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# COMPUTER-AIDED METHODS FOR REDESIGNING THE STABILIZED PITCH CONTROL SYSTEM OF A SEMI-ACTIVE TERMINAL HOMING MISSILE

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**Abstract**—An unstable pitch control system of a terminal homing missile was formerly stabilized using a high order stabilization filter that was realized using active elements. A new dominant-data matching method is presented to redesign the high-order stabilization filter for obtaining reduced-order filters. As a result, the implementation cost is reduced and the reliability increased. An algebraic method is also applied to improve the performance of the redesigned pitch control system. In addition, the proposed dominant-data matching method can be applied to determine a reduced-order model of a high-order system. Unlike most existing model reduction methods, the reduced-order model has the exact assigned frequency-domain specifications of the original system. Computer-aided design methods can also be applied to design general control systems.

## I. INTRODUCTION

The pitch control system of an unstable terminal homing missile[1] was formerly stabilized using a fourth-order series compensator. The compensator had two pairs of complex poles that were realized using active elements. The objective of this paper is to develop a computer-aided method for redesigning the compensator such that the implementation cost of the compensator can be reduced and the performance of the redesigned pitch control system improved. The block diagram of the existing stabilized system is shown in Fig. 1, and its over-all transfer function is

$$T_e(s) = \frac{T_s(s)G_0(s)}{1 + T_s(s)G_0(s)H_g(s)} = \frac{G_e(s)}{1 + G_e(s)} = \frac{b_0 + b_1 s + \cdots + b_9 s^9 + b_{10} s^{10}}{a_0 + a_1 s + \cdots + a_{10} s^{10} + a_{11} s^{11}} \triangleq \frac{N(s)}{D(s)} \tag{1a}$$

where

$$\begin{array}{ll}
a_0 = 8.802158509 \times 10^{18} & b_0 = 8.80215809 \times 10^{18} \\
a_1 = 2.419047424 \times 10^{19} & b_1 = 4.610004670 \times 10^{19} \\
a_2 = 2.911920560 \times 10^{18} & b_2 = 2.926344345 \times 10^{18} \\
a_3 = 2.420405431 \times 10^{18} & b_3 = 5.017212044 \times 10^{16} \\
a_4 = 6.667397031 \times 10^{16} & b_4 = 2.563396371 \times 10^{14} \\
a_5 = 9.749923212 \times 10^{14} & b_5 = 1.494523312 \times 10^{11} \\
a_6 = 9.360329977 \times 10^{12} & b_6 = 0. \\
a_7 = 6.231675318 \times 10^{10} & b_7 = 0. \\
a_8 = 2.976950696 \times 10^{8} & b_8 = 0. \\
a_9 = 9.316239040 \times 10^{5} & b_9 = 0. \\
a_{10} = 1.923554000 \times 10^{3} & b_{10} = 0. \\
a_{11} = 1. &
\end{array} \tag{1b}$$

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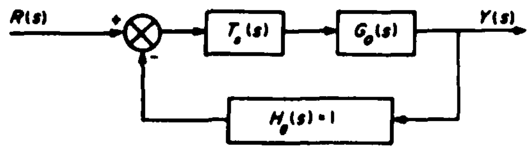


Fig. 1. The block diagram of the existing control system.

and

$T_s(s)$ = The transfer function of the existing series stabilization filter

$$= \frac{1.6\left(\frac{s}{25}+1\right)\left(\frac{s}{125}+1\right)}{\left[\left(\frac{s}{150}\right)^2+\left(\frac{0.6s}{150}\right)+1\right]\left[\left(\frac{s}{200}\right)^2+\left(\frac{0.8s}{200}\right)+1\right]} \triangleq \frac{N_s(s)}{D_s(s)} \quad (1c)$$

$H_g(s)$ = The transfer function of the gyro = 1 (Note that a rate gyro is not available) (1d)

$G_0(s)$ = The transfer function of the actuator and air frame dynamics of the missile system—The plane transfer function

= The open-loop transfer function of the original pitch control system with $T_s(s) = 1$ and $H_g(s) = 1$

$$= \frac{324332.316(s+0.1933)(s+65)(s+1500)}{s(s-2.921)(s+3.175)(s+87.9\pm j95.5)(s+112.5)(s+1385)} \quad (1e)$$

$G_e(s)$ = The open-loop transfer function of the existing stabilized system

$$= T_s(s)G_0(s)H_g(s). \quad (1f)$$

For ease of presentation, we define the dominant data

(a) The real and imaginary parts of the transfer function when $\omega = 0$.
(b) The gain margin.
(c) The phase-crossover frequency.
(d) The phase margin.
(e) The gain crossover frequency.

Nyquist plots of $G_e(s)$ and $G_0(s)$ are shown in Fig. 2. The dominant frequency-response data of $G_e(s)$ are defined as follows

(1) The real ($R_e$) and imaginary ($I_m$) parts of $G_e(s)$ at $s = j\omega = j0$ are

$$R_eG_e(j0) = -2.103817 \text{ and } I_mG_e(j0) = \infty \quad (2a)$$

(2) The gain margin $G_{em}$ of this system is

$$G_{em} = \left|\frac{1}{G_e(j\omega_{ep})}\right| = \left|\frac{1}{R_eG_e(j\omega_{ep})}\right| \cong \left|\frac{1}{-1.5}\right| \quad (2b)$$

where the phase-crossover frequency $\omega_{ep}$ is

$$\angle G_e(j\omega_{ep}) = -180° \text{ or } \omega_{ep} = 1.9 \text{ rad./sec.} \quad (2c)$$

