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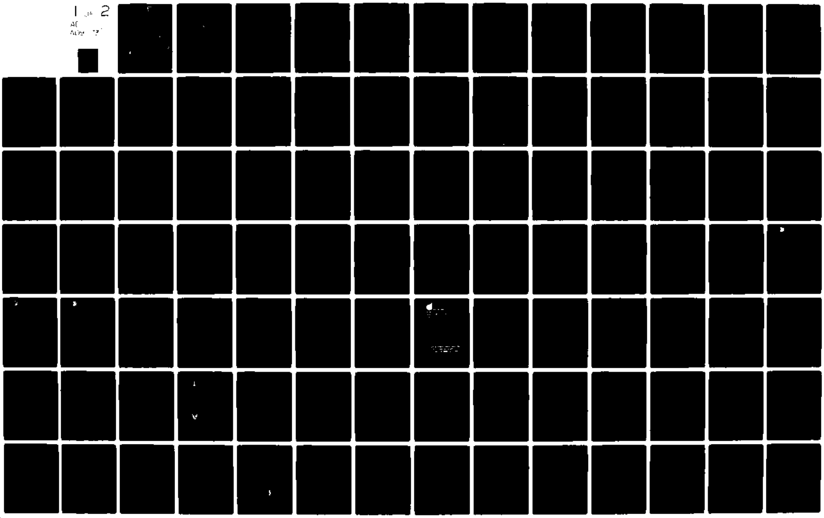

LEVEL II

(2)

APPLICATION OF DISCRETE GUIDANCE AND CONTROL THEORY TO FUTURE ARMY MODULAR MISSILES

By

S. M. Seltzer, Principal Investigator

FINAL TECHNICAL REPORT

This research was performed for
the US Army Missile Command

Redstone Arsenal, Alabama 35898

Under

Contract No. DAAK40-79-C-0213

Control Dynamics Company
221 East Side Square, Suite 1B
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30 September 1980

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This technical report covers the Control Dynamics Company's efforts performed in completing the requirements set forth in Technical Requirement No. T-0208, entitled, "Application of Digital Technology to Guidance and Control Theory." Described is a simple analytical set of tools developed to help the practising control system design engineer. These tools are the Parameter Space technique and the Cross-Multiplication technique. Also described is a comprehensive Digital Design seminar conducted for selected members of the Guidance and Control Directorate. Application of advanced digital theory to the PERSHING II digital

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Block 20 continued:

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ABSTRACT

This technical report covers the Control Dynamics Company's efforts performed in completing the requirements set forth in Technical Requirement No. T-0208, entitled, "Application of Digital Technology to Guidance and Control Theory." Described is a simple analytical set of tools developed to help the practising control system design engineer. These tools are the Parameter Space technique and the Cross-Multiplication technique. Also described is a comprehensive Digital Design seminar conducted for selected members of the Guidance and Control Directorate. Application of advanced digital theory to the PERSHING II digital autopilot is described, as well as the beginning efforts to evaluate the consequences of microprocessor implementation of digital filters.

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SECTION I. INTRODUCTION

A. Study Objective

The purpose of this study is to perform research necessary to develop new discrete design techniques, discrete adaptations of control theory, and in particular extended Parameter Space analysis techniques that will help to insure development of effective guidance and control (G&C) systems for future weapon systems. This research will permit the development of candidate advanced G&C systems for the family of Future Army Modular Missiles (FAMM). The prime output of this technical effort is to be the enhancement of the ability to create and evaluate new discrete G&C systems capable of meeting the elusive threat of the future.

The study effort is composed of seven tasks described in the Scope of Work (as altered by the three modifications to the original contract), contained in Technical Requirement No. T-0208 entitled "Application of Digital Technology to Guidance and Control Theory."[1-4] The requirements imposed by these seven tasks are contained in the referenced contract documents and re-stated (for convenience of the reader) below.

Task I requirements:

"2.1 Investigate to determine if a sensitivity problem due to clustering of the system poles and zeros exists for the parameter plane technique of digital system analysis/design."

"2.2 If a sensitivity problem (i.e., clustering of the system poles and zeros) is found in the investigation of Paragraph 2.1, a technique shall be developed to mathematically spread apart the system poles and zeros via intermediate complex transform techniques to eliminate the sensitivity."

Task II requirements:

"2.3 Utilizing Government supplied data (Section IV, page 19, of contract, reference 1) on such subjects as, but not limited to, projected future targets, missile sensor-tracker technology, airframe performance capabilities/limitations, onboard microprocessor technology, and predicted computer analysis techniques, the contractor shall propose at least two candidate Advanced Theoretical Optimal Guidance and Control Systems."

Task III requirements:

"2.7 The contractor shall investigate the capabilities and limitations of utilizing a relatively new method of analysis and design of digital systems called the Parameter Space technique to analyze guidance and control systems and shall extend and refine the Parameter Space technique through efforts to reduce the number and severity of the limitations so noted. Also, other digital design techniques shall be expanded and restructured to make them usable to the applications engineer. This shall include, but not be limited to, investigating the possibility of not having to obtain the system characteristic equation before applying the Parameter Space technique."

"2.8 The digital system design and analysis techniques considered in sub-section 2.7 shall be fully documented with at least one example for aid in applying the techniques."

Task IV requirements:

Deleted (per Mod. P00002).[3]

Task V requirements:

"3.1 The contractor shall conduct a technical seminar for selected members of the Guidance and Control Directorate. The purpose of the seminar is to develop a design expertise among members of the Directorate to enable them to design and evaluate digital (discrete) guidance and control systems. Both existing and newly-developed sampled-data techniques will be described and discussed in detail with practical applications conducted during the course of the seminar conferences."

"3.2 The contractor shall document a comprehensive outline of the material presented in the seminar defined in sub-section 3.3 as specified on the control data documents list (DD Form 1423)."

Task VI requirements:

"3.3 The contractor shall apply unique digital design and analysis expertise and experience to evaluate planned and proposed digital guidance and control systems for the PERSHING II. When it is deemed appropriate (subject to Government approval), the contractor will recommend modifications to these systems with the aim of obtaining improved system performance and/or decreased system complexity/cost. To analyze promising systems the contractor will (subject to Government approval) either conduct simulations or recommend their conduct by the Guidance and Control Analysis Group (DRSMI-RGN)."

"3.4 The contractor shall document the analyses and evaluations indicated in sub-section 3.3 as specified on the control data requirement list (DD Form 1423)."

Task VII (Unnumbered in contract) requirements:

"a. Investigate present digital filter requirements and recast them for the desired performance rather than emulating analogue forms."

"b. Identify technical areas to be expanded or initiated in order to develop future digital filters more effectively and efficiently."

B. Study Schedule

The requirements of Task I were completed on 30 September 1979. They were documented in a technical report entitled, "A Mathematical Means of Spreading Clustered System Roots."[5] The requirements of Tasks II-VII are to be completed by 30 September 1980.

C. Contents of Report

This technical report covers the efforts performed in completing the requirements of Tasks II-VII of the study. As stated in the Documentation Addendum to Form 1423 as amended by contract modifications, the contractor is required to document the following tasks: Task I, Task III, Task V, and Task VI.[1-4] As indicated in Part B (above) of Section I, the documentation requirement for Task I has been met by the submission of a technical report.[5] Although not required, a number of efforts that were performed under Task II are documented herein. Also, although not required, a summary of the work performed under Task VII is included.

SECTION II. ADVANCED G&C SYSTEMS (Task II)

A. Discussion

The discharge of this task has been centered largely around the efforts to develop and implement a computer simulation that is sufficiently flexible to assess and compare performance of various candidate G&C systems. Additionally, effort has been expended in determining the G&C system aspects associated with the Anti-Tactical Ballistic Missile (ATBM) field. Finally, certain efforts were expended to better define advanced G&C system requirements and potential characteristics.

B. Computer Simulation Development

The development of a suitable computer simulation of a FAMM system has been the subject of numerous meetings with personnel of the G&C Analysis Group and from the Aeroballistics and Propulsion Directorates. An unclassified design point for the missile at the terminus of its trajectory has been selected as Mach 8, 70,000 feet altitude, and lateral acceleration of 12 g's. This is to meet a threat that is aerial in nature and combines high speed and high agility (i.e., high g-maneuver capability) with low radar cross section. It is this threat that leads to the requirement for developing an optimally guided and controlled, highly maneuvering, defensive missile using terminal guidance sensors chosen from across a wide range of the frequency spectrum. The attendant required high level of control authority demands that the airframe and propulsion be developed beyond the present state of the art: hence the involvement of Aeroballistics and Propulsion Directorate personnel in the above referenced discussions. However, in order to have a simulation that is checked out and running, a simplified model (termed FAMM I) has been arrived at. It will initially model the airframe as a standard one that already exists and is coded. The actuators and sensors will be added as program modules and will initially have dynamic transfer functions of unity. An existing Proportional Navigation Guidance (PNG) module will be included with the option of later replacing it with an existing optimal G&C module. The foregoing systems are well documented for the G&C Analysis Group by M&S Computing Company, so they are not repeated in this report.

C. Anti-Tactical Ballistic Missile (ATBM)

In August 1980, it was decided to begin investigating the applicability of the advanced G&C systems being studied to the demanding problems posed by the new Anti-Tactical Ballistic Missile (ATBM) systems. The threat posed by enemy TBM's is significantly more severe, particularly from a dynamical point of view, than that used in developing the design point of Section IIB (above).

As a first step, on 8 July 1980, Dr. Pastrick and Dr. Seltzer were exposed to a review of ATBM G&C that is under investigation by the US Army Ballistic Missile Defense Systems Command (BMSCOM). This was followed by a visit to The Charles Stark Draper Laboratory, Cambridge, Massachussetts, on 24 July by Dr. Pastrick, Mr. Roy Pugh (G&C Directorate), and Dr. Seltzer. The work being done on ballistic missile defense systems for BMSCOM was presented in detail (Appendix A).

Shortly after effort was initiated to study this aspect of the problem, Control Dynamics Company support was provided in a review of the US Navy's High Angle Threat (HAT) program. A review of that program was conducted and the unclassified portion provided Dr. Pastrick (Appendix B). As a result of that effort, the Control Dynamics Company was requested to continue participation in the continuing evaluation of the HAT program with the selection of the most promising concepts to carry into any further studies. The critical issues associated with the HAT defense have been identified; they are shown in Appendix C.

A basis has been developed from which a future study of the ATBM implications on advanced G&C may be studied. Further, the characteristics of such a G&C system may now be developed, should such an effort be desired.

D. Documentation

There are some items of documentation that are not referred to in other portions of this report. They are described in this section.

1. Advanced Analysis for Future Missiles (MICOM TR)

Dr. Seltzer organized, reviewed, and proof-read US Army Missile

Command Technical Report RG-80-8, "Advanced Analysis for Future Missiles," dated 21 November 1979.[6] It describes the work that was completed on the (then) new program to develop an advanced G&C system for Future Army Modular Missiles. It includes the works of members of the G&C Analysis Group, the Aeroballistics Directorate, and several technical specialists on contract from universities and private industry.

2. Future US Army Missile G&C Systems (AIAA Paper)

Dr. Seltzer prepared and presented an invited (by the American Institute of Aeronautics and Astronautics) paper entitled, "Future US Army Missile G&C Systems." It was presented at the 1980 AIAA International Conference, Baltimore, MD, on 5-8 May 1980 (Appendix D).[7]

3. Special Edition of AIAA Journal of Guidance and Control

Dr. Pastrick and Dr. Seltzer were invited by the Editor-in-Chief of the AIAA Journal of Guidance and Control to organize a special edition of that journal. The edition will be devoted primarily to tactical missile G&C. They are also preparing, with Professor M. Warren of University of Florida, a survey paper for that edition.

E. Trips and Meetings

There are some trips and meetings that are not referred to in other portions of this report. They are described in this section.

1. Guidance Test Symposium

On 10-12 October 1980, Dr. Seltzer attended and participated in the Guidance Test Symposium at Holloman Air Force Base, New Mexico.

2. Office of the Under Secretary of Defense for Research and Engineering

On 19-20 November 1979, Dr. Seltzer traveled to Washington, D.C. to meet with Dr. Pastrick (then on one year's duty with the Office of the Under Secretary of Defense for Research and Engineering) on missile G&C matters. While there he also participated in a meeting with Mr. Charles Bernard, the Director of Land Warfare, DDRE, to discuss future tactical missile G&C.

3. Joint Technology Common Intercept Missile

On 15 January 1980, Dr. Seltzer attended a briefing and discussion of a potential Joint Technology Common Intercept Missile held at the Advanced Systems Concepts Office (ASCO). The main briefings were presented by Mssrs. Ben Glatt (NAVAIR), Tom Hamilton (NAVWEPS), and Charles Weaver (NAVSEA). The ASCO personnel responded with briefings on their new Army concept for an Evolutionary Surface-to-Air Missile system. The briefings were followed by a general discussion of system requirements and delineation of possible areas of cooperation or joint development efforts.

4. <u>AFTL/DLO, Eglin Air Force Base</u>

On 29 April 1980, Dr. Seltzer visited the 8th AFATL/DLO, Eglin Air Force Base, Florida with Dr. Pastrick. Discussions took place and design and program information concerning progress and plans for the US MICOM and the USAF advanced G&C systems were exchanged. Key USAF participants were Captain Riggs and Lieutenants McLemoon and Vergez.

5. <u>1980 Tactical Missiles Conference</u>

On 30 April - 1 May, Dr. Pastrick and Dr. Seltzer attended the 1980 Tactical Missiles Conference at Eglin Air Force Base.

SECTION III. PARAMETER SPACE TECHNIQUE (Task III)

A. Accomplishments

The Parameter Space method is a technique for determining stability and dynamic characteristics of a control system in terms of several selected system parameters. The development and use of the technique is described in US Army Missile Command Technical Report T-79-64, entitled, "Sampled-Data Analysis in Parameter Space," dated June 1979 (Appendix E).[8] In essence, the method provides an analytical tool for use in control system design and analysis. Although not necessary, its application is facilitated by augmenting the analytical results with graphical portrayals in a selected multi-parameter space. In its present form, the method requires that the controlled system be described by a characteristic equation which, for sampled-data or digital systems, may be expressed in the z-domain. The method enables the designer to evaluate graphically (or analytically) the locations of roots of the characteristic polynomial. Hence, he may design the control system in terms of his selected performance criteria (also known as "pole placement"). He is able to see the effect on the roots of changing numerical values of several system parameters.

Also developed under an earlier contract with MICOM is the Cross-Multiplication method for determining a digital control system's response to a deterministic input. This method is described in US Army Missile Research Command Technical Report T-79-58, entitled, "Determination of Digital Control System Response by Cross-Multiplication," dated 29 May 1979.[9] This technique assumes that the closed-loop transfer function is available in the z- or modified z-transform domain. The Real Translation Theorem is then applied to the result, yielding a difference equation in the time-domain. This may be solved for the system response in terms of the reference (or other) input(s) to the system as well as in terms of system state initial conditions. The technique also permits the determination of intra-sampling responses of the system by using either the Submultiple Method or the Modified Z-Transform Method. An additional strength of the method is that the input need be specified only at the sampling instants. Hence it is not necessary that the input be described in a continuous (with respect to time) manner, nor is it necessary to obtain the z-transform of the input(s).

The Parameter Space method and Cross-Multiplication technique have each been programmed for usage in the Hewlett-Packard Series 9835A digital computer. The program language is an advanced version of BASIC. A listing of the programs is included in Appendices F and G. To use the Parameter Space program, one must have the characteristic equation in the z-domain. The program then provides contours that portray the stability boundary in terms of a selected two-parameter space. It then determines and plots contours corresponding to the complex and real roots of the system characteristic equation, thereby permitting design by pole placement. The program contains the option of being able to determine the system roots at any location in the selected parameter space (see subroutine entitled "SILJAK" for an efficient root determination routine).

The Cross-Multiplication program requires that the desired closed-loop transfer function be available in the z-domain. Of course, this can be determined from the characteristic equation used for the Parameter Space program. From this transfer function, the output can be provided, at the sampling instants, in terms of any desired input known (deterministically) at the sampling instants. Several typical inputs (unit step, ramp, sinusoidal) are provided by the program itself.

The Parameter Space Program has been structured so that, at a later date, stability regions and maps of system roots can be determined and plotted for continuous-data systems in the Laplace(s) domain. This option will be useful to provide a reference or comparison. Also, often contemporary control system designers (sadly) perform their design analysis in the s-domain and then "merely sample the system fast enough." This modification will permit an analysis of such systems.

Because many designers prefer to design in the w-domain (bilinear transformation from the z-domain), the Parameter Space Program also has been structured to permit its inclusion at a future date. This is contingent on future investigations of the applicability of the Parameter Space method to w-domain analysis.

B. Example

1. System Description

To better demonstrate the capabilities, limitations, and general use of these two techniques and their implementation into two digital programs, an example is provided. For pedagogical purposes, it represents a simplified aerospace vehicle under control of a digital computer. See Figure 1 (Section X) for a description of the system in block diagram form. Capital letters are used to denote matrices, and capital letters with underbars denote vectors.

2. Application of SAM

Suppose the design goal is to specify a digital control system that will result in a stable system whose performance is desirable (in some sense). The approach to be used will be to obtain a mathematical description of the system dynamics that can subsequently be simplified. The intermediate objective is to obtain a closed-loop transfer function in the z-domain. The ultimate objective is to use this closed-loop transfer function to assess system performance, including stability.

Referring to Figure 1, one is able to write the "System Equations" (first column in Table I, below) describing the system dynamics. Applying the "SAM" (Simplified Analytical Method) technique, one may then obtain the "Modified System Equations" (Column 2) and the "Pulsed Equations (Column 3)."

