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ASYMPTOTIC BEHAVIOR OF SOME NONLINEAR HEAT EQUATIONS.(U)
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ASYMPTOTIC BEHAVIOR OF SOME NONLINEAR HEAT EQUATIONS

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ABSTRACT

In this paper a semilinear heat equation with a convex nonlinearity is considered. The asymptotic behavior of the solutions is completely determined and this gives, in particular, a very precise description of the global stability of stationary solutions.

AMS(MOS) Subject Classifications: 35K55, 35P30

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SIGNIFICANCE AND EXPLANATION

Semilinear heat equations (that is heat equations perturbed by a nonlinearity just acting on the solution but not on its derivatives) occur in many applications: for example in combustion theory, or in population genetics ... One of the main problems concerning this type of problem is to determine the asymptotic behavior of solutions (when the time $t \to \infty$). In this paper, assuming that the nonlinearity is convex, a complete description of the asymptotic behavior of solutions is given including in particular a precise determination of the global stability of steady state solutions.

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ASYMPTOTIC BEHAVIOR OF SOME NONLINEAR HEAT EQUATIONS

P. L. Lions(*)

Introduction:

The goal of this paper is to give a complete description of the asymptotic behavior of the solution $u(t,x)$ as $t \to \infty$ of the following nonlinear heat equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u = f(u) & \text{in } (0,\infty) \times \mathcal{O} \\ u(t,x) = 0 \text{ on } \partial\mathcal{O} \ , & u(0,x) = u_0(x) \ ; \end{cases}$$

where f is some convex nonlinearity, and $\mathcal{O}$ is a bounded, regular and connected domain in $\mathbb{R}^N$.

To illustrate our result let us consider the following equation

$$(1) \quad \begin{cases} \dfrac{\partial u}{\partial t} - \Delta u = u^2 & \text{in } (0,\infty) \times \mathcal{O} \\ u(t,x) = 0 \text{ on } \partial\mathcal{O} \ , & u(0,x) = u_0(x) \ ; \end{cases}$$

we denote by K the set of initial data $u_0(x)$ on $W_0^{1,\infty}(\mathcal{O})$ $(= \{v \in W^{1,\infty}(\mathcal{O}),$ $v = 0$ on $\partial\mathcal{O}\})$ such that the solution $u(t,x)$ of (1) exists for all $t \geq 0$ and remains bounded uniformly in $t \geq 0$.

Then we prove

i) K is an unbounded, convex set and $0 \in \overset{\circ}{K}$,

ii) If u is a non-trivial stationary solution i.e. if u satisfies:

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(2) $\quad -\Delta u = u^2$ in $\mathcal{O}$, $u \in C^2(\bar{\mathcal{O}})$, $u = 0$ on $\partial\mathcal{O}$, $u \not\equiv 0$;

then u is an extremal point of K,

iii) If u_0 is in K without being an extremal point of K, then

$$u(t,x) \in \overset{\circ}{K} \text{ for all } t > 0 \text{ and } u(t,x) \underset{t\to\infty}{\longrightarrow} 0 .$$

Other examples are given after the general statement of Theorem II.1 (in particular the case where u^2 is replaced by λe^u).

Obviously this result shows that every non-trivial solution of (2) is highly unstable (in the context of (1)): remark the fact that u is unstable (in the linearized sense at least) is probably well-known(*); but we give here a very precise picture of that instability. In particular the result above shows that generically (with respect to u_0), $u(t,x)$ does not converge to any solution of (2).

Section I is devoted to our main result, while in section II we give some extensions and some variants of our results.

Let us finally indicate that we do not consider here the existence problem of solutions of (2) (or related problems); for these we refer to P. H. Rabinowitz [17]; A. Ambrosetti and P. H. Rabinowitz [1]; H. Brezis and R. E. L. Turner [8]; D. G. De Figueiredo, R. D. Nussbaum and P. L. Lions [11]; H. Berestycki and P. L. Lions [6].

(*) We did not find a precise reference for that, but it is somewhat straightforward to prove.

I Main results.

I.1: Notations and assumptions.

Let $\mathcal{O}$ be a bounded, regular, connected domain in $\mathbb{R}^N$. Let f be a C^2 function from $\mathbb{R}$ into $\mathbb{R}$ satisfying

(3) $\qquad f$ is strictly convex, and $f'(0) < \lambda_1$, $f(0) = 0$,

where λ_1 is the first eigenvalue of $-\Delta$ in $\mathcal{O}$, with Dirichlet boundary conditions.

We will consider the *following nonlinear heat equation:*

$$(4) \quad \begin{cases} \dfrac{\partial u}{\partial t} - \Delta u = f(u) \quad t \geq 0 \quad , \quad x \in \mathcal{O} \\ u(t,x) = 0 \quad \text{on} \quad \partial\mathcal{O} \quad \text{for} \quad t \geq 0 \quad , \quad u(t,0) = u_0(x) \quad \text{in} \quad \bar{\mathcal{O}} \quad ; \end{cases}$$

where u_0 is some given function in $X = W_0^{1,\infty}(\mathcal{O})$.

It is well-known that for any u_0 there exists a unique local solution to (4) (that is for $t \in [0,t_{max})$ and t_{max} depends on u_0) and $u(t,x)$ $C^{2,1,\alpha}(\bar{\mathcal{O}} \times [0,T])$ (for any $T < t_{max}$ and for any $\alpha < 1$).

