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THE FORWARD AND ADJOINT MONTE CARLO ESTIMATION OF THE KILL PROBABILITY OF A CRITICAL COMPONENT INSIDE AN ARMORED VEHICLE BY A BURST OF FRAGMENTS
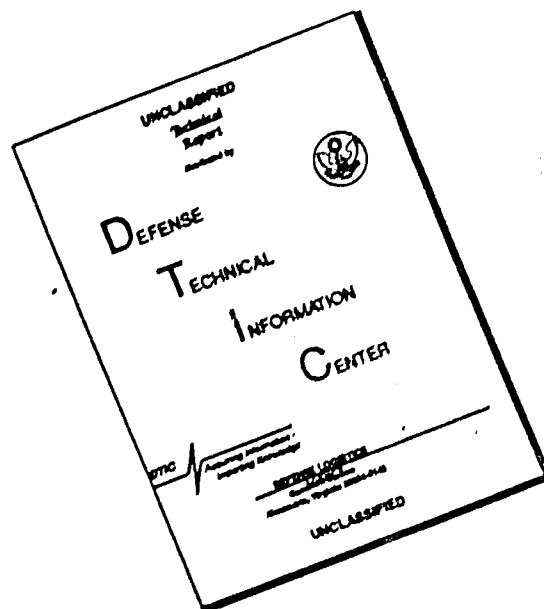
William Beverly

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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Alternate forms of the defining integral which predicts the kill probability of a critical component in a military vehicle by a burst of fragments have been derived. A brief outline of a general Monte Carlo solution of each form is given. The calculational advantages of applying each solution in a ballistic vulnerability methodology are noted. A hypothetical problem is devised and its solution is calculated using each method. The answers to the problem agreed within the calculated standard deviations.

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I. INTRODUCTION

A burst of debris fragments is produced when an AP round or shaped-charge jet perforates the armor of a tank. These fragments can damage interior components and injure crew members and thus contribute to the disruption of the proper operation of the vehicle. Consequently, in Army vehicle vulnerability studies, this spall-burst damage must be evaluated and added to that caused by the primary round or jet to obtain the total incapacitation of the vehicle.

Experimental spall-burst data, gathered at the Ballistic Research Laboratory (BRL)[1,2,3] of the US Army Armament Research and Development Command (USARRADCOM), were given to Systems, Science, and Software (S[3]) to analyze and develop into a spall-burst model. Their primary conclusion[4] was that the available data were so sparse and possessed such huge scatter that the final development of a predictive model was impossible at that time. However, they did conclude that the distribution of fragments, averaged over several similar firings, appeared to be symmetrical about an axis located midway between the normal to the inner-armor surface and the direction of the perforating round or jet (Figure 1).

Calculational studies have been conducted at BRL to determine the increase in the kill rate of vehicles when the contribution of

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[1]B. Rodin, "Documentation of Spall Produced by a 3.2 Inch Precision Shaped Charge," BRL Interim Memorandum Report No. 226, May 1974.

[2]R.J. Pipino, R.C. Brown, and L.T. Brown, "Compilation of Data on Spall Produced from Armor Plate by Detonation of a Shaped Charge," THOR Technical Note No. 138, June 1968.

[3]R.J. Pipino and L.T. Brown, "Spall Decks and Tapes-Identification and Contents, " THOR/Falcon Letter Report dated November 9, 1971.

[4]R.T. Sedgwick, P.L. Anderson, M.S. Chawla, C.R. Hastings, and L.J. Walsh, "Characterization of Behind-the-Armor Debris Resulting from Shaped-Charge Jet Formation," BRL Contract Report No. 249, July 1975.
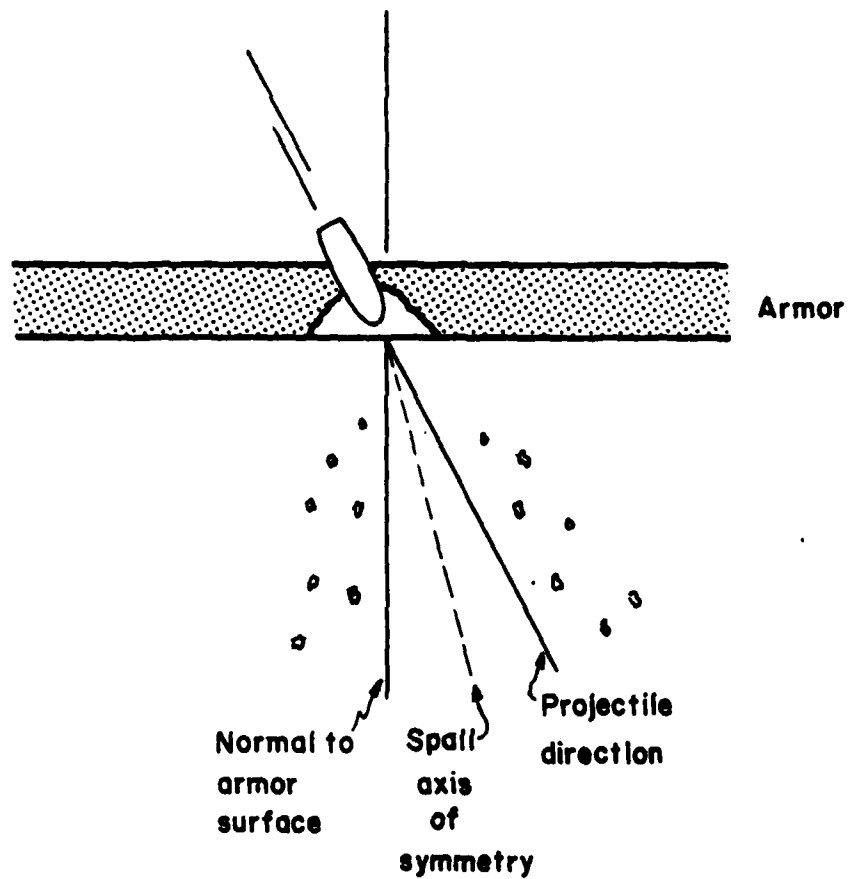
Figure 1.   The Geometry of a Spall Burst

the spall-burst damage is included. A. Hafer[5] has calculated the kill rate of a tank exposed to shaped-charge land mines. T. Hafer[6] has calculated the kill rate of a tank exposed to shaped-charge rounds. Each of these calculations was performed using the VAST[7] assemblage of computer programs. A significant contribution by spall bursts to the incapacitation of vehicles was reported by each study.

The components which are essential to a specified operation of the vehicle and are susceptible to being disabled by the direct impact of representative projectiles are identified as critical. The remaining components of the vehicle are identified as inert and participate in vulnerability studies only insofar as they may shield components. The fractional loss of capability (incapacitation) by a critical component is given by the incapacitation version of the disablement function.[8] An incapacitation function can often be approximated by a kill function which gives the probability of a total loss of capability (kill) per impact.[8,9] This study uses only the kill version of the disablement function.

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[5] A. Hafer, "Analytical Evaluation of Spall Suppression in the Soviet T-55 Tank as a Countermeasure to the U.S. XM-70," Mine-Countermeasure Quarterly Symposium, Picatinny Arsenal, New Jersey, 1 October 1975.

[6] T. Hafer, "The SHUTE Internal Point Burst Vulnerability Assessment Program," Proceedings of the 2nd Symposium on Vulnerability and Survivability, Volume II, January 1977.

[7] "Vulnerable Areas for Surface Targets (VAST); An Internal, Point-Burst Model," compiled by C. Nail, Computer Sciences Corporation, Contract DAAK40-78-D-003/P0009, September 1978.

[8] W.B. Beverly, "The Disablement Functions of a Critical Component and their Use in Vehicle Ballistic Vulnerability Calculations," to be published as a USAARADCOM/BRL Report.

[9] L. Kruse and P. Brizzolara, "An Analytical Method for Deriving Conditional Probabilities of Kill for Target Components," US Army Ballistic Research Laboratories Report No. 1563, December 1971.

In the stochastic model of Beverly,[10] the descriptions (states) of representative samples of fragments which have perforated a given amount of material are assumed to be given by probability density functions (PDF) of incomplete sets of parameters. The killing probability of a spall burst (Figure 2) described by $B^*(\vec{a}_0)$ is calculated in terms of the expectation of the PDF of each impacting fragment and the kill function $R^*(\vec{a}_1)$ of the component. In analytic form, the kill probability P of the spall burst is given by

$$P = 1 - e^{-\lambda} \qquad (1A)$$

where λ is given by either

$$\lambda = \int_\infty B^*(\vec{a}_0) T^*[\vec{a}_0, \vec{g} \rightarrow P^*(\vec{a}_1 | \vec{a}_0)]$$
$$\int_\infty P^*(\vec{a}_1 | \vec{a}_0) R^*(\vec{a}_1) dA_1 \, dA_0, \qquad (1B)$$

or the more concise form

$$\lambda = \int_\infty B^*(\vec{a}_0) \int_\infty P^*(\vec{a}_1 | \vec{a}_0) R^*(\vec{a}_1) dA_1 \, dA_0 \cdot \qquad (1C)$$

The quantities used in the preceding equations are defined as:

$\vec{a}_0$ = a set of parameters which incompletely describes a source fragment,

$\vec{a}_1$ = a set of parameters which incompletely describes a residual fragment at impact with a critical component,

$\vec{g}$ = a set of parameters which describes the medium between the fragment origin and the critical component,

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[10] W.B. Beverly, "The Stochastic Representation of the Perforation of Barriers by Steel Fragments in Military Vehicle Ballistic Vulnerability Studies," to be published as a USAARRADCOM/BRL Report.

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dA_0 = the infinitesimal volume associated with $\vec{a}_0$,

dA_1 = the infinitesimal volume associated with $\vec{a}_1$,

$B*(\vec{a}_0)$ = the spall-burst source term,

$R*(\vec{a}_1)$ = the average killing probability of a representative sample of fragments described by a_1, as they impact the critical component,

$P*(\vec{a}_1/\vec{a}_0)$ = The PDF of residual fragment states derived from a source fragment described by $\vec{a}_0$,

$T*[\vec{a}_0, \vec{g} \rightarrow P*(\vec{a}_1/\vec{a}_0)]$ = The operation which describes the average transport of a representative sample of *source fragments*, all described by $\vec{a}_0$, to a possible impact with the critical component.

The spall-burst source term normalizes to the number of significant fragments N; i.e.,

$$\int_\infty B^*(\vec{a_0})\, dA_0 = N.\qquad (1D)$$

The incomplete parameter sets $\vec{a}_0$ and $\vec{a}_1$ are composed of

$$\vec{a_0} = [\vec{r_0}, \vec{p_0}, (a_0)_7, \cdots (a_0)_K],\qquad (1E)$$

$$\vec{a_1} = [\vec{r_1}, \vec{p_1}, (a_1)_7, \cdots (a_1)_K],\qquad (1F)$$

where $\vec{r}_0$ and $\vec{r}_1$ are position vectors which locate the fragments, $\vec{P}_0$ and $\vec{P}_1$ are the directions of the fragment, $(a_0)_7$ and $(a_1)_7$ are the seventh components of $\vec{a}_0$ and $\vec{a}_1$, and K is the dimensionality of both $\vec{a}_0$ and $\vec{a}_1$. We will use the more explicit form of equation 1B in the following discussion since this form can be more easily related to the Monte Carlo solution which is given later in this report.

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The integral in equation 1B is identified here as the kill integral. The star superscript, sometimes used to identify adjoint quantities, is used here to identify problem-dependent quantities.[10] The dagger superscript is introduced later in this study to identify the adjoint transport operation.

We have assumed in equation 1B that the shape and orientation of each fragment are described in some detail. Since this detail can not be predicted by the current spall-burst and fragment-transport models, we must modify equation 1B to form which is compatible with the present level of detail in these models. The nomenclature of equation 1B is retained insofar as possible, but the form of the functions and operations will change as their parameter lists are shortened. The transport operation is replaced by a simpler operation which merely predicts the average mass and velocity of the largest residual fragment. The spall-burst source term and the critical component kill function are averaged over a representative sample of shapes and orientations and are also given in units of the mass and velocity of the fragment.

We will apply four additional approximations to the description of spall-burst phenomena to bring vulnerability calculations to a reasonable level of effort. The effect of these approximations on the accuracy of vulnerability predictions is not investigated here but should be evaluated in future studies. The first and third of the approximations are suggested in Reference 4. These approximations are briefly tabulated as:

1. All fragments in a burst (source fragments) originate at the same point.

2. All fragments follow straight paths from their birth to their interaction with the critical component.

3. The probability density functions (PDF) describing an average burst are symmetrical about the axis described in Figure 1.

