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SOME EXPERIMENTAL RESULTS ON LINEAR ESTIMATION FOR IMAGE ANALYS--ETC(U)  
JAN 81 C H CHEN, R WU, C YEN  
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LINEAR ESTIMATION FOR IMAGE ANALYSIS\*

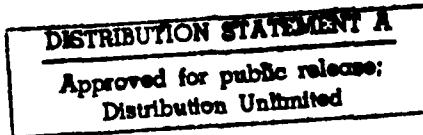
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# SOME EXPERIMENTAL RESULTS ON LINEAR ESTIMATION FOR IMAGE ANALYSIS

C. H. Chen  
Rong-Hwang Wu  
Chihsung Yen

## 1. Introduction

In a previous report (ref. 1), a critical comparison is made on the various statistical image segmentation techniques. Linear estimation plays an important role in image segmentation which may be considered as an estimation problem. Included in the linear estimation methods considered in Ref. 1 are the parameter estimation for ARMA models, Fisher's linear discriminant, maximum likelihood and maximum a posteriori estimations for the regions, boundary estimation via split-and-merge algorithm, and decision-directed estimation using conditional population-mixture model. In this report, experimental results are presented for a textured subimage on the segmentation by maximum likelihood estimation, maximum a posteriori estimation, and Fisher's linear discriminant. Among the three methods, the maximum a posteriori estimation performs the best with 3.96% segmentation error, the Fisher's linear discriminant is a close second with 4.6% segmentation error and requires slightly more computation. The performance of the maximum likelihood estimation is much worse even though it requires less computation than the other two methods.

## 2. Construction of the Textured Subimage

The construction of the 64x64 textured subimage was kindly described by Dr. Charles W. Therrien of MIT Lincoln Laboratory who employed the textured subimage in his study of the linear filtering

models for image segmentation and classification (Ref. 2). The desired subimage is constructed from subimages 5 and 9 (counting from upper left to lower right horizontally) of image B2568-38 of USC data base by using the following procedure. For each point  $(n,m)$ , the following expressions are evaluated.

$$B_1(n, m) = (n-36) + (m-40)^2/50$$

$$B_2(n, m) = ((n-48)/8)^2 + ((m-20)/12)^2 - 1$$

Then if

$B_1 < 0$  or  $B_2 < 0$ , take point from subimage 5

if  $B_1 > 0$  and  $B_2 > 0$ , take point from subimage 9

This results in a new subimage with the texture from subimage 5 above the parabolic boundary and within the ellipse and the texture from subimage 9 everywhere else. The texture from subimage 5 is denoted as type 1 while the texture from subimage 9 is denoted as type 2. Thus the segmentation is to classify for each pixel whether it is from the type 1 or from the type 2 texture. For the results in Ref. 2, the filters were  $4 \times 4$  causal (first quadrant) computed by solving two-dimensional normal equations with an estimated correlation function. Transition probabilities computed in the paper made use of  $5 \times 5$  neighborhoods.

By using the lineprinter display with 16 levels, Fig. 1a, 1b and 1c show respectively the original subimages 5, 9 and the constructed subimage. Two-level displays of these subimages from the Tektronix 4010 terminal are shown in Fig. 2. Fig. 3 shows the ideal segmentation as defined by the parabola and ellipse in the above quations.

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Fig. 1b  
Subimage 9



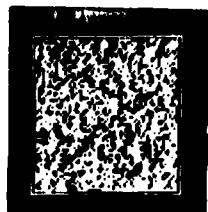


Fig. 2a Subimage 5 (above) in two-level display and histogram of the subimage (right). There are 16 levels in the original image.

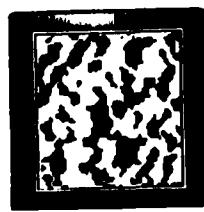
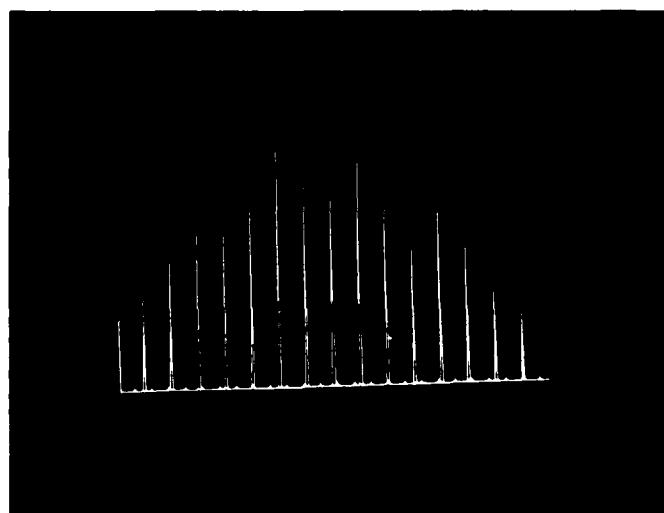


Fig. 2b Subimage 9 (above) in two-level display and histogram of the subimage (right).