Table I. SAM

System Equations	Modified Sys. Eqns.	Pulsed Equations	
$\underline{X} = G_pFG_{h0}\underline{U}^*$	$\underline{X} = G_pFG_{h0}\underline{U}^*$	$\underline{X}^* = (G_pFG_{h0})^*\underline{U}^*$	(1)
$\underline{U} = D\underline{E}^*$	$\underline{U} = D\underline{E}^*$	$\underline{U}^* = D^*\underline{E}^*$	(2)
$\underline{E} = \underline{R} - \underline{Y}$	$\underline{E} = \underline{R} - \underline{Y}$	$\underline{E}^* = \underline{R}^* - \underline{Y}^*$	(3)
$\underline{Y} = C\underline{X}$	$\underline{Y} = CG_pFG_{h0}\underline{U}^*$	$\underline{Y}^* = (CG_pFG_{h0})^*\underline{U}^*$	(4)

Now any desired closed-loop transfer function may readily be obtained from the Table I equations. For the purposes of this example, suppose one wishes to determine the response of the system output vector, $\underline{X}$, in terms of the system input vector, $\underline{R}$. The value of these two vectors at the sampling instants are denoted by the star operator (*), i.e., $\underline{X}^*$ and $\underline{R}^*$. From the third ("Pulsed Equations") column, one readily obtains that relation with minimal mathematical manipulation:

$$\begin{aligned} \underline{E}^* &= \underline{R}^* - \underline{Y}^* \\ &= \underline{R}^* - (CG_pFG_{h0})^*\underline{U}^* \\ &= \underline{R}^* - (CG_pFG_{h0})^*D^*\underline{E}^* \\ &= [I + (CG_pFG_{h0})^*D^*]^{-1}\,\underline{R}^* \ . \end{aligned} \tag{5}$$

This expression for $\underline{E}^*$, in terms of $\underline{R}^*$, may now be substituted into Equation (2) to obtain the control vector $\underline{U}^*$ in terms of $\underline{R}^*$:

$$\begin{aligned} \underline{U}^* &= D^*\underline{E}^* \\ &= D^*\,[I + (CG_pFG_{h0})^*D^*]^{-1}\,\underline{R}^* \ . \end{aligned} \tag{6}$$

Finally, this expression for $\underline{U}^*$ may be substituted into Equation (1) to obtain the sought-after expression for $\underline{X}^*$ in terms of $\underline{R}^*$:

$$\begin{aligned} \underline{X}^* &= (G_pFG_{h0})^*\underline{U}^* \\ &= (G_pFG_{h0})^*D^*\,[I + (CG_pFG_{h0})^*D^*]^{-1}\,\underline{R}^* \end{aligned} \tag{7}$$

In general, pulsed quantities may be expressed directly in the z-domain, i.e.,

$$\mathfrak{z}\,\{\underline{X}^*\} = \underline{X}(z) \tag{8}$$

Taking the z-transform of each side of Equation (7) leads to

$$\underline{X}(z) = G_pFG_{h0}(z)\ D(z)\ [I + CG_pFG_{h0}(z)\ D(z)]^{-1}\ \underline{R}(z), \tag{9}$$

where the following notation has been used:

$$\mathfrak{Z}\{(G_pFG_{h0})^*\} \overset{d}{=} G_pFG_{h0}(z) \tag{9a}$$

and

$$\mathfrak{Z}\{(CG_pFG_{h0})^*\} \overset{d}{=} CG_pFG_{h0}(z)\ . \tag{9b}$$

As previously stated, the expression of Equation (9) will be simplified for pedagogical purposes. It is therefore assumed that only planar rotational dynamics of a rigid body will be investigated. If the system output $\underline{X}$ is assumed to be the time rate of change of attitude, i.e., $\dot{\theta}$, then the actuator output V (assumed scalar in form) is

$$V = I\ddot{\theta}, \tag{10}$$

where I represents the principal moment of inertia along the axis of rotation. In the Laplace domain, this leads to the definition of the plant transfer function, $G_p(s)$

$$G_p(s) \overset{d}{=} \frac{\dot{\theta}(s)}{V(s)} = \frac{1}{Is} \tag{11}$$

If the zero order hold is assumed, then

$$G_{h0}(s) = \frac{1 - e^{-sT}}{s}\ , \tag{12}$$

and

$$G_pG_{h0}(z) = \mathcal{Z}\{G_p(s)\ G_{h0}(s)\}$$

$$= \mathcal{Z}\left\{\frac{1 - e^{-sT}}{Is^2}\right\}$$

$$= \frac{T}{I\ (z-1)} \tag{13}$$

Suppose one assumes, for the form of the digital controller, that olde standby, PID. In analogue form, the controller would be

$$D(s) = \frac{K_p}{s} + \frac{K_I}{s^2} + K_D\ . \tag{14}$$

If one applies the trapezoidal approximation for integration, the s^{-1} is replaced by

$$\frac{T}{2}\ \frac{(z+1)}{(z-1)}\ .$$

This substitution into Equation (14) leads to the z-form of the controller, i.e.,

$$D(z) = \frac{K_p}{2}\frac{(z+1)}{(z-1)} + \frac{K_I T^2}{4}\ \frac{(z+1)^2}{(z-1)^2} + K_D\ . \tag{15}$$

For simplification, let

$$a_p \overset{d}{=}\cdot \frac{K_p T^2}{2I}\ , \tag{16P}$$

$$a_I \overset{d}{=}\cdot \frac{K_I T^3}{4I}\ , \tag{16I}$$

and

$$a_D \overset{d}{=}\cdot \frac{K_D T}{I}\ . \tag{16D}$$

Equations (15) - (16) lead to the expression,

$$\frac{TD(z)}{I} = \frac{(a_P + a_I + a_D)\; z^2 + 2\;(a_I - a_D)\; z + (-a_P + a_I + a_D)}{(z-1)^2} \; . \tag{17}$$

From Equation (13), one may obtain the expression

$$\frac{I G_p G_{h0}(z)}{T} = \frac{1}{z-1} \; . \tag{18}$$

Multiplying Equations (17) and (18) together, one obtains an expression for $G_p G_{h0}(z) D(z)$ for use in Equation (9), i.e.,

$$G_p G_{h0}(z) D(z) = \frac{(a_P + a_I + a_D)\; z^2 + 2\;(a_I - a_D)\; z + (a-_P + a_I + a_D)}{(z-1)^3} \tag{19}$$

The scalar version of vector-matrix Equation (9) leads to the elementary closed loop transfer function,

$$\frac{X(z)}{R(z)} = \frac{G_p F G_{h0}(z) D(z)}{1 + C G_p F G_{h0}(z) D(z)} \; . \tag{20}$$

Letting $X(z) \equiv \dot{\theta}(z)$ and $R(z) \equiv \dot{\theta}_c(z)$ and assuming that $F \equiv 1$ and $C \equiv 1$, one finally obtains, with the use of Equation (19), the desired form of the closed-loop transfer function:

$$\frac{\dot{\theta}(z)}{\dot{\theta}_c(z)} = \frac{\underline{A}^T\; \underline{Z}_N}{\underline{B}^T\; \underline{Z}_D + \underline{A}^T\; \underline{Z}_N} \; , \tag{21}$$

where

$$\underline{A} \stackrel{d.}{=} [(-a_P + a_I = a_D),\ 2\,(a_I - a_D),\ (a_P + a_I + a_D)]^T \ , \tag{22}$$

$$\underline{B} \stackrel{d.}{=} [-1,\ 3,\ -3,\ 1]^T \ , \tag{23}$$

$$\underline{Z}_N \stackrel{d.}{=} [1,\ Z,\ Z^2]^T \ , \tag{24}$$

and

$$\underline{Z}_D \stackrel{d.}{=} [1,\ Z,\ Z^2,\ Z^3]^T \ . \tag{25}$$

Now consider Equation (21). It is in a form to apply both the Parameter Space and the Cross-Multiplication techniques, since its denominator is, when set equal to zero, the system characteristic equation. The only parameter that needs to be specified numerically are a_P, a_I, and a_D. These will determine the response of the system to any specified input. Notice that no knowledge of complex variables has been needed by the system designer: only a blind reliance on some simple rules (SAM). At this point one may (blindly) determine the system response (by Cross-Multiplication) without applying the Parameter Plane method. However, application of the Parameter Space technique (or some other stability and system performance method or methods) will determine system stability and desired performance requirements. If done in terms of the three parameters a_P, a_I, and a_D, one will determine the numerical values for digital controller gains K_P, K_I, K_D (normalized by the moment of inertia I) and the sampling period T.

3. Application of Parameter Space Technique

First the control system designer must specify two parameters of interest. Since he has three (a_P, a_I, a_D) at his disposal, assume he selects a_P and a_D (since he may ultimately choose to not use any integral gain). He must, however, choose a numerical value (which may be iteratively changed in the design sequel) for a_I. Let us assume a value of $a_I = 0.2$ for this example. Then the characteristic equation -- the denominator of Equation (21) -- may be re-cast in the following form:

$$C.E.(z) = \tilde{A}\underline{Z}_D\ P1 + \tilde{B}\underline{Z}_D\ P2 + \tilde{F}\underline{Z}_D = 0 \tag{26}$$

where

$$P1 \overset{d.}{=} a_D \tag{27}$$

and

$$P2 \overset{d.}{=} a_p\ . \tag{28}$$

Grouping the coefficients of the characteristic equation in the form of Equation (26), i.e.,

$$\begin{aligned} \underline{B}^T\underline{Z}_D + \underline{A}^T\underline{Z}^N &= [-1, 3, -3, 1]\ [1, Z, Z^2, Z^3]^T \\ &+ [(-a_p + a_I + a_D), 2\ (a_I - a_D), (a_p + a_I + a_D)]\ [1, Z, Z^2]^T \\ &= (1 - 2Z + Z^2)\ a_D + (-1 + Z^2)\ a_p \\ &+ (-1 + a_I) + (3 + 2a_I)\ Z + (-3 + a_I)\ Z^2 + Z^3\ . \end{aligned} \tag{29}$$

By association, matching Equation (29) with Equation (26) leads to

$$\tilde{A} = [A\ (0), A\ (1), A\ (2), A\ (3)] = [1, -2, 1, 0]\ , \tag{30}$$

$$\tilde{B} = [B\ (0), B\ (1), B\ (2), B\ (3)] = [-1, 0, 1, 0]\ , \tag{31}$$

and

$$\begin{aligned} \tilde{F} &= [F\ (0), F\ (1), F\ (2), F\ (3)] = [(-1 + a_I), (3 + 2a_I), (-3 + a_I), 1] \\ &= [-0.8, 3.4, -2.8, 1]\ . \end{aligned} \tag{32}$$

Recognizing that the order of the characteristic equation is three, one is ready to use the parameter space program, MICPP (Appendix F). For exposition, the computer printout will be included in the following development.

```
Z TRANSFORM METHOD
THE SYSTEM ORDER IS = 3

ELEMENTS OF A MATRIX  A(0),A(1),A(2),...
 1.0000E+00 -2.0000E+00  1.0000E+00  0.0000E+00

ELEMENTS OF B MATRIX  B(0),B(1),B(2),...
-1.0000E+00  0.0000E+00  1.0000E+00  0.0000E+00

ELEMENTS OF F MATRIX  F(0),F(1),F(2),...
-8.0000E-01  3.4000E+00 -2.8000E+00  1.0000E+00
```

First the stability boundaries will be determined (these are usually associated with the unit circle, or $\zeta = 0$ contour, in the z-domain and with the singularities a ($z = +1$ and $z = -1$). The parameter plane boundary associated with $z = -1$ is found to be $P1 \equiv a_D = 2$, and the boundary associated with $z = +1$ is $P2 \equiv a_p = 0$. For plotting convenience, the x and y intercepts of these straight lines are also indicated.

```
ZETA = 0.0000E+00
THE STABILITY BOUNDARIES FOR Z=+1 AND -1 ARE AS FOLLOWS

FOR Z=-1
P1 = 2.0000E+00

THE AXES INTERCEPTS ARE AS FOLLOWS:
AND AT P2=0 , P1= 2.0000E+00
P1 = CONSTANT = 2.0000E+00
```

```
FOR Z=+1
COEFFICIENTS OF P1 & P2 EQUAL TO ZERO

THE AXES INTERCEPTS ARE AS FOLLOWS:
P1 = CONSTANT =-9.9999E+99
P2 = CONSTANT =-9.9999E+99
```

For the boundary associated with $\zeta = 0$, it is specified by the designer to let the angle, $\omega_n T$, vary between zero and pi in 20 equal increments. Printed out are these values of $\omega_n T$ and the associated values of P1 and P2. To investigate the possibility of the existence of the fourth stability boundary associated with the case where the Jacobian becomes identically equal to zero, the Jacobian is printed out in the second column. If it were to change sign, then there obviously would be a stability boundary at the value of $\omega_n T$ where it (the Jacobian) was equal to zero. Also, the sign of the printed out Jacobian indicates on which side of the $\zeta = 0$ contour the stability region lies.

```
INITIAL OMEGA = 0.0000E+00          FINAL OMEGA = 3.1416E+00
NUMBER OF STEPS= 100
WnT = 1.0000E-25
THE DETERMINANT OF THE COEFFICIENT MATRIX IS EQUAL TO ZERO
MATRIX AB =
 0.0000E+00          0.0000E+00

 0.0000E+00          2.0000E-25



MATRIX Fp =
-8.0000E-01         -8.0000E-26
```

WnT= 3.1416E-02	DET=-6.1997E-05	P1= 8.1044E+02	P2= 4.9343E-04
WnT= 6.2832E-02	DET=-4.9561E-04	P1= 2.0251E+02	P2= 1.9733E-03
WnT= 9.4248E-02	DET=-1.6706E-03	P1= 8.9934E+01	P2= 4.4380E-03
WnT= 1.2566E-01	DET=-3.9532E-03	P1= 5.0535E+01	P2= 7.8853E-03
WnT= 1.5708E-01	DET=-7.7039E-03	P1= 3.2302E+01	P2= 1.2312E-02
WnT= 1.8850E-01	DET=-1.3276E-02	P1= 2.2400E+01	P2= 1.7713E-02
WnT= 2.1991E-01	DET=-2.1014E-02	P1= 1.6433E+01	P2= 2.4083E-02
WnT= 2.5133E-01	DET=-3.1252E-02	P1= 1.2563E+01	P2= 3.1417E-02
WnT= 2.8274E-01	DET=-4.4311E-02	P1= 9.9137E+00	P2= 3.9706E-02
WnT= 3.1416E-01	DET=-6.0497E-02	P1= 8.0216E+00	P2= 4.8943E-02
WnT= 3.4558E-01	DET=-8.0104E-02	P1= 6.6251E+00	P2= 5.9119E-02
WnT= 3.7699E-01	DET=-1.0340E-01	P1= 5.5663E+00	P2= 7.0224E-02
WnT= 4.0841E-01	DET=-1.3065E-01	P1= 4.7457E+00	P2= 8.2245E-02
WnT= 4.3982E-01	DET=-1.6209E-01	P1= 4.0980E+00	P2= 9.5173E-02
WnT= 4.7124E-01	DET=-1.9793E-01	P1= 3.5789E+00	P2= 1.0899E-01
WnT= 5.0265E-01	DET=-2.3836E-01	P1= 3.1575E+00	P2= 1.2369E-01
WnT= 5.3407E-01	DET=-2.8355E-01	P1= 2.8116E+00	P2= 1.3926E-01
WnT= 5.6549E-01	DET=-3.3365E-01	P1= 2.5252E+00	P2= 1.5567E-01
WnT= 5.9690E-01	DET=-3.8878E-01	P1= 2.2861E+00	P2= 1.7292E-01
WnT= 6.2832E-01	DET=-4.4903E-01	P1= 2.0854E+00	P2= 1.9098E-01
WnT= 6.5973E-01	DET=-5.1446E-01	P1= 1.9160E+00	P2= 2.0984E-01
WnT= 6.9115E-01	DET=-5.8512E-01	P1= 1.7725E+00	P2= 2.2949E-01
WnT= 7.2257E-01	DET=-6.6102E-01	P1= 1.6506E+00	P2= 2.4989E-01
WnT= 7.5398E-01	DET=-7.4213E-01	P1= 1.5469E+00	P2= 2.7103E-01
WnT= 7.8540E-01	DET=-8.2843E-01	P1= 1.4586E+00	P2= 2.9289E-01
WnT= 8.1681E-01	DET=-9.1982E-01	P1= 1.3835E+00	P2= 3.1545E-01
WnT= 8.4823E-01	DET=-1.0162E+00	P1= 1.3197E+00	P2= 3.3869E-01
WnT= 8.7965E-01	DET=-1.1175E+00	P1= 1.2658E+00	P2= 3.6258E-01
WnT= 9.1106E-01	DET=-1.2235E+00	P1= 1.2204E+00	P2= 3.8709E-01
WnT= 9.4248E-01	DET=-1.3340E+00	P1= 1.1826E+00	P2= 4.1221E-01
WnT= 9.7389E-01	DET=-1.4488E+00	P1= 1.1513E+00	P2= 4.3792E-01
WnT= 1.0053E+00	DET=-1.5677E+00	P1= 1.1259E+00	P2= 4.6417E-01
WnT= 1.0367E+00	DET=-1.6904E+00	P1= 1.1057E+00	P2= 4.9096E-01
WnT= 1.0681E+00	DET=-1.8166E+00	P1= 1.0901E+00	P2= 5.1825E-01
WnT= 1.0996E+00	DET=-1.9460E+00	P1= 1.0786E+00	P2= 5.4601E-01
WnT= 1.1310E+00	DET=-2.0783E+00	P1= 1.0708E+00	P2= 5.7422E-01
WnT= 1.1624E+00	DET=-2.2131E+00	P1= 1.0664E+00	P2= 6.0285E-01
WnT= 1.1938E+00	DET=-2.3500E+00	P1= 1.0649E+00	P2= 6.3188E-01
WnT= 1.2252E+00	DET=-2.4887E+00	P1= 1.0662E+00	P2= 6.6126E-01
WnT= 1.2566E+00	DET=-2.6287E+00	P1= 1.0699E+00	P2= 6.9098E-01
WnT= 1.2881E+00	DET=-2.7695E+00	P1= 1.0758E+00	P2= 7.2101E-01
WnT= 1.3195E+00	DET=-2.9108E+00	P1= 1.0837E+00	P2= 7.5131E-01
WnT= 1.3509E+00	DET=-3.0521E+00	P1= 1.0935E+00	P2= 7.8186E-01
WnT= 1.3823E+00	DET=-3.1929E+00	P1= 1.1049E+00	P2= 8.1262E-01
WnT= 1.4137E+00	DET=-3.3327E+00	P1= 1.1177E+00	P2= 8.4357E-01
WnT= 1.4451E+00	DET=-3.4711E+00	P1= 1.1320E+00	P2= 8.7467E-01
WnT= 1.4765E+00	DET=-3.6075E+00	P1= 1.1474E+00	P2= 9.0589E-01
WnT= 1.5080E+00	DET=-3.7414E+00	P1= 1.1640E+00	P2= 9.3721E-01
WnT= 1.5394E+00	DET=-3.8724E+00	P1= 1.1816E+00	P2= 9.6859E-01
WnT= 1.5708E+00	DET=-4.0000E+00	P1= 1.2000E+00	P2= 1.0000E+00
WnT= 1.6022E+00	DET=-4.1236E+00	P1= 1.2192E+00	P2= 1.0314E+00
WnT= 1.6336E+00	DET=-4.2428E+00	P1= 1.2392E+00	P2= 1.0628E+00
WnT= 1.6650E+00	DET=-4.3570E+00	P1= 1.2597E+00	P2= 1.0941E+00
WnT= 1.6965E+00	DET=-4.4659E+00	P1= 1.2808E+00	P2= 1.1253E+00
WnT= 1.7279E+00	DET=-4.5688E+00	P1= 1.3023E+00	P2= 1.1564E+00
WnT= 1.7593E+00	DET=-4.6654E+00	P1= 1.3243E+00	P2= 1.1874E+00
WnT= 1.7907E+00	DET=-4.7552E+00	P1= 1.3465E+00	P2= 1.2181E+00
WnT= 1.8221E+00	DET=-4.8378E+00	P1= 1.3690E+00	P2= 1.2487E+00
WnT= 1.8535E+00	DET=-4.9128E+00	P1= 1.3917E+00	P2= 1.2790E+00
WnT= 1.8850E+00	DET=-4.9798E+00	P1= 1.4146E+00	P2= 1.3090E+00
WnT= 1.9164E+00	DET=-5.0384E+00	P1= 1.4375E+00	P2= 1.3387E+00
WnT= 1.9478E+00	DET=-5.0882E+00	P1= 1.4605E+00	P2= 1.3681E+00
WnT= 1.9792E+00	DET=-5.1290E+00	P1= 1.4834E+00	P2= 1.3971E+00

