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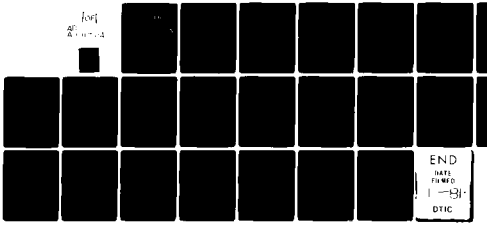

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on

CONTROL OF DYNAMICAL SYSTEMS

for the period

Sept. 1, 1979 - Aug. 31, 1980

Lefschetz Center for Dynamical Systems
Division of Applied Mathematics
Brown University
Providence, R. I. 02912

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September 30, 1980

Report prepared by:
H. T. Banks

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Spline based approximation schemes for nonlinear nonautonomous functional differential equations control systems are discussed and it is shown how these may be employed in parameter estimation techniques. A sample of our numerical findings is also presented.

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ANNUAL PROGRESS REPORT to AFOSR

In addition to the investigations detailed in the semi-annual progress report to the AFOSR for the period September 1, 1979 - March 1, 1980, the following efforts were also made in connection with this grant.

Banks, in pursuing investigations on computational methods for control and identification of delay systems, has obtained general approximation results for nonlinear systems and is now testing the related techniques on numerical examples. Included in these are difficult problems where one wishes to estimate the delay parameters themselves. Kappel, a visitor from Austria, obtained related results on approximation schemes for nonlinear functional differential equations. The efforts by Banks and Kappel have resulted in general spline type approximation schemes (of any desired order) for a general class of nonlinear delay equations.

Banks, in collaboration with K. Kunisch (a post doctoral visitor at Brown during the past year) and J. Crowley (a graduate student at Brown), has begun investigations of the use of some of his approximation ideas (used previously for delay systems) in developing approximation schemes for use in partial differential equation parameter estimation problems. Preliminary theoretical and numerical findings (involving modal or eigenfunction based schemes) are quite encouraging and this work will continue during the next several years.

Banks and Rosen (a graduate student at Brown) have numerically tested their simultaneous time and state discretization methods on parameter estimation examples with linear delay systems. As predicted by their earlier theoretical findings, the methods offer rapidly convergent, easily implemented schemes.

Kunisch has also investigated use of the approximation schemes (in particular the "averaging" and "spline" methods) developed previously by Banks and collaborators in two other areas: (1) nonlinear neutral FDE (systems where $\dot{x}(t)$ depends on $x(t)$, $x(t-r)$, and $\dot{x}(t-r)$) control problems; (2) approximation of the Riccati integral equation associated with the linear regulator problem for delay systems. It appears that his findings on this latter problem will play an important role in the development of numerical methods with the potential for widespread use in computational problems encountered by engineers in a number of areas of application.

Hale (in collaboration with Shui-Nee Chow at Michigan State University and Brown) is completing investigations on the behavior of solutions of ordinary differential equations near an equilibrium point in three dimensions when the associated linear variational equation has one zero and two purely imaginary roots. They have given a complete analysis under mild hypotheses on the quadratic and cubic terms. The results answer important questions on the

role of successive bifurcations in studying motions which have a random or chaotic behavior. For the case of two pairs of purely imaginary roots, partial results have been obtained.

Results have been obtained by Hale and Vegas on the number of solutions of a reaction diffusion equation as a function of the domain. Stability properties of the solutions were also investigated.

Hale with a student, Luis Magelhães, is studying an equation in the theory of rheology which describes the behavior of a viscoelastic material after it has been stretched. This is a singularly perturbed differential integral equation. Partial results on the boundary layer terms have been obtained. These improve the earlier results of Hodge, Nohel and McLeod.

PUBLICATIONS SUPPORTED BY THE AIR FORCE

SEPTEMBER 1, 1979 - AUGUST 31, 1980

BANKS, H.T.

Identification of nonlinear delay systems using spline methods. Invited Lecture, International Conference on Nonlinear Phenomena in the Mathematical Sciences, University of Texas, Arlington, Texas, June 13-20, 1980.

BANKS, H.T. (with Karl Kunisch)

Parameter estimation techniques for nonlinear distributed parameter systems. Invited Lecture, International Conference on Nonlinear Phenomena in the Mathematical Sciences, University of Texas, Arlington, Texas, June 16-20, 1980.

