A Mechanism for Heating the Solar Corona

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# A MECHANISM FOR HEATING THE SOLAR CORONA

The high temperatures measured in the solar corona are explained in terms of heating which results from the work done on the plasma by expanding magnetic flux. This theory, which invokes no wave deposition or ohmic heating processes, is consistent with the strongly magnetized, highly variable nature of the solar atmosphere.
A MECHANISM FOR HEATING THE SOLAR CORONA

The nature of the mechanism which causes temperatures in the solar atmosphere to rise from about 5000 K in the photosphere to over $10^6$ K in the corona is a major unsolved problem. Wave processes have been invoked to transport the necessary energy from the sun's surface (Osterbrock, 1961). Ohmic heating or magnetic reconnection has also been proposed (Vaiana and Rosner, 1978). It is the purpose of this letter to point out that another process, compressional work done on the plasma by expanding magnetic flux, may be important.

Observations made in the last few years (Sheeley, et al., 1975) show that the corona is highly structured, with magnetic fields playing a significant - perhaps dominant - role. They also reveal that the corona is in a continual state of fluctuation on both large and small space-time scales (Wentzel, 1978). In spite of this, elevated coronal temperatures are apparently maintained continuously. It is thus possible to model the coronal heating as the outcome of frequently occurring but discrete events, each of which deposits some fraction of the energy needed to balance the power dissipated by the corona.

Consider a bubble-like object, composed of magnetic flux and plasma, emerging from the surface of the sun. The exact topology of this entity is not critical; it may resemble a jet more than a bubble, for example, or it may have the form of an arch, and the field lines may remain connected to the solar magnetic dipole. The plasma density is less than or equal to that in the surrounding medium, but the initial magnetic pressure $B^2/8\pi$ is assumed to exceed the average ambient pressure, so that the structure expands as it rises. This continues until pressure balance is reached, reconnection annihilates the magnetic flux, or the bubble vents out the top of the atmosphere. In any case the work $\Delta W$ done on the surrounding plasma is essentially $\Delta V B^2/8\pi$, where $\Delta V$ is the initial volume of the bubble. Moreover, a mass $\Delta m = \rho \Delta V$ is injected into the corona at the same time, where $\rho$ is the mass density when the bubble emerges.

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If structures of this type are continuously generated so that the average volume of flux emitted per unit time is \( \dot{V} \) and if they are responsible for heating the corona, then power balance implies that in the quiet sun (Withbroe and Noyes, 1977), on the average,

\[
\dot{V} \frac{B^2}{8\pi} = (p_{\text{rad}} + p_{\text{conv}} + p_{\text{cond}}) A_e
\]

\[
\approx (10^5 + 2 \times 10^5 + 5 \times 10^4) A_e \text{ erg s}^{-1}
\]

\[
= 3.5 \times 10^5 A_e \text{ erg s}^{-1}. \tag{1}
\]

Here \( A_e = 4\pi R_e^2 \) and the three terms on the right hand side of (1) represent losses due to radiation, convection (i.e., the solar wind), and thermal conduction along the field lines to the cooler chromosphere. The structures emanate with a velocity \( v \) from some fraction \( F \) of the sun's bipolar magnetic regions. The latter have an area \( \sim 0.1 A_e \), so \( \dot{V} \sim 0.1 F A_v v \).

If the emerging structure has low density and rises because of buoyancy forces, then \( v \sim (g r_0) v \), where \( g \) is the surface gravitational acceleration and \( r_0 \) is a characteristic dimension for bipolar magnetic regions. If instead it is propelled by magnetic forces, then \( v \approx 1.3 \times 10^7 B \text{ cm s}^{-1} \) (with \( B \) in G). Other mechanisms lead to different estimates of \( v \). Although the choice is not crucial to the theory, we will assume here that \( v = v_A \sim 2 \times 10^6 B \text{ cm s}^{-1} \), where \( v \) has been calculated by taking an ion number density \( \sim 10^{10} \text{ cm}^{-3} \). Then from (1) a fraction \( F \approx 40 \text{ B}^{-3} \) of the sun's magnetically active regions must be emitting flux at any instant of time. The requirement \( F \leq 1 \) implies that \( B \gtrsim 3 \text{ G} \) must hold.

