contours. The intervals however, are maintained in a whole number of feet or fathoms.

Once the celerities at the various depths have been determined, two methods are available for plotting the diagram: the wave front method as described by Weigel (4) and the more commonly used orthogonal method as described by Johnson et al (3).

#### The Wave Front Method

In the wave front method, the position of every nth wave crest is plotted by scaling n times the wavelength at the depth the wave crosses in a direction normal to the wave front. This is done at intervals along the crest until the diagram is advanced to the point of interest. Analytic methods as described by Wilson (6) utilize essentially this method for application to digital computers.

#### The Orthogonal Method

In this method plotting is accomplished by sketching a mid-contour between the selected contours to serve as a point of intersection in the

TABLE 1-7  
COMPUTATIONS FOR VALUES OF  $C_1/C_2$

$T = 10 \text{ seconds}$				
$d$	$\frac{d}{L_0}$	$\tanh \frac{2\pi d}{L}$	$C_1/C_2$ or $C_d/C_s$	$C_2/C_1$ or $C_s/C_d$
(1) (feet)	(2)	(3)	(4)	(5)
6	0.0117	0.268		
12	0.0234	0.374	1.40	0.72
18	0.0352	0.453	1.21	0.83
24	0.0469	0.516	1.14	0.88

Column 1 gives depths corresponding to chart contours. These would extend from 6 feet to a depth equal to  $L_0/2$ .

Column 2 is column 1 divided by  $L_0$  corresponding to the determined period.

Column 3: these values may be found in Table D-1 of Appendix D, as a function of  $d/L_0$ . This term is also  $C/C_0$ .

Column 4 is the quotient of successive terms in column 3.

Column 5 is the reciprocal of column 4.

FIG. 2--Computations for Refraction Diagram (Ref. 1, Table 1-7, p. 69)

orthogonal. The previously computed orthogonal is extended to the mid-contour and the angle  $\alpha$  of the new orthogonal is calculated using Snell's law (equation 7). The outgoing orthogonal is then plotted either with a protractor or a fairly complicated template.

#### THE SIMPLIFIED METHOD

In 1969, the author, faced with the problem of preparing some refraction diagrams in a limited time, conceived that good resolution could be obtained with less work by maintaining a constant ratio,  $R$ , between  $C_s$  and  $C_d$  so that

$$R = \frac{C_s}{C_d} = \text{Constant} \quad (12)$$

This ratio is selected according to the degree of resolution desired. Values of 0.8 or 0.9 yield good results and are demonstrated here. Once the ratio  $R$  has been selected, a universal table of the depths of mid-contours applicable to any wave period can be developed.

First values of  $\frac{C_s}{C_o}$  for each depth interval, are determined by:

$$\frac{C_s}{C_o} = R^{(n-1)} \quad (13)$$

where  $n$  is the number of the interval. The celerity at the mid-contour is then computed as the average of  $C_d$  and  $C_s$  or

$$C_{\text{mid}} = \frac{C_s + C_d}{2} \quad (14)$$

so that

$$\frac{C_{\text{mid}}}{C_o} = \frac{1}{2} (1+R) R^{(n-1)} = K \quad (15)$$

which is still a constant for any value of  $n$ .

At this point, it is well to note that the depth of the mid-contour is selected as the depth corresponding to the average celerity over the depth interval. The traditional approach on the other hand utilizes as a mid-depth a contour placed by eye between the computed depths.

Since:

$$K = \frac{C_{mid}}{C_o} = \tanh \frac{2\pi h_{mid}}{L_{mid}} = \frac{L_{mid}}{L_o} \quad (16)$$

where  $L_{mid}$  is the wavelength at the mid-depth,

$$\frac{2\pi h_{mid}}{L_{mid}} = \tanh^{-1} K \quad (17)$$

and

$$\frac{h_{mid}}{L_o} = \frac{K}{2\pi} \tanh^{-1} K \quad (18)$$

Since

$$L_o = \frac{gT^2}{2\pi}$$

$$\frac{h_{mid}}{gT^2} = \frac{K \tanh^{-1} K}{(2\pi)^2} \quad (19)$$

which is dimensionless and dependent only in the constant ratio  $\frac{C_s}{C_d}$  and the wave period. A more convenient ratio,  $\frac{h_{mid}}{T^2}$  in meters or feet can be obtained by multiplying the expression by  $g$  in the units desired.

