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STEEPEST EDGE ALGORITHMS IN LINEAR AND NONLINEAR PROGRAMMING. (U)
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ALGORITHMS IN LINEAR AND NONLINEAR PROGRAMMING

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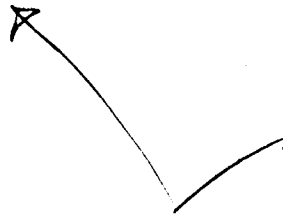
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Several algorithms in linear and nonlinear programming have been developed and analyzed. A worst-case analysis of the steepest edge simplex method showed it to have exponential time-complexity. This algorithm was specialized for solving minimum cost network flow problems and a partial pricing variant was developed. After extensive testing on randomly generated large problems the method was found to be inferior to efficiently coded partial pricing		

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variants of the "standard" simplex method except for the max flow problem and the problem of finding a feasible flow. On the max flow problem the algorithm was compared with the Edmonds-Karp and Dinic algorithms and found to be superior. In quadratic programming, the use of the steepest edge (face) criterion for dropping constraints was found to be helpful. Dual methods were found to be even more efficient and their use in recursive quadratic programming algorithms for nonlinear programming problems is recommended. A numerically stable ellipsoid algorithm for linear programming was developed and analyzed. At present the main promise that this method holds is as a powerful theoretical tool. Several minimization algorithms for taking advantage of negative curvature were developed as was a curvilinear steplength algorithm which ensures convergence to a positive semidefinite point.



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A. PROBLEM STATEMENT

Although we originally only proposed to study steepest edge algorithms in linear and nonlinear programming, exciting new developments and our own scientific curiosity led us to broaden our research into other types of algorithms in linear and nonlinear programming as well. Our researches, however, remained limited to a study of algorithms in these two areas.

In linear programming a number of problems were studied including (i) the computational complexity of the steepest edge simplex algorithm, (ii) the ellipsoid algorithm of Shor and Judin and Nemirovskii, which was used by Khachiyan to show that linear programs could be solved in polynomial time, and (iii) the preservation of sparsity when updating the LU factorization of the basis matrix in the simplex method.

For the subclass of linear programming problems which are subsumed under the category of minimum cost network flow problems, we studied (i) the development of efficient steepest edge simplex algorithms for these problems, (ii) how to choose appropriate data structures to implement these algorithms, (iii) the development of specialized algorithms for the max flow problem and (iv) the avoidance of degenerate pivots.

In the closely related subject of quadratic programming we studied (i) steepest edge (i.e., face) algorithms, and (ii) dual and primal-dual algorithms.

In the area of nonlinear programming we studied the problems of (i) developing algorithms which can take advantage of negative curvature in the objective function, and (ii) how to determine the steplength to take along a curvilinear path in a region where the function being minimized is nonconvex.

B. SUMMARY OF RESEARCH RESULTS

1. Linear Programming Algorithms

(i) Computational Complexity of the Steepest Edge Simplex Algorithm.

We have investigated the computational complexity of the steepest-edge simplex algorithm for general linear programming problems. We have shown how to construct linear programs whose feasible regions are combinatorially equivalent to hypercubes of arbitrary dimension n for which the steepest edge simplex algorithm passes through all 2^n vertices. This "worst-case" analysis extends the well-known analogous result of Klee and Minty for the standard pivot section rule.

(ii) The Ellipsoid Method

A rigorous proof has been given of the fact that among all ellipsoids that contain a given half-ellipsoid the one constructed by the

ellipsoid method is of minimum volume. The deep-cut version of the ellipsoid method was analyzed for a special class of problems and it was shown that this version is only four to five times as fast as the Shor-Khachiyan version. It was also demonstrated that although the volumes of the ellipsoids generated converge to zero, the points (i.e. centers of the ellipsoids) need not converge to a feasible point if the feasible set is not full-dimensional. Also, in the infeasible case it was shown that infeasibility can go undetected.

A survey of past and current work on the ellipsoid method was written in collaboration with Profs. Robert Bland and Michael Todd of Cornell University, and a numerically stable algorithm was developed for solving linear programming problems in collaboration with Professor Michael Todd. Our implementation updates at each iteration the Cholesky factors of the symmetric positive definite matrix which defines the ellipsoids in a way which ensures numerical stability. By using so-called "surrogate" cuts and objective function cuts and by refining both lower and upper bounds on the optimal value of the objective function, our algorithm should be able to terminate with an indication of infeasibility or with a provably good feasible solution in a moderate number of iterations. The method was coded and tested on a small set of problems. Unfortunately, the results obtained were not very promising even though all sorts of devices to accelerate convergence were employed.