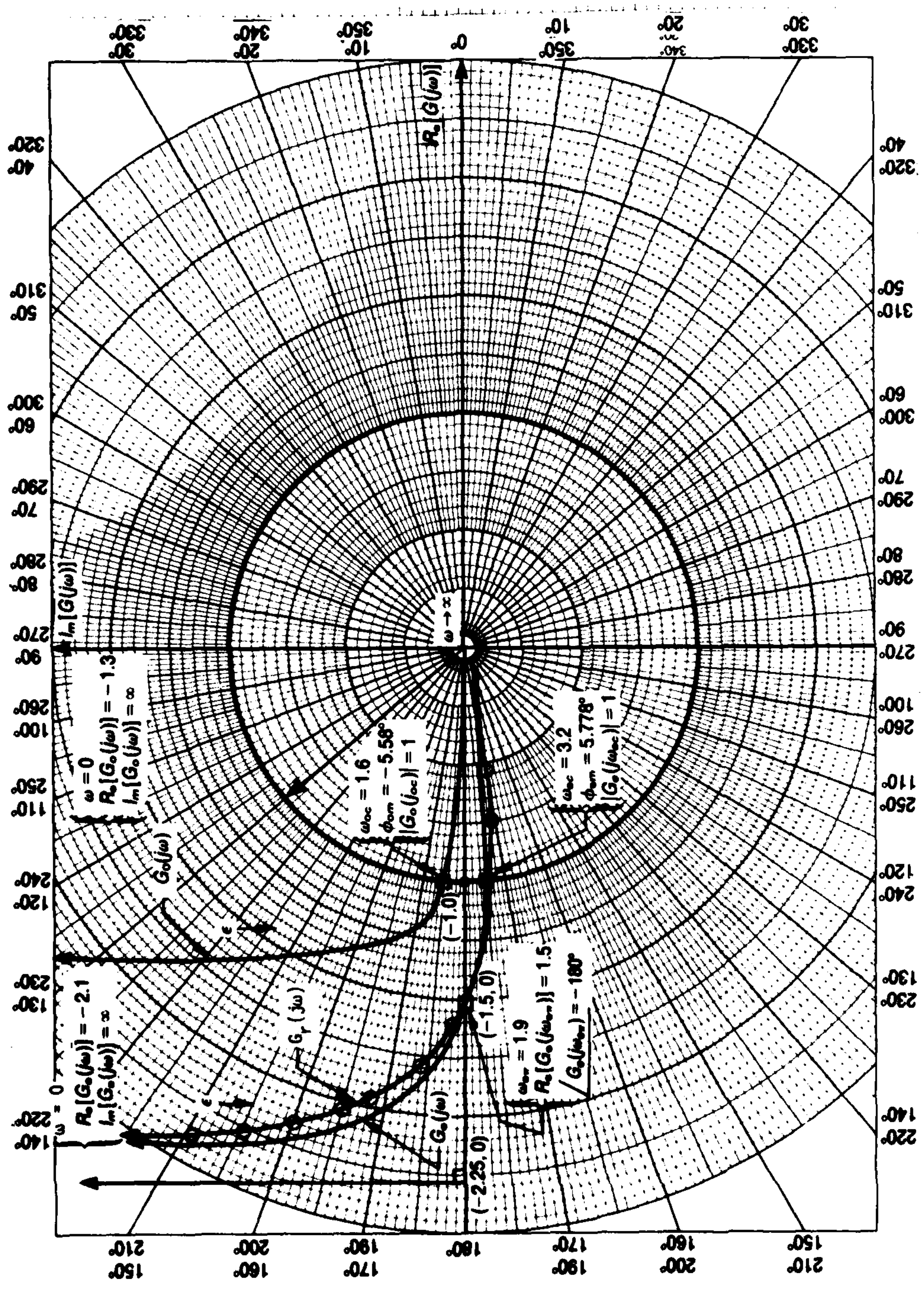


Fig. 2. The Nyquist plots of various open-loop transfer functions.

The equivalent real and imaginary parts of $G_e(j\omega_{e\pi})$ at $\omega_{e\pi} = 1.9$ are

$$R_e G_e(j\omega_{e\pi}) = -1.507944 \tag{2d}$$

$$I_m G_e(j\omega_{e\pi}) = -0.006490205. \tag{2e}$$

(3) The phase margin $\phi_{em}$ of this system is

$$\phi_{em} = 180° + \underline{/G_e(j\omega_{ec})} \cong 5.7787° \tag{2f}$$

where the gain-crossover frequency $\omega_{ec}$ is

$$|G_e(j\omega_{ec})| = 1 \text{ or } \omega_{ec} \cong 3.2 \text{ rad/sec.} \tag{2g}$$

The equivalent real and imaginary parts of $G_e(j\omega_{ec})$ at $\omega_{ec} = 3.2$ are

$$R_e G_e(j\omega_{ec}) = -0.9939143 \tag{2h}$$

$$I_m G_e(j\omega_{ec}) = -0.09547478. \tag{2i}$$

The frequency-response data at $\omega = 0$ in (2a) indirectly indicates the steady-state value of the unit-step response of $T_e(s)$. The data at $\omega = \omega_{e\pi}$ and $\omega = \omega_{ec}$ in (2) represent two control specifications[2]: gain margin and phase margin that characterize the relative stability and the transient response of the existing stabilized system. The dominant frequency response data of the plant $G_0(s)$ and others are as follows

(1) The real and imaginary parts of $G_0(j\omega)$ at $\omega = 0$ are

$$R_e G_0(j0) = -1.304841 \text{ and } I_m G_0(j0) = \infty \tag{3a}$$

(2) The phase margin $\phi_{0m}$ of the plant is

$$\phi_{0m} = 180° + \underline{/G_0(j\omega_{0c})} = -5.58° \tag{3b}$$

where the gain-crossover frequency $\omega_{0c}$ is

$$|G_0(j\omega_{0c})| = 1 \text{ or } \omega_{0c} \cong 1.6 \text{ rad/sec.} \tag{3c}$$

Other frequency-response data at $\omega_{e\pi}$ and $\omega_{ec}$ are

$$(3)\ \left.\begin{array}{l} R_e G_0(j\omega_{e\pi}) = -0.9370766 \\ I_m G_0(j\omega_{e\pi}) = 0.06716120 \end{array}\right\} \text{ for } \omega_{e\pi} = 1.9 \tag{3d}$$

$$(4)\ \left.\begin{array}{l} R_e G_0(j\omega_{ec}) = -0.6181657 \\ I_m G_0(j\omega_{ec}) = 0.01949691 \end{array}\right\} \text{ for } \omega_{ec} = 3.2. \tag{3e}$$

Comparing the dominant data of $G_e(s)$ in (2) and $G_0(s)$ in (3d) and (3e) yields the dominant data required of the stabilization filter $T_s(s)$ as

$$(1)\ R_e T_s(j\omega) = 1.6 \text{ and } I_m T_s(j\omega) = 0 \text{ at } \omega = 0 \tag{4a}$$

$$(2)\ R_e T_s(j\omega_{e\pi}) = 1.600492 \text{ and } I_m T_s(j\omega_{e\pi}) = 0.1216316 \text{ at } \omega_{e\pi} = 1.9 \tag{4b}$$

or

$$|T_s(j\omega_{e\pi})| = 1.605107127 \text{ and } \underline{/T_s(j\omega_{e\pi})} = 4.345918198° \text{ at } \omega_{e\pi} = 1.9 \tag{4c}$$

$$(3)\ R_e T_s(j\omega_{ec}) = 1.601402 \text{ and } I_m T_s(j\omega_{ec}) = 0.2049554 \text{ at } \omega_{ec} = 3.2 \tag{4d}$$

or

$$|T_s(j\omega_{ec})| = 1.614464333 \text{ and } \underline{/T_s(j\omega_{ec})} = 7.293349493^\circ \text{ at } \omega_{ec} = 3.2. \tag{4e}$$

From (1e) and (1f) we observe that $G_0(s)$ and $G_e(s)$ are non-minimum phase functions. From the Nyquist plots in Fig. 2 and the Nyquist stability criterion, we can conclude that: the original missile system (without the stabilization filter) is unstable, the existing stabilized system is asymptotically stable and its time response is oscillatory due to the small positive phase margin in (2f). The purpose of this paper is to develop computer-aided design methods for redesigning the stabilization filter to reduce the implementation cost and improve the flight control performance of the missile system.

