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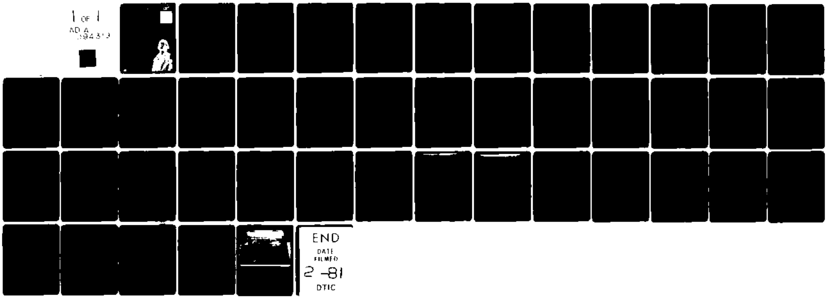

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ON THE AMPLE SERVICE ASSUMPTION OF PALM'S THEOREM IN INVENTORY MODELING

by

⑩ Donald Gross

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Serial T-433
3 October 1980 ⑪

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The George Washington University
School of Engineering and Applied Science
Institute for Management Science and Engineering

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Abstract
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A key assumption of much of the continuous review inventory modeling work is that orders placed do not queue up, so that there is complete order crossing and hence order lead times are strictly independent random variables. This paper investigates the effects of this assumption (which is almost never true).

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by

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1. Introduction

An appropriate inventory policy in many situations is a one-for-one ordering policy [continuous review (s,S) policy where $s = S-1$]. That is, when a demand for an item arises, an order is immediately placed for a replacement. It is desired to find, then, the optimal value of the safety stock needed to support such a policy so that there is a control on both stockout probability and inventory investment.

Such a policy is most often used for items which are expensive and important, so that inventory investment and shortages are significant factors. Also, most repairable item inventory models fall into the one-for-one ordering category, as failed items are usually dispatched immediately to a repair facility upon failure. The METRIC class of models [see Muckstadt (1973)], one of the most useful multi-echelon models currently available, uses such a policy.

A key factor in these types of models is often the "ample server assumption;" that is, orders to be filled or items to be repaired never queue up but go into "service" immediately. Statistically, this means that successive order replenishment times (or repair times if we are talking about repairable items) are *independent*. This assumption allows one to take advantage of Palm's Theorem from queueing theory, which states that if demand is Poisson [or compound Poisson--see Feeney and Sherbrooke (1966)], and there are ample "servers," then regardless of the distribution of order replenishment times, the state probabilities depend on the replenishment time [see Hadley and Whitin (1963), pp. 209 ff., for example]. In fact, letting N represent the steady state number of orders outstanding, λ the mean demand rate assuming the Poisson distribution, and T the mean replenishment lead time,

$$\Pr(N=n) \equiv \pi(n) = \frac{(\lambda T)^n e^{-\lambda T}}{n!} . \tag{1}$$

If we denote the steady state on-hand inventory by Z and assume complete backordering, then we have $Z = S-N$ and

$$\Pr(Z=z) \equiv p(z) = \pi(S-z) . \tag{2}$$

Using this relationship, it is easy to set up cost equations in terms of the decision variable S to be minimized. Since a shortage cost in many cases may be hard to assess, a service level constraint is often used instead. Fill rate (the percentage of requests filled immediately from on-shelf inventory) is one such constraint in wide use. Denoting the fill rate by F, we have

$$\begin{aligned} F &= \frac{\lambda - \Pr(Z\leq 0)}{\lambda} \times 100 \\ &= [1 - \Pr(Z\leq 0)] \times 100 \\ &= \left[1 - \sum_{n=S}^{\infty} \pi(n)\right] \times 100 \\ &= \left[\sum_{n=0}^{S-1} \pi(n)\right] \times 100 . \end{aligned} \tag{3}$$

Now suppose there is not ample service in the order filling (or repair) process. The question we seek to answer is, "What effect does this have on the calculation of S and on the actual F to be realized?" After all, one might argue from the inventory manager's point of view that is, on the average, an order takes τ time units to be received after being placed, what difference does it make if it spends part of its time waiting in a queue to be processed or if it goes into processing immediately? The answer lies in the fact that if queueing occurs, successive replenishment times are correlated and the distribution of $\pi(n)$ can be radically changed.

2. Ample Servers versus Single Server Cases

To see the effect of introducing correlation in successive order replenishment times, let us suppose that instead of a potentially infinite number of "order pickers" (or repair channels), there is only one. Further, let us assume that order filling times are exponentially distributed with mean rate μ. Equation (1) still suffices for the ample server case (with $\tau = 1/\mu$), and in terms of queueing notation we call this the $M/M/\infty$ model with mean arrival rate λ and mean service rate μ $(= 1/\tau)$.

For the single server case, we have an M/M/1 model, still with mean arrival rate λ, but with a mean service rate of $\mu \neq 1/\tau$. Here, τ is equal to the total expected waiting plus service time to process an order (which is usually denoted as W in standard queueing notation). Further, from M/M/1 queueing theory,

$$\tau = W = \frac{1}{\mu-\lambda}, \tag{4}$$

so that the μ to make the M/M/1 "equivalent" in terms of mean lead time to the $M/M/\infty$ is then [rewriting Equation (4)]

$$\mu = \frac{1+\lambda\tau}{\tau}. \tag{5}$$

Now the difference between the two systems can be clearly seen. Denoting the steady state probabilities for the ample server case by $\pi_\infty(n)$ [Equation (1)] and the single server case by $\pi_1(n)$, it can readily be shown [see, for example, Hillier and Lieberman (1980), p. 418] that

$$\pi_1(n) = \left(1 - \frac{\lambda\tau}{1+\lambda\tau}\right)\left(\frac{\lambda\tau}{1+\lambda\tau}\right)^n = \left(\frac{1}{1+\lambda\tau}\right)\left(\frac{\lambda\tau}{1+\lambda\tau}\right)^n . \qquad (6)$$

Note that in the ample server case, the steady state probabilities that n orders are outstanding are Poisson, while for the single server case the steady state probabilities are geometric, even though the mean number of orders outstanding is the same, namely, $\lambda\tau$, the mean leadtime demand. As we shall see later, in certain cases (certain values of $\lambda\tau$), sizable discrepancies in S and F can result from assuming an ample server situation when in reality there is only a single server, even though the mean replenishment times are the same.

3. Ample Servers versus Multiple Server Case

The $M/M/\infty$ and $M/M/1$ cases are the extremes. We consider now "equivalent" $M/M/c$ systems for comparison to $M/M/\infty$. The time in system (waiting plus service) for an $M/M/c$ queue is given as

$$\tau = W = \frac{1}{\mu} + \frac{\mu(\lambda/\mu)^c / (c-1)!(c\mu-\lambda)^2}{\sum\limits_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)} . \qquad (7)$$

It is now necessary to employ numerical solution techniques to find the desired μ which will enable the calculation of the $\pi_c(n)$. We know that

$$\frac{1}{\tau} < \mu < \frac{1+\lambda\tau}{\tau} ; \qquad (8)$$

that is, the resulting μ will be somewhere between the $M/M/\infty$ and $M/M/1$ cases. A Newton-Raphson procedure was easily employed to calculate μ, and once having done so, we have from queueing theory

$$\pi_c(n) = \begin{cases} \dfrac{\lambda^n}{n!\ \mu^n}\, \pi_c(0) , & n < c \\ \\ \dfrac{\lambda^n}{c^{n-c} c!\ \mu^n}\, \pi_c(0) , & n \geq c \end{cases} , \qquad (9)$$

where

$$\pi_c(0) = \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!} \left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right) \right]^{-1} .$$

Thus the resulting fill rate becomes

$$F = \left[\sum_{n=0}^{S-1} \pi_c(n) \right] \times 100 . \tag{10}$$

We can now compare the F's and S's obtained when using an ample server assumption in a situation where service is not truly ample; that is, for part of the replenishment leadtime items may wait in a queue.

4. Numerical Results

The calculations for S are performed by setting a desired fill rate level, say $\hat{F}$, and solving for the S such that

$$100 \sum_{n=0}^{S-1} \pi(n) \overset{\text{just}}{>} \hat{F} . \tag{11}$$

For the ample server case, $\pi_\infty(n)$ [from Equation (1)] is used in Equation (11), while for the "equivalent" c server case, $\pi_c(n)$ [from Equation (9)] is utilized. The respective S's obtained we denote by S_∞ and S_c.

If in reality, we truly had an M/M/c system, but were using the ample service assumption to calculate S, that is we stock S_∞, then the true fill rate $F(S_\infty, \pi_c)$ [call F_∞] is

$$F_\infty = \sum_{n=0}^{S_\infty - 1} \pi_c(n)$$

and may be less than $\hat{F}$, since $S_\infty < S_c$ for all reasonable values of [illegible]. It is this type of "error" which is of interest.

Another "error" of interest involves the expected average backorder level (which is sometimes used instead of fill rate as a service level constraint). For a safety stock level of S units, the expected average backorder level is

$$\begin{aligned} \overline{B} &= \sum_{n=S}^{\infty} (n-S)\pi(n) \\ &= L - S - \sum_{n=0}^{S-1} (n-S)\pi(n) \ , \end{aligned} \tag{12}$$

where L is the expected number of orders outstanding, that is,

$$\begin{aligned} L &\equiv \sum_{n=0}^{\infty} n\pi(n) \\ &= \begin{cases} \lambda\tau \ , & \text{ample service} \ , \\ \dfrac{\lambda}{\mu} + \left[\dfrac{(\lambda/\mu)^c \ \lambda\mu}{(c-1)!(c\mu-\lambda)^2}\right]\left[\displaystyle\sum_{n=0}^{c-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1} \ , & c \text{ servers} \ . \end{cases} \end{aligned} \tag{13}$$

Thus if we provision based on ample servers, that is, stock according to S_∞, our expected average backorder level $\overline{B}(S_\infty,\pi_c)$ [call $\overline{B}_\infty$] is

$$\overline{B}_\infty = L_c - S_\infty - \sum_{n=0}^{S_\infty-1} (n-S_\infty)\pi_c(n) \ ,$$

whereas had we used the "correct" modeling assumptions, accounting for the fact that only c servers are available, our expected average backorder level would have been $\overline{B}(S_c,\pi_c)$ [call $\overline{B}_c$], namely,

$$\overline{B}_c = L_c - S_c - \sum_{n=0}^{S_c-1} (n-S_c)\pi_c(n) \ .$$

If in reality we had ample service, the expected average backorder level would have been $\overline{B}(S_\infty,\pi_\infty)$ [denote by $\overline{B}^*$], specifically,

$$\overline{B}^* = L_\infty - S_\infty - \sum_{n=0}^{S_\infty-1} (n-S_\infty)\pi_\infty(n) \ .$$

Table 1 shows the input and output quantities used for the numerical analyses. We calculate several possible "error" measures on fill rate and backorder level as defined in the table. For fill rate, we look at three quantities. First we compute the percent difference between the actual fill rate attained (F_∞) assuming ample service and using S_∞ and the fill rate we should have gotten (F_c) by stocking S_c had we correctly accounted for the fact that only c servers were available. This we call D_{F_∞}. Next we compute $D_{\hat{F}}$, the percent that the actual fill rate, F_∞, is below our goal $\hat{F}$. For the cases where $S_\infty = S_c$, there is no error and we set $D_{\hat{F}}$ to zero, since for these cases we always either achieve or exceed the goal $\hat{F}$. The final measure on fill rate we compute is D_{F*}, the percent difference between what we *think* we are achieving by assuming ample service $(F*)$ and what we are really achieving (F_∞).

For expected average backorder level, we compute two measures, namely, $D_{\overline{B}_\infty}$, the percent difference in $\overline{B}$ for a c server system if we stock under ample service conditions--that is, using S_∞ instead of the correct S_c --and $D_{\overline{B}*}$, the percent difference in the perceived (believing we have ample service) and actual (with c servers) $\overline{B}$'s.

Figures 1, 2, and 3 show the output of the cases considered, namely, for $\hat{F}$ of 80% (Figure 1), 90% (Figure 2), and 95% (Figure 3), we have computed the error measures for combinations of $\lambda\tau = .25$, .50, 1, 5, 10, 15, ..., 50, and $c = 1,3,5,10,15,20,25$. It appears that the larger errors occur for the larger values of $\lambda\tau$ and smaller values of c, as we would expect. Also, the three fill rate measures seem to track quite closely with each other. Note that the magnitude of the percent error for the backorder measures is much higher than that for the fill rate measures, with $D_{\overline{B}*}$ being an order of magnitude higher than $D_{\overline{B}_\infty}$, which itself is almost an order of magnitude higher than the D_F measures.

TABLE 1

FACTORS IN THE NUMERICAL ANALYSES

Symbol	Definition	Formula
	INPUT	
$\hat{F}$	Desired fill rate	Input
λt	Mean demand over a replenishment lead-time	Input
c	Number of servers (order "pickers" or repair channels)	Input
	OUTPUT	
S_∞	Safety stock required to achieve $\hat{F}$ if ample servers available	$\sum_{n=0}^{S_\infty-1} \pi_\infty(n) \overset{\text{just}}{\geq} \hat{F}$
S_c	Safety stock required to achieve $\hat{F}$ under c servers $(S_c \geq S_\infty)$	$\sum_{n=0}^{S_c-1} \pi_c(n) \overset{\text{just}}{\geq} \hat{F}$
F^*	Actual fill rate achieved using S_∞ if the true state of affairs is ample service	$F^* = \sum_{n=0}^{S_\infty-1} \pi_\infty(n)$
F_c	True fill rate for c servers stocking with S_c $(F_c \geq \hat{F})$	$F_c = \sum_{n=0}^{S_c-1} \pi_c(n)$
F_∞	Actual fill rate achieved for c servers stocking under the assumption of ample service; i.e., using S_∞ $(F_\infty \leq F_c)$	$F_\infty = \sum_{n=0}^{S_\infty-1} \pi_c(n)$
D_{F_∞}	Percent difference actual fill rate is *below* correct fill rate when stocking for a c-server system but using the ample service assumptions	$D_{F_\infty} = \frac{F_c - F_\infty}{F_c} \times 100$
$D_{\hat{F}}$	Percent actual fill rate is *below* fill rate goal when stocking for a c-server system but using the ample service assumption	$D_{\hat{F}} = \max\left[0, \frac{\hat{F} - F_\infty}{\hat{F}} \times 100\right]$

TABLE 1--*continued*

Symbol	Definition	Formula
D_{F*}	Percent actual fill rate is *below* assumed fill rate when stocking for a c-server system but using the ample service assumptions	$D_{F*} = \frac{F* - F_\infty}{F*} \times 100$
$D_{\bar{B}_\infty}$	Percent *increase* in expected average backorder level when stocking for a c-server system but using ample service assumptions	$D_{\bar{B}_\infty} = \frac{\bar{B}_\infty - \bar{B}_c}{\bar{B}_c} \times 100$
$D_{\bar{B}*}$	Percent actual expected average backorder level is *above* assumed expected average backorder level when stocking for a c-server system but using ample service assumptions	$D_{\bar{B}*} = \frac{\bar{B}_\infty - \bar{B}_*}{\bar{B}*} \times 100$

All values for the error measures in Figures 1, 2, and 3 are given in percents so that, for example from Figure 2, for $\hat{F} = 90\%$, $\lambda\tau = 20$, $c = 5$, D_{F_∞} shows a 17.93% error, $D_{\hat{F}}$ a 17.47% error, and D_{F*} a 19.45% error, while $D_{\bar{B}_\infty}$ shows a 170.67% error and $D_{\bar{B}*}$ a 3,113.81% error! Of course, $D_{\bar{B}*}$ shows such large errors because if we think we have ample service, we expect a very low average backorder level (namely, 0.14 units) while in reality we have a level of 4.52 units, which is a large percentage change from 0.14. Had we correctly used the stocking criteria for five servers $(S_c = 45)$, then our expected average backorder level would have been 1.67 units. Perhaps in terms of backorder measures, one should also keep in mind the absolute error as the percentage error is distorted by the small "base" upon which it is calculated.

While errors are larger for larger values of $\lambda\tau$ and smaller values of c , there is not always strict monotonicity which, we believe, is due to the discrete process required in calculating S to satisfy the inequality constraint on fill rate goal $\hat{F}$. For example, from

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                                              FHAT= 85.