WnT= 2.0106E+00	DET=-5.1603E+00	P1= 1.5063E+00	P2= 1.4258E+00
WnT= 2.0420E+00	DET=-5.1821E+00	P1= 1.5291E+00	P2= 1.4540E+00
WnT= 2.0735E+00	DET=-5.1939E+00	P1= 1.5517E+00	P2= 1.4818E+00
WnT= 2.1049E+00	DET=-5.1956E+00	P1= 1.5741E+00	P2= 1.5090E+00
WnT= 2.1363E+00	DET=-5.1870E+00	P1= 1.5963E+00	P2= 1.5358E+00
WnT= 2.1677E+00	DET=-5.1679E+00	P1= 1.6182E+00	P2= 1.5621E+00
WnT= 2.1991E+00	DET=-5.1382E+00	P1= 1.6397E+00	P2= 1.5878E+00
WnT= 2.2305E+00	DET=-5.0978E+00	P1= 1.6609E+00	P2= 1.6129E+00
WnT= 2.2619E+00	DET=-5.0466E+00	P1= 1.6817E+00	P2= 1.6374E+00
WnT= 2.2934E+00	DET=-4.9847E+00	P1= 1.7021E+00	P2= 1.6613E+00
WnT= 2.3248E+00	DET=-4.9119E+00	P1= 1.7220E+00	P2= 1.6845E+00
WnT= 2.3562E+00	DET=-4.8284E+00	P1= 1.7414E+00	P2= 1.7071E+00
WnT= 2.3876E+00	DET=-4.7342E+00	P1= 1.7603E+00	P2= 1.7290E+00
WnT= 2.4190E+00	DET=-4.6295E+00	P1= 1.7787E+00	P2= 1.7501E+00
WnT= 2.4504E+00	DET=-4.5143E+00	P1= 1.7964E+00	P2= 1.7705E+00
WnT= 2.4819E+00	DET=-4.3888E+00	P1= 1.8136E+00	P2= 1.7902E+00
WnT= 2.5133E+00	DET=-4.2533E+00	P1= 1.8301E+00	P2= 1.8090E+00
WnT= 2.5447E+00	DET=-4.1079E+00	P1= 1.8460E+00	P2= 1.8271E+00
WnT= 2.5761E+00	DET=-3.9530E+00	P1= 1.8612E+00	P2= 1.8443E+00
WnT= 2.6075E+00	DET=-3.7888E+00	P1= 1.8757E+00	P2= 1.8607E+00
WnT= 2.6389E+00	DET=-3.6157E+00	P1= 1.8895E+00	P2= 1.8763E+00
WnT= 2.6704E+00	DET=-3.4340E+00	P1= 1.9025E+00	P2= 1.8910E+00
WnT= 2.7018E+00	DET=-3.2441E+00	P1= 1.9148E+00	P2= 1.9048E+00
WnT= 2.7332E+00	DET=-3.0465E+00	P1= 1.9263E+00	P2= 1.9178E+00
WnT= 2.7646E+00	DET=-2.8416E+00	P1= 1.9371E+00	P2= 1.9298E+00
WnT= 2.7960E+00	DET=-2.6298E+00	P1= 1.9470E+00	P2= 1.9409E+00
WnT= 2.8274E+00	DET=-2.4116E+00	P1= 1.9561E+00	P2= 1.9511E+00
WnT= 2.8588E+00	DET=-2.1876E+00	P1= 1.9643E+00	P2= 1.9603E+00
WnT= 2.8903E+00	DET=-1.9583E+00	P1= 1.9718E+00	P2= 1.9686E+00
WnT= 2.9217E+00	DET=-1.7241E+00	P1= 1.9784E+00	P2= 1.9759E+00
WnT= 2.9531E+00	DET=-1.4858E+00	P1= 1.9841E+00	P2= 1.9823E+00
WnT= 2.9845E+00	DET=-1.2438E+00	P1= 1.9889E+00	P2= 1.9877E+00
WnT= 3.0159E+00	DET=-9.9871E-01	P1= 1.9929E+00	P2= 1.9921E+00
WnT= 3.0473E+00	DET=-7.5120E-01	P1= 1.9960E+00	P2= 1.9956E+00
WnT= 3.0788E+00	DET=-5.0183E-01	P1= 1.9982E+00	P2= 1.9980E+00
WnT= 3.1102E+00	DET=-2.5122E-01	P1= 1.9996E+00	P2= 1.9995E+00

WnT = 3.1416E+00

THE DETERMINANT OF THE COEFFICIENT MATRIX IS EQUAL TO ZERO

MATRIX AB =

4.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00

MATRIX Fp =

8.0000E+00	0.0000E+00

The stability boundaries may now be plotted on the a_D - a_P parameter plane (Figure 2). The stable region lies in the first quadrant, bounded (on the left) by the $\zeta = 0$ curve and (on the right) by the $P1 \equiv a_D = 2$ line.

Since the characteristic equation is third order, it must have three roots. ζ-contours, printed out as functions of the argument $\omega_n T$, display the characteristics of complex conjugate pairs of roots. Since the $\zeta = 0$ contour exists, there must be some complex conjugate roots. However, since there are three roots, there also must exist a real root. Hence, at each point in the a_D - a_P parameter plane, one must expect the existence of either one complex conjugate pair of roots plus one real root or three real roots. It will be shown that, for this case, the former prevails.

Zeta contours for zeta values of 0.2, 0.5, 0.707, and 1 are shown as functions of $\omega_n T$ in the following printouts. These contours are also plotted on Figure 2. For clarity, the associated values of $\omega_n T$ are omitted from the figure.

```
ZETA = 2.0000E-01
INITIAL OMEGA = 0.0000E+00        FINAL OMEGA = 3.1416E+00
NUMBER OF STEPS=  50
WnT = 1.0000E-25
THE DETERMINANT OF THE COEFFICIENT MATRIX IS EQUAL TO ZERO
MATRIX AB =
 0.0000E+00          0.0000E+00

 0.0000E+00          1.9596E-25




MATRIX Fp =
-8.0000E-01         -7.8384E-26
```

WnT= 6.2832E-02	DET=-4.7357E-04	P1= 2.0255E+02	P2= 2.5493E+00
WnT= 1.2566E-01	DET=-3.6842E-03	P1= 5.0595E+01	P2= 1.2826E+00
WnT= 1.8850E-01	DET=-1.2069E-02	P1= 2.2483E+01	P2= 8.6341E-01
WnT= 2.5133E-01	DET=-2.7716E-02	P1= 1.2668E+01	P2= 6.6987E-01
WnT= 3.1416E-01	DET=-5.2345E-02	P1= 8.1475E+00	P2= 5.5950E-01
WnT= 3.7699E-01	DET=-8.7299E-02	P1= 5.7120E+00	P2= 4.9466E-01
WnT= 4.3982E-01	DET=-1.3354E-01	P1= 4.2623E+00	P2= 4.5697E-01
WnT= 5.0265E-01	DET=-1.9166E-01	P1= 3.3388E+00	P2= 4.3715E-01
WnT= 5.6549E-01	DET=-2.6186E-01	P1= 2.7222E+00	P2= 4.2992E-01
WnT= 6.2832E-01	DET=-3.4403E-01	P1= 2.2965E+00	P2= 4.3206E-01
WnT= 6.9115E-01	DET=-4.3767E-01	P1= 1.9961E+00	P2= 4.4143E-01
WnT= 7.5398E-01	DET=-5.4203E-01	P1= 1.7813E+00	P2= 4.5653E-01
WnT= 8.1681E-01	DET=-6.5605E-01	P1= 1.6270E+00	P2= 4.7626E-01
WnT= 8.7965E-01	DET=-7.7844E-01	P1= 1.5167E+00	P2= 4.9976E-01
WnT= 9.4248E-01	DET=-9.0768E-01	P1= 1.4392E+00	P2= 5.2636E-01
WnT= 1.0053E+00	DET=-1.0421E+00	P1= 1.3865E+00	P2= 5.5551E-01
WnT= 1.0681E+00	DET=-1.1799E+00	P1= 1.3529E+00	P2= 5.8671E-01
WnT= 1.1310E+00	DET=-1.3191E+00	P1= 1.3341E+00	P2= 6.1957E-01
WnT= 1.1938E+00	DET=-1.4579E+00	P1= 1.3270E+00	P2= 6.5372E-01
WnT= 1.2566E+00	DET=-1.5941E+00	P1= 1.3292E+00	P2= 6.8882E-01
WnT= 1.3195E+00	DET=-1.7258E+00	P1= 1.3386E+00	P2= 7.2458E-01
WnT= 1.3823E+00	DET=-1.8512E+00	P1= 1.3538E+00	P2= 7.6073E-01
WnT= 1.4451E+00	DET=-1.9684E+00	P1= 1.3735E+00	P2= 7.9701E-01
WnT= 1.5080E+00	DET=-2.0756E+00	P1= 1.3967E+00	P2= 8.3319E-01
WnT= 1.5708E+00	DET=-2.1713E+00	P1= 1.4225E+00	P2= 8.6905E-01
WnT= 1.6336E+00	DET=-2.2541E+00	P1= 1.4504E+00	P2= 9.0439E-01
WnT= 1.6965E+00	DET=-2.3227E+00	P1= 1.4796E+00	P2= 9.3903E-01
WnT= 1.7593E+00	DET=-2.3762E+00	P1= 1.5096E+00	P2= 9.7279E-01
WnT= 1.8221E+00	DET=-2.4136E+00	P1= 1.5401E+00	P2= 1.0055E+00
WnT= 1.8850E+00	DET=-2.4345E+00	P1= 1.5706E+00	P2= 1.0370E+00
WnT= 1.9478E+00	DET=-2.4383E+00	P1= 1.6009E+00	P2= 1.0673E+00
WnT= 2.0106E+00	DET=-2.4250E+00	P1= 1.6306E+00	P2= 1.0960E+00
WnT= 2.0735E+00	DET=-2.3947E+00	P1= 1.6594E+00	P2= 1.1233E+00
WnT= 2.1363E+00	DET=-2.3476E+00	P1= 1.6873E+00	P2= 1.1489E+00
WnT= 2.1991E+00	DET=-2.2842E+00	P1= 1.7140E+00	P2= 1.1727E+00
WnT= 2.2619E+00	DET=-2.2053E+00	P1= 1.7393E+00	P2= 1.1947E+00
WnT= 2.3248E+00	DET=-2.1116E+00	P1= 1.7631E+00	P2= 1.2149E+00
WnT= 2.3876E+00	DET=-2.0042E+00	P1= 1.7853E+00	P2= 1.2331E+00
WnT= 2.4504E+00	DET=-1.8844E+00	P1= 1.8058E+00	P2= 1.2493E+00
WnT= 2.5133E+00	DET=-1.7533E+00	P1= 1.8245E+00	P2= 1.2636E+00
WnT= 2.5761E+00	DET=-1.6125E+00	P1= 1.8415E+00	P2= 1.2758E+00
WnT= 2.6389E+00	DET=-1.4633E+00	P1= 1.8565E+00	P2= 1.2860E+00
WnT= 2.7018E+00	DET=-1.3074E+00	P1= 1.8696E+00	P2= 1.2942E+00
WnT= 2.7646E+00	DET=-1.1463E+00	P1= 1.8809E+00	P2= 1.3003E+00
WnT= 2.8274E+00	DET=-9.8168E-01	P1= 1.8902E+00	P2= 1.3045E+00
WnT= 2.8903E+00	DET=-8.1503E-01	P1= 1.8976E+00	P2= 1.3067E+00
WnT= 2.9531E+00	DET=-6.4795E-01	P1= 1.9031E+00	P2= 1.3069E+00
WnT= 3.0159E+00	DET=-4.8196E-01	P1= 1.9068E+00	P2= 1.3053E+00
WnT= 3.0788E+00	DET=-3.1852E-01	P1= 1.9087E+00	P2= 1.3018E+00
WnT= 3.1416E+00	DET=-1.5901E-01	P1= 1.9088E+00	P2= 1.2965E+00

ZETA = 5.0000E-01
INITIAL OMEGA = 0.0000E+00 FINAL OMEGA = 3.1416E+00
NUMBER OF STEPS= 50
WnT = 1.0000E-25

THE DETERMINANT OF THE COEFFICIENT MATRIX IS EQUAL TO ZERO

MATRIX AB =

0.0000E+00	0.0000E+00
0.0000E+00	1.7321E-25

MATRIX Fp =

-8.0000E-01	-6.9282E-26

WnT= 6.2832E-02	DET=-4.0321E-04	P1= 2.0264E+02	P2= 6.3702E+00
WnT= 1.2566E-01	DET=-3.0232E-03	P1= 5.0720E+01	P2= 3.1947E+00
WnT= 1.8850E-01	DET=-9.5505E-03	P1= 2.2637E+01	P2= 2.1445E+00
WnT= 2.5133E-01	DET=-2.1162E-02	P1= 1.2849E+01	P2= 1.6277E+00
WnT= 3.1416E-01	DET=-3.8585E-02	P1= 8.3513E+00	P2= 1.3257E+00
WnT= 3.7699E-01	DET=-6.2161E-02	P1= 5.9358E+00	P2= 1.1322E+00
WnT= 4.3982E-01	DET=-9.1904E-02	P1= 4.5030E+00	P2= 1.0012E+00
WnT= 5.0265E-01	DET=-1.2756E-01	P1= 3.5936E+00	P2= 9.0990E-01
WnT= 5.6549E-01	DET=-1.6864E-01	P1= 2.9881E+00	P2= 8.4533E-01
WnT= 6.2832E-01	DET=-2.1452E-01	P1= 2.5708E+00	P2= 7.9967E-01
WnT= 6.9115E-01	DET=-2.6440E-01	P1= 2.2760E+00	P2= 7.6789E-01
WnT= 7.5398E-01	DET=-3.1743E-01	P1= 2.0643E+00	P2= 7.4656E-01
WnT= 8.1681E-01	DET=-3.7269E-01	P1= 1.9108E+00	P2= 7.3324E-01
WnT= 8.7965E-01	DET=-4.2924E-01	P1= 1.7989E+00	P2= 7.2617E-01
WnT= 9.4248E-01	DET=-4.8615E-01	P1= 1.7176E+00	P2= 7.2401E-01
WnT= 1.0053E+00	DET=-5.4251E-01	P1= 1.6590E+00	P2= 7.2572E-01
WnT= 1.0681E+00	DET=-5.9746E-01	P1= 1.6177E+00	P2= 7.3050E-01
WnT= 1.1310E+00	DET=-6.5020E-01	P1= 1.5895E+00	P2= 7.3767E-01
WnT= 1.1938E+00	DET=-7.0000E-01	P1= 1.5713E+00	P2= 7.4672E-01
WnT= 1.2566E+00	DET=-7.4622E-01	P1= 1.5610E+00	P2= 7.5720E-01
WnT= 1.3195E+00	DET=-7.8830E-01	P1= 1.5566E+00	P2= 7.6875E-01
WnT= 1.3823E+00	DET=-8.2577E-01	P1= 1.5569E+00	P2= 7.8106E-01
WnT= 1.4451E+00	DET=-8.5827E-01	P1= 1.5607E+00	P2= 7.9386E-01
WnT= 1.5080E+00	DET=-8.8549E-01	P1= 1.5672E+00	P2= 8.0695E-01
WnT= 1.5708E+00	DET=-9.0727E-01	P1= 1.5757E+00	P2= 8.2013E-01
WnT= 1.6336E+00	DET=-9.2348E-01	P1= 1.5857E+00	P2= 8.3324E-01
WnT= 1.6965E+00	DET=-9.3410E-01	P1= 1.5966E+00	P2= 8.4616E-01
WnT= 1.7593E+00	DET=-9.3919E-01	P1= 1.6082E+00	P2= 8.5876E-01
WnT= 1.8221E+00	DET=-9.3886E-01	P1= 1.6201E+00	P2= 8.7096E-01
WnT= 1.8850E+00	DET=-9.3328E-01	P1= 1.6321E+00	P2= 8.8268E-01
WnT= 1.9478E+00	DET=-9.2269E-01	P1= 1.6440E+00	P2= 8.9384E-01
WnT= 2.0106E+00	DET=-9.0738E-01	P1= 1.6556E+00	P2= 9.0440E-01
WnT= 2.0735E+00	DET=-8.8765E-01	P1= 1.6669E+00	P2= 9.1432E-01
WnT= 2.1363E+00	DET=-8.6385E-01	P1= 1.6777E+00	P2= 9.2357E-01
WnT= 2.1991E+00	DET=-8.3637E-01	P1= 1.6880E+00	P2= 9.3211E-01
WnT= 2.2619E+00	DET=-8.0558E-01	P1= 1.6976E+00	P2= 9.3995E-01
WnT= 2.3248E+00	DET=-7.7190E-01	P1= 1.7066E+00	P2= 9.4708E-01
WnT= 2.3876E+00	DET=-7.3572E-01	P1= 1.7148E+00	P2= 9.5348E-01
WnT= 2.4504E+00	DET=-6.9746E-01	P1= 1.7224E+00	P2= 9.5918E-01
WnT= 2.5133E+00	DET=-6.5750E-01	P1= 1.7293E+00	P2= 9.6417E-01
WnT= 2.5761E+00	DET=-6.1625E-01	P1= 1.7354E+00	P2= 9.6848E-01
WnT= 2.6389E+00	DET=-5.7407E-01	P1= 1.7409E+00	P2= 9.7212E-01
WnT= 2.7018E+00	DET=-5.3133E-01	P1= 1.7456E+00	P2= 9.7511E-01
WnT= 2.7646E+00	DET=-4.8835E-01	P1= 1.7497E+00	P2= 9.7748E-01
WnT= 2.8274E+00	DET=-4.4547E-01	P1= 1.7531E+00	P2= 9.7926E-01
WnT= 2.8903E+00	DET=-4.0296E-01	P1= 1.7559E+00	P2= 9.8047E-01
WnT= 2.9531E+00	DET=-3.6110E-01	P1= 1.7581E+00	P2= 9.8116E-01
WnT= 3.0159E+00	DET=-3.2013E-01	P1= 1.7597E+00	P2= 9.8134E-01
WnT= 3.0788E+00	DET=-2.8026E-01	P1= 1.7608E+00	P2= 9.8106E-01
WnT= 3.1416E+00	DET=-2.4167E-01	P1= 1.7614E+00	P2= 9.8035E-01

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ZETA = 7.0700E-01
INITIAL OMEGA = 0.0000E+00          FINAL OMEGA = 3.1416E+00
NUMBER OF STEPS=  50
WnT = 1.0000E-25
THE DETERMINANT OF THE COEFFICIENT MATRIX IS EQUAL TO ZERO
MATRIX AB =
 0.0000E+00          0.0000E+00