BANKS, H.T.

Computational difficulties in the identification and optimization of control systems. Invited Lecture, Workshop on Applied Control Theory to Renewable Resource Management and Ecology, January 7-11, 1980, University of Canterbury, Christchurch, New Zealand. To appear in Lecture Notes in Biomathematics, Springer-Verlag.

BANKS, H.T. (with J.A. Burns and E.M. Cliff)

Parameter estimation and identification for systems with delays. SIAM J. Optimization and Control, to appear.

BANKS, H.T. (with J.A. Burns and E.M. Cliff)

A comparison of numerical methods for identification and optimization problems involving control systems with delays. LCDS Technical Report TR 79-7, November 1979, Brown University.

BANKS, H.T. (with Franz Kappel)

Spline approximations for functional differential equations. Journal of Differential Equations, vol. 34, no. 3, December 1979, pp. 496-522.

BANKS, H.T.

Approximation of nonlinear functional differential equation control systems. Journal of Optimization Theory and Applications, vol. 29, no. 3, November 1979, pp. 383-408.

HALE, J.K.

Stability and bifurcation in a parabolic equation. To be submitted.

HALE, J.K.

Topics in Dynamic Bifurcation Theory. Lectures presented at the NSF-CBMS Conference, Arlington, Texas, June 16-20, 1980.

HALE, J.K.

Generic properties of an integro-differential equation. Presented at the Conference on Differential Equations in Analysis and Geometry, April 24, 25, 1980, Baltimore, Maryland on the occasion of P. Hartman's Retirement.

HALE, J.K.

Asymptotic behavior of an integro-differential equation. To be submitted.

HALE, J.K.

Stability from the bifurcation function. Presented at the 8th Fall Conference on Differential Equations, Oklahoma State University, October 18-19, 1979. Differential Equations, Academic Press, Inc., 1980, pp. 23-30.

HALE, J.K.

Remarks on bifurcation theory in differential equations. New Approaches to Nonlinear Problems in Dynamics, P.J. Homes, ed., SIAM Philadelphia 1980.

HALE, J.K. (with S-N. Chow and J. Mallet-Paret)

An example of bifurcation to homoclinic orbits. To appear in the Journal of Differential Equations.

HALE, J.K. (with J.C. deOliveira)

Dynamic behavior from bifurcation equations. Tohoku Mathematical Journal, vol. 32, no. 4, 1980.

HALE, J.K. (with J.C. deOliveira)

Hopf bifurcation for functional equations. JMAA, vol. 74, no. 1, March 1980, pp. 41-59.

HALE, J.K. (with C.E. Avellar)

On the zeros of exponential polynomials. JMAA, vol. 73, no. 2, February 1980, pp. 434-452.

INFANTE, E.F. (with L.A.V. Carvalho and J.A. Walker)

On the existence of simple Liapunov functions for linear retarded difference differential equations. Tohoku Mathematical Journal, vol. 32, no. 2, June 1980, pp. 283-297.

INFANTE, E.F. (with J.A. Walker)

A stability investigation for an incompressible simple fluid with fading memory. Archives for Rational Mechanics, vol. 72, no. 3, 1980.

KAPPEL, Franz

Spline approximation for autonomous nonlinear functional differential equations. To be submitted.

KAPPEL, Franz

An approximation scheme for delay equations. Invited Lecture, International Conference on Nonlinear Phenomena in Mathematical Sciences, University of Texas, Arlington, Texas, June 16-20, 1980.

KUNISCH, Karl

The Riccati integral equation arising in optimal control of delay differential equations. Invited Lecture, International Conference on Nonlinear Phenomena in Mathematical Sciences, University of Texas, Arlington, Texas, June 16-20, 1980.

KUNISCH, Karl

Approximation schemes for nonlinear neutral optimal control systems, to appear in Journal of Mathematical Analysis and Applications.

KUNISCH, Karl

Approximation schemes for the linear-quadratic optimal control problem associated with delay-equations, to appear in the SIAM J. Control and Optimization.

KUNISCH, Karl

A semigroup approach to partial differential equations with delay, to appear in Pitman Research Notes in Mathematics.

