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**PROBABILISTIC ESTIMATES
WITH LIMITED DATA.**

FINAL REPORT



Conrad W. Faber

November 1980

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Decision makers require and need more than a point estimate of the cost and schedule for new or on-going programs. Much has been written about how to quantify program risks based upon historical data. However, estimates for new systems frequently depend upon expert opinion from a few knowledgeable persons because an applicable historical data base is not available. The method proposed in this report utilizes a PERT type network, beta distribution parameters for expert opinion inputs, and convolution of activity distributions by simulation.		

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PREFACE

For the 56 major Army programs* covered by the selected acquisition reports (SAR's) as of September 30, 1979, there was a cost growth of 19.6 billion dollars or 49% over the baseline estimate as reported by the General Accounting Office.¹ Although 70% of this cost growth was attributed to changes in quantities of items procured and inflation, \$2.9 billion dollars or 16% was attributed to cost estimating. The objective of this report is to provide analysts with an additional tool for estimating cost and/or time when data bases are not available with which to develop traditional parametric statistical relationships.

* Defense defined major programs are those with an estimated research, development, test, and evaluation (RDT&E) cost exceeding \$75 million or an estimated production cost exceeding \$300 million. The 56 Army programs conform to this definition.

INTRODUCTION

Much has been written about developing cost estimating relations (CER's) based upon historical data. The decision-maker requires and needs more than a point estimator to evaluate the associated risks of developing and fielding a new system. Therefore, methodology has been developed for estimating the range of future system costs and their associated probability distributions based upon this historical data. However, with the rapid development of exotic weapon systems, new manufacturing methods, and new composite materials, historical data bases are not sufficiently analogous to the new system and/or the physical/performance characteristics of the new system are beyond the reliable ranges of the data base. Consequently, estimates for the new system frequently depend upon expert opinion of a very few knowledgeable persons.

When CER's are "somewhat" analogous but inadequate (for reasons mentioned above), the method frequently used is to get expert opinion adjustment factors, e.g., 1.4 times CER estimate. Although this may be the best point estimate available, the decision-maker still needs an indicator of the risks involved, i.e., some measure of how much the estimate could be in error. The method proposed in this paper provides this information.

I would like to emphasize that the method presented is a complimentary procedure. When sufficient historical or engineering data is available, standard statistical procedures should be used or complemented by the proposed method.

PROPOSED METHOD

GENERAL:

a. Uncertainty (risk) of an activity* can be described by a probability distribution. This uncertainty relates to potential technical risks and economic factors. Another major source of uncertainty, not covered in this paper, is requirements uncertainty, e.g., changes in performance requirements and quantity procured. For a given risk assessment, requirements are assumed to be fixed. Program uncertainty is a convolution of the activities risks.

b. To determine program uncertainty, a PERT type network must be developed displaying the activities and events and their major interdependencies. This network should be correlated to the elements of the program work breakdown structure. For each activity, a distribution describes the uncertainty involved in that activity. When applicable historical data is available or factors assumed, appropriate distributions should be used. This paper describes a method of estimating activity distributions when the above is not available.

c. Once the activity distributions and parameters are specified, a total program (or intermediate milestone) probability distribution can be derived by Monte Carlo simulation. Three models (RISCA, SOLVNET and VERT) are described in DARCOM Handbook 11-1.1-79, Army Programs: Decision Risk Analysis Handbook² Of the models known by the author, VERT (Venture Evaluation and Review Technique) is the most

* Activity is defined as the time or cost to complete a task whereas an event is a point in time, e.g., start of flight testing.

versatile and is used in the sample case in a subsequent section.

Also reference article from Defense Management Journal, "VERT: A risk analysis tool for program management", contained herein as Appendix A.

d. A major shortcoming of most risk models is the limited number of distributions built into the basic program and/or the amount of subjective probabilistic data to be requested from the "expert." Since many activity distributions are skewed to the right, i.e., the possible range of an overrun exceeds that of an underrun, the standard normal distribution is not appropriate. Also, the frequently used triangular distribution can easily misrepresent the probability densities as shown by an example in Appendix B. The VERT program allows the analyst a choice of over a dozen density functions. These can be used to describe activities in terms of time and/or cost.

e. For a program with many activities, the Central Limit Theorem (CLT) provides an unbiased estimate of the expected mean and variance of the total program cost* by simply adding the expected values of the activity means and variances since the limiting distribution of additive variables (even from skewed distributions) is normal. However, for a few skewed activities, or domination by a few skewed activities, or intermediate milestone distributions based upon a small number of skewed activities, application of the CLT can give misleading results regarding variance and skewness.

* Estimates of program time or cost as a function of time generally cannot use the CLT. While the expected mean for time can be computed along the critical path, the distribution for time or cost for several activities requires more sophisticated techniques, e.g., simulation.

SELECTION OF DISTRIBUTION(S):

a. When the analyst must determine the activity distributions from subjective inputs, the analyst usually can only zero in on the general shape of the distribution and its associated parameters. Consequently the following criteria was used to select a distribution or distributions:

1. Simplicity
2. Could be symmetric, skewed left, or skewed right
3. Could have varying degrees of kurtosis, i.e., concentration around mean or mode
4. Could be normalized for computer simulation

b. During the 1960's, several theoretical papers (See References 3 thru 8) were written regarding cost uncertainty. All of the authors of these papers chose the beta probability function to describe activities because of its versatility and simplicity. Because of the large amount of computer core and central processing unit (CPU) time required to run a Monte Carlo simulation, most of these earlier authors advocated the convolution of beta distributions by the method of moments. This method provides a total program cost distribution profile but suffers from the same shortcomings as using the Central Limit Theorem discussed earlier. During the decade of the 1970's, little use has been made of this research. However, with today's high speed computers, a complex network can be simulated via Monte Carlo techniques much more efficiently. (Using the VERT program, a complex network can be simulated 1000 times with under 240K core and under 2 minutes CPU time.)

c. This author evaluated several distributions (triangular, gamma, weibull, beta, et al.) and reached the same conclusion that the beta function could adequately describe most activity distributions and was generally superior to other probability functions vis-a-vis the criteria listed above. The preceding statement is not meant to suggest the exclusive use of the beta distribution when acquiring subjective inputs. There are situations where other distributions maybe more appropriate, e.g., the Poisson distribution for the expected life of a component or the binomial distribution for either/or situations.

d. The beta probability density function (pdf) is:

$$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{(a-1)} (1-x)^{(b-1)}, \text{ where } 0 < x < 1$$

The parameters "a" and "b" determine the degree of skewness and kurtosis. The following transformation of actual high (H) and low (L) points of the range conform to the beta pdf range of 0 thru 1.

$$x = (X-L)/(H-L) - L, \text{ where } X \text{ is the actual data value.}$$

The computer program, given the beta pdf parameters, randomly selects x and then transforms it to X for each iteration through the network.

e. When obtaining subjective probabilistic information, the author has experienced the best results when the choices available to the experts have the following characteristics:

1. Finite end points which exclude extremely unlikely probabilities
2. Unimodal
3. Continuous rather than discrete
4. Few input parameters required
5. A finite set, with visual illustrations, from which to choose

The beta distribution also met the first four elements of this criteria. To meet criteria five, nine representative beta distributions were selected. The first four are skewed to the right with modes 25% and 40% of the way through the range. Distributions five through seven are symmetric and distributions eight and nine slightly skewed to the left with the modes 60% of the way through the range. These nine distributions are displayed in Appendix C.

INPUT REQUIRED:

a. To determine program or subprogram uncertainties, the activities and events must be defined and their interrelationships established. This is best depicted in a PERT type network. Although it is not the purpose of this paper to describe how to construct this network, the following general comments indicate the flexibility available.

1. Branching probability paths can be constructed, e.g., probabilities of failure causing program stop, sufficient problems to cause major redesign, or adequate success to continue work as originally planned. This branching may be activated by cost and/or time constraints.

2. Activities can be described in terms of time or cost risk. Care should be taken to include the interdependancies of events. Activities may have to be subdivided for this purpose, e.g., design of item A into preliminary design and final design of A because the preliminary design of A is required before item B can be designed.

3. Time uncertainty usually assumes a normal work pace (e.g., a 40 hour work week). Analysis can then determine critical activities which allows management the option of selected overtime or reallocation of resources and awareness of which activities/events to monitor closely.

4. Cost is frequently determined as a linear function of time, i.e., $\text{cost} = a + bx$, where a is a base constant and b is a cost per unit of time. This is based on the close relationship between cost and time where time is a function of technical uncertainty.