4. The PDF of states for a fragment emerging from a transport operation is replaced by the mean value of the used parameters. The uncertainty introduced into vulnerability calculations by this approximation is being investigated at BRL.[11]

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[11] W.B. Beverly, "The Use of Correlated Sampling in the Monte Carlo Calculation of Ballistic-Vulnerability Predictions for Military Vehicles," to be published as a USAARRADCOM/BRL Technical Report.
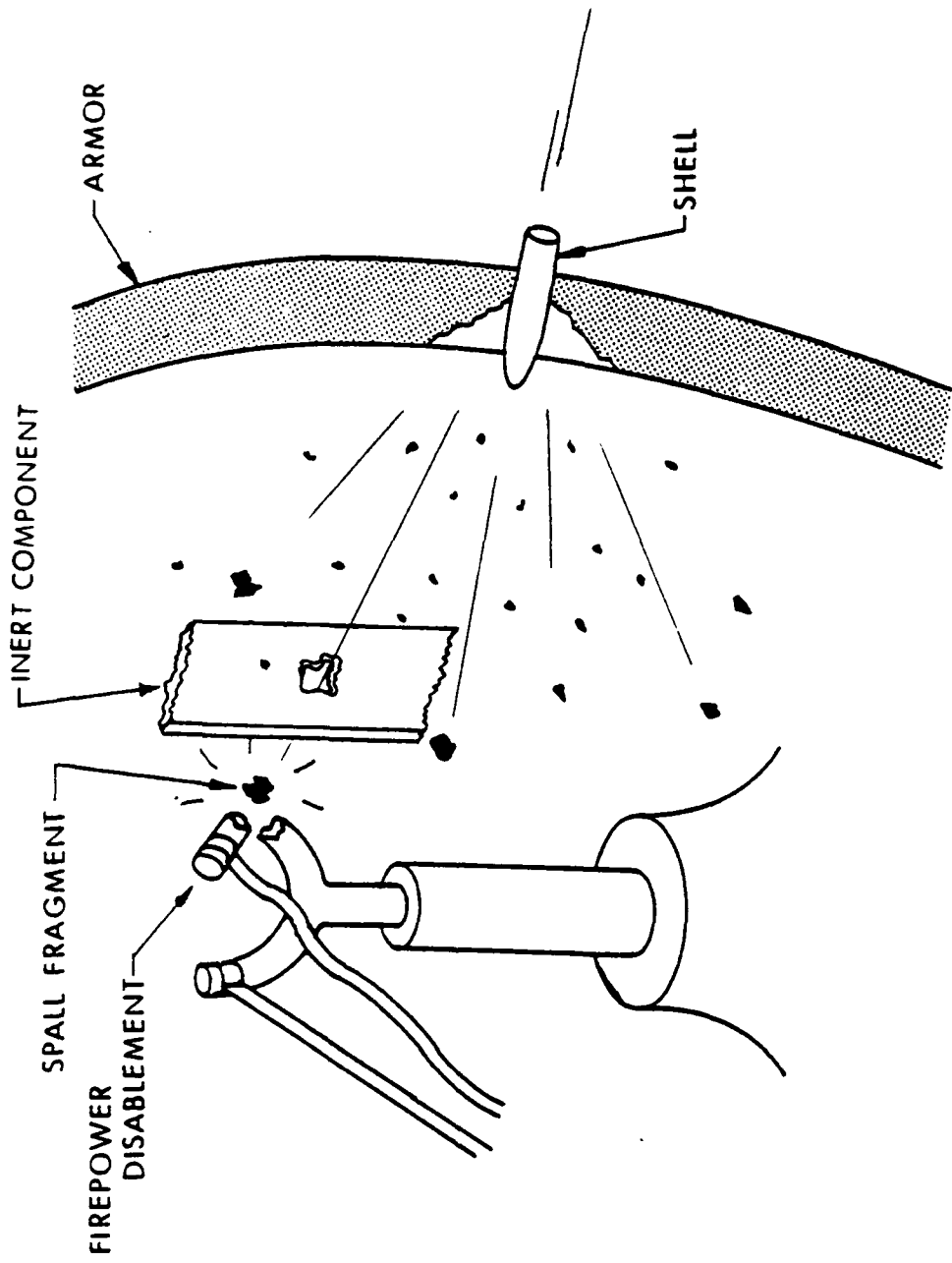
Figure 2. An Example of a Firepower Kill

ARMOR

SHELL

INERT COMPONENT

SPALL FRAGMENT

FIREPOWER DISABLEMENT

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Applying these approximations, equation 1B becomes

$$\lambda = \int_0^{\pi/2} \int_0^\infty \int_0^\infty B^*(\bar{r}_0, m_0, v_0, \theta)$$
$$T^*(\bar{r}_0, m_0, v_0, \theta, \bar{g} - \bar{r}_1, \bar{m}_1, \bar{v}_1)$$
$$R^*(\bar{r}_1, \bar{m}_1, \bar{v}_1) \, dm_0 \, dv_0 \, d\theta \, , \qquad (2)$$

where

m_0 = the mass of a source fragment,

v_0 = the velocity of a source fragment,

θ = the angle made by the fragment path with the spall-burst axis of symmetry,

$\bar{m}_1$ = the average expected mass of the residual fragment at impact with a critical component for all source fragments having mass m_0 and velocity v_0,

$\bar{v}_1$ = the average expected velocity of the residual fragment at impact with a critical component for all source fragments having mass m_0 and velocity v_0.

The transport of steel fragments through inert material (non-critical components) is currently described by the THOR parametric equations[12] given below. The coefficients used in these equations have been evaluated for different materials using a modified least-squares technique to fit the data gathered from firing experiments. The coefficients are averaged over fragment

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[12]"The Resistance of Various Metallic Materials to Perforation by Steel Fragments; Empirical Relationships for Fragment Residual Velocity and Resident Weight," Project THOR Technical Report No. 47, April 1961, The Johns Hopkins University, Baltimore, Maryland.

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shapes to the extent that separate sets of coefficients are calcu-
lated for chunky fragments and for fragments having no particular
shape. The THOR perforation equations are:

$$\bar{m}_1 = m_0 - 10^{C6}(AT)^{C7}m_0^{C8}(Sec\,\eta)^{C9}v_0^{C0}, \qquad (3A)$$

$$\bar{v}_1 = v_0 - 10^{C1}(AT)^{C2}m_0^{C3}(Sec\,\eta)^{C4}v_0^{C5}, \qquad (3B)$$

where

$\bar{m}_1$ = the average mass (grains) of the largest perforating
fragment,

$\bar{v}_1$ = the average velocity (ft/sec) of the residual fragment,

m_0 = the mass of a source projectile (the projectiles used
here are not actual fragments.),

v_0 = the velocity of the source projectile,

$C1, \cdots, C0$ = the fitting coefficients (A set of values for the
coefficients was derived for each target material.),

A = average impact area of the fragment (in^2),

T = target thickness (in),

η = the angle between the fragment shot line and the
normal to the target material.

The general outlines of three different Monte Carlo methods
for evaluating the kill integral in equation 2 are developed in
Section II of this report. The necessary transformations and
changes are developed and explained for each method. The three
methods are:

1. The Forward Transport of Sample Fragments from the Spall-
Burst Origin to A Possible Impact with the Critical Component.
Importance sampling (Appendix A) is not used in the picking of sample
path rays. The PDF of sample fragments in this method is approxi-
mately that of fragments in an actual burst.

2. The Forward Transport of Sample Fragments from the Spall-
Burst Origin to A Possible Impact with the Critical Component.
A method of importance sampling was suggested by Reisinger[13] where

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[13] *M. Reisinger, USARRADCOM/BRL, Private Communication.*

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the path of a sample fragment is constructed through a point picked with equal probability at any location within a rectangular volume which barely encloses the critical component. An approximation implementation of this method is used in the current BRL vehicle vulnerability methodology.[14] An exact analytical representation of this method is derived here and its use in vulnerability studies is outlined. The PDF of sample fragments in this method differs from the PDF of fragments in an actual burst.

3. The Adjoint Transport of Sample Residual Fragments from the Outer Surface of the Critical Component to the Spall-Burst Origin. The importance sampling used in the second method is also used here. The sample events used here are calculational in their nature and do not correspond to any actual events.

In Section III of this report, a test problem is devised and its Monte Carlo solution is calculated using each of the preceding approaches. In each solution, the transport of fragments is assumed to be described by the THOR equations 3A and 3B. In these solutions, the geometrical and distributional symmetries of the test problem are utilized to simplify the calculations. However, the solutions demonstrate the essential features of vehicle vulnerability calculations for spall bursts.

II. THE MONTE CARLO CALCULATION OF THE KILL PROBABILITY OF A CRITICAL COMPONENT BY A BURST OF FRAGMENTS

A. The Forward Transport of Fragments Without Importance Sampling

The general solution of equations 2A and 2B are outlined below. In this solution, each calculational event will correlate, within the accuracy of the vehicle vulnerability model,[10] to a real event.

1. Pick a set of parameter values for a source fragment by sampling the source term (Appendix A). These parameter values are identified as $[\vec{r}_o,(m_o)_i,(v_o)_i,\theta_i]$ where i is the running index over sample fragments. The burst origin $\vec{r}_o$ is included as an argument although all fragments in a burst are assumed to originate at the same point.

2. Transport the fragment through any intervening inert components to a possible impact with the critical component. The

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[14] L. Losie and T. Hafer, "RIP", to be published as a USAARRADCOM/ BRL Report.

parameter values of an impacting fragment are identified as
$[(\vec{r}_1)_i, (\overline{m}_1)_i, (\overline{v}_1)_i, \theta_i]$.

3. Calculate the score S_i for the sample event as
$P^*[(\vec{r}_1)_i, (\overline{m}_1)_i, (\overline{v}_1)_i]$. In this instance, the score is the kill probability of the impact. A fragment which does not impact the component is given a score of 0.

4. Repeat steps 1 through 3 for a total of I sample fragments. The value of the kill integral is then estimated using

$$\overline{\lambda} = (\overline{N} \sum_{i}^{I} S_i) / I = \overline{N}\, \overline{S}. \tag{4A}$$

A bar is placed over a variable to identify a mean value. We have assumed in equation 4A that a Monte Carlo estimate is made of N in a secondary calculation which parallels the main calculation. The details of such a secondary calculation are not discussed in this report but can be easily derived by using the procedures outlined in Appendix A.

5. The quantity $\overline{\lambda}$ will converge toward the true value of λ as an increasing number of fragment histories are conducted and their kill probabilities are calculated and assimilated into equation 4A. The standard deviation of $\overline{S}$, used as a measure of the statistical accuracy of $\overline{S}$, is given by[15]

$$\delta\overline{S} = \left[\frac{\sum S_i^2 - I\,\overline{S}^2}{I(I-1)} \right]^{1/2}, \tag{4B}$$

and the standard deviation of $\overline{\lambda}$ is given by

$$\delta\overline{\lambda} = \left[(\overline{S}\, \delta\overline{N})^2 + (\overline{N}\, \delta\overline{S})^2 \right]^{1/2}, \tag{4C}$$

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[15] Y. Beers, "Introduction to the Theory of Errors," Addison-Wesley Publishing Company, Inc., 1953.

where

$\delta\overline{N}$ = the standard deviation of $\overline{N}$,

$\delta\overline{S}$ = the standard deviation of $\overline{S}$,

$\delta\overline{\lambda}$ = the standard deviation of $\overline{\lambda}$.

6. An estimate of the kill probability of the component by the burst of fragments is given by

$$\overline{PK} \simeq 1 - e^{-\overline{\lambda}} \bullet \tag{1A}$$

The standard deviation of the kill-probability estimate is given by

$$\delta\overline{PK} = e^{-\overline{\lambda}}\delta\overline{\lambda} , \tag{5}$$

where $\delta\overline{P}$ is the standard deviation of $\overline{P}$.

Step 6 completes a brief outline of the calculation of an estimate of P where sample fragments are picked by sampling the unbiased spall-burst term. In this solution, each calculated score S_i is an approximation of the average kill probability of all fragments described by $[\vec{r}_0, (\overline{m}_1)_i, (\overline{V}_1)_i]$.

B. The Forward Transport of Fragments With Importance Sampling

The preceding procedure will often result in the picking of a large number of sample-fragment paths which miss the critical component. In these cases, the calculational effort wasted by the generation and tracking of misses can often be reduced by changing the direction variables of source fragments to new variables whose region of integration permits the picking of sample fragments which impact the critical component with a higher probability (importance sampling). We choose these new variables, whose region of integration RP is the interior of the rectangular parallelepiped which barely encloses the critical component (Figure 3), to be the coordinates of the position vector $\vec{r}_2$.

A sample-fragment path is constructed by first picking a point $\vec{r}_2$ with equal probability at any location in the region RP. A ray is then constructed from the burst origin $\vec{r}_0$ through the sample point $\vec{r}_2$. An alternate region of integration RC can be defined as that part of region RP which will always yield rays which intersect the critical component (Figure 4). The quantities used in the transformation are illustrated in Figure 5 and are developed below in Equations 6A-6H:

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X — RPP WALL & FRAGMENT PATH INTERSECTION.
● — SAMPLE POINT
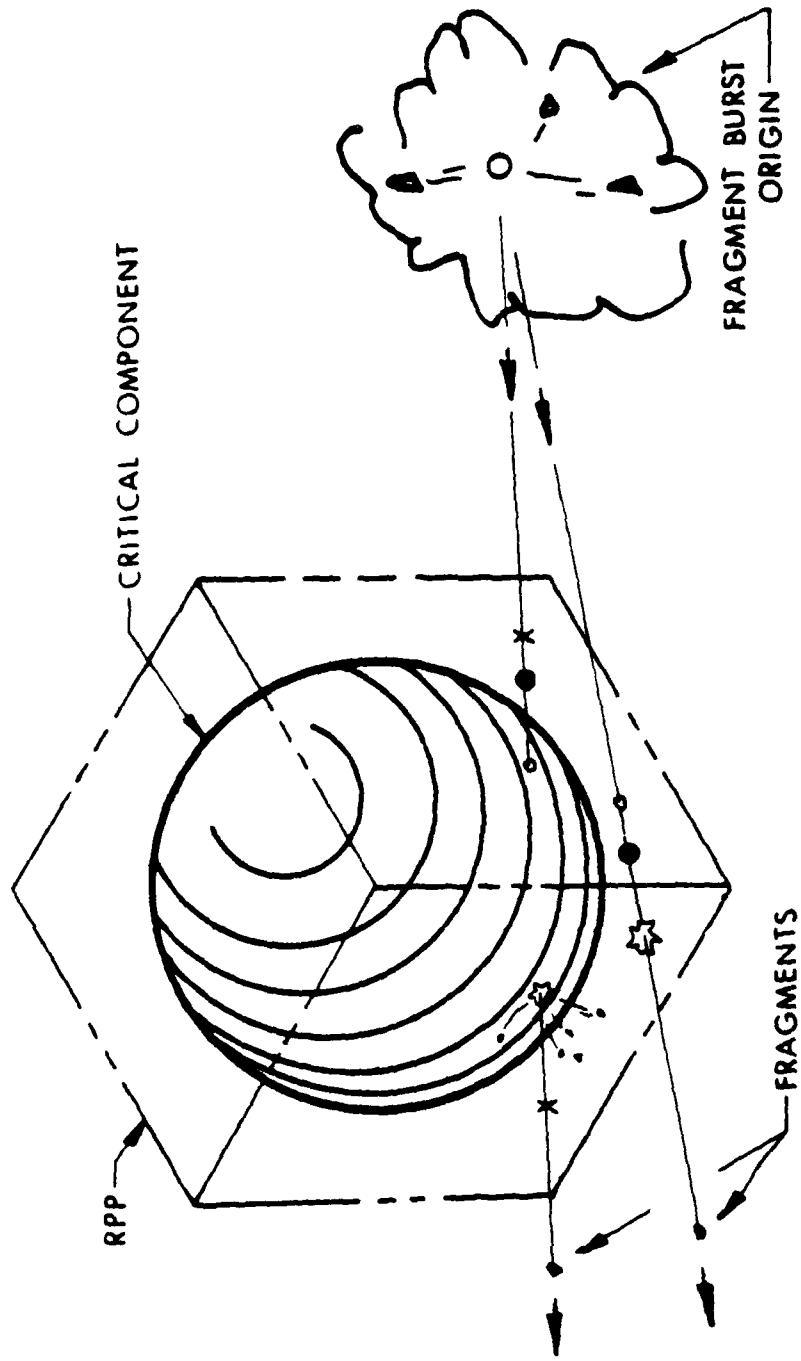
CRITICAL COMPONENT

