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High resolution power spectra of the Zürich sunspot numbers for the period 1849 to 1978 are obtained using the maximum entropy technique of Burg. The monthly means are first smoothed using a digital, least-squares, band-pass filter with 193 weights, and then decimated to obtain a more manageable series. An overall power spectrum, which displays multiple structure and harmonic series is then obtained. In order to explain the complications in the spectrum, a dynamic spectrum is displayed. This is obtained by finding the spectra of data samples 66 years long,

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repeatedly slipped by 2 years for a total of 78 spectra. The spectra are then much simpler and vary smoothly with epoch. The nominal 11-year line varies in period from about 10 to 12½ years. In the process of obtaining a power spectrum a prediction error filter is derived. This linear filter may then be used to make predictions as follows: using sets of data containing 5 solar cycles of unsmoothed monthly values beginning with cycles 9 through 15, predictions of the next 12 months of the time series are made and compared to the observed values. RMS errors vary between 5 and 33 and lead to an expectation that the error of prediction for cycle 21 will be about 20. The entire cycle 21 is then predicted using 50, 100, 150 prediction error coefficients. The maximum of cycle 21 is predicted to be 130 ± 20 at 1980.1 ± 0.2.

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SUNSPOTS: POWER SPECTRA AND A FORECAST

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High resolution power spectra of the Zürich sunspot numbers for the period 1849 to 1978 are obtained using the maximum entropy technique of Burg. The monthly means are first smoothed using a digital, least-squares, band-pass filter with 193 weights, and then decimated to obtain a more manageable series. An overall power spectrum, which displays multiple structure and harmonic series is then obtained.

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Introduction

The subject of this paper is the time series of Zürich sunspot numbers, R_z. The basic data set consists of monthly mean values of R_z beginning in 1749 and running without gaps to October 1978, the latest available value. See Chernosky and Hagan (1958) for the earlier data. There is some smoothing built into the way in which sunspot counts are obtained, because each daily count uses the entire visible hemisphere, which is rotating with a period of 27 days, or a rate of 13 1/3 degrees per day. Thus the daily count is really a 13½-day running mean of those spots and groups which could be observed on a 13 1/3 degree lune centered at central medidian. Despite the low-pass filter, one of the striking features of the series of monthly means of R_z is the large amplitude, high frequency noise which rides

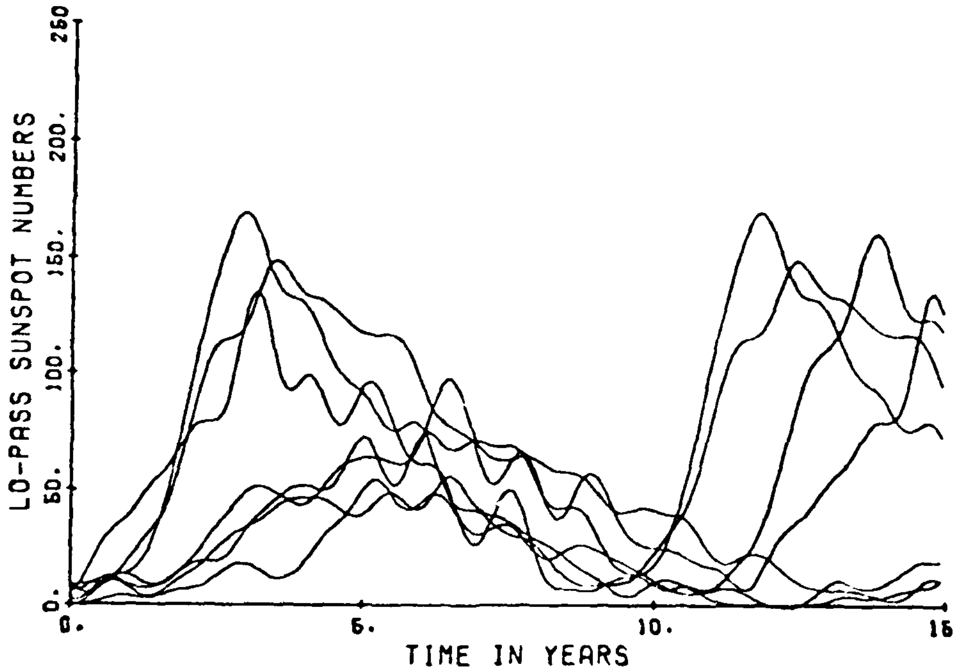

FIGURE 1. SOLAR CYCLES 1-7. DIGITAL LOW-PASS FILTER WITH 13 WEIGHTS HAS BEEN USED. EACH CYCLE BEGINS AT TIME OF SMOOTHED MINIMUM.

