







REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS
1. REPORT NUMBER 2. GOVT ACCESSION N	D. 3. RECIPIENT'S CATALOG NUMBER
AFGL-TR-80-0221 AP A0 94/23	· [
8. TITLE (and Subtilie)	5. TYPE OF REPORT & PERIOD COV
NEAR FIELD STATIC TILT FROM SURFACE LOADS	Scientific. Interim.
	6. PERFORMING ONG. REPORT NUME ERP No. 710
7 AUTHOR(a)	. CONTRACT OR GRANT NUMBER(+)
James F. Lewkowicz Gerry H. Cabaniss	
PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT. PROJECT, T AREA & WORK UNIT NUMBERS
Hanscom AFB	611025
Massachusetts 01731	2309G201
1. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Air Force Geophysics Laboratory (LWH)	8 July 1980
Hanscom AFB	13. NUMBER OF PAGES
MASSACHUSELLS U1731 MONITORING AGENCY NAME & ADDRESS(I dillerent from Controlling Office)	15. SECURITY CLASS. (of this report)
	Unalegnified
	Unclassified
	SCHEDULE
approved for public release; distribution unimi	ted.
7. DISTRIBUTION STATEMENT of the aberract entered in Black 20, if Jillerent in	ted.
7. DISTRIBUTION STATEMENT of the aberract entered in Block 20, it different to	om Report)
7. DISTRIBUTION STATEMENT of the aberract entered in Block 20, it different fr 8. SUPPLEMENTARY NOTES	ted,
 DISTRIBUTION STATEMENT of the aberract entered in Black 20, if different in SUPPLEMENTARY NOTES * Boston College, Chestnut Hill, MA 02167 	ted,
 DISTRIBUTION STATEMENT of the about act onlored in Black 20, if different if SUPPLEMENTARY NOTES Boston College, Chestnut Hill, MA 02167 KEY WORDS (Continue on reverse side if necessary and identify by block number 	om Report)
 DISTRIBUTION STATEMENT of the aberract entered in Block 20, if different in SUPPLEMENTARY NOTES BOSTON College, Chestnut Hill, MA 02167 KEY WORDS (Continue on reverse side if necessary and identify by block number Tilt 	ted, on Report)
 DISTRIBUTION STATEMENT of the abarract entered in Block 20, if different fr SUPPLEMENTARY NOTES Boston College, Chestnut Hill, MA 02167 KEY WORDS (Continue on reverse side if necessary and identify by block number Tilt Tilt meter Earth tilt 	ted, om Report)
 PUPPLEMENTARY NOTES * Boston College, Chestnut Hill, MA 02167 KEY WORDS (Continue on reverse side if necessary and identify by block number Tilt Tilt Earth tilt 	ted, om Report)
 DISTRIBUTION STATEMENT of the abarract entered in Block 20, if different in SUPPLEMENTARY NOTES * Boston College, Chestnut Hill, MA 02167 XEY WORDS (Continue on reverse side if necessary and identify by block number Tilt Tilt Tilt Tilt Earth tilt 	ted, om Report)
 DISTRIBUTION STATEMENT of the abstract entered in Black 20, if different in SUPPLEMENTARY NOTES * Boston College, Chestnut Hill, MA 02167 * KEY WORDS (Continue on reverse side if necessary and identify by block number) Tilt Tiltmeter Earth tilt A field a vioce niment words conformed to dominity by block number) 	ted,
 DISTRIBUTION STATEMENT of the abetract entered in Block 20, if different in SUPPLEMENTARY NOTES * Boston College, Chestnut Hill, MA 02167 * KEY WORDS (Continue on reverse side if necessary and identify by block number Tilt Tiltmeter Earth tilt > A field experiment was performed to determ meter to static loads produced by a vehicle. Th 	nine the response of a tilt
 DISTRIBUTION STATEMENT of the abatract entered in Black 20, if different in SUPPLEMENTARY NOTES Boston College, Chestnut Hill, MA 02167 KEY WORDS (Continue on reverse side if necessary and identify by block number) Tilt Tiltmeter Earth tilt ABSTRACT (Continue on reverse side if necessary and identify by block number) A field experiment was performed to determ meter to static loads produced by a vehicle. Th sults of a theoretical approximation of the vehicle 	nine the response of a tilt is report compares the re le load, by four point forc
 DISTRIBUTION STATEMENT of the abstract entered in Block 20, if different in SUPPLEMENTARY NOTES *Boston College, Chestnut Hill, MA 02167 XEV WORDS (Continue on reverse side if necessary and identify by block number Tilt Tiltmeter Earth tilt ABSTRACT (Continue on reverse side if necessary and identify by block number Earth tilt ABSTRACT (Continue on reverse side if necessary and identify by block number Earth tilt ABSTRACT (Continue on reverse side if necessary and identify by block number Earth tilt 	nine the response of a tilt is report compares the re le load, by four point forc bus half space, with the e
 DISTRIBUTION STATEMENT of the abstract entered in Block 20, if different in SUPPLEMENTARY NOTES Boston College, Chestnut Hill, MA 02167 KEY WORDS (Continue on reverse side if necessary and identify by block number Tilt Tiltmeter Earth tilt ABSTRACT (Continue on reverse side if necessary and identify by block number Earth tilt ABSTRACT (Continue on reverse side if necessary and identify by block number Earth tilt ABSTRACT (Continue on reverse side if necessary and identify by block number Earth tilt ABSTRACT (Continue on reverse side if necessary and identify by block number) A field experiment was performed to determ meter to static loads produced by a vehicle. Th sults of a theoretical approximation of the vehicl acting on a linear elastic single layer homogeneer perimental data. Results indicate that this mode mation to the field data. In order to model the field 	mine the response of a tilt is report compares the re le load, by four point forc bus half space, with the end of the space of the spac
 DISTRIBUTION STATEMENT of the abstract entered in Block 20, if different in SUPPLEMENTARY NOTES * Boston College, Chestnut Hill, MA 02167 XEY WORDS (Continue on reverse side if necessary and identify by block number Tilt Tilt Tiltmeter Earth tilt A field experiment was performed to deterr meter to static loads produced by a vehicle. Th sults of a theoretical approximation of the vehicl acting on a linear elastic single layer homogenee perimental data. Results indicate that this mode mation to the field data. In order to model the finite element meter to static homose. 	mine the response of a tilt is report compares the re- le load, by four point forc- bus half space, with the ex- el gives a reasonable appr- ield conditions more reali-