LANGENHOP, C.E.

Controllability and stabilizability of regular singular linear systems with constant coefficients. To appear in the SIAM Journal on Control and Optimization.

MASSATT, Paul

Attractivity properties of α-contractions. To appear in the Journal of Differential Equations.

MASSATT, Paul

Asymptotic behavior for a strongly damped nonlinear wave equation. Invited Lecture, International Conference on Nonlinear Phenomena in Mathematical Sciences, University of Texas, Arlington, Texas, June 16-20, 1980.

MASSATT, Paul

Properties of condensing maps and dissipative systems. Ph.D. Dissertation, June 1980.

MASSATT, Paul

Stability and fixed points of point dissipative systems. To appear in the Journal of Differential Equations.

ROSEN, I.G.

A discrete approximation framework for hereditary systems. To appear in the Journal of Differential Equations.

TURYN, L.

Functional difference equations and an epidemic model. Presented at the International Conference on Nonlinear Phenomena in Mathematical Sciences, June 16-20, 1980, University of Texas, Arlington, Texas.

IDENTIFICATION OF NONLINEAR DELAY SYSTEMS USING SPLINE METHODS

H.T. Banks

Abstract

Spline based approximation schemes for nonlinear nonautonomous functional differential equation control systems are discussed and it is shown how these may be employed in parameter estimation techniques. A sample of our numerical findings is also presented.

PARAMETER ESTIMATION TECHNIQUES

for

NONLINEAR DISTRIBUTED PARAMETER SYSTEMS

H.T. Banks and K. Kunisch

ABSTRACT

Methods for estimating system parameters are discussed for a class of partial differential equations. We develop schemes based on modal subspace approximations in some detail and include numerical examples.

COMPUTATIONAL DIFFICULTIES IN THE IDENTIFICATION AND OPTIMIZATION OF CONTROL SYSTEMS

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ABSTRACT

As more realistic models for resource management are developed, the need for efficient computational techniques for parameter estimation and optimal control involving nonlinear vector systems will grow. We discuss some of the difficulties associated with such computational schemes and also report on results available for identification and control of several classes of systems which are of increasing importance in a number of areas of applications.

PARAMETER ESTIMATION AND IDENTIFICATION FOR SYSTEMS WITH DELAYS

H. T. Banks, J. A. Burns and E. M. Cliff

Abstract

Parameter identification problems for delay systems motivated by examples from aerodynamics and biochemistry are considered. The problem of estimation of the delays is included. Using approximation results from semigroup theory a class of theoretical approximation schemes is developed and two specific cases ("averaging" and "spline" methods) are shown to be included in this treatment. Convergence results, error estimates, and a sample of numerical findings are given.

A COMPARISON OF NUMERICAL METHODS FOR IDENTIFICATION AND OPTIMIZATION PROBLEMS INVOLVING CONTROL SYSTEMS WITH DELAYS

H.T. Banks, J.A. Burns and E.M. Cliff

ABSTRACT

In this report we present numerical results for two approximation techniques for functional differential control systems. One technique is based on an averaging scheme, the other on spline approximations. A number of examples are considered and the techniques are applied to parameter estimation problems and optimal control problems where the systems are given by differential equations with hereditary terms.

SPLINE APPROXIMATIONS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

H. T. Banks and F. Kappel

Abstract: We develop an approximation framework for linear hereditary systems which includes as special cases approximation schemes employing splines of arbitrary order. Numerical results for first and third order spline based methods are presented and compared with results obtained using a previously developed scheme based on averaging ideas.

STABILITY AND BIFURCATION IN A PARABOLIC EQUATION

by

Jack K. Hale

Abstract

Recent results on the stability of equilibrium solutions of a parabolic equation are given with indications of the proofs. Particular attention is devoted to dependence of the stability properties on the shape of the domain and the manner in which nonhomogeneous stable equilibria can occur through a bifurcation induced by varying the domain.