on top of the other striking feature, the 11-year period. Accordingly, it has become customary, for example in the monthly Solar Geophysical Data bulletin, to further smooth the monthly averages by using a running mean of 13 monthly values with the 2 extreme months weighted at ½.

So dominant is the appearance of the 11-year cycle that the individual cycles have been assigned numbers beginning with cycle 1 in 1749 to cycle 21, the current cycle, which started in mid-1976. Since the time of minimum can be located with re~~sponsible~~ reasonable accuracy, the cycles begin and end at a minimum. Figure 1 shows the first 7 cycles with the time origin set at the time of minimum. These data have been smoothed using a 13-point digital least-squares, low-pass filter following the method of Behannon and Ness (1966). This simple filter has a more nearly accurate low-pass response than that of the 13-point running mean.

Despite the use of the filter, the extreme variability of the data is evident. All of the cycles are bumpy; cycle length, from minimum to minimum, varies between 9 and 15 years; time from minimum to maximum varies from about 2½ to 6 years, and the size of the maximum varies by a factor of 4. These early 7 cycles have been criticized in the classical paper on sun spot prediction by McNish and Lincoln (1949). According to that paper the early data are considerably less reliable and even belong to a different statistical population from the modern data beginning in 1834.

Cycles 8 through 20, the modern data, are shown in Figure 2. For these data, the length of the cycle varies less, between 10 and 12½ years. The

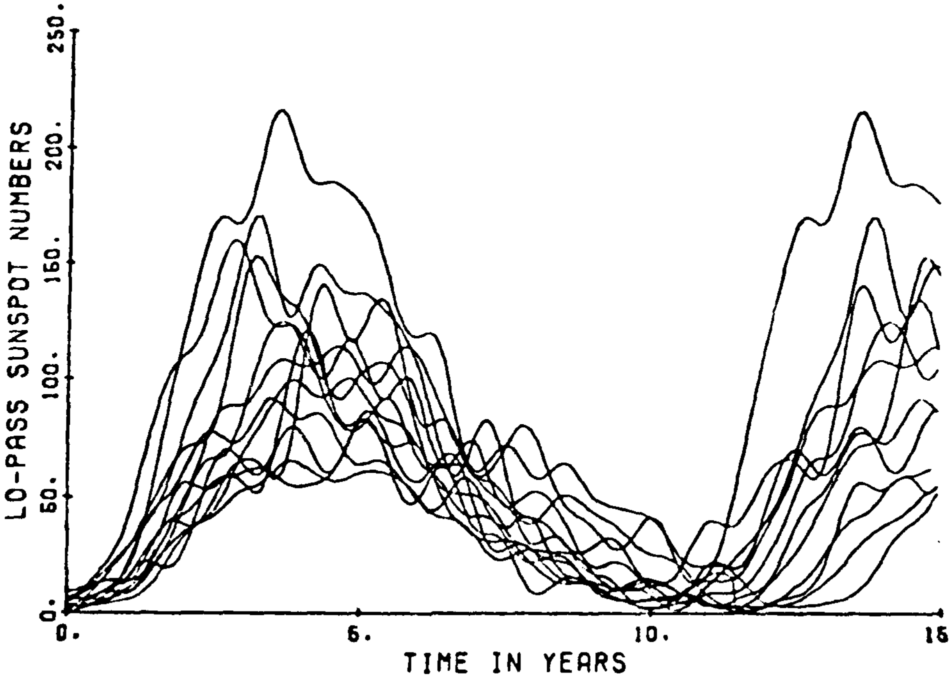

FIGURE 2. SOLAR CYCLES 8-20. DIGITAL LOW-PASS FILTER WITH 13 WEIGHTS HAS BEEN USED. EACH CYCLE BEGINS AT TIME OF SMOOTHED MINIMUM.

time to maximum is now about 2½ to 4½ years, but the height of the maximum still varies by a factor of 4: from about 50 to more than 200.