•

Preface

The authors should like to acknowledge several interesting and helpful discussions with Mr. J. Battis. They also thank Mr. H.A. Ossing for supplying the dispersion data.

3

Assession For
Set J T B St
Jusification
P
Distributami/
Inveiled flity Codes
Avera and or
Dist Special
M

. 4

Contents

1.	INTRODUCTION	7
2.	FIELD EXPERIMENT	8
3.	COMPUTATION OF TILTS	9
4.	MODEL	9
5.	EXPRESSIONS FOR TILTS	10
6.	THEORETICAL RESULTS	12
7.	MODEL VS EXPERIMENT	16
8.	DISCUSSION	18
9.	CONCLUSION	19
AP	PENDIX A: Tiltmeter Measurement in the Field	21

5 N

1

Illustrations

21

۰.

1.	Experimental Tilt Data	8
2.	Experimental Configurations	10
3.	Displacements Due to a Point Force	13
4.	Analytic Tilts Due to a Point Force	13

5

11

Illustrations

. •,

5.	Analytic Tilts and Pipe Tilt Due to a Point Force	14
6.	Pipe Tilt and Displacements Due to a Point Force	14
7.	Analytic Tilts Due to a Point Force and Four Point Forces	15
8.	Pipe Tilt Due to Two Front and Rear Wheels	16
9.	Pipe Tilt Due to Four About Equally Spaced Point Forces and the Experimental Tilt (Depth = -4.0 ft, $E = 4.32 \times 10^6 \text{ lb/ft}^2$, $\delta = 0.25$, $P = 3.2 \times 10^3 \text{ lb}$)	17
10.	Pipe Tilt Due to Four About Equally Spaced Point Forces. (Pipe tilt on a smaller distance scale)	17
11	Comparison of Pine Tilt and Experimental Data	18

6

۲,

Near Field Static Tilt From Surface Loads

1. INTRODUCTION

1. Sec. 1. Sec

Tiltmeters are usually placed on horizontal surfaces (a mine tunnel floor) or attached to vertical surfaces (the wall of a vertical borehole), measuring the tilt of initially horizontal and vertical line elements, respectively. While the tilt of a horizontal and of a vertical surface is not the same quantity, at the free surface of an elastic body they become equal (Appendix A). Historically, tilt measurements have been made in mine tunnels to avoid contamination of the data by meteorological effects near the surface. These instruments were practically always placed on a horizontal platform, but because it was assumed that they were near enough to the free surface, horizontal and vertical tilts were considered equal. Therefore, no confusion arose when comparing theoretical tilt with measured tilt. There are occasions, however, when care must be taken in making the foregoing comparison. This report discusses the theoretical calculation of tilt due to a stationary vehicle and compares results with the observed tilt measured in the field.