5. Activities/events become more specific as a program is defined in more detail. E.g., to monitor the risk in an ongoing program, the activities/events for the next 6 months are in more detail than those farther in the future whereas past activities are now given a fixed number.

6. A well constructed program network may combine elements of all the above.

The inputs required for risk analysis can be readily seen from the developed network. The suggested procedure which follows assumes activity estimates cannot be obtained by traditional parametric statistical relationships.

b. Parameters required to describe an activity's uncertainty, using the beta distribution, are: the lower and upper bounds, the most likely value, and a choice of one of the nine beta distributions shown in Appendix C. Note that there is a redundancy between the most likely value and the beta distribution selected. This redundancy provides a check on the consistency of the information provided.

1. The high (H) or pessimistic bound assumes significant aspects of the activity develop problems but excludes extremely unlikely or catastrophic occurrences such as a tornado destroying a prototype or a national transportation strike. There should be little chance of exceeding this bound - a workable guideline is no more than one chance in a hundred.

2. The low (L) or optimistic bound is defined similarly to the high estimate, except the most favorable conditions exist.

3. The most likely value or mode (M) is that estimate which has the greatest possibility of occurring.

4. Unless the distribution is symmetric around the mean, the mode is different from the mean. The above terms are illustrated by a hypothetical example in Appendix D.

DATA COLLECTION:

a. Unless the person providing the information has experience in this method of estimating, the personal interview method is preferred. Although it may require more time and money, the analyst has more confidence in the reliability of the inputs. When two or more estimators are available, the Delphi technique may be used. Other data collection techniques are discussed in DARCOM Handbook 11-1.1-79, Reference 2.

b. Some general points the interviewer should consider are:

1. He(s) must understand and be able to describe the program, scope of work, and the network in adequate detail to answer questions by the estimator and to ask the right questions.

2. Allow sufficient time for the interview. Try to pick a setting which minimizes interruptions.

3. The mode is the point most likely, i.e., the point with the most chance of being correct. It may not be the mean or expected value. To assist the interviewer, the modes and means are given in Appendix C with the nine beta distributions. Also given are the areas under the decile and quartile tails of the distributions.

4. The low and high points of the range should be reasonable. This includes the possibility that many events could be favorable or unfavorable but nothing catastrophic would happen.

5. Because of the redundancy in input, the interviewer can quickly check for consistency. However, an atmosphere of cooperation should be promoted to minimize defensive reactions. Also, the interviewer should be careful to not introduce bias into the the estimates received.

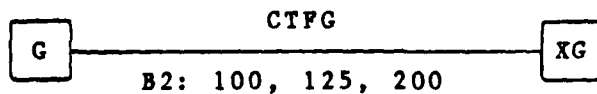
6. The interviewer should remain alert to the estimator's understanding of the process and his knowledge of what is being estimated.

7. As a result of additional data acquired, the program network may need to be revised.

SAMPLE CASE:

a. Situation: A missile system is to be developed using an existing proven system as the base. The only major change will be in the guidance subsystem. The system is composed of five subsystems: airframe (A), propulsion (P), guidance (G), peculiar ground support equipment (PG) and common ground support equipment (CG). No major problems in subsystem interfaces is expected. The first four subsystems will be designed, fabricated or modified (DFM). These four subsystems will then each be component tested and fixed (CTF) as necessary. Meanwhile CG will be acquired (ACG). Next all subsystems will be integrated and fixed (IF) as necessary, followed with a complete system test (ST).

b. The above relationships are shown in Exhibit 1. Event names follow the abbreviations above. Figures below each line indicate the beta type distribution plus the low, mode and high cost estimates. E.g., for the activity CTFG



the guidance subsystem is component tested and fixed as necessary. The cost uncertainty is described by the beta type 2 distribution with a range from 100 to 200 and a mode of 125. Exhibit 2 portrays the same data in tabular form.

SAMPLE CASE NETWORK

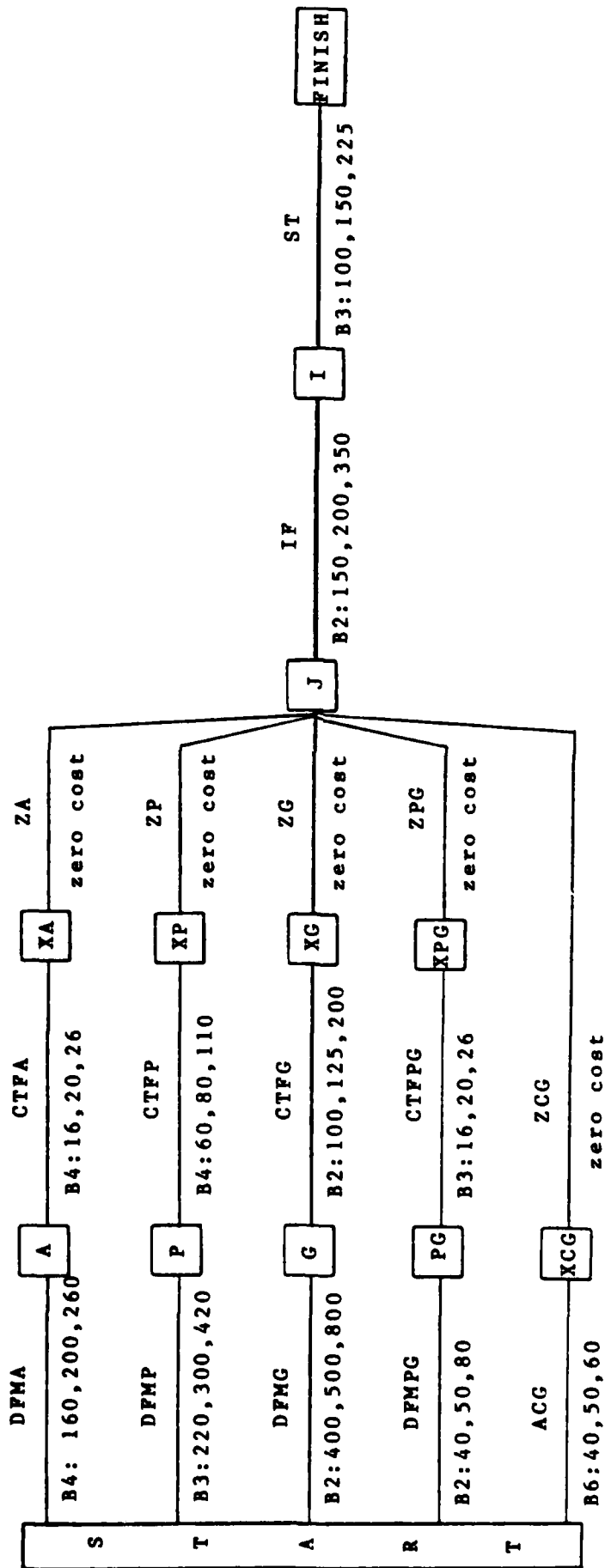


EXHIBIT I

SAMPLE CASE DATA

Activity	Beta Parameters			E[x] [*]	Sim ^{**}		
	a	b	L			M	H
DFMA	3.	4.	160.	200.	260.	203.	203.
DFMP	2.	2.5	220.	300.	420.	309.	301.
DFMG	2.	4.	400.	500.	800.	533.	536.
DFMPG	2.	4.	40.	50.	80.	53.	53.
ACG	3.	3.	40.	50.	60.	50.	50.
CTFA	3.	4.	16.	20.	26.	20.	20.
CTFP	3.	4.	60.	80.	110.	81.	82.
CTFG	2.	4.	100.	125.	200.	133.	132.
CTFPG	2.	2.5	16.	20.	26.	20.	20.
Subtotal at event "J"			1052.	1345.	1982.	1402.	1397.
IF	2.	4.	150.	200.	350.	217.	215.
ST	2.	2.5	100.	150.	225.	156.	150.
TOTAL PROGRAM			1302.	1695.	2557.	1775.	1762.

* Expected value for the normalized beta distribution is $a/(a+b)$, which is converted by multiplying by the range and adding the lower limit.

** Activity mean values resulting from simulating the activities by 1000 iterations through the network using the VERT model.

EXHIBIT 2

c. Analysis:

1. Although the mode is that value which occurs most often, it is not the expected value for an activity. (Reference example in Appendix D.) In the Sample Case, the point estimate for the total program, determined by adding the modes, is 1695 whereas the sum of the activity expected values is 1775. The mode method underestimates the costs by 80 units or 5 percent. The mode method typically underestimates a program's cost or time when the program component activities are skewed to the right, i.e., the range of an overrun exceeds that of an underrun. The program total mean value, as determined by simulation, will normally not equal the expected mean value because of the random selection of activity values during simulation; however, the two methods should have totals within ± 1 percent. Although the point estimate as determined by the expected value or by simulation is superior to the mode method, the decision maker still has no quantification of the uncertainties about the point estimate, i.e., some measure of how much the estimate could be in error.

