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A GENERALIZATION OF THE KREISS MATRIX THEOREM.(U)
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A GENERALIZATION OF THE KREISS MATRIX THEOREM

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ABSTRACT

Let $\mathcal{A}$ be a set of $n \times n$ complex matrices A which satisfy the condition $\|(I - zA)^{-1}\| \leq K/(1 - |z|)^{\alpha+1}$ for some $\alpha \geq 0$ and all $|z| < 1$. Then it is shown here that there exists a constant $\rho(\alpha,n)$ such that $\|A^{\nu}\| \leq K\rho(\alpha,n)\nu^{\alpha}$, $\nu = 0,1,\ldots$. This forms a generalization of the Kreiss resolvent condition $(\alpha = 0)$.

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SIGNIFICANCE AND EXPLANATION

Let A be an $n \times n$ complex valued matrix. A standard and useful result in matrix theory claims that all powers of A are bounded if and only if the spectral radius-$\rho(A)$ - is less or equal to one and for all eigenvalues λ of A such that $|\lambda| = 1$ the matrix $(I - zA)^{-1}$ has a simple pole at $z = \bar{\lambda}$. If we consider a more general problem, namely when the powers A^{ν}, $\nu = 0,1,\ldots,$ grow at most as ν^{α}, where α is a positive integer, then this condition holds if and only if $\rho(A) \leq 1$, and for all eigenvalues λ of A such that $|\lambda| = 1$ the matrix $(I - zA)^{-1}$ has at most a pole of order $\alpha + 1$ at $z = \bar{\lambda}$.

In the early sixties H. O. Kreiss, while studying stability of numerical schemes for partial differential equations, considered a generalization of ~~the~~ a first problem, described above. Namely, given a set $\mathcal{A}$ of $n \times n$ complex valued matrices, when all powers of $A \in \mathcal{A}$ are uniformly bounded. These sets - called the stable sets - were completely characterized by Kreiss by giving three equivalent conditions.

In this paper we consider α-stable sets $\mathcal{A}(\alpha \geq 0)$, such that for any $A \in \mathcal{A}$ the powers A^{ν} are uniformly bounded by $K\nu^{\alpha}$. We generalize the Kreiss resolvent condition for α-stable sets. It seems that α-stable sets are related to the concept of weakly stable numerical schemes for partial differential equations.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

A GENERALIZATION OF THE KREISS MATRIX THEOREM

Shmuel Friedland

1. Introduction

In various instances one deals with iterative systems of equations

(1.1) $$x^{(i+1)} = Ax^{(i)}, \qquad i = 0,1,2,\ldots .$$

Here $x^{(i)} \in C^n$, $A \in M_n$, where C^n is the set of n column complex vectors and M_n is the set of $n \times n$ complex matrices. Clearly

(1.2) $$x^{(i)} = A^i x^{(0)}$$

and thus in order to investigate the behaviour of $x^{(i)}$ for large i one needs to study the powers A^i, $i = 0,1,\ldots$. Let $\mathcal{A}$ be a set of $n \times n$ matrices. $\mathcal{A}$ is called an α-stable set if

(1.3) $$\|A^\nu\| \le K\nu^\alpha, \qquad \nu = 0,1,2,\ldots .$$

Here α is a nonnegative number and $\|\cdot\|$ is a norm on M_n. The concept of stability of the numerical schemes for solutions of partial differential equations is intimately connected with the notion of stable sets. Consult for example Kreiss [1962], Richtmyer and Morton [1967] and others. It seems that α-stable sets are related to the concept of weakly stable numerical schemes for partial differential equations. See Kreiss [1962] and Forsyth and Wasow [1960]. The stable sets were characterized completely by Kreiss [1962]. In this paper we generalize the Kreiss result to α-stable sets.

Theorem 1. *Let α be a nonnegative number and $\mathcal{A}$ be a set of $n \times n$ complex valued matrices. Then the following two conditions are equivalent.*

(A) *There exists a constant $K(\ge 1)$ such that for all $A \in \mathcal{A}$ (1.3) holds.*

(R) *There exists a constant $K(\ge 1)$ such that for all $A \in \mathcal{A}$*

(1.4) $$\|(I - zA)^{-1}\| \le K(1 - |z|)^{-(\alpha+1)}, \qquad |z| < 1 .$$

The implication (A) ==> (R) is obvious. The implication (R) ==> (A) is a consequence of Theorem 2 which estimates the Maclaurin coefficients of a certain family of rational

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functions in terms of the growth of their moduli. We were not able to give conditions analogous to the conditions (S) and (H) of Kreiss.