 LAM*TAU   C   SC  SINF     FC     FINF    FSTAR   DFINF   DFHAT  DFSTAR   BBARC  BBRINF  BBRSTR  DBRINF   DBRSTR

    0.25   1    2    2   96.00    96.00    97.35    0.0     0.0     1.39  0.0100  0.0100  0.0023    0.0    334.42
    0.25   3    2    2   97.35    97.35    97.35    0.0     0.0     0.00  0.0024  0.0024  0.0023    0.0      4.72
    0.25   5    2    2   97.35    97.35    97.35    0.0     0.0    -0.00  0.0023  0.0023  0.0023    0.0      0.12
    0.25  10    2    2   97.35    97.35    97.35    0.0     0.0    -0.00  0.0023  0.0023  0.0023    0.0      0.17
    0.25  15    2    2   97.35    97.35    97.35    0.0     0.0    -0.00  0.0023  0.0023  0.0023    0.0      0.17
    0.25  20    2    2   97.35    97.35    97.35    0.0     0.0    -0.00  0.0023  0.0023  0.0023    0.0      0.17
    0.25  25    2    2   97.35    97.35    97.35    0.0     0.0    -0.00  0.0023  0.0023  0.0023    0.0      0.12
    0.50   1    2    2   88.89    88.89    90.98    0.0     0.0     2.30  0.0556  0.0556  0.0163    0.0    240.28
    0.50   3    2    2   91.00    91.00    90.98    0.0     0.0    -0.02  0.0179  0.0179  0.0163    0.0      9.47
    0.50   5    2    2   90.98    90.98    90.98    0.0     0.0    -0.00  0.0163  0.0163  0.0163    0.0      0.09
    0.50  10    2    2   90.98    90.98    90.98    0.0     0.0    -0.00  0.0163  0.0163  0.0163    0.0      0.02
    0.50  15    2    2   90.98    90.98    90.98    0.0     0.0    -0.00  0.0163  0.0163  0.0163    0.0      0.02
    0.50  20    2    2   90.98    90.98    90.98    0.0     0.0    -0.00  0.0163  0.0163  0.0163    0.0      0.02
    0.50  25    2    2   90.98    90.98    90.98    0.0     0.0    -0.00  0.0163  0.0163  0.0163    0.0      0.02
    1.00   1    3    3   87.50    87.50    91.97    0.0     0.0     4.86  0.1250  0.1250  0.0233    0.0    435.64
    1.00   3    3    3   91.75    91.75    91.97    0.0     0.0     0.23  0.0389  0.0389  0.0233    0.0     66.55
    1.00   5    3    3   91.97    91.97    91.97    0.0     0.0    -0.00  0.0239  0.0239  0.0233    0.0      2.28
    1.00  10    3    3   91.97    91.97    91.97    0.0     0.0    -0.00  0.0233  0.0233  0.0233    0.0      0.01
    1.00  15    3    3   91.97    91.97    91.97    0.0     0.0    -0.00  0.0233  0.0233  0.0233    0.0      0.01
    1.00  20    3    3   91.97    91.97    91.97    0.0     0.0    -0.00  0.0233  0.0233  0.0233    0.0      0.01
    1.00  25    3    3   91.97    91.97    91.97    0.0     0.0    -0.00  0.0233  0.0233  0.0233    0.0      0.01
    3.00   1    7    6   86.65    62.20    91.61    5.13    3.29   10.27  0.4005  0.5339  0.0507   33.33   953.06
    3.00   3    6    6   85.69    85.69    91.61    0.0     0.0     6.46  0.3009  0.3009  0.0507    0.0    493.48
    3.00   5    6    6   89.83    89.83    91.61    0.0     0.0     1.94  0.1265  0.1265  0.0507    0.0    149.48
    3.00  10    6    6   91.61    91.61    91.61    0.0     0.0    -0.00  0.0510  0.0510  0.0507    0.0      0.58
    3.00  15    6    6   91.61    91.61    91.61    0.0     0.0    -0.00  0.0507  0.0507  0.0507    0.0      0.03
    3.00  20    6    6   91.61    91.61    91.61    0.0     0.0    -0.00  0.0507  0.0507  0.0507    0.0      0.03
    3.00  25    6    6   91.61    91.61    91.61    0.0     0.0    -0.00  0.0507  0.0507  0.0507    0.0      0.03
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    5.00   3   10    8   86.36    78.72    86.66    8.86    7.39    9.17  0.5468  0.8535  0.1221   56.09   598.99
    5.00   5    9    8   86.41    81.68    86.66    5.47    3.90    5.75  0.3913  0.5273  0.1221   34.74   331.79
    5.00  10    8    8   86.65    86.65    86.66    0.0     0.0     0.01  0.1386  0.1386  0.1221    0.0     13.53
    5.00  15    8    8   86.66    86.66    86.66    0.0     0.0    -0.00  0.1222  0.1222  0.1221    0.0      0.06
    5.00  20    8    8   86.66    86.66    86.66    0.0     0.0    -0.00  0.1221  0.1221  0.1221    0.0      0.00
    5.00  25    8    8   86.66    86.66    86.66    0.0     0.0    -0.00  0.1221  0.1221  0.1221    0.0      0.00
   10.00   1   20   14   85.14    73.67    86.45   13.47   13.33   14.78  1.4864  2.6333  0.1869   77.16  1308.71
   10.00   3   19   14   85.03    74.56    86.45   12.31   12.28   13.75  1.3393  2.2752  0.1869   69.88  1117.12
   10.00   5   18   14   85.22    75.86    86.45   10.99   10.75   12.25  1.1322  1.8494  0.1869   63.36   889.38
   10.00  10   16   14   82.52    81.06    86.45    7.38    4.64    6.23  0.5385  0.8171  0.1869   51.73   337.10
   10.00  15   14   14   86.17    86.17    86.45    0.0     0.0     0.32  0.2628  0.2628  0.1869    0.0     40.57
   10.00  20   14   14   86.45    86.45    86.45    0.0     0.0    -0.00  0.1890  0.1890  0.1869    0.0      1.11
   10.00  25   14   14   86.45    86.45    86.45    0.0     0.0    -0.00  0.1870  0.1870  0.1869    0.0      0.02
   15.00   1   30   20   85.57    72.49    87.52   15.29   14.71   17.17  2.1638  4.1259  0.2123   90.67  1843.46
   15.00   3   29   20   85.56    73.04    87.52   14.63   14.07   16.54  2.0117  3.7546  0.2123   86.64  1668.59
   15.00   5   28   20   85.79    73.83    87.52   13.95   13.15   15.65  1.7912  3.2991  0.2123   84.19  1454.03
   15.00  10   25   20   86.29    76.76    87.52   11.05    9.70   12.30  1.2310  2.0872  0.2123   69.54   883.15
   15.00  15   22   20   86.83    81.52    87.52    6.11    4.10    6.86  0.7142  1.0020  0.2123   40.29   371.97
   15.00  20   20   20   86.69    86.69    87.52    0.0     0.0     0.95  0.3632  0.3632  0.2123    0.0     71.09
   15.00  25   20   20   87.51    87.51    87.52    0.0     0.0     0.01  0.2218  0.2218  0.2123    0.0      4.46
   20.00   1   39   26   85.08    71.88    88.78   15.52   15.44   19.04  2.9830  5.6250  0.2186   88.57  2472.85
   20.00   3   38   26   85.05    72.27    88.78   15.03   14.98   18.60  2.8278  5.2465  0.2186   85.54  2299.76
   20.00   5   37   26   85.20    72.81    88.78   14.55   14.34   17.99  2.6015  4.7806  0.2186   83.77  2086.65
   20.00  10   34   26   85.52    74.73    88.78   12.62   12.08   15.83  2.0088  3.5056  0.2186   74.51  1503.45
   20.00  15   31   26   86.15    77.72    88.78    9.79    8.56   12.46  1.3871  2.2320  0.2186   60.92   920.92
   20.00  20   28   26   86.70    82.19    88.78    5.20    3.31    7.43  0.8456  1.1325  0.2186   33.93   417.99
   20.00  25   26   26   87.24    87.24    88.78    0.0     0.0     1.74  0.4449  0.4449  0.2186    0.0    103.48
```

Figure 1.--Output for $\hat{F}$ = 85% .

```
                                         FHAT= 85.

 LAM*TAU  C    SC  SINF     FC      FINF    FSTAR   DFINF   DFHAT   DFSTAR   BBARC   BBRINF  BBRSTR  DBRINF  DBRSTR

  25.00   1    49   31    85.37    70.35    86.33   17.59   17.23   18.51   3.6584   7.4115  0.3152  102.59 2251.69
  25.00   3    48   31    85.36    70.62    86.33   17.27   16.92   18.20   3.5016   7.0262  0.3152  100.66 2129.43
  25.00   5    47   31    85.51    71.00    86.33   16.97   16.47   17.76   3.2685   6.5427  0.3152  100.17 1976.03
  25.00  10    43   31    85.09    72.23    86.33   15.11   15.02   16.33   2.8050   5.2227  0.3152   86.19 1557.18
  25.00  15    40   31    85.53    74.01    86.33   13.47   12.93   14.27   2.1518   3.8652  0.3152   79.63 1126.44
  25.00  20    37   31    86.20    76.65    86.33   11.08    9.82   11.21   1.5065   2.5488  0.3152   69.19  708.76
  25.00  25    33   31    85.00    80.54    86.33    5.26    5.25    6.71   1.0765   1.3973  0.3152   29.89  343.36
  30.00   1    58   37    85.07    70.28    88.04   17.39   17.32   20.17   4.4790   8.9173  0.2953   99.09 2919.72
  30.00   3    57   37    85.05    70.50    88.04   17.12   17.06   19.92   4.3209   8.5298  0.2953   97.41 2788.49
  30.00   5    56   37    85.17    70.61    88.04   16.87   16.70   19.57   4.0843   8.0418  0.2953   96.89 2623.22
  30.00  10    53   37    85.39    71.78    88.04   15.95   15.56   18.47   3.4743   6.7143  0.2953   93.25 2173.68
  30.00  15    49   37    85.13    73.13    88.04   14.09   13.96   16.93   2.9440   5.3185  0.2953   80.66 1701.03
  30.00  20    46   37    85.68    75.05    88.04   12.41   11.70   14.75   2.2500   3.9203  0.2953   74.23 1227.54
  30.00  25    42   37    85.21    77.72    88.04    8.79    8.56   11.72   1.7316   2.6086  0.2953   50.65  783.35
  35.00   1    68   42    85.27    69.37    86.31   18.65   18.39   19.63   5.1537  10.7206  0.3789  108.02 2729.34
  35.00   3    67   42    85.27    69.54    86.31   18.45   18.19   19.44   4.9948  10.3297  0.3789  106.81 2626.18
  35.00   5    66   42    85.38    69.77    86.31   18.28   17.91   19.16   4.7570   9.8362  0.3789  106.77 2495.93
  35.00  10    62   42    85.11    70.50    86.31   17.16   17.06   18.32   4.2808   8.4814  0.3789   98.13 2138.40
  35.00  15    59   42    85.44    71.45    86.31   16.37   15.94   17.22   3.6026   7.0648  0.3789   96.10 1764.51
  35.00  20    55   42    85.27    72.74    86.31   14.69   14.42   15.72   3.0382   5.6221  0.3789   85.05 1383.78
  35.00  25    52   42    85.85    74.50    86.31   13.21   12.35   13.68   2.3338   4.2047  0.3789   80.16 1009.68
  40.00   1    77   48    85.06    69.43    88.04   18.37   18.31   21.14   5.9747  12.2268  0.3448  104.64 3446.18
  40.00   3    76   48    85.06    69.59    88.04   18.19   18.12   20.95   5.8089  11.8252  0.3448  103.57 3329.70
  40.00   5    75   48    85.15    69.79    88.04   18.03   17.89   20.73   5.5755  11.3395  0.3448  103.38 3188.84
  40.00  10    72   48    85.31    70.40    88.04   17.48   17.17   20.03   4.9573   9.9896  0.3448  101.51 2797.33
  40.00  15    68   48    85.17    71.24    88.04   16.36   16.19   19.09   4.4040   8.5413  0.3448   93.94 2377.26
  40.00  20    65   48    85.55    72.27    88.04   15.53   14.98   17.92   3.6928   7.0901  0.3448   92.00 1956.36
  40.00  25    61   48    85.45    73.64    88.04   13.83   13.37   16.36   3.1090   5.6338  0.3448   81.21 1534.00
  45.00   1    87   53    85.22    68.80    86.72   19.27   19.05   20.66   6.6494  14.0385  0.4178  111.12 3259.73
  45.00   3    86   53    85.23    68.94    86.72   19.12   18.90   20.51   6.4823  13.6337  0.4178  110.32 3162.
  45.00   5    85   53    85.29    69.08    86.72   19.01   18.74   20.35   6.2572  13.1587  0.4178  110.30 3049.
  45.00  10    81   53    85.09    69.58    86.72   18.24   18.15   19.77   5.7751  11.7876  0.4178  104.11 2721.[illegible]
  45.00  15    78   53    85.36    70.20    86.72   17.75   17.41   19.04   5.0809  10.3361  0.4178  103.47 2374.14
  45.00  20    74   53    85.27    71.00    86.72   16.73   16.46   18.12   4.4968   8.8496  0.4178   96.80 2017.90
  45.00  25    70   53    85.17    72.02    86.72   15.44   15.27   16.95   3.8998   7.3563  0.4178   88.63 1660.52
  50.00   1    96   58    85.06    68.29    85.51   19.71   19.66   20.14   7.4708  15.8551  0.4922  112.23 3121.15
  50.00   3    95   58    85.06    68.40    85.51   19.59   19.53   20.01   7.3036  15.4492  0.4922  111.53 3038.69
  50.00   5    94   58    85.11    68.51    85.51   19.51   19.40   19.88   7.0773  14.9719  0.4922  111.55 2941.72
  50.00  10    91   58    85.26    68.92    85.51   19.17   18.92   19.41   6.4476  13.5947  0.4922  110.85 2661.92
  50.00  15    87   58    85.16    69.44    85.51   18.45   18.30   18.79   5.8850  12.1174  0.4922  105.90 2361.79
  50.00  20    83   58    85.05    70.05    85.51   17.63   17.59   18.08   5.3060  10.6288  0.4922  100.31 2059.36
  50.00  25    80   58    85.41    70.82    85.51   17.08   16.68   17.18   4.5578   9.1158  0.4922  100.00 1751.98
```

Figure 1.--*continued.*

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                                          FHAT= 90.

LAM*TAU   L    SC  SINF     FL     FINF    FSTAR    DFINF    DFHAT  DFSTAR    BBARC   BBRINF   BBRSTR   DBRINF   DBRSTR

  0.25    1    2    2    96.00   96.00   97.35    0.0      0.0      1.39   0.0100   0.0100   0.0023    0.0     334.42
  0.25    3    2    2    97.35   97.35   97.35    0.0      0.0      0.00   0.0024   0.0024   0.0023    0.0       4.72
  0.25    5    2    2    97.35   97.35   97.35    0.0      0.0     -0.00   0.0023   0.0023   0.0023    0.0       0.12
  0.25   10    2    2    97.35   97.35   97.35    0.0      0.0     -0.00   0.0023   0.0023   0.0023    0.0       0.17
  0.25   15    2    2    97.35   97.35   97.35    0.0      0.0     -0.00   0.0023   0.0023   0.0023    0.0       0.17
  0.25   20    2    2    97.35   97.35   97.35    0.0      0.0     -0.00   0.0023   0.0023   0.0023    0.0       0.17
  0.25   25    2    2    97.35   97.35   97.35    0.0      0.0     -0.00   0.0023   0.0023   0.0023    0.0       0.12
  0.50    1    3    2    96.30   88.89   90.98    7.69     1.23     2.30   0.0185   0.0556   0.0163  199.98    240.28
  0.50    3    2    2    91.00   91.00   90.98    0.0      0.0     -0.02   0.0179   0.0179   0.0163    0.0       9.47
  0.50    5    2    2    90.98   90.98   90.98    0.0      0.0     -0.00   0.0163   0.0163   0.0163    0.0       0.09
  0.50   10    2    2    90.98   90.98   90.98    0.0      0.0     -0.00   0.0163   0.0163   0.0163    0.0       0.02
  0.50   15    2    2    90.98   90.98   90.98    0.0      0.0     -0.00   0.0163   0.0163   0.0163    0.0       0.02
  0.50   20    2    2    90.98   90.98   90.98    0.0      0.0     -0.00   0.0163   0.0163   0.0163    0.0       0.02
  0.50   25    2    2    90.98   90.98   90.98    0.0      0.0     -0.00   0.0163   0.0163   0.0163    0.0       0.02
  1.00    1    4    3    93.75   87.50   91.97    6.67     2.78     4.86   0.0625   0.1250   0.0233  100.00    435.64
  1.00    3    3    3    91.75   91.75   91.97    0.0      0.0      0.23   0.0389   0.0389   0.0233    0.0      66.55
  1.00    5    3    3    91.97   91.97   91.97    0.0      0.0     -0.00   0.0239   0.0239   0.0233    0.0       2.28
  1.00   10    3    3    91.97   91.97   91.97    0.0      0.0     -0.00   0.0233   0.0233   0.0233    0.0       0.01
  1.00   15    3    3    91.97   91.97   91.97    0.0      0.0     -0.00   0.0233   0.0233   0.0233    0.0       0.01
  1.00   20    3    3    91.97   91.97   91.97    0.0      0.0     -0.00   0.0233   0.0233   0.0233    0.0       0.01
  1.00   25    3    3    91.97   91.97   91.97    0.0      0.0     -0.00   0.0233   0.0233   0.0233    0.0       0.01
  3.00    1    9    6    92.49   82.20   91.61   11.12     8.66    10.27   0.2253   0.5339   0.0507  137.04    953.06
  3.00    3    7    6    90.30   85.69   91.61    5.11     4.79     6.46   0.2039   0.3009   0.0507   47.57    493.48
  3.00    5    7    6    94.36   89.83   91.61    4.80     0.19     1.94   0.0701   0.1265   0.0507   80.38    149.48
  3.00   10    6    6    91.61   91.61   91.61    0.0      0.0     -0.00   0.0510   0.0510   0.0507    0.0       0.58
  3.00   15    6    6    91.61   91.61   91.61    0.0      0.0     -0.00   0.0507   0.0507   0.0507    0.0       0.03
  3.00   20    6    6    91.61   91.61   91.61    0.0      0.0     -0.00   0.0507   0.0507   0.0507    0.0       0.03
  3.00   25    6    6    91.61   91.61   91.61    0.0      0.0     -0.00   0.0507   0.0507   0.0507    0.0       0.03
  5.00    1   13    9    90.65   80.62   93.19   11.07    10.42    13.49   0.4673   0.9690   0.0540  107.36   1694.00
  5.00    3   12    9    91.26   82.96   93.19    9.09     7.82    10.97   0.3503   0.6832   0.0540   95.01   1164.78
  5.00    5   11    9    92.51   86.41   93.19    6.60     3.99     7.28   0.2155   0.3913   0.0540   81.55    624.44
  5.00   10    9    9    93.02   93.02   93.19    0.0      0.0      0.18   0.0688   0.0688   0.0540    0.0      27.45
  5.00   15    9    9    93.19   93.19   93.19    0.0      0.0      0.00   0.0541   0.0541   0.0540    0.0       0.14
  5.00   20    9    9    93.19   93.19   93.19    0.0      0.0     -0.00   0.0540   0.0540   0.0540    0.0       0.01
  5.00   25    9    9    93.19   93.19   93.19    0.0      0.0     -0.00   0.0540   0.0540   0.0540    0.0       0.01
 10.00    1   25   15    90.77   76.06   91.65   16.21    15.49    17.01   0.9230   2.3939   0.1035  159.37   2213.64
 10.00    3   23   15    90.20   77.12   91.65   14.50    14.31    15.86   0.8765   2.0464   0.1035  133.46   1877.76
 10.00    5   22   15    90.95   78.45   91.65   13.53    12.62    14.19   0.6931   1.6359   0.1035  136.04   1481.05
 10.00   10   18   15    91.77   84.62   91.65    7.79     5.97     7.67   0.3549   0.6633   0.1035   86.91    541.08
 10.00   15   15   15    90.94   90.94   91.65    0.0      0.0      0.78   0.1722   0.1722   0.1035    0.0      66.39
 10.00   20   15   15    91.65   91.65   91.65    0.0      0.0      0.01   0.1055   0.1055   0.1035    0.0       1.94
 10.00   25   15   15    91.65   91.65   91.65    0.0      0.0     -0.00   0.1035   0.1035   0.1035    0.0       0.04
 15.00    1   36   21    90.21   74.21   91.70   17.73    17.54    19.07   1.4691   3.8680   0.1293  163.29   2890.72
 15.00    3   35   21    90.47   74.85   91.70   17.27    16.83    18.38   1.3271   3.5031   0.1293  163.98   2608.61
 15.00    5   33   21    90.30   75.75   91.70   16.11    15.83    17.40   1.2228   3.0566   0.1293  149.97   2263.36
 15.00   10   28   21    90.01   79.09   91.70   12.14    12.13    13.76   0.8968   1.8780   0.1293  109.41   1352.08
 15.00   15   24   21    90.61   84.40   91.70    6.86     6.23     7.97   0.5090   0.8459   0.1293   66.18    554.07
 15.00   20   21   21    90.26   90.26   91.70    0.0      0.0      1.57   0.2658   0.2658   0.1293    0.0     105.53
 15.00   25   21   21    91.66   91.66   91.70    0.0      0.0      0.04   0.1384   0.1384   0.1293    0.0       7.02
 20.00    1   48   27    90.39   73.21   92.21   19.00    18.65    20.60   1.9229   5.3571   0.1407  178.60   3706.20
 20.00    3   46   27    90.10   73.66   92.21   18.25    18.15    20.12   1.8728   4.9832   0.1407  166.08   3440.50
 20.00    5   45   27    90.49   74.27   92.21   17.93    17.47    19.45   1.6712   4.5233   0.1407  170.67   3113.81
 20.00   10   40   27    90.46   76.43   92.21   15.51    15.08    17.11   1.3230   3.2699   0.1407  147.15   2223.24
 20.00   15   35   27    90.54   79.74   92.21   11.92    11.40    13.52   0.9480   2.0294   0.1407  114.08   1341.91
 20.00   20   30   27    90.07   64.61   92.21    6.06     5.99     8.24   0.6313   0.9785   0.1407   55.00    595.25
 20.00   25   27   27    90.08   90.08   92.21    0.0      0.0      2.31   0.3457   0.3457   0.1407    0.0     145.63
```