(Received for publication 8 July 1980)

2. FIELD EXPERIMENT

The purpose of the experiment was to ascertain if under field conditions a tiltmeter or an array of tiltmeters could determine the vehicle load distributions.

The experiment included the installation of a single tiltmeter at a shallow depth (~6 ft). The installation procedure was essentially the one used by the U.S. Geological Survey.¹ Briefly, the installation involved augering a vertical hole into the alluvium and placing the tiltmeter, which is housed in a 48-inch long capsule, into the augered hole. A mixture of dry sand was then compacted around the tiltmeter case.

The top of the capsule was approximately 2 ft below the ground surface. After completion of the installation, the tiltmeter was allowed to operate for a sufficient length of time to stabilize.

The experiment involved driving a vehicle (standard sedan) of known weight up to and away from the tiltmeter on a line parallel to one of the sensitive axes of the instrument. The vehicle stopped at predetermined distances from the tiltmeter and the tilt was recorded. The vehicle then moved to the next position, and so on.

The data showed that the derivative of the measured tilt underwent a sign reversal, as can be seen from Figure 1.



Figure 1. Experimental Tilt Data

1. Johnston, M.J.S., and Mortensen, C.E. (1974) Tilt precursors before earthquakes on the San Andreas Fault, California, Science 186:1031-1034.

3. COMPUTATION OF TILTS

The usual computation of the horizontal tilt of a free surface, for an infinitesimal line segment, is not able to explain the sign reversal tilt observed in the field data. But, the tiltmeter used in this experiment is not in fact measuring the horizontal tilt of the free surface, but rather the vertical tilt at depth. Furthermore, the tilt sensed by the tiltmeter (a 4-ft capsule) is approximately the tilt of a finite vertical line element, which is not the same as the tilt of an infinitesimal line element, in an inhomogeneous strain field.

Based on the foregoing considerations, we now calculate the vertical tilts sensed by a tiltmeter emplaced in a finite capsule.

4. MODEL

We make the following two initial assumptions:

- 1. The soil responds elastically to the loading.
- The capsule behaves as a rigid body compared to the soil, and can be treated as a finite length line element of negligible width.

The first assumption appears justified due to the fact that the tiltmeter returned to its initial position when the load was removed.

The second assumption is based on a comparison of Young's Moduli (E), of steel and soil, approximately 3.0×10^7 lb/ft² and 4.32×10^6 lb/ft², respectively, and the length to width ratio (32:1) for the capsule.

The elastic constants chosen for the model are based on compressional and shear wave velocities derived from an empirical dispersion curve for the local area.² Using reasonable values for soil densities, we found Young's modulus to be 4.32×10^{6} lb/ft² and Poisson's ratio (δ) was chosen to be 0.25.

Based on the geometry shown in Figure 2a, we calculated the vertical tilt at depth of the capsule as a function of distance from the vehicle. To gain insight into the problem, we approximated the four loads of the vehicle's wheels as a single point force, the vehicle's weight, 3.2×10^3 lb. The more realistic calculation which computes the sum of the four tilts caused by the loads of the vehicle's wheels is presented later.

2. Ossing, H.A. (1979) Personal communication.



Figure 2. Experimental Configurations

5. EXPRESSIONS FOR TILTS

Farrell³ gives expressions for the vertical and horizontal displacements at depth due to a single unit point force on the surface of an elastic half space. Farrell uses a cylindrical coordinate system with basis vectors e_z , e_r , and e_{θ} and lets $z \leq 0$ be the volume occupied by the half space. The problem is axially symmetric, so that there is no θ dependence in the solution.