2. Coefficient Estimates for Certain Analytic Functions

Let D be a unit disc $|z| < 1$. Suppose that $f(z)$ is an analytic function in D. Consider the Maclaurin expansion of f

$$f(z) = \sum_{\nu=0}^{\infty} a_\nu z^\nu, \qquad |z| < 1 . \tag{2.1}$$

Suppose that

$$|a_\nu| \le K\nu^\alpha, \qquad \alpha = 0,1,2,... \tag{2.2}$$

for $\alpha > -1$. It is a standard result in theory of special functions (e.g. Olver [1974, p. 119]) that

$$\nu^\alpha \approx (-1)^\nu \binom{-(\alpha+1)}{\nu} = \frac{\Gamma(\alpha+\nu+1)}{\Gamma(\nu+1)\Gamma(\alpha+1)} . \tag{2.3}$$

Here two positive sequences $\{u_m\}$ and $\{v_m\}$ are called equivalent $u_m \approx v_m$ if

$$\lim_{m\to\infty} \frac{u_m}{v_m} = \beta, \qquad 0 < \beta < \infty .$$

Thus (2.3) implies

$$|f(z)| \le K\rho(\alpha)(1 - |z|)^{-(\alpha+1)} \tag{2.4}$$

for some positive constant $\rho(\alpha)$ with $\alpha > -1$. Conversely we have a weaker result.

__Lemma 1.__ __Let__ $f(z)$ __be analytic in__ D. __Assume that__

$$|f(z)| \le K(1 - |z|)^{-\alpha} , \tag{2.5}$$

__for some__ $\alpha > 0$ __and all__ $|z| < 1$. __Then__

$$|a_\nu| \le K\left(1 + \frac{\alpha}{\nu}\right)^\nu \left(\frac{\nu+\alpha}{\alpha}\right)^\alpha < Ke(\nu + 1)^\alpha \tag{2.6}$$

__and this inequality is sharp.__

__Proof.__ As

$$a_\nu = (2\pi i)^{-1} \int_{|z|=r<1} f(z) z^{-(\nu+1)} dz \tag{2.7}$$

we get

$$(2.8) \qquad |a_\nu| \le [\max_{|z|=r} |f(z)|]r^{-\lambda} \le K(1-r)^{-\alpha}r^{-\nu} .$$

Note that

$$\min_{0\le r\le 1} (1-r)^{-\alpha}r^{-\nu} = (1-r)^{-\alpha}r^{-\nu}\Big|_{r=\frac{\nu}{\nu+\alpha}} = \left(1+\frac{\alpha}{\nu}\right)^{\nu}\left(\frac{\nu+\alpha}{\alpha}\right)^{\alpha} .$$

This establishes the first inequality in (2.6). To obtain the second inequality in (2.6) choose in (2.8) $r = \frac{\nu}{\nu+1}$ and use the well known fact that $\left(1+\frac{1}{\nu}\right)^{\nu} < e$. To see that (2.6) is sharp for each ν consider the polynomial

$$(2.9) \qquad p(z) = K\left(1+\frac{\alpha}{\nu}\right)^{\nu}\left(\frac{\nu+\alpha}{\alpha}\right)^{\nu} z^{\nu} . \qquad \square$$

Let B be a Banach space with a norm $\|\cdot\|$. Assume that $A : B \to B$ is a bounded linear operator. Suppose that the spectrum of A lies in the unit disc. Then expanding $(I - zA)^{-1}$ in power series

$$(2.10) \qquad (I - zA)^{-1} = \sum_{\nu=0}^{\infty} z^\nu A^\nu$$

we get

$$(2.11) \qquad A^\nu = (2\pi i)^{-1} \int_{|z|=r<1} (I - zA)^{-1} dz .$$

Thus if

$$(2.12) \qquad \|(I - zA)^{-1}\| \le K(1 - |z|)^{-\alpha}, \qquad |z| < 1$$

applying the results of Lemma 1 we obtain

$$(2.13) \qquad \|A^\nu\| < Ke(\nu+1)^{\alpha} .$$

It is an open problem whether the estimate (2.13) is sharp in some infinite dimensional Banach space. The following result enables one to improve the inequality (2.13) for matrices (i.e. B is finite dimensional).