FRAGMENT BURST
ORIGIN

RPP

FRAGMENTS

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Figure 3. An Example of a Critical Component Inside An RPP
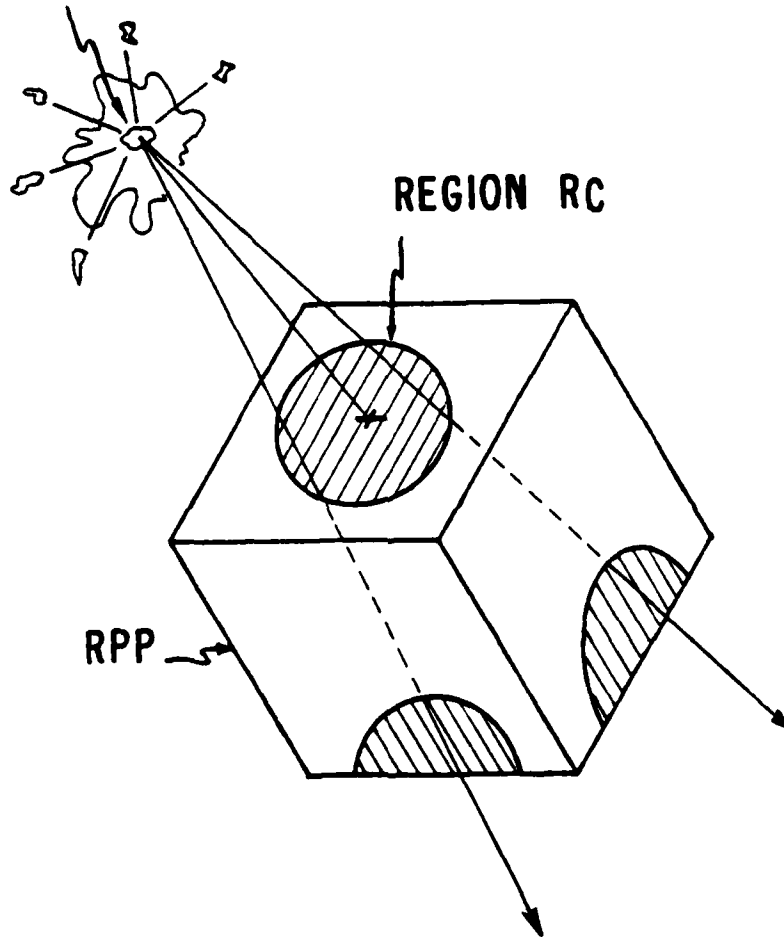
Figure 4. Region RC For A Spherical Critical Component
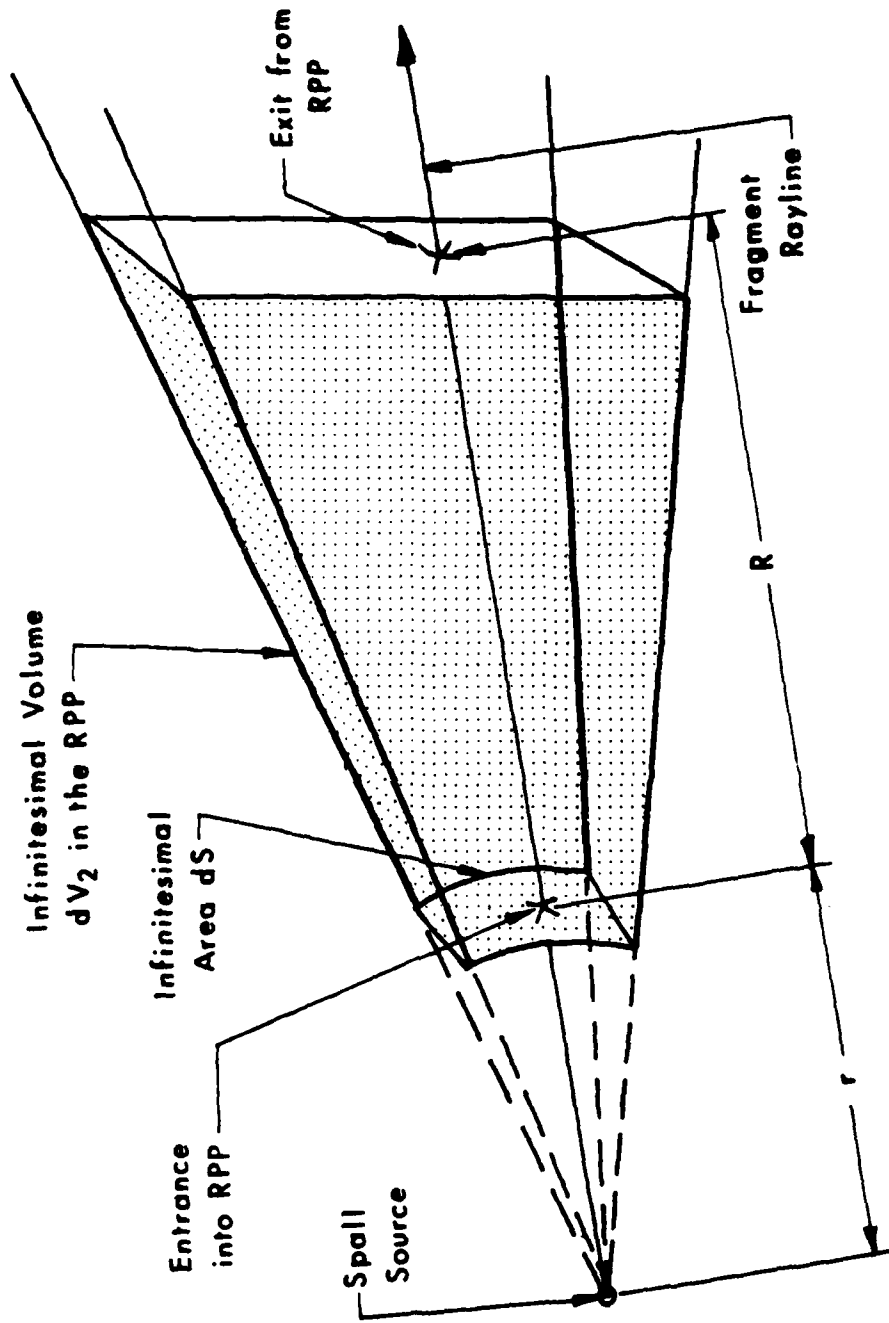
Figure 5. The Infinitesimal Quantities Used In The Transformation Of Spatial Variables

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$$\lambda =$$

$$\int_0^{\pi}\int_0^{\infty}\int_0^{\infty} B^*(\vec{r}_0,m_0,v_0,\theta)\, T^*(\vec{r}_0,m_0,v_0,\theta,\vec{g}-\vec{r}_1,\vec{m}_1,\vec{v}_1)$$

$$R^*(r_1,m_1,v_1)\, dm_0 dv_0 d\theta \ , \tag{6A}$$

$$= \int_0^{2\pi}\int_0^{\infty}\int_0^{\infty} N^*(\vec{r}_0,m_0,v_0,\theta)\, T^*(\vec{r}_0,m_0,v_0,\theta,\vec{g}-\vec{r}_1,\vec{m}_1,\vec{v}_1)$$

$$R^*(\vec{r}_1,\vec{m}_1,\vec{v}_1)\, dm_0 dv_0 d\Omega \ , \tag{6B}$$

$$= \int_{S_C}\int_0^{\infty}\int_0^{\infty} \frac{N^*(\vec{r}_0,m_0,v_0,\theta)\, T^*\left[\vec{r}_0,m_0,v_0,\theta(\vec{r}_2),\vec{g}-\vec{r}_1,\vec{m}_1,\vec{v}_1\right]}{}$$

$$\frac{R^*(\vec{r}_1,\vec{m}_1,\vec{v}_1)\, dm_0 dv_0 \dfrac{dS}{r^2}}{} \ , \tag{6C}$$

$$= V_{RP}\int_{RP}\int_0^{\infty}\int_0^{\infty} \frac{N^*\left[\vec{r}_0,m_0,v_0,\theta(\vec{r}_2)\right]\, T^*\left[\vec{r}_0,m_0,v_0,\theta(\vec{r}_2),\vec{g}-\vec{r}_1(\vec{r}_2),\vec{m}_1,\vec{v}_1\right]}{r^2 F(r,R)}$$

$$\frac{R^*\left[\vec{r}_1(\vec{r}_2),\vec{m}_1,\vec{v}_1\right] dm_0 dv_0\left(\dfrac{dV_2}{V_{RP}}\right)}{} \ , \tag{6D}$$

$$= V_{RC}\int_{RC}\int_0^{\infty}\int_0^{\infty} \frac{N^*\left[\vec{r}_0,m_0,v_0,\theta(\vec{r}_2)\right]\, T^*\left[\vec{r}_0,m_0,v_0,\theta(\vec{r}_2),\vec{g}-\vec{r}_1(\vec{r}_2),\vec{m}_1,\vec{v}_1\right]}{r^2 F(r,R)}$$

$$\frac{R^*\left[\vec{r}_1(\vec{r}_2),\vec{m}_1,\vec{v}_1\right] dm_0 dv_0\left(\dfrac{dV_2}{V_{RC}}\right)}{} \ , \tag{6E}$$

$$N^*(\vec{r}_0,m_0,v_0,\theta) = \frac{B^*(\vec{r}_0,m_0,v_0,\theta)}{2\pi \sin\theta} \ , \tag{6F}$$

$$2\pi r^2 \sin\theta\, d\theta = r^2 d\Omega = dS = \frac{dV}{F(r,R)} \ , \tag{6G}$$

$$F(r,R) = \frac{R}{3}\left[1 + \frac{r+R}{r} + \left(\frac{r+R}{r}\right)^2\right] \ . \tag{6H}$$

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The new quantities introduced in the preceding equations are defined as:

r = the distance along a ray from the spall-burst origin to the first intersection with the RPP,

R = the length of the ray intercepted by the RPP,

dS = an infinitesimal area on the sphere whose center is the spall-burst origin and whose radius is r (This S is used only in equations 6C and 6G to represent area and should not be confused with the S used elsewhere to represent scores.),

$d\Omega$ = the solid angle subtended by dS at the spall-burst origin.

$\vec{r}_2$ = the position vector which locates a point in region RP or RC,

dV_2 = the infinitesimal volume in the RPP which is associated with dS (Figure 5),

S_C = the surface of the critical component which can be impacted by fragments from the spall burst,

RP = the region enclosed by the RPP,

RC = the region inside the RPP which barely encloses all possible fragment rays which intersect the critical component,

V_{RP} = the volume of the region RP,

V_{RC} = the volume of the region RC.

In its present form, the source term $N^*[\vec{r}_o, m_o, v_o, \theta(\vec{r}_2)]$ in equation 6E is not tractable to the efficient picking of sample fragments. Using the procedure outlined in Appendix A, we will first change the kill integral to a form where N^* is given in terms of the one dimensional function $G(\vec{r}_2)$ and the two dimensional function $Q(m_o, v_o, \vec{r}_2, \vec{r}_2')$. Sample paths can now be picked by sampling $1/V_{RC}'$, and their associated mass and velocity values are picked by sampling Q. These changes in the kill integral are described by:

$$\lambda = V_{RC}' \int_{RC} \frac{G(\vec{r_2'})}{r^2 F(r,R)} \left\{ \int_{RC} \int_0^\infty \int_0^\infty Q(m_0, v_0, \vec{r_2}, \vec{r_2'}) \right]$$

$$T^*[\vec{r_0}, m_0, v_0, \theta(\vec{r_2}), \vec{g} \rightarrow \vec{r_1}(\vec{r_2}), \overline{m_1}, \overline{v_1}]$$

$$R^*[\vec{r_1}(\vec{r_2}), \overline{m_1}, \overline{v_1}] \, dm_0 \, dv_0 \, dV_2 \right\} \frac{dV_2'}{V_{RC}'} \quad , \tag{7A}$$

$$G(\vec{r_2}) = \int_0^\infty \int_0^\infty N^*[\vec{r_0}, m_0, v_0, \theta(\vec{r_2})] \, dm_0 \, dv_0 \quad , \tag{7B}$$

$$Q(m_0, v_0, \vec{r_2}, \vec{r_2'}) =$$

$$\frac{N^*[\vec{r_0}, m_0, v_0, \theta(\vec{r_2})] \, \delta(\vec{r_2} - \vec{r_2'})}{\int_0^\infty \int_0^\infty N^*[\vec{r_0}, m_0, v_0, \theta(\vec{r_2})] \, dm_0 \, dv_0} \quad , \tag{7C}$$

$$\delta(\vec{r_2} - \vec{r_2'}) = \delta[(r_2)_1 - (r_2')_1] \, \delta[(r_2)_2 - (r_2')_2] \, \delta[(r_2)_3 - (r_2')_3] \quad , \tag{7D}$$

where

$\vec{r_2'}$ = a dummy value for $\vec{r_2}$,

dV_2' = a dummy value for dV_2,

RC' = a dummy representation of region RC,

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$\delta[(r_2)_1 - (r_2')_1]$ = the **Dirac** delta function for the x-component of $\vec{r}_2$ and $\vec{r}_2'$.

A brief outline of a Monte Carlo evaluation of the preceding kill integral is given below:

1. Construct an RPP which barely encloses the critical component.

2. Pick a point with equal probability at any location within the enclosing RPP. A running tally U is maintained of the sample points, and is used at the end of all sampling to calculate an estimate of V_{RC}.

3. Construct a ray from the spall-burst origin through the sample point and determine if the ray intersects the critical component. Discard any ray which misses the component and pick another ray until an intersection is obtained. A sample point which leads to an intersection is identified as $(\vec{r}_2')_i$ where i is the sample-fragment index.