Two computer-aided methods are developed in this paper and subsequently used to redesign the pitch control system. In Section II a dominant-data matching method for modeling a transfer function (called a standard transfer function $T_r(s)$) that matches the assigned specifications shown in (2) is developed. The obtained standard transfer function $T_r(s)$ is a reduced-order model of the existing stabilized system $T_e(s)$ in (1a). The time and frequency-response curves of $T_e(s)$ and $T_r(s)$ will be compared to verify that the data in (2) are dominant. The dominant-data matching method is then applied to obtain the reduced-order model of the existing stabilization filter $T_s(s)$. Also, the method is used to fit a low-order model that satisfies the specifications shown in (4). Thus, two low-order stabilization filters are obtained. In Section III, we apply the dominant-data matching method and the algebraic method due to Shieh[3] and Chen[4] to redesign the pitch control system. In order to simplify the design process, the dominant-data matching method is first applied to obtain an unstable reduced-order model of the original unstable high-order system $G_0(s)$. Then, the algebraic method is applied to redesign the pitch control system that has a series filter in the feed forward loop and a parallel filter in the feedback loop. Thus, the advantages of a compensator in the feedback structure can be fully used.

## II. THE DOMINANT-DATA MATCHING METHOD

The design goals and/or the nature of the transient response of a control system are often characterized by a set of control specifications[2]. These specifications are commonly classified as: (1) time-domain specifications, e.g. rise time and overshoot; (2) frequency-domain specifications, e.g. gain and phase margin; (3) complex-domain specifications, e.g. damping ratio and underdamped natural angular frequency. Rules of thumb that represent the relationships among the above three control specifications have been proposed by Axelby[5] and Seshadri *et al.*[6]. Given the above guidelines, it is obvious that the gain margin, phase margin, phase-crossover frequency and gain-crossover frequency are the most important specifications. These data are called the dominant frequency-response data and shown in (2b) through (2i). Other important frequency-response data is the steady-state value of a closed-loop system that is indirectly represented by the value of $G_e(j\omega)$ at $\omega = 0$ in (2a). These dominant data in (2) may be considered as the design goal. In order to verify that the data in (2) are dominant ones we need to construct a transfer function $T_r(s)$ that is the reduced-order model of $T_e(s)$ and has the exact assigned specifications shown in (2). The time response curve and the corresponding time-domain specifications of this practical system $T_e(s)$ are difficult to obtain because $T_e(s)$ is a high-order transfer function with large coefficients. Furthermore, it is a stiff function. The latter can be verified from its small coefficient $a_{10}$ (the sum of all poles of the system) and its large constant $a_0$ (the product of all poles of the system) in (1a). As a result, many numerical integration methods (for example, the Runge–Kutta method[7]) require large data size and high precision calculation for time response determination. However, the frequency response curve and the corresponding frequency-domain specifications of this system can be easily determined by a digital computer. Thus, a frequency-domain approach or a dominant-data matching method is proposed to determine the reduced-order model $T_r(s)$ and to redesign the pitch control system. There exists several frequency-domain methods for model reductions[8–10]. However, the reduced-order models obtained from the proposed method gives the exact assigned frequency-domain specifications in contrast to other techniques which do not.

Let the desired reduced-order model of $T_e(s)$ or the standard model that represents the design goal in (2) have the following form

$$T_r(s) = \frac{a_0 + b_1 s + b_2 s^2}{a_0 + a_1 s + a_2 s^2 + s^3} = \frac{G_r(s)}{1 + G_r(s)} \tag{5a}$$

where the open-loop transfer function $G_r(s)$ is

$$G_r(s) = \frac{a_0 + b_1 s + b_2 s^2}{s[(a_1 - b_1) + (a_2 - b_2)s + s^2]}. \tag{5b}$$

The unknown constants $a_i$ and $b_i$ are to be determined from the conditions in (2). Following the basic definitions and knowing the required values from (2) yields a set of nonlinear equations $f_i(a_0, a_1, a_2, b_1, b_2) = 0$ for $i = 1, 2, \ldots, 5$ as follows

(1) The requirement of (2a), or $R_e G_r(j0) = -2.1$, gives

$$f_1(a_0, a_1, a_2, b_1, b_2) = a_1 b_1 - b_1^2 - a_0 a_2 + a_0 b_2 + 2.1(a_1 - b_1)^2 = 0 \tag{6a}$$

(2) The requirement of (2b), or $R_e G_r(j\omega_{c\pi}) = -1.5$ at $\omega_{c\pi} = 1.9$ gives

$$\begin{aligned} f_2(a_0, a_1, a_2, b_1, b_2) = {} & (a_2 - b_2)(a_0 - 3.61 b_2) - b_1(a_1 - b_1 - 3.61) \\ & - 1.5[3.61(a_2 - b_2)^2 + (a_1 - b_1 - 3.61)^2] = 0 \end{aligned} \tag{6b}$$

(3) The requirement in (2c), or $\angle G_r(j\omega_{c\pi}) = -180°$ at $\omega_{c\pi} = 1.9$, gives

$$f_3(a_0, a_1, a_2, b_1, b_2) = 3.61 b_1(a_2 - b_2) + (a_0 - 3.61 b_2)(a_1 - b_1 - 3.61) = 0 \tag{6c}$$

(4) The requirement of (2f), or $\phi_{cm} = 5.7°$ at $\omega_{cc} = 3.2$, yields

$$\begin{aligned} f_4(a_0, a_1, a_2, b_1, b_2) = {} & 10.24 b_1(a_2 - b_2) + (a_0 - 10.24 b_2)(a_1 - b_1 - 10.24) \\ & - 0.31940224[(a_2 - b_2)(a_0 - 10.24 b_2) \\ & - b_1(a_1 - b_1 - 10.24)] = 0 \end{aligned} \tag{6d}$$

(5) The requirement of (2g), or $|G_r(j\omega_{cc})| = 1$ at $\omega_{cc} = 3.2$, gives

$$\begin{aligned} f_5(a_0, a_1, a_2, b_1, b_2) = {} & (a_0 - 10.24 b_2)^2 + 10.24 b_1^2 - 104.8576(a_2 - b_2)^2 \\ & - 10.24(a_1 - b_1 - 10.24)^2 = 0. \end{aligned} \tag{6e}$$

The above set of high-order, nonlinear, algebraic simultaneous eqns (6) are difficult to solve. The Newton-Raphson method[11] that is available as a library computer program package (called the $Z$ system[12] in many digital computers can be used to solve these nonlinear equations. However, it is well known that the Newton-Raphson method will only converge to a desired solution for a small range of starting values or initial estimates. In order to improve the convergent speed, the following methods are suggested for good initial estimates.

(1) Initial estimate by using the model reduction method due to Shieh and Chen[3, 8].