Theorem 2. Consider all polynomials $p(z)$ and $q(z)$ of degrees m and n respectively such that the function $f(z) = p(z)/q(z)$ satisfies (2.5). Suppose that $\alpha > 1$. Then there exists a positive constant $\rho(\alpha,m,n)$ such that

$$|a_\nu| \leq K\rho(\alpha,m,n)\nu^{(\alpha-1)} . \tag{2.14}$$

To prove this theorem we need the following lemma.

Lemma 2. Let $p(z)$ be a polynomial of degree m. Then there exists a constant $K(m)$ such that

$$\max_{|z|=r} |p(z)| \leq K(m)\left(\max_{|\theta|\leq\pi/4} |p(re^{i\theta})|\right) . \tag{2.15}$$

Proof. It is enough to consider the case $r = 1$ with $p(z)$ of the form

$$p(z) = \prod_{i=1}^{m} (z - \zeta_i), \qquad |\zeta_1| \leq |\zeta_2| \cdots \leq |\zeta_m| .$$

For $m = 1$ it suffices to choose $K(1) = 5$. Let $m > 1$. Define

$$K'(m) = \max_{0\leq|\zeta_1|\leq\cdots\leq|\zeta_m|\leq 3} \left(\max_{|z|=1} |p(z)| / \max_{|\theta|\leq\pi/4} |p(e^{i\theta})|\right) .$$

In case that $|\zeta_m| > 3$ let $q(z) = \prod_{i=1}^{m-1} (z - \zeta_i)$. Then

$$\max_{|z|=1} |p(z)| \leq (|\zeta_m| + 1) \max_{|z|=1} |q(z)|$$

$$\leq 2(|\zeta_m|-1)K(m - 1) \max_{|\theta|\leq\pi/4} |q(e^{i\theta})| \leq 2K(m - 1) \max_{|\theta|\leq\pi/4} |p(e^{i\theta})| .$$

Put

$$K(m) = \max(K'(m), 2K(m - 1))$$

and the lemma follows. $\square$

Proof of Theorem 1. Without restriction in generality we may assume that $p(z)$ and $q(z)$ do not have common zeros. Also it is enough to consider the case $K = 1$. The inequality (2.5) implies that we can choose q and p of the form

$$p(z) = z^{\ell} A \prod_{i=1}^{m-\ell} (1 - z\omega_i), \quad q(z) = \prod_{i=1}^{n} (1 - z\zeta_i) . \tag{2.16}$$

The inequality (2.5) yields $|\zeta_i| < 1$, $i = 1,\ldots,n$. Put

$$g(z) = A \prod_{i=1}^{m-\ell} (z - \omega_i) \Big/ \prod_{i=1}^{n} (z - \zeta_i) , \tag{2.17}$$

$$|g(z)| < |z|^{m-n+\alpha}/(|z| - 1)^{\alpha}, \quad |z| > 1 . \tag{2.18}$$

Also

$$g(z) = \sum_{\nu=0}^{\infty} a_\nu z^{-(\nu+n-m)}, \quad |z| > 1 . \tag{2.19}$$

Note that

$$a_\nu = (2\pi i)^{-1} \int_{|z|=R>1} g(z) z^{(\nu+n-m-1)} dz .$$

Let $D_1,\ldots,D_p$ be p-mutually disjoint, open and bounded domains with the boundary $\Gamma_1,\ldots,\Gamma_p$ respectively. Assume that $\zeta_i \in \bigcup_{j=1}^{p} D_j$, $i = 1,\ldots,n$. Then we obtain

$$a_\nu = \sum_{j=1}^{p} (2\pi i)^{-1} \int_{\Gamma_j} g(z) z^{(\nu+n-m-1)} dz . \tag{2.20}$$

To obtain the estimate (2.14) we are going to choose the domains $D_1,\ldots,D_p$ according to the configuration of $\zeta_1,\ldots,\zeta_n$ and the value of ν. First we group the points $S_1,\ldots,S_s$ following Morton [1964]. Let ζ_{i_1} be one of the points with the largest modulus, $|\zeta_{i_1}| = 1 - \delta_1 \geq |\zeta_i|$, $i = 1,\ldots,n$. Then we form S_1 from all those points which can be joined to ζ_{i_1} by a chain of points, each link of which has length $\leq \delta_1$.