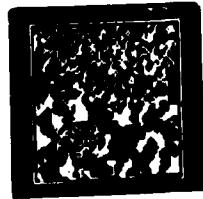
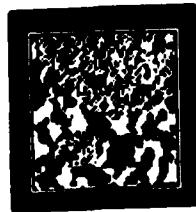
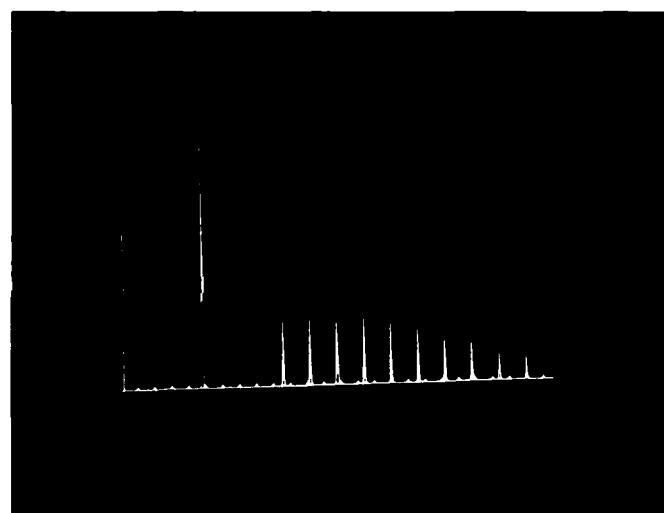
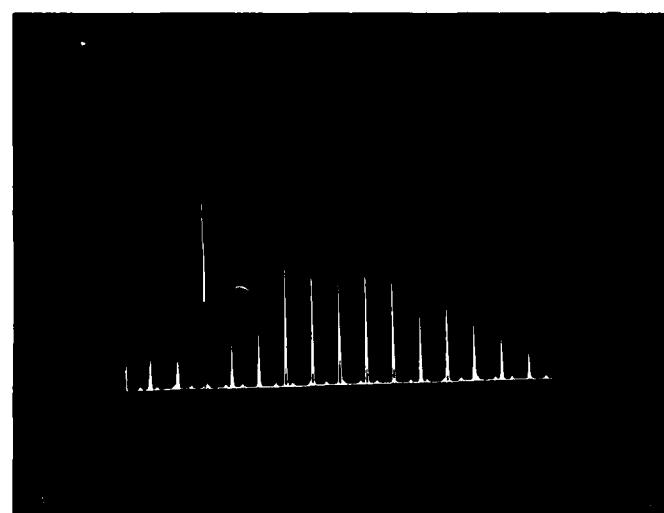


Fig. 2c Subimage constructed (two shown above) and the histogram of the constructed image (right).



### 3. The Maximum Likelihood Estimation

By assuming a first-order autoregressive model, we followed closely the mathematical development in Ref. 2. The maximum likelihood estimate for the regions is obtained by the minimization of the expression (10) in Ref. 2. Fig. 4 shows the maximum likelihood (ML) segmentation obtained by assigning pixels to white and black texture type 1 and 2 respectively. This result is very close to Fig. 2 (b) of Ref. 2 which employed a higher order autoregressive model as described in the previous section. Our first-order model made use of our earlier work reported in Ref. 3.

### 4. The Maximum a Posteriori Estimation

The maximum a posteriori estimation which uses the combination of the Markov chain with a Gaussian autoregressive process is to minimize the expression (11) in Ref. 2. To begin with, the image segmentation is considered as an assignment of the pixels to 0 and 1. Such an assignment for a point ( $n, m$ ) is referred to as its "state". The state interdependence complicates the computation of the estimate and an iterative procedure must be employed. The transition probability that plays a major role in the maximum a posteriori (MAP) segmentation is counted from the numbers of state assignment of 0 and 1 for a  $5 \times 5$  neighborhood. It is noted that MAP segmentation starts with the ML segmentation result. For clarity the sequence of lineprinter outputs is shown in Fig. 5. They include the ML segmentation (Fig. 5a), Iteration 1 (Fig. 5b), Iteration 4 (Fig. 5c), Iteration 7 (Fig. 5d) and Iteration 11 (Fig. 5e). At Iteration 11, the small false region in the upper left corner starts to disappear. It is noted that such false region was not