Figure 2.--Output for $\hat{F} = 90\%$.

```
                                          FHAT= 90.

LAM*TAU   C    SC  SINF      FC     FINF    FSTAR    DFINF    DFHAT   DFSTAR    BBARC   BBINF   BBRSTR   DBRINF   DBRSTR

 25.00    1    59   33    90.11    72.59    92.85    19.45    19.34    21.82   2.4715  6.8523   0.1436   177.26  4671.29
 25.00    3    58   33    90.28    72.93    92.85    19.22    18.97    21.46   2.3246  6.4734   0.1436   178.48  4407.48
 25.00    5    56   33    90.19    73.41    92.85    18.61    18.44    20.94   2.2121  5.9991   0.1436   171.20  4077.17
 25.00   10    51   33    90.15    74.97    92.85    16.84    16.70    19.26   1.8534  4.7087   0.1436   154.06  3178.70
 25.00   15    46   33    90.21    77.18    92.85    14.44    14.24    16.88   1.4562  3.3935   0.1436   133.03  2262.88
 25.00   20    41   33    90.28    80.41    92.85    10.94    10.66    13.41   1.0610  2.1390   0.1436   101.60  1389.40
 25.00   25    37   33    91.10    85.00    92.85     6.69     5.55     8.45   0.6590  1.0765   0.1436    69.47   649.55
 30.00    1    71   38    90.25    71.23    91.10    21.07    20.85    21.80   2.9246  8.6297   0.2063   195.08  4083.41
 30.00    3    69   38    90.06    71.48    91.10    20.63    20.57    21.53   2.8732  8.2446   0.2063   186.95  3896.75
 30.00    5    68   38    90.33    71.83    91.10    20.48    20.19    21.15   2.6625  7.7601   0.2063   191.45  3661.84
 30.00   10    63   38    90.32    72.91    91.10    19.27    18.98    19.96   2.3017  6.4434   0.2063   179.95  3023.57
 30.00   15    58   38    90.45    74.42    91.10    17.72    17.31    18.30   1.8893  5.0627   0.2063   167.97  2354.25
 30.00   20    52   38    90.11    76.55    91.10    15.05    14.95    15.97   1.5540  3.6857   0.2063   137.18  1686.74
 30.00   25    47   38    90.18    79.47    91.10    11.88    11.70    12.76   1.1494  2.4033   0.2063   109.09  1065.04
 35.00    1    82   44    90.07    71.05    92.09    21.12    21.06    22.85   3.4740 10.1332   0.1948   191.69  5102.44
 35.00    3    81   44    90.19    71.26    92.09    21.00    20.83    22.62   3.3250  9.7463   0.1948   193.12  4903.80
 35.00    5    79   44    90.14    71.55    92.09    20.62    20.50    22.30   3.2095  9.2584   0.1948   188.47  4653.29
 35.00   10    74   44    90.12    72.45    92.09    19.61    19.50    21.32   2.8402  7.9209   0.1948   178.88  3966.61
 35.00   15    69   44    90.20    73.62    92.09    18.38    18.19    20.05   2.4241  6.5266   0.1948   169.23  3250.78
 35.00   20    64   44    90.38    75.20    92.09    16.79    16.44    18.33   1.9842  5.1142   0.1948   157.75  2525.66
 35.00   25    58   44    90.06    77.34    92.09    14.13    14.07    16.02   1.6393  3.7376   0.1948   128.00  1818.92
 40.00    1    96   49    90.18    70.18    90.75    22.18    22.02    22.67   3.9266 11.9286   0.2523   203.79  4627.56
 40.00    3    92   49    90.05    70.36    90.75    21.87    21.83    22.47   3.8697 11.5287   0.2523   197.92  4469.10
 40.00    5    91   49    90.25    70.57    90.75    21.80    21.59    22.24   3.6608 11.0453   0.2523   201.72  4277.50
 40.00   10    86   49    90.24    71.26    90.75    21.04    20.83    21.48   3.2941  9.7022   0.2523   194.53  3745.21
 40.00   15    80   49    90.03    72.17    90.75    19.83    19.81    20.47   2.9596  8.2631   0.2523   179.19  3174.84
 40.00   20    75   49    90.16    73.31    90.75    18.69    18.54    19.22   2.5160  6.8232   0.2523   171.19  2604.19
 40.00   25    70   49    90.36    74.82    90.75    17.20    16.87    17.56   2.0601  5.3820   0.2523   161.24  2033.01
 45.00    1   105   55    90.05    70.14    91.84    22.11    22.06    23.62   4.4768 13.4348   0.2314   200.10  5707.03
 45.00    3   104   55    90.15    70.30    91.84    22.02    21.88    23.45   4.3212 13.0330   0.2314   201.60  5533.36
 45.00    5   102   55    90.09    70.48    91.84    21.77    21.69    23.26   4.2157 12.5613   0.2314   197.97  5329.49
 45.00   10    97   55    90.08    71.09    91.84    21.09    21.01    22.59   3.8414 11.2019   0.2314   191.61  4741.90
 45.00   15    92   55    90.16    71.65    91.84    20.31    20.17    21.76   3.4133  9.7670   0.2314   186.15  4121.68
 45.00   20    87   55    90.31    72.82    91.84    19.37    19.09    20.71   2.9572  8.7970   0.2314   180.57  3486.23
 45.00   25    81   55    90.15    74.03    91.84    17.89    17.74    19.39   2.5865  6.8270   0.2314   163.95  2850.90
 50.00    1   117   60    90.14    69.52    90.77    22.88    22.75    23.41   4.9291 15.2394   0.2835   209.17  5274.72
 50.00    3   115   60    90.03    69.65    90.77    22.64    22.61    23.27   4.8714 14.8361   0.2835   204.56  5132.47
 50.00    5   114   60    90.18    69.80    90.77    22.61    22.45    23.11   4.6674 14.3615   0.2835   207.70  4965.08
 50.00   10   109   60    90.19    70.29    90.77    22.06    21.90    22.56   4.2921 12.9938   0.2835   202.73  4482.71
 50.00   15   103   60    90.04    70.93    90.77    21.22    21.19    21.86   3.9508 11.5286   0.2835   191.81  3965.97
 50.00   20    98   60    90.14    71.67    90.77    20.49    20.37    21.05   3.4973 10.0542   0.2835   187.48  3445.95
 50.00   25    92   60    90.00    72.60    90.77    19.33    19.33    20.02   3.1229  8.5591   0.2835   174.08  2918.66
```

Figure 2.--*continued.*

```
                                          FHAT= 95.

LAM*TAU   C   SC  SINF    FC     FINF    FSTAR   DFINF   DFHAT   DFSTAR   BBARC   BBKINF  BBRSTR  DBRINF  DBRSTR

  0.25    1    2    2   96.00   96.00   97.35    0.0     0.0     1.39   0.0100  0.0100  0.0023    0.0   334.42
  0.25    3    2    2   97.35   97.35   97.35    0.0     0.0     0.00   0.0024  0.0024  0.0023    0.0     4.72
  0.25    5    2    2   97.35   97.35   97.35    0.0     0.0    -0.00   0.0023  0.0023  0.0023    0.0     0.12
  0.25   10    2    2   97.35   97.35   97.35    0.0     0.0    -0.00   0.0023  0.0023  0.0023    0.0     0.17
  0.25   15    2    2   97.35   97.35   97.35    0.0     0.0    -0.00   0.0023  0.0023  0.0023    0.0     0.17
  0.25   20    2    2   97.35   97.35   97.35    0.0     0.0    -0.00   0.0023  0.0023  0.0023    0.0     0.17
  0.25   25    2    2   97.35   97.35   97.35    0.0     0.0    -0.00   0.0023  0.0023  0.0023    0.0     0.12
  0.50    1    3    3   96.30   96.30   98.56    0.0     0.0     2.30   0.0185  0.0185  0.0019    0.0   855.24
  0.50    3    3    3   98.51   98.51   98.56    0.0     0.0     0.05   0.0030  0.0030  0.0019    0.0    52.83
  0.50    5    3    3   98.56   98.56   98.56    0.0     0.0     0.00   0.0020  0.0020  0.0019    0.0     0.69
  0.50   10    3    3   98.56   98.56   98.56    0.0     0.0    -0.00   0.0019  0.0019  0.0019    0.0     0.25
  0.50   15    3    3   98.56   98.56   98.56    0.0     0.0    -0.00   0.0019  0.0019  0.0019    0.0     0.25
  0.50   20    3    3   98.56   98.56   98.56    0.0     0.0    -0.00   0.0019  0.0019  0.0019    0.0     0.25
  0.50   25    3    3   98.56   98.56   98.56    0.0     0.0    -0.00   0.0019  0.0019  0.0019    0.0     0.25
  1.00    1    5    4   96.88   93.75   98.10    3.23    1.32    4.44   0.0313  0.0625  0.0043  100.00 1337.19
  1.00    3    4    4   97.36   97.36   98.10    0.0     0.0     0.76   0.0125  0.0125  0.0043    0.0   186.34
  1.00    5    4    4   98.09   98.09   98.10    0.0     0.0     0.01   0.0048  0.0048  0.0043    0.0     9.69
  1.00   10    4    4   98.10   98.10   98.10    0.0     0.0    -0.00   0.0044  0.0044  0.0043    0.0     0.07
  1.00   15    4    4   98.10   98.10   98.10    0.0     0.0    -0.00   0.0044  0.0044  0.0043    0.0     0.07
  1.00   20    4    4   98.10   98.10   98.10    0.0     0.0    -0.00   0.0044  0.0044  0.0043    0.0     0.07
  1.00   25    4    4   98.10   98.10   98.10    0.0     0.0    -0.00   0.0044  0.0044  0.0043    0.0     0.07
  3.00    1   11    7   95.78   86.65   96.65    9.53    8.79   10.34   0.1267  0.4005  0.0172  216.05 2228.92
  3.00    3    9    7   95.55   90.30   96.65    5.49    4.95    6.57   0.0936  0.2039  0.0172  117.77 1085.89
  3.00    5    8    7   96.88   94.36   96.65    2.59    0.67    2.37   0.0389  0.0701  0.0172   80.37  307.83
  3.00   10    7    7   96.65   96.65   96.65    0.0     0.0     0.00   0.0175  0.0175  0.0172    0.0     1.55
  3.00   15    7    7   96.65   96.65   96.65    0.0     0.0    -0.00   0.0172  0.0172  0.0172    0.0     0.11
  3.00   20    7    7   96.65   96.65   96.65    0.0     0.0    -0.00   0.0172  0.0172  0.0172    0.0     0.11
  3.00   25    7    7   96.65   96.65   96.65    0.0     0.0    -0.00   0.0172  0.0172  0.0172    0.0     0.13
  5.00    1   17   10   95.49   83.85   96.82   12.19   11.74   13.39   0.2254  0.8075  0.0222  258.33 3539.76
  5.00    3   15   10   95.52   86.36   96.82    9.59    9.09   10.80   0.1796  0.5468  0.0222  204.39 2364.67
  5.00    5   13   10   95.88   89.91   96.82    6.22    5.36    7.13   0.1187  0.2904  0.0222  144.62 1209.00
  5.00   10   10   10   96.53   96.53   96.82    0.0     0.0     0.29   0.0342  0.0342  0.0222    0.0    54.09
  5.00   15   10   10   96.82   96.82   96.82    0.0     0.0     0.00   0.0223  0.0223  0.0222    0.0     0.32
  5.00   20   10   10   96.82   96.82   96.82    0.0     0.0    -0.00   0.0222  0.0222  0.0222    0.0     0.03
  5.00   25   10   10   96.82   96.82   96.82    0.0     0.0    -0.00   0.0222  0.0222  0.0222    0.0     0.03
 10.00    1   32   16   95.26   78.24   95.13   17.87   17.65   17.75   0.4735  2.1703  0.0547  359.50 3476.52
 10.00    3   30   16   95.33   79.42   95.13   16.69   16.40   16.51   0.4174  1.8406  0.0547  340.93 3263.15
 10.00    5   27   16   95.10   81.11   95.13   14.71   14.62   14.73   0.2755  1.4470  0.0547  285.59 2544.01
 10.00   10   21   16   95.60   87.52   95.13    8.45    7.88    8.00   0.1899  0.5385  0.0547  183.60  883.94
 10.00   15   17   16   96.11   94.06   95.13    2.13    0.99    1.12   0.0739  0.1128  0.0547   52.61  106.11
 10.00   20   16   16   95.11   95.11   95.13    0.0     0.0     0.01   0.0566  0.0566  0.0547    0.0     3.44
 10.00   25   16   16   95.13   95.13   95.13    0.0     0.0    -0.00   0.0548  0.0548  0.0547    0.0     0.07
 15.00    1   47   23   95.18   77.34   96.73   18.75   18.59   20.05   0.7223  3.3996  0.0435  370.66 7720.16
 15.00    3   45   23   95.24   78.11   96.73   17.99   17.78   19.25   0.6634  3.0495  0.0435  359.71 6914.89
 15.00    5   42   23   95.12   79.18   96.73   16.75   16.65   18.14   0.6151  2.6238  0.0435  326.59 5935.50
 15.00   10   35   23   95.23   83.07   96.73   12.77   12.56   14.12   0.4283  1.5205  0.0435  255.05 3397.65
 15.00   15   28   23   95.23   88.88   96.73    6.67    6.44    8.11   0.2586  0.6030  0.0435  133.15 1287.01
 15.00   20   24   23   96.18   94.78   96.73    1.45    0.23    2.01   0.1042  0.1424  0.0435   36.64  227.52
 15.00   25   23   23   96.64   96.64   96.73    0.0     0.0     0.09   0.0511  0.0511  0.0435    0.0    17.48
 20.00    1   62   29   95.14   75.70   96.57   20.43   20.31   21.60   0.9711  4.8591  0.0539  400.38 8910.86
 20.00    3   60   29   95.19   76.24   96.57   19.91   19.75   21.05   0.9105  4.4954  0.0539  393.73 8236.45
 20.00    5   57   29   95.11   76.97   96.57   19.07   18.98   20.30   0.8604  4.0496  0.0539  370.68 7409.76
 20.00   10   50   29   95.24   79.49   96.57   16.54   16.32   17.68   0.6596  2.8450  0.0539  331.31 5175.83
 20.00   15   42   29   95.14   83.25   96.57   12.49   12.37   13.79   0.4871  1.6778  0.0539  244.46 3011.35
 20.00   20   35   29   95.22   88.51   96.57    7.05    6.83    8.34   0.3041  0.7306  0.0539  140.23 1254.92
 20.00   25   30   29   95.35   94.01   96.57    1.40    1.04    2.65   0.1623  0.2088  0.0539   28.67  287.18
```