At this stage values of the shoaling coefficient  $D_d$  or  $\frac{H}{H_o}$  can be computed for each selected ratio of  $\frac{h}{L}$ .

Tables 1 and 2 illustrate the relationship described above for two values of  $\frac{C_s}{C_d}$  which will cover a reasonable range of wave periods.

TABLE 1: COMPUTATIONS FOR MID-CONTOURS WITH  $C_s/C_d = 0.80$ 

$C_s/C_o$	$C_{mid}/C_o$	$h_{mid}/gT^2$	$h_{mid}/T^2$ (meters/second <sup>2</sup> )	$h_{mid}/T^2$ (feet/second <sup>2</sup> )	$H/H_o$
(1)	(2)	(3)	(4)	(5)	(6)
1.0000	.9000	.0336	.3289	1.0791	.92
.8000	.7200	.0166	.1622	.5322	.93
.6400	.5760	.0096	.0939	.3079	.99
.5120	.4608	.0058	.0570	.1870	1.08
.4096	.3686	.0036	.0354	.1161	1.19
.3277	.2949	.0023	.0223	.0730	1.32
.2621	.2359	.0014	.0141	.0462	1.47
.2097	.1887	.0009	.0090	.0294	1.64

TABLE 2: COMPUTATIONS FOR MID-CONTOURS WITH  $C_s/C_d = 0.90$ 

$C_s/C_o$ (1)	$C_{mid}/C_o$ (2)	$h_{mid}/gT^2$ (3)	$h_{mid}/T^2$ (meters/second <sup>2</sup> ) (4)	$h_{mid}/T^2$ (feet/second <sup>2</sup> ) (5)	$H/H_o$ (6)
1.0000	.9500	.0441	.4320	1.4173	.94
.9000	.8550	.0276	.2705	.8874	.91
.8100	.7695	.0199	.1947	.6387	.92
.7290	.6926	.0150	.1466	.4810	.94
.6561	.6233	.0115	.1130	.3708	.97
.5905	.5610	.0090	.0883	.2898	1.00
.5314	.5049	.0071	.0697	.2285	1.04
.4783	.4544	.0056	.0553	.1814	1.09
.4305	.4089	.0045	.0441	.1447	1.14
.3874	.3680	.0036	.0353	.1158	1.19
.3487	.3312	.0029	.0283	.0929	1.25
.3138	.2981	.0023	.0228	.0746	1.32
.2824	.2683	.0019	.0183	.0601	1.38
.2542	.2415	.0015	.0148	.0484	1.45
.2288	.2173	.0012	.0119	.0391	1.53
.2059	.1956	.0010	.0096	.0316	1.61

Actually only the third column is necessary for the computation of the depth of the mid-contour, the fourth and fifth columns having been added for convenience. The first two columns are shown only to demonstrate the rationale of the method. The sixth column provides ready reference to the shoaling coefficient. These tables are universal in application and are sufficiently brief to be placed on a refraction template as will be seen later.

In utilizing Table 1 for instance, for a wave having a period of 10 seconds, the depths of the mid-contours in feet could be: 108, 53, 31, 19, 12, 7 and 5. These depths would be plotted on the chart to be used. If Table 2 were used for better resolution, the depths would be 142, 89, 64, 48, 37, 29, 23, 18, 14, 12, 9, 7, 6 and 5. These 14 depths can be compared to Figure 2 which would require 43 depths to be plotted.

When the mid-contours have been plotted, either the wave front method or the orthogonal method can be used with great facility to plot the refraction diagram.