In the same way S_2 is formed from the remaining points, and so on until all the points have been included in some S_β. For each S_β we denote by $1 - \delta_\beta$ and $1 - \varepsilon_\beta$ the modulus of the largest and the smallest $|\zeta_i|$, $\zeta_i \in S_\beta$. We rename $\zeta_1,\ldots,\zeta_n$ so that

(2.21) $$0 \leq \delta_1 \leq \cdots \leq \delta_s \, .$$

Consider any particular S_β and let us denote its members by λ_i, $i = 1,2,\ldots,k$, where $1 - {}_\beta \leq |\lambda_i| \leq 1 - \delta_\beta$, $i = 1,\ldots,k$. Let us also denote the points not in S_β by μ_j, $j = 1,2,\ldots,n-k$. We claim

(2.22) $$\delta_\beta \leq 1 - |\lambda_i| \leq k\delta_\beta, \quad |\lambda_i - \lambda_j| \leq (k-1)\delta_\beta, \quad |\lambda_i - \mu_j| > \delta_\beta \, .$$

Indeed, the first two inequalities follow immediately from the assumption that there exists a chain of at most k points between λ_i and λ_j such that the distance of any link $\leq \delta_\beta$. The last inequality is a consequence of μ_j not being in S_β. Let

(2.23) $$h(z) = A \prod_{i=1}^{m-\ell} (z - \omega_i) \, .$$

For $\lambda_t \in S_\beta$ put

(2.24) $$\eta = (1 + 2\delta_\beta)\lambda_t/|\lambda_t| \, .$$

Then

(2.25) $$h(z) = \sum_{j=0}^{m-\ell} h_j (z - \eta)^j \, .$$

We now estimate h_j. Let Γ be a circle $|z - \eta| = \delta_\beta$. Then

$$\begin{aligned} |z - \zeta_i| &\leq |z - \lambda_t| + |\lambda_t - \zeta_i| \leq |z - \eta| + |\eta - \lambda_t| + |\lambda_t - \zeta_i| \\ &= \delta_\beta + 1 + 2\delta_\beta - |\lambda_t| + |\lambda_t - \zeta_i| \leq (k+3)\delta_\beta + |\lambda_t - \zeta_i| \end{aligned}$$

where the last inequality follows from (2.22). In particular

$$|z - \lambda_j| \leq 2(k+1)\delta_\beta$$

in view of (2.22). Apply the Cauchy formula for h_j and use (2.18) to get

$$|h_j| = (2\pi)^{-1}\left|\int_\Gamma h(z)dz(z-\eta)^{-(j+1)}\right| \le \delta_\beta^{-(j+\alpha)}4^{m+\alpha}\prod_{i=1}^{n}[(k+3)\delta_\beta + |\lambda_t-\zeta_i|]$$

(2.26)

$$\le [2(k+1)]^k 4^{m+\alpha}\delta_\beta^{-(j+\alpha)+k}\prod_{i=1}^{n-k}[(k+3)\delta_\beta + |\lambda_t - \mu_i|]$$

$$\le [2(k+2)]^n 4^{m+\alpha}\delta_\beta^{-(j+\alpha)+k}\prod_{i=1}^{n-k}|\lambda_t - \mu_i| .$$

We now consider the following three cases

(i) $\delta_\beta \ge 1/(2^{n+2}n\nu)$,

(ii) $\delta_\beta \le 1/(4n\nu)$,

(iii) neither (i) nor (ii) holds.

Here ν is a positive integer and $\nu \ge m - n + \alpha$.