 0.0000E+00          1.4144E-25

MATRIX Fp =
-8.0000E-01         -5.6577E-26


WnT= 6.2832E-02     DET=-3.2092E-04      P1= 2.0273E+02      P2= 9.0067E+00
WnT= 1.2566E-01     DET=-2.3468E-03      P1= 5.0831E+01      P2= 4.5141E+00
WnT= 1.8850E-01     DET=-7.2351E-03      P1= 2.2766E+01      P2= 3.0250E+00
WnT= 2.5133E-01     DET=-1.5656E-02      P1= 1.2991E+01      P2= 2.2888E+00
WnT= 3.1416E-01     DET=-2.7895E-02      P1= 8.5046E+00      P2= 1.8547E+00
WnT= 3.7699E-01     DET=-4.3945E-02      P1= 6.0977E+00      P2= 1.5725E+00
WnT= 4.3982E-01     DET=-6.3578E-02      P1= 4.6710E+00      P2= 1.3776E+00
WnT= 5.0265E-01     DET=-8.6406E-02      P1= 3.7655E+00      P2= 1.2376E+00
WnT= 5.6549E-01     DET=-1.1194E-01      P1= 3.1619E+00      P2= 1.1342E+00
WnT= 6.2832E-01     DET=-1.3961E-01      P1= 2.7448E+00      P2= 1.0566E+00
WnT= 6.9115E-01     DET=-1.6885E-01      P1= 2.4486E+00      P2= 9.9772E-01
WnT= 7.5398E-01     DET=-1.9905E-01      P1= 2.2341E+00      P2= 9.5285E-01
WnT= 8.1681E-01     DET=-2.2964E-01      P1= 2.0765E+00      P2= 9.1867E-01
WnT= 8.7965E-01     DET=-2.6008E-01      P1= 1.9594E+00      P2= 8.9278E-01
WnT= 9.4248E-01     DET=-2.8987E-01      P1= 1.8719E+00      P2= 8.7341E-01
WnT= 1.0053E+00     DET=-3.1857E-01      P1= 1.8065E+00      P2= 8.5920E-01
WnT= 1.0681E+00     DET=-3.4577E-01      P1= 1.7575E+00      P2= 8.4911E-01
WnT= 1.1310E+00     DET=-3.7116E-01      P1= 1.7211E+00      P2= 8.4231E-01
WnT= 1.1938E+00     DET=-3.9445E-01      P1= 1.6943E+00      P2= 8.3816E-01
WnT= 1.2566E+00     DET=-4.1543E-01      P1= 1.6750E+00      P2= 8.3611E-01
WnT= 1.3195E+00     DET=-4.3394E-01      P1= 1.6614E+00      P2= 8.3576E-01
WnT= 1.3823E+00     DET=-4.4987E-01      P1= 1.6523E+00      P2= 8.3674E-01
WnT= 1.4451E+00     DET=-4.6315E-01      P1= 1.6467E+00      P2= 8.3876E-01
WnT= 1.5080E+00     DET=-4.7377E-01      P1= 1.6437E+00      P2= 8.4158E-01
WnT= 1.5708E+00     DET=-4.8174E-01      P1= 1.6429E+00      P2= 8.4501E-01
WnT= 1.6336E+00     DET=-4.8712E-01      P1= 1.6436E+00      P2= 8.4888E-01
WnT= 1.6965E+00     DET=-4.8999E-01      P1= 1.6455E+00      P2= 8.5306E-01
WnT= 1.7593E+00     DET=-4.9045E-01      P1= 1.6483E+00      P2= 8.5742E-01
WnT= 1.8221E+00     DET=-4.8864E-01      P1= 1.6518E+00      P2= 8.6188E-01
WnT= 1.8850E+00     DET=-4.8469E-01      P1= 1.6557E+00      P2= 8.6636E-01
WnT= 1.9478E+00     DET=-4.7875E-01      P1= 1.6598E+00      P2= 8.7079E-01
WnT= 2.0106E+00     DET=-4.7099E-01      P1= 1.6641E+00      P2= 8.7512E-01
WnT= 2.0735E+00     DET=-4.6159E-01      P1= 1.6684E+00      P2= 8.7932E-01
WnT= 2.1363E+00     DET=-4.5070E-01      P1= 1.6727E+00      P2= 8.8334E-01
WnT= 2.1991E+00     DET=-4.3850E-01      P1= 1.6769E+00      P2= 8.8717E-01
WnT= 2.2619E+00     DET=-4.2516E-01      P1= 1.6810E+00      P2= 8.9078E-01
WnT= 2.3248E+00     DET=-4.1084E-01      P1= 1.6849E+00      P2= 8.9416E-01
WnT= 2.3876E+00     DET=-3.9571E-01      P1= 1.6885E+00      P2= 8.9731E-01
WnT= 2.4504E+00     DET=-3.7991E-01      P1= 1.6919E+00      P2= 9.0022E-01
WnT= 2.5133E+00     DET=-3.6359E-01      P1= 1.6951E+00      P2= 9.0288E-01
WnT= 2.5761E+00     DET=-3.4689E-01      P1= 1.6980E+00      P2= 9.0530E-01
WnT= 2.6389E+00     DET=-3.2994E-01      P1= 1.7007E+00      P2= 9.0748E-01
WnT= 2.7018E+00     DET=-3.1285E-01      P1= 1.7032E+00      P2= 9.0944E-01
WnT= 2.7646E+00     DET=-2.9574E-01      P1= 1.7054E+00      P2= 9.1117E-01
WnT= 2.8274E+00     DET=-2.7869E-01      P1= 1.7073E+00      P2= 9.1268E-01
WnT= 2.8903E+00     DET=-2.6181E-01      P1= 1.7090E+00      P2= 9.1399E-01
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WnT= 2.9531E+00	DET=-2.4516E-01	P1= 1.7105E+00	P2= 9.1510E-01
WnT= 3.0159E+00	DET=-2.2883E-01	P1= 1.7118E+00	P2= 9.1603E-01
WnT= 3.0788E+00	DET=-2.1287E-01	P1= 1.7129E+00	P2= 9.1679E-01
WnT= 3.1416E+00	DET=-1.9733E-01	P1= 1.7139E+00	P2= 9.1739E-01

ZETA = 9.9999E-01 ≈1
INITIAL OMEGA = 0.0000E+00   FINAL OMEGA = 3.1416E+00
NUMBER OF STEPS= 50
WnT = 1.0000E-25
THE DETERMINANT OF THE COEFFICIENT MATRIX IS EQUAL TO ZERO
MATRIX AB =

0.0000E+00	0.0000E+00
0.0000E+00	8.9442E-28

MATRIX Fp =

-8.0000E-01	-3.5777E-28

WnT= 6.2832E-02	DET=-1.9573E-06	P1= 2.0290E+02	P2= 1.2738E+01
WnT= 1.2566E-01	DET=-1.3823E-05	P1= 5.1023E+01	P2= 6.3815E+00
WnT= 1.8850E-01	DET=-4.1211E-05	P1= 2.2978E+01	P2= 4.2714E+00
WnT= 2.5133E-01	DET=-8.6348E-05	P1= 1.3218E+01	P2= 3.2245E+00
WnT= 3.1416E-01	DET=-1.4917E-04	P1= 8.7422E+00	P2= 2.6037E+00
WnT= 3.7699E-01	DET=-2.2815E-04	P1= 6.3416E+00	P2= 2.1964E+00
WnT= 4.3982E-01	DET=-3.2088E-04	P1= 4.9179E+00	P2= 1.9114E+00
WnT= 5.0265E-01	DET=-4.2450E-04	P1= 4.0126E+00	P2= 1.7029E+00
WnT= 5.6549E-01	DET=-5.3602E-04	P1= 3.4067E+00	P2= 1.5455E+00
WnT= 6.2832E-01	DET=-6.5250E-04	P1= 2.9853E+00	P2= 1.4237E+00
WnT= 6.9115E-01	DET=-7.7119E-04	P1= 2.6831E+00	P2= 1.3277E+00
WnT= 7.5398E-01	DET=-8.8963E-04	P1= 2.4612E+00	P2= 1.2510E+00
WnT= 8.1681E-01	DET=-1.0057E-03	P1= 2.2951E+00	P2= 1.1890E+00
WnT= 8.7965E-01	DET=-1.1175E-03	P1= 2.1687E+00	P2= 1.1385E+00
WnT= 9.4248E-01	DET=-1.2236E-03	P1= 2.0712E+00	P2= 1.0970E+00
WnT= 1.0053E+00	DET=-1.3229E-03	P1= 1.9953E+00	P2= 1.0627E+00
WnT= 1.0681E+00	DET=-1.4144E-03	P1= 1.9355E+00	P2= 1.0342E+00
WnT= 1.1310E+00	DET=-1.4975E-03	P1= 1.8880E+00	P2= 1.0105E+00
WnT= 1.1938E+00	DET=-1.5718E-03	P1= 1.8502E+00	P2= 9.9074E-01
WnT= 1.2566E+00	DET=-1.6372E-03	P1= 1.8198E+00	P2= 9.7416E-01
WnT= 1.3195E+00	DET=-1.6935E-03	P1= 1.7953E+00	P2= 9.6025E-01
WnT= 1.3823E+00	DET=-1.7410E-03	P1= 1.7754E+00	P2= 9.4859E-01
WnT= 1.4451E+00	DET=-1.7797E-03	P1= 1.7593E+00	P2= 9.3879E-01
WnT= 1.5080E+00	DET=-1.8101E-03	P1= 1.7462E+00	P2= 9.3057E-01
WnT= 1.5708E+00	DET=-1.8326E-03	P1= 1.7356E+00	P2= 9.2367E-01
WnT= 1.6336E+00	DET=-1.8475E-03	P1= 1.7269E+00	P2= 9.1789E-01
WnT= 1.6965E+00	DET=-1.8553E-03	P1= 1.7198E+00	P2= 9.1306E-01
WnT= 1.7593E+00	DET=-1.8566E-03	P1= 1.7140E+00	P2= 9.0903E-01
WnT= 1.8221E+00	DET=-1.8519E-03	P1= 1.7093E+00	P2= 9.0568E-01
WnT= 1.8850E+00	DET=-1.8416E-03	P1= 1.7055E+00	P2= 9.0290E-01
WnT= 1.9478E+00	DET=-1.8262E-03	P1= 1.7024E+00	P2= 9.0062E-01
WnT= 2.0106E+00	DET=-1.8064E-03	P1= 1.6999E+00	P2= 8.9875E-01
WnT= 2.0735E+00	DET=-1.7825E-03	P1= 1.6980E+00	P2= 8.9723E-01
WnT= 2.1363E+00	DET=-1.7550E-03	P1= 1.6964E+00	P2= 8.9600E-01
WnT= 2.1991E+00	DET=-1.7244E-03	P1= 1.6952E+00	P2= 8.9503E-01
WnT= 2.2619E+00	DET=-1.6910E-03	P1= 1.6942E+00	P2= 8.9428E-01
WnT= 2.3248E+00	DET=-1.6554E-03	P1= 1.6935E+00	P2= 8.9370E-01
WnT= 2.3876E+00	DET=-1.6177E-03	P1= 1.6930E+00	P2= 8.9328E-01
WnT= 2.4504E+00	DET=-1.5784E-03	P1= 1.6927E+00	P2= 8.9298E-01

WnT= 2.5133E+00	DET=-1.5379E-03	P1= 1.6924E+00	P2= 8.9279E-01
WnT= 2.5761E+00	DET=-1.4962E-03	P1= 1.6923E+00	P2= 8.9269E-01
WnT= 2.6389E+00	DET=-1.4539E-03	P1= 1.6923E+00	P2= 8.9266E-01
WnT= 2.7018E+00	DET=-1.4110E-03	P1= 1.6923E+00	P2= 8.9269E-01
WnT= 2.7646E+00	DET=-1.3678E-03	P1= 1.6924E+00	P2= 8.9277E-01
WnT= 2.8274E+00	DET=-1.3244E-03	P1= 1.6926E+00	P2= 8.9289E-01
WnT= 2.8903E+00	DET=-1.2812E-03	P1= 1.6927E+00	P2= 8.9305E-01
WnT= 2.9531E+00	DET=-1.2381E-03	P1= 1.6929E+00	P2= 8.9322E-01
WnT= 3.0159E+00	DET=-1.1954E-03	P1= 1.6931E+00	P2= 8.9342E-01
WnT= 3.0788E+00	DET=-1.1532E-03	P1= 1.6934E+00	P2= 8.9363E-01
WnT= 3.1416E+00	DET=-1.1116E-03	P1= 1.6936E+00	P2= 8.9385E-01

Now a tentative design point may be selected. Let us suppose that, for the first design iteration, the control system designer selects a location of $\zeta = 0.707$ and $\omega_n T = \pi/2$ for the placement of the poles. This corresponds to a point in the parameter space at $a_P = 0.845$, $a_I = 0.2$, and $a_D = 1.643$. Notice that this establishes the value for the real root, i.e., $\delta \simeq 0.02$. If other values of δ were desired, the value of $\zeta = 0.707$ could be maintained and the value of $\omega_n T$ varied, or vice versa. These values for root locations may be independently checked by using the programmed polynomial rootfinder termed, by Hewlett-Packard, "Siljak's Method" (Appendix F).

```
COEFFICIENTS OF THE CHARACTERISTIC EQUATION WITH
P1= 1.6430E+00                 P2= 8.4500E-01

C(0) + C(1)*S + C(2)*S^2 + C(3)*3^3 + ...
C( 0)=-2.000000E-03
C( 1)= 1.140000E-01
C( 2)=-3.120000E-01
C( 3)= 1.000000E+00

ROOTS:
       REAL           IMAGINARY        DAMPING RATIO          WnT

   1.467913E-01     -2.950344E-01     7.074111E-01      1.569207E+00
   1.841740E-02      1.608759E-13     1.000000E+00      3.994460E+00
   1.467913E-01      2.950344E-01     7.074111E-01      1.569207E+00
```

Since the ζ-contours by interpolation span the entire stable region, there must exist (in that region) only one real root at each point. If real root locations in the z-domain are designated by the symbol δ, the δ-contours (straight lines) are solved for values of δ = 0.8, 0.7, 0.5, 0.2, -0.1, -0.5, and -0.9 (notice of course, that the z = -1 stability boundary is synonymous with the δ = -1 contour).

```
FOR REAL ROOTS OF THE CHARACTERISTIC EQUATION

DELTA = 8.0000E-01
P2= 1.1111E-01 *P1 +  1.7778E+00

THE AXES INTERCEPTS ARE AS FOLLOWS:
AT P1=0 , P2= 1.7778E+00
AND AT P2=0 , P1=-1.6000E+01


FOR REAL ROOTS OF THE CHARACTERISTIC EQUATION

DELTA = 7.0000E-01
P2= 1.7647E-01 *P1 +  1.0804E+00

THE AXES INTERCEPTS ARE AS FOLLOWS:
AT P1=0 , P2= 1.0804E+00
AND AT P2=0 , P1=-6.1222E+00


FOR REAL ROOTS OF THE CHARACTERISTIC EQUATION

DELTA = 5.0000E-01
P2= 3.3333E-01 *P1 +  4.3333E-01

THE AXES INTERCEPTS ARE AS FOLLOWS:
AT P1=0 , P2= 4.3333E-01
AND AT P2=0 , P1=-1.3000E+00


FOR REAL ROOTS OF THE CHARACTERISTIC EQUATION

DELTA = 2.0000E-01
P2= 6.6667E-01 *P1 + -2.3333E-01

THE AXES INTERCEPTS ARE AS FOLLOWS:
AT P1=0 , P2=-2.3333E-01
AND AT P2=0 , P1= 3.5000E-01
```

```
FOR REAL ROOTS OF THE CHARACTERISTIC EQUATION

DELTA =-1.0000E-01
P2= 1.2222E+00 *P1 + -1.1808E+00

THE AXES INTERCEPTS ARE AS FOLLOWS:
AT P1=0 , P2=-1.1808E+00



AND AT P2=0 , P1= 9.6612E-01



FOR REAL ROOTS OF THE CHARACTERISTIC EQUATION

DELTA =-5.0000E-01
P2= 3.0000E+00 *P1 + -4.4333E+00

THE AXES INTERCEPTS ARE AS FOLLOWS:
AT P1=0 , P2=-4.4333E+00
AND AT P2=0 , P1= 1.4778E+00



FOR REAL ROOTS OF THE CHARACTERISTIC EQUATION

DELTA =-9.0000E-01
P2= 1.9000E+01 *P1 + -3.6089E+01

THE AXES INTERCEPTS ARE AS FOLLOWS:
AT P1=0 , P2=-3.6089E+01
AND AT P2=0 , P1= 1.8994E+00
```

4. Application of Cross-Multiplication Technique

Suppose now the control system designer wishes to assess the system response, using the numerical values for a_P, a_I, and a_D. Since he already has the closed-loop transfer function of Equation (20), he may go directly to the Cross-Multiplication method. The digital program for this method is shown in Appendix G. He loads in the order of numerator (2) and denominator (3) and selects a value for sampling interval (here, T = 1s).

```
ORDER OF NUMERATOR IS  2          ORDER OF DENOMINATOR IS  3
SAMPLE INTERVAL = 1

CALCULATIONS MADE ACCORDING TO FOLLOWING EQUATION DEFINITION

        C(Z)/R(Z) = NUMERATOR/DENOMINATOR
        NUMERATOR = A(0)*Z^0 + A(1)*Z^1 + A(2)*Z^2 . . . A(N)*Z^N
      DENOMINATOR = B(0)*Z^0 + B(1)*Z^1 + B(2)*Z^2 . . . B(D)*Z^D
```

Next, the numerical values for the coefficients in the numerator and denominator are loaded.

```
NUMERATOR COEFFICIENTS  A(0),A(1),A(2),...A(N)
 .998 -2.886  2.688


DENOMINATOR COEFFICIENTS   B(0),B(1),B(2),...B(N)
-.002  .114 -.312  1
```

Then, the number of sampling instants to be used is selected. In this case, time was varied from zero to 10T. The test input selected for $\dot{\theta}_c$ (here designated "INPUT") is a unit step function. The output $\dot{\theta}$ is printed out as "RESPONSE." It is plotted as Figure 3. The R's indicate values for the input at sampling instants, and the asterisks indicate value for the response at the same instants.

```
NUMBER OF STEPS FOR RESPONSE = 10
THE INPUT IS A UNIT STEP FUNCTION
```

K	TIME	$\dot{\theta}_c$ INPUT	$\dot{\theta}$ RESPONSE
0	0.000E+00	1.000E+00	0.000E+00
1	1.000E+00	1.000E+00	2.688E+00
2	2.000E+00	1.000E+00	6.407E-01
3	3.000E+00	1.000E+00	6.935E-01
4	4.000E+00	1.000E+00	9.487E-01
5	5.000E+00	1.000E+00	1.018E+00
6	6.000E+00	1.000E+00	1.011E+00
7	7.000E+00	1.000E+00	1.001E+00
8	8.000E+00	1.000E+00	9.992E-01
9	9.000E+00	1.000E+00	9.996E-01
10	1.000E+01	1.000E+00	1.000E+00

At this point the designer may wish to iterate upon his selected design point, with his frequency and time-domain information now in hand.

SECTION IV. DIGITAL SYSTEM SEMINAR (Task V)

During the December 1979 - September 1980 portion of the contracted period, a Digital Design Seminar was conducted for selected members of the Guidance and Control Directorate. The text, Digital Control Systems, by B. C. Kuo was provided the participants by the G&C Directorate.[10] A detailed outline of the topics covered follows.

