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GOAL AND AVERAGE COST PROBLEMS IN DECISION PROCESSES WITH FINIT--ETC(U)
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I. Research Objectives

The objective of the supported research was a continuation of the principal investigator's analysis of decision processes with arbitrary decision sets, with special emphasis on two classical payoff functions. The first was a process with single fixed goal in which the objective is to maximize the probability of reaching the goal, and to minimize the expected time to the goal. A specific objective was to determine whether one can do as well with stationary strategies as he can with strategies which take the whole past into account. The second object of study was the average reward payoff in finite state decision processes, a specific objective being to determine whether or not strategies based only on the current time and state are as good as those which take the whole past into account.

II. Status of the Research

A complete, affirmative answer was found to the first question; in every dynamic programming or gambling problem with single fixed goal and finite state space, there exists a stationary strategy which not only uniformly (nearly) maximizes the probability of reaching the goal, but also uniformly (nearly) minimizes the expected time to the goal. Techniques in the proof of this result were considerably generalized to answer several questions about decision processes with arbitrary state spaces and total-cost criteria. It was shown that in a countable state decision process with non-negative costs depending on the current state, the action taken, and the following state, there is always available a Markov strategy which uniformly (nearly) minimizes the expected total cost. If the costs are strictly positive and depend only on the current state, there is even a stationary strategy with the same property.

Investigations of these results led peripherally to several results in optimal stopping theory, and in classical probability theory. Universal, best possible constants were found which compared the optimal expected return of a decision maker with the expected supremum of a sequence of random variables. For example, it was found that for every sequence of independent random variables taking only values between zero and one, the difference between the optimal stop rule expectation and the expected value of the supremum of the random variables is no more than one-fourth. Results in classical probability stemming from this research include a stronger form of the Borel-Cantelli Lemma, and a very general conditioning principle for strong laws which conclude the partial sums converge almost surely.

For the question of existence of good Markov strategies in decision processes with average reward criteria, various partial results have been obtained, including examples showing that the limit of good strategies for the discounted reward payoff is not necessarily average-reward good. This research is still in progress and it is hoped that a complete answer to the finite state case is not far away.

III. List of Publications

1. "On Reaching a Goal Quickly," Technical Report.

2. "Ratio Comparisons of Supremum and Stop Rule Expectations," (with R. Kertz), submitted to Z. Wahrscheinlichkeitstheorie.

3. "Decision Processes with Total-Cost Criteria," (with S. Demko), accepted for publication in Annals of Probability.

4. "Additive comparisons of Stop Rule and Supremum Expectations of Uniformly Bounded Independent Random Variables," (with R. Kertz), submitted to Proceedings of the A.M.S.

5. "A Stronger Form of the Borel-Cantelli Lemma," submitted to Pacific Journal of Mathematics.

6. "Comparisons of Stop Rule and Supremum Expectations of i.i.d. Random Variables," (with R. Kertz), submitted to Annals of Probability.

7. "Conditional Generalizations of Strong Laws Which Conclude the Partial Sums Coverage Almost Surely," submitted to Annals of Probability.

IV. Spoken Papers

A. Presented

1. "Betting Against a Prophet," seminar in Mathematics Department at University of California at Berkeley, August 1979.

2. "A Stronger Form of the Borel-Cantelli Lemma," seminar in the Mathematics Department at University of California at Berkeley, September 1979.

3. "On the Existence of Good Markov Strategies," Colloquium Presentation, Statistics Department, University of California at Berkeley, October 1979.

4. "Markov Decision Processes, Gambling, and Dynamic Programming," Colloquium Presentation, Mathematics Department, University of Hawaii, April 1980.

5. "Conditional Generalizations of Strong Laws," seminar in the Mathematics Department, University of California at Berkeley, June 1980.

6. ________________________________, Probability Seminar, Mathematics Department, Georgia Institute of Technology, October 1980.

7. "A Stronger Form of the Borel-Cantelli Lemma," Probability Seminar, Mathematics Department, Georgia Institute of Technology, November 1980.

B. Scheduled

1. "Conditional Generalizations of Strong Laws," Annual American Mathematical Society Meeting, San Francisco, January 1981.

2. "Finite State Decision Processes," Operations Research Seminar, Georgia Institute of Technology, February 1981.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 80-1342	2. GOVT ACCESSION NO. AD-A093 876	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) GOAL AND AVERAGE COST PROBLEMS IN DECISION PROCESSES WITH FINITE STATE SPACES		5. TYPE OF REPORT & PERIOD COVERED Final rept.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Theodore P. Hill		8. CONTRACT OR GRANT NUMBER(s) F49620-79-C-0123
9. PERFORMING ORGANIZATION NAME AND ADDRESS ~~Georgia Tech Research Institute~~ School of Mathematics, Georgia Inst. of Technology Atlanta, Georgia 30332		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A3
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research Air Force Systems Command, USAF Bolling A.F.B., Washington, D. C. 20332		12. REPORT DATE 1980
		13. NUMBER OF PAGES 3
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Decision processes, gambling theory, dynamic programming, Markov processes, stationary strategy

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The principal investigator carried out research in probability theory, and, in particular, in the theory of decision processes. First, it was shown that in every finite state decision process (gambling problem) with single fixed goal, there always exsits a stationary strategy which not only (nearly) maximizes the probability of reaching the goal, but (nearly) minimizes the expected time to the goal. This result was considerably generalized to include decision processes with arbitrary state spaces and total cost criteria.

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Investigations of these processes led to results in optimal stopping theory, and in classical probability theory. Universal, best possible constants were found which compared the optimal expected return of a decision maker with the expected supremum of a sequence of independent random variables. A generalization of the classical Borel-Cantelli Lemma was found, as was a very general conditioning principle for strong laws of several forms.

The question of existence of good Markov strategies in finite state decision processes with average reward criteria was addressed, and various partial results were obtained, although the general case was not settled.