Shieh[3] and Chen[8] have proposed a continued fraction method for model reduction. The method is as follows. The $N(s)$ and $D(s)$ in (1a) are arrayed into ascending order and expanded into the continued fraction of the second Cauer form by performing repeated long divisions, i.e.

$$T_e(s) = \frac{N(s)}{D(s)} = \frac{b_0 + b_1 s + \cdots + b_{10} s^{10}}{a_0 + a_1 s + \cdots + a_{11} s^{11}}$$

$$= \cfrac{1}{h_1 + \cfrac{s}{h_2 + \cfrac{s}{h_3 + \cfrac{s}{h_4 + \cfrac{s}{\ddots}}}}} \tag{7a}$$

where $h_1 = 1$, $h_2 = -0.401749$, $h_3 = -0.475321$, $h_4 = 25.1998$, $h_5 = -0.0322195$, $h_6 = -24.1061$, $h_7 = \cdots, \cdots$, and $h_{22} = \cdots$ for the example problem. The reduced-order can be obtained by retention of the first several dominant quotients $(h_i)$ for $i = 1, 2, \ldots$ as follows

$$T_e(s) \cong \frac{1}{h_1 + \dfrac{s}{h_2}} = \frac{h_2}{h_1 h_2 + s} \tag{7b}$$

$$\cong \frac{1}{h_1 + \dfrac{s}{h_2 + s/h_3}} = \frac{h_2 h_3 + s}{h_1 h_2 h_3 + (h_1 + h_3)s} \tag{7c}$$

$$\cong \frac{h_2 h_3 h_4 + (h_2 + h_4)s}{h_1 h_2 h_3 h_4 + (h_1 h_2 + h_1 h_4 + h_3 h_4)s + s^2} \tag{7d}$$

$$\cong \frac{h_2 h_3 h_4 h_5 + (h_2 h_3 + h_2 h_5 + h_4 h_5)s + s^2}{h_1 h_2 h_3 h_4 h_5 + (h_1 h_2 h_3 + h_1 h_2 h_5 + h_1 h_4 h_5 + h_3 h_4 h_5)s + (h_1 + h_3 + h_5)s^2} \tag{7e}$$

$$\cong \frac{h_2 h_3 h_4 h_5 h_6 + (h_2 h_3 h_4 + h_2 h_3 h_6 + h_2 h_5 h_6 + h_4 h_5 h_6)s + (h_2 + h_4 + h_6)s^2}{h_1 h_2 h_3 h_4 h_5 h_6 + (h_1 h_2 h_3 h_4 + h_1 h_2 h_3 h_6 + h_1 h_2 h_5 h_6 + h_1 h_4 h_5 h_6 + h_3 h_4 h_5 h_6)s + (h_1 h_2 + h_1 h_4 + h_1 h_6 + h_3 h_4 + h_3 h_6 + h_5 h_6)s^2 + s^3} \tag{7f}$$

$$\cong \ldots$$

where (7f) is the 3rd order approximate model of the original 11th order system, or in our problem

$$T_r^*(s) = \frac{3.7376 + 19.4692s + 0.6920s^2}{3.7376 + 10.1661s + 0.9488s^2 + s^3}. \tag{8a}$$

Using the coefficients in (8a) as initial estimates: $a_0^* = 3.7376$, $a_1^* = 10.1661$, $a_2^* = 0.9488$, $b_1^* = 19.4692$ and $b_2^* = 0.6920$ and applying the Newton-Raphson method [12] to solve the nonlinear equations in (6) yields the desired solution: $a_0 = 6.37807$, $a_1 = 10.46222$, $a_2 = 1.259008$, $b_1 = 20.55661$ and $b_2 = 0.243466$ at the 8th iteration with the error tolerance of $10^{-6}$. The desired reduced-order or the standard model is

$$T_r(s) = \frac{6.37807 + 20.55661s + 0.243466s^2}{6.37807 + 10.46222s + 1.259008s^2 + s^3} = \frac{G_r(s)}{1 + G_r(s)} \tag{8b}$$

where

$G_r(s)$ = The open-loop transfer function of the standard model

$$= \frac{6.37807 + 20.55661s + 0.243466s^2}{s(-10.09439 + 1.015542s + s^2)}. \tag{8c}$$

The Nyquist plot of $G_r(s)$ is shown in Fig. 2, the unit-step responses of $T_e(s)$ in (1a) and $T_r(s)$ in (8b) are shown in Fig. 3. The approximate results are satisfactory. Thus, we verify that the data in (2) are dominant. It is obvious that although the $T_r^*(s)$ in (8a) may have a good overall approximation of $T_e(s)$, only $T_r(s)$ in (8b) has the exact assigned frequency-domain specifications as required in (2) that is essential in the design of a control system in the frequency domain. It may also be noticed that the $T_r^*(s)$ obtained by the continued fraction

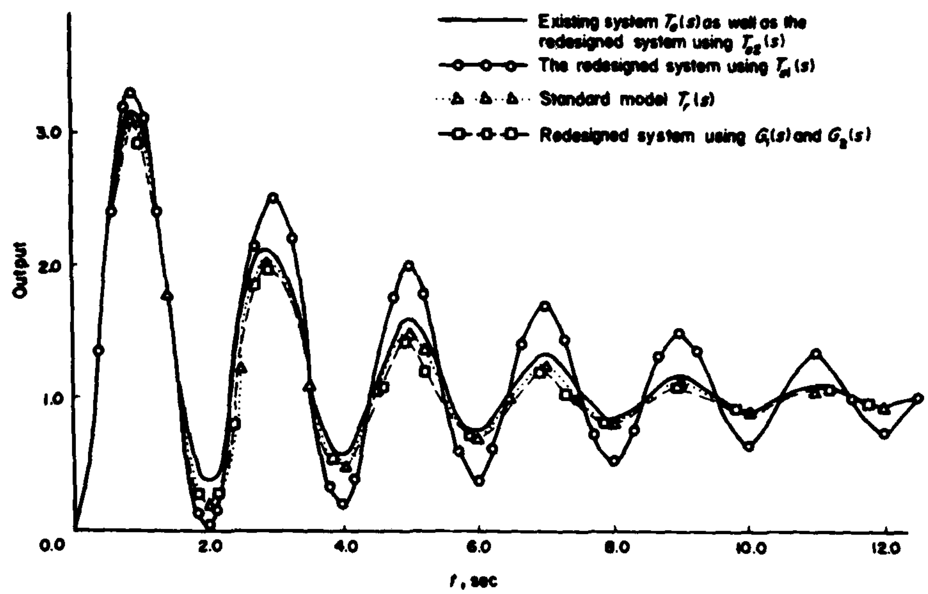


Fig. 3. Time response of various models.

method (7) may be unstable even if the original system $T_r(s)$ is stable. The following mixed method is suggested for obtaining a stable reduced-order model.

(2) Initial guess by using the mixed method.

The mixed method has both advantages of the continued fraction method[3, 8], the dominant-pole method[13] or the equivalent dominant-pole method[9]. It can be applied to determine a stable reduced-order model from which a good initial estimate can be determined. The mixed method is as follows. The relationship between the quotients $h_i$ and the coefficients $a_i$ and $b_i$ in (7a) can be expressed in the following matrix equation[3, 4]:

$$[b] = [H][a] \tag{9}$$

where

$$\begin{aligned} [a]^T &= [a_0, a_1, a_2, \ldots, a_{n-1}] \\ [b]^T &= [b_0, b_1, b_2, \ldots, b_{n-1}] \\ [H] &= [H_2]^{-1}[H_1] \end{aligned}$$