GENERIC PROPERTIES OF AN INTEGRO-DIFFERENTIAL EQUATION

Jack K. Hale

Abstract: Consider the functional differential equation

$$\dot{x}(t) = - \int_{-1}^{0} a(-\theta) g(x(t+\theta)) d\theta$$

where a, g are continuous, $a \geq 0$, $a(1) = 0$, $g(0) = 0$, $g'(0) = 1$, $xg(x) > 0$ for $x \neq 0$. The linear function $a_0(s) = 4\pi^2(1-s)$ is such that the characteristic equation

$$\lambda + \int_{-1}^{0} a_0(-\theta) e^{\lambda\theta} d\theta = 0$$

has two eigenvalues on the imaginary axis and the remaining ones with negative real parts. In spite of this, it is shown there is no generic Hopf bifurcation for any g. The nature of the bifurcation is characterized under hypotheses which appear to be generic in g.

ASYMPTOTIC BEHAVIOR OF AN INTEGRO-DIFFERENTIAL EQUATION

by

Jack K. Hale

Abstract: Consider the functional differential equation

$$\dot{x}(t) = -\int_{-1}^{0} a(-\theta)g(x(t+\theta))d\theta$$

where $a \geq 0$, g are continuous, $a(1) = 0$. If $\ddot{a} > 0$ and $G(x) = \int_0^x g \to \infty$ as $|x| \to \infty$, then it is known that every solution either approaches a zero of g as $t \to \infty$ or has an ω-limit set which is a 1-periodic solution of the equation $\ddot{x} + a(0)g(x) = 0$. If there are only a finite number of equilibrium points and 1-periodic orbits, then there is a maximal compact invariant set $A_{a,g}$ in $C([-1,0],\mathbb{R})$ which is uniformly asymptotically stable. When a is convex, the topologically structure of $A_{a,g}$ and the flow on $A_{a,g}$ are discussed as a function of g. When g is fixed, $xg(x) > 0$, $x \neq 0$, the complete bifurcation diagram is given for a in a neighborhood of a linear function.

STABILITY FROM THE BIFURCATION FUNCTION

Jack K. Hale

ABSTRACT

This paper contains an extension to C^k-vector fields of the classical results of Liapunov on the stability of an equilibrium point in the critical case of one zero root. The transformation theory of Liapunov is not applicable to this case. We exploit fundamental relations from bifurcation theory.

AN EXAMPLE OF BIFURCATION TO HOMOCLINIC ORBITS

by

S-N. Chow, Jack K. Hale and John Mallet-Paret

ABSTRACT

Consider the equation

$$\ddot{x} - x + x^2 = -\lambda_1 \dot{x} + \lambda_2 f(t)$$

where $f(t+1) = f(t)$ and $\lambda = (\lambda_1, \lambda_2)$ is small. For $\lambda = 0$, there is a homoclinic orbit Γ through zero. For $\lambda \neq 0$ and small, there can be strange attractors near Γ. The purpose of this paper is to determine the curves in λ-space of bifurcation to strange attractors and to relate this to hyperbolic subharmonic bifurcations.

SPLINE APPROXIMATION FOR AUTONOMOUS NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

by

F. Kappel

ABSTRACT:

Based on abstract approximation results in semigroup theory we develop an approximation scheme for nonlinear autonomous functional-differential equations with globally Lipschitzean right-hand side. The scheme can be realized by using spline approximation of the state.

AN APPROXIMATION SCHEME FOR DELAY EQUATIONS

by

F. Kappel

ABSTRACT:

We present a general approximation scheme for autonomous FDE's with Lipschitzean right-hand side. Our approach is based on approximation results in semigroup theory. The results are applied to nonlinear autonomous FDE's in the state space $\mathbb{R}^n \times L^2$ and to linear autonomous FDE's of neutral type in the state spaces $\mathbb{R}^n \times L^2$ and $W^{1,2}$.

THE RICCATI INTEGRAL EQUATION ARISING IN OPTIMAL CONTROL OF DELAY DIFFERENTIAL EQUATIONS

by

K. Kunisch

Abstract:

In this paper we discuss the linear regulator problem associated with delay-differential equations. Formulas for the optimal feedback control and optimal trajectory are derived; this naturally leads to a Riccati integral equation in the state space of the delay equation. Finally, the linear regulator problem for the delay equation is approximated by sequences of regulator problems associated with ordinary differential equations in Euclidean space.