A. Introduction
   1. Objective of seminar
   2. Philosophy of seminar
   3. Why digital control?
   4. Problem definition
   5. Description of the sampling operation
   6. The Z-transform
   7. Examples of analysis of digital systems by "cook book recipes"
      a. SAM
      b. Stability
      c. Response by Cross-Multiplication

B. The sampling process
   1. Motivation
   2. Mathematical description by Pulse Amplitude Modulation
      a. Time-domain approach
      b. Frequency spectrum approach
      c. Complex convolution approaches

C. The Sampling Theorem
   1. Limits on sampling frequency
   2. Shannon's Theorem
   3. Fogel

D. The "Ideal Sampler": Impulse sampling approximation
   1. Motivation
   2. Mathematical descriptions
      a. Time-domain
      b. Frequency domain using Fourier analysis

c. Frequency domain using Laplace analysis

(1) Series forms

(2) Closed form

3. Zero Order Hold

4. S-domain properties

E. Z-transform analysis of linear sampled-data systems

1. Introduction

a. Series form

b. Closed form

c. Advantages versus disadvantages

2. Evaluation of z-transforms

3. Mapping the s-plane onto the z-plane

4. Inverse z-transform

a. Inversion

b. Methods

(1) Partial Fraction

(2) Power Series

(3) Inversion Formula

(4) Cross-Multiplication

5. Theorems of the z-transform

a. Linearity

b. Real translation

c. Complex translation

d. Final value

e. Initial value

f. Partial differentiation

6. Pulse transfer functions

7. Limitations of z-transform method

8. Response of open-loop sampled-data systems between sampling instants

a. Discussion

b. Submultiple sampling method

c. Modified z-transform

d. The inverse modified z-transform

(1) Introduction

(2) Power series expansion series method

(3) Inversion formula

(4) Cross-multiplication method

9. Modified z-transform theorems

a. Real translation

b. Complex translation

c. Initial value theorem

d. Final value theorem

F. Data reconstruction

1. Introduction

2. Data reconstruction by polynomial extrapolation

3. Zero-order hold

4. First-order hold

5. Other holds

6. Summary

G. Sampled-data systems

1. Introduction

2. Block diagram analysis and transfer functions of closed-loop sampled-data systems

a. Introduction

b. Transfer functions and output response of open-loop systems with cascaded elements

c. Closed-loop sampled-data systems

d. Summary of some appropriate observations

e. Additional comments: parallel paths

f. Motivation for Signal Flow Graph (SFG)/SAM methods

3. Signal Flow Graphs (SFG)

a. Introduction

b. The sampled SFG method

c. "Formal" SFG method

d. Modified SFG method

e. SAM: "Systematic Analytical Method"

4. Modified z-transforms of outputs of closed-loop sampled-data systems

H. System response

1. Introduction

2. Comparison of transient response and sampled systems

3. Stability of sampled-data systems

a. Introduction

b. Hidden oscillations

c. Review of continuous-data system stability

d. Sampled-data system stability

e. Characteristic equation

f. Schur-Cohn stability criterion

g. Jury's stability test

(1) Description

(2) Strengths and weaknesses

(3) Singular cases

h. Raible's tabular form

(1) Description

(2) Strengths and weaknesses

(3) Singular cases

i. Extension of Routh-Hurwitz Criterion to sampled-data systems

(1) In Laplace (s) domain

(2) Bilinear transformations

(a) w-transform

(b) r-transform

(c) Relation between real frequency and warped w-- or r-domain frequencies

4. Effect of pole-zero configurations (of starred and z functions) upon system transient response

5. Effect of pole-zero configurations on maximum overshoot and peak time of transient response of sampled-data systems

6. Root Locus method for sampled-data systems

a. In Laplace (s) domain
b. In z-domain
c. Rules of construction
d. Stability determination
e. Transient response and relative stability
f. Discussion of method

7. System response between sampling instants
   a. Introduction
   b. Submultiple sampling method
      (1) Formal method
      (2) Cross-multiplication method
   c. Modified z-transform method
      (1) Power series method
      (2) Inversion Formular method
      (3) Cross-multiplication method
   d. Determination of hidden instabilities and oscillations
8. Steady-state response and steady-state error analysis

I. Frequency response of sampled-data systems
   1. Introduction
   2. Nyquist criterion and plot
      a. 2- or 3-term approximation method (s-domain)
      b. z-transform method
      c. Bilinear transform method(s)
   3. Bode diagram (w and r domains)
   4. Gain phase plots (z-domain)
   5. Parameter Space method (z-domain)
      a. Stability determination
      b. Transient response determination
      c. Computer mechanization
      d. Advantages and disadvantages

J. State variable (modern) approaches
   1. Introduction
   2. Continuous-data systems
      a. State equations

b. State transition equations

3. Digital systems

a. Analogies to continuous-data system approaches

b. Discrete state equation

c. Discrete state transition equations

d. Solution of time-invariant discrete state equation by z-transform methods

e. Discussion of further applications

K. Summary

SECTION V. PERSHING II DIGITAL G&C (Task VI)

The applicability of applying advanced digital technology to the PERSHING II G&C system was investigated. In concert with Mr. James McLean (DRSMI-RGN), the digital autopilot was selected as the point of focus. The results of these investigations were provided to Mssrs. Russell Gambill and McLean in the form of oral reports.

Dr. Seltzer attended and participated in the PERSHING II digital autopilot design review held at The Martin Company in Orlando in late March 1980. Written comments were provided to Mr. McLean (Appendix G). An important part of this (Appendix G) document is the concluding portion which is comprised of six recommendations, repeated below for continuity.

1. Provide copies of the proposed presentation material to all reviewers approximately one week before the next technical review.
2. Investigate the possible existence of dynamically significant nonlinearities. Predict their dynamic effect by mathematical analysis to be verified by computer simulation. Where this is deemed impractical, include in the hybrid simulation the actual hardware that gives rise to the nonlinearity.
3. Investigate analytically the dynamic (and accuracy) effects of digital computer quantization.
4. Investigate the sensitivity of the control system to variations in numerical values of bending parameters. View the 5% accuracy figure associated with the NASTRAN program with skepticism. This should have a strong effect on the planned "notch filters."
5. Investigate the sensitivity of the control system to realistic variations in numerical values of system parameters involved in any pole-zero cancellation techniques.
6. Initiate action to decrease computer capacity requirements and/or strongly limit future additional demands upon the onboard computer.

SECTION VI. DIGITAL FILTERS (Task VII)

In June 1980, effort was begun to evaluate the consequences of microprocessor implementation of digital filters and the attendant control logic. Three key issues were identified:

1. Accuracy due to word length,
2. Computational capability, and
3. Memory requirements.

It was determined that requirements associated with the identified key issues must be determined. The requirements would be of a form that would then enable one to determine hardware and control implementation requirements. Emphasis will be from a digital perspective, as opposed to attempting to emulate analog systems (as is apparently the current thrust in contemporary literature).

An investigation of implementation techniques and required analytical tools required for digital filtering was begun, as was a review of current microprocessor state-of-the-art and hardware characteristics. This was interrupted by the arising emphasis on ATBM G&C (Section IIC).

SECTION VII. SUMMARY

During the contracted period of 31 July 1979 through 30 September 1980, the study objective set forth in the Contract was met, i.e., to perform research necessary to develop new discrete design techniques, discrete adaptations of control theory, and in particular extended Parameter Space analysis techniques that will help to ensure development of effective G&C systems for future weapon systems. The study effort was composed of seven tasks. The requirements established in Task I were performed and documented on 30 September 1979 in a separate technical report entitled, "A Mathematical Means of Spreading Clustered System Roots." The requirements of the remaining tasks were performed and are documented in this Final Report.

One of the most significant technical achievements to be accomplished in this contracting period was the development and documentation of a simple digital system design tool. This tool is a combination of the z-domain Parameter Space technique and the Cross-Multiplication technique. With this tool, the designer can determine system stability. He may then specify system performance by pole placement. Finally, he may verify that performance, in the time domain, with the Cross-Multiplication technique, for any specified deterministic reference input the designer might wish to select. This has all been programmed in a digital computer to ease the computational burden of the designer. Indeed, the two theoretical methods have been deliberately cast in a form that is readily amenable to digital implementation.

Although long term in nature, the efforts of the Guidance and Control Analysis Group that were supported by the Control Dynamics Company are going along smoothly. The usual "blind alleys" were encountered (but eventually circumvented!) as the model of the plant was being defined.

The PERSHING II G&C efforts were funneled into a study of the autopilot design being implemented. A number of recommendations have been offered, where they seemed appropriate.

An effort to evaluate the consequences of microprocessor implementation of digital filters was begun in June 1980. This effort was interrupted by the emergence of added emphasis of ATBM G&C.

Finally, a Digital Design Seminar was conducted for selected members of the Guidance and Control Directorate. It appears that those whose perserverance permitted found the techniques they learned to be useful and the seminar to be technically rewarding.

SECTION VIII. RECOMMENDED FUTURE EFFORTS

The technical efforts completed in this contracting period have shed light on new technical work that should be undertaken. These efforts are natural extensions of the work that is reported upon in this Final Report and are in three general categories:

A. Development of Discrete Technology

In Section III of this report, the Parameter Space technique was expanded. Several powerful future extensions to the technique became apparent as this work progressed. First, it probably can be extended to the popular w-domain, but even more important, the technique would be even more powerful if a means of obviating the requirement for the characteristic equation were developed. These two thrusts toward extending the technique should be investigated and, if feasible, implemented. A third thrust that became apparent is the need to incorporate modern sampled-data techniques into the present optimal G&C techniques, thereby investigating and developing techniques for optimizing digital G&C systems.

B. Design of Advanced Discrete G&C System

In Section II of this report, Advanced G&C Systems were investigated. The major thrust was toward the development of a simulation tool with which to test and evaluate various candidate G&C systems and their subsystems. In Section VI of this report, advanced digital filters were investigated briefly. The results of these two efforts focus toward the need for the design and development of an advanced discrete (digital) G&C system that uses advanced digital filter techniques. Such a system would be implemented on microprocessors, where possible.

C. Evaluation of PERSHING II Digital G&C System

As the design of the PERSHING II G&C system continues, the Control Dynamics Company's unique digital design and analysis expertise and experience would be applied to evaluate the developing G&C system. When deemed appropriate, modification to that system would be recommended with the aim of obtaining improved system performance and/or decreased system complexity/cost.

SECTION IX. REFERENCES

1. Contract No. DAAK40-79-0-0213, dated 31 July 1979, US Army Missile Command.

2. Mod. P00001 to above Contract, dated 22 October 1979.

3. Mod. P00002 to above Contract, dated 8 February 1980.

4. Mod. P00003 to above Contract, dated 12 June 1980.

5. Technical Report for Task I: "A Mathematical Means of Spreading Clustered System Roots," Control Dynamics Company Report CDC-79-3, dated 30 September 1979.

6. Guidance and Control Directorate, US Army Missile Command, "Advanced Analysis for Future Missiles," Technical Report RG-80-8, 21 November 1979.

7. Seltzer, S. M., "Development of US Army Advanced Missile G&C System," AIAA Paper 80-0908, AIAA International Meeting and Technical Display "Global Technology 2000," May 6-8, 1980, Baltimore, MD.

8. Seltzer, S. M., "Sampled-Data Analysis in Parameter Space," Technical Report T-79-64, US Army Missile Command, June 1979.

9. Seltzer, S. M., "Determination of Digital Control System Response by Cross-Multiplication," Technical Report T-79-58, US Army Missile Command, 29 May 1979.

10. Kuo, B. C., Digital Control Systems, Systems Research Laboratory, Champaign, IL, 1977.

11. Seltzer, S. M., "SAM: An Alternative to Sampled-Data Signal Flow Graphs," Technical Report T-79-49, US Army Missile Research and Development Command, May 1979.

SECTION X. FIGURES

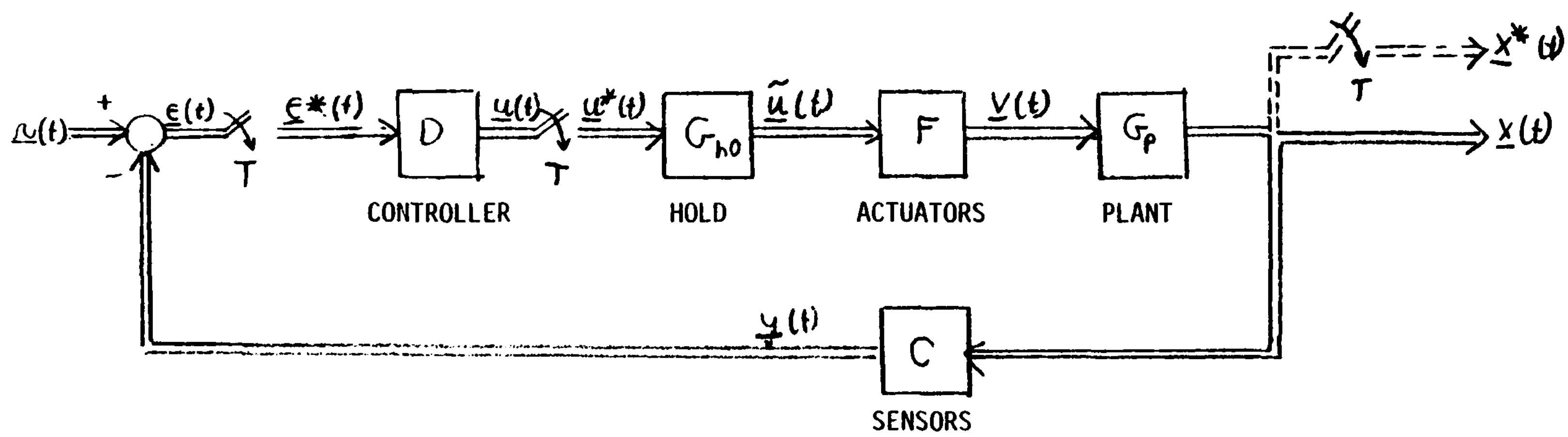


Figure 1. Typical Digitally-Controlled Aerospace Vehicle.

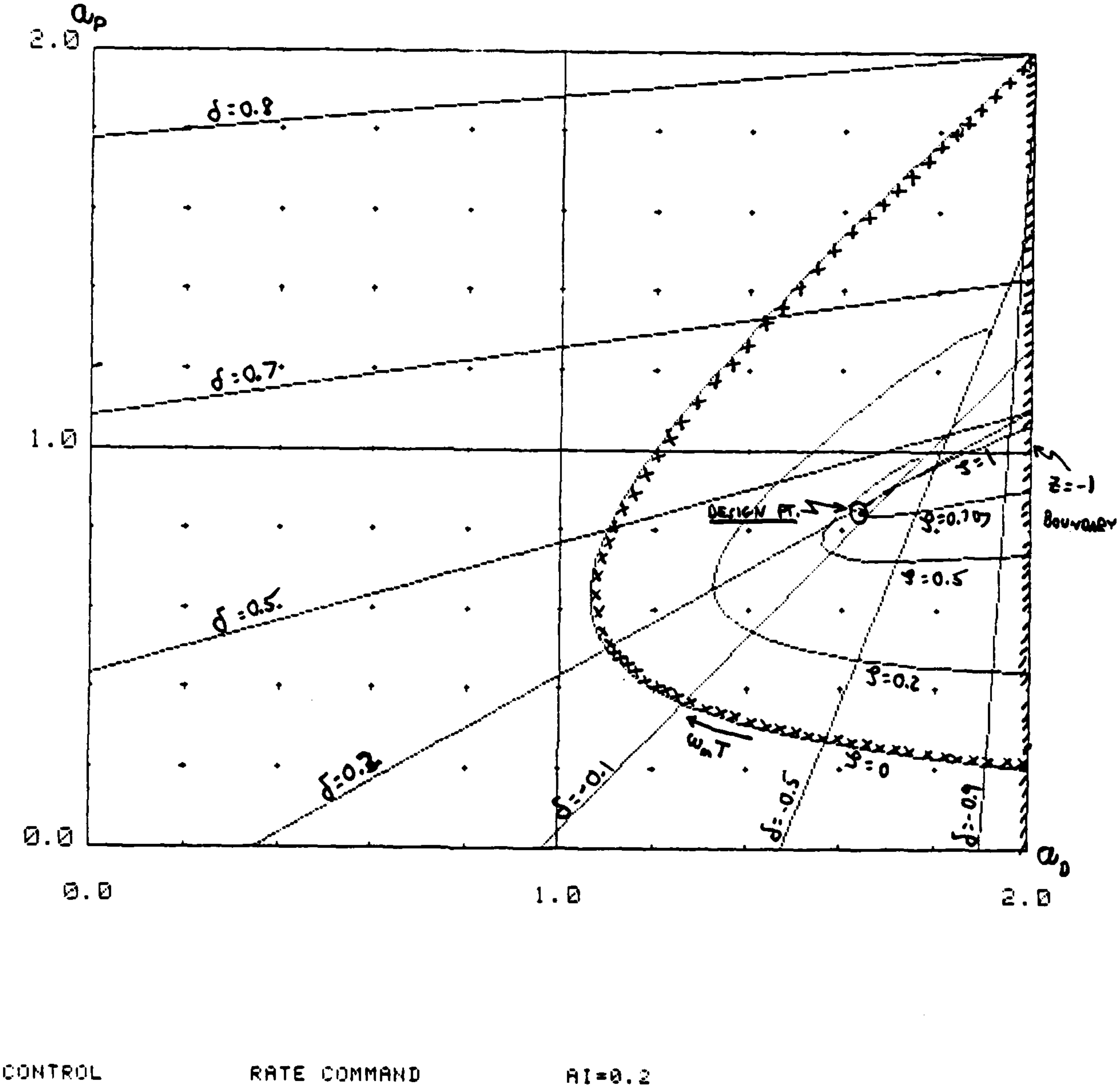
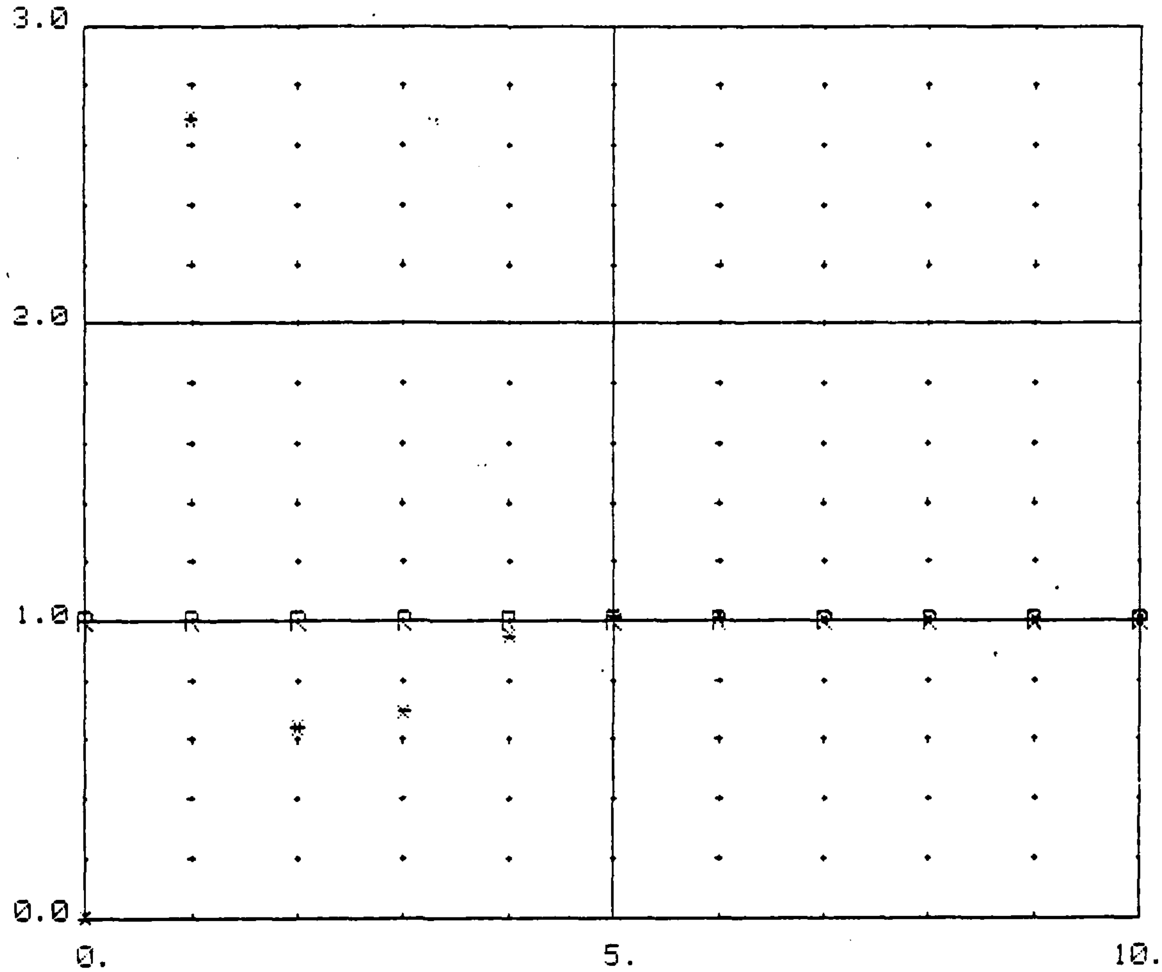


Figure 2. a_D - a_P Parameter Plane.