^{3.} Farrell, W.E. (1972) Deformation of the earth by surface loads, <u>Rev. Geophys.</u> and Space Phys. 10(No. 3):767.

$$u(z, r) = -\frac{1}{4\pi} \frac{1}{\mu R} \left(\frac{\sigma}{\eta} + \frac{z^2}{R^2} \right)$$

$$v(z, r) = -\frac{1}{4\pi\eta r} \left(1 + \frac{z}{R} + \frac{\eta r^2 z}{\mu R^3} \right)$$
(1)

where

$$\sigma = \lambda + 2\mu$$
$$\eta = \lambda + \mu$$
$$B^{2} = r^{2} + z^{2}$$

with λ and μ being Lamé parameters.

Using the same coordinate system, we define the horizontal and vertical tilt of an infinitesimal line element, respectively, as

$$\lambda_{\rm H} = \frac{\partial u}{\partial r}$$

$$\lambda_{\rm V} = \frac{\partial \mathbf{v}}{\partial z} .$$
(2)

The quantities u and v are the vertical and horizontal displacements, respectively; that is, the tilt $\lambda_{\rm H}$ is equal to the change in the vertical displacement between two points, as that distance approaches zero, and similarly for $\lambda_{\rm V}$.

At the surface, these two tilts become equal in magnitude but, opposite in sign, though the same is not true at depth (Appendix A).

Performing the operations indicated by the defining equations for the tilt shown above on Farrell's Eq. (1), we obtain the expressions for the horizontal and vertical tilts of an infinitesimal line element.

$$\lambda_{\rm H} = \frac{\Pr}{4\pi\,\mu} \left[\frac{\sigma}{\eta} \left(r^2 + z^2 \right)^{-1.5} + 3 z^2 \left(r^2 + z^2 \right)^{-2.5} \right]$$

$$\lambda_{\rm V} = \frac{-\Pr}{4\pi\,\eta\,r} \left[\left(r^2 + z^2 \right)^{-0.5} + \left(r^2 + z^2 \right)^{-1.5} \frac{\eta\,r^2}{\mu} \right]$$

$$- \left(r^2 + z^2 \right)^{-1.5} z^2 - \frac{3\eta r^2 z^2}{\mu} \left(r^2 + z^2 \right)^{-2.5} \right]$$
(3)

where P = force.

For finite line elements, λ_{V} can be approximated by

$$\lambda_{\rm V} \stackrel{\sim}{=} \overline{\lambda_{\rm V}} = \frac{\Delta v}{\Delta z} = \frac{v_2 - v_1}{z_2 - z_1} \tag{4}$$

where the subscripts refer to the extremities of the capsule. Subscript 2 designates the deeper of the two extremities; subscript 1, the shallower; and z represents the depth below the surface; v_1 and v_2 are the horizontal displacements of the shallow and deep extremities of the capsule. In the present problem z_2 and z_1 are -6 ft and -2 ft, respectively; therefore Eq. (4) becomes

$$\overline{\lambda}_{V} = \frac{\Delta v}{\Delta \overline{z}} = \frac{v_{2} - v_{1}}{-4.0 \text{ ft}} .$$

6. THEORETICAL RESULTS

We first examine the vertical and horizontal displacements at a depth of -4 ft due to a point force (see Figure 3). Qualitatively, the vertical displacement is negative infinity at r = 0; it approaches zero as r approaches infinity, but never becomes positive. The horizontal displacement also is negative near the load; it goes through zero and becomes positive approximately 8 ft from the force. A positive horizontal displacement represents a displacement away from the force in the positive r direction.

We next examine the analytic horizontal and vertical tilts (Appendix A) shown in Figure 4. A positive tilt is a clockwise rotation in the region of positive r. Note that the horizontal tilt has a turning point at ~4.0 ft, approaches zero but never becomes positive. The vertical tilt also has a turning point at ~4.0 ft, then crosses zero and becomes positive.

In Figure 5, the vertical numerical (Appendix A) pipe tilt, at a depth of -4.0 ft is plotted with the analytic horizontal and vertical tilts. Clearly the capsule and vertical tilt have the same qualitative signature, but differ in magnitude. They also cross zero at different distances.

Figure 6 shows the relationship between the horizontal displacements at a depth of -2 ft and -6 ft, and the vertical capsule tilt. When $v_2 = v_1$ [Eq. (4)], the capsule tilt is zero as shown at approximately $r_0 = 4.0$ ft. Now note the part of the figure to the left of the point where $v_2 = v_1$ ($r = r_0$). For values of $r < r_0$, we see that v_1 is always more negative than v_2 . This means that the top extremity of the capsule is displaced closer to the origin than the bottom extremity, resulting in a negative tilt. For values of $v > r_0$, the opposite is true and the capsule experiences a positive tilt.