<u>Case (i)</u>. Let C_i be a disc $|z - \zeta_i| < \delta_\beta/2$, for $\zeta_i \in S_\beta$. Then

(2.27)
$$D = \bigcup_{i=1}^{n} C_i = \bigcup_{j=1}^{p} D_j$$

where each D_j contains a subset of some S_β and $D_j \cap D_k = \phi$ for $j \ne k$. Let Γ_j be the boundary of D_j. Then $\ell(\Gamma_j)$ - the length of Γ_j - satisfies the inequality

(2.28)
$$\ell(\Gamma_j) \le 2\pi n(D_j)\delta_\beta ,$$

where $n(D_j)$ is the number of points $\zeta_1,\ldots,\zeta_n$ in D_j. Let $z \in \Gamma_j$. Clearly $z = \lambda_t + \rho$, $|\rho| = \delta_\beta/2$, $S_\beta = \{\lambda_1,\ldots,\lambda_k\}$. By the definition of D_j, $|z - \lambda_j| \ge \delta_\beta/2$ for $1 \le j \le k$. Also

$$|z - \mu_j| = |\lambda_t - \mu_j + \rho| \ge |\lambda_t - \mu_j| - \delta_\beta/2 \ge \frac{1}{2}|\lambda_t - \mu_j| .$$

Thus

(2.29)
$$\left|\prod_{i=1}^{n}(z - \zeta_i)^{-1}\right| \le 2^n\delta_\beta^{-k}\prod_{j=1}^{n-k}|\lambda_t - \mu_j|^{-1} .$$

Also for η of the form (2.24) we have

$$|z - \eta| \le |z - \lambda_t| + |\lambda_t - \eta| \le \frac{\delta_\beta}{2} + 1 + 2\delta_\beta - |\lambda_t| < (k + 3)\delta_\beta .$$

Combine (2.25)-(2.26) with the above equality to deduce

$$(2.30) \qquad |h(z)| \le [2(k + 2)]^{n+m} m\, 4^{m+\alpha}\delta_\beta^{k-\alpha} \prod_{i=1}^{n} |\lambda_t - \mu_i| .$$

Finally we deduce

$$(2.31) \qquad |g(z)| \le [16(n + 2)]^{n+m+\alpha}\delta_\beta^{-\alpha} .$$

Using the equality (...20) and the inequalities (2.28), (2.31) for $\nu > m - n$ we get

$$|a_\nu| \le \sum_{j=1}^{p} (2\pi)^{-1} \int_{\Gamma_j} |g(z)||z|^{(\nu+m-n-1)}|dz| \le n[16(n + 2)]^{n+m+\alpha}(\min_{1\le\beta\le s} \delta_\beta)^{-\alpha+1}$$

$$\le n^\alpha[16(n + 2)]^{n+m+\alpha}2^{(n+2)(\alpha-1)}\nu^{\alpha-1} ,$$

as $\alpha \ge 1$. Thus we have shown (2.14) $(K = 1)$.

Case (ii). Let C_i *be an open disc with center at* $\zeta_i/|\zeta_i|$ and radius $1/2\nu$. Form D by (2.27). Assume that $z \in \Gamma_j$. So

$$(2.32) \qquad z = \zeta_i/|\zeta_i| + \rho, \quad |\rho| = \frac{1}{2\nu} .$$

We now estimate

$$K(\Gamma) = \max_{z\in\Gamma} |h(z)|,\ \Gamma = \{z, z = \frac{\zeta_i}{|\zeta_i|}(1 + \frac{1}{2\nu}e^{i\theta}), \quad |\theta| \le \frac{\pi}{4}\} .$$

According to (2.18)

$$K(\Gamma) \le e(4\nu)^\alpha[\max_{z\in\Gamma} \prod_{t=1}^{n} |z - \zeta_t|] ,$$

for $\nu \ge m - n + \alpha$. Let $\eta_i = (1 + \frac{1}{2\nu})\zeta_i/|\zeta_i|$. Clearly $\eta_i \in \Gamma_j$. We claim that for $z \in \Gamma$ or z of the form (2.32) which is in Γ_j we have

$$\frac{|\eta_i - \zeta_t|}{5} \le |z - \zeta_t| \le 3|\eta_i - \zeta_t| .$$

Indeed it is easy to see that for such z the following inequalities hold

$$|z - \zeta_t| \geq \frac{1}{4\nu}, \quad |\eta_i - \zeta_t| \geq \frac{1}{2\nu}, \quad |z - \eta_i| \leq \frac{1}{\nu} .$$