PID Control Rate command, Rate output AP=0.945,AI=0.2,AD=1.643,T=1

Figure 3. Rate Commanded Response.

APPENDIX A

The Charles Stark Draper Laboratory, Inc.
555 Technology Square, Cambridge, Massachusetts 02139 Telephone (617) 258-

Briefing to U.S. Army MICOM

CSDL Board Room (7103)

Thursday, 24 July 1980

MICOM ATTENDEES:

Dr. Harold L. Pastrick	Chief, Guidance and Control Analysis
Roy E. Pugh	Aerospace Engineer
Dr. Sherman Seltzer	Consultant

CSDL ATTENDEES:

Byong Ahn	Leader, Design and Analysis Div.
Edward V. Bergmann	Spacecraft Control Staff
Robert A. Booth	Chief, Systems Development Section
Timothy J. Brand	Leader, Guidance and Navigation Analysis Div.
Robert G. Brown	Consultant
Dr. Raymond Carroll	Project Manager, Optical Reference Gyro
William J. Delaney	Assoc. Head, Air Force Program Dept.
William G. Denhard	Head, Air Force Program Dept.
Dr. John Deyst	Chief, Integrated System Section Section
Robert A. Duffy	President
Daniel F. Dunn, Jr.	Staff Engineer, Inertial Sub-Systems Div.
Dr. Donald C. Fraser	Leader, Control and Flight Dynamic Div.
Jerold P. Gilmore	Leader, Inertial Sub-Systems Div.
Peter K. Kampion	Leader, Computer Science Div.
Karen Koehler	Staff Engineer
Richard A. McKern	Chief, Requirements Analysis Section
Dr. James E. Potter	Staff Scientist
Ralph R. Ragan	Head, Planning Staff
Dr. Rudrapatna Ramnath	Staff Scientist
Michele S. Sapuppo	Head, Component Development Dept.
Roy Schluntz	Program Manager, Molded Instrument Development
Robert R. Strunce	Spacecraft Control Staff

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ROOM 7103

AGENDA

0900	CSDL Overview	Bob Duffy
0920	Summary of BMDATC Programs @ CSDL	Dr. Jim Potter
0930	Divide into 2 Groups - Analysis & Hardware	
0930	HOE Autopilot	Dr. Jim Potter
1000	Quadratic Synthesis for Homing Guidance	Bob Strunce
1030	Break	
1045	Phase Space Autopilot	Ed Bergmann
1115	Fuel Depletion Guidance	Tim Brand
1145	Multiple Time Scales	Dr. Ram Ramnath
1215	Lunch	
1300	JTIDS/POD and Real Time Software	Pete Kampion
1330	Laser Gyro Tests and PRLG	Jerry Gilmore
1400	Fault Tolerant Systems	Dr. John Deyst
1430	Tour, demonstration of Fault Tolerant Aircraft, FBM, MX, Block 5D, APTS, etc.	
1630	Depart	

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ROOM 7100

HARDWARE AGENDA

0930	Low Cost System Overview	Jerry Gilmore
1030	Plastic Technology for SDOF Gyros	Roy Schluntz
1100	ORG(Optical Reference Gyro)	Ray Carroll
1130	DTG(Dry Tuned Gyro)	Ray Carroll
1145	DTG(LC Version)	Karen Koehler
1215	Rejoin Analysis Group for Lunch	

APPENDIX B

(WITHDRAWN)

APPENDIX C

HAT DEFENSE CRITICAL ISSUES
(AN ATBM PERSPECTIVE)

I. SYSTEM ISSUES

a. Early warning scheme/vehicle
   1. Vulnerability
   2. Reliability
   3. Sensitivity
   4. Redundancy/autonomy
   5. Reaction time capability

b. Battle management system
   1. Saturation level & performance near saturation
   2. Field worthiness
   3. Optimum utilization of resources
   4. Redundancy
   5. Kill assessment
   6. Sensitivity to off-nominal threats
   7. What is "Achilles' heel"?
   8. Requirement for manual participation
   9. Threat discrimination capability
   10. Reaction time requirement
   11. Threat prioritization

c. Reassess Navy threat scenario for sensitivity

II. GUIDANCE, NAVIGATION AND CONTROL

a. Determination of controllability and observalibility required by kill mechanis

b. Guidance Law

1. Conventional or new development?
2. Accuracy potential
3. Calculation of "time to go"
4. Succeptability to threat maneuvers
5. Flexibility
6. Computer implementation

c. Control Law (Vehicle autopilot)

1. Authority
2. Stability (rigid body + flexible structure)
3. Performance
4. Flexibility
5. Redundancy (logic and actuators)
6. Computer implementation
7. Sensitivity
8. Number of degrees of control required
9. Consideration of additional control authority (e.g."bank-to-turn")

APPENDIX D

AIAA-80-0908

Development of U.S. Army Advanced Missile Guidance and Control System

SHERMAN M. SELTZER, Control Dynamics Co., Huntsville, Ala.

AIAA INTERNATIONAL MEETING & TECHNICAL DISPLAY "GLOBAL TECHNOLOGY 2000"

May 6-8, 1980/Baltimore, Md.

DEVELOPMENT OF US ARMY ADVANCED MISSILE G&C SYSTEM*

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Huntsville, Alabama 35802

Abstract

In 1978 the US Army Missile Command embarked upon a task to develop an advanced guidance and control system for future missiles. It was intended to "leapfrog" systems currently under development in order to meet the stringent demands and constraints imposed by targets with predicted characteristics of the 1990s and beyond and by the predicated battlefield environment of that time.

In this paper the problem is redefined and the latest development program presented. Results that have been achieved to date are described, particularly in the areas of mathematical models of the missiles and their guidance and control (G&C) systems being used for analyses and simulations; aerodynamics; propulsion; guidance laws being developed and analyzed; status of development of Disturbance Accommodating Control; signal processing to locate and track the target(s); and digital design tool development. The paper is concluded with a section of future plans, to include contractor support.

I. Introduction

The US Army Missile Command (MICOM) recently began a task to develop an advanced G&C system for Future Army Modular Missiles. The intent is to "leapfrog" systems currently under development. The purpose of this paper is to describe the work that has been completed within this new task and to provide an indication of future efforts that are now planned.

The first step in implementing this task was to conduct a literature survey to establish a technology base starting point. Following this survey, guidance laws were placed in five categories and defined mathematically. The implementation and predicted performance of each category was then investigated and compared in light of current and predicted hardware and software capabilities.[1] This work was subsequently updated in 1979.[2]

The program objectives are three-fold:

- To develop and prove a G&C system that is capable of guiding and controlling future US Army missiles (generally defined as air defense and surface-to-surface general support) to destroy prescribed lines of future targets. This must be accomplished under the predicted severe battlefield environment of the future.
- To "leapfrog" systems under current development.
- To broaden and deepen the existing G&C system technology and design base within the G&C Directorate of MICOM.

In the sequel the expected threat and its characteristics will be summarized. This is followed by a description of the development plan for the advanced G&C system. The progress to date is presented, and the paper is concluded with a section on future plans.

II. Expected Threat

Theater defense typically is provided by a mixture of ground-based and airborne defense systems supported by radars, command and control systems, electronic warfare equipment, and passive measures such as camouflage, decoys, and equipment dispersion. The air defense objective of ground based systems is to limit the opponent's effectiveness by attacking his critical assets so that land forces may maneuver with a minimum of interference from the enemy air weaponry.

For many years now, the enemy doctrine has emphasized large mass and brute force, and his air attacks will provide no exception. It is entirely feasible to assume that an attack in the Central Europe area will be accompanied by several thousand combat aircraft.[3] In addition, his doctrine calls for the massing of large quantities of artillery fire on a section selected for a tank-led breakthrough. It is unlikely that NATO forces either now or in the near future will match the Warsaw Pact forces in terms of numbers of weapons, nor is it the intent to aim toward that end. Rather, it is important to optimize the effectiveness of our smaller force to meet the anticipated threat.[4]

The Army is attempting to maximize the effectiveness of its current family of air defense weapons while concurrently developing a new family to meet the threat of the 1990's. In the near term, there will be continued modification of current systems as necessary, and while still feasible, to overcome qualitative and quantitative deficiencies. Longer-term replacements continue in development or procurement for all the major field army air defense systems. Examples of this strategy include the following: high to medium altitude missile systems - PATRIOT for NIKE HERCULES and HAWK; short range missile systms - U. S. ROLAND for CHAPARRAL; manportable missiles - STINGER for REDEYE; mobile gun systems - DIVAD given to VULCAN. The systems will provide the effective aerial umbrella needed by our forces to not only survive, but to fight effectively.

For security reasons, it is impossible in this forum to describe specifically the air threat that will be encountered in the scenario described above. However, in order to quantify the problem somewhat, we shall attempt to attribute vehicle characteristics to the enemy based on our current technology in the field of air defense targets. The Targets Management Office, US Army Missile Command since

*This paper is declared a work of the US Government and therefore is in the public domain. A portion of this paper was prepared under US Army Missile Command Contract DAAK40-79-C-0213 dated 31 July 1979.

**Consulting Engineer, Associate Fellow, AIAA.

1964 has published an extensive library of target program reports. In particular, they have classified a variety of aerial targets as test and evaluation (T&E) targets used for air defense weapon systems. Needless to say, targets used for this purpose must exercise an air defense weapon system to the limits of its capabilities.[5]

A particularly interesting target is known as HAHST, an acronym for High Altitude High Speed Target. It is designed to achieve speeds up to Mach 4 at altitudes up to 100,000 feet. Additional performance characteristics for HAHST and other existing targets are given in References 2 and 5.

The effectiveness of any missile system is conditioned on its ability to function in an Electronic Countermeasures environment. Stand off jammers, barrage jammers and even dispensed chaff will be used to deny the air defense tracking radars the capabilities needed to be effective in their sector. The jammers are intended to reduce the acquisition range of the radars and, if perfected, will eliminate accurate tracking entirely. Additionally, the threat, aircraft and missiles will be fabricated to present the smallest possible radar cross section. The state-of-the-art in this field is beyond the scope of presentation in this paper.

From a defensive viewpoint, the effect of enemy jamming of the air defense radars and the minimization of enemy attack aircraft radar cross sections have a profound impact on the air defense missile system. The rationale is reasonable and straightforward. If the enemy does indeed have aircraft and attack missiles and remote pilotless vehicles (RPVs) either with or better than the characteristics attributed to them via the method above, and if the enemy minimizes his radar cross section to the current state-of-the-art, his attack vehicles will be extremely difficult to acquire at long range. The effect of the combined high speed and high agility (i.e., high g-maneuver capability) with low radar cross section yields precious little reaction time to the air defense system. The close-in acquisition will seriously degrade the existing air defense missile's G&C system's performance, since most are based on a proportional navigation and guidance (PNG) law. An environment such as presented above, however, can be better addressed in terms of guidance laws explicitly tailored to this type of threat. Thus, the optimally guided and controlled, highly maneuvering, defensive missile using terminal guidance sensors chosen from across a wide range of the frequency spectrum must be initiated into the development cycle. As a necessary first step, the research described in the sequel addresses that problem.

III. Development Program Plan

From an overall systems viewpoint, this program shall address the issue of creating new theory in the G&C area to meet the high performance threat of the future as one of the leading technology items. Closely associated with it and in parallel with the G&C effort, weapon system work shall be undertaken to modify airframe and propulsion to be capable of engaging the threat of the 1990's. General support weapons shall be viewed initially as a subset of the air defense system(s), whereas previously, these two classes of weapons were developed independently. This research shall attempt to view them as potentially similar systems that utilize different modules such as propulsion, guidance, warhead, etc.

A program plan initiated by the G&C Directorate, MICOM, was undertaken approximately two years ago with the program objectives enumerated above. The program plan contains six intermediate objectives, the end of which each constitutes a program milestone. They are:

- Define overall program;
- Collect elements that will form the candidate G&C systems;
- Define candidate G&C systems;
- Evaluate the candidate G&C systems: select best system;
- Design and fabricate the selected system;
- Demonstrate "Proof-of-Concept" of the selected system.

These separate program elements (or intermediate) objectives are described below in more detail. An accompanying milestone chart is provided as Fig. 1.

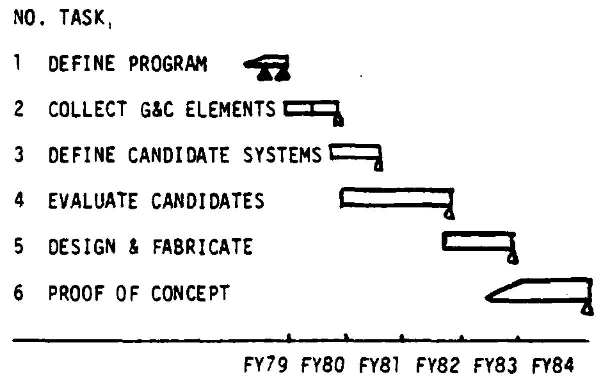

Figure 1. Program Milestone Chart.

Task 1. Define Overall Program

In this first task it is necessary to define the program objectives, goals, and constraints. This is followed by a survey of representative missile plants and the characteristics of sensors and effectors that might be used (either available or under development). Target dynamics and initial conditions must be defined, as must missions for the missile system(s) using the advanced G&C systems that emanates from this program. Also included in the first program element is the beginning of coordination with other missile-developing agencies and services. Finally, the time frame predicted usage of the G&C system must be defined. As described in this paper, most of Task 1 has been completed.

Task 2. Collect Elements for Candidate G&C Systems

In this task detailed definitions of the characteristics of candidate guidance laws, candidate autopilots (control laws), state estimation techniques, state truncation techniques, and Disturbance Accommodation Control (DAC) shall be accomplished.[6] Further, characteristics of all expected disturbances, both external (to the missile) and internal, must be collected and evaluated. The need for DAC and for state estimation will be evaluated. Characteristics of all expected significant system nonlinearities will be collected and their dynamic importance evaluated.

Task 3. Define Candidate G&C Systems

The components and subsystems that have been identified in the earlier tasks will now be combined into candidate G&C systems. Missile structural dynamics will be determined (if not already completed within the plant definition of the earlier tasks) or refined, as necessary. The simulation program objectives will be defined; the simulation program will then be defined; and finally, development of the final system simulation will begin. Investigations of the dynamics of the candidate G&C systems will begin using both mathematical analysis and computer simulation. A figure of merit (cost functional) will be developed during this task, as will model error effects and criteria.

Task 4. Evaluate Candidate G&C Systems

The development of the system simulation and the investigation of the candidate G&C systems, begun in Task 3, will be completed. The candidate G&C systems then will be evaluated with respect to the Figure of Merit developed in Task 3, using mathematical analysis and computer simulation. The strengths and weaknesses of distributed versus centralized controllers will be assessed, probably as enhanced by the use of microcomputers. Finally, the best G&C system will be selected and integrated into a missile airframe.

Task 5. Design and Fabrication

A detailed design of the selected G&C system will be performed. The testbed(s) selected for use in the Proof-of-Concept phase will be fabricated and/or assembled.

Task 6. Proof-of-Concept

The Proof-of-Concept will be demonstrated with hardware firings during this task. These firings will be augmented by simulations (computer and hardware-in-the-loop as deemed necessary) and analyses as necessary.

IV. Progress to Date

The accomplishments to date have been achieved by elements of MICOM and through the use of research contracts. It has been supplemented through coordination with the US Air Force Armaments Laboratory at Eglin Air Force Base, the US Army Ballistic Missile Defense Systems Command at Huntsville, Alabama, and the Office of the Under Secretary of Defense. This coordination has been and will continue to be carried out to eliminate duplicated development effort on similar projects within the Department of Defense. To augment the in-house research and engineering capability, several research contracts have been initiated in specialized areas currently including: the Computer Sciences Corporation, Huntsville, Alabama; the Dynamic Systems Research and Training Corporation, Huntsville, Alabama; the University of Florida; Western Kentucky University; and the Control Dynamics Company, Huntsville, Alabama. Additional contracts with other organizations are anticipated as the scope of the program grows.

The technical work comprising Tasks 1 and 2 has been apportioned to various members of the MICOM and contractor team. As indicated, most of Task 1 has been completed, and effort on Task 2 work is underway.

A. Target Definition

A comprehensive investigation of predicted future targets and their dynamics has been completed. This investigation included reviewing and discussing material available within the sources of the US Army and US Air Force and included inputs from several industrial organizations. Finally, the collected information was discussed with elements of the Office of the Under Secretary of Defense, Research and Engineering. While most of the collected information is classified for security reasons, the results of this most important phase of the study lead to emphasizing targets in three categories: highly maneuverable aircraft, cruise missiles, and tactical ballistic missiles. Not addressed in this study are RPV's and surface targets. It is felt that use of a sophisticated missile system to engage numerous RPV's would not be cost effective and that missiles currently under development will be able to combat surface targets. Except for periodic updating, this subtask is now complete.

B. Missile Plant Characteristics

In order to get the other subtasks underway, standard equations of motion for missile bodies with some structural flexibility have been used. The missiles are assumed to be acted upon by the usual aerodynamic forces and torques. Currently, variations of the SPRINT missile family are among the principal contenders for the airframe and propulsion system.