and

$$[H_2] = \begin{bmatrix} h_1 & 0 & 0 & \cdot & 0 & 0 \\ 1 & h_2 & 0 & \cdot & 0 & 0 \\ 0 & 1 & h_3 & \cdot & 0 & 0 \\ 0 & 0 & 1 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 1 & h_n \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \cdot & 0 & 0 \\ 0 & h_1 & 0 & \cdot & 0 & 0 \\ 0 & 1 & h_2 & \cdot & 0 & 0 \\ 0 & 0 & 1 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 1 & h_{n-1} \end{bmatrix} \cdots \begin{bmatrix} 1 & 0 & 0 & \cdot & 0 & 0 \\ 0 & 1 & 0 & \cdot & 0 & 0 \\ 0 & 0 & 1 & \cdot & 0 & 0 \\ 0 & 0 & 0 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 0 & h_1 \end{bmatrix}$$

$$[H_1] = \begin{bmatrix} 1 & 0 & 0 & \cdot & 0 & 0 \\ 0 & h_2 & 0 & \cdot & 0 & 0 \\ 0 & 1 & h_3 & \cdot & 0 & 0 \\ 0 & 0 & 1 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 1 & h_n \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \cdot & 0 & 0 \\ 0 & 1 & 0 & \cdot & 0 & 0 \\ 0 & 0 & h_2 & \cdot & 0 & 0 \\ 0 & 0 & 1 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 1 & h_{n-1} \end{bmatrix} \cdots \begin{bmatrix} 1 & 0 & 0 & \cdot & 0 & 0 \\ 0 & 1 & 0 & \cdot & 0 & 0 \\ 0 & 0 & 1 & \cdot & 0 & 0 \\ 0 & 0 & 0 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 0 & h_2 \end{bmatrix}$$

$T$ in (9) designates transpose. The desired reduced-order model may be

$$T_r(s) = \frac{d_0 + d_1 s + \cdots + d_{r-1}s^{r-1}}{e_0 + e_1 s + \cdots + e_{r-1}s^{r-1} + e_r s^r},\ e_r = 1. \tag{10}$$

The coefficients $e_i$ in (10) can be determined from the coefficients of the polynomial that is the product of the dominant poles of $T_e(s)$ in (7a). Replacing $a_i$ in (9) by $e_i$ in (10) and solving the matrix equation in (9) yields the desired coefficients $d_i$ in (10). The obtained $T_r(s)$ has the dominant poles and quotients of $T_e(s)$ and is always stable. When the roots of $D(s)$ in (7a) are not available, the approximate equivalent dominant poles and the resulting coefficients $e_i$ can be determined from the Routh table as suggested by Hutton and Friedland[9]. Because the method uses the dominant quotients of the original system and the equivalent dominant poles from the Routh table, this method may be conveniently called the mixed method of the continued fraction approximation[8] and the Routh approximation[9]. Using the mixed method we can obtain another stable reduced-order model. Thus, a good initial estimate can be determined.

Since the transfer function of the existing stabilization filter $T_s(s)$ is available, we will use the proposed dominant-data matching method for determining the reduced-order of $T_s(s)$. The $T_s(s)$ in (1c) can be considered the closed-loop transfer function of a control system

$$T_s(s) = \frac{N_s(s)}{D_s(s)} = \frac{G_s(s)}{1 + G_s(s)} = \frac{460800s^2 + 69120000s + 144 \times 10^7}{s^4 + 250s^3 + 76900s^2 + 72 \times 10^5 s + 9 \times 10^8} \tag{11a}$$

where the open-loop transfer function $G_s(s)$ is

$$G_s(s) = \frac{460800s^2 + 69120000s + 144 \times 10^7}{s^4 + 250s^3 - 383900s^2 - 61920000s - 5.4 \times 10^8}. \tag{11b}$$

The dominant frequency-response data of this system are

(1) $G_s(j0) \simeq -2.667$ (12a)

(2) $R_e G_s(j\omega_{s\pi}) = -1.032833$ (12b)
$I_m G_s(j\omega_{s\pi}) = 0.002017351$ (12c)

where $\omega_{s\pi}$ = the phase-crossover frequency = 140 rad/sec.

(3) $R_e G_s(j\omega_{sc}) = -1.002941$ (12d)
$I_m G_s(j\omega_{sc}) = -0.03668759$ (12e)

where $\omega_{sc}$ = the gain-crossover frequency = 200 rad/sec.

The reduced-order stabilization filter $T_{s1}(s)$ is assumed to be

$$T_{s1}(s) = \frac{b_0 + b_1 s}{a_0 + a_1 s + s^2} = \frac{G_{s1}(s)}{1 + G_{s1}(s)} \tag{13a}$$

where

$$G_{s1}(s) = \frac{b_0 + b_1 s}{(a_0 - b_0) + (a_1 - b_1)s + s^2} \tag{13b}$$

and $a_i$ and $b_i$ are unknown constants to be determined.

Using the coefficients $a_i$ and $b_i$ of $G_{s1}(s)$ and following the basic definitions of the data shown in (12a), (12b), (12c) and (12d) results in the following nonlinear equations, respectively

(1) (12b)

$$f_1(a_0, a_1, b_1) = 1.6a_0(-0.6a_0 - 19600) + 19600b_1(a_1 - b_1) + 1.032833[(0.6a_0 + 19600)^2 + 19600(a_1 - b_1)^2] = 0 \tag{14a}$$

(2) (12c)

$$f_2(a_0, a_1, b_1) = 140b_1(-0.6a_0 - 19600) - 224a_0(a_1 - b_1) - 0.002017351 \\ [(0.6a_0 + 19600)^2 + 19600(a_1 - b_1)^2] = 0 \quad (14b)$$

(3) (12d)

$$f_3(a_0, a_1, b_1) = 1.6a_0(-0.6a_0 - 40000) + 40000b_1(a_1 - b_1) + 1.002941 \\ [(0.6a_0 + 40000)^2 + 40000(a_1 - b_1)^2] = 0 \quad (14c)$$

where $b_0 = 1.6a_0$ as obtained from (12a).

The initial estimates can be obtained from the reduced-order model of $T_s(s)$ in (1c) using the mixed method of the continued fraction approximation and the Routh approximation. The reduced-order model is

$$T^*_{r1}(s) = \frac{1281.40525s + 29937.62994}{s^2 + 52.4375s + 18711.01871}. \quad (15)$$

Using the coefficients in $T^*_{r1}(s)$ as initial estimates and applying the Newton-Raphson method[12] we have the desired $a_i$ and $b_i$ in (13) for the 7th iteration with an error tolerance of $10^{-6}$. The desired low-order stabilization filter is therefore

$$T_{s1}(s) = \frac{957.260014s + 33467.93525}{s^2 + 29.981293s + 20917.459536}. \quad (16)$$

The unit-step response of the existing stabilized pitch control system in (1a) and the redesigned pitch control system using $T_{s1}(s)$ in (16) and $G_0(s)$ in (1e) are shown in Fig. 3. The result is considered to be satisfactory.