- 3a -

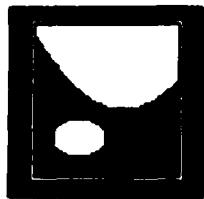


Fig. 3 Ideal segmentation



Fig. 4 ML segmentation

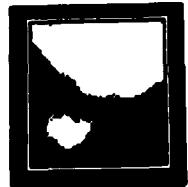


Fig. 6a MAP segmentation with 14 iterations



Fig. 6b Smoothing of Fig. 6a

Note: Learning samples for ML and MAP segmentations are based on 480 pixels selected from each type of texture. The following are parameters computed for each texture. Type 1 has mean of 108 and Type 2 has mean of 81.

$$\text{Mean}_1 = 108 \quad \text{Mean}_2 = 81$$

$$\text{Std Dev}_1 = 12.5 \text{ pixels} \quad \text{Std Dev}_2 = 0.17 \text{ pixels}$$

$$\text{Correlation}_1 = 0.94 \text{ pixels} \quad \text{Correlation}_2 = 0.19 \text{ pixels}$$

$$\text{Variance}_1 = 150.2 \text{ pixels}^2 \quad \text{Variance}_2 = 6.48 \text{ pixels}^2$$

$$\text{Skewness}_1 = 0.0001 \text{ pixels}^3 \quad \text{Skewness}_2 = 0.0001 \text{ pixels}^3$$

Fig. 5a  
ML  
segmentation





-3e-



removed in Ref. 2 even after 16 iterations. At Iteration 14, the Tektronix display is shown in Fig. 6a which is then smoothed by using a procedure due to Sklansky with the result shown in Fig. 6b. The percentage segmentation error of Fig. 6b is computed as 3.96%. It is important to note that for all linear estimation methods considered in this report, the supervised learning samples are selected from regions of type 1 and type 2.

### 5. Fisher's Linear Discriminant

Our initial effort of using Fisher's linear discriminant follows the procedure employed in Ref. 4. The features are the gray levels of the pixels. Fig. 7a is a tabulation of the scattered matrices for Type 1(target) and Type 2 (background) textures. The two scatter matrices are nearly proportional by a factor of 3. The Fisher's linear discriminant simply cannot classify the two types of textures. A new feature selection procedure is used that extracts three features  $x(i)$ ,  $i=1,2,3$  for a  $3 \times 3$  neighborhood

A	B	C
D	E	F
G	H	I

by computing

$$x(1) = E$$

$$x(2) = (A+B+C+D+E+F+G+H+I)/9$$

$$x(3) = (B+D+F+H) - 4E$$

The learning samples are taken from a  $24 \times 28$  area with each type of texture region. With each  $3 \times 3$  neighborhood represented by the feature vector ( $x(1)$ ,  $x(2)$ ,  $x(3)$ ), the two scatter matrices as tabulated in Fig. 7b are quite different. The Fisher's linear discriminant can then successfully segment the subimage as shown in

## THEORETICAL APPROXIMATIONS

It is apparent that the theoretical approximation is in good agreement with the experimental data. The theoretical values are slightly higher than the experimental values.

## THEORETICAL APPROXIMATIONS

Theoretical values of  $\sigma_{\text{tot}}$  and  $\sigma_{\text{sc}}/\sigma_{\text{tot}}$  are given below. The values of  $\sigma_{\text{tot}}$  are in good agreement with the experimental values.

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Fig. 7b

## THEORETICAL APPROXIMATIONS

$\sigma_{\text{tot}}$  (Eq. 5)  
—  $\sigma_{\text{sc}}$  (Eq. 5)  
—  $\sigma_{\text{tot}}$  (Eq. 6)

## THEORETICAL APPROXIMATIONS

$\sigma_{\text{tot}}$  (Eq. 5)  
—  $\sigma_{\text{sc}}$  (Eq. 5)  
—  $\sigma_{\text{tot}}$  (Eq. 6)