The extreme variability of these data is further illustrated in Figure 3, showing a 3-dimensional representation of all 20 cycles. Cycle 19, the largest ever, which began in 1954, is so large that it completely hides cycle 20.

Power Spectra

Now we would like to determine the power spectrum of this time series using the maximum entropy method (Burg, 1975) which works admirably on rather short-term series. The original 2758 monthly means constitute a rather long series. We can easily remedy this by decimation but only after smoothing to prevent aliasing. Accordingly, a low-pass filter with a cutoff period of 2½ years was designed. At the same time, a high-pass filter was used to remove the mean and very long time trends. The response of the resulting band-pass filter is shown in Figure 4. The digital least-squares filter used 193 weights to achieve this response. Notice the filter beginning to cut off at about 2½ years, as designed, so that the response at 2 years and shorter times (high frequencies) is close to zero. Thus we can sample the output safely once a year and introduce no aliasing because all frequencies higher than the new Nyquist frequency of 0.5 cycles per year have been removed.

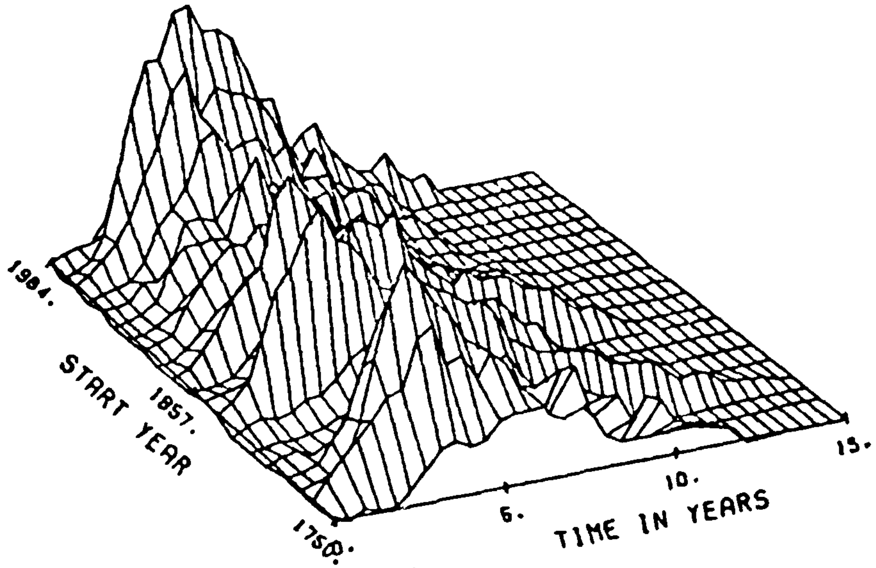

FIGURE 3. PERSPECTIVE PLOT OF CYCLES 1-20.