(iii) Updating of Sparse LU Factorization of an LP Basis Matrix:

A new way to preserve as much sparsity as possible when updating the LU factorization of a basis matrix (in the simplex method, for example) has been developed. By proper choice of row and column permutations it produces a block upper triangular matrix with small diagonal blocks which is then triangularized by the Bartels-Golub algorithm.

2. Minimum Cost Network Flow Algorithms

(i) Steepest Edge Simplex Algorithms. / (ii) Implementation.

A steepest edge simplex algorithm for network flow problems has been developed which updates the number of arcs in each nonbasic cycle, (i.e., flow augmenting path), after each simplex pivot from properties of the basic tree. A partial pricing variant of this network steepest edge simplex method has also been developed. A network simplex code using list structures for storing the basic tree and including several partial pricing strategies as options has been written in FORTRAN.

A novel feature of the code is the use of a postorder thread to represent the basis tree. This code was subjected to extensive

computational testing on thirty randomly generated large problems (thousands of nodes and tens of thousands of arcs) - to determine the most efficient level of partial pricing.

Our principal finding is that the decrease in iterations that results from using the steepest edge column selection rule in either a full or partial pricing algorithm does not compensate sufficiently for the increase in work per iteration over that required by the "minimum reduced cost" rule on most network flow problems. Exceptions to this are the max flow problem and the problem of finding a feasible flow.

(iii) Max Flow Algorithm:

A partial pricing variant of the steepest edge simplex algorithm for minimum cost network flow problems was found to be very efficient for solving max flow problems. A FORTRAN program for solving max flow problems by the steepest edge simplex method was developed in collaboration with Dr. Michael Grigoriadis on the basis of his RNET code. This was compared computationally with our own FORTRAN codes for the Edmonds-Karp algorithm and Dinic's algorithm. Our results indicate that our method is superior to the latter method and that both the steepest edge and Dinic algorithms are vastly superior to the Edmonds-Karp algorithm.

(iv) Avoiding Degenerate Pivots.

Two techniques for avoiding degenerate pivots in network flow problems have been developed, but not yet computationally tested. They are based upon the use of special list structures.

3. Quadratic Programming Algorithms.

(i) Steepest Edge (Face) Algorithms.

The steepest edge(face)criterion for dropping constraints from an active set in primal quadratic programming algorithms has been implemented in a FORTRAN code.

Extensive computational testing indicated that use of the steepest face criterion results in fewer steps and basis changes. However because of the increased computational requirements per iteration, there is practically no change in the total work required when compared with algorithms using standard column selection criteria.

(ii) Dual and Primal-Dual Algorithms.

A numerically stable dual method for positive definite quadratic programs has been developed and fully implemented in FORTRAN. It uses both Cholesky and QR factorizations. Extensive computational testing indicates that it is three to four times as efficient as competitive primal codes. When used as a subroutine in the Harwell

code of Powell's recursive quadratic programming algorithm for solving nonlinear programming problems a similar speed-up in performance was observed.

4. Nonlinear Programming Algorithms

(i) Minimization Algorithms which Make Use of Negative Curvature.

Several minimization algorithms which make use of negative curvature have been developed. These include versions of the Goldfeld-Quandt-Trotter method and gradient path methods which incorporate negative curvature information through the numerically stable Bunch-Parlett factorization.

(ii) Curvilinear Path Steplength Algorithms

A curvilinear path steplength algorithm for use in minimization methods was developed. In non-convex regions the path followed is initially tangent to the direction of steepest descent, (or some other suitable downhill direction), eventually becoming tangent to a direction of negative curvature. In strictly convex regions a straight path is followed. Under fairly modest conditions we have proved that our steplength algorithm ensures convergence of the iterates generated by the minimization algorithm to a critical point at which the Hessian of the objective function is positive semidefinite.

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D. PARTICIPATING SCIENTIFIC PERSONNEL:

Principal Investigator: Professor Donald Goldfarb

Graduate Research Assistant: Dr. Ashok Idnani (Ph.D. awarded by The City University of New York, May 1980)

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