C. Aerodynamics

A detailed plan is being generated for future actions in gathering and generating aerodynamic data to be used in this program. Novel aerodynamic shapes are under consideration and evaluation for use in developing control authority for missiles using advanced G&C systems. These include missiles using configurations shown in Table 1 (along with their characteristics).

D. Error Sources

Expected error sources have been categorized into five detailed groups, to include predicted mean and standard deviation values. Several are presented as representative of the groups. These will, of course, change as the choices are narrowed and the program develops.

1. Missile. Included in this group are parameter uncertainties such as thrust magnitude and misalignment, C.G. location and offset, mass, and transverse and axial moments of inertia.

2. Aerodynamic. Normal and axial forces; pitch, yaw, and roll moments; pitch and yaw damping derivatives; roll damping coefficient due to fin deflection; and axial drag are factors.

Table 1. Aerodynamic Configurations and Characteristics.

Configuration	Advantages	Disadvantages	Mach No.	Altitude (ft.)	Maneuverability (g's)
1. Deployable wings with all-moveable tails	a. Simple controls b. Good roll control	a. Control force in opposite direction from maneuver b. High angle of attack	2-4	0-100 K	10-15
2. Low aspect ratio, long-chord delta wing with tab controls	a. Simple controls b. Good roll control	a. Control force in opposite direction from maneuver b. High angle of attack	4-7	0-140 K	20-30
3. Flared skirt-stabilized missile with all moveable wings	a. Control force in direction of maneuver b. Lower angle of attack	Possible higher drag and hinge moment	4-7	0-140 K	20-30
4. Reentry body shape with moveable wedge controls *	a. Simple shape b. Good roll control	Roll control surfaces separate from maneuvering controls	5-10	?	?
5. Lifting body *	a. Lower angle of attack b. Control force in direction of maneuver	Complicated aerodynamic shape	3-6	0-90 K	10

* Comment. Must use bank-to-turn guidance.

3. Instruments

- Accelerometers - scale factor stability, bias stability, non-orthogonality, "g^2" - scale factor, third-order scale factor, cross-axis sensitivity, cross-coupling, scale factor asymmetry, and rectification error.

- Gyroscopes - scale factor stability; bias stability; non-orthogonality; anisoelastic drift; drift rates in pitch, yaw, and roll due to input axis and spin axis accelerations as well as those (rates) independent of acceleration, due to torquer nonlinearities, and due to electronic noise; and mass unbalance.

- Porro-prism azimuth alignment.

- Laser inertial measurement unit misalignment with respect to missile body axes.

- Accelerometer triad origin displacement.

- Uplink/downlink bias and calibration errors.

- Optical correlator errors.

- Radar - range track and angle track noise and accuracy, ground clutter noise, and target glint noise.

4. External. Wind magnitude and direction (initial azimuth, elevation, and roll alignment of the missile; target velocity and illumination jitter; semiactive laser pointing accuracy and beam divergence; gravity bias; atmospheric effects (e.g., upon radio range).

5. Subsystems

- Common nonlinearities - saturation, coulomb (and other) friction, backlash, and bang-bang with dead zone.

- Computer - quantization, truncation, and fixed word length.

- Seeker - boresight error (in pitch and yaw) due to servo noise; channel crosscoupling; coupling between the seeker head and the airframe; effects of the radome and irdome on angle linearity; angle bias (the electrical equivalent of mechanical BSE); gain stability; angle noise; and boresight error in pitch and yaw due to clutter, receiver, and jamming noise.

- Autopilot - bias errors, time delays, and gain stability.

- Guidance - errors in initial position, velocity, and acceleration.

As the program progresses, these error sources will be analyzed further to determine which might be amenable to cancellation by appropriate disturbance accommodation design theory and techniques.

E. Guidance Laws

This is a major thrust area within the program. As indicated above, an extensive literature search has been completed and documented. This was followed by placing guidance laws in five categories and describing each mathematically. The implementation and predicted performance of each category has been initially reported in Reference 1 and subsequently updated in Reference 2. A summary of the latter is shown in Table 2.

Investigations continue into areas of optimal guidance, in particular, emphasizing digital aspects in anticipation of the expected use of on-board digital controllers. Constant effort is made to reduce implementation complexity, where complexity is defined as requirements for hardware and software. Other innovative techniques are under investigation. They include assessing the potential application of Singular Perturbation Theory, Disturbance Accommodating Theory, and means of determining or predicting the very important (for optimal applications) quantity, time-to-go, i.e., remaining time of flight at any instant (see Table 3).

1. Optimal Control. A conventional implementation of Linear-Quadratic Optimal Control is portrayed graphically in Figure 2. The optimal control authority, u^o_{LQ}, is selected to be

$$u^o_{LQ} = -R^{-1}B^T K_x \dot{x} \ . \tag{1}$$

The matric Riccati equation is solved to obtain the control gain, $K_x(t)$, while minimizing a quadratic performance index.

2. Disturbance Accommodating Control. The Disturbance Accommodating Control (DAC) theory is described for the continuous-time domain in Reference 6. The general nature of the DAC controller is to generate a real-time on-line estimate of the actual (instantaneous) disturbance waveform and create a special control action that exactly

Table 2. Conventional Guidance

Approach	Advantages	Limitations
1. Attitude Pursuit	a. Simplest implementation b. Fixed targets	Sensitive to target velocity, disturbances
2. Velocity Pursuit	a. Simple implementation b. Non-maneuvering targets	Sensitive to target acceleration, disturbances
3. Proportional Navigation Guidance (PNG)	a. Simple implementation b. Maneuvering targets	Sensitive to high end game maneuvers

Table 3. New Guidance Approaches

Approach	Advantages	Limitations
1. Linear Quadratic (LQ) Regulator	Better than PNG against maneuvering target	a. T_{GO} estimate required b. Must compute time-varying control gain (K_x) c. Disturbances ignored
2. Linear Quadratic Gaussian (LQG) Regulator	Better than LQ against noise-type disturbances	a. T_{GO} estimate required b. Must compute time-varying control gain (K_x) c. Must compute Kalman gain for estimator
3. Disturbance-Utilizing Control (DUC)	Better than LQ or LQG against waveform-type disturbances	a. T_{GO} estimate required b. Must compute 2 time-varying control gains (K_x, K_{xz})
4. Singular Perturbations	Computational efficiency	An approximation to optimal control

cancels-out the disturbance effect on the missile. The DAC theory will be extended into a discrete-time domain. To date, the class of systems and disturbances amenable to a discrete-time version of DAC have been defined. Work is underway on a description of the state-reconstructor that will be associated with a digital DAC controller. Also under consideration is the possibility of applying DAC to various missile subsystem or component outputs, such as sensors, whose normal outputs have been modified by the influence of the disturbances upon the sensors.

An innovative modification of DAC is Disturbance-Utilizing Optimal Control (DUC)[7]. In this case waveform-type disturbances are exploited optimally. Examples of such disturbances are drag, target maneuvers, wind gusts, any effects of the gravitational field. A graphical portrayal of DUC implementation is shown in Figure 3. In this case, the optimal control authority is specified as

$$u^o_{DUC} = -R^{-1}B^T(K_x\hat{x} + K_{xz}\hat{z}) , \qquad (2)$$

where $K_x(t)$ is found by solving a matric Riccati equation, and $K_{xz}(t)$ is found by solving a linear differential equation. To date, computer simulations have shown DUC to be quite effective when compared to the performance achieved by using conventional LQ controllers.

3. Singular Perturbations. Application of Singular Perturbation Theory to missile control may be attractive if it is deemed necessary for the control law to account for high order model terms.[8] The standard approach is to approximate the model with relatively lower order equations. However, the neglected higher order terms may be dynamically significant. Because their inclusion might create computational problems, a possible alternative might be the application of Singular Perturbation Theory. The application is particularly amenable to controller design where the open-loop plant has a wide eigenvalue dispersion, slow and fast modes, or parasitic parameters.

F. Sensor Characteristics

Various existing and predicted sensors have been characterized. Trade studies to aid in their selection have been identified. It now appears that a sensor using some form of pattern recognition may be required. The problem is to find or begin developing a sensor that can provide a guidance signal with a superior signal-to-noise ratio from data that has been deliberately modified, for example, by high powered jamming equipment.

G. Computer-Aided Design Tools

A number of computer programs are available (such as root locus and other frequency domain

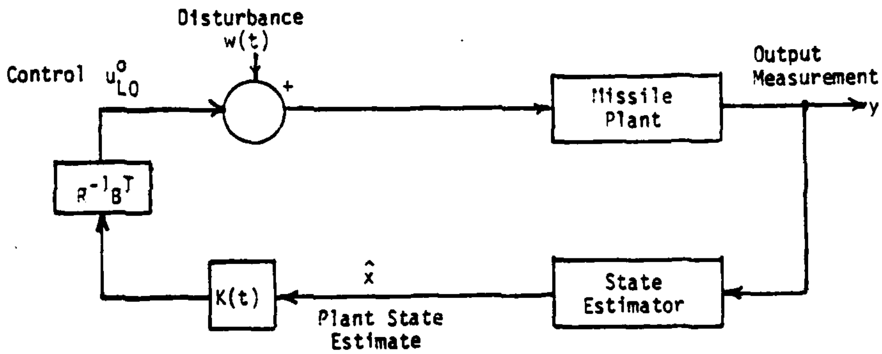

Figure 2. Conventional Linear Quadratic Optimal Control.

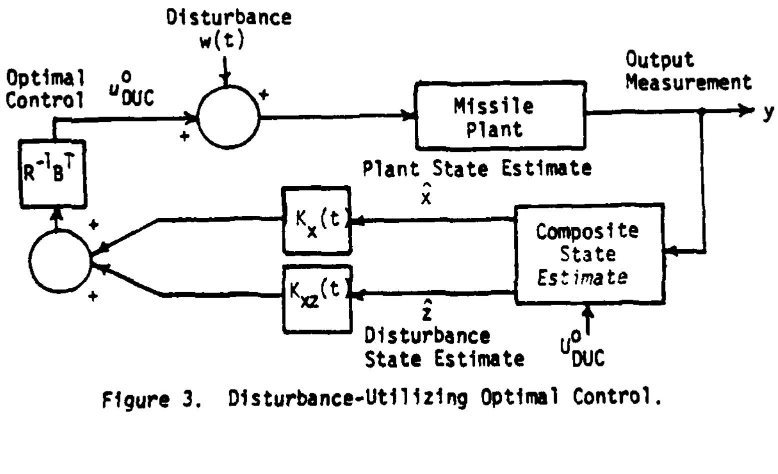

Figure 3. Disturbance-Utilizing Optimal Control.

techniques) to aid in design of control systems. Most of them are in the continuous-time domain, although there does exist a z-transform manipulation program. An executive routine currently is being coded to manipulate these programs efficiently.

H. Microcomputer State-of-the-Art

A continuing assessment of microcomputer state-of-the-art has been instigated within this program. Microcomputers will become an integral part of advanced G&C systems, with their small size, light weight, and relatively low cost. Presently, microcomputers software capabilities are being investigated with a Tektronix 8002 Microprocessor Laboratory. This investigation will be followed by an analysis of the computational capability to meet the requirements imposed by an advanced G&C system.

I. Digital Design Tool Development

The probability that an advanced G&C system will be implemented digitally seems to be nearly assured. It is for this reason that the foregoing work has been oriented strongly in the direction of digital implementation and its consequences. A review of analysis and design tools available to the G&C system designer is being conducted. Where the need for a new tool appears to be warranted, it will be developed within this program when possible. Emphasis is placed on both simplicity of application and being able to draw on practising engineers existing engineering training and experience.

One example of such a tool is "SAM" (acronym for Systematic Analytical Method). SAM provides an alternative to the use of signal flow graphs and the application of Mason's Gain Rule to determine selected states of the missile.[9] The technique is particularly useful for analysis of complicated sampled-data control systems. An advantage of using SAM is that the cumbersome application of Mason's Gain Formula can be avoided. Further, the entire method of constructing signal flow graphs may be circumvented. Since only the equations describing the system are needed for SAM, even the customary block diagram is not needed. The technique is analytical in nature and makes use of a systematic manipulation of the system algebraic equations. These manipulations follow prescribed rules set forth in the technique.

A second simplified technique is the determination of digital control system response by cross-multiplication.[10] This technique permits the analyst to obtain the response (at the sampling instants) of any system state from its closed-loop transfer function expressed in the complex z-domain. If it is desired to know the response between sampling instants, either the submultiple method or the modified z-transform method may be adapted to the cross-multiplication technique.

A third, more sophisticated technique is the Parameter Space Method. It is being developed for determining the stability and dynamic characteristics of a digital control system in terms of several selected system parameters.[11] The method requires that the system characteristic equation be available in the complex z-domain. Although not necessary, its application is facilitated by augmenting the analytical results with graphical portrayals in a selected multiparameter space. The method is based on analysis and synthesis methods for linear and nonlinear control system design which are amply described in Siljak's excellent monograph on the subject.[12] In essence, the parameter space method permits the designer to evaluate graphically the effects of the locations of the roots of the characteristic equation. Hence, he may design the control system in terms of his selected performance criteria; e.g., absolute stability, damping ratio, settling time. He is able to see the effects on the characteristic equation roots (and hence on system dynamics) of changing several adjustable parameters. The method has been extended to portray the effect of varying the sampling period, thereby permitting one to observe the effect of the choice of various values assigned to the sampling period on absolute and relative stability. Also, simple recursive formulas have been derived so that the resulting formulation is deliberately cast in a form particularly amenable to solution by a digital computer or a desk calculator, emphasizing the interplay between analysis and computing machines.

J. Documentation

A major portion of the progress reported above has been documented in a comprehensive US Army Missile Command report.[13]

V. Future Plans

Plans for the next fiscal year revolve primarily about implementating Task 3 (define candidate G&C systems) of the development program as well as completing any partially-completed portions of Task 2 now underway. Close communication will be maintained with the intelligence community to become aware of any changes to the presently predicted targets as contrasted to the attribution mentioned above. Detailed analytical models of the missile plant(s), effectors, and autopilot(s) will be developed. The possibility of modifying plant characteristics by making innovative use of aerodynamics is to be investigated. Projected advances in the field of propulsion will also be investigated. The comparison of various digitally-implemented guidance laws will be continued, including those that incorporate DAC. Applications (other than to guidance laws) of DAC theory to improve system performance will be investigated. Development of a modular guidance simulation to implement this investigation has already begun. Trade studies concerning identified existing and future sensors will be conducted, with the possibility of developing a new sensor with capabilities not yet in existence. Advances in microcomputer state-of-the-art will be watched closely. Finally, tools to aid the G&C system designer to handle digital implementation will continue to be both assessed and developed.

VI. Conclusions

As indicated within this paper, there is a clear need for the development of an advanced G&C system for US Army future tactical missiles. This need is dictated by the predicted targets of the future, the anticipated battlefield of the future, and the characteristics of tactical missiles that are either in the inventory or under development at this time. The nature of the future G&C system will be digital so that the overall missile system may be availed of present and predicted advantages that are implicit in digital controllers. The form of the guidance law probably will be optimal, since the performance criteria that must be minimized must take more into account than miss distance.

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APPENDIX E

TECHNICAL REPORT T-79-64

SAMPLED-DATA ANALYSIS IN PARAMETER SPACE

S.M. Seltzer
Technology Laboratory

June 1979

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35809

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This report describes a technique for determining the stability and dynamic characteristics of a digital control system in terms of several selected system parameters. The method requires that the system characteristic equation be available in the complex z-domain. An example is provided to further elucidate the technique. It is this capability that makes the method more powerful than most design techniques (which describe stability in terms of only one variable parameter or gain).

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1. INTRODUCTION

The parameter space method provides an analytical tool developed for use in control system analysis and synthesis. Although not necessary, its application is facilitated by augmenting the analytical results with graphical portrayals in a selected multi-parameter space. The method requires that the control system be described by a characteristic equation which, for sampled-data or digital systems, may be expressed in the z-domain. The technique is based on the analysis and synthesis methods for linear and nonlinear control system design which are amply described in Siljak's excellent monograph on the subject.[1] In another work, Siljak describes the application of the technique to the analysis and synthesis of linear sampled-data control systems.[2]

Once the system characteristic equation has been obtained, the parameter plane method enables the designer to evaluate graphically the locations of roots of the equation. Hence, he may design the control system in terms of the chosen performance criteria; e.g. absolute stability, damping ratio, and setting time. He is able to see the effect on the characteristic equation roots of changing two adjustable parameters. Siljak further simplified the design by introducing Chebyshev functions into the equations, thereby putting them in a form that is amenable to their solution by a digital computer.

The method has been extended to portray the effect of varying the sampling period.[3,4] The extended method permits one to see the effect of the choice of values assigned to the sampling period on absolute and relative stability. Also, the recursive formulas shown therein are simpler in form than the Chebyshev functions of Reference 2. The resulting formulation is deliberately cast in a form that makes it particularly amenable to solution by a digital computer or a desk calculator, again emphasizing the interplay between analysis and computing machines. When portrayed graphically, the results show the dynamic relation between the selected parameters and the characteristic equation roots, as a function of the nondimensional independent argument, $\omega_n T$. Hence one readily can deduce the dynamic effect upon the system of various combinations of values of the selected parameters defining the parameter space.

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1. D.D. Siljak, *Nonlinear Systems*, Wiley, New York, 1969.
2. D.D. Siljak, Analysis and Synthesis of Feedback Control Systems in the Parameter Plane, Part II-Sampled Data Systems," *Transactions of the Institute of Electrical and Electronics Engineers, Part II Applications and Industry*, Vol. 83, November 1964, pp. 458-466.
3. S.M. Seltzer, "Sampled-Data Control System Design in the Parameter Plane," *Proceedings of the 8th Annual Allerton Conference Circuits and System Theory*, 1970, pp. 454-463.
4. S.M. Seltzer, "Enhancing Simulation Efficiency with Analytical Tools," *Computers and Electrical Engineering*, Vol. 2, 1975, pp. 35-44.