#### The Wave Front Method

In using the wave front method, a set of proportional dividers set to the ratio  $\frac{C_s}{C_d}$  can be used to plot the positions of the nth successive wave fronts without the necessity for any measuring or scaling except for the crests in deep water in case it is desired to show actual crest position instead of simply the shape of the wave front.

Although it has not been tried this method would appear to be well adapted to numerical analysis, by digital computer, and its simplicity should greatly reduce computation time.

### The Orthogonal Method

In utilizing the orthogonal method the constant relationship between  $\sin \alpha_d$  and  $\sin \alpha_s$  permits construction of a very simple template. Table 3 shows the relationships between orthogonal angles, for ratios of  $\frac{C_s}{C_d} = 0.9$  and 0.8.

An example of such a template for  $\frac{C_s}{C_d} = 0.9$  is shown in Figure 3. Since the template has graduations in both sides, there is no need to have it on transparent material.

In using the template, the depths of the mid-contour are determined from Table 2 or the slightly abbreviated table on the template. When the mid-contours have been plotted on the chart the template is used to plot the orthogonals in the following manner as demonstrated by Figure 4:

1. The template is placed so that the tangent line is on the tangent to the mid-contour at the point where the incoming orthogonal intersects it. This places the baseline of the template on the normal to the contour which is drawn using the base as a straight edge.
2. The angle of the incoming orthogonal with the normal is noted on the scale for deeper or shallower water as the case may be.
3. The template is then rotated so that the angle on the other side of the template corresponding to the incoming angle is on the normal line. The new orthogonal is then drawn using the base of the template as a straight edge.

TABLE 3: SIMPLIFIED METHOD COMPUTATION FOR REFRACTION TEMPLATES

$\alpha_d$	$C_s/C_d = .9$	$C_s/C_d = .8$
$\alpha_s$	$\alpha_s$	$\alpha_s$
(1)	(2)	(3)
<u>degrees</u>	<u>degrees</u>	<u>degrees</u>
0	0	0
10	9.0	8.0
20	17.9	15.9
30	26.7	23.6
40	35.3	30.9
50	43.6	37.8
60	51.2	43.9
70	57.7	48.7
80	62.4	52.0
90	64.2	53.1