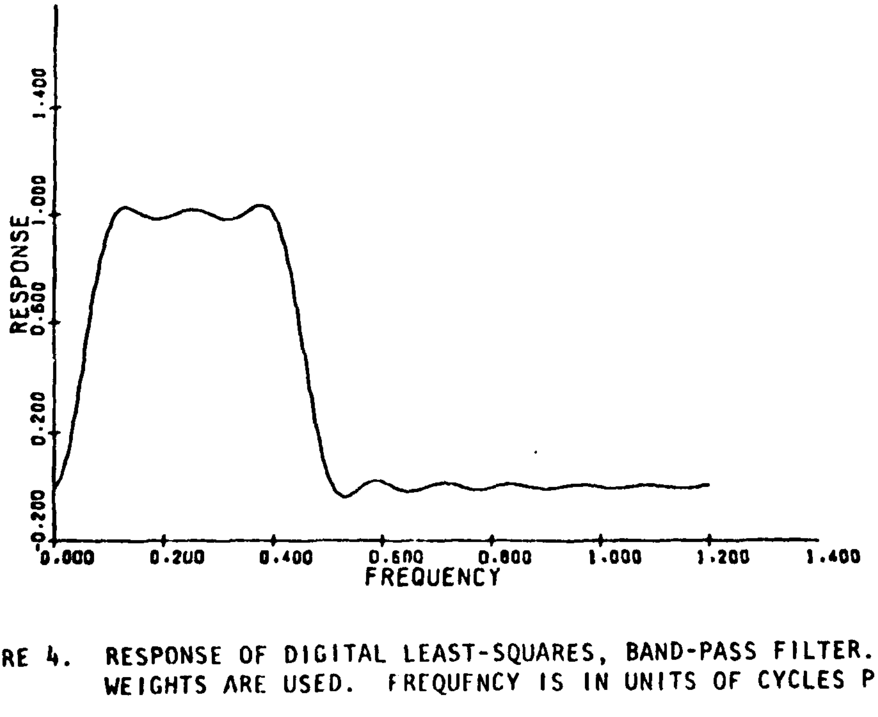

FIGURE 4. RESPONSE OF DIGITAL LEAST-SQUARES, BAND-PASS FILTER. 193 WEIGHTS ARE USED. FREQUENCY IS IN UNITS OF CYCLES PER YEAR.

The results are shown in Figure 5. The original unsmoothed, monthly means are given in the bottom panel with values running from 0 to 250 and the dates running from the year 1750 to the year 2000. The band-pass filtered output is shown in the top panel, and the differences between the original and band-pass data, the residuals, are given in the middle panel. Thus the original data set in the bottom panel is the sum of the top 2 panels, month by month. The top panel shows a smooth, zero-mean time series which is still not stationary because the amplitudes are quite variable. The residual series, in the central panel, contains essentially all of the high-frequency noise plus the slowly-varying mean or DC level. It is now quite safe to decimate the band-pass filtered series by 12 to obtain 211 yearly values.

The maximum entropy method of Burg (1975) was applied to these 211 numbers and the number of prediction error filter weights, analogous to the number of lags in a Blackman-Tukey spectrum, was varied from 4 to 130. The 3-dimensional representation of these spectra is shown in Figure 6. The nominal 11-year line, which carries most of the power, splits into two very stable lines with periods of 10.9 and 9.93 years. There is also quite clear evidence of power peaking at about 12 years and again at about 8½ years. The second harmonic of the 11-year doublet is also a doublet at about 5.5 and 4.8 years. The richness of structure in the spectrum is simply another manifestation of the extreme variability of the time series.

The complicated spectrum also can be explained by taking small overlapping segments of data, performing spectral analysis on each and watching the spectrum change with time. Such a dynamical spectrum is shown in Figure 7. Each spectrum is based on a segment of data 66 years long.