Figure 3.--Output for $\hat{F} = 95\%$.

```
                                        FHAT= 95.

LAM*TAU  C   SC  SINF     FC     FINF    FSTAR   DFINF   DFHAT  DFSTAR   BBARC  BBRINF  BBRSTR  DBRINF  DBRSTR

 25.00   1   77   34   95.12   73.64   95.02   22.58   22.48   22.50  1.2200  6.5888  0.0938  440.06 6921.17
 25.00   3   75   34   95.15   74.02   95.02   22.21   22.09   22.10  1.1584  6.2136  0.0938  436.38 6521.41
 25.00   5   72   34   95.10   74.54   95.02   21.62   21.54   21.56  1.1050  5.7444  0.0938  419.84 6021.39
 25.00  10   65   34   95.23   76.23   95.02   19.95   19.76   19.78  0.8975  4.4710  0.0938  398.19 4664.45
 25.00  15   57   34   95.21   78.62   95.02   17.43   17.24   17.26  0.7118  3.1796  0.0938  346.71 3288.31
 25.00  20   49   34   95.18   82.05   95.02   13.79   13.63   13.65  0.5263  1.9595  0.0938  272.30 1908.13
 25.00  25   42   34   95.36   86.84   95.02    8.94    8.59    8.61  0.3329  0.9449  0.0938  183.79  906.88
 30.00   1   92   40   95.10   73.06   95.37   23.19   23.09   23.40  1.4690  8.0819  0.0952  450.17 8392.16
 30.00   3   90   40   95.13   73.36   95.37   22.89   22.78   23.08  1.4068  7.7026  0.0952  447.54 7993.55
 30.00   5   87   40   95.09   73.77   95.37   22.42   22.35   22.65  1.3523  7.2259  0.0952  434.35 7492.72
 30.00  10   80   40   95.19   75.06   95.37   21.16   20.99   21.30  1.1429  5.9340  0.0952  419.20 6135.24
 30.00  15   72   40   95.21   76.82   95.37   19.31   19.13   19.45  0.9476  4.5875  0.0952  384.09 4720.36
 30.00  20   64   40   95.28   79.27   95.37   16.81   16.56   16.89  0.7413  3.2579  0.0952  339.50 3323.28
 30.00  25   56   40   95.30   82.58   95.37   13.36   13.08   13.42  0.5498  2.0399  0.0952  271.03 2043.50
 35.00   1  107   46   95.09   72.63   95.75   23.62   23.54   24.14  1.7178  9.5781  0.0938  457.5910111.64
 35.00   3  105   46   95.12   72.88   95.75   23.38   23.28   23.89  1.6550  9.1959  0.0938  455.63 9704.20
 35.00   5  102   46   95.08   73.22   95.75   22.99   22.93   23.53  1.5997  8.7145  0.0938  444.76 9190.95
 35.00  10   94   46   95.01   74.27   95.75   21.83   21.82   22.43  1.4335  7.3974  0.0938  416.02 7786.69
 35.00  15   86   46   95.00   75.63   95.75   20.39   20.39   21.01  1.2361  6.0294  0.0938  387.78 6328.25
 35.00  20   78   46   95.04   77.44   95.75   18.52   18.48   19.12  1.0227  4.6522  0.0938  354.89 4859.91
 35.00  25   70   46   95.10   79.85   95.75   16.03   15.94   16.60  0.8089  3.3225  0.0938  310.75 3442.26
 40.00   1  122   52   95.08   72.31   96.13   23.95   23.89   24.78  1.9668 11.0768  0.0906  463.2012123.13
 40.00   3  120   52   95.11   72.53   96.13   23.74   23.65   24.55  1.9008 10.6833  0.0906  462.0411688.82
 40.00   5  117   52   95.08   72.81   96.13   23.42   23.36   24.26  1.8477 10.2075  0.0906  452.4411163.81
 40.00  10  109   52   95.01   73.67   96.13   22.47   22.46   23.36  1.6831  8.8886  0.0906  428.12 9708.43
 40.00  15  101   52   95.03   74.81   96.13   21.28   21.26   22.18  1.4761  7.4815  0.0906  406.84 8155.77
 40.00  20   93   52   95.07   76.21   96.13   19.83   19.78   20.72  1.2611  6.0813  0.0906  382.23 6610.59
 40.00  25   85   52   95.15   78.04   96.13   17.97   17.85   18.81  1.0376  4.6921  0.0906  352.21 5077.64
 45.00   1  137   57   95.08   71.43   95.27   24.87   24.81   25.03  2.2157 12.8570  0.1215  480.2710478.73
 45.00   3  135   57   95.10   71.61   95.27   24.70   24.62   24.83  2.1490 12.4588  0.1215  479.7510151.07
 45.00   5  132   57   95.06   71.82   95.27   24.45   24.40   24.62  2.0997 11.9911  0.1215  471.10 9766.30
 45.00  10  124   57   95.02   72.52   95.27   23.67   23.66   23.88  1.9304 10.6453  0.1215  451.46 8658.98
 45.00  15  116   57   95.02   73.40   95.27   22.75   22.73   22.95  1.7257  9.2275  0.1215  434.72 7492.35
 45.00  20  108   57   95.08   74.51   95.27   21.63   21.57   21.79  1.5026  7.7789  0.1215  417.70 6300.47
 45.00  25  100   57   95.16   75.90   95.27   20.24   20.11   20.33  1.2726  6.3359  0.1215  397.86 5113.17
 50.00   1  152   63   95.07   71.28   95.76   25.02   24.97   25.56  2.4646 14.3605  0.1133  482.6812575.11
 50.00   3  150   63   95.09   71.44   95.76   24.87   24.80   25.39  2.3977 13.9617  0.1133  482.2912223.15
 50.00   5  147   63   95.06   71.62   95.76   24.65   24.61   25.20  2.3480 13.4922  0.1133  474.6311808.74
 50.00  10  139   63   95.02   72.24   95.76   23.97   23.96   24.56  2.1782 12.1418  0.1133  457.4110616.83
 50.00  15  131   63   95.04   73.02   95.76   23.17   23.14   23.75  1.9669 10.6987  0.1133  443.95 9343.07
 50.00  20  123   63   95.08   73.93   95.76   22.24   22.17   22.79  1.7457  9.2500  0.1133  429.88 8064.39
 50.00  25  114   63   95.00   75.07   95.76   20.98   20.98   21.60  1.5614  7.7871  0.1133  398.72 6773.20
```

Figure 3.--*continued.*

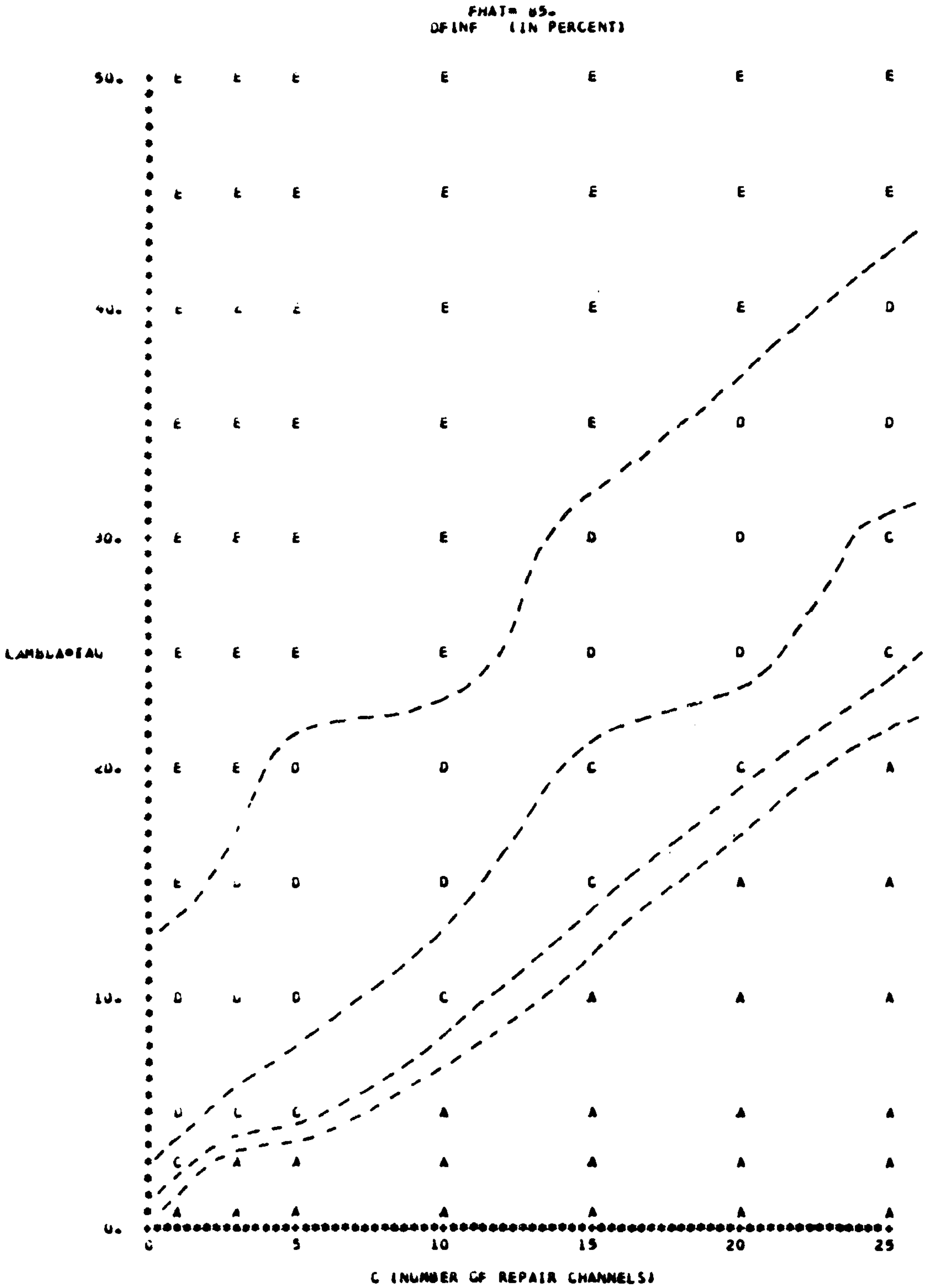
Figure 2, $\lambda\tau = 25$, we see that the percentage error in $D_{\bar{B}_\infty}$ increases slightly when going from $C = 1$ to $C = 3$, although this is a rather rare situation and the general trend is decreasing. The "nonmonotonicity" is somewhat more common when fixing c and observing the errors as $\lambda\tau$ increases. Again, even in cases where there is not strict monotonicity, the violations are small and there still remains a general trend.

Figures 4 through 18 show graphs of the error ranges of the D_F and $D_{\bar{B}}$ measures on the $\lambda\tau$ versus c space. A definite pattern emerges even though in a few cases the nonmonotonicity shows up. The error band lines are purposely plotted as "fuzzy," since the grid is not fine enough to obtain precise boundaries.

These graphs do show clearly that for large $\lambda\tau$ and small c , the percentage error can be sizable. Also, errors for comparable cases (same $\lambda\tau$ and c) become larger as $\hat{F}$ is increased. This can be seen by looking at comparable measures for the three $\hat{F}$ situations; for example, by comparing Figures 4, 9, and 14, or Figures 7, 12, and 17, and so forth.

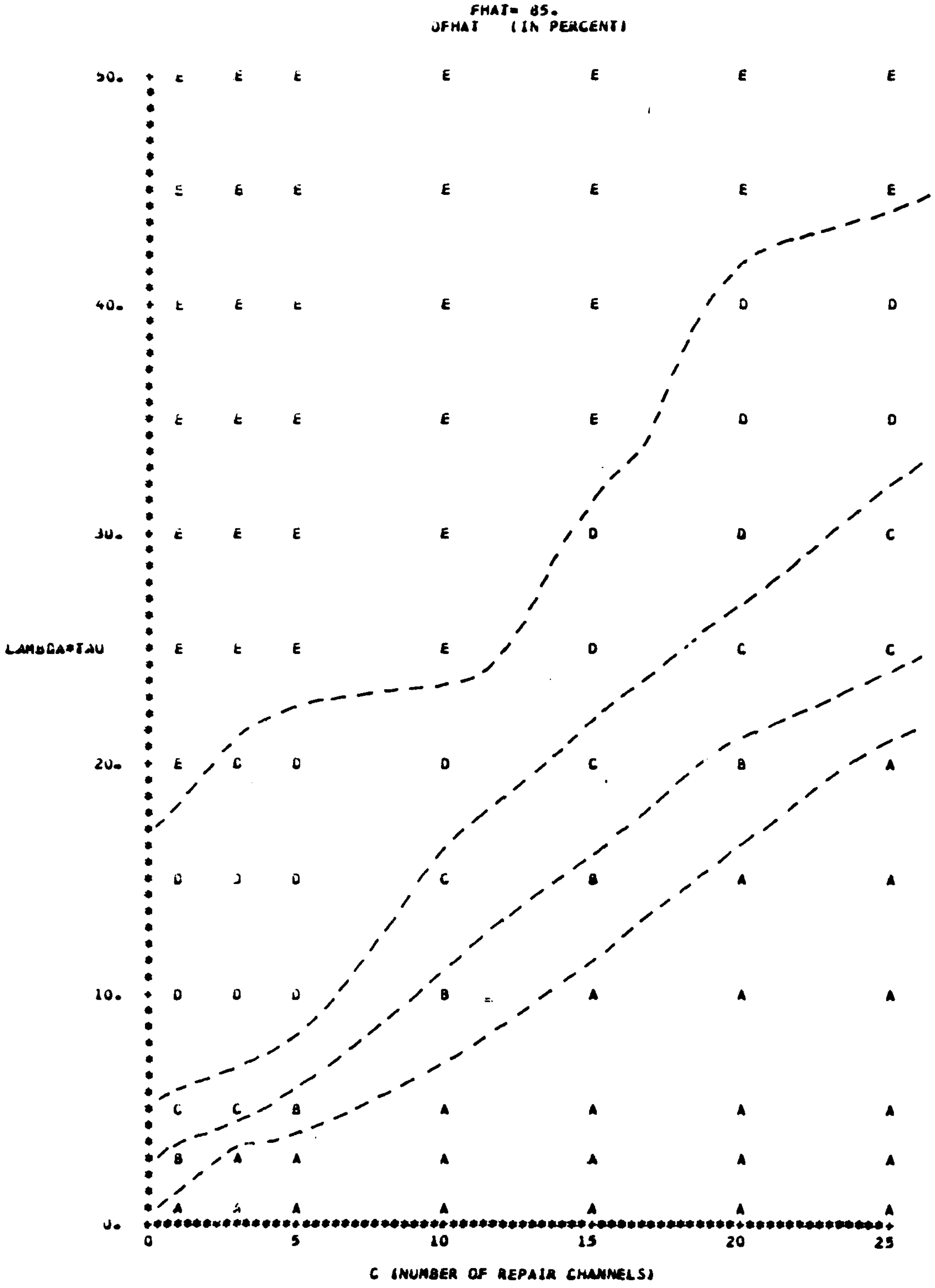
While the general direction of large errors (larger $\lambda\tau$, smaller c , larger $\hat{F}$) may not be surprising, the actual magnitude might be. Certainly, one should give careful thought prior to employing the ample service assumption.

Figures 19 through 24 give plots of D_{F_∞} and $D_{\bar{B}_\infty}$ versus $\lambda\tau$ for various c values, and $\hat{F}$'s of 85%, 90%, and 95%, respectively. These sets of curves can be used to find the error (or approximate error if interpolation is necessary) in assuming ample service when in reality it is not. Keep in mind that these results assume exponential lead times in the nonample service model, and that the stockage criteria are based on fill rate control.



KEY:	DFINF		
A:	-	0.	%
B:	0.-	5.	%
C:	5.-	10.	%
D:	10.-	15.	%
E:	15.-	20.	%
F:	20.-		%

Figure 4.--D_{F_∞} for $\hat{F} = 85\%$.



KEY: DFHAT
A: - 0. %
B: 0.- 5. %
C: 5.- 10. %
D: 10.- 15. %
E: 15.- 20. %
F: 20.- %

Figure 5.--$D_{\hat{F}}$ for $\hat{F}$ = 85% .

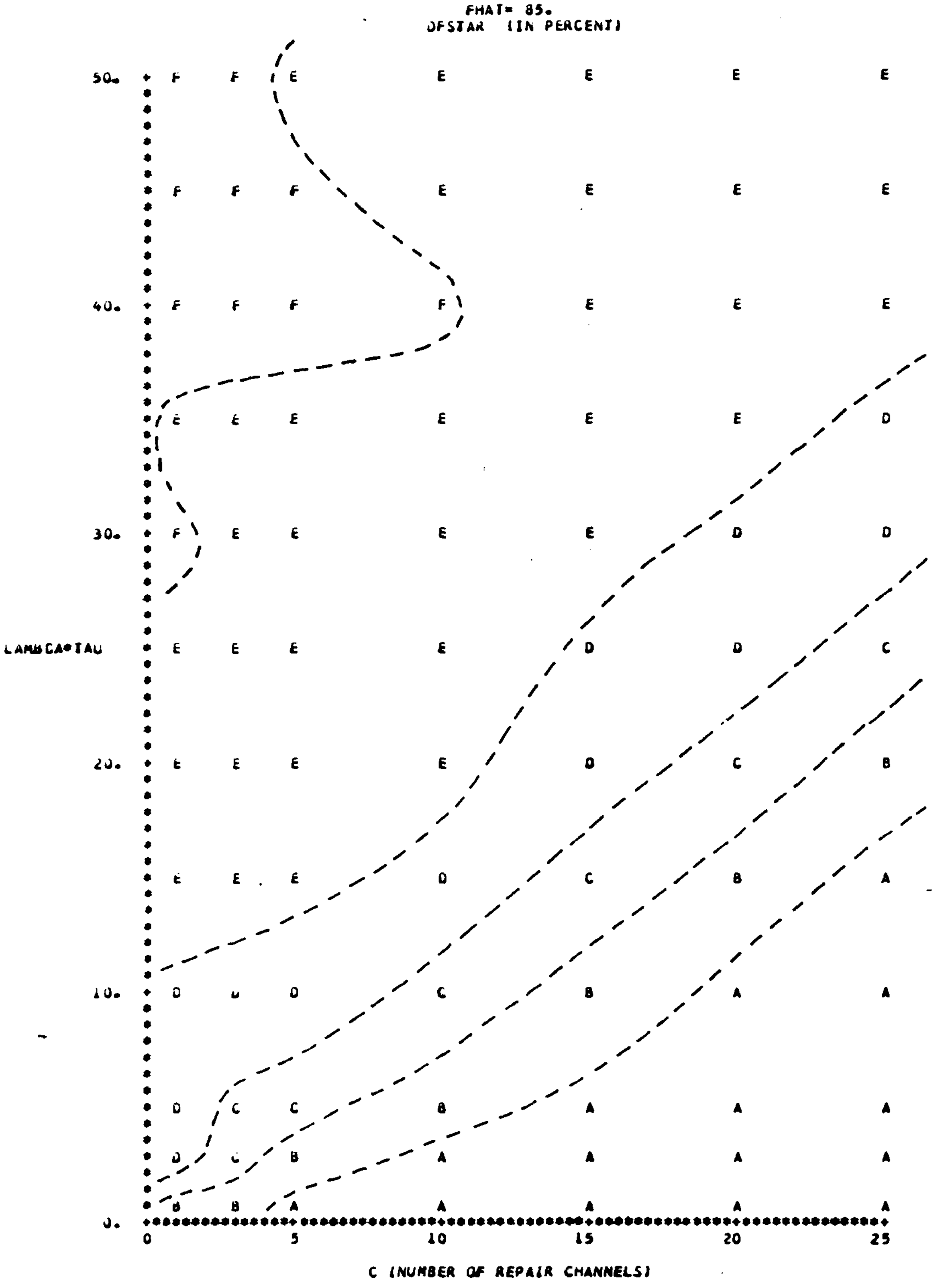
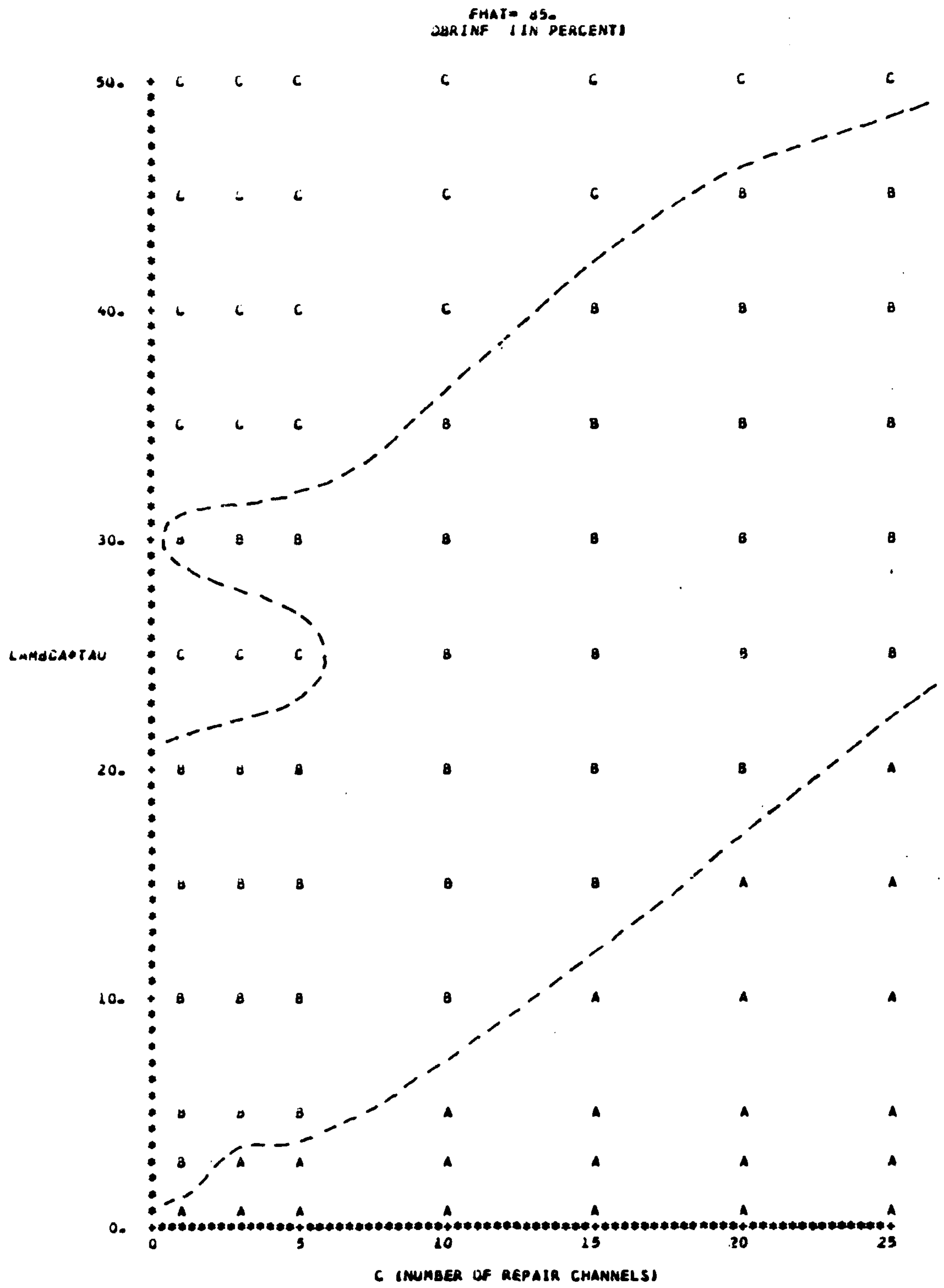