2. Simulating the Sample Case network by Monte Carlo techniques provides a convolution of the activity probability distributions and provides a measure of the uncertainty around the point estimate. Exhibit 3 displays probabilities and costs for selected events. E.g., using the beta distribution, there is a 75 percent chance that the cost of the program will equal or be less than 1831 units or a 25 percent chance that the cost will exceed 1831 units. Exhibits 4.1 thru 4.3 display the VERT output for the Sample Case for the same events summarized on Exhibit 3. Using the VERT model, output can be generated at any event.

3. The triangular distribution is frequently used to describe activity uncertainty. The reason usually given for its use is a more descriptive distribution is not justified because of the lack of data. There are occasions when this argument is valid. However, in this writer's opinion, this rationale is often used as rationalization to avoid unfamiliar and/or more arduous methods. Appendix B compares the two distributions. The Sample Case was simulated using triangular distributions in place of the beta distributions. The same ranges and modes for each activity were used. The results, using the triangular vis-a-vis the beta distributions, are summarized on Exhibit 3.

SAMPLE CASE

Probability Points for Convoluted Distributions

Event*	Probability	Beta Dist.	Triangular Dist.
XG	.10	576	598
	.25	610	638
	mean	668	705
	.75	719	763
	.90	778	837
J	.10	1285	1331
	.25	1330	1385
	mean	1397	1457
	.75	1459	1525
	.90	1524	1595
FINISH	.10	1632	1712
	.25	1689	1764
	mean	1762	1848
	.75	1831	1927
	.90	1899	1994

* Costs are for all activities leading to the event.

XG: Completion of guidance subsystem prior to integration with other subsystems.

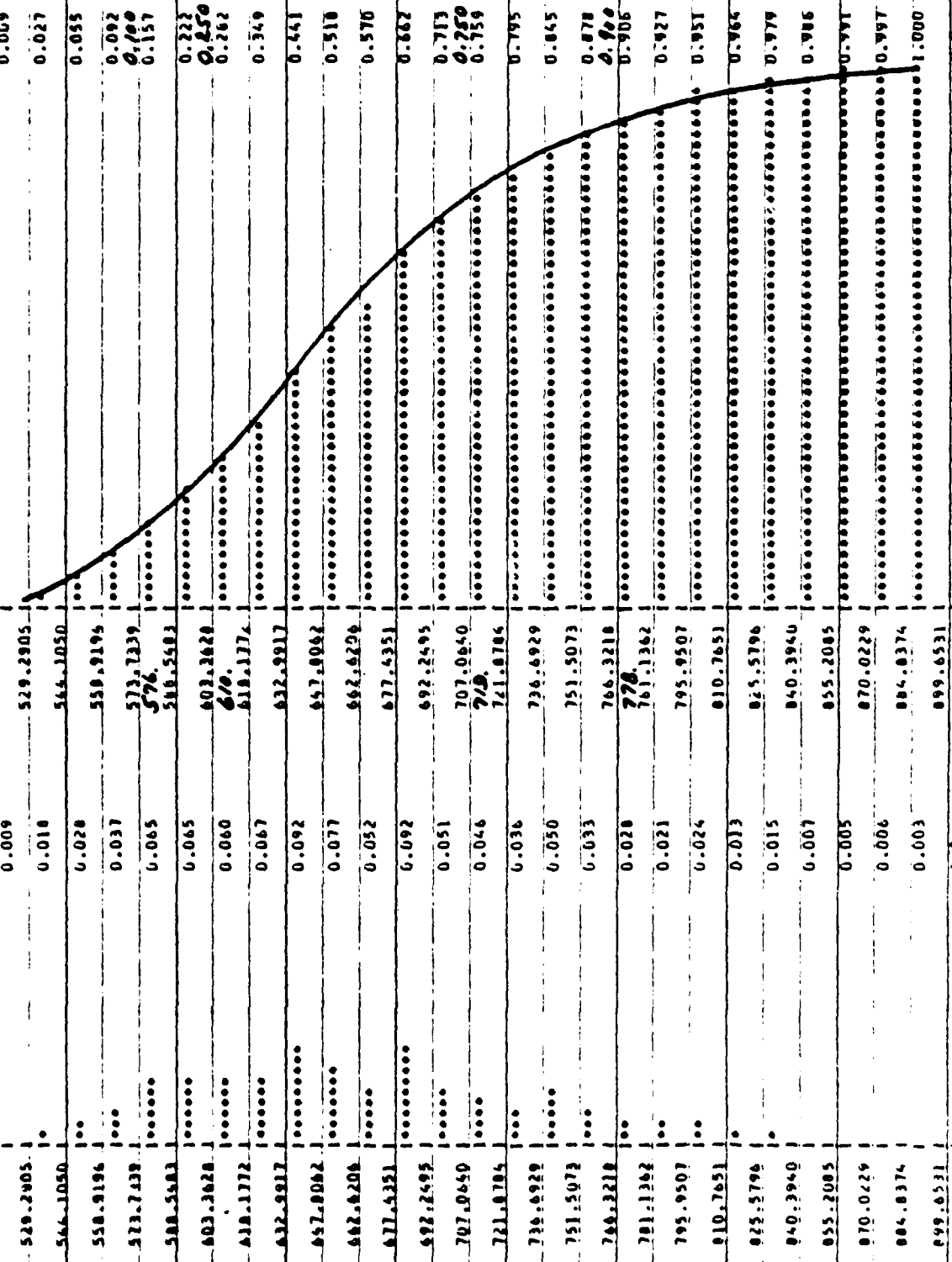
J: Cost of all subsystems before system integration.

FINISH: Cost of total program.

PATH COST FOR NODE X6

0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00

MIN	0.0	0.009	0.018	0.028	0.037	0.065	0.065	0.060	0.067	0.092	0.077	0.052	0.092	0.051	0.046	0.036	0.050	0.033	0.028	0.021	0.024	0.013	0.015	0.007	0.005	0.006	0.003	0.0
514.4761	514.4761	529.2905	544.1050	559.9194	573.7339	588.5483	602.3628	616.1772	632.9917	647.8062	662.6206	677.4351	692.2495	707.0640	721.8784	736.6929	751.5073	766.3218	781.1362	795.9507	810.7651	825.5796	840.3940	855.2085	870.0229	884.8374	899.6518	914.4663
0.0	0.009	0.018	0.028	0.037	0.065	0.065	0.060	0.067	0.092	0.077	0.052	0.092	0.051	0.046	0.036	0.050	0.033	0.028	0.021	0.024	0.013	0.015	0.007	0.005	0.006	0.003	0.0	