, •.



Figure 5. Analytic Tilts and Pipe Tilt Due to a Point Force. (Depth = -4.0 ft, E = $4.32 \times 10^6 \text{ lb/ft}^2$ δ = 0.25, P = $3.2 \times 10^3 \text{ lb}$)



Figure 6. Pipe Tilt and Displacements Due to a Point Force. (Depths of horizontal displacements are at -2.0 and -4.0 ft, $E = 4.32 \times 10^6 \text{ lb/ft}^2$, $\delta = 0.25$, $P = 3.2 \times 10^3 \text{ lb}$)

٠.

Figure 7 shows the difference in the capsule tilt when 4 point forces, representing the vehicle's wheels are contrasted with a single point force. In both cases the total force is 3200 lb.



Figure 7. Analytic Tilts Due to a Point Force and Four Point Forces. (Depth = -4.0 ft, $E = 4.32 \times 10^6$ lb/ft², $\delta = 0.25$, $P = 3.2 \times 10^3$ lb for both one and four point forces)

The tilt due to 4 point forces is the sum of the projections of the tilts due to each wheel along the sensitive axis of the tiltmeter. The force used for each of the front wheels was 950 lb and for each of the rear wheels 650 lb. Actually, the distance from the force is the distance to the midpoint of the front axle in the case of the 4 point forces; otherwise, it is simply the distance to the point force. The geometry due to the 4 wheels is shown in Figure 2b with the dimensions used in the computations. The tilt due to 4 point forces shows only one turning point and qualitatively does not even resemble the tilt due to a single force, except at large distances ($r \ge 20$ ft). At small r the tilt due to the 4 point forces is not zero, as the rear wheels still contribute (recall that distance is measured to the front axle). Figure 8 shows the separate contributions of the 2 front and the 2 rear wheels to the total tilt.



Figure 8. Pipe Tilt Due to Two Front and Rear Wheels. (Depth -4.0 ft, $E = 4.32 \times 10^6$ lb/ft², $\delta = 0.25$, $P = 3.2 \times 10^3$ lb)

7. MODEL VS EXPERIMENT

If we now halve the distance between the wheels of the vehicle (the quantity a, shown in Figure 2), we approximate the dimensions of the vehicle used in the field experiment. Figure 9 shows the computed pipe tilt and the tilt measured in the field. Comparison of the pipe tilt shown in Figures 7 and 9 shows that as the point forces move closer to each other, the pipe tilt more closely resembles the tilt due to a single point force. Comparison with the field data is qualitatively good, but the peaks of the curves do not match well.

Varying the elastic constants has a negligible effect on the position of the peak of the computed tilt curve.

. 4



Figure 9. Pipe Tilt Due to Four About Equally Spaced Point Forces and the Experimental Tilt. (Depth = -4.0 ft, E = 4.32×10^{6} lb/ft², $\delta = 0.25$, P = 3.2×10^{3} lb)



Figure 10. Pipe Tilt Due to Four About Equally Spaced Point Forces. (Pipe tilt on a smaller distance scale)

. "A

We then attempted to simulate the influence of the stiff electrical cable at the top of the tiltmeter. We increased the length of the capsule to 6 ft, letting the top be at the ground surface. These changes moved the peak to 3 ft (the experimental peak is at approximately 2.5 ft). Also, increasing Young's modulus to 3 times its estimated value $(4.32 \times 10^6 \text{ to } 1.296 \times 10^7 \text{ lb/ft}^2)$ makes the amplitude of the computed curve agree well with the experimental curve. The resulting tilt from both of these changes can be seen in Figure 11.



Figure 11. Comparison of Pipe Tilt and Experimental Data (top of tiltmeter is at surface of ground, $E = 1,296 \times 10^7 \text{ lb/ft}^2, \delta = 0,25, P = 3,2 \times 10^3 \text{ lb}$)

8. DISCUSSION

We have shown that the correct tilt to compute is the numerical vertical tilt, at depth, experienced by a finite length (4 ft) capsule. We approximated the loads of the vehicle's 4 wheels as point forces and computed the total tilt due to these forces. This result is shown in Figure 11, along with a plot of the experimental data.