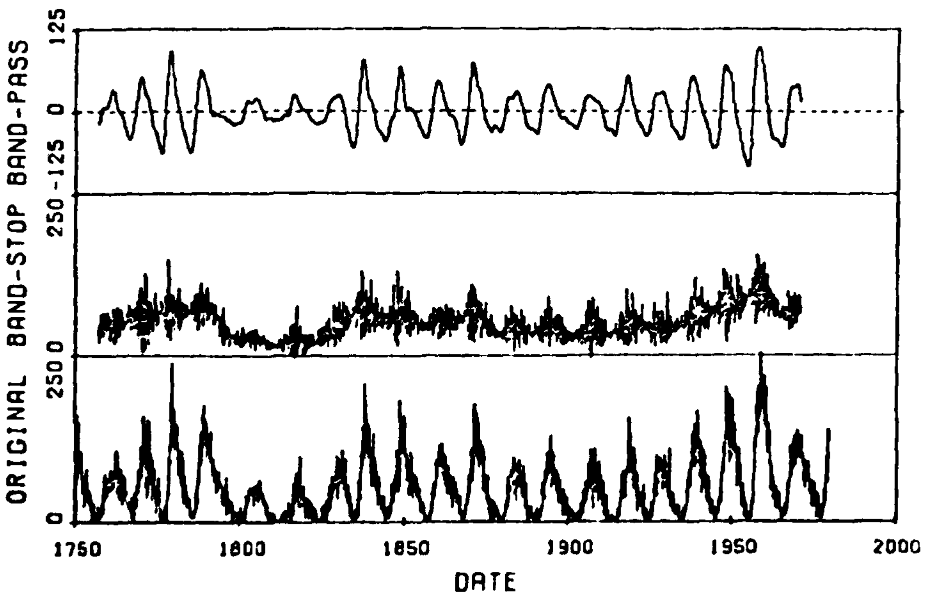

FIGURE 5. SUNSPOT NUMBERS VERSUS DATA FROM 1750 TO PRESENT (OCT. 1978).
TOP PANEL: BAND-PASS FILTERED DATA (193 WEIGHTS).
MIDDLE PANEL: ORIGINAL MINUS BAND-PASS.
BOTTOM PANEL: ORIGINAL, UNSMOOTHED MONTHLY MEANS.

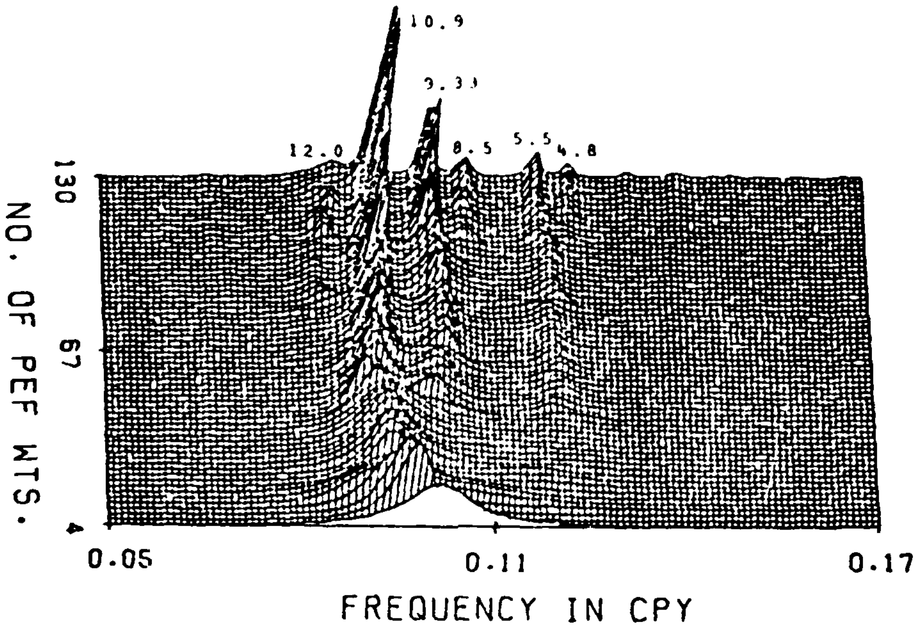

FIGURE 6. PERSPECTIVE PLOT OF MAXIMUM ENTROPY SPECTRA OF BAND-PASS FILTERED DATA SAMPLED ONCE PER YEAR. THE NUMBER OF PREDICTION ERROR FILTER (PEF) WEIGHTS RANGES BETWEEN 4 AND 130 IN STEPS OF 2. PERIODS IN YEARS ARE GIVEN AT APPROPRIATE PEAKS.