So

$$|\eta_i - \zeta_t| \leq |z - \eta_i| + |z - \zeta_t| \leq 4|z - \zeta_t| + |z - \zeta_t| = 5|z - \zeta_t| ,$$

$$|z - \zeta_t| \leq |z - \eta_i| + |\eta_i - \zeta_t| \leq 2|\eta_i - \zeta_t| + |\eta_i - \zeta_t| = 3|\eta_i - \zeta_t| .$$

Therefore

$$K(\Gamma) \leq e3^n(4\nu)^\alpha \prod_{t=1}^{n} |\eta_i - \zeta_t| .$$

Let $z = \zeta_i/|\zeta_i| + \rho \in \Gamma_j$, $|\rho| = \frac{1}{2\nu}$. Then by Lemma 2 and the above inequalities

$$|g(z)| \leq \frac{K(\Gamma)K(m - \ell)}{\prod_{t=1}^{n} |z - \zeta_t|} \leq K(m - \ell)(15)^n(4\nu)^\alpha e ,$$

and

$$|g(z)z^{\nu+m-n-1}| \leq K(m - \ell)(15)^n(4\nu)^\alpha e(1 + \frac{1}{2\nu})^{\nu+m-n-1} \leq K(m - \ell)(15)^n(4\nu)^\alpha e^2$$

for $\nu \geq m - n + \alpha$. As the length of the boundary of D does not exceed $\pi n/\nu$ from (2.20) we get

$$|a_\nu| \leq K(m - \ell)n(15)^n 4^\alpha e^2 \nu^{\alpha-1} .$$

Case (iii). In this case we claim that there exists $1 \leq \gamma \leq s$ such that

(2.33) $$\delta_{\beta+1} < \frac{1}{2^{n+2}\nu n} + \max_{0\leq j\leq \beta} \varepsilon_j , \quad \beta = 0,\dots,\gamma - 1 \quad (\varepsilon_0 = 0)$$

and

(2.34) $$\delta_{\gamma+1} \geq \frac{1}{2^{n+2}\nu n} + \max_{0\leq j\leq \gamma} \varepsilon_j ,$$

otherwise either (i) or (ii) hold. (Note the inequality 2.21). Put

(2.35) $$r = (\max_{0\leq j\leq \gamma} \varepsilon_j) + \frac{1}{2^{n+3}\nu n} .$$

It is not difficult to show that $r \leq \frac{1}{2\nu}$. Let $\zeta_i \in S_\beta$. For $\beta \leq \gamma$ denote by C_i a disc with center at $\zeta_i/|\zeta_i|$ and radius r. For $\beta > \gamma$ let C_i be a disc with center at ζ_i and radius $\delta_\beta/2$. As before define D by (2.27). Now estimate a_ν from the equality (2.20) using the arguments of the Cases (i) and (ii) in accordance with $\beta > \gamma$ or $\beta \leq \gamma$ to deduce (2.14). This concludes the proof of Theorem 2. $\square$

Remark 1. A special case of Theorem 2, namely $\alpha = 1$ and $m = n - 1$ was established in Morton [1964].

Proof of Theorem 1.

(A) ==> (R). Follows immediately from (2.3).

(R) ==> (A). Let $(I - zA)^{-1} = (f_{ij}(z))_1^n$.

Then $f_{ij}(z) = p_{ij}(z)/q_{ij}(z)$, where the degrees of p_{ij} and q_{ij} are $n - 1$ and n respectively. Now (1.3) follows from Theorem 2.

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Let $\mathcal{A}$ be a set of $n \times n$ complex matrices A which satisfy the condition $\|(I - zA)^{-1}\| \le K/(1-|z|)^{\alpha+1}$ for some $\alpha \ge 0$ and all $|z| < 1$. Then it is shown here that there exists a constant $\rho(\alpha,n)$ such that $\|A^\nu\| \le K\rho(\alpha,n)\nu^\alpha$, $\nu = 0,1,\ldots$. This forms a generalization of the Kreiss resolvent condition ($\alpha = 0$).

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