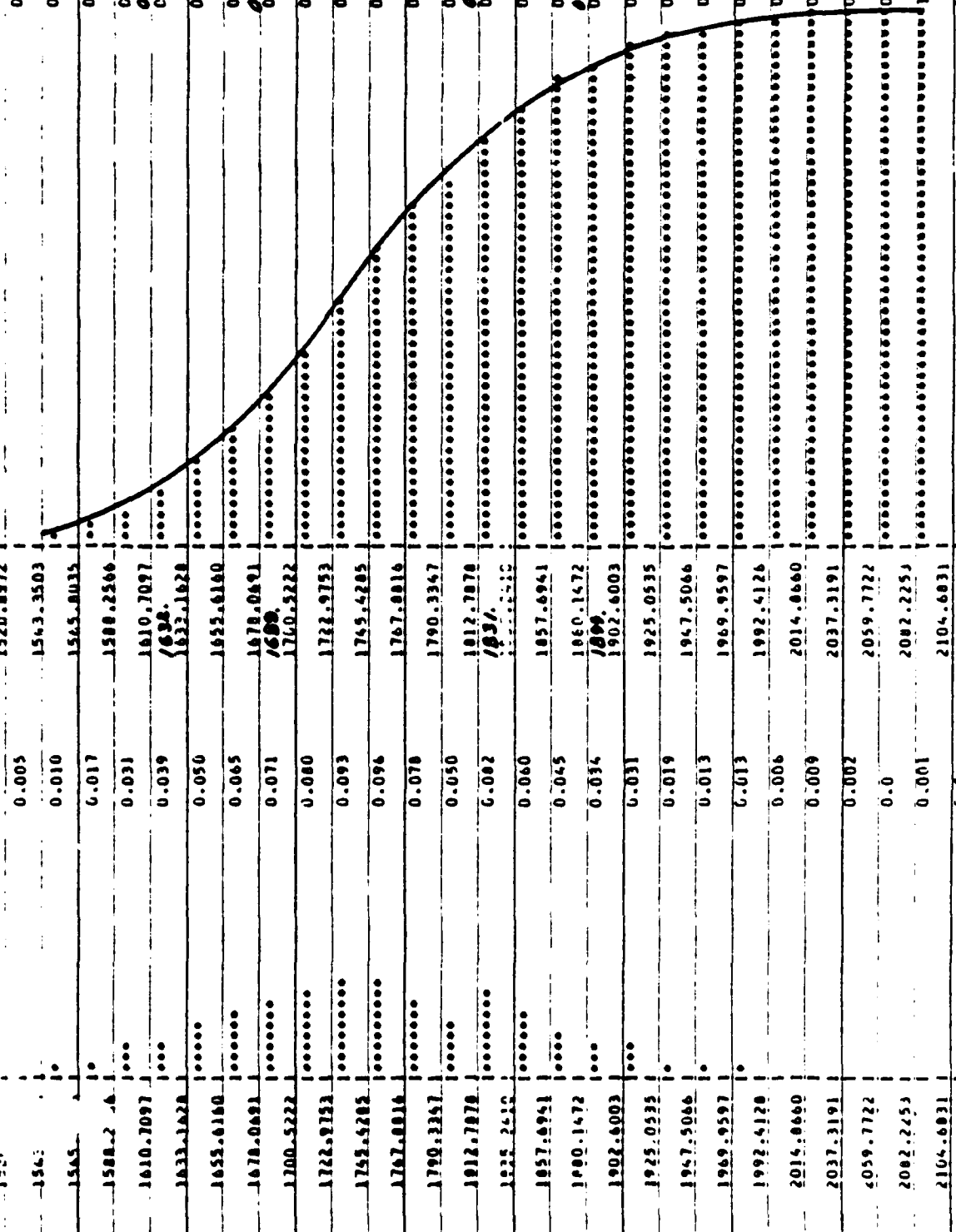
EX: 139 4

999.6531 MAX 77.4072 (COEFF OF VARIATION * 0.12 KURTOSIS (BETA 2)) * 2.73

PATH AND OVERALL COST ARE THE SAME FOR ALL FINISH

0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00

MIN	0.0	0.005	0.010	0.017	0.031	0.039	0.050	0.065	0.071	0.080	0.093	0.096	0.078	0.050	0.002	0.060	0.045	0.034	0.031	0.019	0.013	0.013	0.006	0.009	0.002	0.0	0.001	0.0
1520.8974	1520.8974	1543.3503	1565.8033	1588.2566	1610.7097	1633.1628	1655.6160	1678.0691	1700.5222	1722.9753	1745.4285	1767.8816	1790.3347	1812.7878	1835.2410	1857.6941	1880.1472	1902.6003	1925.0535	1947.5066	1969.9597	1992.4128	2014.8660	2037.3191	2059.7722	2082.2253	2104.6831	
0.005	0.005	0.010	0.017	0.031	0.039	0.050	0.065	0.071	0.080	0.093	0.096	0.078	0.050	0.002	0.060	0.045	0.034	0.031	0.019	0.013	0.013	0.006	0.009	0.002	0.0	0.001	0.0	



MU. 785. + 1000 MEAN * 1740.6361 PEARSONIAN SEM * 0.15
 MAX * 2104.6831
 C-EFF OF VARIATION * 0.06 KURTOSIS (BETA 2) * 2.00
 MIN * 1520.8974

CONCLUDING REMARKS:

The desirability and even the necessity for quantifying the uncertainty around the point estimates (time or cost) for new or ongoing programs is becoming a standard procedure within the Department of Army. The professionalism of analysts dictate that they maintain awareness of new and revised techniques. With the increasing availability of efficient and fast computer equipment, former analytical methods can now be performed economically. Although the method proposed in this paper is not new, its use has been limited by unawareness, availability of computer models with random number generators for many probability distributions, and limited computer capability. The later reasons are no longer true and a major purpose of this paper is to promote more widespread awareness of the capabilities available to analysts and decision makers.

Although this method does not reduce the amount of uncertainty in a program, it does attempt to quantify them in a more precise manner. Provided with this additional knowledge, the decision maker should be able to make better decisions and allocations of our available resources.

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APPENDIX A

"VERT: A Risk Analysis Tool for Program Management"

Defense Management Journal

May-June 1979



VERT
A Risk Analysis
Tool for Program
Management



VERT: **A risk analysis tool for program management**

By Major Greg A. Mann, USAF

So far, it has not taken a strong hold, but the Venture Analysis and Review Technique is proving its value for program managers who need to assess the risk of changes in cost, schedule, or specifications.

The weapons-system acquisition process has been subject to a great deal of criticism in the last decade. Poor forecasting has contributed to cost and schedule overruns which often affect our national defense capabilities and create adverse public opinion.¹ Faced with public and Congressional scrutiny, managers can no longer fall back on "cost growth" as an excuse for such overruns, and will be tasked more than ever to buy the best available system for the least possible cost within the prescribed time frame. For each program decision, the program manager must determine the best balance among three parameters: cost, schedule, and performance. In the weapons-system acquisition process, as contrasted with other areas of management, such determinations are more frequent and more complex, and are made with less of the essential information.² This is because of the inherent uncertainty involved in identifying and resolving the technological unknowns of developing programs.

Uncertainty creates risk,³ but risk can be controlled to some extent by risk analysis. In particular, one recently developed quantitative risk-analysis method, the Venture Evaluation and Review Technique, is proving to be a powerful program-management tool and has been applied satisfactorily to several system-development programs.

Background

Studies of weapons-development projects indicate that most cost and time estimates made early in the acquisition cycle eventually prove to be lower than the actual cost and time for development. This cost growth and time delay can be attributed principally to two factors of the initial estimates.⁴ First, the inability to accurately predict inflationary trends creates an inherent cost-estimating error. This error, however, tends to be small in relation to the second factor—requirements errors, which result from contractual changes in the scope of work. As a project develops, operational considerations and technical innovation necessitate changes in performance specifications, which in turn affect the schedule and cost. Such changes are most pronounced in a technically complex research and development project. A RAND Corporation study found that requirements uncertainty contributes as much as 30 percent to the variations in cost estimates.⁵

These technical-requirements errors, schedule overruns, and cost overruns, together with the rapid increase in the potential enemy's technical capability, influenced DoD's decision in 1970 to accomplish formal risk analysis as an integral part of the development process.⁶ This directive raises a question: how is the program manager to implement formal risk analysis?

Risk analysis is not new. It has always been conducted to varying degrees based on subjective judgment, experience, and qualitative inputs. Over the past 20 years, numerous risk-analysis techniques have been developed. However, most risk analyses are intuitive and incomplete: intuitive in that the structured quantitative approach often gives way to hunches and blackboard analysis; incom-

plete in that detailed analyses of isolated aspects of the problem are rarely integrated into a comprehensive analysis.

Because the three parameters of cost, time, and performance are highly interrelated, it is impossible to work with each factor independently without introducing errors.⁷ But past techniques could not mathematically represent the three parameters and their interrelationships in a way that provided the program manager with accurate risk information on all three parameters simultaneously.

Furthermore, in the past, military procurement of major weapon systems often sacrificed the cost and schedule parameters in order to maintain prescribed performance requirements. In the 1960s attempts to alleviate the imbalance led to changes in procurement strategy. Today, top managers in the Air Force Systems Command consider cost to be as important as schedule and performance.

As this change in emphasis was evolving, decision-management techniques were also changing. The Critical Path Method and the Program Evaluation and Review Technique were developed in the late 1950s. These original networking techniques were useful in the basic managerial functions of planning, scheduling, and controlling. They were also beneficial in laying out tasks and in making gross estimates for material, equipment, and manpower. However, both techniques assumed unrealistically that all activities would be completed successfully.

In the mid-1960s, the Graphical Evaluation and Review Technique was developed as the first computer-oriented networking methodology. From this evolved the Mathematical Network Analyser, developed by the U.S. Army. MATHNET provided the capability for events, activities, activity times, and cost to be modeled probabilistically.

This program was subsequently modified by Army Logistics Management Center personnel and renamed the Risk Information System and Cost Analysis. RISCA provides for the analysis of event uncertainty, but it does not evaluate the risk of failing to attain the performance

¹ Herbert L. Bevelhymmer, *A Proposed Methodology for Weapon Systems Development Risk Analysis*, thesis, Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, June 1973, p. 2.