Figure 6.--D_{F*} for $\hat{F}$ = 85% .



KEY: DBRINF
A: - 0. %
B: 0.- 100. %
C: 100.- 200. %
D: 200.- 300. %
E: 300.- 400. %
F: 400.- %

Figure 7.--$D_{\overline{B}_\infty}$ for $\hat{F}$ = 85% .

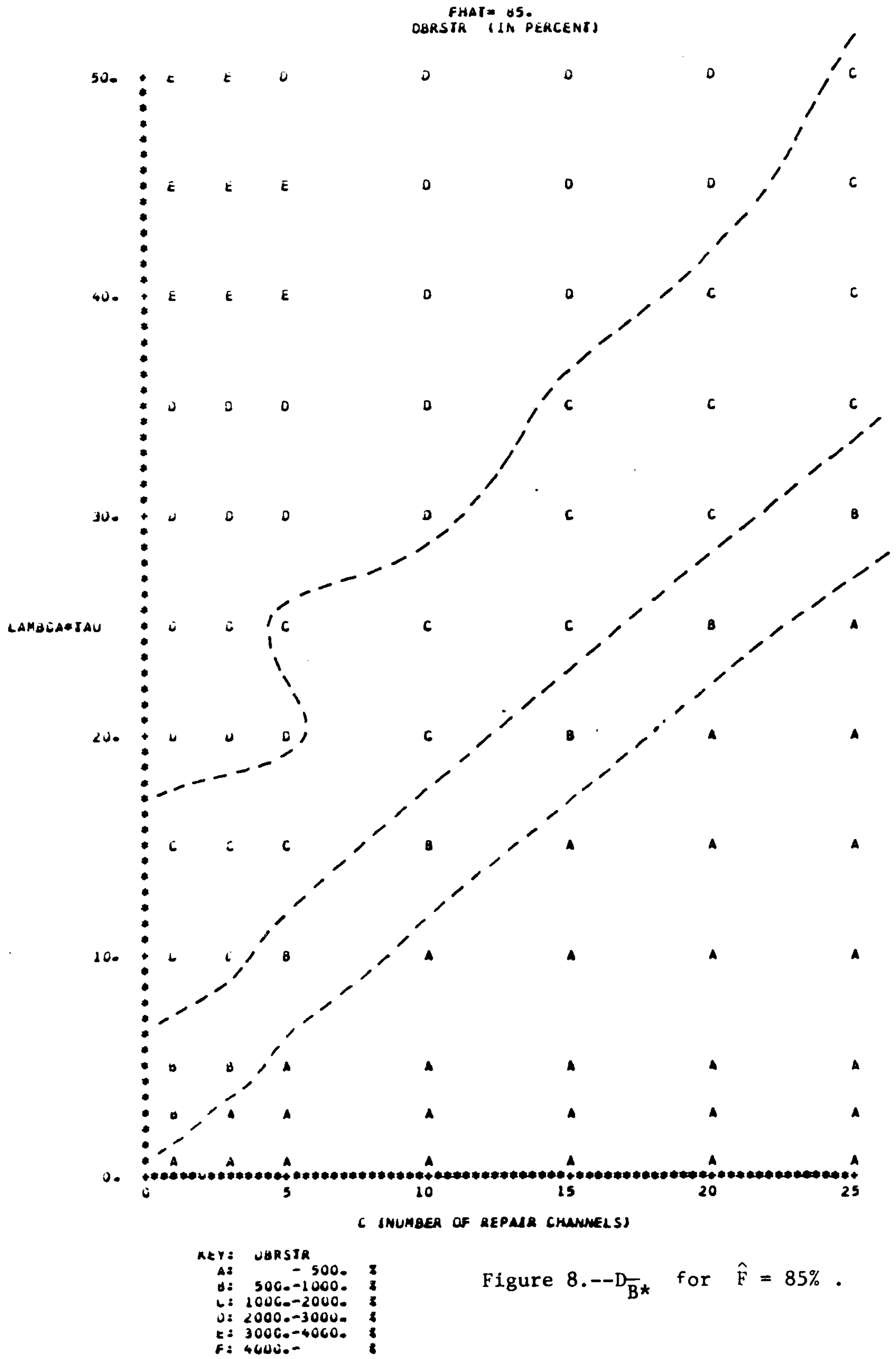
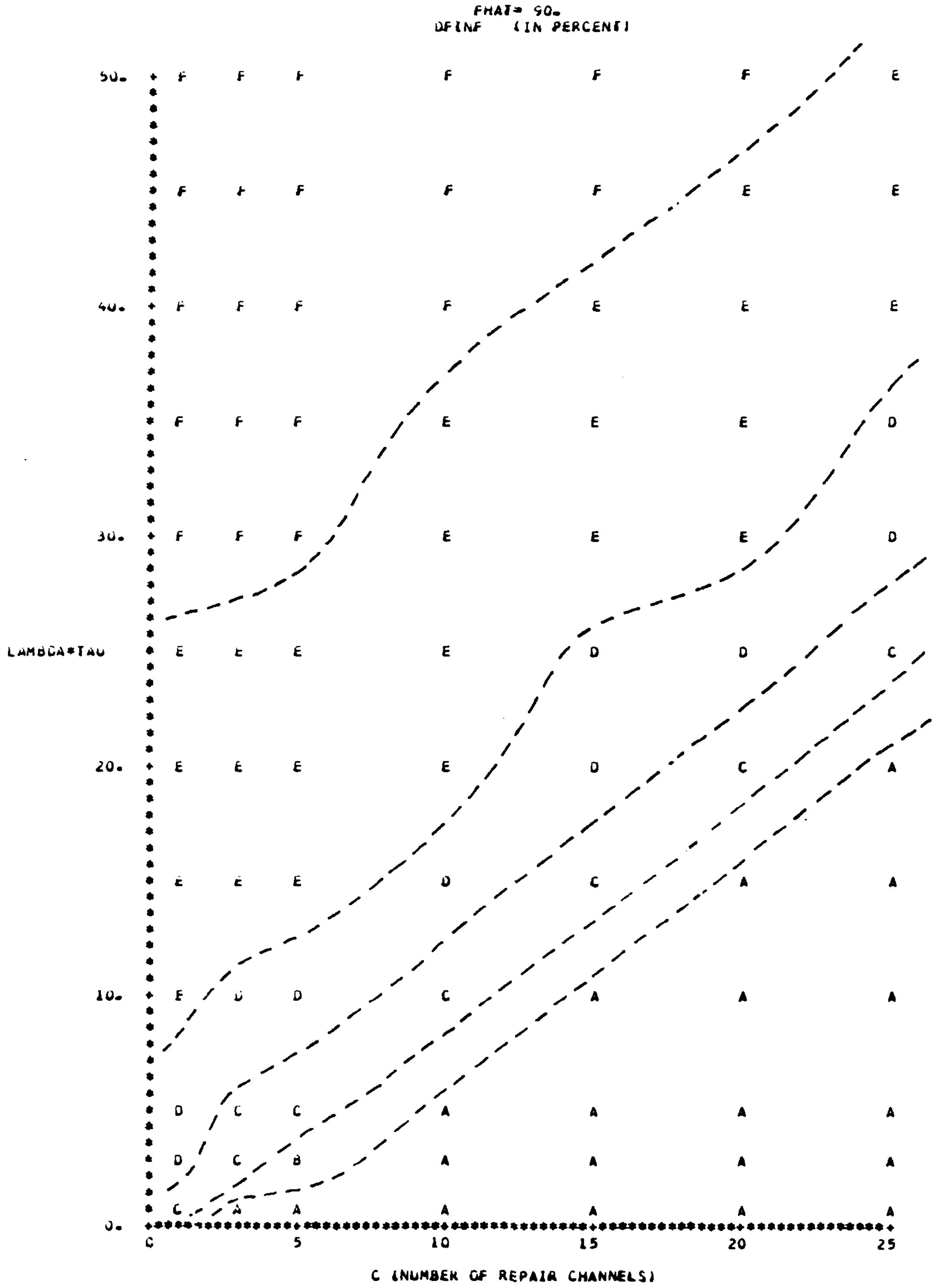
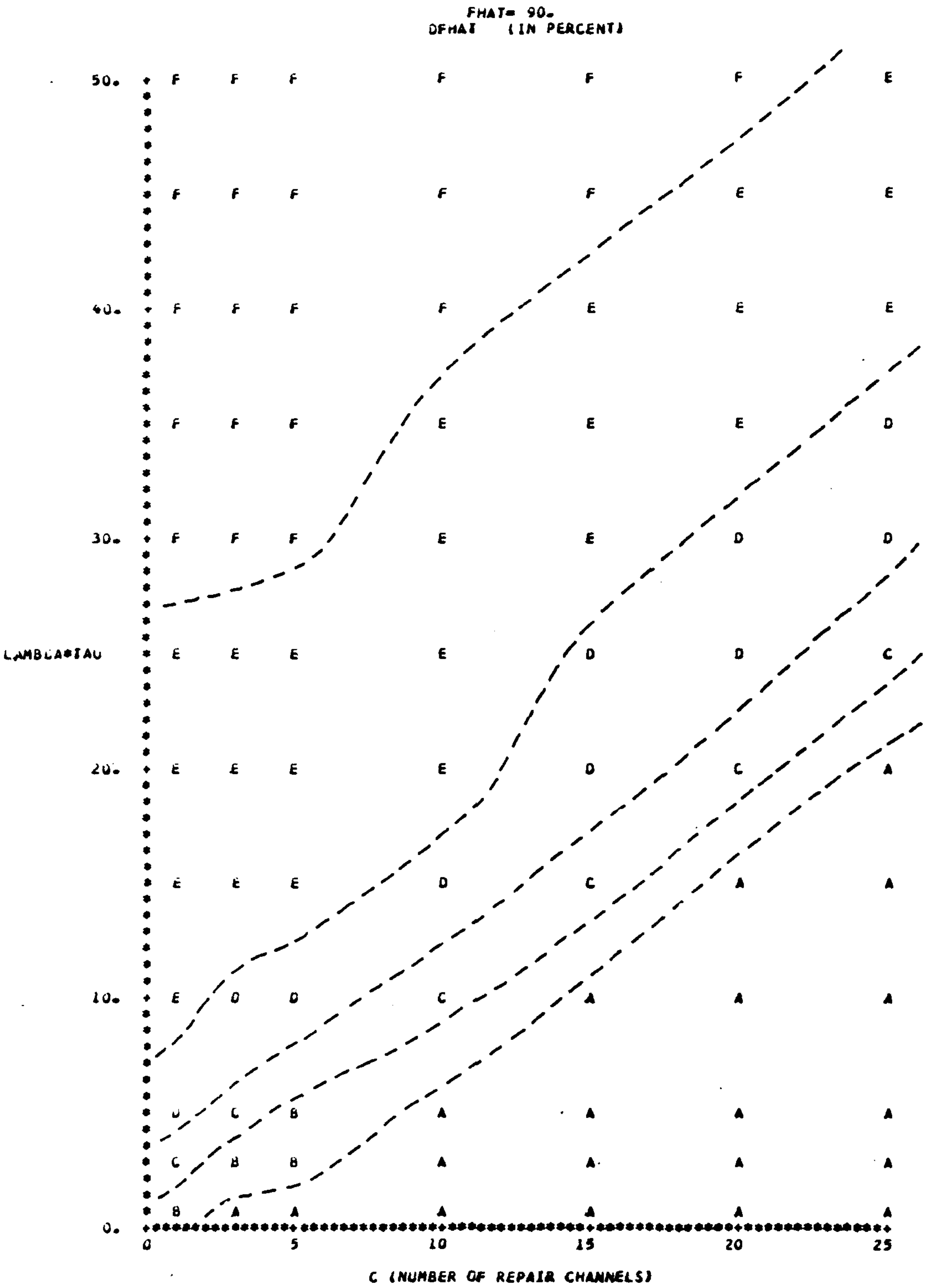


Figure 8.--$D_{\overline{B}*}$ for $\hat{F}$ = 85% .



KEY.	DFINF		
A:	-	0.	%
B:	0.-	5.	%
C:	5.-	10.	%
D:	10.-	15.	%
E:	15.-	20.	%
F:	20.-		%

Figure 9.--D_{F_∞} for $\hat{F}$ = 90% .



KEY: DFHAT
A: - 0. %
B: 0.- 5. %
C: 5.- 10. %
D: 10.- 15. %
E: 15.- 20. %
F: 20.- %

Figure 10.--$D_{\hat{F}}$ for $\hat{F}$ = 90% .