## THEORETICAL APPROXIMATIONS

$\sigma_{\text{tot}}$ (Eq. 5)	$\sigma_{\text{tot}}$ (Eq. 6)	$\sigma_{\text{tot}}$ (Eq. 7)
$0.140 \times 10^{-24} \text{ cm}^2$	$0.130 \times 10^{-24} \text{ cm}^2$	$0.130 \times 10^{-24} \text{ cm}^2$
$0.945 \times 10^{-24}$	$0.945 \times 10^{-24}$	$0.945 \times 10^{-24}$
$\sigma_{\text{sc}}/\sigma_{\text{tot}}$	$\sigma_{\text{sc}}/\sigma_{\text{tot}}$	$\sigma_{\text{sc}}/\sigma_{\text{tot}}$
$0.700 \times 10^{-24}$	$0.700 \times 10^{-24}$	$0.700 \times 10^{-24}$

## THEORETICAL APPROXIMATIONS

$\sigma_{\text{tot}}$ (Eq. 5)	$\sigma_{\text{tot}}$ (Eq. 6)	$\sigma_{\text{tot}}$ (Eq. 7)
$0.140 \times 10^{-24} \text{ cm}^2$	$0.130 \times 10^{-24} \text{ cm}^2$	$0.130 \times 10^{-24} \text{ cm}^2$
$0.945 \times 10^{-24}$	$0.945 \times 10^{-24}$	$0.945 \times 10^{-24}$
$\sigma_{\text{sc}}/\sigma_{\text{tot}}$	$\sigma_{\text{sc}}/\sigma_{\text{tot}}$	$\sigma_{\text{sc}}/\sigma_{\text{tot}}$
$0.700 \times 10^{-24}$	$0.700 \times 10^{-24}$	$0.700 \times 10^{-24}$

## THEORETICAL APPROXIMATIONS

$\sigma_{\text{tot}}$ (Eq. 5)	$\sigma_{\text{tot}}$ (Eq. 6)	$\sigma_{\text{tot}}$ (Eq. 7)
$0.100 \times 10^{-24} \text{ cm}^2$	$0.100 \times 10^{-24} \text{ cm}^2$	$0.100 \times 10^{-24} \text{ cm}^2$
$0.840 \times 10^{-24}$	$0.840 \times 10^{-24}$	$0.840 \times 10^{-24}$
$\sigma_{\text{sc}}/\sigma_{\text{tot}}$	$\sigma_{\text{sc}}/\sigma_{\text{tot}}$	$\sigma_{\text{sc}}/\sigma_{\text{tot}}$
$0.700 \times 10^{-24}$	$0.700 \times 10^{-24}$	$0.700 \times 10^{-24}$

## THEORETICAL APPROXIMATIONS

$\sigma_{\text{tot}}$ (Eq. 5)	$\sigma_{\text{tot}}$ (Eq. 6)	$\sigma_{\text{tot}}$ (Eq. 7)
$0.100 \times 10^{-24} \text{ cm}^2$	$0.100 \times 10^{-24} \text{ cm}^2$	$0.100 \times 10^{-24} \text{ cm}^2$
$0.840 \times 10^{-24}$	$0.840 \times 10^{-24}$	$0.840 \times 10^{-24}$
$\sigma_{\text{sc}}/\sigma_{\text{tot}}$	$\sigma_{\text{sc}}/\sigma_{\text{tot}}$	$\sigma_{\text{sc}}/\sigma_{\text{tot}}$
$0.700 \times 10^{-24}$	$0.700 \times 10^{-24}$	$0.700 \times 10^{-24}$



Fig. 8a Segmentation by Fisher's linear discriminant

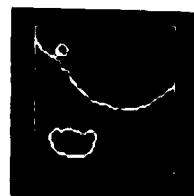


Fig. 8b Sobel gradient operation on Fig. 8a. (threshold is 3.162)

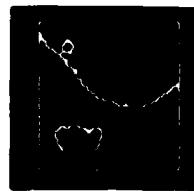


Fig. 8c Robert gradient operation on Fig. 8a. (threshold is 1)

Fig. 8a. The small false region, however, cannot be removed. The results of Sobel and Robert gradient operations on Fig. 8a are shown in Fig. 8b and Fig. 8c respectively. Due to the false region, the percentage segmentation error of Fig. 8a is 4.64%. The Fisher's linear discriminant requires slightly more computation time because of the scatter matrix and feature projection computations.

As a concluding remark, it is interesting to note that while both the maximum a posteriori estimation and the Fisher's linear discriminant are feasible linear estimation methods for image analysis, they employ quite different procedures for utilizing the statistical contextual information.

Acknowledgment We thank Dr. Therrien for helpful communications. The results reported in Sections 3 and 4 are due to R. H. Wu and the results reported in Section 5 are due to C. Yen.

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