An alternate approach is proposed for redesigning the stabilization filter as follows. Because the function of a stabilization filter is to convert the dominant data at $\omega = 0$, $\omega_{e\pi} = 1.9$ and $\omega_{ec} = 3.2$ of the original unstable system $G_0(s)$ in (3) to the assigned dominant data of $G_e(s)$ in (2), we can directly apply the dominant-data matching method to fit a low-order stabilization filter that satisfies the specifications assigned in (4). Assume the desired low-order model is

$$T_{s2}(s) = \frac{b_0 + b_1 s}{a_0 + a_1 s + s^2}. \quad (17a)$$

From the definition of (4a) we have $b_0 = 1.6a_0$, and (17a) can be rewritten as

$$T_{s2}(s) = \frac{1.6a_0 + b_1 s}{a_0 + a_1 s + s^2}. \quad (17b)$$

When $s = j\omega_{e\pi} = j1.9$ the respective values of $|T_{s2}(j\omega_{e\pi})|$ and $\angle T_{s2}(j\omega_{e\pi})$ in (17b) match the values of $|T_s(j\omega_{e\pi})|$ and $\angle T_s(j\omega_{e\pi})$ in (4c), or the corresponding non-linear equations are

$$f_1(a_0, a_1, b_1) = 2.56a_0^2 + 3.61b_1^2 - 2.576368889[(a_0 - 3.61)^2 + 3.61a_1^2] = 0 \quad (18a)$$

and

$$f_2(a_0, a_1, b_1) = 1.9b_1(a_0 - 3.61) - 3.04a_0a_1 - 0.0759963811[1.6a_0(a_0 - 3.61) \\ + 3.61a_1b_1] = 0. \quad (18b)$$

When $s = j\omega_{ec} = j3.2$ the value of $\angle T_{s2}(j\omega_{ec})$ in (17b) matches the value of $\angle T_s(j\omega_{ec})$ in (4e), or the nonlinear equation is

$$f_3(a_0, a_1, b_1) = 3.2b_1(a_0 - 10.24) - 5.12a_0a_1 - 0.1279849782[1.6a_0(a_0 - 10.24) \\ + 10.24a_1b_1] = 0. \quad (18c)$$

Applying the Newton-Raphson method[12] and using the initial estimates obtained in (15) we have the desired solution of the nonlinear equations of (18) at the 9th iteration with the error tolerance of $10^{-6}$. The desired low-order stabilization filter is

$$T_{s2}(s) = \frac{856.628596s + 21283.19886}{s^2 + 3.318051s + 13301.999297}. \tag{19}$$

The unit-step response curves of the existing stabilized pitch control system $T_e(s)$ in (1a) and the redesigned system that uses the low-order filter $T_{s2}(s)$ in (19) and the $G_0(s)$ in (1e) are shown in Fig. 3. The result is practically identical. From the response curves in Fig. 3 we observe that $T_{s2}(s)$ in (19) is a better filter than the $T_{s1}(s)$ in (16) as far as duplicating the performance of the original pitch control system is concerned. This implies that the existing stabilization filter $T_s(s)$ in (1c) might be overdesigned since we can duplicate performance with a lower filter. Obviously, the implementation cost of the filter $T_{s2}(s)$ is less than that of $T_s(s)$ in (1c).

## III. AN ALGEBRAIC METHOD

The original fourth-order stabilization filter $T_s(s)$ may be replaced by two second-order filters, $T_{s1}(s)$ and $T_{s2}(s)$, using the dominant-data matching method. It is observed that all three stabilization filters have complex roots and that all are placed in the feed forward loop. Therefore, they are sensitive to external disturbances. If alternate filters can be designed and placed in both the feed forward and feedback loops, then (a) the designed filters may result in simple transfer functions with positive real roots that may be easily synthesized using RC type passive elements, and (b) the reliability and cost of the designed system may be improved. The compensators in the feedback loop enable the designed system to be less sensitive to parameter variations and modeling errors. In addition, it will reduce the effects of many external disturbances[14].

The algebraic method given by Shieh[3] and Chen[4] is extended and modified to redesign this pitch control system. The steps of the algebraic method are summarized as follows:

*Step* 1

Assign the design goals using frequency-domain specifications and model a standard transfer function using the dominant-data matching method.

*Step* 2

Expand the obtained standard transfer function into the continued fraction expansion shown in (7) to obtain the dominant quotients and to formulate the matrix equation in (9).

*Step* 3

Assume the fixed configuration compensators with unknown parameters and determine the overall transfer function that consists of the unknown parameters.

*Step* 4

Substitute the coefficients of the obtained over-all transfer function in Step 3 into the vectors $[a]$ and $[b]$ in (9) and expand the matrix equation to obtain a set of equations.

*Step* 5

Solve the set of equations to determine the unknown constants assigned in the compensators.

The designed system using the algebraic method has the exact dominant quotients of the standard models and is a good approximation of the standard model.

Before we design the pitch control system using the algebraic method we apply the dominant-data matching method to determine a reduced-order model of the original unstable system $G_0(s)$ in (1e) which had a transfer function with large coefficients. The unstable transfer function $G_0(s)$ in (1e) can be decomposed into a stable portion and an unstable portion as follows

$$G_0(s) = \frac{1}{s(s-2.921)} T_0(s) \tag{20a}$$

where the stable portion is

$$T_0(s) = \frac{324332.316(s+0.1933)(s+65)(s+1500)}{(s+3.175)(s+87.9\pm j95.5)(s+112.5)(s+1385)}. \tag{20b}$$

The pole at the origin and the unstable pole at $s = 2.921$ are considered as dominant poles. Therefore, they are retained in the simplified model $G_0^*(s)$, or

$$G_0(s) \cong G_0^*(s) = \frac{1}{s(s-2.291)} T_0^*(s) \tag{20c}$$

where $T_0^*(s)$ is the reduced order model of $T_0(s)$ using the dominant-data matching method. The frequency-response data of the gain margin, phase margin, phase-crossover frequency, gain-crossover frequency and the final value at $\omega = 0$ are used as the dominant data for the transfer function fitting. The resulting $T_0^*(s)$ is

$$T_0^*(s) = \frac{496.854897s^2 + 192897.961011s + 37103.33375}{s^3 + 117.073733s^2 + 16552.300003s + 50595.685093}. \tag{20d}$$

The desired $T_0^*(s)$ is a low-order model with small coefficients. Thus, the design process has been greatly simplified.

Following the steps in the algebraic method we assign the series compensator $G_1(s)$ and the parallel compensator $G_2(s)$ with unknown parameters $X_i$, $i = 1, 2, \ldots, 7$ as

$$G_1(s) = \frac{X_6 s + X_7}{s + X_5} \tag{21a}$$

and

$$G_2(s) = \frac{X_3 s^2 + X_4 s + X_2}{s^2 + X_1 s + X_2}. \tag{21b}$$

The block diagram of this redesigned system is shown in Fig. 4(1). The overall transfer function

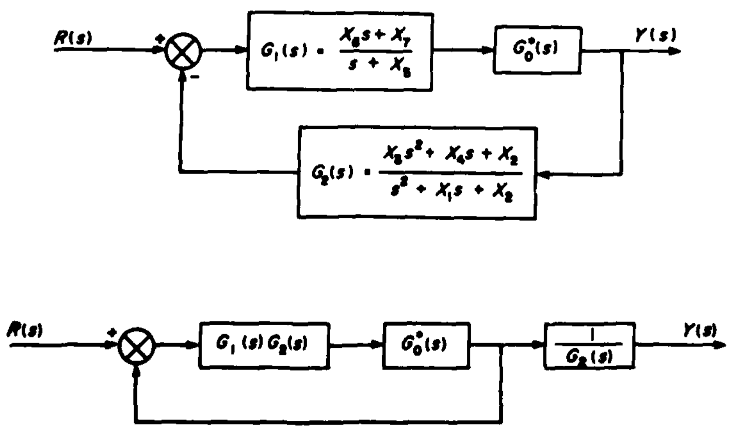


Fig. 4. The block diagrams of the redesigned system using algebraic method.