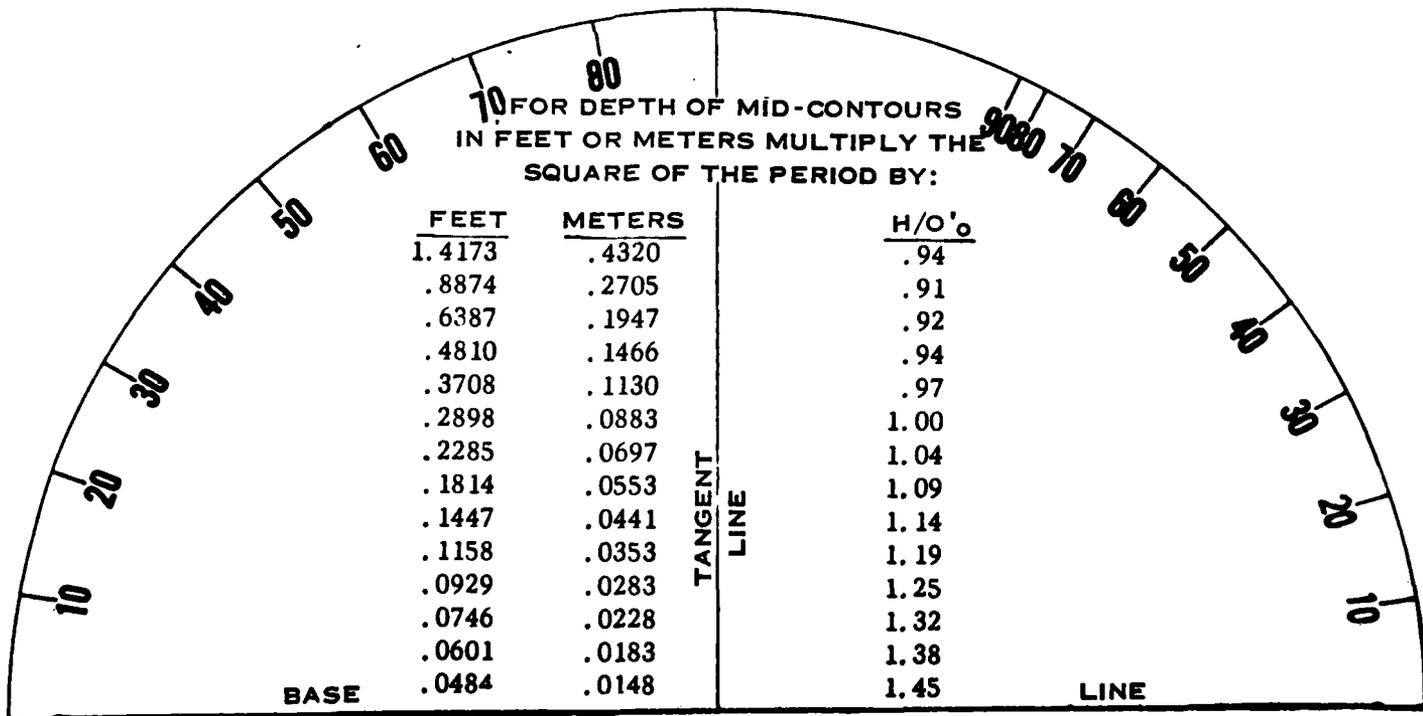
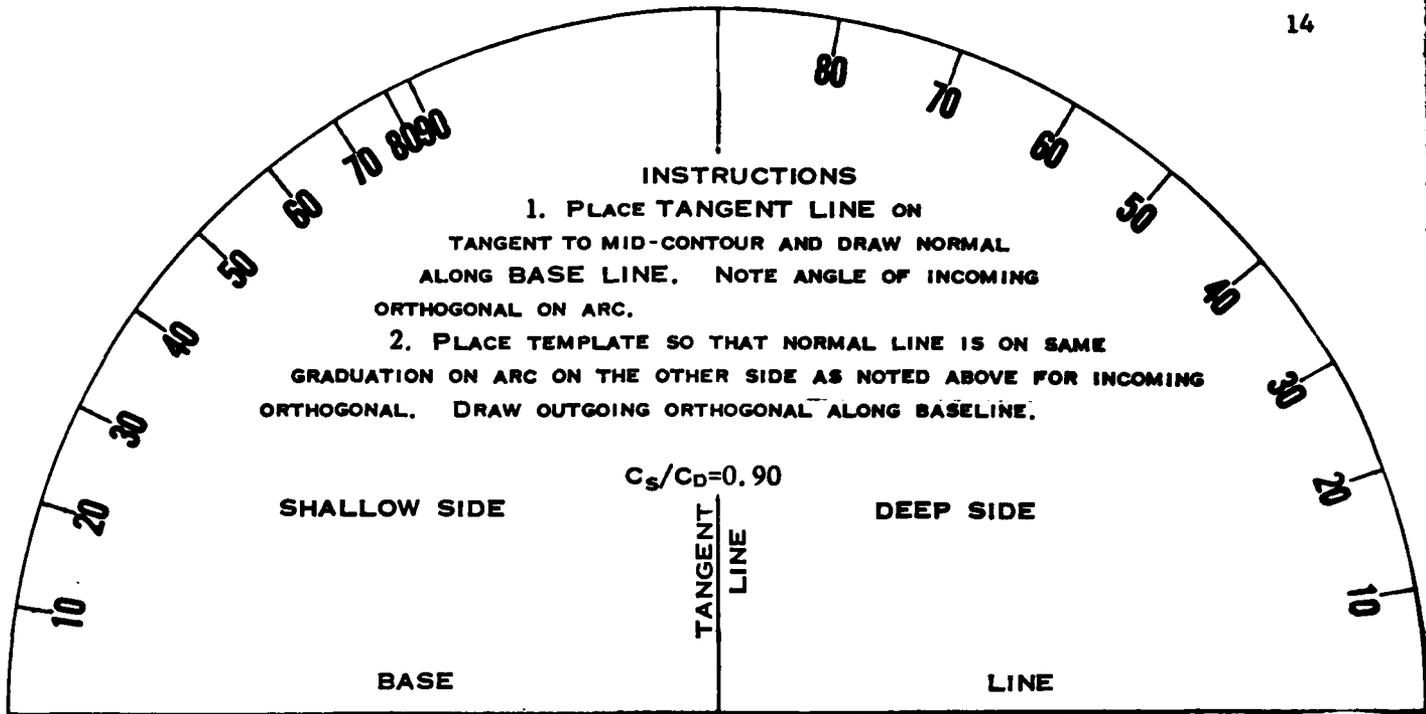