4. Calculate the quantities r_i, R_i, (Figure 5) and θ_i for the sample ray.

5. Pick values **for** the mass and velocity of the sample fragment by sampling $Q[m_0, v_0, \vec{r}_2, (\vec{r}_2')_i]$. In practice, Q is changed to one-dimensional functions in a manner similar to that used in the picking of path rays, but we will not outline the operation here. The full set of values is identified as $[\vec{r}_0, (m_0)_i, (v_0)_i, \theta_i]$.

6. Calculate the transport of the sample fragment from the spall-burst origin to a possible impact with the critical component. The parameters of the residual fragment at impact are identified as $[(\vec{r}_1)_i, (\vec{m}_1)_i, (\vec{v}_1)_i]$.

7. Calculate the score S_i of the event. A fragment which is defeated before impacting the component is given a score of 0. The score of a fragment **which** impacts the component is given by

$$S_i = \frac{R^*\{\vec{r}_i[(\vec{r}_2)_i], [\vec{m}_1]_i, [\vec{v}_1]_i\} \, G[(\vec{r}_2)_i]}{r_i^2 \, F[r_i, R_i]} \quad . \tag{8A}$$

8. Repeat steps 2-7 until a total of I sample fragments have been picked whose paths intersect the critical component.

9. Calculate an estimate of λ using

$$\overline{\lambda} = \left(\overline{V}_{RC} \overset{I}{\sum} S_i \right) / I = \overline{V}_{RC} \, \overline{S} \; , \tag{8B}$$

$$\overline{V}_{RC} = \left(I \, \overline{V}_{RP} \right) / U \; , \tag{8C}$$

where U is the number of sample points picked in region RP.

10. Using the appropriate form of equation 4B, calculate the standard deviation of $\overline{V}_{RC}$ and $\overline{S}$. The standard deviation of $\overline{\lambda}$ is then calculated using

$$\delta \overline{\lambda} = \left[\left(\overline{V}_{RC} \delta \overline{S} \right)^2 + \left(\overline{S} \, \delta \overline{V}_{RC} \right)^2 \right]^{1/2} \; , \tag{8D}$$

where $\delta \overline{V}_{RC}$ is the standard deviation of $\overline{V}_{RC}$.

11. An estimate of the kill probability of the component by the burst of fragments is given by

$$\overline{PK} = 1 - e^{-\overline{\lambda}} \; . \tag{1A}$$

The standard deviation of the kill-probability estimate is given by

$$\delta \overline{PK} = e^{-\overline{\lambda}} \delta \overline{\lambda} \; . \tag{5}$$

Step 11 completes the outline of a general procedure for calculating a Monte Carlo estimate of the kill probability of a critical component by a burst of fragments where importance sampling is used during the picking of paths for the sample fragments. The two preceding solutions are similar in that sample fragments are birthed at the burst origin and then transported through any intervening inert material to a possible impact with the critical component. The two solutions differ in that the paths of the sample fragments are picked from different probability density functions.

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C. The Adjoint Transport of Fragments With Importance Sampling

In the preceding method, we changed the direction variables in the kill integral (equation 2) from θ to the coordinates of a point in region RC (equation 6E). In the following method, we will also change the remaining integration variables of equation 6E from m_o and v_o to $\overline{m}_1$ and $\overline{v}_1$. This transformation can be regarded as interchanging the roles of the source function N* and the kill function P*. In a Monte Carlo evaluation of the changed kill integral, a residual fragment is picked at the critical component by sampling P* and then transported back to the burst origin (Figure 6). The event score is then proportional to N* evaluated for the parameter values of the transported fragment.

We assume that the parameters of a source fragment are defined in terms of the parameters of the associated residual fragment by

$$m_0 = m_0\left[\vec{r_1}(\vec{r_2}), \overline{m}_1, \overline{v}_1, \theta(\vec{r_2}), \vec{g}\right] , \tag{9A}$$

$$v_0 = v_0\left[\vec{r_1}(\vec{r_2}), \overline{m}_1, \overline{v}_1, \theta(\vec{r_2}), \vec{g}\right] , \tag{9B}$$

$$\vec{r_2} = \vec{r_2} , \tag{9C}$$

and have continuous derivatives over the region of interest.[16] We also assume that the inverse functions

$$\overline{m}_1 = \overline{m}_1\left[\vec{r_0}, m_0, v_0, \theta(\vec{r_2}), \vec{g}\right] , \tag{9D}$$

$$\overline{v}_1 = \overline{v}_1\left[\vec{r_0}, m_0, v_0, \theta(\vec{r_2}), \vec{g}\right] , \tag{9E}$$

$$\vec{r_2} = \vec{r_2} , \tag{9F}$$

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16. *W. Kaplan, "Advanced Calculus," Addison-Wesley Publishing Company, Inc., 1953*

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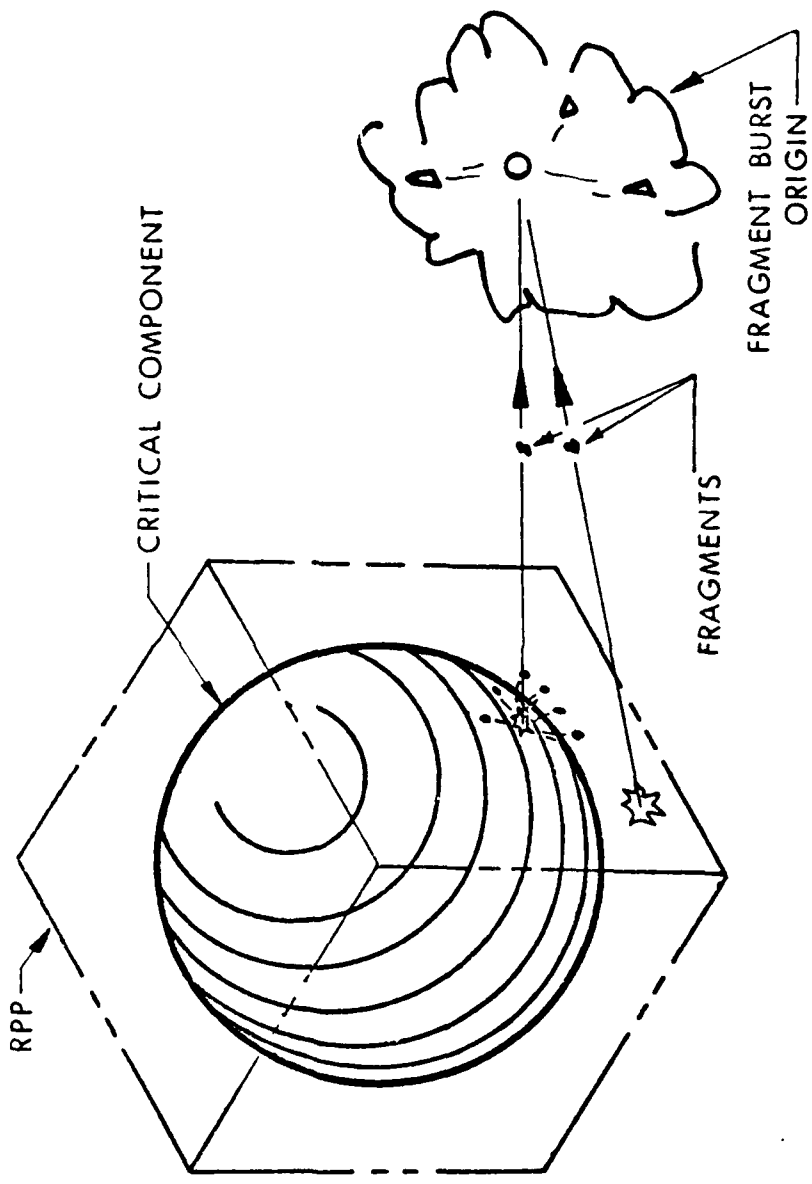
Figure 6. An Example Of The Adjoint Transport Of Fragments

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are defined and have continuous derivatives over the same region. Equations 9D-9F are identified in the integral of equation 6E as the (forward) transport operation, and equations 9A-9C will be identified in the following development as the adjoint transport operation.

The kill integral now becomes

$$\lambda =$$

$$\int_{RC}\int_0^\infty\int_0^\infty \frac{R^*[\vec{r_1}(\vec{r_2}),\overline{m}_1,\overline{v}_1]T^\dagger[\vec{r_1}(\vec{r_2}),\overline{m}_1,\overline{v}_1,\vec{g}\rightarrow\vec{r}_0,m_0,v_0,\theta(\vec{r_2})]}{r^2 F(r,R)}$$

$$\frac{N^*[\vec{r}_0,m_0,v_0,\theta(\vec{r_2})]J(\overline{m}_1,\overline{v}_1)d\overline{m}_1\,d\overline{v}_1,dV_2}{} \quad , \quad (10A)$$

$$J(\overline{m}_1,\overline{v}_1) = \left|\frac{\partial(m_0,v_0)}{\partial(\overline{m}_1,\overline{v}_1)}\right| \quad , \quad (10B)$$

where $T^\dagger[\vec{r}_1(\vec{r}_2),\overline{m}_1,\overline{v}_1,\vec{g}\rightarrow\vec{r}_0,m_0,v_0,\Theta(\vec{r}_2)]$ is the adjoint transport operation. The terms in the integrand are rearranged so that the source term for residual fragments P* is leftmost in the integrand.

In its present form, the kill integral in equation 10A is not tractable to a Monte Carlo solution. Applying the procedure outlined in Appendix A and used in the preceding forward solution, the integral is changed to the more tractable form:

$$\lambda = V'_{RC} \int_{RC'} \frac{H(\vec{r_2}')}{\iint_{RC} \int_0^\infty \int_0^\infty \frac{L(\overline{m}_1, \overline{v}_1, \vec{r_2}, \vec{r_2}')}{r^2 F(r,R)}}$$

$$\underline{T^\dagger[\vec{r_1}(\vec{r_2}), \overline{m}_1, \overline{v}_1, \vec{g} \rightarrow \vec{r_0}, m_0, v_0, \theta(\vec{r_2})]}$$

$$\underline{N^*[\vec{r_0}, m_0, v_0, \theta(\vec{r_2})] J(\overline{m}_1, \overline{v}_1) d\overline{m}_1 d\overline{v}_1 dV_2} \} \frac{dV_2'}{V'_{RC}} \quad , \quad (11A)$$

$$H(\vec{r_2}') = \int_0^\infty \int_0^\infty R^*[\vec{r_1}(\vec{r_2}), \overline{m}_1, \overline{v}_1] d\overline{m}_1 d\overline{v}_1 \quad , \quad (11B)$$

$$L(\overline{m}_1, \overline{v}_1, \vec{r_2}, \vec{r_2}') = \frac{R^*[\vec{r_1}(\vec{r_2}), \overline{m}_1, \overline{v}_1] \delta(\vec{r_2} - \vec{r_2}')}{\int_0^\infty \int_0^\infty P^*[\vec{r_1}(\vec{r_2}'), \overline{m}_1, \overline{v}_1] d\overline{m}_1 d\overline{v}_1} \quad , \quad (11C)$$

$$\int_{RC} \int_0^\infty \int_0^\infty L(\overline{m}_1, \overline{v}_1, \vec{r_2}, \vec{r_2}') d\overline{m}_1 d\overline{v}_1 dV_2 = 1 \quad . \quad (11D)$$

This form of the integral can be evaluated by applying the procedure outlined for the preceding solution. However, the reader should note that the sample events used here are strictly calculational in their nature and do not correspond to any actual ballistic events. A brief outline of such a solution is given below:

1. Construct an RPP which barely encloses the critical component.

2. Pick a **sample-fragment** path $(\vec{r_2})_i$ using the method outlined in the preceding forward solution.

3. Calculate the quantities r_i, R_i, and θ_i.

4. Pick a birth set of values for the mass and velocity of the residual fragment by sampling $L(\overline{m}_1, \overline{v}_1, \vec{r_2}, \vec{r_2}')$. The sample fragment

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is identified as $[(\vec{r}_1)_i, (\bar{m}_1)_i, (\bar{v}_1)_i]$. The fragment is assumed to be birthed on the outer surface of the critical component which is nearest to the spall-burst origin.

5. Transport the sample fragment through any intervening inert material to the spall-burst origin.

6. Calculate the score of the event using

$$S_i = \frac{H[(\vec{r}_2)_i] \, N^*[\vec{r}_0, (m_0)_i, (v_0)_i, \theta_i] \, J[(\bar{m}_1)_i, (\bar{v}_1)_i]}{r_i^2 \, F(r_i, R_i)} \qquad (12)$$

A fragment after transport whose mass $(m_0)_i$ or velocity $(v_0)_i$ does not fall within the region of non-zero values of B^* is assigned a score of 0.

7. Repeat steps 2-6 until I events have been conducted.

8. Calculate an estimate of λ using

$$\bar{\lambda} = \left(\bar{V}_{RC} \sum^{I} S_i \right) \Big/ I = \bar{V}_{RC} \bar{S} \qquad (8B)$$

9. Using the appropriate forms of equations 4C, calculate the standard deviation of $\bar{V}_{RC}$ and $\bar{S}$. Calculate the standard deviation of $\bar{\lambda}$ using

$$\delta \bar{\lambda} = \left[(\bar{V}_{RC} \delta \bar{S})^2 + (\bar{S} \delta \bar{V}_{RC})^2 \right]^{1/2} \qquad (8D)$$

10. Calculate an estimate of the kill probability of the critical component using

$$\overline{PK} = 1 - e^{-\bar{\lambda}} \qquad (1A)$$

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11. Calculate the standard deviation of $\bar{P}$ using

$$\delta\overline{PK} = e^{-\bar{\lambda}}\,\delta\bar{\lambda}\,.$$ (5)

Step 11 completes the outline of a general procedure for calculating a Monte Carlo estimate of the kill probability of a component by using the adjoint transport of fragments. An adjoint solution of the test problem is described in the next section of this report.

III. THE TEST PROBLEM AND ITS SOLUTIONS

A. The Test Problem

A cross section of the test problem is illustrated in Figure 7. A spherical critical component having a radius of 10 inches is located 20 inches from a point spall burst. The burst has an axis of symmetry which lies along the line connecting the burst origin to the center of the critical component. An inert component of varying thickness (not shown in Figure 7) lies between the burst origin and the critical component. The inert component is positioned so that it degrades or defeats all fragments before they impact the critical component.