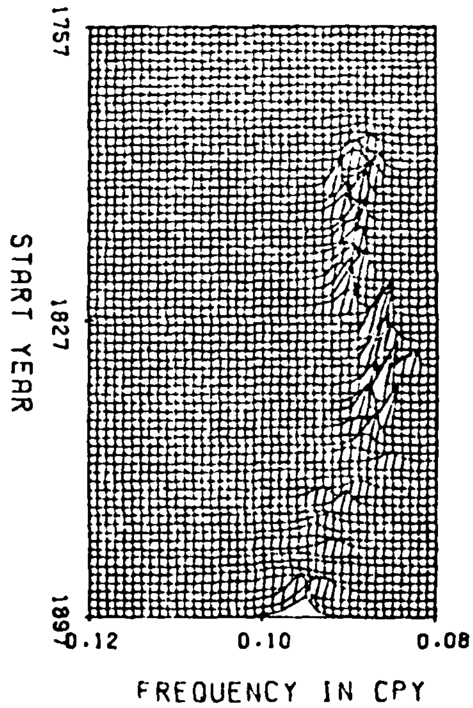

FIGURE 7. DYNAMIC MAXIMUM ENTROPY SPECTRUM OF 66 YEARS (APPROXIMATELY 6 CYCLES) OF BAND-PASS FILTERED DATA SAMPLED ONCE PER YEAR. THE START YEAR ADVANCES BY 2 YEARS FOR EACH SPECTRUM SHOWN.

The segments are overlapped by 64 points, so that the start year moves by 2 years for each new segment. We are now concentrating on a narrow band of periods between 8 and 12½ years. The peaks in the earliest data are too small to be seen in these linear power spectral density plots. The nominal 11-year line shows an instantaneous period varying between 10 and 12½ years while the power spectral density varies by over two orders of magnitude.

Predictions

At the heart of the maximum entropy method is the determination of a prediction error filter. This is a linear filter whose scaler product with a segment of data yields the error in a one-step-ahead prediction. We find the filter essentially by minimizing the sum of squares of such prediction errors over the set of observations. Once we have such a filter we can use it to make predictions off both ends of the series as far as we wish to go simply by applying the filter and moving over the data and the newly acquired predictions.

The predictions of the sunspot numbers were tested as follows. Beginning in 1843, 5 cycles of original unsmoothed monthly values, containing 708 monthly values, constitute a data set for which spectra are obtained for a number of prediction-error filter-weights ranging from 50 to 300 in steps of 50. For each filter, predictions were made for the following 12 months and the RMS prediction error was determined. This is possible because predictions and observed values which had not been used to obtain the predictions are available. The entire procedure was repeated seven times, using 5 cycles of monthly data beginning with cycles 9, 10, 11 through cycle 15, and making predictions into cycles 14 through 20 respectively.

The mean square prediction errors, all quite reasonable numbers from a low of 5 to a high of 33, are shown in Figure 8. Here are plotted the predicted cycle numbers from 14 through 21 and the RMS prediction errors for 12 predictions running from 0 through 40. Curiously, the RMS error seems to alternate in magnitude, being low for even cycles, and appreciably higher for odd-numbered cycles. Accordingly a guess is made that the RMS prediction error for the real predictions for cycle 21 will be around 20.

Predictions were made using 50 to 300 weights for the entire cycle 21 and plotted. The most consistent behavior for the entire cycle comes from sets with 100, 150, and 200 weights and these are shown in Figure 9. Smoothing these results, the predictions of the maximum and its date become:

$$\text{MAX}\ (R_z) = 130 \pm 20$$

$$\text{DATE} = 1980.1 \pm 0.2$$

The entire cycle seems quite reasonable with a minimum around 1986.2.

In summary, I have shown some pictures of the sunspot cycles and discussed their extreme variability. I have filtered these data with a digital band-pass filter, and decimated the resulting series. I have shown maximum entropy power spectra varying in a stable manner as the number of weights is varied and also a dynamical spectrum showing that the