FIG. 3--Refraction Template

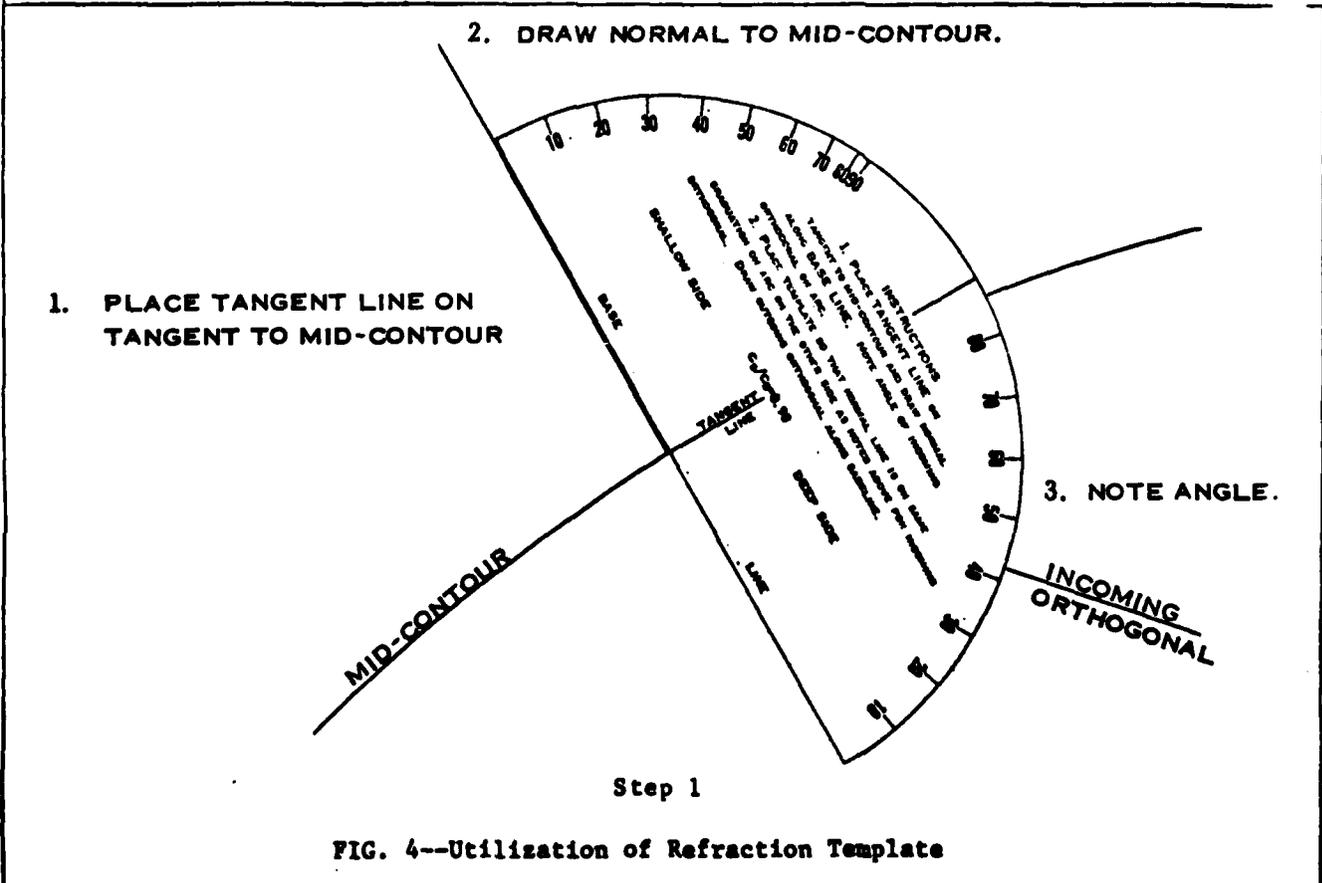