On the other hand $u(t,x)$ may not exist for all $t \geq 0$ since there may be blow-up in finite time (see for example J. M. Ball [3]). Thus, of particular interest for the asymptotic behavior of $u(t,x)$ is the following set of initial conditions:

$$K = \{u_0 \in X, \text{ such that there exists a unique solution of (4) for all } t \geq 0 \text{ and } |u(t,x)| \leq C_{u_0} \text{ (indep. of } t \geq 0 \text{ and } x \text{ in } \bar{\mathcal{O}})\} .$$

Then, we have

Theorem I.1: Under assumption (3), we have

i) K is convex, unbounded; $0 \in \overset{\circ}{K}$; if $u_0 \in K$, then for all $v \leq u_0$, $v \in K$; in addition, if we denote by $S(v) = \int_{\mathcal{O}} \frac{1}{2} |\nabla v|^2 \, dx - \int_{\mathcal{O}} F(v) dx$ where $F(t) = \int_0^t f(s) ds$, then we have

$$S(v) > 0, \text{ for all } v \text{ in } K - \{0\} ;$$

ii) If u is a non-trivial stationary solution of (4) i.e. if u satisfies

(5) $\quad -\Delta u = f(u)$ in $\mathcal{O}$, $u = 0$ on $\partial\mathcal{O}$, $u \in C^2(\bar{\mathcal{O}})$, $u \not\equiv 0$;

then $u(x) > 0$ for x in $\mathcal{O}$ and u is an extremal point of K.

iii) If $u_0 \in K$ and if u_0 is not an extremal point of K then the corresponding solution $u(t,x)$ of (4) belongs to $\overset{\circ}{K}$, for all $t > 0$.

iv) Moreover if $u_0 \in \overset{\circ}{K}$, then $u(t,x) \xrightarrow[t\to\infty]{} 0$ (in $C^2(\bar{\mathcal{O}})$).

Remark I.1: The assumption of convexity for f is essential (except for some arguments of the proof of (iv)), and we will explain in section II what happens if we no longer assume $f(0) = 0$ or $f'(0) < \lambda_1$.

Remark I.2: This result shows that the only way to approach a non-trivial solution of (5) via the evolution problem (4) is to start with u_0 being an extremal point of K and to stay for all $t \geq 0$ in the set of extremal points of K. In particular generically (with respect to u_0 in X) $u(t,x)$ does not converge to a solution of (5): indeed $\overset{\circ}{K} \cup (X - \bar{K})$ is a dense open set of X on which $u(t,x)$ either goes to 0, or is unbounded.

Remark I.3: We may extend the above result, by replacing $-\Delta$ by a more general second-order elliptic operator and the Dirichlet boundary conditions by other types of boundary conditions. Finally one can allow f to depend also on x; but we will not consider such obvious extensions.

While the proof of statements i) - iii) is fairly easy, the proof of iv) will involve some technicalities. In I.2 below, we prove i) - iii); and in I.3 some preliminary results are proved; finally in I.4 we prove iv).

I.2: <u>Geometrical properties of</u> K:

For u_0 in K, we will denote by $S(t)u_0 = u(t,x)$ the solution of (4).

<u>Proof of i)</u>:

Let u_0, v_0 be in K and let $0 < \theta < 1$, since f is convex one has

$$\frac{d}{dt}(\theta S(t)u_0 + (1-\theta)S(t)v_0) - \Delta(\theta S(t)u_0 + (1-\theta)S(t)v_0) =$$

$$= \theta f(S(t)u_0) + (1-\theta)f(S(t)v_0) \geq f(\theta S(t)u_0 + (1-\theta)S(t)v_0))$$

thus if $w(t,x)$ is the maximal solution of (4) with $\theta u_0 + (1-\theta)v_0$ as initial data, one has by well-known comparison theorems $w(t,x) \leq \theta S(t)u_0 + (1-\theta)S(v)v_0 \leq C$ for all x in $\mathcal{O}$ and $t \leq t_{max}$.

Now since $f'(0) < \lambda_1$, we have

$$\frac{dw}{dt} - \Delta w = f(w) \geq f'(0)w$$

and this implies $w(t,x) \geq -C$. And this proves that K is convex.

To prove that if $u_0 \in K$ and if $v \leq u_0$ then $v \in K$, one just needs to remark that by the above proof for all v one has a bound from below for the solution $v(t,x)$ of (4) with initial data v, while if $v \leq u_0$ with $u_0 \in K$ then

$$v(t,x) \leq S(t)u_0 \quad ,$$

applying again comparison results.

Now let us prove that $0 \in \overset{\circ}{K}$, indeed if v_1 satisfies:

$$-\Delta v_1 = \lambda_1 v_1 \text{ in } \mathcal{O} \ , \ v_1 \in C^2(\bar{\mathcal{O}}) \ , \ v_1 > 0 \text{ in } \mathcal{O} \ , \ v_1 = 0 \text{ on } \partial\mathcal{O} \ ,$$

for ε small enough, we deduce

$$-\Delta\varepsilon v_1 = \lambda_1(\varepsilon v_1) \geq f'(\varepsilon v_1)\varepsilon v_1 \geq f(\varepsilon v_1) \quad .$$

Thus, $S(t)(\varepsilon v_1) \geq \varepsilon v_1$ for all $t \geq 0$ and $\varepsilon v_1 \in K$.

Applying what we proved above, we get that

$$I = \{w \in X \ , \ w \leq \varepsilon v_1\} \subset K \quad .$$

And this set I is a neighborhood (in X) of 0 (since $v_1(x) > 0$ in $\mathcal{O}$ and by *Hopf maximum principle* $\frac{\partial v_1}{\partial n} \leq -\alpha < 0$ *on* $\partial\mathcal{O}$, *where* n *is the unit* outward normal to $\partial\mathcal{O}$).