The burst is described by a truncated bivariate normal distribution of fragment mass and velocity. The non-zero values of the distribution are given by

$$B\,(r_0,m_0,v_0,\theta\,) =$$

$$\frac{25\,N(\theta)\sin\theta\,\mathrm{Exp}\!\left[-0.5Z_1^2,-0.5Z_2^2\right]}{(0.9973)^2\,M(\theta)\,V(\theta)}\,,$$ (13A)

$$Z_1 = \left[m_0 - M(\theta)\right]/0.2M(\theta)\,,$$ (13B)

$$Z_2 = \left[v_0 - V(\theta)\right]/0.2V(\theta)\,,$$ (13C)

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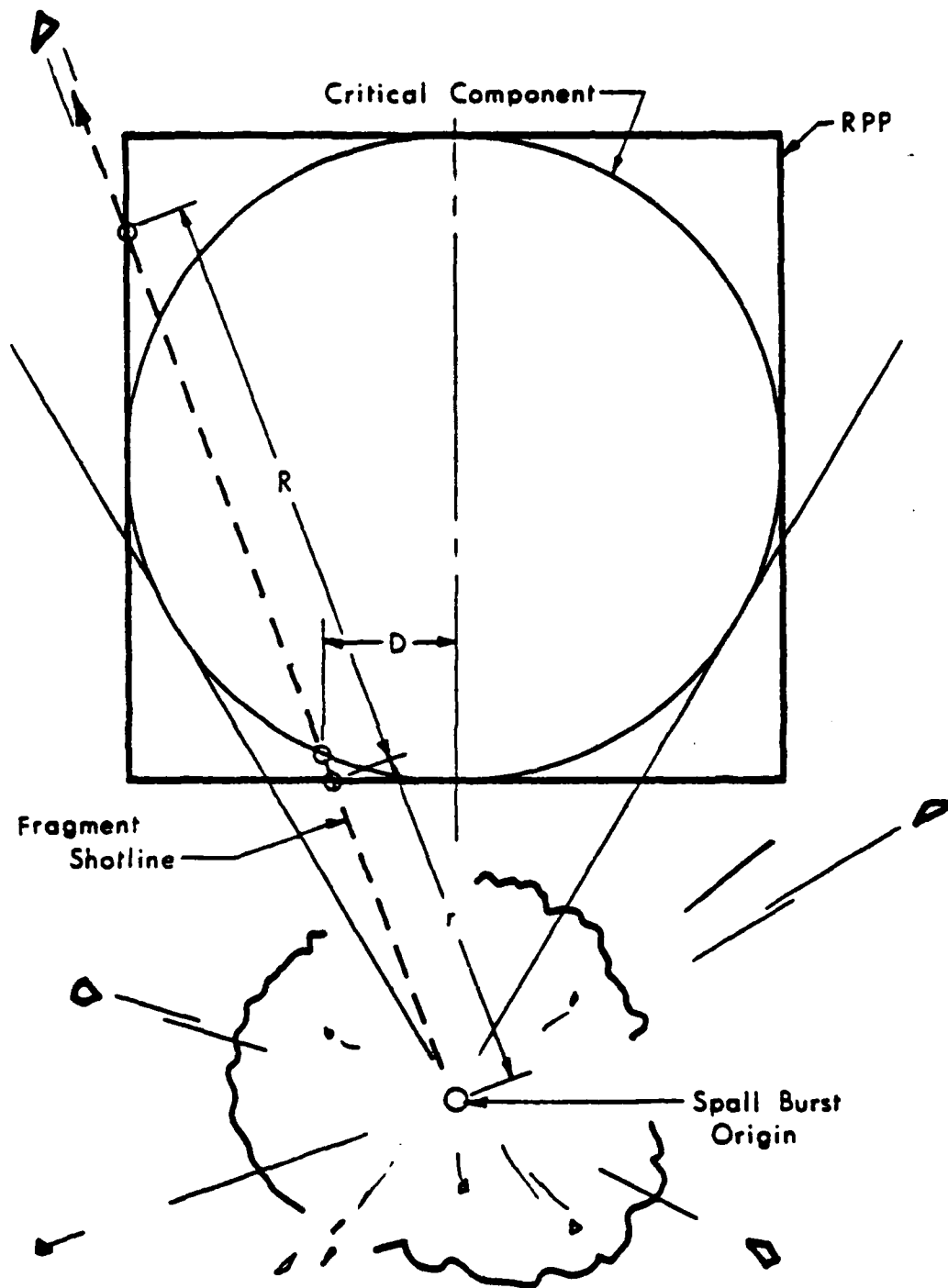
Figure 7. Cross Section Of Test Problem

when

$$0.4\, M(\theta) \leq m_0 \leq 1.6\, M(\theta) \quad , \tag{13D}$$

$$0.4\, V(\theta) \leq v_0 \leq 1.6\, V(\theta) \quad , \tag{13E}$$

where $M(\theta)$ is the mean mass and $V(\theta)$ is the mean velocity of fragments emitted at angle θ with the burst axis. The mean mass is given by

$$M(\theta) = 500 - 3000\, \theta / \pi \quad , \tag{13F}$$

and the mean velocity is given by

$$V(\theta) = 500 + 54000\, \theta / \pi \quad . \tag{13G}$$

The function $N(\theta)$ is given by

$$N(\theta) = 5 + 180\, \theta / \pi \quad , \tag{13H}$$

The star superscript is dropped from the problem dependent quantities is this example. We assume that no correlation exists between the mass and velocity of fragments.

The distribution of inert material between the burst source and the critical component is described by a truncated normal distribution at each emission angle θ. The probability $P(t)$ of a thickness is given by

$$P(t) = 5\, \mathrm{Exp}\left[-0.5\, Z_3^2\right] / \left[0.9973\, T(\theta)(2\pi)^{1/2}\right], \tag{13I}$$

$$T(\theta) = 0.3 - 1.2\, \theta / \pi \quad , \tag{13J}$$

$$Z_3 = \left[t - T(\theta)\right] / \left[0.2\, T(\theta)\right] \quad , \tag{13K}$$

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$$0.4\,T(\theta) \leq t \leq 1.6\,T(\theta)\,,\tag{13L}$$

where $T(\theta)$ is the average thickness of inert material at angle θ. The quantity $P(t)$ is used only in the preceding equation to identify the material thickness probability, and should not be mistaken for the kill function P^* which is used elsewhere in this report.

The kill function of the critical component is given by

$$R(\vec{r}_1,\overline{m}_1,\overline{v}_1) = 10^{-8}[20\overline{m}_1 + \overline{v}_1 + \overline{m}_1\,\overline{v}_1][30 - D(\theta)]\tag{13M}$$

where D is the distance from the burst axis to the point of impact with the critical component (Figure 7). The use of the average quantities $\overline{m}_1$ and $\overline{v}_1$ would be expected to introduce an error into actual vulnerability calculations, but we will ignore this effect here as it is being investigated in a current BRL study.[11]

The THOR perforation equations (equations 3A and 3B) are used to describe the transport of fragments. The material coefficients used by these equations are given in statements 30 and 35 of computer program PROB3 (Appendix D) and are not repeated here. All fragments are assumed to be normally incident on the inert component.

B. The Forward Solution Without Importance Sampling

The BASIC computer program PROB1 (Appendix B) is used to calculate the kill probability of the critical component when importance sampling is not used during the picking of sample-fragment paths. The procedures developed and outlined in Section II are used here with minor modifications. Much of the calculational procedure outlined below is given in Section II, but is repeated here for the sake of completeness.

1. Pick the emission angle θ_i of the sample source fragment by sampling the PDF $\Theta(\theta)$

$$\Theta(\theta) = 2\pi \int_0^\infty \int_0^\infty B(m_0, v_0, \theta) \, dm_0 \, dv_0 = 2\pi \sin\theta \, N(\theta) \,. \qquad (14)$$

The picked values lie between 0 and $\pi/2$. The rejection technique[17] is used to perform the sampling. The θ-symmetry of the problem permits the calculation to be performed in the xz - plane.

2. Calculate the average fragment mass M_i (equation 13F) and the average fragment velocity V_i (equation 13G) at emission angle θ_i.

3. Pick a mass $(m_0)_i$ and velocity $(v_0)_i$ by sampling the appropriate normal distribution contained within equation 13A. The rejection technique is used to conduct the sampling.

4. Calculate the average thickness of material lying between the burst origin and the critical component at angle θ_i (equation 13J). Pick a thickness t_i by sampling $P(t)$ (equation 13I). The rejection technique is used to conduct the sampling.

5. Calculate the transport of the sample fragment to a possible impact with the critical component. The residual fragment at impact with the critical component is identified as

$$[(\vec{r}_1)_i, (\overline{m_1})_i, (\overline{v_1})_i].$$

6. Calculate the score (kill probability) of the event as $P[(\vec{r}_1)_i, (\overline{m_1})_i, (\overline{v_1})_i]$ (equation 13M). The quantity D_i is derived from $(\vec{r}_1)_i$. The score is identified as S_i.

[17] Y. A. Shreider, "The Monte Carlo Method," Pergamon Press, Long Island City, New York, 1966.

7. Repeat steps 1 through 6 until I events have been conducted.

8. Calculate an estimate of λ using

$$\bar{\lambda} = (\bar{N} \sum_{i}^{I} S_i)/I = \bar{N}\,\bar{S}. \tag{4A}$$

As noted in equation 1D, the quantity N is the number of significant fragments in the burst.

9. Using the appropriate form of equation 4B, calculate an estimate of the standard deviation of $\bar{N}$ and $\bar{S}$. Calculate the standard deviation of $\bar{\lambda}$ using

$$\delta\bar{\lambda} = \left[(\bar{S}\,\delta\bar{N})^2 + (\bar{N}\,\delta\bar{S})^2\right]^{1/2}. \tag{4C}$$

10. Calculate an estimate of the kill probability of the component by the burst of fragments using

$$\overline{PK} \simeq 1 - e^{-\bar{\lambda}}. \tag{1A}$$

11. Calculate the standard deviation of the kill probability estimate using

$$\delta\overline{PK} = e^{-\bar{\lambda}}\,\delta\bar{\lambda}. \tag{5}$$

Step 11 completes an outline of a Monte Carlo forward calculation of the kill probability of the test component where sample-fragment paths are picked from their natural distribution. The results are tabulated, along with the results obtained using the other two methods of solution, in Table I at the end of Section III.

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C. The Forward Solution With Importance Sampling

The BASIC computer program PROB2 (Appendix C) is used to calculate the kill probability of the test component by using the forward transport of fragments and importance sampling in the picking of sample-fragment paths. The procedures developed and outlined in Section II are used here with minor modification. The calculational procedure used in PROB2 is outlined below:

    1. Construct an RPP which barely encloses the critical component. In this instance, the RPP is the cube whose faces lie within the planes $x=10$, $x=-10$, $y=10$, $y=-10$, $z=10$, $z=-10$.

    2. Pick a point with equal probability at any location within the upper right quarter of the enclosing RPP. Because of the symmetry of the test problem, sample-fragment paths constructed through these points are representative of fragment paths for the total burst.

    3. Construct a ray from the spall-burst origin through the sample point. The ray is used as a fragment path if it intersects the critical component. Continue picking sample points until an intersection is obtained. A running tally U is maintained of the sample points and is used at the completion of all sampling to calculate an estimate of the volume V_{RC}.

    4. Calculate the fragment emission angle θ_i.

    5. Calculate the average mass M_i (equation 13F), average velocity V_i (equation 13G), and average material thickness T_i (equation 13J) at angle θ_i. Pick a mass $(m_o)_i$, velocity $(v_o)_i$, and material thickness t_i by sampling the appropriate normal distribution.

    6. Transport the sample fragment to a possible impact with the critical component. Assign a score of 0 and go to step 9 if the fragment is defeated during transport.

    7. Calculate the coordinates of the point of intersection of the fragment path with the RPP and with the critical component. Calculate D_i, r_i, and R_i.

    8. Calculate the score of the event using

$$S_i = \frac{R\{\vec{r_i}[(\vec{r_2})_i],[\overline{m}_i]_i,[\overline{v}_i]_i\}\,G[(\vec{r_2})_i]}{r_i^2\,F[r_i,R_i]} \cdot \qquad (8A)$$

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9. Repeat steps 2 through 8 for a total of I events.

10. Calculate an estimate of λ using

$$\overline{\lambda} = \left(\overline{V}_{RC} \overset{I}{\sum} S_i \right) / I = \overline{V}_{RC} \overline{S} , \tag{8B}$$

$$\overline{V}_{RC} = \left(I \overline{V}_{RP} \right) / U . \tag{8C}$$

11. Calculate the standard deviation of $\overline{\lambda}$ using

$$\delta \overline{\lambda} = \left[\left(\overline{V}_{RC} \delta \overline{S} \right)^2 + \left(\overline{S} \, \delta \overline{V}_{RC} \right)^2 \right]^{1/2} . \tag{8D}$$

12. Calculate an estimate of the kill probability of the component using

$$\overline{PK} = 1 - e^{-\overline{\lambda}} . \tag{1A}$$

13. Calculate the standard deviation of the kill probability estimate using

$$\delta \overline{PK} = e^{-\overline{\lambda}} \delta \overline{\lambda} . \tag{5}$$

Step 13 completes an outline of the calculation of the kill probability of a component by a burst of fragments where fragment paths are picked using importance sampling. The results are tabulated, along with the results obtained using the other two methods of solution, in Table I at the end of Section III.

D. The Adjoint Solution With Importance Sampling

The BASIC Computer Program PROB3 (Appendix D) is used to calculate the kill probability of the component by using the adjoint transport of fragments and importance sampling in the picking of rays.

Values for the mass and velocity of a sample residual fragment are not picked by sampling P as described in Section 2C. Instead, the integral in equation 11A is changed to the sum of K new integrals where the integration range of each kth new integral is taken over an equal part of the original integral. The total $\bar{m}_1$ and $\bar{v}_1$ range of integration is assumed to extend from (0,0) to the largest m_o and v_o values at which N is zero. This new representation of the kill integral is given by

$$\lambda = \sum^K \int_{RC} \int_{\Delta k} \int \frac{R T^\top N J\, d\bar{m}_1\, d\bar{v}_1\, dV_2}{r^2 F(r,R)} \tag{15}$$

where Δ_k is the integration range of the $\bar{m}_1$ and $\bar{v}_1$ variables in the kth new integral. Equation 15 is made less cumbersome by not including the parameter lists.