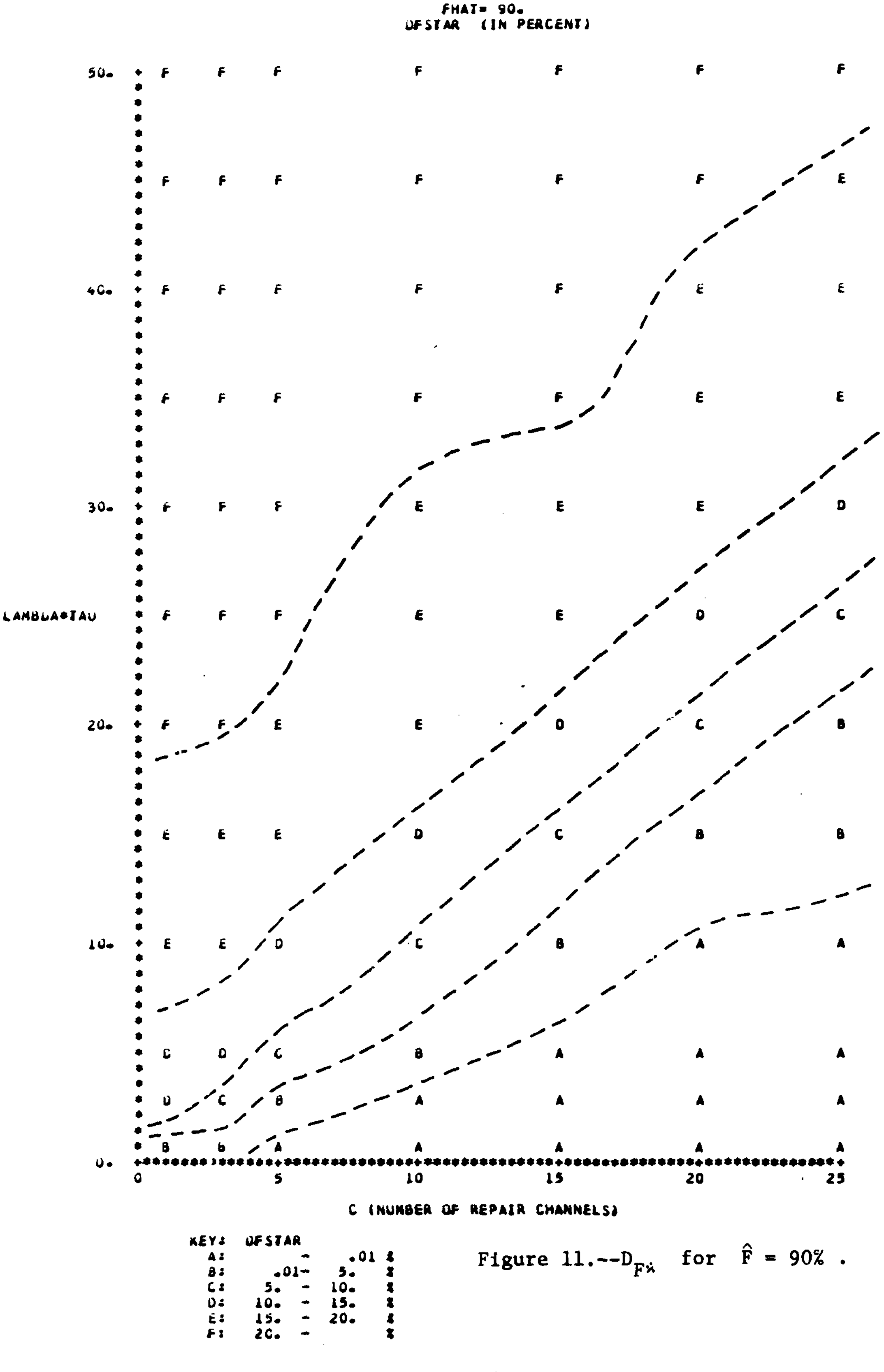
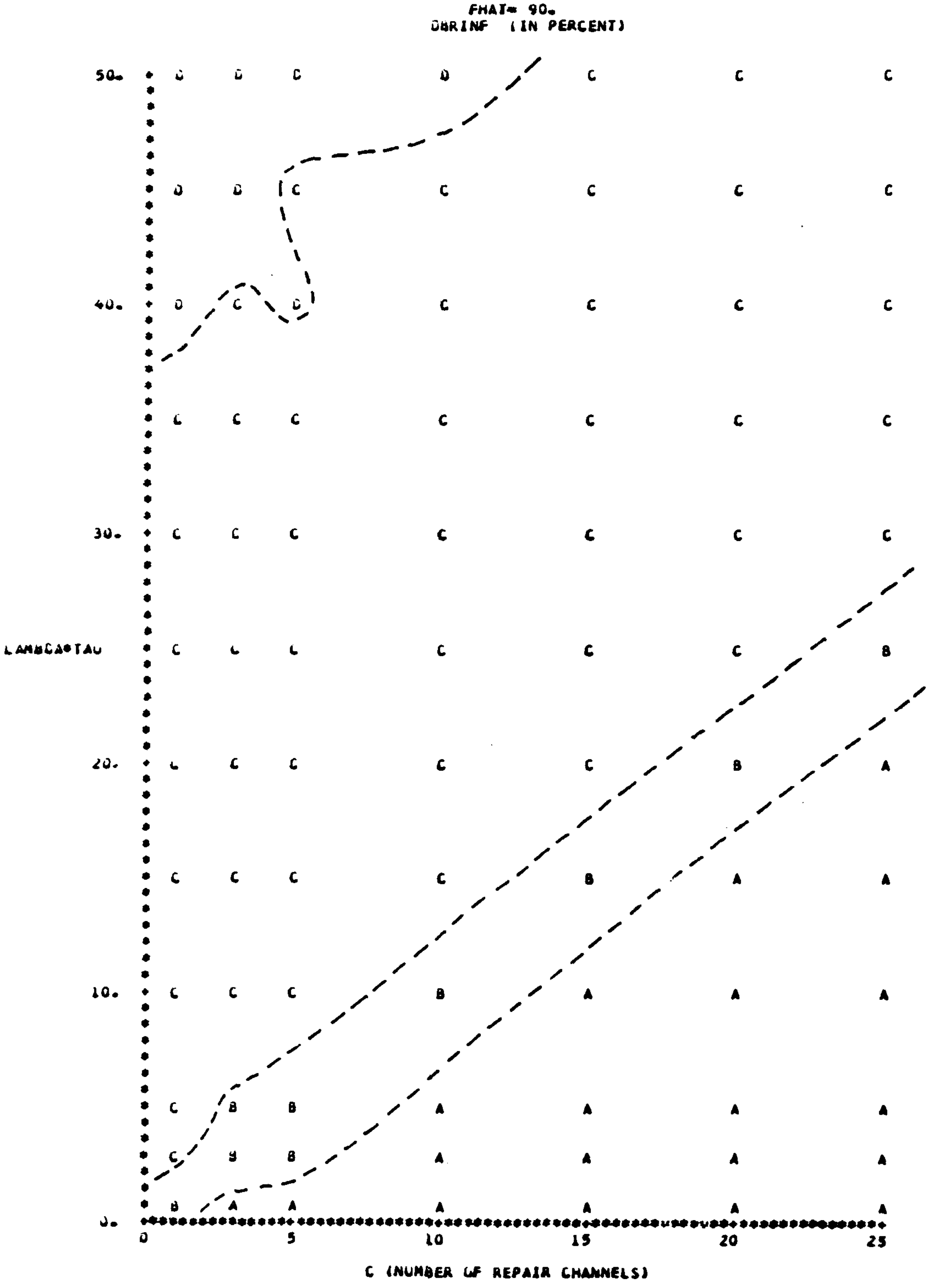


Figure 11.--D_{F*} for $\hat{F}$ = 90% .



KEY: DBRINF
A: - 0. %
B: 0.- 100. %
C: 100.- 200. %
D: 200.- 300. %
E: 300.- 400. %
F: 400.- %

Figure 12.--$D_{\bar{B}_\infty}$ for $\hat{F}$ = 90% .

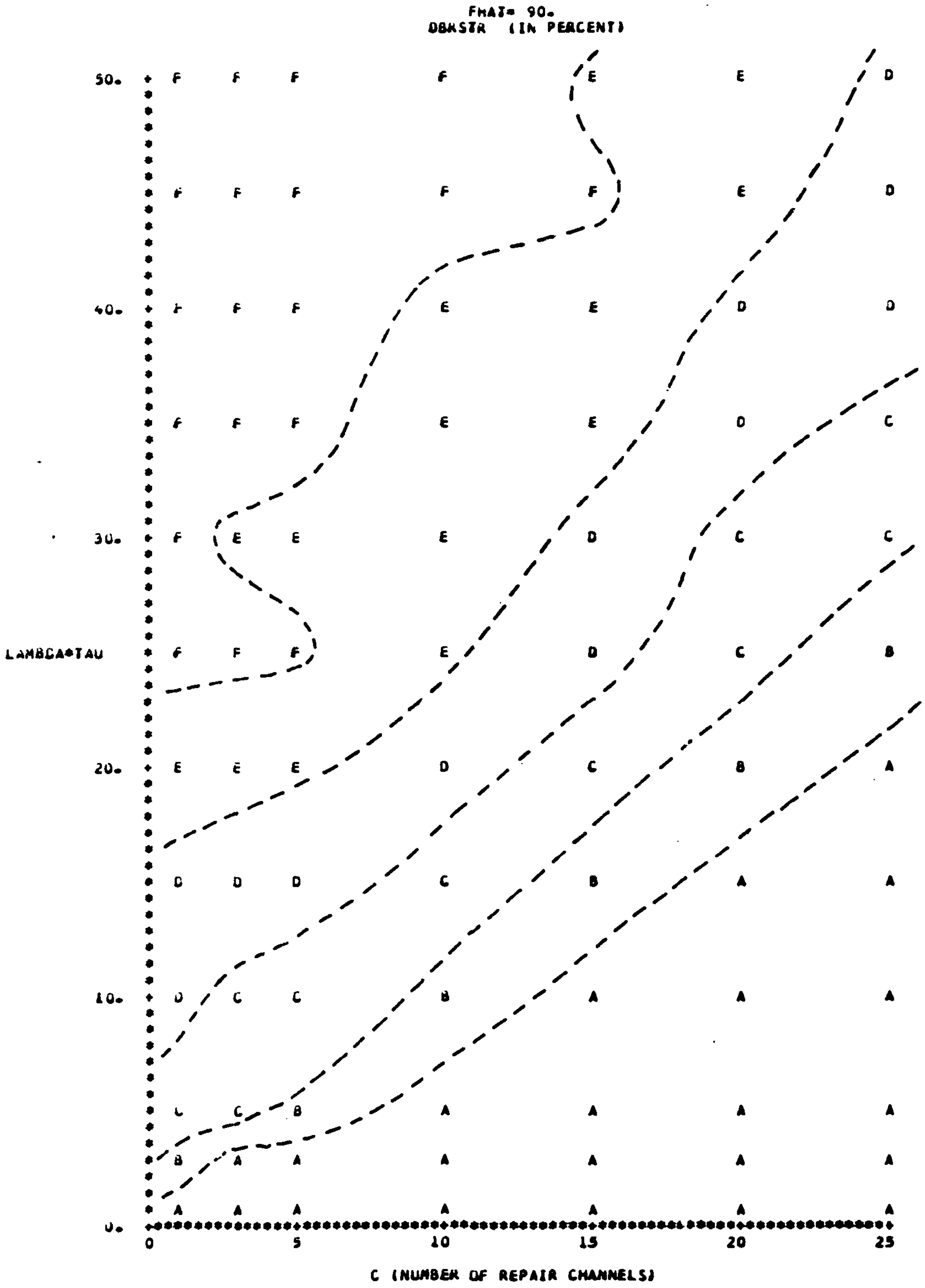


Figure 13.--$D_{\bar{B}*}$ for $\hat{F}$ = 90% .

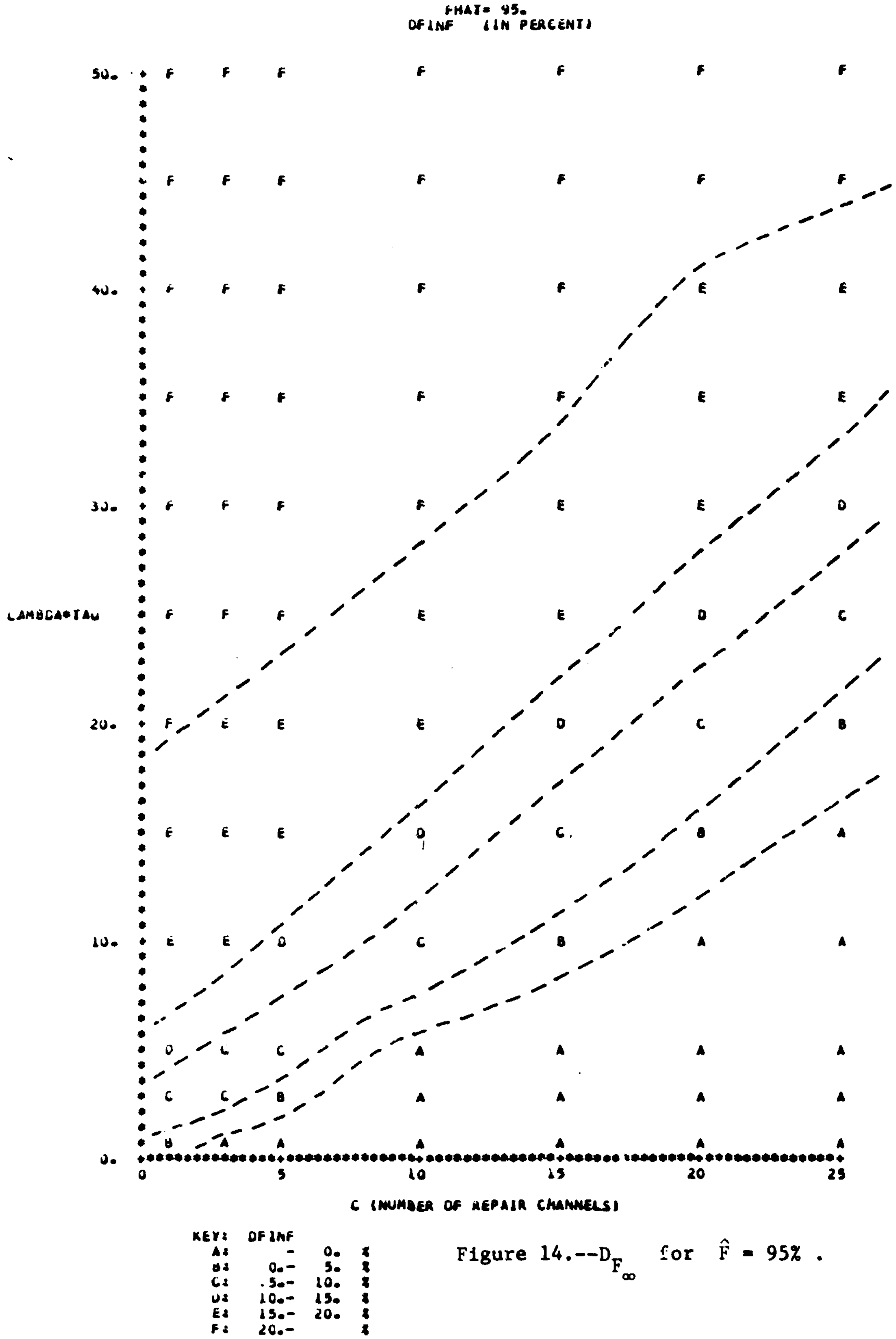


Figure 14.--D_{F_∞} for $\hat{F}$ = 95% .

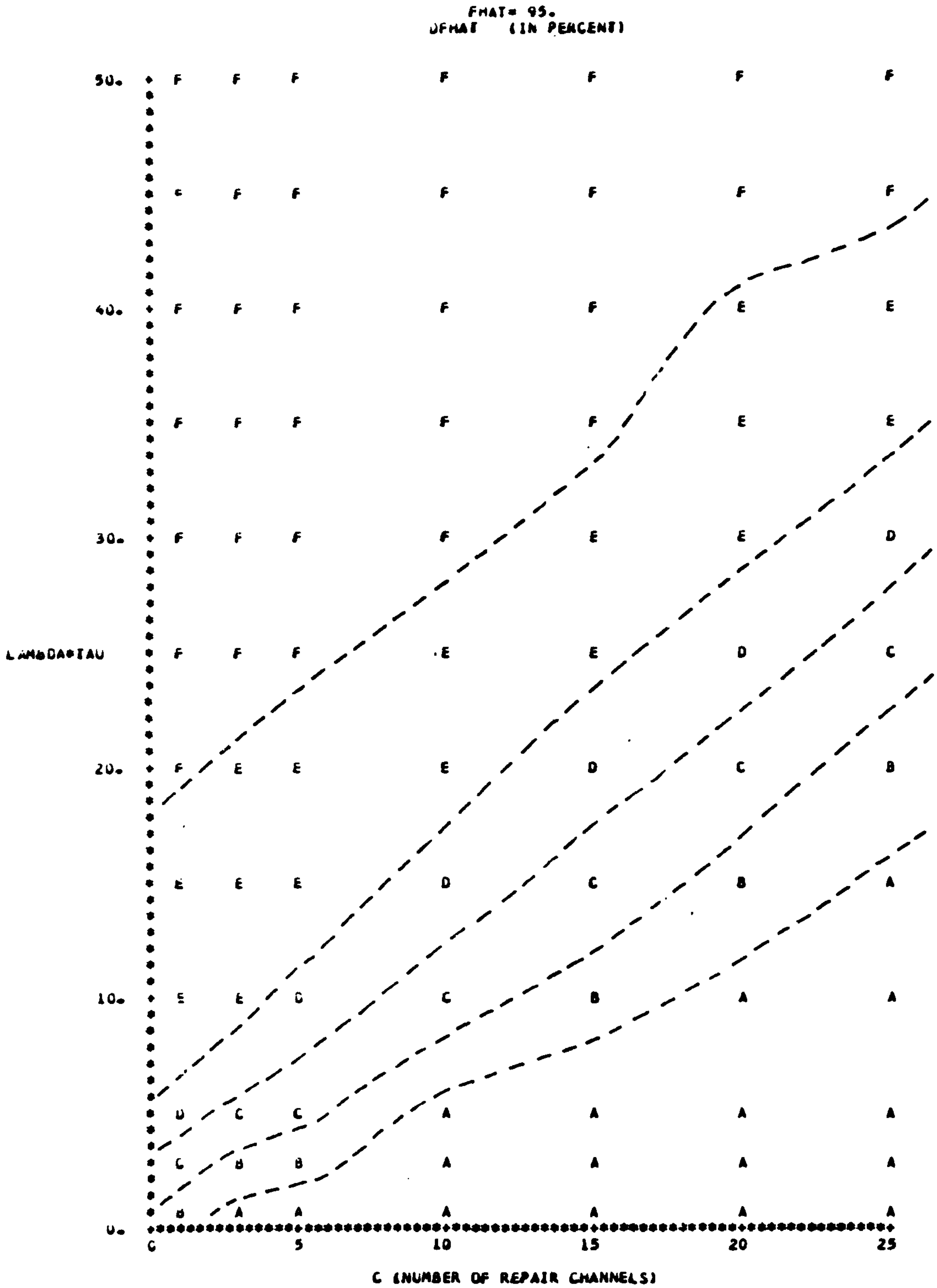
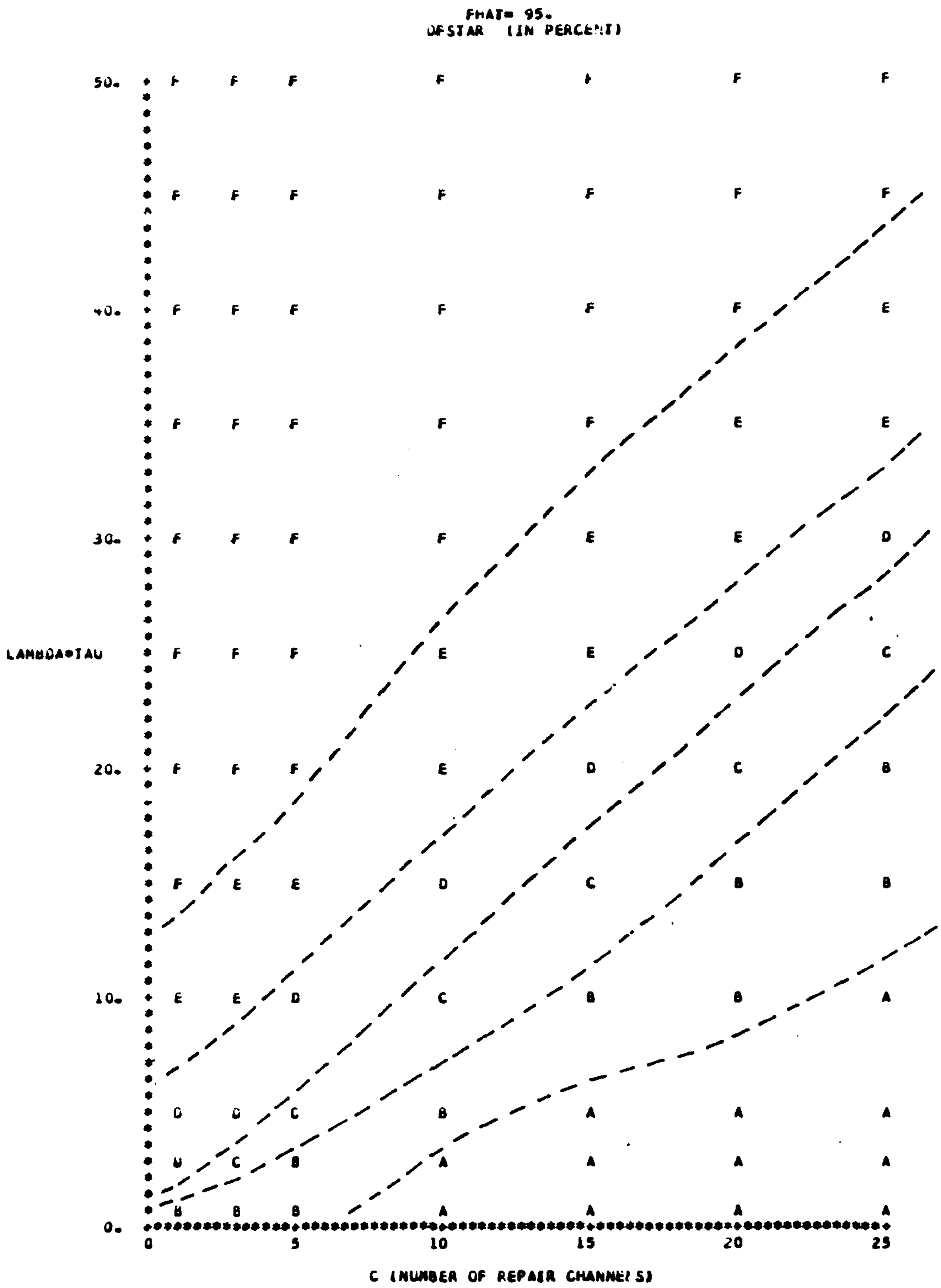


Figure 15.--$D_{\hat{F}}$ for $\hat{F} = 95\%$.