To prove that $S(v) > 0$, for all v in $K - \{0\}$; we first show that $S(v) \geq 0$ for v in $\bar{K}$. Indeed if we admit iv) and if we remark that $S(u(t,x))$ is nonincreasing (multiply (4) by $\frac{\partial u}{\partial t}$), then for all u_0 in $\overset{\circ}{K}$ we have

$$S(u_0) \geq 0 \quad .$$

Since $\bar{K} = \bar{\overset{\circ}{K}}$, the claim is proved. Now, suppose that for some v in K, $S(v) = 0$: obviously $S(S(t)v) = 0$ and thus $\frac{d}{dt} S(t)v = 0$. Hence v is a stationary solution. But if $v \not\equiv 0$, we have, since f is convex

$$\int_{\mathcal{O}} |\nabla v|^2 \, dx = \int_{\mathcal{O}} f(v)v \, dx > 2\int_{\mathcal{O}} F(v)dx \quad ;$$

and we conclude.

Proof of ii): Let $u(x)$ be a solution of (5) and let us prove first that $u \geq 0$. Indeed multiply (5) by $u^-(= \max(-u,0))$ and integrate by parts, we obtain

$$-\int_{\mathcal{O}} |\nabla u^-|^2 \, dx = \int_{\mathcal{O}} f(-u^-)u^- \, dx \quad ,$$

but $f(t) \geq f'(0)t$ and the above equality yields:

$$\int_{\mathcal{O}} |\nabla u^-|^2 dx \leq f'(0) \int_{\mathcal{O}} |u^-|^2 \, dx$$

since we assume $f'(0) < \lambda_1$, this implies $u^- = 0$ that is $u \geq 0$.

Next, we prove u is an extremal point of K: we argue by contradiction. There exist u_0, v_0 in K, $\theta \in (0,1)$ such that:

$$u = \theta u_0 + (1 - \theta) v_0 \quad .$$

We already saw that $\theta S(t)u_0 + (1 - \theta)S(t)v_0 = w(t,x)$ satisfies:

$$\frac{\partial w}{\partial t} - \Delta w \geq f(w) \quad \text{in} \quad (0,\infty) \times \mathcal{O} , \tag{5}$$

actually since f is strictly convex, this inequality is strict.

On the other hand $w(0,x) = u(x)$ and u satisfies

$$\frac{\partial u}{\partial t} - \Delta u = - \Delta u = f(u) \quad ,$$

thus we know not only that $w(t,x) \geq u(x)$, but also by the strong maximum principle and Hopf principle:

$$\begin{cases} w(t,x) > u(x) & \text{in} \quad (0,\infty) \times \mathcal{O} \\ \dfrac{\partial w}{\partial n}(t,x) < \dfrac{\partial u}{\partial n}(x) & \text{in} \quad (0,\infty) \times \partial\mathcal{O} . \end{cases} \tag{6}$$

Next, multiply (5) by $u (u \geq 0)$ and (4) by w;

$$\frac{d}{dt}\int_{\mathcal{O}} w(t,x)u(x)dx \geq \int_{\mathcal{O}} f(w(t,x))u(x) - f(u(x))w(t,x)dx =$$

$$= \int_{\mathcal{O}} \left(\frac{f(w(t,x))}{w(t,x)} - \frac{f(u(x))}{u(x)}\right)w(t,x)u(x)dx$$

and this quantity is nonnegative since f is convex and $w \geq u \geq 0$. Hence $\int_{\mathcal{O}} w(t,x)u(x)dx \underset{t\nearrow+\infty}{\nearrow} M$ and $M > \int_{\mathcal{O}} u^2(x)dx$.

To conclude, we admit for the moment the two lemmas which follow:

<u>Lemma I.1</u>: <u>If</u> $u_0 \in K$, <u>then</u> $(S(t)u_0, t \geq 0)$ <u>is relatively compact in</u> X. <u>And if we denote by</u> $\omega(u_0)$ <u>the</u> w-<u>limit set of</u> u_0 <u>that is the set of</u> u <u>in</u> X <u>such that there exists a sequence</u> $t_n \nearrow +\infty$ <u>satisfying</u>

$$S(t_n)u_0 \underset{X}{\to} u \quad ;$$

<u>then</u> $\omega(u_0)$ <u>is a compact, connected subset of</u> X <u>and for all</u> u <u>in</u> $\omega(u_0)$ u <u>satisfies</u>:

$$-\Delta u = f(u) \text{ in } \mathcal{O}, \quad u \in C^2(\bar{\mathcal{O}}), \quad u = 0 \text{ on } \partial\mathcal{O}.$$

<u>In addition</u>, $\frac{d}{dt}(S(t)u_0) \xrightarrow[C(\bar{\mathcal{O}})]{} 0$.

<u>Lemma I.2</u>: <u>If</u> $u,v \in C^2(\bar{\mathcal{O}})$ <u>satisfy</u>: $u \geq v \geq 0$, $v \not\equiv 0$

$$(7) \qquad \begin{cases} -\Delta u \geq f(u) \text{ in } \mathcal{O}, & u = 0 \text{ on } \partial\mathcal{O} \\ -\Delta v \leq f(v) \text{ in } \mathcal{O}, & v = 0 \text{ on } \partial\mathcal{O} \end{cases}$$

<u>then</u> $v = u$.

Now, if we apply Lemma I.1, we find that there exists $t_n \to \infty$ such that $S(t_n)u_0(x) \underset{X}{\to} \tilde{u}(x)$, $S(t_n)v_0(x) \underset{X}{\to} v(x)$; and $\tilde{u}$, v are stationary solutions of (4). Therefore $w(t_n,x) \underset{X}{\to} \theta\tilde{u} + (1-\theta)v$ and $M = \int_{\mathcal{O}} (\theta\tilde{u} + (1-\theta)v)u\,dx$, $\theta\tilde{u} + (1-\theta)v \geq u$. Since $M > \int_{\mathcal{O}} u^2\,dx$, $\theta\tilde{u} + (1-\theta)v \not\equiv u$. Finally, we just need to remark that $-\Delta(\theta\tilde{u} + (1-\theta)v) = \theta f(\tilde{u}) + (1-\theta)f(v) \geq f(\theta\tilde{u} + (1-\theta)v)$; and a straightforward application of Lemma I.2 yields the desired contradiction. The proofs of the above Lemmas are given in I.3. We will not give the proof of iii) since it is the same as the argument which enables us to prove (6) above. Let us finally observe that the use of the convexity in the arguments above is somewhat reminiscent of H. Berestycki [5].