$T_f(s)$ of this feedback system is

$$T_f(s) = \frac{b_0 + b_1 s + \cdots + b_7 s^7}{a_0 + a_1 s + \cdots + a_8 s^8} \tag{21c}$$

where

$$\begin{aligned}
a_0 &= 37103.33375 X_2 X_7 \\
a_1 &= 192897.961011 X_2 X_7 + 37103.33375(X_2 X_6 + X_4 X_7) - 147789.9961 X_2 X_5 \\
a_2 &= 496.854897 X_2 X_7 + 192897.961011(X_2 X_6 + X_4 X_7) + 37103.33375(X_4 X_6 \\
&\quad + X_3 X_7) + 2246.41679 X_2 X_5 - 147789.9961(X_2 + X_1 X_5) \\
a_3 &= 496.854897(X_2 X_6 + X_4 X_7) + 192897.961011(X_4 X_6 + X_3 X_7) + 37103.33375 \\
&\quad \cdot X_3 X_6 - 147789.9961(X_1 + X_5) + 2246.41679(X_2 + X_1 X_5) + 16210.32763 X_2 X_5 \\
a_4 &= 496.854897(X_4 X_6 + X_3 X_7) + 192897.961011 X_3 X_6 - 147789.9961 + 2246.41673 \\
&\quad \cdot (X_1 + X_5) + 16210.32763(X_2 + X_1 X_5) + 114.152733 X_2 X_5 \\
a_5 &= 496.854897 X_3 X_6 + 2246.41679 + 16210.32763(X_1 + X_5) \\
&\quad + 114.152733(X_2 + X_1 X_5) + X_2 X_5 \\
a_6 &= 16210.32763 + 114.152733(X_1 + X_5) + X_2 + X_1 X_5 \\
a_7 &= 114.152733 + X_1 + X_5 \\
a_8 &= 1 \\
b_0 &= 37103.33375 X_2 X_7 \\
b_1 &= 192897.961011 X_2 X_7 + 37103.33375(X_2 X_6 + X_1 X_7) \\
b_2 &= 496.854897 X_2 X_7 + 192897.961011(X_2 X_6 + X_1 X_7) + 37103.33375(X_1 X_6 + X_7) \\
b_3 &= 496.854897(X_2 X_6 + X_1 X_7) + 192897.961011(X_1 X_6 + X_7) + 37103.33375 X_6 \\
b_4 &= 496.854897(X_1 X_6 + X_7) + 192897.961011 X_6 \\
b_5 &= 496.854897 X_6 \\
b_6 &= 0 \\
b_7 &= 0.
\end{aligned}$$

In order to match the seven unknown parameters $X_i$ in (21) for this type "1" system we need eight quotients, $h_i$ in (9). Therefore, the third order standard model in (8b) that has six quotients: $h_i = 1$, $h_2 = -0.631845015$, $h_3 = -0.476189214$, $h_4 = 14.799589050$, $h_5 = -0.102867450$, $h_6 = -13.924278040$ should be increased to a fourth order model by inserting $h_7 = 100$ and $h_8 = 0.1$ where the values of $h_7$ and $h_8$ are selected based on rules developed in[15]. The resulting amplified model is

$$T_r(s) = \frac{6.37807 + 20.55661 s + 0.243466 s^2}{6.37807 + 10.46222 s + 1.259008 s^2 + s^3}$$

$$= \cfrac{1}{h_1 + \cfrac{s}{h_2 + \cfrac{s}{h_3 + \cfrac{s}{h_4 + \cfrac{s}{h_5 + \cfrac{s}{h_6}}}}}} \cong \cfrac{1}{h_1 + \cfrac{s}{h_2 + \cfrac{s}{\ddots \, h_5 + \cfrac{s}{h_6 + \cfrac{s}{h_7 + \cfrac{s}{h_8}}}}}}$$

$$= T_a(s) = \frac{63.78098007 + 211.8989926 s + 22.87561717 s^2 + 0.34346 s^3}{63.78098007 + 110.9545225 s + 23.00917551 s^2 + 11.30110515 s^3 + s^4}. \tag{22}$$

It[15] has been shown that (22) is a good approximation of the original model in (8b). Substituting the $a_i$, $i = 0, 1, \ldots, 7$ and $b_i$, $i = 0, \ldots, 7$ in (21) and using the quotients $h_i$, $i = 1, \ldots, 8$ in (22) into (9) yields the following set of high-order nonlinear simultaneous algebraic equations.

$$f_1(X_1, \ldots, X_7) = X_2 X_7 + 0.6318422396[X_7(X_4 - X_1) - 3.98319992 X_2 X_5] = 0 \tag{23a}$$

$$f_2(X_1,\ldots,X_7) = X_7(8.22822291X_2 + 8.522553136X_4 - 6.939879587X_1 + X_3 - 1) + X_2(1.582676549X_6 - 13.17807554X_5) + X_6(X_4 - X_1) - 3.983199922(X_2 + X_1X_5) = 0 \tag{23b}$$

$$f_3(X_1,\ldots,X_7) = X_2(-12.71361621X_6 - X_5) + X_7(13.58355291X_1 + 1.820964317X_2 - 26.29716913X_4) + 10.79844539(X_1X_6 + X_7) - 13.31248704(X_4X_6 + X_3X_7) + 6.327224282(X_1 + X_5) + 1.588477708X_6(1 - X_3) + 20.03527143(X_2 + X_1X_5) = 0 \tag{23c}$$

$$f_4(X_1,\ldots,X_7) = X_7(X_2 + 668.4670071X_4 - 281.48094X_1) + X_6(386.9860673X_2 + 362.767005 - 456.258273X_3) - 647.2403649(X_4X_6 + X_3X_7) - 57.53603068X_2X_5 - 548.5188427(X_2 + X_1X_5) + 235.861385(X_1X_6 + X_7) + 235.2945185 + 590.5096275(X_1 + X_5) = 0 \tag{23d}$$

$$f_5(X_1,\ldots,X_7) = 2357.408023(X_1X_6 + X_7) + X_6(1598.839931X_2 + 17096.15228 - 32881.95043X_3) + X_7(4.16745091X_2 + 1599.83991X_1 - X_4) - 472.6735322(X_4X_6 + X_3X_7) + 24996.98242 - 939.0287936(X_1 + X_5) - 2765.323026(X_2 + X_1X_5) - 52.07771943X_2X_5 = 0 \tag{23e}$$

$$f_6(X_1,\ldots,X_7) = X_6(-99.4209415X_2 + 11132.91981X_3 - 57256.87822) + X_7(X_4 - 100.4209415X_1) + 411.4274907(X_4X_6 + X_3X_7) + 67006.93001(X_1 - X_5) + 1234.567433(X_2 + X_1X_5) + 42.69011171X_2X_5 + 23203.53455 - 39112.69694(X_1X_6 + X_7) = 0 \tag{23f}$$

$$f_7(X_1,\ldots,X_7) = 496.854897(X_2X_6 + X_1X_7) + 198512.9704(X_1X_6 + X_7) + 2228495.695X_6 - 170.6497831(X_4X_6 + X_3X_7) - 77618.59617X_3X_6 - 3442861.087 - 395845.4335(X_1 + X_5) - 8390.812346(X_2 + X_1X_5) - 62.08251489X_2X_5 = 0. \tag{23g}$$

Equation (23) is a high order nonlinear equation which is difficult to solve. The Newton-Raphson method[12] is applied to determine the unknown parameters $X_i$. The following method is suggested for obtaining the initial estimates.