KEY: DFSTAR
A: - .01 %
B: .01- 5. %
C: 5. - 10. %
D: 10. - 15. %
E: 15. - 20. %
F: 20. - %

Figure 16.--D_{F*} for $\hat{F}$ = 95% .

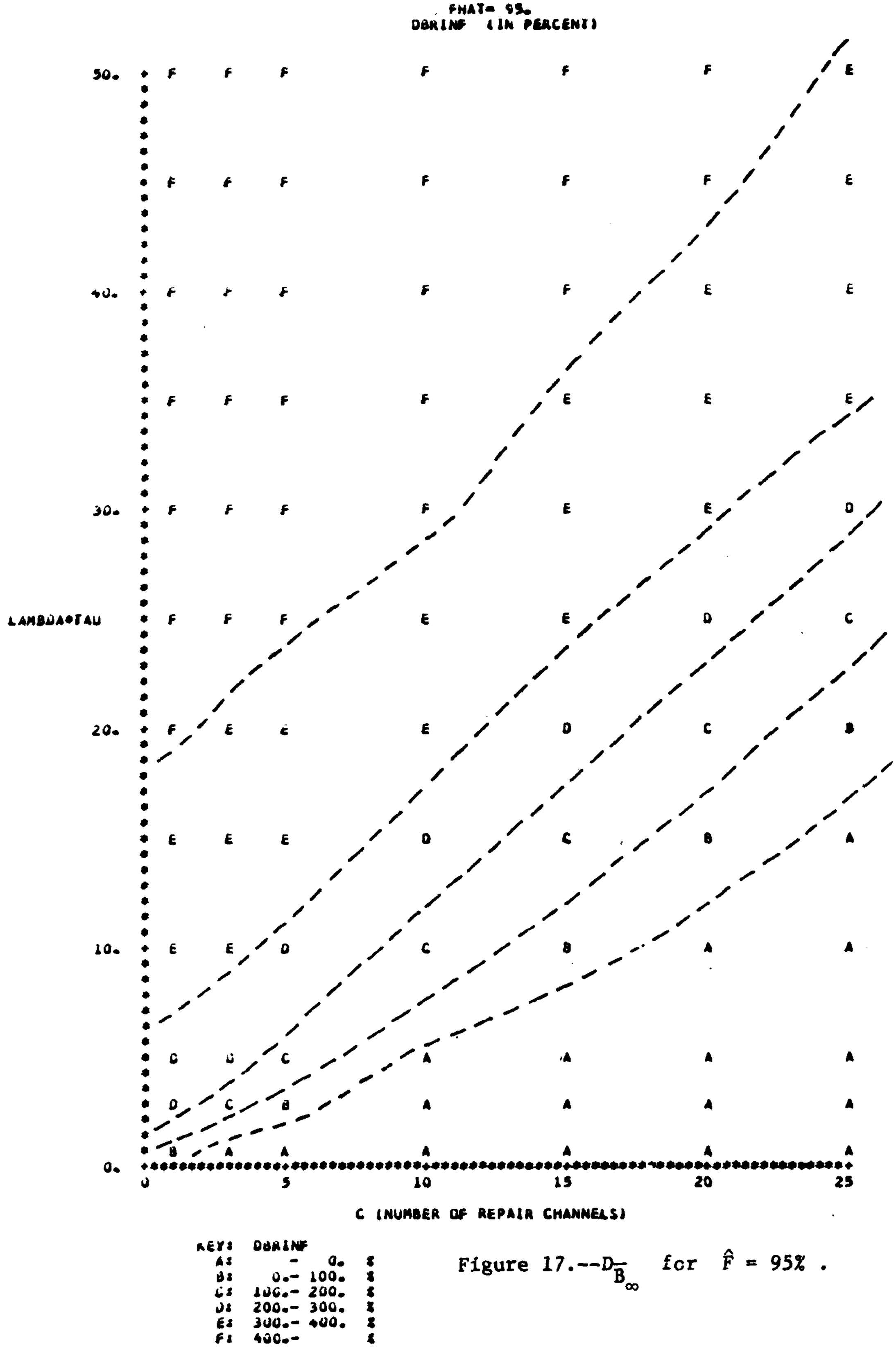


Figure 17.--$D_{\overline{B}_\infty}$ for $\hat{F}$ = 95% .

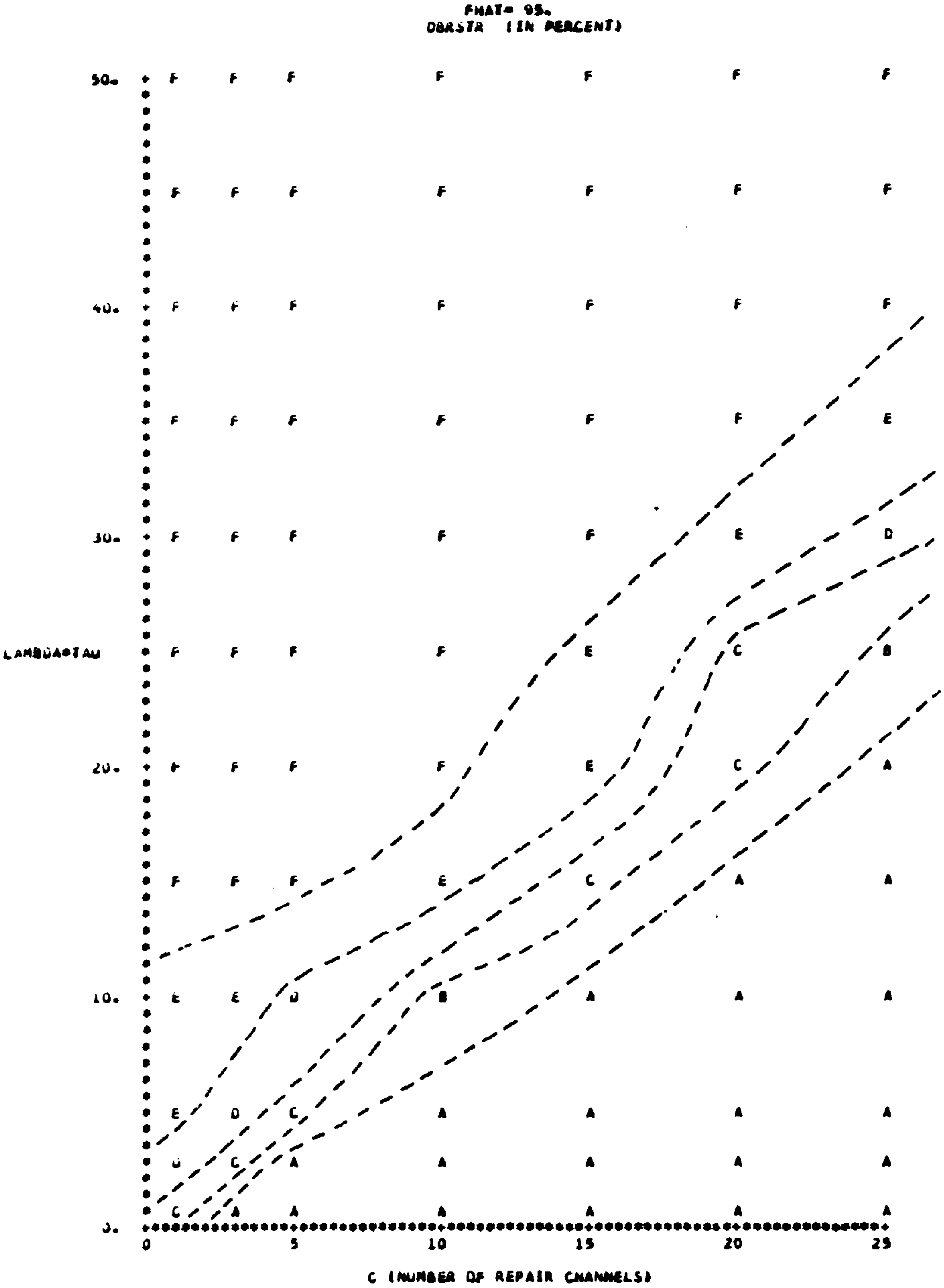


Figure 18.--$D_{\bar{B}*}$ for $\hat{F}$ = 95% .

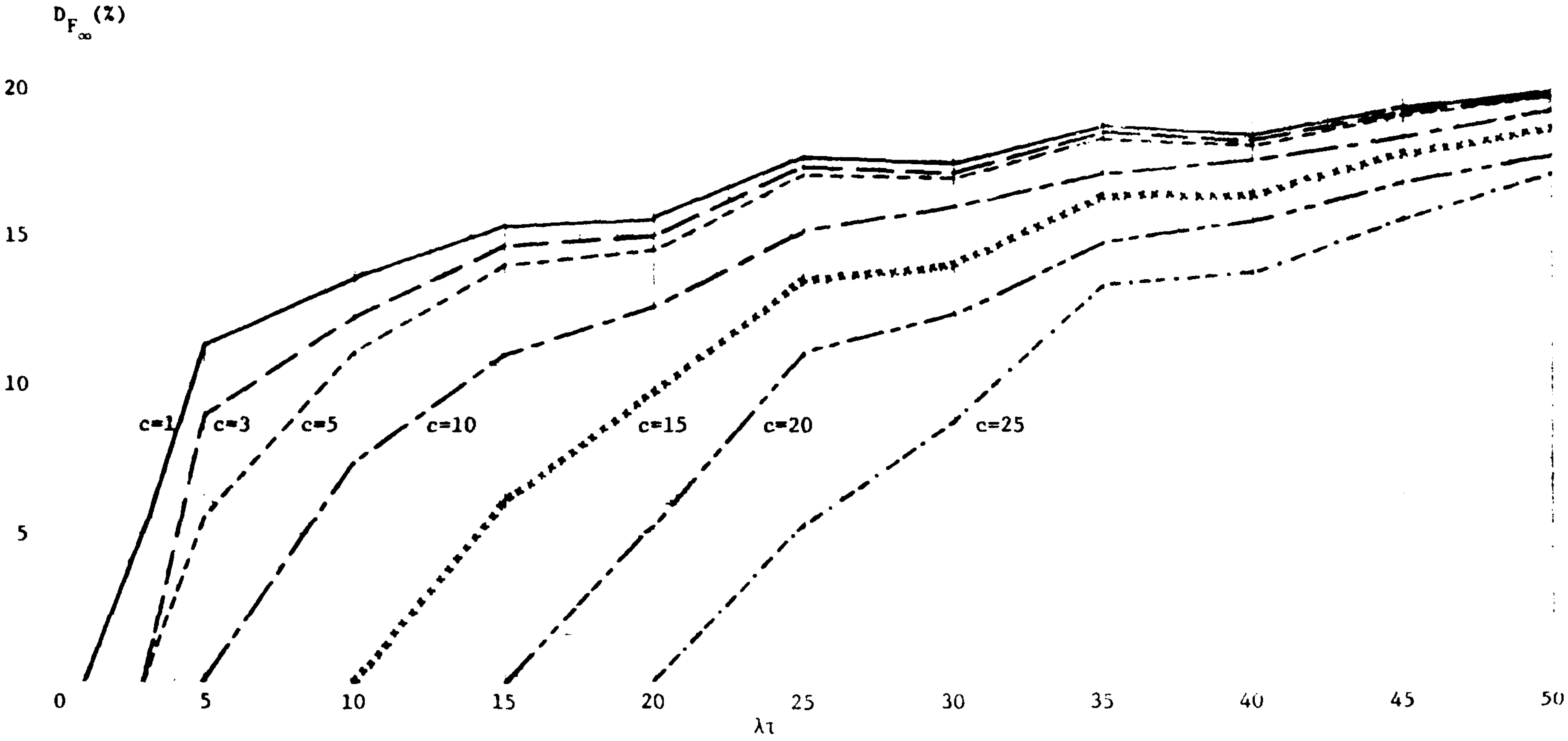


Figure 19.--D_{F_∞} vs. $\lambda\tau$ for $c=1,3,5,10,15,20,25$ and $\hat{F} = 85\%$.

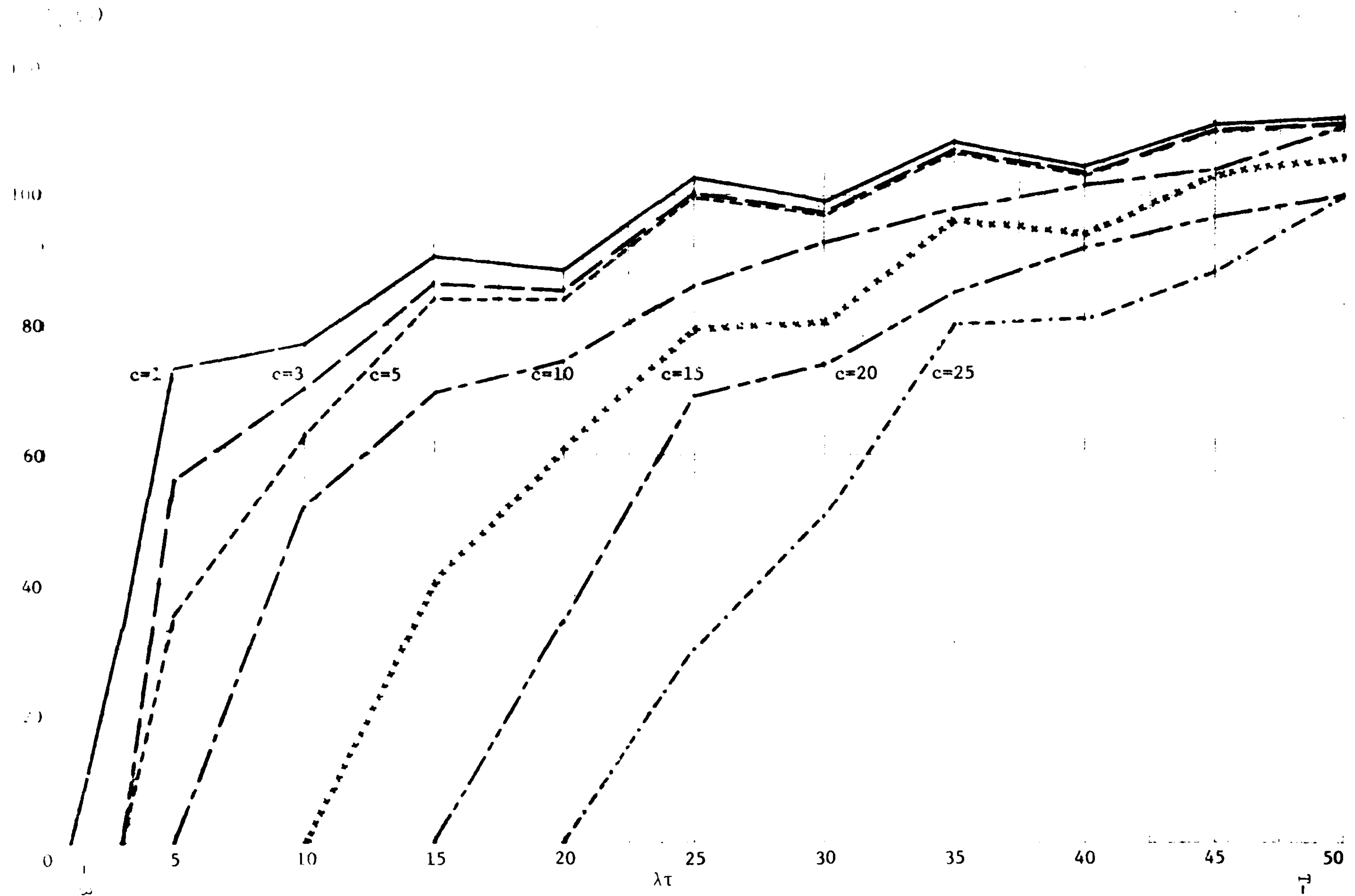


Figure 2'.--$D_{\bar{B}_\infty}$ vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 85% .