I.3: <u>Some preliminary results</u>:

<u>Proof of Lemma I.1</u>: The first part of Lemma I.1 is well-known (see for example C. M. Dafermos [10]) since $u_0 \in K$ implies (by definition $\|u(t,x)\|_{C^2(\bar{\mathcal{O}})} \leq C$ (for example) for $t \geq 1$). Thus, we just need to prove that $\frac{d}{dt}(S(t)u_0) \xrightarrow[t\to\infty]{} 0$. Indeed, remark first that we have (setting $u = S(t)u_0$) $\|\frac{\partial u}{\partial t}\|_{C^{0,\alpha}(\bar{\mathcal{O}})}$, $\|\frac{\partial^2 u}{\partial t^2}\|_{C^{0,\alpha}(\bar{\mathcal{O}})} \leq C$, for $t \geq 1$ and $\alpha < 1$.

On the other hand, we have

$$\frac{d}{dt}\left\{\int_{\mathcal{O}} \frac{1}{2}|\nabla u|^2 - \int_{\mathcal{O}} F(u)\right\} = -\left|\frac{du}{dt}\right|_{L^2}^2 ,$$

where $F(t) = \int_0^t f(s)ds$. In other words $\frac{1}{2}|\nabla u|_{L^2}^2 - \int_{\mathcal{O}} F(u)$ is a Liapunov functional.

Thus $\int_0^\infty \left|\frac{du}{dt}\right|^2_{L^2} ds < \infty$, and since by the above estimates $\left|\frac{du}{dt}\right|^2_{L^2}$ is a uniformly continuous function on $[0,\infty)$, we deduce:

$$\frac{du}{dt} \xrightarrow[t\to\infty]{L^2} 0 \quad .$$

Since $\frac{du}{dt}$ is bounded in $C^{0,\alpha}(\bar{\mathcal{O}})$ for any $\alpha < 1$, this implies: $\frac{du}{dt} \xrightarrow[t\to\infty]{C^{0,\alpha}(\bar{\mathcal{O}})} 0$ (for $\alpha < 1$).

<u>Proof of Lemma I.2</u>: This result is well-known but we make the proof for the sake of completeness. Multiply by v, u (7):

$$\int_{\mathcal{O}} f(u)v\,dx \le \int_{\mathcal{O}} f(v)u\,dx$$

or

$$\int_{\mathcal{O}} \left(\frac{f(u)}{u} - \frac{f(v)}{v}\right)uv\,dx \le 0$$

(when v or $u = 0$, $\frac{f(v)}{v}$ is to be understood as $f'(0)$), since $0 \le v \le u$, $v \not\equiv 0$ and f is strictly convex, this implies $u \equiv v$.

Before going into the proof of iv), we state and prove some preliminary results of independent interest:

<u>Lemma I.3</u>: <u>Let</u> u_0 <u>be in</u> K, <u>if</u> $0 \in \omega(u_0)$ <u>then</u> $\omega(u_0) = \{0\}$ i.e.

$$u(t,x) = S(t)u_0(x) \xrightarrow[X]{t\to\infty} 0 \quad .$$

<u>Proof</u>: It is well-known (see for example H. Brézis and R. E. L. Turner [8]) that since $f'(0) < \lambda_1$ there exists $\alpha > 0$, such that

$$\|u\|_{L^\infty(\mathcal{O})} > \alpha \quad \text{for any } u \text{ solution of (5)} \quad .$$

Now remarking that $u \in \omega(u_0)$ implies $u \equiv 0$ or u is a solution of (5), we just need to invoke Lemma I.1 and the fact that $\omega(u_0)$ is connected.

We need some notations in order to state the next result: if $c(x) \in C(\bar{\mathcal{O}})$ we will denote by $\lambda_1(c)$ the first eigenvalue of the problem:

$$\begin{cases} -\Delta u = cu + \lambda_1 u \quad \text{in} \quad \mathcal{O}, \quad u \in W^{2,p}(\mathcal{O})\ (p < \infty) \\ u = 0 \quad \text{on} \quad \partial\mathcal{O} \ . \end{cases}$$

And $v_1(c)$ will be the corresponding positive normalized eigenfunction:

$$\begin{cases} -\Delta v_1(c) = cv_1(c) + \lambda_1 v_1(c) \quad \text{in} \quad \mathcal{O}, \quad v_1(c) \in W^{2,p}(\mathcal{O})\ (p < \infty) \\ v_1(c) = 0 \quad \text{on} \quad \partial\mathcal{O}, \quad v_1(c) > 0 \quad \text{in} \quad \mathcal{O}, \quad |v_1(c)|_{L^2(\mathcal{O})} = +1 \ . \end{cases}$$

It is well-known that if $c_n \xrightarrow[C(\bar{\theta})]{n\to\infty} c$, then $\lambda_1(c_n) \xrightarrow{n\to\infty} \lambda_1(c)$ and

$$v_1(c_n) \xrightarrow{n\to\infty} v_1(c) \quad \text{in} \quad W^{2,p}(\mathcal{O}) \quad \text{weakly} \quad (p < \infty) \ .$$

We need the following result

<u>Lemma I.4</u>: <u>Let</u> $c(t,x) \in C_b([0,\infty[\times \bar{\mathcal{O}})$ (*), <u>we assume</u>

(8) $\qquad c(\cdot,x) \in C^1([0,\infty[)$ <u>and</u> $\frac{\partial}{\partial t} c(t,x) \in C_b([0,\infty[\times \bar{\mathcal{O}})$.