² *Ibid.*

³ For purposes of this article, risk will be defined as the "probability of not being able to acquire a weapon system of specified performance characteristics within an allotted time, under a given cost and by following a specific course of action." R.R. Lochry et al., *Final Report of the USAF Academy Risk Analysis Study Team*, Denver, Colorado: U.S. Air Force Academy, August 1971.

⁴ *Ibid.*

⁵ Fisher, G.H., *A Discussion of Uncertainty in Cost Analysis*, The Rand Corporation, April 1962.

⁶ Deputy Secretary of Defense Memorandum, May 28, 1970, subject: *Policy Guidance on Major Weapon System Acquisition*.

⁷ Hamilton T. Lenox, *Risk Analysis*, thesis, Wright-Patterson Air Force Base, Ohio: Air Force Institute of Technology, June 1973, p. 71.

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objectives. Thus there was still a need to include the performance variables in the total risk-analysis methodology. This was accomplished in 1973 with the development of the Venture Evaluation and Review Technique. Since then, VERT has been used almost exclusively by Army program managers, who have accepted it as a flexible and valuable tool.⁸

The Venture Evaluation and Review Technique uses a network-simulation approach. In brief, this approach determines risk analysis through two steps. The first step entails constructing a graphic representation of the network—the ordered series of activities leading to specific events. The second step consists of analyzing that network using a computer program. The following example illustrates the process.

The F-X, a hypothetical fighter under development, has three major components: an airframe, an engine, and an avionics system. The desired course of action is to build each subsystem concurrently and integrate them later. A model of the essential features of this process as applied to the F-X is depicted in the Figure. The nodes (decision points) in the network represent alternatives which determine the next arc (activity) to be undertaken in the network. Additionally, the size of the problem has a bearing on how the network is structured. If the problem is large and complex, it is often advisable to construct lower level networks or subnetworks of major subsystems.⁹

Once developed, the network is converted to VERT program terminology. The program has a variety of input capabilities that make it possible for decision events and activities occurring in the network to be described. Numerical values for an activity's time, cost, and performance are assigned to each arc. At each node the next arc is determined by probabilities or by some criteria specified by a mathematical relationship.

The process involves a Monte Carlo simulation in which the design of a network flow across the entire network or subnetwork from the beginning to an appropriate end point leads to a trial solution of the problem being modeled. On the F-X fighter, for example, simulation could assess the activity flow across the total development program, or could focus on the flow across the wing-development subnetwork.

The process is repeated as many times as requested by the user in order to create a large sample of possible outcomes. Slack time, completion time, cost, and performance results are generated as output data for each node. A relative frequency distribution depicts the range

and concentration of values observed at a given node. Also, the probability of exceeding certain value levels can be obtained from the cumulative frequency distributions, and confidence levels can be inferred.

The computer program produces pictorial histogram approximations for selected nodes. Thus, a program manager would have an integrated risk analysis for a particular point of interest in his program. For example, the analysis of the cost, schedule, and performance risk for the F-X program with respect to meeting the scheduled Defense Systems Acquisition Review Council milestones could be expressed in the following manner.

Schedule Risk. The probability or confidence level of being within eight weeks of the scheduled DSARC is 90 percent; the probability of a schedule overrun of 20 weeks or more is 5 percent.

Cost Risk. The total cost of the program will be within \$100 million of the target cost, with a 90 percent confidence level; there is only a 5 percent probability of a cost overrun exceeding \$225 million.

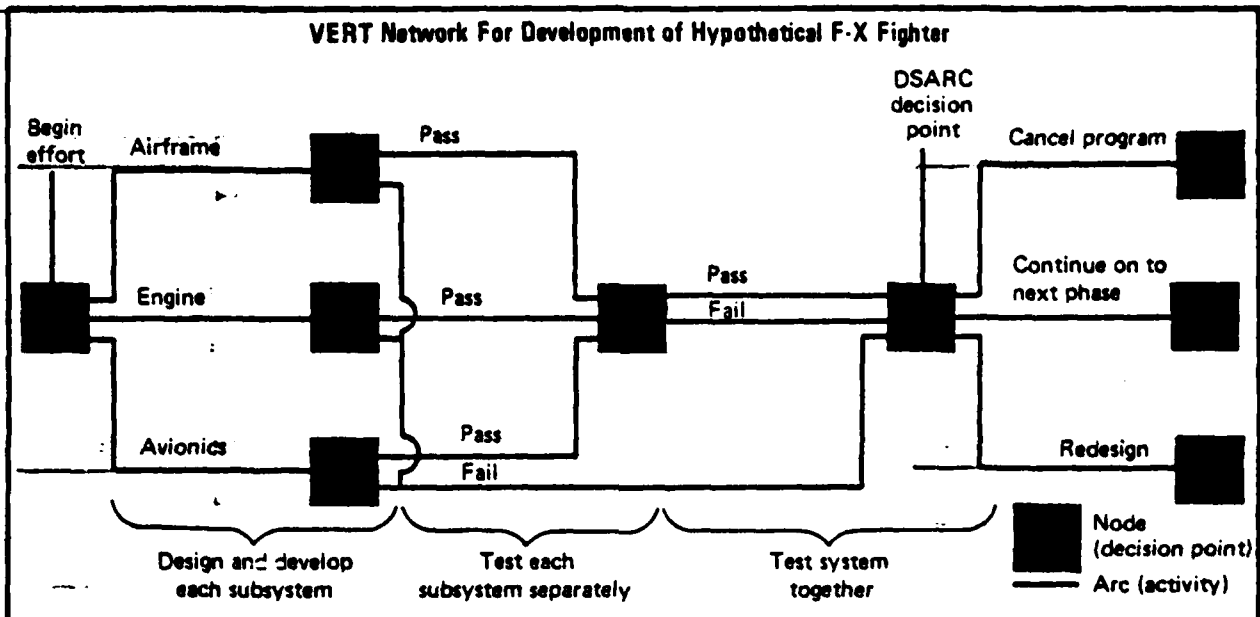
Performance Risk. The confidence level of being within 500 pounds of the static sea-level thrust specifications is 90 percent; performance risk could be indexed to other specifications such as speed, weight, reliability, and maintainability.

The conclusions of the above analysis could vary as key input parameters change. By modifying the values of the input data, one can easily rerun the model. This sensitivity-analysis capability provides the decision maker with the answers to many hypothetical questions. For example, what if the delivery of critical avionics components on the F-X were to take three weeks longer than originally expected? This contingency could be evaluated quickly. By substituting the "what if" data and rerunning the simulation, the decision maker is provided with new information. Although the program manager is the ultimate user of the VERT analysis, the majority of simulations have been developed and run by the systems analysis or program control offices supporting the manager. Yet VERT is not a difficult risk-analysis technique requiring the services of a computer programmer or systems analyst. All that is needed is an individual who is familiar with basic mathematics and computer programming and who can devote about a week of continuous study and effort to master the model's capabilities.¹⁰ However, such proficiency would be required only in simulating the most complex or unusual risk situations. The extent to which a project needs to be segmented into activities and events is a function of the available data and the results desired. Breaking down complex situations into subnetworks simplifies the programming greatly. Some managers

⁸ T.N. Thomas, VERT: A Risk Analysis Technique for Program Managers, Defense Systems Management College, May 1977, p. 21.

⁹ Gerald Moeller, VERT Documentation, Rock Island, Illinois: U.S. Army Armament Command, 1976. Moeller developed VERT in 1973.

¹⁰ *Ibid.*, p. 4.



prefer to estimate parameters for the smaller elemental items rather than for the entire system or for higher-level work packages.

If the results achieved in the analysis are not satisfactory, the program manager must analyze the situation and come up with results that agree with his subjective judgment. When the proper relationships are determinable and mathematically tractable, most analysts and decision makers prefer the quantitative approach.¹¹ In the VERT network-analyzer program, emphasis must be placed on establishing proper relationships. Actual conditions must be represented if creditable analytical results are to be produced. The desire for a quantitative answer or analysis should not force the analyst to disregard or alter critical relationships or facts. The analyst must recognize not only his own limitations but those of VERT as well.

Program applications

The Venture Evaluation and Review Technique has been used in support of several Army programs and at least one Navy project. One of the most noteworthy applications of VERT occurred during the 1975 demonstration and validation phase of the Army's XM-1 Tank development program. The study was structured to examine the XM-1 program manager's question: given a decision to proceed into full-scale engineering development, what is the risk of experiencing unfavorable schedule, cost, or system performance variances? The study was refined to address the following specific objectives:

- Schedule risk expressed as a time distribution for meeting the Army System Acquisition Review Council milestone.