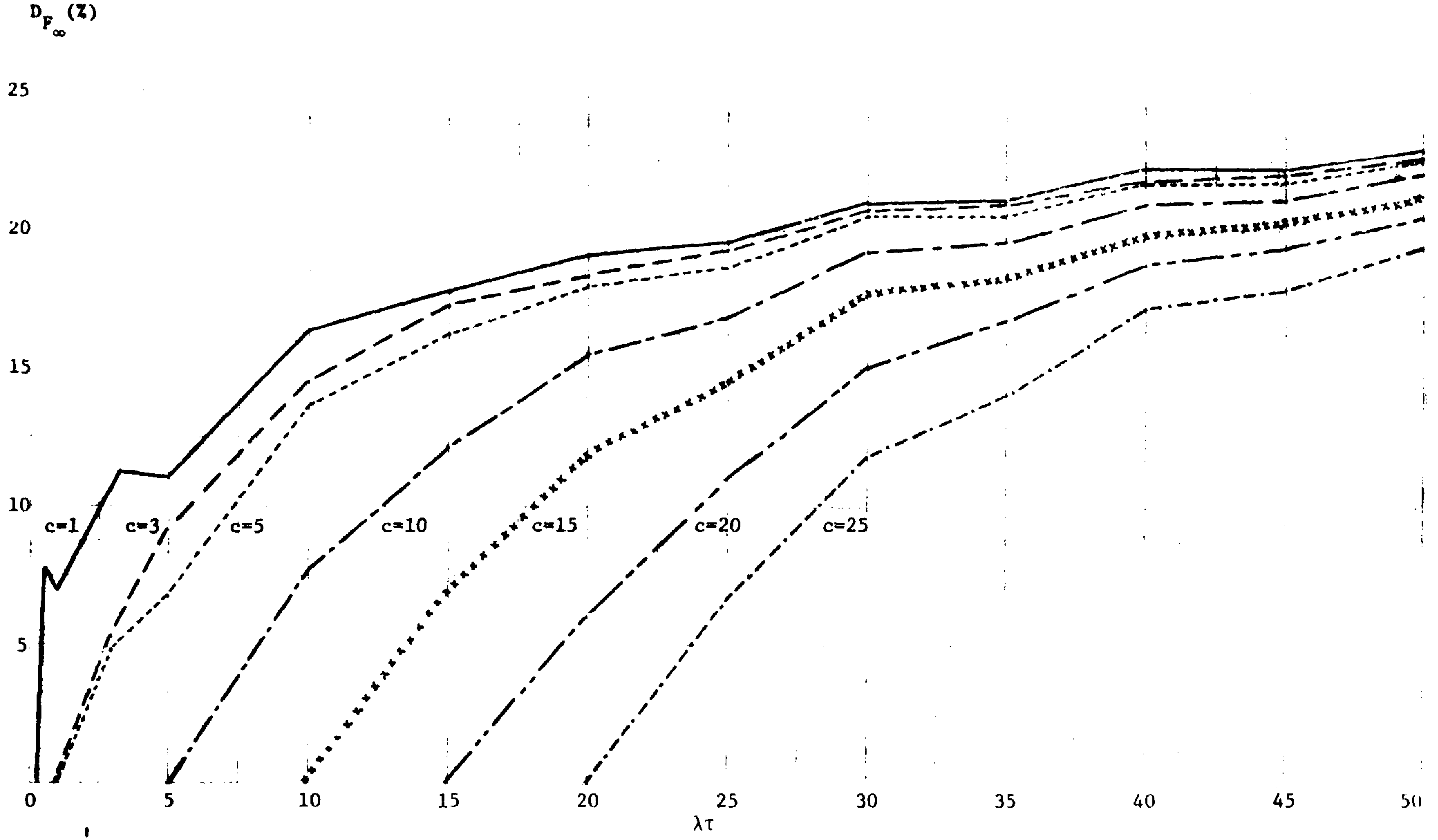


Figure 21.--D_{F_∞} vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 90% .

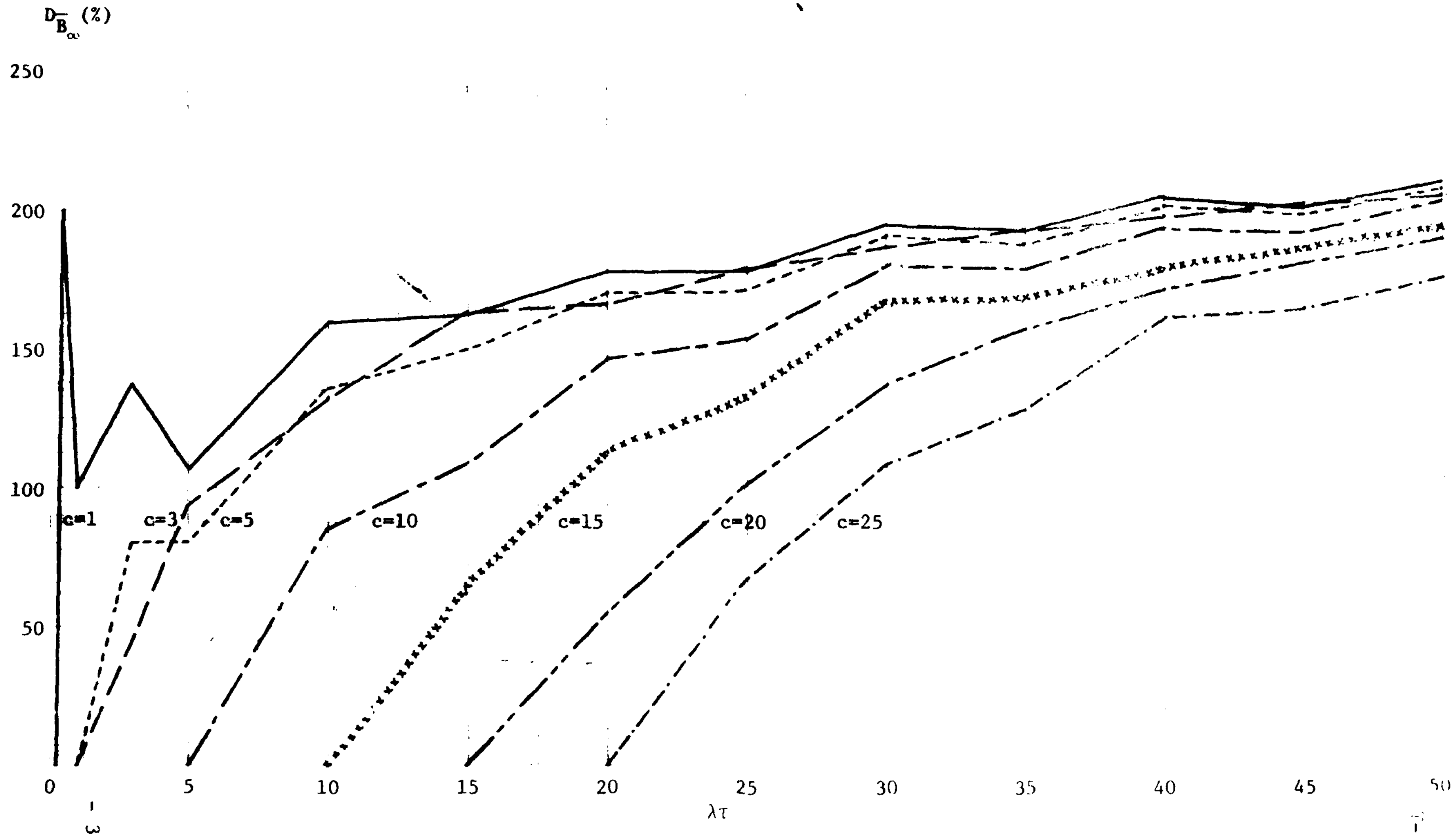


Figure 22.--$D_{\bar{B}_\infty}$ vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 90% .

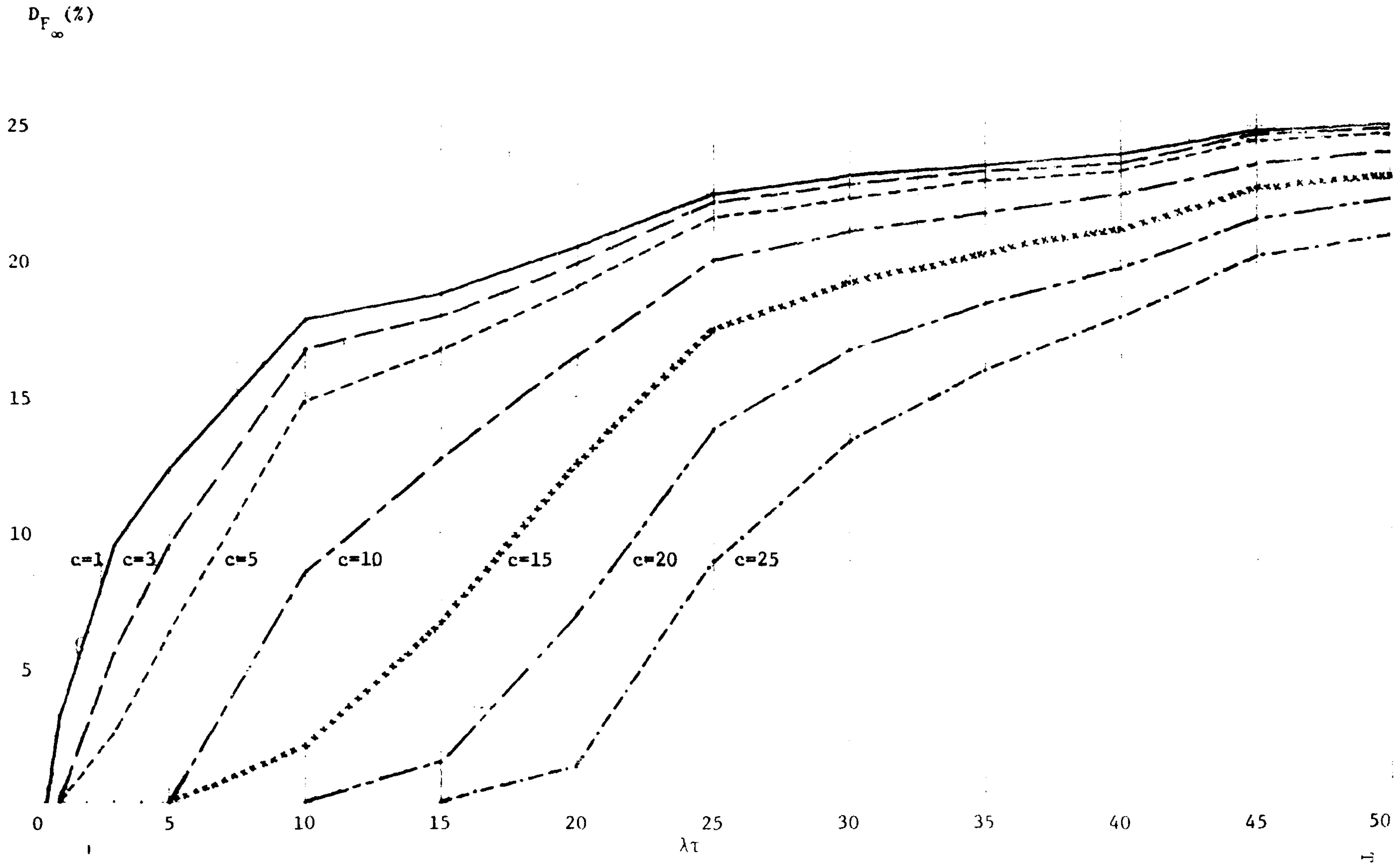


Figure 23.--D_{F_∞} vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 95% .

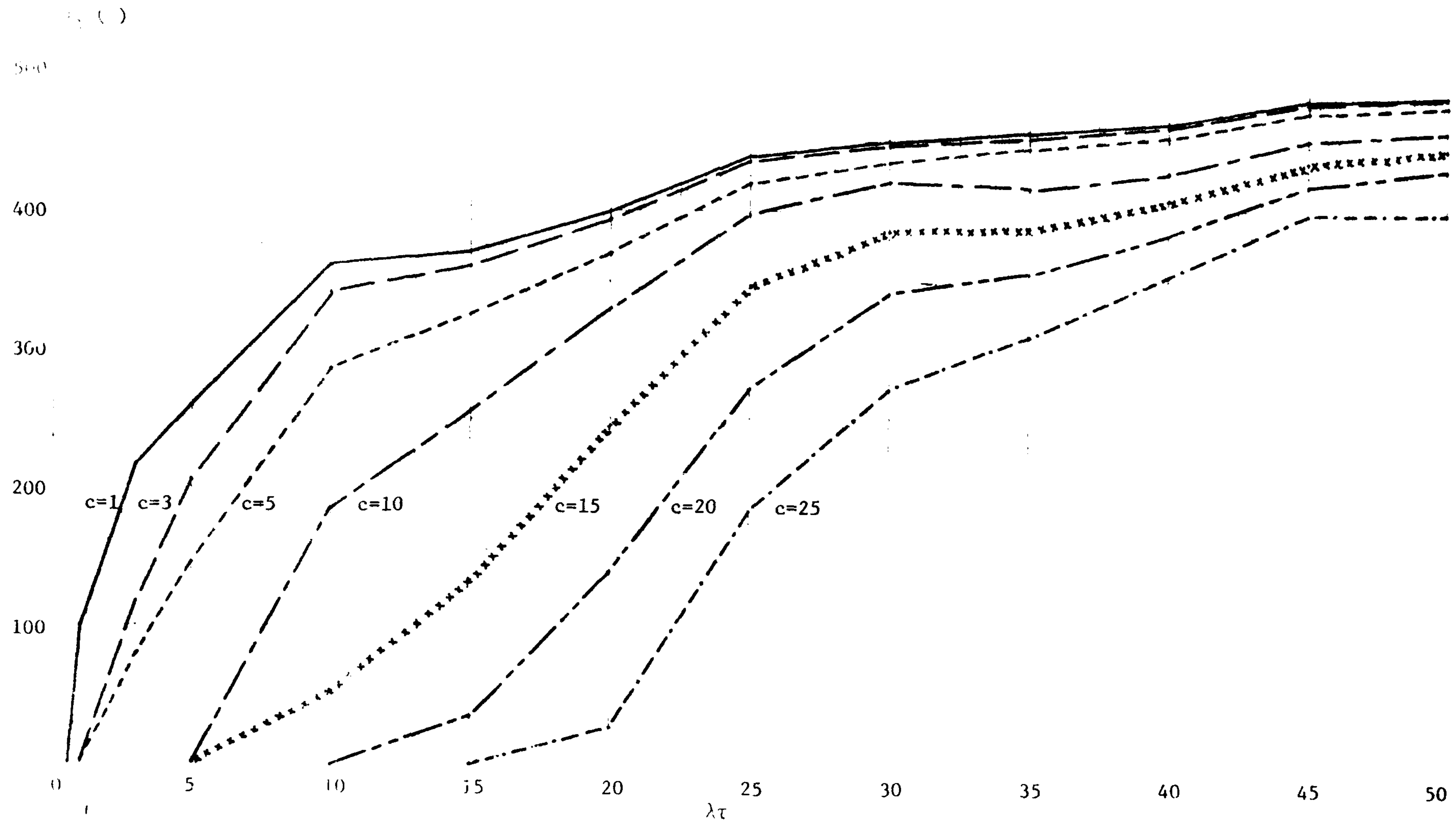


Figure 24.--$D_{\bar{B}_\alpha}$ vs. $\lambda\tau$ for c=1,3,5,10,15,20,25 and $\hat{F}$ = 95% .

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REFERENCES

FEENEY, G. J. and C. P. SHERBROOKE (1966). The (s-1,s) inventory policy under compound Poisson demand. *Management Sci.*, **12**, 391-411.

HADLEY, G. and T. M. WHITIN (1963). *Analysis of Inventory Systems.* Prentice-Hall, New Jersey.

HILLIER, F. S. and G. J. LIEBERMAN (1980). *Introduction to Operations Research*, 3 Ed. Holden-Day, San Francisco.

MUCKSTADT, J. A. (1973). A model for a multi-item, multi-echelon, multi-indenture inventory system. *Management Sci.*, **20**, 472-481.

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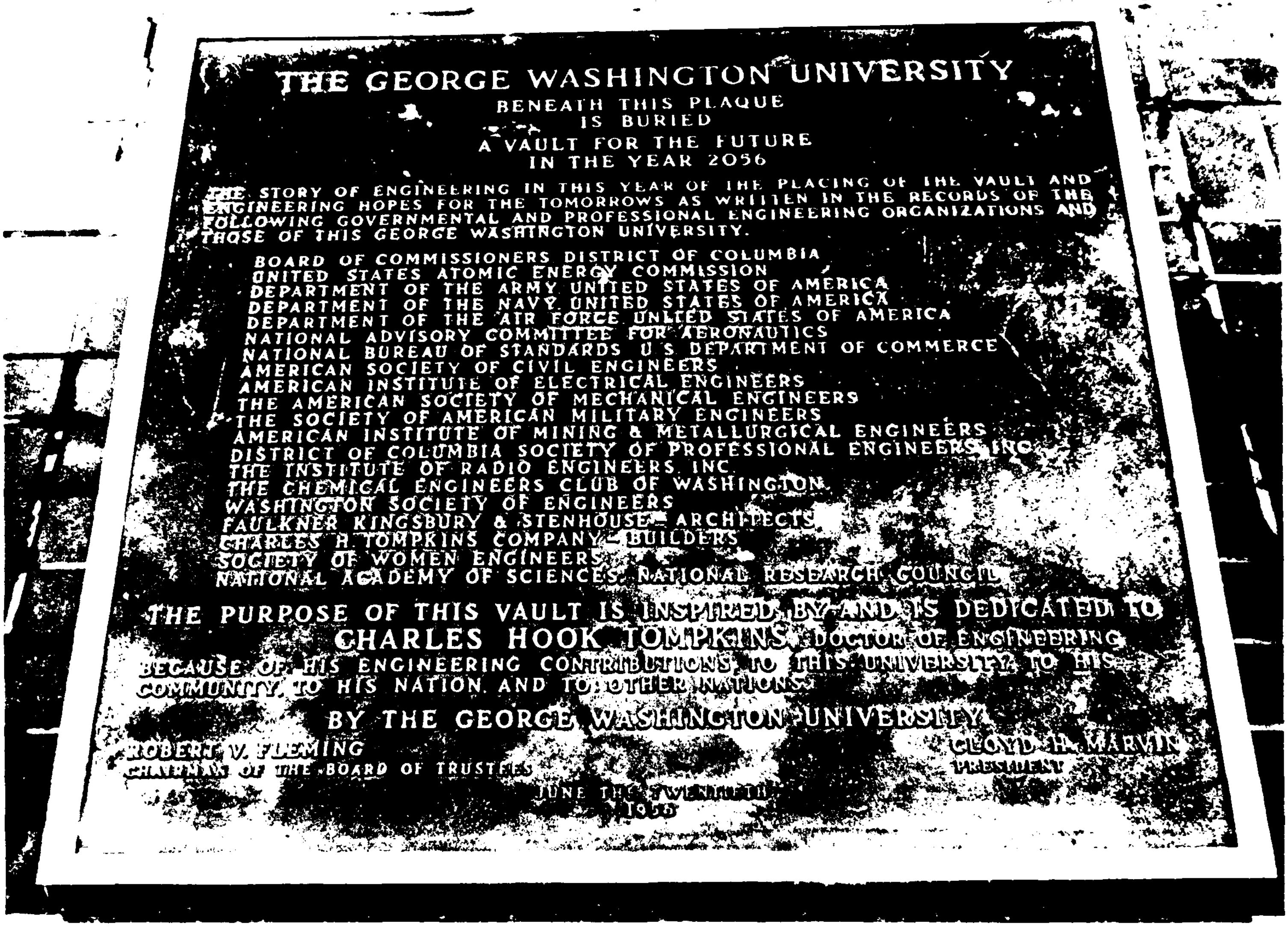

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