APPROXIMATION SCHEMES FOR NONLINEAR NEUTRAL OPTIMAL CONTROL SYSTEMS

by

Karl Kunisch

Abstract:

We discuss methods of approximating stable neutral functional differential equations and associated optimal control problems by sequences of optimal control problems for ordinary differential equations. By introducing a class of "mollified" neutral functional differential equations, convergence of the linear interpolating spline and the averaging approximation scheme is proved. A number of numerical examples is included.

APPROXIMATION SCHEMES FOR THE LINEAR-QUADRATIC OPTIMAL CONTROL PROBLEM ASSOCIATED WITH DELAY-EQUATIONS

Karl Kunisch

Abstract: The linear regulator problem for delay equation is discussed. We propose a (theoretical) solution involving Riccati integral equations and then axiomatically discuss a general approximation scheme. The details are given for spline-and averaging approximations.

A SEMIGROUP APPROACH TO PARTIAL DIFFERENTIAL EQUATIONS WITH DELAY

by

Karl Kunisch

Abstract: In this paper we discuss a semigroup approach to the equation

$$\frac{d}{dt}\, x(t) = f(x(t)) + g(x(t), x_t)$$

in the space $Y \times L^p(-\infty,0;Y)$, where Y is a reflexiv Banach space, f an m-dissipative operator and g satisfies local "Borisovich Turbabin" conditions. Representation formulas for the local semigroups are derived and the relationship between mild and strong solutions is discussed.

CONTROLLABILITY AND STABILIZABILITY OF REGULAR SINGULAR LINEAR SYSTEMS WITH CONSTANT COEFFICIENTS

by

C. E. Langenhop

Abstract. A concept of controllability for systems $A\dot{x} + Bx = Cu(t)$ in which A may be singular is introduced. When $\det(As+B) \not\equiv 0$, $s \in \mathbb{C}$, this is shown to be equivalent to the condition that $c^T(As+B)^{-1}C \equiv 0$ implies $c = 0$. It is also shown that when such a system is controllable and C is a column vector, then there exists a feedback $u = g_0^T x + g_1^T \dot{x}$ such that $A - Cg_1^T$ is non-singular and all solutions of $A\dot{x} + Bx = C(g_0^T x + g_1^T \dot{x})$ decay exponentially.

ASYMPTOTIC BEHAVIOR FOR A STRONGLY DAMPED NONLINEAR WAVE EQUATION

Paul Massatt

In this paper, we consider the limiting behavior of equations of the form

$$u_{tt} - \alpha\Delta u_t - \Delta u = \mathcal{F}(t,x,u,\nabla u,u_t,\nabla u_t) \tag{1}$$

with $\alpha > 0$ and $\mathcal{F}$ periodic in t. We also consider special cases where $\mathcal{F}$ may depend on fewer terms. We are interested in solutions of the form $u(t,x): \mathbb{R}^+ \times \Lambda \to \mathbb{R}$ where Λ is a bounded domain in $\mathbb{R}^n$ with smooth boundary, $u + \beta \frac{\partial u}{\partial \nu} = 0$ for all $x \in \partial\Lambda$ and some $\beta \geq 0$, and $u(0,x) = \phi(x)$ and $u_t(0,x) = \psi(x)$ for $\phi(x),\psi(x)$ in some appropriate space of initial functions. The case of Neumann boundary conditions could also be considered but is slightly more complicated and so we have omitted this case from discussion in this paper. (See [14]).

This paper is a specific application of my recent paper, "Limiting Behavior for Strongly Damped Nonlinear Wave Equations" [14] where results of Webb [16] and Fitzgibbon [4] were extended by applying results of a few recent papers of mine ([11], [12], [13], [14]). I am deeply grateful to Professor Jack K. Hale for his help and supervision in the preparation of this paper.

STABILITY AND FIXED POINTS OF POINT DISSIPATIVE SYSTEMS

Paul Massatt

Abstract: It is known that if $T: X \to X$ is completely continuous or if there exists an $n_0 > 0$ such that T^{n_0} is completely continuous, then T point dissipative implies there is a maximal compact invariant set which is uniformly asymptotically stable, attracts bounded sets, and has a fixed point (see Billotti and LaSalle [1]). The result is used, for example, in studying retarded functional differential equations, or parabolic partial differential equations.