- Cost risk expressed as cost-variance distributions derived from schedule analysis.

- Performance risk expressed as the probability of experiencing a hardware problem that would significantly delay completion of the test program.

VERT simulation was also used in the Cannon-Launched Guided Projectile program to examine the probability that the development effort would successfully reach the production phase. The simulation indicated that there was a 95 percent probability of at least one manufacturer qualifying for full production. It also indicated that the total cost of the program would run about \$9 million over baseline cost if there were a 9-month extension in the schedule.¹²

The technique has also been used in support of the Army's Platoon Early Warning System, the M110E1 self-propelled howitzer, and the Advanced Attack Helicopter program. On the helicopter program, VERT was used to evaluate the validation-phase schedules through the second Defense Systems Acquisition Review Council milestone. At this early stage of development there was considerable risk in many areas. The analysis allowed early identification of possible impacts caused by activities having high probabilities of not occurring as planned. The benefits were so great that the program manager requested continuous tracking of the program by VERT simulation.

To explore the capabilities of the risk-assessment technique, the Navy ran a test application of VERT on the radar system for the F-18 aircraft. The risks were related to new performance requirements, and the simulation examined the amount of testing to be conducted in the laboratory versus aboard a flight-test aircraft. Again, the

¹¹ Lenox, p. 72.

¹² James B. Besson, *Risk Analysis of the 155MM Cannon-Launched Guided Projectile*, Rock Island, Illinois: U.S. Army Armament Command, 1976, p. 4.

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analysis provided the program manager with valuable information.

Problems with VERT

Some minor problems have arisen with VERT, but none are considered major obstacles to its effective use. The most frequent problem is related to the collection of data needed to describe the probabilistic behavior of the variables of time, cost, and performance. Although the VERT program is capable of using many different distributions, most data are represented by a triangular distribution indicating, for example, most pessimistic, most likely, and most optimistic. This is not necessarily wrong, but it does not really use the capabilities of the model, and it thus reduces the accuracy of the simulation output.¹³

Another common data problem is the inability to obtain from the experts accurate estimates of the time and cost. The experts tend to be overly optimistic in their estimates, but this problem is waning as they are coming to realize that the data are being used only for a risk-analysis simulation and will not cause them embarrassment by appearing in other documents.

More can be done

Although VERT appears to be quite promising and devoid of major problems, it has not enjoyed wide use. One reason for this lies not with VERT, but with the inadequate understanding of risk-analysis concepts in general.¹⁴ Many program managers are handicapped by a lack of familiarity with quantitative risk-assessment techniques, and few people in the military services are experienced enough to perform the analysis. In Air Force acquisition programs, for example, such techniques have not been used. Similarly, few managers are accustomed to using the outputs of a risk analysis. For instance, probability distributions depict the risk of development more accurately than do point estimates; yet there is widespread resistance to probability distributions because of their unfamiliarity.¹⁵

¹³ Thomas, p. 17.

¹⁴ Lochry, p. 107.

¹⁵ Ibid.

Consequently, an education program is needed to instruct analysts and managers in the preparation and use of formal, quantitative risk analysis. The program needs to be designed to emphasize risk analysis for high-level officials who deal with uncertainties in program management and program approval.

Another reason that VERT is not used often is the systems-acquisition community's failure to publicize or offer significant training in VERT. Consequently, program-management personnel are unaware of the technique and its possible applications in the program-development environment. The Army recognized this shortfall and started a comprehensive course of instruction on risk-analysis techniques, primarily oriented toward the RISCA methodology. Now, because of increasing interest and confidence in VERT, the Army Logistics Management Center intends to emphasize it in advanced risk-analysis courses.

Yet another reason VERT is not used more frequently is the problem of limited numbers of personnel and a high rate of personnel turnover in program offices. No agency outside the program office can effectively perform a risk analysis of that program, since only the program office has the necessary data to work with the program manager and has access to him in selecting alternative courses of action. Thus, a risk-analysis team is needed at the product-division staff level to provide the corporate memory necessary to implement a quantitative risk analysis. This team would marry the mechanics of VERT with the data source in the program office.

As the use of VERT increases, knowledge of its applications will grow. Further applications and research are necessary to confirm its validity as a risk-assessment technique. Users need to be encouraged to express their reactions to the technique. These reactions should be analyzed to ascertain the actual benefits being achieved. This investigation could lead to the development of a data bank to determine the degree to which actual program events were substantiated by the model's predictions.

The Venture Evaluation and Review Technique is not necessarily better than any other technique, but it does provide the program manager an accessible tool for integrating cost, schedule, and performance parameters. With VERT, the program manager can add a new dimension to the analysis of program decisions, improving the perspective on alternative courses of action. **DMJ**

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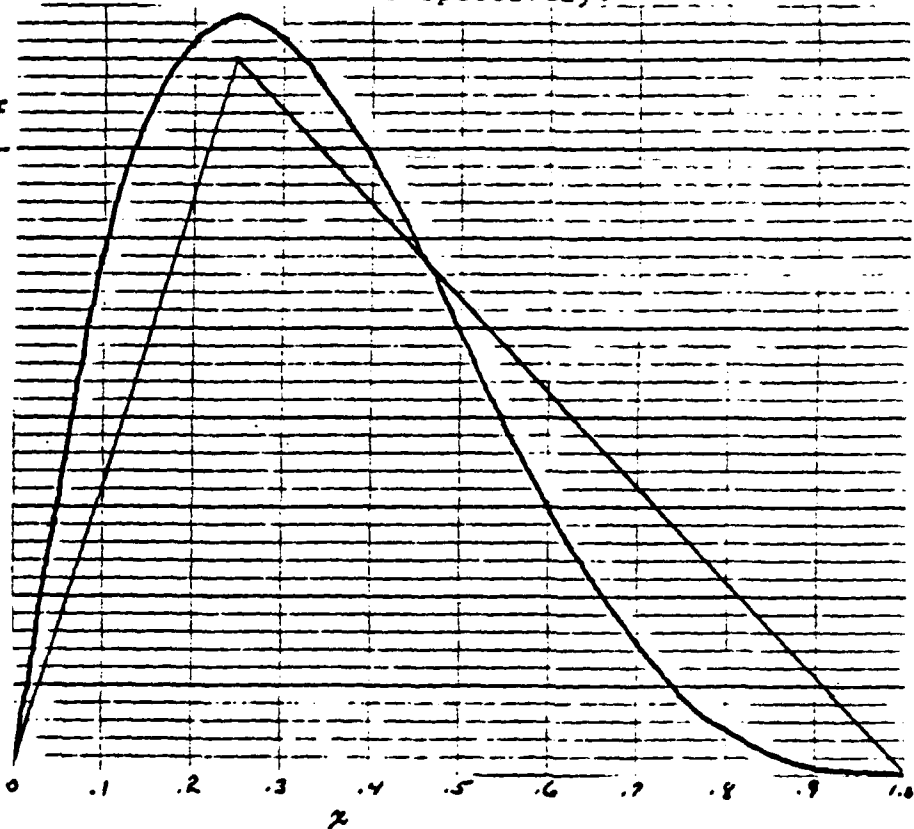
APPENDIX B

Beta Vis-a-Vis Triangular Distributions

BETA VIS-A-VIS TRIANGULAR DISTRIBUTIONS

SPECIFIC EXAMPLE: Compared are the beta type 2 distribution and a triangular distribution, both with the same range and mode. For this case, as shown on the graph and chart below, the triangular distribution has significantly less area in the low range and more in the high range. Also, the expected value or mean of the beta and triangular distributions shown are 0.333 and 0.417 respectively.

x	Cum. Prob Beta	Triangular
.00	.000	.000
.10	.081	.040
.25	.263	.160
.30	.367	.250
.40	.472	.347
.50	.663	.520
.60	.813	.667
.70	.913	.787
.75	.969	.880
.80	.984	.917
.90	.993	.947
.99	.999	.987
1.00	1.000	1.000



GENERAL: The differences described above are for a specific case and will change with different shaped beta distributions. Whereas both distributions include the parameters of range and mode, the beta parameters include a shape parameter which allows greater discretion in describing the uncertainty in an activity. However, under certain conditions, the triangular distribution maybe as accurate as experience will justify.