<u>Then</u> $\lambda_1(c(t)) \in C_b^1([0,\infty[)$ <u>and</u>

(*) $C_b(\bar{\mathcal{O}})$ denotes the space of bounded continuous functions on $\bar{\mathcal{O}}$.

(9) $$\frac{d}{dt}(\lambda_1(c(t))) = -\int_{\mathcal{O}} \left(\frac{\partial}{\partial t} c(t)\right) |v_1(c(t))|^2 \, dx \quad .$$

<u>In addition</u> $\frac{\partial}{\partial t} v_1(c(t))$ <u>exists, is bounded independently of</u> $t \geq 0$ <u>and is continuous in</u> $t \geq 0$: $v_1' = \frac{\partial}{\partial t} v_1(c(t))$ <u>solves the problem</u>

(10) $$\begin{cases} -\Delta v_1' = \frac{\partial}{\partial t}(\lambda_1(c(t)) + c(t)) v_1(c(t)) + (\lambda_1(c(t)) + c(t)) v_1' \ \underline{\text{in}}\ \mathcal{O} \\ (v_1', v_1)_{L^2(\mathcal{O})} = 0 \quad , \quad v_1' \in W^{2,p}(\mathcal{O}) \ (p < \infty) \quad , \quad v_1' = 0 \ \underline{\text{on}}\ \partial\mathcal{O} \ . \end{cases}$$

<u>Finally, if we assume in addition</u>: $\frac{\partial}{\partial t}(c(t,x)) \xrightarrow[C(\bar{\mathcal{O}})]{t\to\infty} 0$; <u>then</u>

$$\frac{d}{dt}\lambda_1(c(t)) \xrightarrow{t\to\infty} 0 \quad , \quad v_1' \xrightarrow{t\to\infty} 0 \quad (\underline{\text{at least in}}\ C^1(\bar{\mathcal{O}})) \quad .$$

Before going into the proof of Lemma I.4, we state and prove a simple application

<u>Corollary I.1</u>: <u>Let</u> $u_0 \in X$, $u_0 \geq 0$, $u_0 \not\equiv 0$ <u>and let</u> $c(t,x)$ <u>in</u> $C_b([0,\infty) \times \bar{\mathcal{O}})$ <u>satisfying</u> (8) <u>and</u> $\frac{\partial}{\partial t} c(t,x) \xrightarrow[C(\bar{\mathcal{O}})]{t\to\infty} 0$.

<u>Let</u> $u(t,x)$ <u>satisfy</u>:

(11) $$\begin{cases} \frac{\partial u}{\partial t} - \Delta u \geq c(t,x) u \ \underline{\text{in}}\ (0,\infty) \times \mathcal{O}, \ u \in W^{2,1,p}((0,T) \times \mathcal{O}) \ \underline{\text{for}}\ T < \infty,\ p < \infty \quad . \\ u(t,x) = 0 \ \underline{\text{on}}\ \partial\mathcal{O} \quad , \quad u(0,x) = u_0(x) \quad . \end{cases}$$

<u>If we assume that</u> $\lambda_1(c(t)) \leq -\alpha < 0$ <u>for</u> $t \geq t_0$; <u>then</u> $u(t,x) \xrightarrow{t\to\infty} +\infty$ <u>uniformly on compact subsets of</u> $\mathcal{O}$.

<u>Proof of Corollary I.1</u>: A tedious (but straightforward) argument yields that, by the strong maximum principle, there exists $\beta > 0$ such that

$$v_1(t,x) \geq \beta v_1(0,x) \quad \text{in } \bar{\mathcal{O}}, \quad \text{where } v_1(t,x) = v_1(c(t))(x) .$$

By assumption and by Lemma I.4: $\frac{\partial}{\partial t} v_1(t,x) \xrightarrow[C^1(\bar{\mathcal{O}})]{t\to\infty} 0$. Thus, for $t \geq t_1$, we have $\frac{\partial v_1}{\partial t} \leq \frac{\alpha\beta}{2} v_1(0,x)$ or

$$\frac{\partial v_1}{\partial t}(t,x) \leq \frac{\alpha}{2} v_1(t,x) .$$

Now let $T = \max(t_0,t_1)$, by the strong maximum principle and Hopf principle we may assume that

$$u(T,x) \geq \gamma v_1(T,x) \quad (\text{for small enough } \gamma > 0) .$$

We finally introduce $\theta(t,x) = \gamma e^{\alpha(t-T)/2} v_1(t,x)$ (for $t \geq T$) and we compute:

$$\frac{d\theta}{dt} - \Delta\theta - c(t,x)\theta = \frac{\alpha}{2}\theta + \gamma e^{\alpha(t-T)/2} \frac{\partial v_1}{\partial t} + \lambda_1(c(t))\theta$$

$$\leq -\frac{\alpha}{2}\theta + \gamma e^{\alpha(t-T)/2} \frac{\partial v_1}{\partial t} \leq 0 \quad , \quad \text{for } t \geq T$$

and $\theta(T,x) = \gamma v_1(T,x) \leq u(T,x)$.

Therefore $u(t,x) \geq \theta(t,x)$, for $t \geq T$, x in $\bar{\mathcal{O}}$. In particular $u(t,x) \geq \beta\gamma e^{\alpha(t-T)/2} v_1(0,x)$ for $t \geq T$.