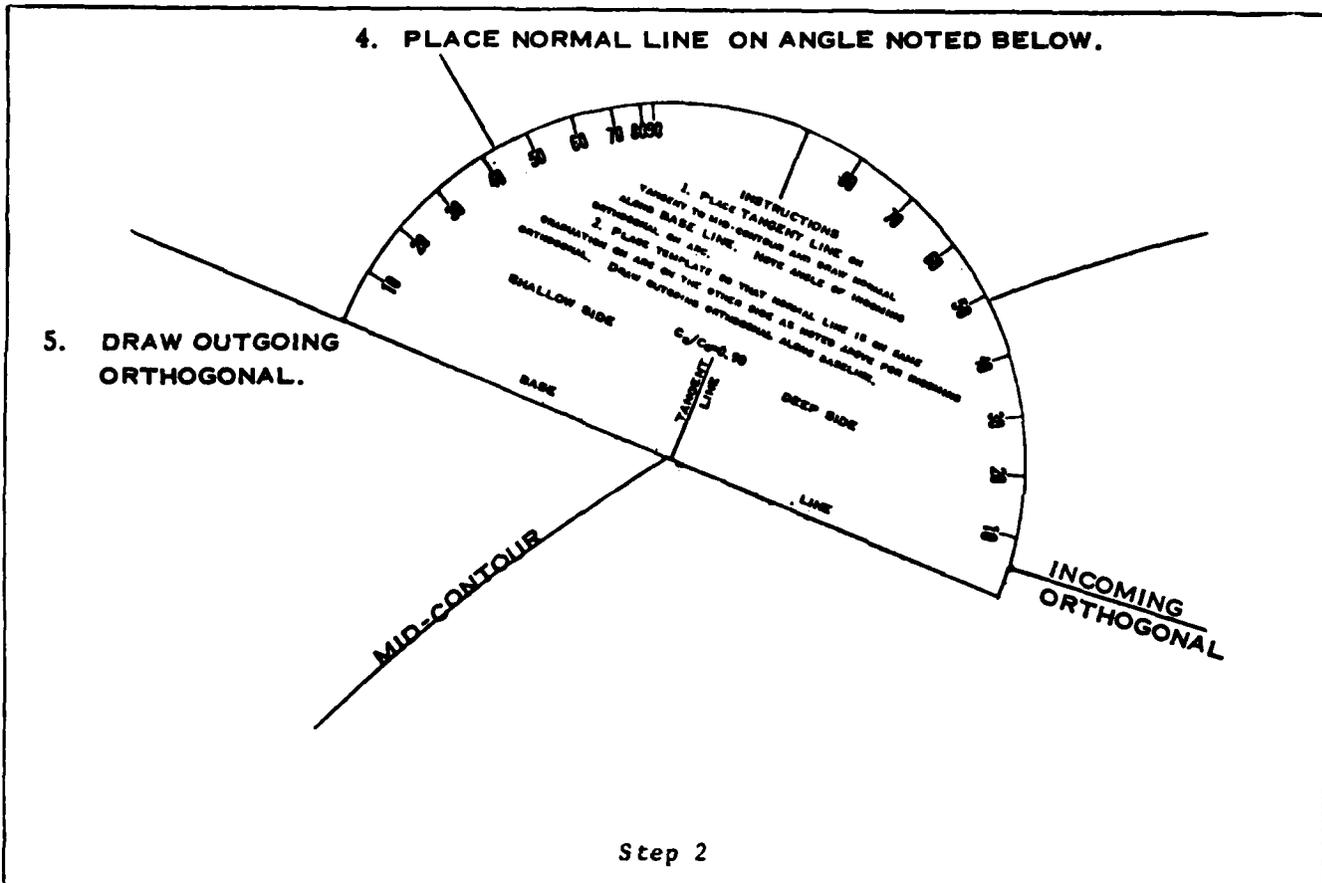