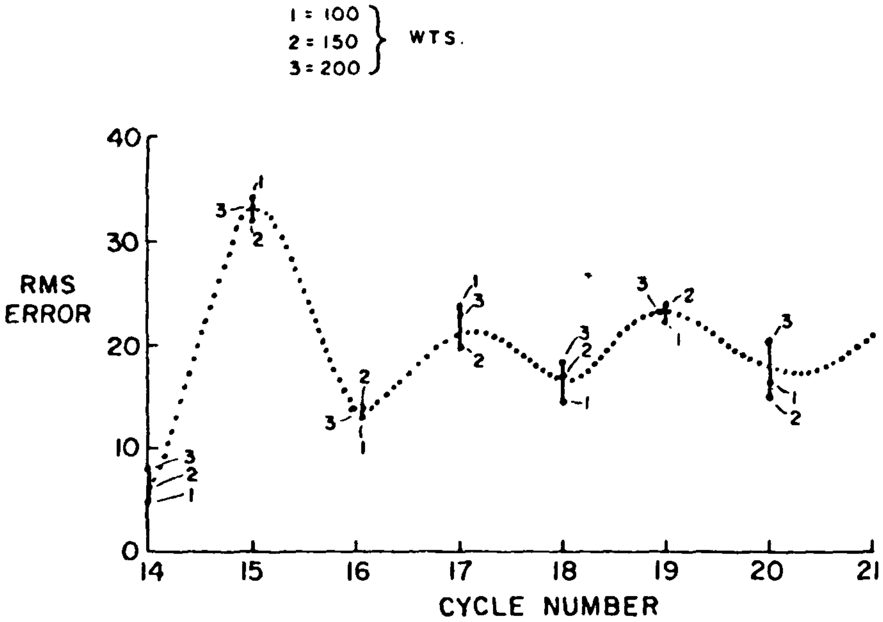

FIGURE 8. RMS ERRORS, FOR 100, 150 AND 200 WEIGHTS. FIVE SOLAR CYCLES OF ORIGINAL UNSMOOTHED MONTHLY MEAN ARE USED AS INPUT TO THE MAXIMUM ENTROPY POWER SPECTRAL ANALYSIS PROGRAM. NUMBER OF OBSERVATIONS WAS 708, 708, 684, 684, 660, 648 AND 624 RESPECTIVELY.

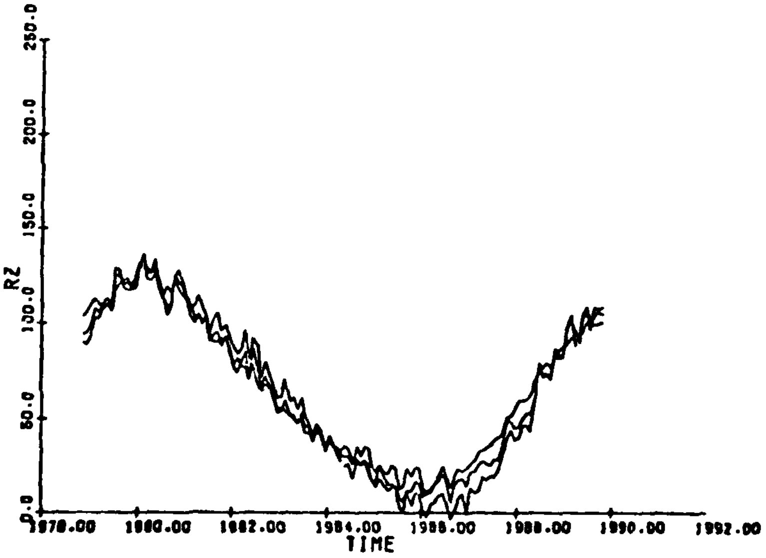

FIGURE 9. PREDICTION OF UNSMOOTHED SUNSPOT NUMBERS MADE USING THREE DIFFERENT SETS OF PREDICTION ERROR FILTERS WITH 100, 150 AND 200 WEIGHTS. DATA USED ARE 670 ORIGINAL UNSMOOTHED MONTHLY MEANS STARTING IN JANUARY, 1923.

nominal 11-year line varies in period from 10 to 12½ years. I have tested a prediction technique with seven separate data sets, each 5 cycles long, to find an expected RMS prediction error of about ± 20. Finally I have made predictions for the remainder of cycle 21 which should have a maximum of 130 ± 20 at 1980.1 ± 0.2.

References

Behannon, K.W. and N.F. Ness, (1966), The Design of Numerical Filters for Geomagnetic Data Analyses, NASA TN D-3341.

Burg, J.P., (1975), Maximum Entropy Spectral Analysis, Ph.D. Thesis, Stanford University.

Chernosky, E.J. and M.P. Hagan, (1958), The Zürich Sunspot Number and Its Variations for 1700-1957, _J. Geophys. Res._, _63_, (4).

McNish, A.G. and J.V. Lincoln, (1949), Prediction of Sunspot Numbers, _Trans. AGU_, _30_, 673-685.

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