We next turn to the proof of Lemma I.4:

<u>Proof of Lemma I.4</u>: We will denote by $\lambda_1(t) = \lambda_1(c(t))$ and $v_1(t,x) = v_1(c(t))(x)$. Recall that $\lambda_1(t)$ is given by

$$\lambda_1(t) = \min_{\substack{|v|_{L^2}=+1 \\ v \in H_0^1}} \left\{ \int_{\mathcal{O}} |\nabla v|^2 - \int_{\mathcal{O}} c(t,x)v^2 \, dx \right\} .$$

Therefore for $h > 0$ (for example)

$$\frac{\lambda_1(t+h) - \lambda_1(t)}{h} \leq \frac{1}{h}\left\{\int_{\mathcal{O}} c(t,x)(v_1(t,x))^2\,dx - \int_{\mathcal{O}} c(t+h,x)(v_1(t,x))^2\,dx\right\}$$

and the right-hand side term goes to $-\int_{\mathcal{O}} \frac{\partial c}{\partial t}(v_1(t))^2\,dx$ as $h \to 0$. On the other hand

$$\frac{\lambda_1(t+h) - \lambda_1(t)}{h} \geq \frac{1}{h}\left\{\int_{\mathcal{O}} c(t,x)(v_1(t+h,x))^2\,dx - \int_{\mathcal{O}} c(t+h,x)(v_1(t+h,x))^2\,dx\right\}$$

and again the right-hand side term goes to $-\int_{\mathcal{O}} \frac{\partial c}{\partial t}(v_1(t))^2\,dx$ as $h \to 0$, since $v_1(t+h,x) \xrightarrow[h\to 0]{} v_1(t,x)$. This proves (9) and the first part of Lemma I.4.

We next prove that $v_1' = \frac{\partial v_1}{\partial t}$ exists and is given by (10). Let $h > 0$ and let $t > 0$, we denote by $v_1 = v_1(t,x)$ and $v_1^h = v_1(t+h,x)$. We have obviously

$$(10')\quad\left\{\begin{aligned} &-\Delta\left(\frac{v_1^h - v_1}{h}\right) = \left\{\frac{\lambda_1(t+h) - \lambda_1(t)}{h} + \frac{c(t+h) - c(t)}{h}\right\}\left(\frac{v_1^h + v_1}{2}\right) + \\ &\qquad + \left(\frac{\lambda_1(t+h) + c(t+h)}{2}\right)\left(\frac{v_1^h - v_1}{h}\right) + \left(\frac{\lambda_1(t) + c(t)}{2}\right)\left(\frac{v_1^h - v_1}{h}\right) \\ &\frac{v_1^h - v_1}{h} \in H_0^1(\mathcal{O}) \quad , \quad \left(\frac{v_1^h - v_1}{h} \quad , \quad \frac{v_1^h + v_1}{2}\right)_{L^2} = 0 \quad . \end{aligned}\right.$$

Since

$$\frac{\lambda_1(t+h) + \lambda_1(t)}{2} = \lambda_1\left(\frac{c(t+h) + c(t)}{2}\right) \quad ,$$

$$\frac{v_1^h + v_1}{2} = v_1\left(\frac{c(t+h) + c(t)}{2}\right) \quad ,$$

we deduce easily

$$\left| \frac{v_1^h - v_1}{h} \right|_{L^2} \leq \frac{C}{\lambda_2^h - \lambda_1^h} ,$$

where

$$\lambda_1^h = \lambda_1\left(\frac{c(t + h) + c(t)}{2}\right) \quad \text{and} \quad \lambda_2^h \quad \text{is the second eigenvalue of}$$

the problem

$$- \Delta v = \lambda v + \frac{c(t + h) + c(t)}{2} v \quad \text{in} \quad \mathcal{O} \quad , \quad v \in H_0^1(\mathcal{O}) \quad .$$

If we prove that $\lambda_2^h - \lambda_1^h$ is bounded away from zero when $h \to 0$, by (10) we deduce that $\frac{v_1^h - v}{h}$ is bounded in $H^2(\mathcal{O})$ and by a bootstrap argument in $W^{2,p}(\mathcal{O})$ $(p < \infty)$, it is then obvious to pass to the limit and to obtain (10). Finally proving the remaining part of the Lemma is straightforward provided we show quantities like $\lambda_2^h - \lambda_1^h$ is bounded away from 0.

In other words, we want to prove that if $c^n \in C(\bar{\mathcal{O}})$, $c^n \xrightarrow[n\to\infty]{} c$ in $C(\bar{\mathcal{O}})$ then $\lambda_2(c^n) - \lambda_1(c^n) \geq \alpha > 0$, indep. of n.

Let us argue by contradiction: replacing if necessary c^n by a subsequence we may assume that $\lambda_2(c^n) - \lambda_1(c^n) \xrightarrow[n\to\infty]{} 0$, and thus

$$\lambda_2(c^n) \xrightarrow[n\to\infty]{} \lambda_1(c) \quad .$$

Now let H^n be the 2-dimensional subspace of H_0^1 generated by v_1^n, v_2^n where v_1^n is an eigenfunction corresponding to $\lambda_1(c^n)$ and v_2^n to $\lambda_2(c^n)$. Then we have,

$$\max_{\substack{v \in H^n \\ |v|_{L^2}=1}} \int_{\mathcal{O}} |\nabla v|^2 - c(x) v^2 \, dx \leq \lambda_2(c^n) + \|c^n - c\|_\infty \quad ;$$

by the variational characterization of $\lambda_2(c)$, this yields:

$$\lambda_2(c) \le \lambda_2(c^n) + \|c^n - c\|_\infty ,$$

which contradicts the fact that $\lambda_2(c) > \lambda_1(c)$.

I.4: Asymptotic behavior:

We now give the proof of part iv) of Theorem I.1: let $u_0 \in \overset{\circ}{K}$, we denote by $u(t,x) = S(t)u_0(x)$. Since $u_0 \in \overset{\circ}{K}$, there exists $v_0 \in K$, $v_0 \ge u_0$ and $v_0 \not\equiv u_0$. We denote by $v(t,x) = S(t)v_0(x)$.