The structure of the desired fixed configuration control system in Fig. 4(1) can be modified as shown in Fig. 4(2). The overall transfer function is

$$T_1(s) = T_2(s)\frac{1}{G_2(s)} \tag{24a}$$

where

$$T_2(s) = \frac{G_1(s)G_2(s)G_0^*(s)}{1 + G_1(s)G_2(s)G_0^*(s)}.$$

The design objective is to determine $G_1(s)$ and $G_2(s)$ such that the response of $T_1(s)$ is close to that of the standard model $T_r(s)$ in (8b). Replacing the series compensator $G_1(s)G_2(s)$ in Fig. 4(2) by the designed stabilization filter $T_{s2}(s)$ in (19) and equating the resulting transfer function $T_1(s)$ in (24a) to the standard model $T_r(s)$ in (8b), we can solve the approximate transfer function $G_2^*(s)$ of $G_2(s)$, or

$$G_2^*(s) = G_2(s) = \frac{T_2(s)}{T_r(s)} = \frac{G_1(s)G_2(s)G_0^*(s)}{1 + G_1(s)G_2(s)G_0^*(s)T_r(s)} = \frac{T_{s2}(s)G_0^*(s)}{1 + T_{s2}(s)G_0^*(s)T_r(s)}$$
$$= [5.036619205 \times 10^9 + 3.46495752 \times 10^{10}s + 4.540060393 \times 10^{10}s^2 + 7.840679235 \times 10^9s^3 + 4.363076841 \times 10^9s^4 + 1.763524302 \times 10^8s^5 + 4.256201128 \times 10^5s^6]/[5.036619205 \times 10^9 + 3.008227329 \times 10^{10}s + 4.613716606 \times 10^{10}s^2 + 6.124169121 \times 10^9s^3 + 4.498497844 \times 10^9s^4 + 8.512494768 \times 10^7s^5 + 9.985459768 \times 10^5s^6 + 9.698650697 \times 10^3s^7 + 4.91568119 \times 10s^8 + 0.243466s^9]. \tag{24b}$$

Expanding (24b) into the form of (7a) yields a set of dominant quotients $h_1 = 1$, $h_2 = -1.102755917$, $h_3 = -0.1287948973$, $h_4 = 5.593229805$, $h_5 = 0.1338916858$, $h_6 = \cdots, \cdots, h_{18} = \cdots$. Substituting the first five quotients into (7e) gives a second order approximate model $G_2^{**}(s)$ of the approximate parallel filter $G_2^\dagger(s)$ in (24b) yielding

$$G_2^{\dagger*}(s) = \frac{0.994929057s^2 + 0.7394973923s + 0.1058245527}{s^2 + 0.643533679s + 0.1058245527}. \tag{24c}$$

$G_2^{\dagger*}(s)$ is an approximate model of the assigned parallel compensator $G_2(s)$ in (21b). The approximate series compensator ($G_1^\dagger(s)$) of $G_1(s)$ is

$$G_1^\dagger(s) = \frac{T_{s2}(s)}{G_2^{\dagger*}(s)} = \frac{2252.284999 + 13787.1076s + 21834.46821s^2}{1407.678125 + 9837.144919s + 13237.10512s^2}$$

$$\frac{+ 856.628596s^3}{+ 4.040722745s^3 + 0.994929057s^4}. \tag{25a}$$

Equation (25a) can be expanded into the form of (7a) to obtain a set of dominant quotients $h_1 = 0.625$, $h_2 = 1.845828612$, $h_3 = 0.0839039052$, $h_4 = \cdots, \cdots, h_8 = \cdots$. Substituting the first three quotients into (7c) yields the reduced-order model of $G_1^\dagger(s)$ in (25a)

$$G_1^{\dagger*}(s) = \frac{1.410628426s + 0.2184671685}{s + 0.1365419803}. \tag{25b}$$

$G_1^{\dagger*}(s)$ is an approximate model of the assigned series compensator $G_1(s)$ in (21a). Comparing (21b) and (24c) and (21a) and (25b) we have a set of initial estimates as $X_1^\dagger = 0.643533679$, $X_2^\dagger = 0.1058245527$, $X_3^\dagger = 0.994929057$, $X_4^\dagger = 0.7394973923$, $X_5^\dagger = 0.1365419803$, $X_6^\dagger = 1.410628426$ and $X_7^\dagger = 0.2184671685$. Using these initial estimates and the Newton-Raphson method[12] to solve the nonlinear simultaneous algebraic equations in (23) yields the solution $X_1 = 0.503850$, $X_2 = 0.059928$, $X_3 = 1.051503$, $X_4 = 0.580016$, $X_5 = 4.831826$, $X_6 = 1.885577$, and $X_7 = 6.744450$ at the 14th iteration with the error tolerance of $10^{-6}$. The desired compensators are

$$G_1(s) = \frac{1.885577s + 6.744450}{s + 4.831826} = \frac{1.885577(s + 3.57688)}{s + 4.831826} \tag{26a}$$

and

$$G_2(s) = \frac{1.051503s^2 + 0.580016s + 0.059928}{s^2 + 0.503850s + 0.059928}$$

$$= \frac{1.051503(s + 0.13769)(s + 0.41391)}{(s + 0.19244)(s + 0.311405)}. \tag{26b}$$

The unit-step response curves of the existing stabilized system $T_e(s)$ in (1a) and the redesigned system using the compensators in (26) and the $G_0(s)$ in (1e) are shown in Fig. 3. The result is satisfactory. Note that $G_1(s)$ and $G_2(s)$ in (26) are positive real functions with positive real poles and zeros that can be realized using RC type passive elements. Thus, the advantage of a feedback controller system[14] may be fully utilized.

## IV. CONCLUSION

Two computer-oriented methods: a dominant-data matching method and an algebraic method have been presented to redesign an existing stabilized pitch control system[1]. Thus, an alternate method to the trial-and-error approach that is traditionally used has been given. The resulting low-order stabilization filters that were obtained using the above methods reduce the implementation cost of the pitch control of the missile system.

The dominant-data matching method can be applied to general control system design to determine the reduced-order model of a high-order system. The proposed model reduction method is superior to most existing methods in that it provides the exact frequency-domain specifications.

An algebraic method has been applied to determine fixed configuration filters such that the performances of the redesigned pitch control system of an example missile has been greatly improved. The algebraic method can be applied to design any desired control system structure. Several methods have been given for estimating good initial guesses for solving nonlinear equations. In summary, the proposed computer-aided design methods present an attractive alternative to trial-and-error methods which can be used to design control systems.

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