This new formulation permits a set of residual fragments to be picked for each sample path. Each fragment in the set is transported back to the burst origin and its score is calculated. The scores of all fragments in a set are totaled to obtain the event score. A step-by-step outline of this calculational procedure is given below:

1. Construct an RPP which barely encloses the critical component. In this instance, the RPP is the cube whose faces lie within the planes x=10, x=-10, y=10, y=-10, z=10, z=-10.

2. Pick a point with equal probability at any location within the upper right quarter of the enclosing RPP. Because of the symmetry of the test problem, sample-fragment paths constructed through these points are representative of fragment paths for the total burst.

3. Construct a ray from the spall-burst origin through the sample point. The ray is used as a fragment path if it intersects the critical component. Continue picking sample points until an intersection is obtained. A running tally U is maintained of the sample points and is used at the completion of all sampling to calculate an estimate of the volume V_{RC}.

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4. Calculate the fragment emission angle θ_i for the picked ray.

5. Calculate the average mass M_i (equation 13F) and average velocity V_i (equation 13G) of a source fragment at angle θ_i. Calculate the average inert material thickness t_i (equation 13J) at angle θ_i.

6. Calculate the coordinates of the point of intersection of the fragment path with the RPP and with the critical component. Calculate D_i, r_i, and R_i.

7. Pick a sample value for the mass and velocity of the kth residual fragment with equal probability at any point within the integration range Δ_k. This fragment is identified as

$$[(\vec{r}_2)_i, (\overline{m}_1)_{ik}, (\overline{v}_1)_{ik}].$$

8. Transport the fragment back to the burst origin using the adjoint transport operation $T^{\dagger}$. As noted in equations 9A through 9C, the inverse of the forward transport equations would ordinarily be used to perform this operation. However, these inverses are difficult to obtain explicitly for the THOR equations 3A and 3B and an alternate method is used. First, we combine the two THOR equations to obtain the monotonically increasing function

$$\overline{m}_1 = \left[\frac{(v_0 - \overline{v}_1)\, v_0^{-\overline{C5}}}{10^{\overline{C1}}\,(aT)^{\overline{C2}}}\right]^{\frac{3}{2\overline{C2}+3\overline{C3}}} -$$

$$\left[\frac{10^{\overline{C6}}\,(aT)^{\overline{C7}}\, v_0^{\overline{C0}}}{}\right]\left[\frac{(v_0 - \overline{v}_1)\, v_0^{-\overline{C5}}}{10^{\overline{C1}}\,(aT)^{\overline{C2}}}\right]^{\frac{2\overline{C7}+3\overline{C8}}{2\overline{C2}+3\overline{C3}}}, \qquad (16A)$$

$$AT = aT\, m_0^{2/3}. \qquad (16B)$$

A bar is placed over the THOR material coefficients to clarify their identification as single quantities. This use of the bar should not be confused with its use elsewhere of specifying mean values.

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We then conduct a binary search over the range of possible values to obtain a value of $(v_o)_{ik}$ to a specified accuracy. The associated $(m_o)_{ik}$ is then evaluated by using

$$m_o = \left[\frac{(v_o - \bar{v}_1)\, v_o^{-\overline{C5}}}{10^{\overline{C1}}\,(aT)^{\overline{C2}}} \right]^{\frac{3}{2\overline{C2}+3\overline{C3}}} . \qquad (16C)$$

9. Calculate the Jacobian for the event. The Jacobian used here is given by

$$j = \left| \frac{\partial(\bar{m}_1, \bar{v}_1)}{\partial(m_o, v_o)} \right| , \qquad (17A)$$

and is related to the Jacobian used in equation 11A by

$$jJ = 1 . \qquad (17B)$$

The components of the Jacobian are derived to be

$$\frac{\partial \bar{m}_1}{\partial m_o} = 1 - \frac{(2\overline{C7}+3\overline{C8})10^{C6}(aT)^{\overline{C7}} v_o^{CO} m_o^{\frac{2\overline{C7}+3\overline{C8}}{3}}}{3 m_o} , \qquad (18A)$$

$$\frac{\partial \bar{m}_1}{\partial v_o} = \overline{CO}\; 10^{\overline{C6}}\,(aT)^{\overline{C7}}\, v_o^{\overline{CO}-1}\, m_o^{\frac{2\overline{C7}+3\overline{C8}}{3}} , \qquad (18B)$$

$$\frac{\partial \bar{v}_1}{\partial m_o} = -(2\overline{C2}+3\overline{C3})10^{\overline{C1}}\,(aT)^{\overline{C2}}\, v_o^{\overline{C5}}\, m_o^{\frac{2\overline{C2}+3\overline{C3}}{3}} , \qquad (18C)$$

$$\frac{\partial \bar{v}_1}{\partial v_o} = 1 - \overline{C5}\; 10^{\overline{C1}}\,(aT)^{\overline{C2}}\, v_o^{\overline{C5}-1}\, m_o^{\frac{2\overline{C2}+3\overline{C3}}{3}} . \qquad (18D)$$

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10. Calculate the score for the fragment. The score is given by

$$S_{ik} =$$

$$\frac{\Delta \bar{m}_1 \, \Delta \bar{v}_1 \, R\left[D_i, (\bar{m}_1)_{ik}, (\bar{v}_1)_{ik}\right] \, N\left[(m_0)_{ik}, (v_0)_{ik}, \theta_i\right]}{j\left[(m_0)_{ik}, (v_0)_{ik}\right] \, r_i^2 F\left(r_i, R_i\right)} \qquad (19)$$

where $\Delta \bar{m}_1$ and $\Delta \bar{v}_1$ are the widths of the region Δ_k.

11. Trends are identified during the sweep over fragments associated with a particular path which can be used to predict zero scores for the remaining fragments associated with the path. If non-zero scores are possible for the remaining fragments, return to Step 7 and conduct the calculation for a fragment picked from the next Δ_k. If zero scores are predicted, continue to the next step.

12. Total the scores for all fragments associated with the path to obtain the event score using

$$S_i = \sum^{K} S_{ik} \ . \qquad (20)$$

13. Pick a total of I paths, where each path has an associated set of sample fragments, and calculate the total score for each path.

14. Calculate an estimate of λ using

$$\bar{\lambda} = \left(\overline{V_{RC}} \sum^{I} S_i\right) / I = \overline{V_{RC}} \, \overline{S} \ . \qquad (8B)$$

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15. Using the appropriate forms of equation 4C, calculate the standard deviation of $\overline{V}_{RC}$ and $\overline{S}$. Calculate the standard deviation of $\overline{\lambda}$ using

$$\delta \overline{\lambda} = \left[\left(\overline{V}_{RC} \delta \overline{S} \right)^2 + \left(\overline{S} \delta \overline{V}_{RC} \right)^2 \right]^{1/2} .$$

(8D)

16. Calculate an estimate of the kill probability of the critical component using

$$\overline{PK} = 1 - e^{-\overline{\lambda}} .$$

(1A)

17. Calculate the standard deviation of $\overline{P}$ using

$$\delta \overline{PK} = e^{-\overline{\lambda}} \delta \overline{\lambda} .$$

(5)

Step 17 completes the outline of a Monte Carlo adjoint calculation of the kill probability of the test component by a burst of fragments. These results are tabulated, along with the results obtained using the other two methods in Table I in Section IIIE.

E. Test Problem Results

The BASIC Computer Programs PROB1, PROB2, and PROB3 were installed on a WANG 2200 computer and used to calculate the kill probability of the test component by a spall burst. A sufficient number of fragment histories were calculated to obtain a small standard deviation for each answer. The answers and their standard deviations are given below:

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TABLE I. Test Problem Results

METHOD	SURVIVAL RATE	NO. HISTORIES
1. Forward Transport without Importance Sampling	0.3641 ± 0.001	7000
2. Forward Transport with Importance Sampling	0.3644 ± 0.003	5000
3. Adjoint Transport with Importance Sampling	0.3724 ± 0.014	2000

IV. CONCLUSION

The procedures for calculating the kill probability of a component by using three different sampling methods have been outlined. The defining equations have been derived for each method. A test problem was devised and is solved using each method. The answers obtained by the three methods agree within the standard deviations of the calculations.

Each of the preceding forward Monte Carlo methods is more efficient for a particular type of problem. The first method is more efficient for those cases where fragments from the burst impact the critical component with a high probability. The second method is more efficient for those cases where fragments from the burst impact the critical component with a low probability. The installation of both methods in a vulnerability methodology, where the most efficient method is selected for a particular burst and component, should be considered. An approximation of the second method is apparently used in the present version of the VAST methodology.[7,14,18] The third method (adjoint transport of fragments with importance sampling) is not, to the author's knowledge, used in any vehicle vulnerability methodology.

In many vulnerability studies, a comparison of the kill probability of a particular vehicle by the spall from different munitions is desired. This comparison is presently obtained by calculating the disabling probabilities of spall bursts produced by the different munitions for a

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[18]D.F. Haskel and M.J. Reisinger, "Armored Vehicle Vulnerability Analysis Model - First Version, Introduction," USARRADCOM/BRL Report No. 1857, February 1976.

common set of spall-burst origins and a common set of sample-fragment rays from each origin. A detailed description of the passage of each ray through the vehicle is calculated and stored on magnetic tape. The loss of capability in a component due to a particular munition can then be obtained in a second calculation by using the retreived ray descriptions in conjunction with the spall-burst source term of the munition.

A comparison of the kill probability of the spall bursts from different munitions can also be calculated using the adjoint transport of fragments. An adequate number of sample residual fragments are picked and transported to define the adjoint fragment flux at the burst origin. This adjoint flux can then be convoluted with any spall-burst source term to obtain its kill probability. An advantage of the adjoint method over the forward method is that less information is stored for the second calculation and the second calculation is shorter. Additionally, in the adjoint method, sample fragments can be picked more efficiently to obtain a specified statistical confidence level for the calculated kill probabilities of a large number of different burst descriptions.

APPENDIX A. THE MONTE CARLO EVALUATION OF DEFINITE INTEGRALS

The Monte Carlo method can often be effectively used to evaluate complex, multi-dimensional definite integrals which aren't amenable to evaluation by other methods. This method is outlined below for a two-dimensional integral, and the terminology commonly used is defined. The outlined method can be easily extended to integrals of a higher order of dimensionality by iterating the outlined procedures.

We wish to evaluate the integral

$$\lambda = \int_{x_1}^{x_2} \int_{y_1}^{y_2} F(x,y)\, Q(x,y)\, dy\, dx \qquad \text{(A-1)}$$

where $F(x,y)$ and $Q(x,y)$ are continuous and single valued within the specified region R of integration. This integral is normally interpreted as the volume under the function $F(x,y)Q(x,y)$ which is located within the integration limits. However, if we normalize $Q(x,y)$ over R to 1, equation A-1 becomes:

$$\lambda = C \int_{x_1}^{x_2} \int_{y_1}^{y_2} F(x,y)\, P(x,y)\, dy\, dx \;, \qquad \text{(A-2)}$$

$$C = \int_{x_1}^{x_2} \int_{y_1}^{y_2} Q(x,y)\, dy\, dx \;, \qquad \text{(A-3)}$$

$$P(x,y) = Q(x,y) / C \;, \qquad \text{(A-4)}$$

and the integral in equation A-2 can now be interpreted as the average expected value of $F(x,y)$ over R where the values of x and y are predicted by the probability density function (PDF) $P(x,y)$.

The quantity λ can now be easily estimated by using the second interpretation of the integral. Several sets of values are picked for x and y by sampling $P(x,y)$, and the mean value of $F(x,y)$ is calculated for these values. An estimate of λ is given by

$$\bar{\lambda} = C \sum_{i}^{I} F(x_i, y_i) / I = C \bar{F} \qquad \text{(A-5)}$$

where i is the sample index and I is the number of sample sets picked. A bar is placed over a quantity to identify its mean value. The quantity $F(x,y)$ is often identified as the estimator, and $F(x_i, y_i)$ is identified as the score. The quantity $\bar{\lambda}$ converges toward the true value of λ as the number of sample events is increased.

Various sampling procedures[19] are available for picking sample values for a variable (variant) from a one-dimensional PDF. Therefore, we will change equation A-2 to a form where the two-dimensional PDF $P(x,y)$ is replaced by a pair of one-dimensional PDF's. This change is given by:

$$\lambda = C \int_{x_1'}^{x_2'} \left\{ \int_{x_1}^{x_2} \int_{y_1}^{y_2} F(x,y) Y(x,x',y) \, dy \, dx \right\} X(x') \, dx' , \qquad \text{(A-6)}$$

$$X(x') = \int_{y_1}^{y_2} P(x',y) \, dy , \qquad \text{(A-7)}$$

$$Y(x,x',y) = \frac{P(x,y) \, \delta(x-x')}{\int_{y_1}^{y_2} P(x',y) \, dy} , \qquad \text{(A-8)}$$

where $\delta(x-x')$ is the Dirac delta function and x' is a dummy value of x.

[19] E.D. Cashwell and C.J. Everette, "A Practical Manual on the Monte Carlo Method for Random Walk Problems," Pergamon Press (1959).