FIG. 4--Utilization of Refraction Template

### SUMMARY

The method of plotting wave refraction diagrams shown here is extremely efficient and simple to use because the ratio of shallower to deeper water celerity is maintained constant. The use of universal tables and simple templates replaces a considerable amount of computation and tedious plotting utilized in the traditional method. It also provides better resolution by utilizing mid-contours that are closer together in shallower water. The simplified method can be used either with the wave front method or the orthogonal method of plotting. It may possibly facilitate in the same manner the utilization of numerical methods with digital computers.

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## APPENDIX - NOTATION

The following symbols are used in this paper:

$b$	=	distance between orthogonals;
$b_o$	=	distance between orthogonals in deep water;
$C$	=	celerity, a phase velocity of a wave;
$C_1, C_2$	=	celerity of a wave in depths $h_1$ and $h_2$ ;
$C_s, C_d$	=	celerity of a wave in shallower or deeper water;
$C_{mid}$	=	celerity of a wave at mid-contour;
$C_o$	=	celerity of a wave in deep water;
$D_d$	=	shoaling coefficient or ratio of unrefracted wave height to deep water wave height;
$g$	=	gravitational constant;
$h$	=	depth of water to still water level;
$h_1, h_2$	=	depths corresponding to $C_1, C_2$ ;
$h_{mid}$	=	the depth at a mid-contour;
$H$	=	wave height from trough to crest;
$H_o'$	=	unrefracted wave height in deep water;
$H/H_o'$	=	$D_d$ see above;
$K$	=	a constant ratio of $C_{mid}/C_o$ ;
$L_1, L_2$	=	wave length in depths $h_1, h_2$ ;
$L$	=	wave length measured from crest to crest;
$n$	=	the number of a depth interval;
$R$	=	a constant ratio of $C_s/C_d$ ;
$T$	=	the period of a wave;
$\alpha$	=	the angle between a wave crest and a bathymetric contour;
$\alpha_s, \alpha_d$	=	angles between wave crests and contours in shallower or deeper water.

## APPENDIX - COMPUTER PROGRAM

```

V M←N REF R:K;X:SC
[1] M←'9990.9990'$0.0001×[0.5+10000×q(5,M)p(R+0.1N-1).K,X,(9.8×X),9.8+0.3048+X+K+(02)+2]+70K+0.5×(1+H)×R+0.1N-1
[2] M←M,(N,7)p'9990.90'$0.01×[0.5+100×SC-(+X×1+2×(-70K)+502×70K)×0.5

```

This function written in APL program language generates a table of information similar to Tables 1 and 2 shown in this paper. The argument N is the number of mid-contours desired and R is the selected ratio of shallow to deeper wave celerity  $\frac{C}{C_d}$ . The matrix M has N rows and 6 columns as follows: (1)  $\frac{C}{C_o}$ , (2)  $\frac{C_{mid}}{C_o}$ , (3)  $\frac{C_{mid}}{C_o}$ , (4)  $\frac{C_{mid}}{T^2}$  for meters, (5)  $\frac{C_{mid}}{T^2}$  for feet and (6)  $\frac{H}{H_o}$  the shoaling coefficient.

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