Let us argue by contradiction: $u(t,x) \underset{t\to\infty}{\not\to} 0$; then by Lemma I.3 $0 \notin \omega(u_0)$.

In addition $w(t,x) = v(t,x) - u(t,x)$ satisfies:

$$\begin{cases} \dfrac{dw}{dt} - \Delta w = \left(\dfrac{f(v) - f(u)}{v - u}\right)w & \text{in } (0,\infty) \times \mathcal{O} \\ w(0,x) = v_0 - u_0 \quad , \quad w(t,x) = 0 & \text{on } (0,\infty) \times \partial\mathcal{O} \ . \end{cases}$$

We are going to apply Corollary I.1 with $c(t,x) = f'(u(t,x))$.

Indeed $v(t,x) \ge u(t,x)$, and therefore $\left(\dfrac{f(v) - f(u)}{v - u}\right)w \ge f'(u)w$. In addition

$$\frac{\partial}{\partial t} c(t,x) = f''(u(t,x)) \frac{\partial u}{\partial t} \xrightarrow{t\to\infty} 0 \quad \text{in} \quad C(\bar{\mathcal{O}})$$

in view of Lemma I.1.

Therefore in order to apply Corollary I.1, we need to check that

(12) $$\lambda_1(f'(u(t,x))) \le -\alpha < 0 \quad , \quad \text{for} \quad t \ge t_0 \ .$$

Assume this is proved; then by Corollary I.1, $w(t,x)$ cannot be bounded which contradicts the definition of w.

Now to prove (12), we need the following well-known lemma that we admit for the moment:

Lemma I.5: Let C be a compact set in X consisting of solutions of (5). Then there exists $\alpha > 0$ such that

$$\lambda_1(f'(u(x))) \leq -\alpha < 0 \quad , \quad \text{for every } u \text{ in } C.$$

In particular we may take $C = \omega(u_0)$, and by continuity there exists an open neighborhood $\tilde{C}$ of C such that

$$\lambda_1(f'(u(x)) \leq -\frac{\alpha}{2} < 0 \quad \text{for every } u \text{ in } \tilde{C} \quad .$$

Now by Lemma I.1, we deduce that for t large enough $u(t,x) \in \tilde{C}$. Indeed $\bigcap_{t \geq 0} (\overline{\{u(s,x), s \geq t\}} \cap (X - \tilde{C})) = \emptyset$ and therefore $\{u(s,x), s \geq t\} \subset \tilde{C}$ for $t \geq T$.

We conclude the proof of Theorem I.1 with the proof of Lemma I.5.

Proof of Lemma I.5: First, let us remark that for every solution u of (5) one has

$$\lambda_1\left(\frac{f(u(x))}{u(x)}\right) = 0 \quad .$$

Then, by the well-known comparison theorems on eigenvalues, this implies

$$\lambda_1(f'(u(x))) < 0 \quad .$$

This proves Lemma I.5, since $\lambda_1(f'(u(x)))$ depends continuously on u.

II Some extensions and related results.

II.1: An extension of (3).

Instead of (3), we now assume that $f \in C^2(\mathbb{R})$ and satisfies:

(13) $$f \text{ is strictly convex} \quad , \quad \lim_{t\to-\infty} f'(t) < \lambda_1 \quad .$$

We then have

Theorem II.1: Under assumption (13), we have

i) $\dot{K}$ is non-empty if and only if there exists a solution of (5)

(5) $$-\Delta u = f(u) \text{ in } \mathcal{O} \;, \quad u \in C^2(\bar{\mathcal{O}}) \;, \quad u = 0 \text{ on } \partial\mathcal{O} \;;$$

moreover, if $K \neq \emptyset$, then there exists a minimum solution $\underline{u}$ of (5). In that case K is convex, $\overset{\circ}{K} \neq 0$, and if $u_0 \in K$, $v \in X$ with $v \leq u_0$ then $v \in K$. Furthermore we have $S(\underline{u}) < S(v)$, for all v in $K - \{\underline{u}\}$.

ii) If $K \neq \emptyset$, then $\underline{u} \in \overset{\circ}{K}$ as soon as there exists a solution of (5) distinct from $\underline{u}$, or as soon as $\lambda_1(f'(\underline{u})) > 0$.

If $\underline{u} \in \partial K$, then $\underline{u}$ is an extremal point of K and for all u_0 in K

$$S(t)u_0 \xrightarrow[X]{t\to\infty} \underline{u} \;.$$

(this last statement also holds if $\underline{u}$ is the only solution of (5)).

iii) If $K \neq \emptyset$ and $\underline{u} \in \overset{\circ}{K}$, then for every solution u of (5) distinct from $\underline{u}$, one has: u is an extremal point of K .

Furthermore if $u_0 \in K$ and if u_0 is not an extremal point of K, then $S(t)u_0 \in \overset{\circ}{K}$ and $S(t)u_0 \xrightarrow{t\to\infty} \underline{u}$.

Remark II.1: The proof of Theorem II.2 is very similar to the proof of Theorem I.1 and we will not give it. A variant of the proof of ii) in Theorem I.1 gives that if f'' is positive and if $\lambda_1(f'(\underline{u})) = 0$, then $\underline{u}$ is an extremal point of K .

Remark II.2: Theorem II.2 holds if we replace $f(u)$ by $f(x,u)$ assuming that $f(x,\cdot) \in C^2(\mathbb{R})$ (for x in $\bar{\mathcal{O}}$), $f(\cdot,t) \in C^{0,\alpha}(\bar{\mathcal{O}})$ (for some $0 < \alpha < 1$ and for all t in $\mathbb{R}$) and that f satisfies:

(14) $f(x,\cdot)$ is strictly convex, for x in $\bar{\mathcal{O}}$; $\lim_{t\to-\infty} \frac{\partial f(x,t)}{\partial t} < \lambda_1$, uniformly in $x \in \bar{\mathcal{O}}$.