Similarly, the upward motion produces a surge which injects mass into the corona at a rate

\[
\dot{m} = \rho \dot{V} \sim 1.3 \times 10^{-7} B^{-2} \text{ g s}^{-1}. \tag{2}
\]
The requirement that the velocity of the material sprayed out of the solar atmosphere not exceed the sum of the observed solar wind speed and the escape speed implies \( v \lesssim 10^8 \text{ cm s}^{-1} \), or \( B \lesssim 50 \text{ G} \). At this upper limit, the total mass injection rate is \( \dot{m} \sim 4 \times 10^{-11} \text{ g s}^{-1} \). This is comparable with observed solar wind mass losses (Withbroe and Noyes, 1977).

To obtain more information about how an expanding magnetic "bubble" heats the corona as a function of, e.g. altitude, we need to adopt a specific model. The following discussion illustrates this. The model chosen, although unrealistic in some respects, makes it clear that wave and current dissipation are not required by the process. Consider the time-dependent axisymmetric system given by

\[
\rho = \rho_0 \, f^{-9};
\]

\[
p = \frac{3}{32} \rho \, z^2/t^2;
\]

\[
u = r/t;
\]

\[
v = z/4t;
\]

\[
B_r = \phi \left( \frac{r}{L} \right) \sin \left( \frac{2z}{L} \right)/u_0^2 \xi^2;
\]

\[
B_\phi = \phi \left( \frac{r}{L} \right) \cos \left( \frac{2z}{L} \right)/u_0^2 \xi^2;
\]

\[
B_z = \phi \cos \left( \frac{2z}{L} \right)/u_0^2 \xi^2.
\]

Here \( r \) and \( z \) are distances measured from and along the axis of symmetry; \( f = (t/t_0)^k \); \( L = k \); and \( \rho_0, u_0, t_0, k \) and \( \phi \) are constants. Equations (3)-(9), which can readily be shown to satisfy the ideal MHD equations with adiabatic index \( \gamma = 5/3 \), describe a force-free self-similar configuration with similarity variables \( \xi = r/u_0 t \) and \( \zeta = 2z/L \), corresponding to independent uniform homologous expansions in the transverse and vertical directions with velocities \( u \) and \( v \), respectively. All three components of the magnetic induction \( B \) are proportional to the flux \( \phi \) contained within a tube of radius...
\[ R(z,t) = u_0 t \sec \frac{L}{2}(2z/L), \] and hence can be chosen arbitrarily large without affecting the material pressure \( p \), the density or flow field.

Equations (3) - (9) become singular at \( t = 0 \), and so must be initialized at some finite time. If we take \( t_0 \) to be the initial time, then \( \rho_0, r_0 = u_0 t_0, A, \) and \( B_0 = \phi/r_0^2 \) can be interpreted as the initial values of the density, characteristic horizontal and vertical length scales, and characteristic magnetic field strength. Likewise, the flux tube radius, and therefore \( B_r \) and \( B_\phi \) on the flux tube boundary, diverge at \( z = \pm L/4 \), so the solution must be restricted to a region \( 0 \leq z \leq Z = z_0 f \), where \( z_0 \) is a constant (to be specified below) satisfying \( z_0 < \pm L/4 \). For \( z > Z \) the system is not describable by a similarity solution, or it must violate the assumption of ideal force-free flow.