At the present time, there appear to be five likely sources of error that could contribute to the discrepancy between the computed tilt and measured tilt. These

are errors in the estimation of the elastic parameters of the soil, possible errors introduced by the 4 to 6 inches of asphalt at the ground surface, that is, not considering a layered half space, the sand-filled cavity into which the tiltmeter capsule was installed, a "stiff" electrical cable connected to the tiltmeter, the approximation of the wheel loads as point forces, and finally geologic effects. We based our estimates of the elastic constants on reasonable values, in agreement with standard references,⁴ but no attempt was made to measure these at the field site. The possible errors introduced by approximating a 2-layer medium (4 in. to 6 in. of asphalt over a soil half space) should be small, as the effect of surface layers dies off as a function of layer thickness. In this problem, the thickness of the asphalt is small compared to the scale of the problem. The sand filled cavity may introduce errors that are quite difficult to estimate. The most practical approach would be to model the situation utilizing the finite element method, and the same approach is suggested for estimating more realistically the effects of a stiff cable, and also geologic inhomogeneities. The expressions for the displacements due to a point force and a force uniformly distributed over a circle of radius b are equivalent at distances of approximately $r \ge 5b$. It therefore seems reasonable to use the expressions for the point force in this study.

9. CONCLUSION

We have shown that a reasonable approximation to the observed tilt can be made using a simple linear isotropic elastic model. This preliminary work suggests that the finite element method be utilized in any further study, in order to model the physical problem more accurately. The study could also be extended to include a moving vehicle, which is the more probable field situation. Also, Figure 4 shows that monitoring horizontal tilt should increase the amplitude of the measured tilt signal by approximately 4.

4. Lambe, T. W., and Whitman, R. W. (1969) Soil Mechanics, Wiley & Sons, New York, New York.

Appendix A

Tiltmeter Measurement in the Field

In this section we discuss briefly the quantity that a tiltmeter measures in the field. First, tiltmeters are usually coupled either to horizontal or to vertical surfaces. It is also assumed that the installation is close enough to the free surface boundary conditions are applicable, namely, that the shear stress is zero; hence the shear strain vanishes.

We now consider a right-handed Cartesian coordinate system and define vertical and horizontal tilts about the x_2 axis to be, respectively,

λ 、 =	$\frac{\partial \mathbf{u}_1}{\partial \mathbf{x}_3}$		(A A)
х _н	$\frac{\partial u_3}{\partial x_1}$	(4	11)

where u_i is the displacement field.

We disregard rotations about the x_1 and x_3 axes for simplicity. The vertical and horizontal tilts can be interpreted as rotations of lines initially vertical and horizontal before deformation. While in general, these two quantities are not equal, for the special case when the shear strain is zero, they become equal in magnitude and opposite in sign. Thus at the free surface

$$\epsilon_{13} = \frac{1}{2} \left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right) = 0$$
 (A2)

and algebra yields

$$\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} . \tag{A3}$$

This shows that it makes no difference whether one monitors vertical or horizontal tilt at the free surface; they are equivalent.

The foregoing discussion refers to infinitesimal line segments. If the strain field is uniform, the above equations for tilt would be applicable, even if we employed a finite length tiltmeter. Because the tiltmeter we are dealing with is not infinitesimal in length, and the strain is not uniform close to the source, the Eqs. (A2) and (A3) must be written in the following form:

$$\lambda_{V} = \frac{\Delta u_{1}}{\Delta x_{3}} = \frac{u_{1}^{2} - u_{1}^{1}}{\Delta x_{3}}$$

$$\lambda_{H} = \frac{\Delta u_{3}}{\Delta x_{1}} = \frac{u_{3}^{2} - u_{3}^{1}}{\Delta x_{1}}$$
(A4)

where the subscripts refer to the coordinate axes directions, the superscripts refer to the extremities of the capsule, and Δx_i is a finite length, in this case the length of the capsule.

Equations (A1) and (A4) become equal as $\Delta x_i \rightarrow 0$ and as the distance from the source at which the tilt is calculated approaches infinity (the distance from the source at which the strain becomes uniform). Equations (A1) and (A4) are designated the analytic and numerical tilt, respectively.

Care must be taken to use the appropriate expression for the displacement in Eqs. (A1) and (A4), depending on whether or not the tiltmeter is located at the free surface or below the free surface.

In the present study we have used the equations for displacement and tilt at depth as, strictly speaking, the tiltmeter is not located at the free surface.

. 6,