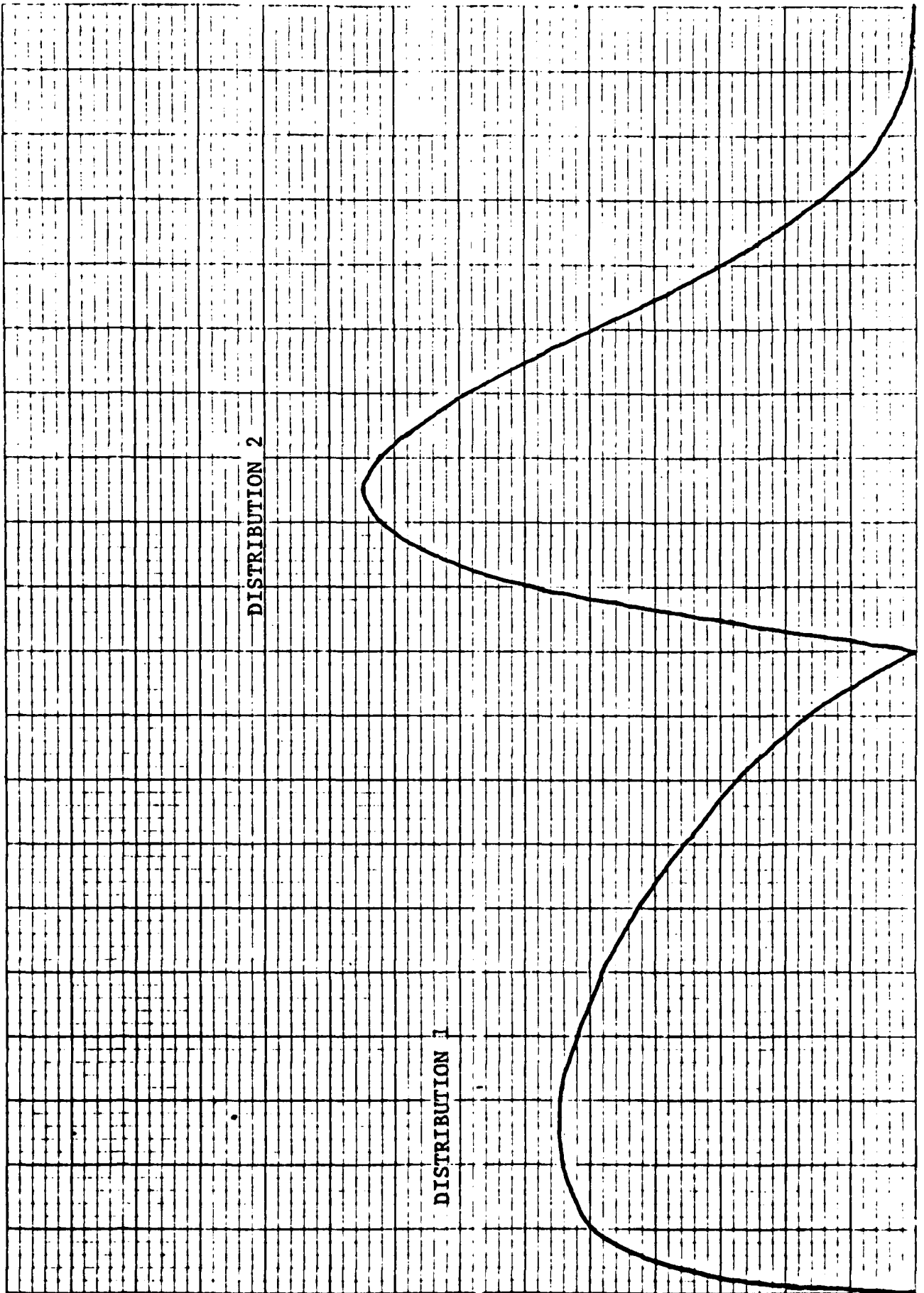
APPENDIX C

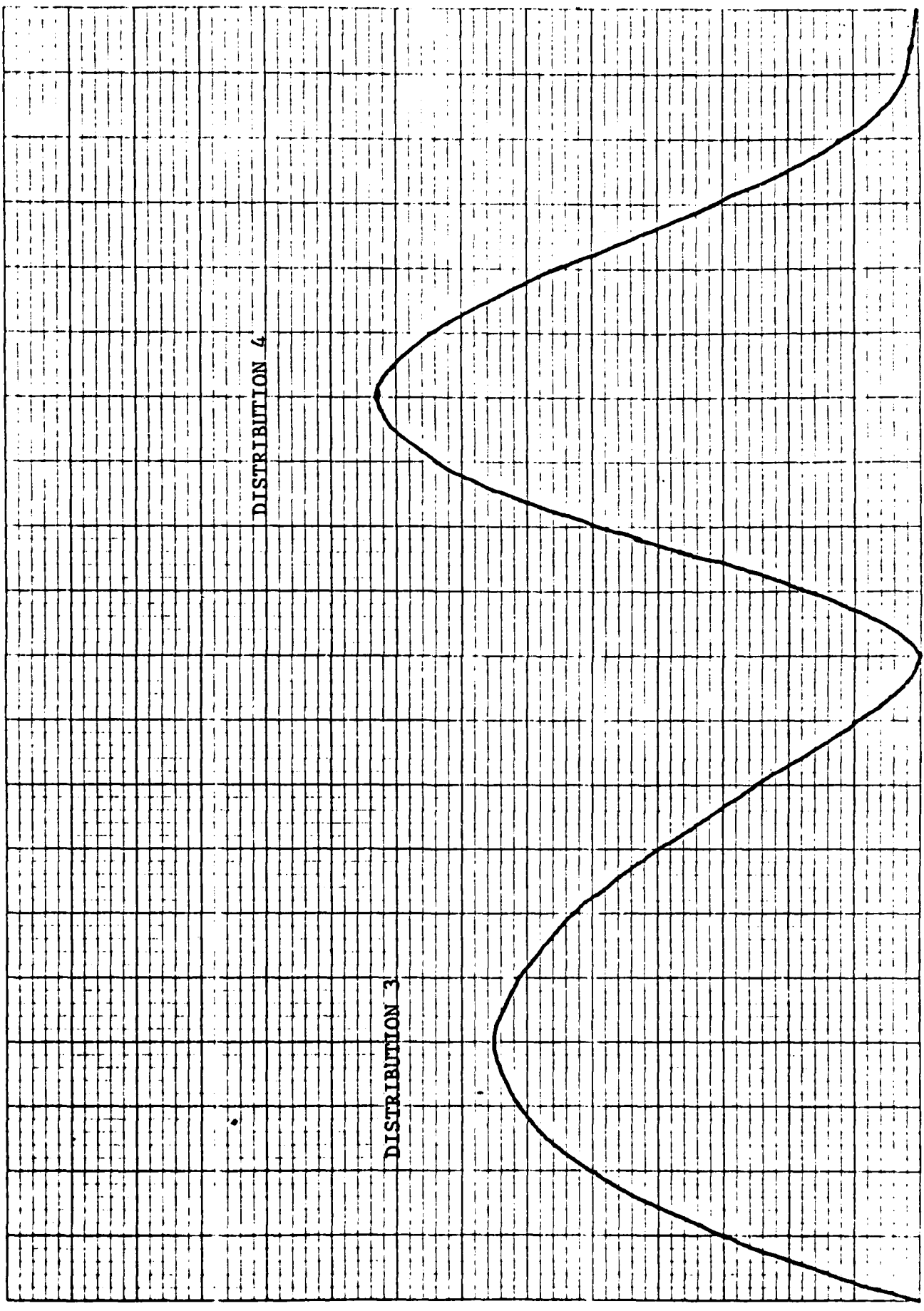
Representative Reta Distributions

Parameters and Data of Beta Distributions

Dist. Code	Alpha	Beta	Mode	Mean	Pr(.10)*	Pr(.25)*	Pr(.75)*	Pr(.90)* Pr(.190*)
1	1.25	1.75	.25	.417	.098	.300	.114	.024
2	2	4	.25	.333	.081	.367	.016	.001
3	2	2.5	.4	.444	.039	.208	.090	.010
4	3	4	.4	.429	.016	.169	.038	.001
5	2	2	.5	.500	.028	.156	.156	.028
6	3	3	.5	.500	.009	.104	.104	.009
7	5	5	.5	.500	.001	.049	.049	.001
8	2.5	2	.6	.556	.010	.090	.208	.039
9	4	3	.6	.571	.001	.038	.169	.016