We will overlook the problems with the magnetic field above \( z = Z \), as well as the neglect in the model of the sun's curvature and gravity, and introduce the further simplifying assumption that the plasma in the exterior region \( r > R \) is unmagnetized. At the flux tube boundary a kinematic condition and the condition of pressure balance, \( p_{\text{ext}} = p + B^2/8\pi \approx B^2/8\pi \), must be satisfied, where in consequence of our motivating physical picture we ignore \( p \) compared with \( B^2/8\pi \). It is unnecessary to construct a complete model of the flow in the exterior region. Instead, we limit ourselves to calculating the work done by the expanding flux tube on the surrounding medium.

The power per unit height generated by the expansion is

\[
\frac{\partial P}{\partial z} = 2n R \left[ 1 + \left( \frac{\partial \phi}{\partial \phi} \right)^2 \right] B_\phi \cdot \text{\textbf{X}} \ast p_{\text{ext}} = \frac{R}{4} \left[ u(R)-v(R) \frac{\partial \phi}{\partial \phi} \right] B^2
\]

\[ = (\phi^2/u_0^2 \xi^3) \left[ 1 - \xi/8 \tan \xi \right] \left( (u_0 \xi/L)^2 + \cos^2 \xi \right) \sec^2 \xi, \quad (10) \]

where \( \xi \) is the velocity of the flux tube boundary. We note that (10) becomes negative for \( \xi \) sufficiently close to \( \pm 1 \). There the tube, which flares out almost horizontally, is locally receding from the plasma. To avoid this, we choose \( \xi \) so that \( (\xi/4) \tan (2\xi/4) = 4 \), i.e., \( \xi = 0.6994 \).
We will assume that, whatever the actual form of the configuration for \( z \gg Z \), the exterior medium does as much work on it as it does on the medium. Then if the expansion work done at a given altitude remains there in the form of thermal energy, the heat deposited as a function of \( z \) is given by \( H(z) = \int_{0}^{z} \frac{dP}{dz} \). In Fig. 1, \( H = H \alpha^2 \frac{r^2}{s^2} \) is plotted against \( z \) for three different values of the aspect ratio \( \alpha = A_0 r_0^2 \). It can be seen that \( H \) scales roughly as \( r_0^2 B_0^2 \alpha^2 \). For \( \alpha \sim 1 \) it has a maximum at \( s \sim 4r_0 \) as \( \alpha \to 0 \), the heating becomes strongest at the bottom of the structure. In either case, most of the heat is deposited in a distance of order \( r_0 \) (the initial horizontal scale of the bipolar magnetic regions) from the bottom of the atmosphere. Integration of (10) over both \( z \) and \( t \) shows that the total heat evolved in the expansion process is equal to the magnetic energy initially contained in the configuration (proportional to \( B_0^2 r_0^2 A \)), as it should. The actual manner in which the unmagnetized atmosphere is heated depends on convection, conduction and radiation processes, and is of course not local. Evidently the rapid expansion of the magnetic bubble can drive shocks, particularly if the expansion velocities are as large as \( v_A \). Numerical models are currently being developed at the Naval Research Laboratory in order to make global quantitative predictions of the heating to be expected from the mechanism discussed here, together with the effects of wave and ohmic heating and other plasma processes.

In summary, bubbles or other structures containing magnetic flux with field strengths \( \sim 20G \) and having transverse dimensions like those of the initial stages of bipolar magnetic regions, can supply heat to the solar atmosphere through expansion in amounts sufficient to explain the observed elevated coronal temperatures. If magnetic forces are responsible for expelling this flux, the bubbles should be emerging from a fraction \( F \sim 0.03\% \) of the bipolar magnetic regions at any time, with characteristic speeds \( v_A \lesssim 500 \text{ km s}^{-1} \) and lifetimes \( R_0/v_A \gtrsim 10^3 \text{ s} \), and should deposit their energy principally within vertical distances \( \sim 10^4 - 10^5 \text{ km} \) from the surface of the sun.
Fig. 1 — Reduced heating function $\tilde{H}$ vs. altitude for three values of the aspect parameter $\alpha$. 

\begin{align*}
\alpha &= 1.0 \\
\alpha &= 10.0 \\
\alpha &= 5.0
\end{align*}
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