We will now outline a step-by-step procedure for calculating a Monte Carlo estimate of λ. This procedure is:

1. Pick a value of x' by sampling X(x'). This value is identified as x'_i.

2. Pick an associated value of y by sampling the one-dimensional PDF Y(x,x',y). This value is identified as y'.

3. Calculate the score $F(x'_i, y_i)$ for the event. This quantity is identified as F_i.

4. Calculate the scores for a total of I events by repeating steps 1-3.

5. An estimate of λ is calculated as

$$\bar{\lambda} = C \sum^{I} F_i = C\bar{F} .$$

(A-5)

6. A measure of the confidence level of $\bar{\lambda}$ is given by the standard deviation of $\bar{\lambda}$. This quantity is given by

$$\delta\lambda = \left[\frac{\sum^{I}(CF_i)^2 - I\bar{\lambda}^2}{I(I-1)} \right]^{1/2} .$$

(A-9)

Step 6 completes a step-by-step outline of the Monte Carlo evaluation of the integral in equation A-2. Often an integral is more amenable to evaluation, or the statistical convergence of $\bar{\lambda}$ can be attained more readily, if the integrand is rearranged from

the form in which it normally occurs. This importance sampling is developed below for equation A-1:

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2} F(x,y)\, Q(x,y)\, dy\, dx =$$

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[F(x,y)\, G(x,y)\right]\frac{Q(x,y)}{G(x,y)}\, dy\, dx = \qquad (A-10)$$

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2} F'(x,y)\, Q'(x,y)\, dy\, dx = \qquad (A-11)$$

$$C'\int_{x_1}^{x_2}\int_{y_1}^{y_2} F'(x,y)\, P'(x,y)\, dy\, dx, \qquad (A-12)$$

$$C' = \int_{x_1}^{x_2}\int_{y_1}^{y_2} Q'(x,y)\, dx\, dy, \qquad (A-13)$$

$$P'(x,y) = (Q'(x,y)\,/\,C'\, . \qquad (A-14)$$

Equation A-12 can now be evaluated by using the same procedure used to evaluate equation A-2. However, the sets of variable values (x_i, y_i) and the scores F'_i as well as the normalization constant C' will differ between the two calculations.

```
5 REM PROGRAM PROB1
10 REM FORWARD SPALL TEST PROBLEM
11 REM NO IMPORTANCE SAMPLING
12 SELECT R
13 SELECT PRINT 215
15 INPUT "RANDOM NUMBER SEED",Z8
20 INPUT "NUMBER OF HISTORIES",I1
25 P1=0   P2=0   I=0
30 INPUT "THOR VELOCITY COEFFICIENTS",C1,C2,C3,C4,C5
35 INPUT "THOR MASS COEFFICIENTS",C6,C7,C8,C9,C0
36 INPUT "DATE",X1
37 PRINT "DATE=",X1
38 PRINT "NUMBER OF HISTORIES=",I1
40 F1$="C1="   F2$="C2="   F3$="C3="   F4$="C4="   F5$="C5="
41 PRINT "THOR VELOCITY COEFFICIENTS"
42 PRINTUSING 48,F1$,C1,F2$,C2,F3$,C3,F4$,C4,F5$,C5
43 F1$="C6="   F2$="C7="   F3$="C8="   F4$="C9="   F5$="C0="
44 PRINTUSING 48,F1$,C6,F2$,C7,F3$,C8,F4$,C9,F5$,C0
48 %### -## ### ### -## ### ### -## ### ### -## ### ### -## ###
49 PRINT
50 B1=3000/3.14159
55 B2=54000/3.14159
60 B3=180/3.14159
65 B6=ARCSIN(0.5)
70 B4=(B3*B6+5)*SIN(B6)
75 B5=SQR(2*3.14159)
80 B7=1.2/3.14159
99 SELECT PRINT 005
100 FOR I=1TO I1
105 REM PICK A1
110 J=J+1
115 GOSUB '99
120 A1=Z9*B6
125 GOSUB '99
130 X1=Z9*B4
135 X2=(B3*A1+5)*SIN(A1)
140 IF X2<X1THEN 110
145 REM PICK M
150 M0=-B1*A1+505
155 GOSUB '10(M0)
160 M=X2
165 REM PICK V
170 V0=B2*A1+500
175 GOSUB '10(V0)
180 V=X2
180 V=X2
185 REM PICK T
190 T0=-B7*A1+0.3
195 GOSUB '10(T0)
200 T=X2
205 REM CALCULATE TRANSPORT
```

```
207 GOSUB 30
210 GOSUB 20(1)
215 IF M<0THEN 227
220 GOSUB 20(2)
225 IF V<0THEN 200
227 P=0  GOTO 250
230 GOSUB 68
235 P=(20+M+V+M*V)*(20-P)/100000000
236 IF P<1THEN 245
240 P=1 0
245 P1=P1+P  P2=P2+P*P
250 PRINT I,J,P
255 NEXT I
260 SELECT PRINT 215
265 P=P1/I1
275 P2=(P2-I1*P*P)/(I1*(I1-1))
280 X=2*#PI*I1+B4*E6/J
285 X2=(I1*J-I1*I1)/(J*J*(J-1))
287 X2=SQR(X2)
288 X2=X2+X
290 P2=(P*X2)^2+(P2*X)^2
295 P2=SQR(P2)
300 P=P+X  P=EXP(-P)
305 P2=P+P2
310 PRINT "SURVIVIAL RATE =",P
311 PRINT "S. R. STANDARD DEVIATION =",P2
315 END
2000 DEFFN 20(X1)
2005 REM CALCULATES TRANSPORT(THOR)
2010 IF X1>1THEN 2025
2014 M1=M
2015 M=M-(10^C6)*((T+A0)^C7)*(M^C8)*(V^C0)
2020 GOTO 2030
2025 V=V-(10^C1)*((T+A0)^C2)*(M1^C3)*(V^C5)
2030 RETURN
2500 DEFFN 30
2505 REM CALCULATES A0
2510 X1=M/14 4I2
2515 X2=X1^7 7641
2520 X3=0 75*X2/3 14159^0 333333
2525 A0=3 14159*X2*X3
2526 A0=A0^(2 54*2 54)
2530 RETURN
3100 DEFFN 10(X1)
3105 REM SAMPLE FROM NORMAL DISTRIBUTION
3110 GOSUB 99
3115 X2=X1*(0 4+1 2*29)
3120 GOSUB 99
3125 X3=B5*X1+0 972
3130 X2=9+29/X2
```

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```
5110 X4=5 0*(X2-X1)/X1
5135 X4=EXP(-0 5*X4^2)
5140 X4=5*X4/(0 972+B5*X1)
5145 IF X4<X3THEN 5110
5150 RETURN
6200 DEFFN 68
6205 REM CALCULATE R
6210 X1=-TAN(A1)
6212 X2=20
6215 A9=1+X1*X1
6220 B9=-2*X2*X1*X1
6225 C9=(X1*X2)^2-100
6230 R=B9*B9-4*A9*C9
6235 IF R>0THEN 6250
6240 PRINT "NEGATIVE R",R
6245 GOTO 6265
6250 R=SQR(R)
6255 R=0 5*(R-B9)/A9
6260 R=X1*(R-20)
6265 RETURN
7000 DEFFN 77
7005 REM DEBUGGING SR
7010 A1$="I=": A2$="J=": A3$="A1=": A4$="R=": A5$="P1="
7015 A6$="R2=": A7$="M=": A8$="V=": A9$="T=": A0$="P="
7020 PRINTUSING 7100,U1$, I, A2$, J, A3$, A1, A4$, R, A5$, P1
7025 PRINTUSING 7100,A6$, R2, A7$, M, A8$, V, A9$, T, A0$, P
7100:### #### ## ### ####. ## ### ####. ## ### #### ## ###
#### ##
7105 RETURN
9000 DEFFN 99
9005 REM RANDOM NUMBER MAIN SR
9010 Z8=25*Z8
9015 GOSUB 98
9020 Z8=5*Z8
9025 GOSUB 98
9030 Z9=Z8/67108864
9035 RETURN
9050 DEFFN 98
9055 REM RANDOM NUMBER SECONDARY SR
9060 X5=Z8/67108864
9065 X6=67108864*INT(X5)
9070 Z8=Z8-X6
9075 RETURN
```

```
1 REM PROGRAM PROB2
2 DIM S1(3),S2(3),W(3)
10 REM FORWARD SPALL TEST PROBLEM
11 REM NO IMPORTANCE SAMPLING
12 SELECT R
13 SELECT PRINT 215
15 INPUT "RANDOM NUMBER SEED",Z8
20 INPUT "NUMBER OF HISTORIES",I1
25 P1=0: P2=0: J=0
30 INPUT "THOR VELOCITY COEFFICIENTS",C1,C2,C3,C4,C5
35 INPUT "THOR MASS COEFFICIENTS",C6,C7,C8,C9,C0
36 INPUT "DATE",X1
37 PRINT "DATE=",X1
38 PRINT "NUMBER OF HISTORIES=",I1
40 F1$="C1=": F2$="C2=": F3$="C3 =": F4$="C4=": F5$="C5="
41 PRINT "THOR VELOCITY COEFFICIENTS"
42 PRINTUSING 48,F1$,C1,F2$,C2,F3$,C3,F4$,C4,F5$,C5
43 F1$="C6=": F2$="C7=": F3$="C8=": F4$="C9=": F5$="C0="
44 PRINTUSING 48,F1$,C6,F2$,C7,F3$,C8,F4$,C9,F5$,C0
48 %### -## ### ### -## ### ### -## ### ### -## ### ### -## ###
49 PRINT
50 B1=3000/3 14159
55 B2=54000/3 14159
60 B3=180/3 14159
75 B5=50*(2*3 14159)
80 B7=1 2/3 14159
85 C=10: S1(1)=20: S1(2)=0: S1(3)=0
99 SELECT PRINT 005
100 FOR I=1TO I1
105 GOSUB 767
107 REM PICK M
110 M0=-A1+B1+505
115 GOSUB 10(M0)
120 M=O2
125 REM PICK V
130 V0=A1+B2+500
135 GOSUB 10(V0)
140 V=O2
145 REM PICK T
150 T0=-A1+B7+0 3
155 GOSUB 10(T0)
160 T=O2
165 REM CALCULATE TRANSPORT
170 GOSUB 30
175 GOSUB 20(1)
180 IF M 0THEN 195
185 GOSUB 20(2)
190 IF V 0THEN 200
195 P=0: GOTO 235
200 P=(20+M+V+M*V)+(30-R)/100000000
```

```
205 IF P<1THEN 215
210 P=1
215 P=P+(B3*H1+5)
220 O1=(R2+R1)/R2
225 P=P/(1+O1+O1*O1)
230 P=3*P/(R1+R2+R2)
235 PRINT I,J,P
240 P1=P1+P  P2=P2+P*P
241 SELECT PRINT 215
242 GOSUB 77
245 NEXT I
250 P=P1/I1
255 P2=(P2-I1*P*P)/(I1*(I1-1))
260 P2=SQR(P2)
265 O2=(I1*J-I1*I1)/(J*J*(J-1))
270 O2=SQR(O2)
275 O=(2+O)^3*I1/J
280 O2=O2+O
285 P2=(P+O2)^2+(O*P2)^2
290 P2=SQR(P2)
295 P=P+O
297 P=EXP(-P)
298 P2=P*P2
300 SELECT PRINT 215
310 PRINT "SURVIVIAL RATE =",P
311 PRINT "S.R. STANDARD DEVIATION =",P2
315 END
2000 DEFFN'20(X1)
2005 REM CALCULATES TRANSPORT(THOR)
2010 IF X1>1THEN 2025
2014 M1=M
2015 M=M-(10^C6)*((T*A0)^C7)*(M^C8)+(V^C9)
2020 GOTO 2030
2025 V=V-(10^C1)*((T*A0)^C2)*(M1^C3)+(V^C5)
2030 RETURN
2500 DEFFN'30
2505 REM CALCULATES A0
2510 O1=M/14.432
2515 O2=O1/7.7641
2520 O3=(0.75*O2/3.14159)^0.333333
2525 A0=3.14159*O3*O3
2526 A0=A0/(2.54*2.54)
2530 RETURN
5100 DEFFN'10(O1)
5105 REM SAMPLE FROM NORMAL DISTRIBUTION
5110 GOSUB 99
5115 O2=O1*(0.4+1.2+29)
5120 GOSUB 99
5125 O3=85*O1+0.973
5126 O3=5*29/O3
```

```
5130 O4=5.0*(O2-O1)/O1
5135 O4=EXP(-0.5*O4^2)
5140 O4=5*O4/(0.973+B5*O1)
5145 IF O4<O3THEN 5110
5150 RETURN
6000 DEFFN'67
6005 REM PICK AND DEFINE SHOT LINES
6007 J=J+1
6010 GOSUB '99
6012 S2(1)=C*(2*Z9-1)
6015 GOSUB '99
6020 S2(2)=C*Z9
6025 GOSUB '99
6030 S2(3)=C*Z9
6035 R9=(S1(1)-S2(1))^2+(S1(2)-S2(2))^2+(S1(3)-S2(3))^2
6040 R9=SQR(R9)
6045 W(1)=(S2(1)-S1(1))/R9
6050 W(2)=(S2(2)-S1(2))/R9
6055 W(3)=(S2(3)-S1(3))/R9
6057 GOSUB '68
6058 IF F1=-1THEN 6007
6060 X1=C
6065 Y1=S1(2)+W(2)*(C-S1(1))/W(1)
6070 Z1=S1(3)+W(3)*(C-S1(1))/W(1)
6075 X2=-C
6080 Y2=S1(2)-W(2)*(C+S1(1))/W(1)
6085 Z2=S1(3)-W(3)*(C+S1(1))/W(1)
6090 Y3=C
6095 X3=S1(1)+W(1)*(C-S1(2))/W(2)
6100 Z3=S1(3)+W(3)*(C-S1(2))/W(2)
6105 Z4=C
6110 X4=S1(1)+W(1)*(C-S1(3))/W(3)
6115 Y4=S1(2)+W(2)*(C-S1(3))/W(3)
6117 REM CALCULATES R2 AND A1
6120 R2=(S1(1)-X1)^2+(S1(2)-Y1)^2+(S1(3)-Z1)^2
6125 R2=SQR(R2)
6130 A1=ARCCOS(10/R2)
6135 REM CALCULATE R1
6140 O1=(S1(1)-X2)^2+(S1(2)-Y2)^2+(S1(3)-Z2)^2
6145 O2=(S1(1)-X3)^2+(S1(2)-Y3)^2+(S1(3)-Z3)^2
6150 O3=(S1(1)-X4)^2+(S1(2)-Y4)^2+(S1(3)-Z4)^2
6155 O4=O1
6160 IF O2>O4THEN 6170
6165 O4=O2
6170 IF O3>O4THEN 6180
6175 O4=O3
6180 R1=SQR(O4)
6185 R1=R1-R2
6190 RETURN
6200 DEFFN'68
```