Remark II.3: If we no longer assume that $\lim_{t\to-\infty} f'(t) < \lambda_1$, we do not know if the result still holds (actually we do not even know that K is convex).

Let us give now a few examples:

Example 1: Take $f(t) = \lambda(1 + |t|^p)$ or $f(t) = \lambda e^t$ ($\lambda > 0$, $1 < p < \infty$). For these kinds of nonlinearities, a rather detailed study of solutions of (5) is given in I. M. Gelfand [13], D. D. Joseph and T. S. Lundgren [14], M. G. Crandall and P. H. Rabinowitz [9], C. Bandle [4], F. Mignot and V. P. Puel [16], P. L. Lions [15]. In particular we know there exists $\lambda^* \in (0,\infty)$ such that (5) has a minimum solution $\underline{u}_\lambda$ for $\lambda \in (0,\lambda^*)$ satisfying $\lambda_1(f'(\underline{u}_\lambda)) > 0$ and (5) has no solution for $\lambda > \lambda^*$. Thus if $\lambda \in (0,\lambda^*)$ iii) applies, while for $\lambda > \lambda^*$ $K = \emptyset$. In addition (there, the result depends on the dimension N) in many cases it is known that for $\lambda = \lambda^*$, (5) has a unique solution $\underline{u}_{\lambda^*}$ and ii) (and Remark II.1) applies ($\underline{u}_{\lambda^*} \in \partial K$ and $S(t)u_0 \to \underline{u}_{\lambda^*}$, for u_0 in K).

Example 2: Take $f(x,t) = f(t) + g(x)$ with f satisfying (13) and $\lambda_1 < \lim_{t\to\infty} f'(t) < \lambda_2$, then (see [5], and [2] for another version) there exists a closed convex set C in $C^{0,\alpha}(\bar{\mathcal{O}})$ with $\mathring{C} \neq \emptyset$ such that 1) if $g \notin C$, then (5) has no solution and thus by Theorem II.1 $K = \emptyset$; 2) if $g \in \partial C$, then (5) has a unique solution u and thus by Theorem II.1 (part ii)), u is an extremal point of K and $S(t)u_0 \xrightarrow[t\to\infty]{} u$; 3) if $g \in \mathring{C}$, then (5) has exactly two distinct solutions $\underline{u} \leq \bar{u}$ and by Theorem II.1 (parts ii) and iii)): $\underline{u} \in \overset{\circ}{K}$, $\bar{u}$ is an extremal point of K and if u_0 is in K and is not an extremal point of K then $S(t)u_0 \xrightarrow[t\to\infty]{} \underline{u}$. Remark also that Theorem II.1 applies also to the extension of [5], [2] given in [7] (where we relax the assumption on f at $+\infty$).

Example 3: Take $f(t) = \lambda t + t^2$ $(\lambda \in \mathbb{R})$. If $\lambda < \lambda_1$, then Theorem I.1 applies.

If $\lambda = \lambda_1$, then obviously 0 is the only solution of

$$-\Delta u = \lambda u + u^2 \quad \text{in } \mathcal{O} \;, \quad u \in C^2(\bar{\mathcal{O}}) \;, \quad u = 0 \quad \text{on } \partial\mathcal{O} \;.$$

Thus i) and the last part of ii) applies: 0 is an extremal point of K and for all u_0 in K, $S(t)u_0 \xrightarrow[t\to\infty]{} 0$.

Now for $\lambda > \lambda_1$, it is quite easy to prove there exists a minimum negative solution $\underline{u}$ which satisfies $\lambda_1(f'(\underline{u})) > 0$. Then ii) and iii) apply and for all u_0 in K, u_0 being not an extremal point, $S(t)u_0 \xrightarrow[t\to\infty]{} \underline{u}$.

II.2: Iterative schemes.

We are now concerned with the convergence of schemes like

$$(15)\quad -\Delta u^{n+1} + \lambda u^{n+1} = \lambda u^n + f(u^n) \;, \quad u^{n+1} \in C^2(\bar{\mathcal{O}}) \;, \quad u^{n+1} = 0 \quad \text{on} \quad \mathcal{O} \;;$$

u^0 is given and we assume (3), $\lambda > 0$ and

$$(16)\qquad f(t) + \lambda t \quad \text{is nondecreasing for} \quad t \in \mathbb{R} \;.$$

(Again we could replace (3) by more general assumptions, but we will not do it here for the sake of simplicity).

The scheme (15) is an implicit one, "approximating (4) for $t \in (0,\infty)$" with λ being the inverse of a time-step; therefore it is quite natural to ask if one has results for (15) which are similar to Theorem I.1.

We introduce again:

$$K = \{u^0 \in X \quad , \quad |u^n(x)| \le C_{u^0} \quad \text{indep. of } n \text{ and } x\} \quad .$$

Then we have:

Theorem II.2: Under assumptions (3) and (16), we have

i) K is convex, unbounded; $0 \in \overset{\circ}{K}$ and if $u^0 \in K$, $v \in X$, $v \le u^0$ then $v \in K$; and we have $S(v) > 0$, for all v in $K - \{0\}$.

ii) If u is a non-trivial solution of (4) then u is an extremal point of K.

iii) If $u^0 \in K$ and if u^0 is not an extremal point of K then $u^n \in \overset{\circ}{K}$, for $n \ge 1$.

iv) Moreover if $u^0 \in \overset{\circ}{K}$, then $u^n \underset{n\to\infty}{\longrightarrow} 0$ (in $C^2(\bar{\mathcal{O}})$) .

The proof of this result is very similar to the one of Theorem I.1 and we will omit it.

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In this paper a semilinear heat equation with a convex nonlinearity is considered. The asymptotic behavior of the solutions is completely determined and this gives, in particular, a very precise description of the global stability of stationary solutions.

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