This result has been extended by Hale and Lopes [8]. They get the result that if T is an α-contraction and compact dissipative then there is a maximal compact invariant set which is uniformly asymptotically stable, attracts neighborhoods of compact sets, and has a fixed point.

The above result requires the stronger assumption of compact dissipative. The principle result of this paper is to get similar results under the weaker assumption of point dissipative. To do this we must make additional assumptions. We will show these assumptions are naturally satisfied by stable neutral functional differential equations and retarded functional differential equations with infinite delay. The result has applications to many other dynamical systems, of course.

Abstract

PROPERTIES OF CONDENSING MAPS AND DISSIPATIVE SYSTEMS

by Paul Massatt, Ph.D., Brown University, June 1980

α-condensing maps are the generalization of α-contractions, which have proved of much usefulness in the study of many types of evolution equations with infinite-dimensional state space. It is also interesting to look at other measures of noncompactness and see what properties are common for condensing maps with these measures of noncompactness as well. Part I deals with these questions. In this part we prove an asymptotic smoothness property, a continuous dependence result, several fixed point thereoms, and that a linear condensing map is an α-contraction under some equivalent norm. We also compare the results of Cooperman in his Ph.D. dissertation with the new results here.

In Part II we study the limiting behavior of dissipative systems. A dissipative system is essentially a system where there is some fixed bounded set into which all trajectories enter. In this section we prove the existence of a maximal compact invariant set along with its stability and attractivity properties under the weakest dissipative conditions, i.e. that all points enter in and remain in the bounded set, by assuming more about the dynamical system. We show the assumptions on the dynamical system are natural enough to be satisfied by stable neutral functional differential equations and retarded functional differential equations of infinite delay. In this section we also survey some of the main results known about dissipative systems and use results of Part I to make it more general.

ATTRACTIVITY PROPERTIES OF α-CONTRACTIONS

Paul Massatt

Abstract: It is known that if $T: X \to X$ is completely continuous where X is a Banach space, then point dissipative and compact dissipative are equivalent, and imply the existence of a maximal compact invariant set which is uniformly asymptotically stable and attracts bounded sets uniformly. If T is an α-contraction it is not known whether point dissipative and compact dissipative are equivalent. However, T is compact dissipative, then there exists a maximal compact invariant set which is uniformly asymptotically stable and attracts neighborhoods of compact sets uniformly.

Since, in practice, it is much easier to verify that a map is point dissipative rather than compact dissipative, it is desirable to say more about the limiting behavior when T is only assumed to be point dissipative. In this paper, we show, with the addition of only a few general assumptions, that point dissipative and compact dissipative are equivalent. The assumptions seem to be general enough to include almost all of the practical applications. Applications are given, or referenced, to stable neutral functional differential equations, retarded functional differential equations of infinite delay, and strongly damped nonlinear wave equations.

A DISCRETE APPROXIMATION FRAMEWORK FOR HEREDITARY SYSTEMS

I.G. Rosen

Abstract

A discrete approximation framework for initial-value problems involving certain classes of linear functional differential equations (FDE) of the retarded type is constructed. An equivalence between the FDE and abstract evolution equations (AEE) in an appropriately chosen Hilbert space is established. This equivalence is then employed in the development of discrete approximation schemes in which the infinite-dimensional AEE is replaced by a finite-dimensional system of difference equations. Convergence and rates of convergence are demonstrated via the properties of rational functions with operator arguments and both classical and recent results from linear semigroup theory. Two examples of families of approximation schemes which are included in the general framework and which may be implemented directly on high-speed computing machines are developed. A numerical study of examples which illustrates the application and feasibility of the approximation techniques in a variety of problems together with a summary and analysis of the numerical results are also included.

FUNCTIONAL DIFFERENCE EQUATIONS AND AN EPIDEMIC MODEL

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ABSTRACT

We consider an epidemic model of the form $S \to I \to S$ with history on $(-\infty,0]$. The well-known threshold phenomenon is discussed in terms of the stability of a functional difference equation, also known as the translation-invariant renewal equation. Since the difference equation has infinite delay, the work of other authors on finite-delay problems is extended. Also, epidemic models with spatial effects are discussed by extension of the results to difference equations in a Banach space.