```
6205 REM CALCULATE R
6207 F1=1
6210 A9=1+(W(2)^2+W(3)^2)/(W(1)^2)
6215 B9=2*W(2)*(S1(2)-W(2)*S1(1)/W(1))/W(1)
6220 B9=B9+2*W(3)*(S1(3)-W(3)*S1(1)/W(1))/W(1)
6225 C9=S1(2)^2-S1(1)*W(2)*(2*S1(2)-W(2)*S1(1)/W(1))/W(1)
6230 C9=C9+S1(3)^2-S1(1)*W(3)*(2*S1(3)-W(3)*S1(1)/W(1))/W(1)
6235 C9=C9-C*C
6240 Q1=B9*B9-4*A9*C9
6245 IF Q1>=0THEN 6260
6250 F1=-1
6255 GOTO 6340
6260 Q1=SQR(Q1)
6265 X5=0.5*(-B9+Q1)/A9
6270 X6=0.5*(-B9-Q1)/A9
6275 Y5=S1(2)+W(2)*(X5-S1(1))/W(1)
6280 Y6=S1(2)+W(2)*(X6-S1(1))/W(1)
6285 Z5=S1(3)+W(3)*(X5-S1(1))/W(1)
6290 Z6=S1(3)+W(3)*(X6-S1(1))/W(1)
6295 R3=(S1(1)-X5)^2+(S1(2)-Y5)^2+(S1(3)-Z5)^2
6300 R4=(S1(1)-X6)^2+(S1(2)-Y6)^2+(S1(3)-Z6)^2
6305 IF R3<R4THEN 6315
6310 X5=X6: Y5=Y6: Z5=Z6
6315 R=Y5*Y5+Z5*Z5
6320 R=SQR(R)
6340 RETURN
7000 DEFFN'77
7005 REM DEBUGGING SR
7010 A1$="I=": A2$="J=": A3$="A1=": A4$="R=": A5$="R1="
7015 A6$="R2=": A7$="M=": A8$="V=": A9$="T=": A0$="P="
7020 PRINTUSING 7100, U1$, I, A2$, J, A3$, A1, A4$, R, A5$, R1
7025 PRINTUSING 7100, A6$, R2, A7$, M, A8$, V, A9$, T, A0$, P
7100%### ####.## ### ####.## ### ####.## ### #### ## ###
#### ##
7105 RETURN
9000 DEFFN'99
9005 REM RANDOM NUMBER MAIN SR
9010 Z8=25*Z8
9015 GOSUB '98
9020 Z8=5+Z8
9025 GOSUB '98
9030 Z9=Z8/67108864
9035 RETURN
9050 DEFFN'98
9055 REM RANDOM NUMBER SECONDARY SR
9060 Q5=Z8/67108864
9065 Q6=67108864*INT(Q5)
9070 Z8=Z8-Q6
9075 RETURN
```

```
1 REM PROGRAM PROB3
2 DIM S1(3),S2(3),W(3)
10 REM ADJOINT SPALL TEST PROBLEM
11 REM NO IMPORTANCE SAMPLING
12 SELECT P
13 SELECT PRINT 215
15 INPUT "RANDOM NUMBER SEED",Z8
20 INPUT "NUMBER OF HISTORIES",I1
25 P1=0: P2=0: J=0
30 C1=6.399: C2=0.889: C3=-0.945: C4=1.262: C5=0.019
35 C6=-2.507: C7=0.138: C8=0.835: C9=0.143: C0=0.761
36 INPUT "DATE",X1
37 PRINT "DATE=",X1
38 PRINT "NUMBER OF HISTORIES=",I1
40 F1$="C1=": F2$="C2=": F3$="C3=": F4$="C4=": F5$="C5="
41 PRINT "THOR VELOCITY COEFFICIENTS"
42 PRINTUSING 48,F1$,C1,F2$,C2,F3$,C3,F4$,C4,F5$,C5
43 F1$="C6=": F2$="C7=": F3$="C8=": F4$="C9=": F5$="C0="
44 PRINTUSING 48,F1$,C6,F2$,C7,F3$,C8,F4$,C9,F5$,C0
48:### -##.### ### -##.### ### -##.### ### -##.### ### -##.###
49 PRINT
50 B1=1000/3.14159
55 B2=54000/3.14159
60 B3=180/3.14159
65 C1=10^C1: C6=10^C6: J0=1
67 E2=(2*C7+3*C8)/3
69 E1=(2*C2+3*C3)/3
71 E3=E2/E1
73 E4=1/E1
75 B5=SQR(2*3.14159)
80 B7=1.2/3.14159
85 C=10: S1(1)=20: S1(2)=0: S1(3)=0
90 A=4*#PI*14.432*7.7641
92 A=(3/A)^.666667
94 A=#PI*A/(2.54*2.54)
99 SELECT PRINT 005
100 FOR I=1TO I1
105 REM PICK ADJOINT ORIGIN
110 GOSUB '67
114 M0=-B1*A1+500
116 V0=B2*A1+500
120 M2=1.6*M0: M1=0.4*M0
140 V2=1.6*V0: V1=0.4*V0
155 REM PICK T
160 T0=-A1*B7+0.3
165 GOSUB '10(T0)
170 T=O2
175 M9=75: V9=1250
180 E5=(A*T)^C2
185 E6=(A*T)^C7
```

59

```
200 GOSUB ´00
300 NEXT I
310 P=P1/I1
320 P2=(P2-I1*P*P)/(I1*(I1-1))
330 P2=SQR(P2)
340 Q2=(I1*J-I1*I1)/(J*J*(J-1))
350 Q2=SQR(Q2)
360 Q=((2*C)^3)*I1/J
370 Q2=Q2*Q
380 P2=(P*Q2)^2+(Q*P2)^2
390 P2=SQR(P2)
400 P=P*Q
405 P=EXP(-P)
406 P2=P2*P
410 SELECT PRINT 215
420 PRINT "SURVIVIAL RATE =",P
430 PRINT "SR STANDARD DEVIATION =",P2
450 END
1000 DEFFN´00
1004 SELECT PRINT 005
1005 REM DIRECT MASS AND VELOCITY SAMPLING AND SCORING
1006 O1=(R1+R2)/R2
1007 O1=R1*(1+O1+O1*O1)/3
1008 F9=25+M9*V9*(B3*A1+5)/(2**#PI*M0*V0*R2+R2*O1)
1010 K=-1: V4=V2: P6=0: F9=100
1015 GOSUB ´99: Z7=Z9
1016 GOSUB ´99: Z6=Z9
1020 K=K+1: L=-1
1025 V8=(K+Z7)*V9
1026 IF V8>V2THEN 1215
1030 V2=V8
1040 L=L+1
1045 M8=(L+Z6)*M9
1050 GOSUB ´77
1055 V=Q7
1060 REM CALCULATE M
1065 O1=C1*E5*(V^C5)
1070 O1=(V-V8)/O1
1075 M=O1^(3/(2*C2+3*C3))
1112 O1=(R1+R2)/R2
1114 O1=R1*(1+O1+O1*O1)/3
1150 IF M>M2THEN 1190
1155 IF M<M1THEN 1190
1160 IF V>V2THEN 1190
1165 IF V<V1THEN 1190
1166 O1=5*(V-V0)/V0
1167 O1=0.5*O1*O1
1168 F8=EXP(-O1)
1170 O1=5*(M-M0)/M0
1175 O1=0.5*O1*O1
```

60

```
1176 P7=EXP(-Q1)
1180 P7=P7*(20*M8+V8+M8*V8)*(30-R)/100000000
1181 GOSUB 17
1185 GOTO 1195
1190 P7=0
1195 P=P7+P8+P9/J0
1200 PRINT 100000*I+J, 100000*K+L, P
1201             PRINT 5*(M-M0)/M0, 5*(V-V0)/V0, J0
1202             PRINT
1205 P6=P6+P
1206 IF L>0THEN 1235
1210 IF M<M2THEN 1235
1215 P6=P6/(.9973*.9973)
1216 P1=P1+P6
1220 P2=P2+P6*P6
1230 GOTO 1300
1235 IF M>M2THEN 1265
1236 IF L=C9THEN 1020
1240 IF V<V1THEN 1020
1245 GOTO 1040
1265 F9=L-1  GOTO 1020
1300 RETURN
4000 DEFFN 77
4005 REM CALCULATE IF V>V2
4006 REM CALCULATE V
4010 S1=0
4015 S1=S1+1
4020 S2=S1*V2
4025 GOSUB 33(S2)
4030 IF Q4<0THEN 4015
4035 GOSUB 34
4040 RETURN
5100 DEFFN 10(Q1)
5105 REM SAMPLE FROM NORMAL DISTRIBUTION
5110 GOSUB 99
5115 Q2=Q1*(0.4+1.2*Z9)
5120 GOSUB 99
5125 Q3=B5*Q1*0.9973
5126 Q3=5+Z9/Q3
5130 Q4=5.0*(Q2-Q1)/Q1
5135 Q4=EXP(-0.5*Q4^2)
5140 Q4=5*Q4/(0.9973*B5*Q1)
5145 IF Q4<Q3THEN 5110
5150 RETURN
6000 DEFFN 67
6005 REM PICK AND DEFINE SHOT LINES
6007 J=J+1
6010 GOSUB 99
6012 S2(1)=C*(2*Z9-1)
6015 GOSUB 99
```

```
6020 S2(2)=C*Z9
6025 GOSUB 199
6030 S2(3)=C*Z9
6035 R9=(S1(1)-S2(1))^2+(S1(2)-S2(2))^2+(S1(3)-S2(3))^2
6040 R9=SQR(R9)
6045 W(1)=(S2(1)-S1(1))/R9
6050 W(2)=(S2(2)-S1(2))/R9
6055 W(3)=(S2(3)-S1(3))/R9
6057 GOSUB 68
6058 IF F1=-1THEN 6007
6060 X1=C
6065 Y1=S1(2)+W(2)*(C-S1(1))/W(1)
6070 Z1=S1(3)+W(3)*(C-S1(1))/W(1)
6075 X2=-C
6080 Y2=S1(2)-W(2)*(C+S1(1))/W(1)
6085 Z2=S1(3)-W(3)*(C+S1(1))/W(1)
6090 Y3=C
6095 X3=S1(1)+W(1)*(C-S1(2))/W(2)
6100 Z3=S1(3)+W(3)*(C-S1(2))/W(2)
6105 Z4=C
6110 X4=S1(1)+W(1)*(C-S1(3))/W(3)
6115 Y4=S1(2)+W(2)*(C-S1(3))/W(3)
6117 REM CALCULATES R2 AND A1
6120 R2=(S1(1)-X1)^2+(S1(2)-Y1)^2+(S1(3)-Z1)^2
6125 R2=SQR(R2)
6130 A1=ARCCOS(10/R2)
6135 REM CALCULATE R1
6140 O1=(S1(1)-X2)^2+(S1(2)-Y2)^2+(S1(3)-Z2)^2
6145 O2=(S1(1)-X3)^2+(S1(2)-Y3)^2+(S1(3)-Z3)^2
6150 O3=(S1(1)-X4)^2+(S1(2)-Y4)^2+(S1(3)-Z4)^2
6155 O4=O1
6160 IF O2>O4THEN 6170
6165 O4=O2
6170 IF O3>O4THEN 6180
6175 O4=O3
6180 R1=SQR(O4)
6185 R1=R1-R2
6190 RETURN
6200 DEFFN 68
6205 REM CALCULATE R
6207 F1=1
6210 A9=1+(W(2)^2+W(3)^2)/(W(1)^2)
6215 B9=2*W(2)*(S1(2)-W(2)*S1(1)/W(1))/W(1)
6220 B9=B9+2*W(3)*(S1(3)-W(3)*S1(1)/W(1))/W(1)
6225 C9=S1(2)^2-S1(1)*W(2)*(2*S1(2)-W(2)*S1(1)/W(1))/W(1)
6230 C9=C9+S1(3)^2-S1(1)*W(3)*(2*S1(3)-W(3)*S1(1)/W(1))/W(1)
6235 C9=C9-C*C
6240 O1=B9*B9-4*A9*C9
6245 IF O1>=0THEN 6260
6250 F1=-1
```

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```
6255 GOTO 6340
6260 Q1=SQR(Q1)
6265 X5=0.5*(-B9+Q1)/A9
6270 X6=0.5*(-B9-Q1)/A9
6275 Y5=S1(2)+W(2)*(X5-S1(1))/W(1)
6280 Y6=S1(2)+W(2)*(X6-S1(1))/W(1)
6285 Z5=S1(3)+W(3)*(X5-S1(1))/W(1)
6290 Z6=S1(3)+W(3)*(X6-S1(1))/W(1)
6295 R3=(S1(1)-X5)^2+(S1(2)-Y5)^2+(S1(3)-Z5)^2
6300 R4=(S1(1)-X6)^2+(S1(2)-Y6)^2+(S1(3)-Z6)^2
6305 IF R3<R4THEN 6315
6310 X5=X6: Y5=Y6: Z5=Z6
6315 R=Y5*Y5+Z5*Z5
6320 R=SQR(R)
6340 RETURN
7000 DEFFN'33(Q1)
7005 REM CALCULATE F(V)
7010 Q2=C1*E5*(Q1^C5)
7015 Q2=(Q1-V8)/Q2
7020 Q3=Q2^E4
7025 Q4=Q2^E3
7030 Q4=C6*E6*(Q1^C0)*Q4
7035 Q4=Q3-Q4
7040 Q4=M8-Q4
7050 RETURN
7200 DEFFN'34
7205 REM BINARY SEARCH
7210 Q6=1.000001*S2
7212 IF S1>1THEN 7215
7214 Q5=0.99999*V8: GOTO 7220
7215 Q5=0.99999*(S1-1)*V2
7220 Q7=0.5*(Q5+Q6)
7222 IF (Q6-Q5)/Q6<0.000001THEN 7270
7225 GOSUB '33(Q7)
7230 IF Q4<0.0THEN 7240
7235 Q6=Q7
7238 GOTO 7220
7240 Q5=Q7
7245 GOTO 7220
7270 RETURN
8000 DEFFN'17
8005 REM CALCULATE JACOBIAN
8010 Q1=M^E2
8015 Q2=E6
8020 Q3=V^C0
8025 Q4=C6
8030 Q5=Q1/M
8035 Q6=E5
8040 Q7=M^E1
8045 Q8=C1
```

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```
8050 O9=Q7/M
8055 O0=V^O5
8060 J1=1-E2*O4*O2*O5*O3
8065 J2=-C0*O4*O2*O1*O3/V
8070 J3=-E1*O8*O6*O9*O0
8075 J4=1-C5*O8*O6*O7*O0/V
8080 J0=J1*J4-J2*J3 : J0=ABS(J0)
8150 RETURN
9000 DEFFN`99
9005 REM RANDOM NUMBER MAIN SR
9010 Z8=25*Z8
9015 GOSUB `98
9020 Z8=5*Z8
9025 GOSUB `98
9030 Z9=Z8/67108864
9035 RETURN
9050 DEFFN`98
9055 REM RANDOM NUMBER SECONDARY SR
9060 O5=Z8/67108864
9065 O6=67108864*INT(O5)
9070 Z8=Z8-O6
9075 RETURN
```

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