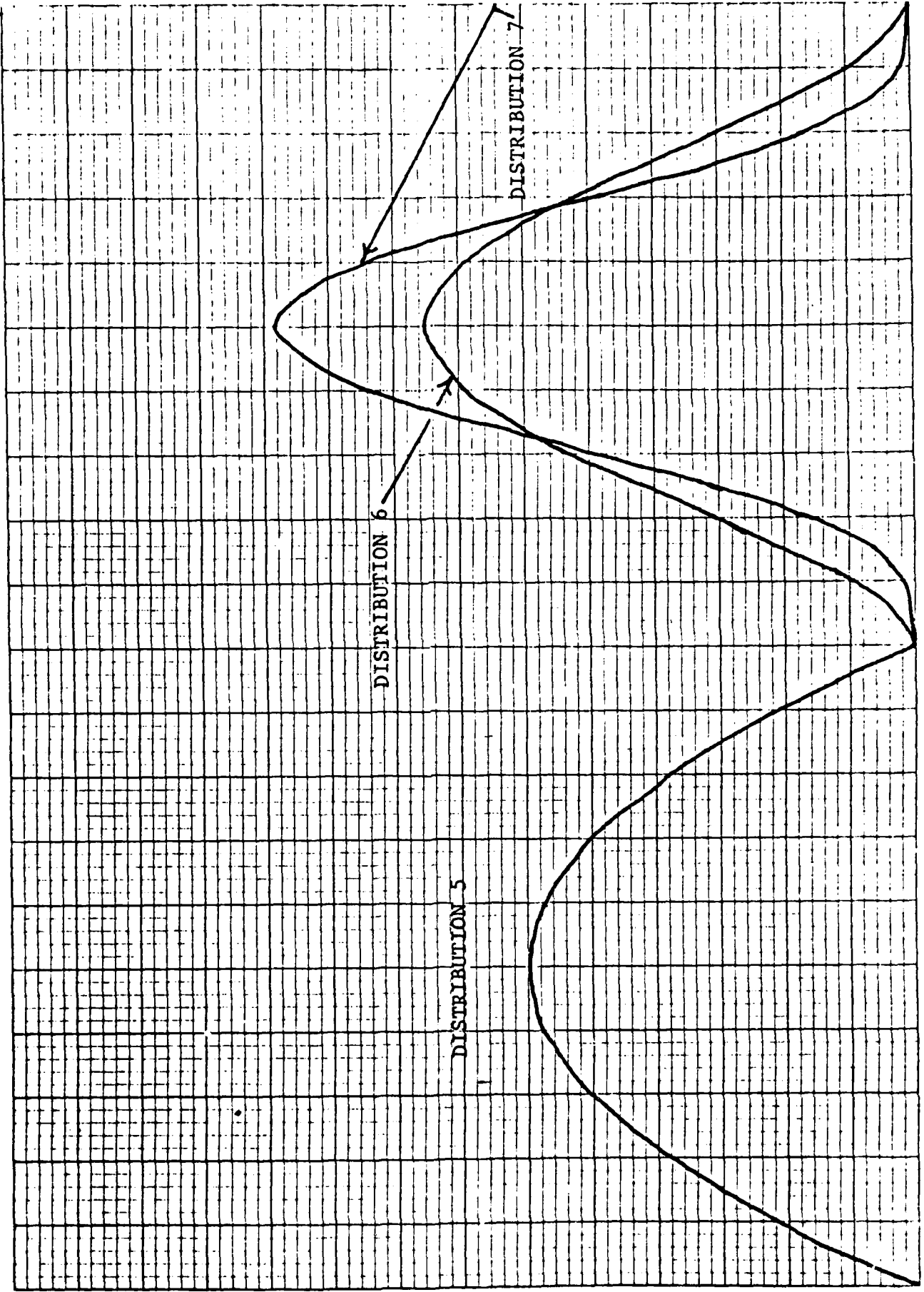
* Denotes the areas in the tails of the distributions from the deciles and quartiles of the range, e.g., Pr(.10) is the probability that the cost or time will be in the lower 10% of the distribution range.





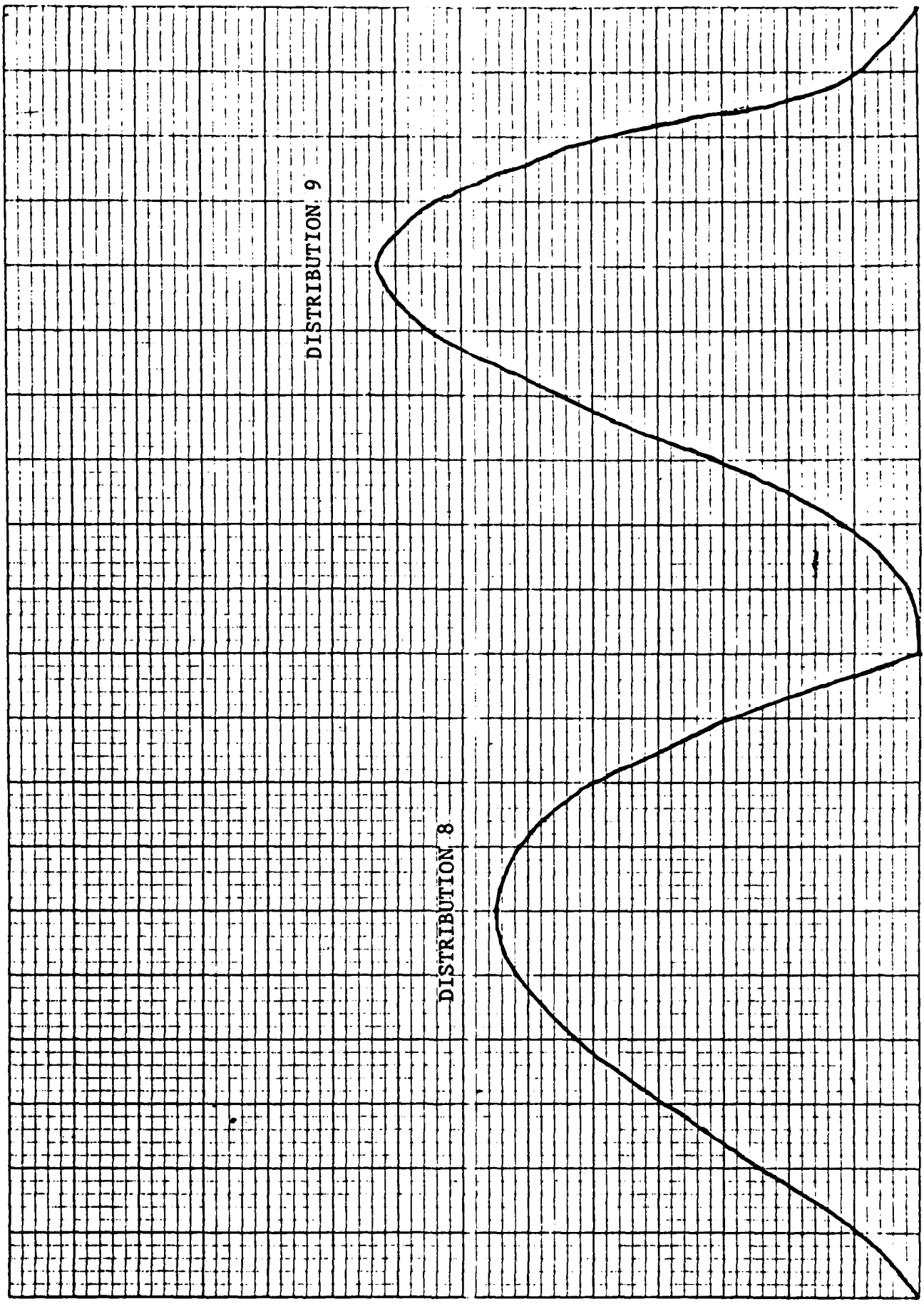
DISTRIBUTION 4

DISTRIBUTION 3



DISTRIBUTION 9

DISTRIBUTION 8



APPENDIX D

Hypothetical Activity Described by a Beta Distribution

HYPOTHETICAL EXAMPLE

Mode vis-a-vis Mean

PROBLEM: How much should you pay the neighbor boy for mowing your yard?

SITUATION: The price is normally a fixed price arrangement. You consider \$2. an hour a fair price and most of the time it takes 2 hours to do the job. Many times there is little rain and the resulting shorter grass can be mowed more quickly. However, if there is some wind, small branches fall on the lawn and the boy must pick them up before he mows. Occasionally, the wind blows down many branches. Extremely rare occurrences are ignored, e.g., extended draught or a tornado. You estimate the job will take from 1.5 to 3.5 hours with the most likely time of 2.0 hours and a distribution shaped like that shown below.

CONCLUSION: Since the distribution is a beta distribution with a/b parameters of 2/4, the average expected time is 2.17 hours or \$4.34 at \$2. per hour.