The history of the continuous-time domain version of the parameter plane technique is well-described with suitable references in Reference 1. Briefly summarizing that history, in 1876 I.A. Vishnegradsky of the Leningrad School of Theoretical and Applied Mechanics developed and used the first version of the parameter plane technique to portray system stability and transient characteristics of a third order system on a two-parameter plane. In 1949 Professor Yu I. Neimark of the Russian School of Automatic Control generalized Vishnegradsky's approach to permit the decomposition of a two-parameter domain (D) describing an nth order system into stable and unstable regions. The technique was called D-decomposition. During the period 1959-1966 Professor D. Mitrovic, founder of a Belgrade group of automatic control, extended the method to enable the analyst to relate the system's variable parameters to the system response, using the last two coefficients of an nth order characteristic equation. Beginning in 1964. Professor D.D. Siljak, then a student of Mitrovic's at the University of Belgrade, generalized the method and called it the Parameter Plane method. His method permitted the analyst to select an arbitrary pair of characteristic equation coefficients (or parameters appearing within the coefficients) and portray both graphically and analytically the dependence of the system response upon the selected parameters. The method was extended subsequently by Siljak and others to encompass a host of related problems. In 1967 Professor J. George modified the D-decomposition method to enable the portrayal of the absolute stability region in a multi-parameter space. George also showed how to portray contours of relative stability, as did Siljak. All of the foregoing work is carefully and completely referenced within Reference 1. In 1966 and subsequent years, Seltzer has applied the parameter space method to the design of missile, aircraft, and satellite controllers, including systems containing one or two nonlinearities; the analysis of the dynamic effects of the nonliner "Solid Friction" (Dahl) model for systems with ball bearings, such as control moment gyroscopes and reaction wheels; and the specification by the system designer of the dynamic structural flexibility constraints to the structural designer. Most of the work has appeared in the technical journals of the Institute of Electrical and Electronic Engineers (IEEE), the American Institute of Astronautics and Aeronautics (AIAA), the International Journal of Control, and the journal, Computers & Electrical Engineering. A portion of the history that has not been reported upon previously (with one exception to be noted) is the control system work conducted by the German rocket scientists in the early 1940's at Peenemunde. There, Dr. W. Haeussermann and others applied the D-decomposition technique to the design of the V-2 Rocket, following Dr. Haeussermann's earlier (pre-World War II) application to the control of an underwater torpedo. This work was not published in the open literature because of national security constraints. When the group came to the United States to work with the Army Ballistic Missile Agency (first at Ft. Bliss, Texas, then at Redstone Arsenal, Huntsville, Alabama). Dr. Haeussermann and his associates continued to apply the method to US Army missiles (and later to NASA space vehicles). Again, national (this time, another nation!) security precluded publication in the open literature until 1957.[5]

In 1964 Professor Siljak published the first application of the parameter plane technique to sampled-data systems.[2] As mentioned above, this was extended in References 3 and 4. In 1971 Seltzer presented an algorithm for systematically solving the Popov Criterion applied to sampled-data systems.[6] Applications of these sampled-data parameter space techniques are found in References 3, 4 and 6.

2. ANALYTICAL PRELIMINARIES

The technique requires that the control system be described by a characteristic equation in the z-domain. Two adjustable parameters (k_0, k_1) are selected, and the characteristic equation (CE) is recast in terms of them; i.e.

$$CE = \sum_{j=0}^{n} \gamma_j z^j = 0, \tag{1}$$

$$\gamma_j = \gamma_j \, (d_j, \; k_o, \; k_1) \tag{2}$$

$$z = e^{Ts} = re^{i\theta}, \tag{3}$$

$$r = r(\zeta, \omega_n, T) = e^{-\zeta\omega_n T}, \tag{4}$$

$$B = B(\zeta, \omega_n, T) = \cos\theta = \cos\,(\omega_n T \sqrt{1-\zeta^2}) = \cos\omega T \tag{5}$$

where ζ, ω_n and T represent the damping ratio, natural frequency, and sampling period, respectively. The symbol d_i represents all system parameters other than k_o, k_1. To transform the characteristic equation from an nth order polynomial into an algebraic equation, z^j may be defined in terms of its real and imaginary parts, R_j and I_j:

$$z^j = R_j + i\, I_j \; . \tag{6}$$

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5. W. Haeussermann, "Stability Areas of Missile Control Systems," *Jet Propulsion*, July 1957, pp. 787-795.
6. S.M. Seltzer, "An Algorithm for Solving the Popov Criterion Applied to Sampled-Data Systems," *Proceedings of the Third Southeastern Symposium on System Theory*, 5-6 April 1971, pp. G5-0 through G5-7.

It readily follows from Equation (3) that the values of R_j and I_j are

$$R_j = r^j \cos j\Theta \tag{7R}$$

and

$$I_j = r^j \sin j\Theta \ . \tag{7I}$$

While equations (7R) and (7I) are satisfactory for determining the values of R_j and I_j, a set of recursive formulas can be derived that are particularly amenable to implementation on a desktop calculator or digital computer. They are shown below as Equation (8) and are derived in the appendix.

$$X_{j+1} = 2rBX_j - r^2X_{j-1} \quad , \tag{8}$$

where X_j may be either R_j or I_j. Only two values of X_j (i.e., two values each of R_j and I_j) are needed to obtain iteratively, one at a time, all other values of R_j, I_j. These are obtained from the definition of z in Equation (3). When j=0, the value of z^j is unity and, from Equation (6),

$$z^0 = R_0 + i\ I_0 \tag{9}$$

or

$$1 = R_0 \tag{10R}$$

$$0 = I_0 \ . \tag{10I}$$

When j=1, the value of z^j is, in Euler form,

$$\begin{aligned} z^1 = z &= r \cos \Theta + ir \sin \Theta \\ &= R_1 + i\ I_1 \ . \end{aligned} \tag{11}$$

Hence,

$$R_1 = rB \tag{12R}$$

and

$$I_1 = r\sqrt{1-B^2}\ . \tag{12I}$$

It will turn out to be useful in the sequel to observe from Equation (8) that the radical, $\sqrt{1-B^2}$, appears as a factor in each value of I_j. Solely for simplicity, it may be factored out by defining a new term, I_j' as

$$I_j' = I_j/\sqrt{1-B^2}\ . \tag{13}$$

It may be observed that R_j and I_j are functions of r and B or of ζ, ω_n, and T. If Equation (6) is substituted into Equation (1) and the real and imaginary parts of the resulting equation are separated, two simultaneous algebraic equations are obtained.

These two equations contain the adjustable parameters, or variables, k_0, k_1. Hence it may be observed that the two equations may be solved explicitly for k_0 and k_1 as functions of the other system parameters (d_j) and, in particular, as functions of the independent argument, $\omega_n T$. It is the latter observation that forms the basis of the parameter space method for determining stability regions and system dynamic response in terms of selected system parameters.

3. STABILITY DETERMINATION

The method involves the definition of stability boundaries on a multi-parameter (to include k_0, k_1) space. These boundaries are found from the pair of simultaneous algebraic equations that result from the following operations on the system characteristic equation written in terms of the complex variable, z. Equation (6) is substituted into characteristic Equation (1), and the resulting real and imaginary parts are separated and equated to zero. The two resulting algebraic equations may be solved for any two parameters (such as k_0 and k_1) within them.

There may be as many as four stability boundaries for any system described by characteristic Equation (1), although it is not necessary for all four to exist.

One stability boundary separates the stable complex conjugate pairs of roots from the unstable ones. It consists of a map of the unit circle from the complex z-plane onto a selected (such as k_0, k_1) parameter space. Initially one may consider the oft-occurring case where the coefficients γ_j of the powers of z in the characteristic equation are linear combinations of k_0, k_1, i.e.,

$$\gamma_j = a_j\ k_o + b_j\ k_1 + c_j\ , \tag{14}$$

where a_j, b_j, c_j represent all system parameters other than k_0, k_1. In this case the two simultaneous equations resulting from the real and imaginary, respectively, parts of the characteristic equation assume the form,

$$\text{Re}\ \{\text{C.E.}\} = A_1\ a + B_1\ b - C_1 = 0, \tag{15R}$$

$$\text{Im}\ \{\text{C.E.}\} = A_2\ a + B_2\ b - C_2 = 0, \tag{15I}$$

and may be solved for k_0 and k_1 readily by applying Cramer's Rule, yielding

$$k_o = \frac{B_1C_2 - B_2C_1}{J} \tag{16}$$

$$k_1 = \frac{A_2C_1 - A_1C_2}{J}, \tag{17}$$

$$J = A_1B_2 - A_2B_1 \tag{18}$$

$$A_1 = \sum_{j=0}^{n} a_jR_j,\quad B_1 = \sum_{j=0}^{n} b_jR_j,\quad C_1 = \sum_{j=0}^{n} c_jR_j,$$

$$A_2 = \sum_{j=0}^{n} a_jI_j,\quad B_2 = \sum_{j=0}^{n} b_jI_j,\quad C_2 = \sum_{j=0}^{n} c_jI_j, \tag{19}$$

where J represents the Jacobian associated with the pair of algebraic equations. To find the stability boundary in question, one merely sets ζ equal to zero in the definitions of R_j and I_j used in obtaining k_0 and k_1 in Equations (16) and (17). The result is a boundary in the k_0-k_1 parameter plane that is defined in terms of system parameters d_j [in the general case, see Equation (2)] or a_j, b_j, c_j in the special case of Equation (14) and the independent argument, $\omega_n T$. The independent argument is varied in value between zero and π (assuming that the system being designed possesses low-pass filter characteristics so that only the "primary strip" need be considered), thereby defining the boundary in terms of k_0 and k_1. If the coefficients a_j, b_j, and c_j contain exponential terms with T appearing in the exponents, then each exponent may be replaced by its power series and truncated according to the accuracy that is desired. If the two parameters k_0 and k_1 do not appear linearly as expressed in Equation (14), they may be solved for by substituting Equation (6) into the characteristic Equation (1) and separating the result into the real and imaginary parts. This still results in two simultaneous algebraic equations which may be solved for the two variables, k_0 and k_1, although not as handily as expressed in Equations (16) and (17).

The next two stability boundaries are those separating the stable real roots from the unstable ones. These comprise a mapping of the $z=1$ and $z=-1$ points from the z-plane onto the selected parameter space. These are found readily by substituting $z=1$ and $z=-1$, respectively, into the characteristic Equation (1). Each of the two resulting equations results in a definition of the two real root boundaries in the selected parameter space.

The fourth boundary is a mapping of the conditions that cause the two simultaneous algebraic equations (used to define the complex conjugate root boundary) to become dependent. In the case where k_0 and k_1 appear linearly as in Equation (14), this case is found by determining the conditions that cause the Jacobian of Equation (18) to become identically equal to zero.

For a linear sampled-data control system to be stable, it is necessary that all roots of the characteristic equation lie within the unit circle on the z-plane. If the system is low-pass in nature, the stable region is bounded by the semi-circle defined by the upper half of the unit circle (as discussed above) and the singularities associated with $z = \pm 1$ (as discussed above) and $J=0$ (as discussed above). The mapping of these boundaries onto the parameter space will bound the stable region, if one exists. That region is determined by applying a shading criterion or using a test point.[1] If the Jacobian is greater than zero, then the stable region (if it exists) lies to the left of the complex conjugate root boundary as $\omega_n T$ increases; the left side of the line is double cross-hatched to indicate a boundary associated with double, or complex conjugate roots. (If the Jacobian is less than zero, the stable region lies to the right). Single cross-hatching is used on the two contours associated with the real roots. The side of the contour on which to place the cross-hatching is determined by the requirement that cross-hatching be continuous, or on the same side, of the contours as the intersections corresponding to $z= +1$ and $z= -1$ are approached along either the complex root or real root stability boundary. In the unusual (but physically possible) singular case when $J=0$, the shading of the resulting boundary is determined in a similar manner, i.e., the side on which to shade the boundary is determined by the physical requirement that the number of stable roots must never become less than zero as one moves across a stability boundary. This $J=0$ case may arise for a particular frequency, ω, or for a particular combination of system parameters. The latter situation is included in the work concerning a spinning Skylab.[7]

4. SYSTEM DYNAMICS

Once the stable region, if it exists, has been determined in the selected parameter space, the dynamic or transient characteristics of the system can be specified in terms of the locations of the roots of the characteristic equation (pole placement). For the complex conjugate roots, these locations are defined in terms of damping ratio (ζ) and system natural frequency (ω_n). Contours of constant ζ are determined as functions of ω_n and T in precisely the same manner that the complex conjugate stability boundary was determined except that ζ is not set equal to zero.

The real root locations corresponding to values of z when θ equals 0° and 180° also may be plotted on the parameter plane by setting z equal to a positive or negative real constant, substituting that value into Equation (1), and solving for k_1 as a function of k_0. Each resulting contour corresponds to a location of a real root in the z-domain. When $z=+\alpha$

$$\sum_{j=0}^{n} \gamma_j \alpha^j = 0, \tag{20}$$

and when $z=-\alpha$,

$$\sum_{j=0}^{n/2} \gamma_{2j} \alpha^{2j} - \sum_{j=0}^{(n-2)/2} \gamma(2_j+1_j)\alpha^{(2j+1i)} = 0, \text{ n even} \tag{21}$$

$$\sum_{j=0}^{(n-1)/2} (\gamma_{2j}\alpha^{2j} - \gamma_{(2j+1)}\alpha^{(2j+1)} = 0, \text{ n odd} \tag{22}$$

where α is a positive real number. (The two associated stability boundaries may be found by setting α equal to unity). Values of α corresponding to desired real root locations may be substituted into Equations (20) through (22) to obtain parameter space contours corresponding to these locations.

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[7] S. M. Seltzer "Passive Stability of a Spinning Skylab." *Journal of Spacecraft and Rockets*, Vol. 9, No. 9, September 1972, pp. [illegible]

Now the dynamic effect of any chosen design point in the parameter space may be specified in terms of ζ, ω_n, α, and T. The analytical technique developed permits the designer to observe the effect of simultaneously changing three control parameters and the sampling period. Most existing conventional techniques permit the observation of the effect of changing only one control parameter and don't show the effect of various sampling periods. If the designer is clever, sometimes the system parameters may be manipulated within the equations defining the stability and dynamic contours so that more than two parameters (such as k_0, k_1) may be used to define a parameter space. An example of a three-parameter space is provided in the sequel.

Sometimes it is specified that the setting time of the system be less than a prescribed value. This corresponds to requiring that the real part of the roots of the characteristic equation be less than a prescribed negative real constant. A boundary corresponding to this requirement can be drawn on the parameter plane by mapping a circle of constant radius (for a chosen values of ω_n, ζ, and T) from the z-plane onto the parameter plane.[7] Relations exist for estimating the maximum overshoot and peak time of transient response when it is valid to assume a second order system.[8] However, a simple estimate sometimes can be made merely by looking at the difference equation representing the system response, estimating when the overshoot will occur, and plotting a corresponding line on the parameter plane. Steady state responses may be found from the open loop transfer function and the assumed forcing *functions in the conventional manner.*

5. EXAMPLE

Consider the planar model that portrays controlled rigid body rotational dynamics. It is shown in block diagram form in *Figure 1*. The plant to be controlled is represented by the transfer functions,

$$G_4(s) = \Phi(s)/T_C(s) = 1/J_V s \tag{23}$$

and

$$G_5(s) = \Phi(s)/\Phi(s) = 1/s \tag{24}$$

8. B.C. Kuo, *Analysis and Synthesis of Sampled-Data Control Systems*, Prentice-Hall, New Jersey, 1963.

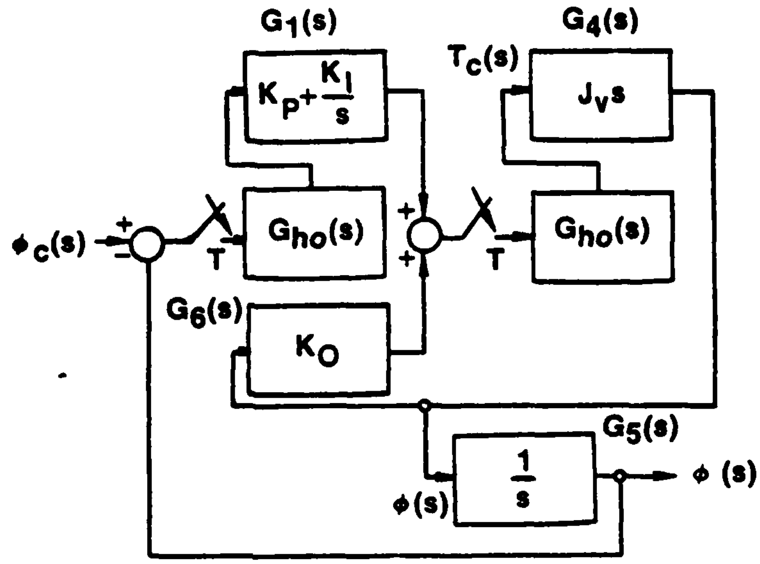


Figure 1. Block diagram of controlled rigid body motion.

where s represents the Laplace operator, ϕ represents the rigid body attitude [in the time domain, $\phi(t)$; in the Laplace domain, $\phi(s)$], T_c represents the commanded torque, and the overdot represents the derivative with respect to time. The two sensors that measure attitude (ϕ) and attitude rates ($\dot{\phi}$) are assumed to be perfect with unity transfer functions. The on-board digital controller develops the commanded control torque from the input states ϕ and $\dot{\phi}$ and the commanded attitude signal ϕ_c; $G_{ho}(s)$ represents a zero-order hold in the computer:

$$Gh_o(s) = (1-e^{-Ts})/s, \tag{25}$$

where T is the sample period of the on-board digital computer. The PID control algorithm is represented by $G_1(s)$ and $G_6(s)$

$$G_1(s) = K_P + K_I/s \tag{26}$$

where K_P and K_I are the position and integral feedback gains and

$$G_6(s) = K_D \tag{27}$$

where K_D is the derivative feedback gain (more commonly called the rate gain). The third order characteristic equation is Equation (1) with n=3, where the coefficients of z^j are in the form of Equation (14), i.e.,

$$\gamma_0 = (a - b + 2c - 1), \tag{28}$$

$$\gamma_1 = (-2a + 2c + 3), \tag{29}$$

$$\gamma_2 = (a + b - 3), \tag{30}$$

$$\gamma_3 = 1. \tag{31}$$

Modified gains a, b, c are defined as

$$a \equiv K_R T/J_v, \tag{32}$$

$$b \equiv K_p T^2/2J_v, \tag{33}$$

$$c \equiv K_I T^3/4J_v, \tag{34}$$

The stability boundaries are presented in terms of a, b, c, and $\omega_n T$ by observing that the roots (in this case three) of a characteristic equation representing a stable linear sampled-data control system must all lie within the unit circle in the z-plane. It is assumed that the system possesses low-pass filter characteristics so that only the primary strip (corresponding to $0 \leqslant \theta \leqslant \pi$) need be considered. The stable region may be defined by mapping its three boundaries from the z-plane onto the a, b, c parameter space by first considering parameters a, b to be k_0, k_1 in Equation (14). The boundary at $z = +1$ is found by substituting that value for z into the C.E., yielding the stability boundary

$$c = 0 \tag{35}$$

The boundary at $z = -1$ is found by substituting that value for z into the C.E., yielding

$$a = 2. \tag{36}$$

The complex conjugate root stability boundary may be found by setting z equal to $\cos \omega_n T + i \sin \omega_n T$ in the C.E. However, at this point it is convenient to use the recursive formulas of Equation (8) and transform the C.E. into two algebraic equations by separating the real and imaginary parts. If they are solved for a and b and the associated Jacobian J, one obtains as the complex conjugate root stability boundary,

$$a = [(1 + B)/(1 - B)]\, c + (1 - B), \tag{37}$$

$$b = c + (1 - B), \tag{38}$$

and

$$J = -4\,(1 - B) < 0 \;\forall\; 0 < \theta < \pi. \tag{39}$$

Examination of Equation (39) reveals that the stability boundary associated with the singular case $J=0$ only occurs when $B=1$, which is the already considered $z=1$ case. Now the stability boundaries of Equations (35), (36), (37) and (38) can be plotted on a three-dimensional plot with axes a, b, c. A sketch of these boundaries is shown in *Figure 2.* The stability region is found by applying the "cross-hatching" rules or by using one or more test points (known to be stable, to lie on a stability boundary, or to be unstable). If $J>0$ the stable region (if it exists) lies to the left of the complex conjugate root stability boundary of Equations (37) and (38), as θ (or $\omega_n T$) increases; the left side of the line is double cross-hatched to indicate a boundary

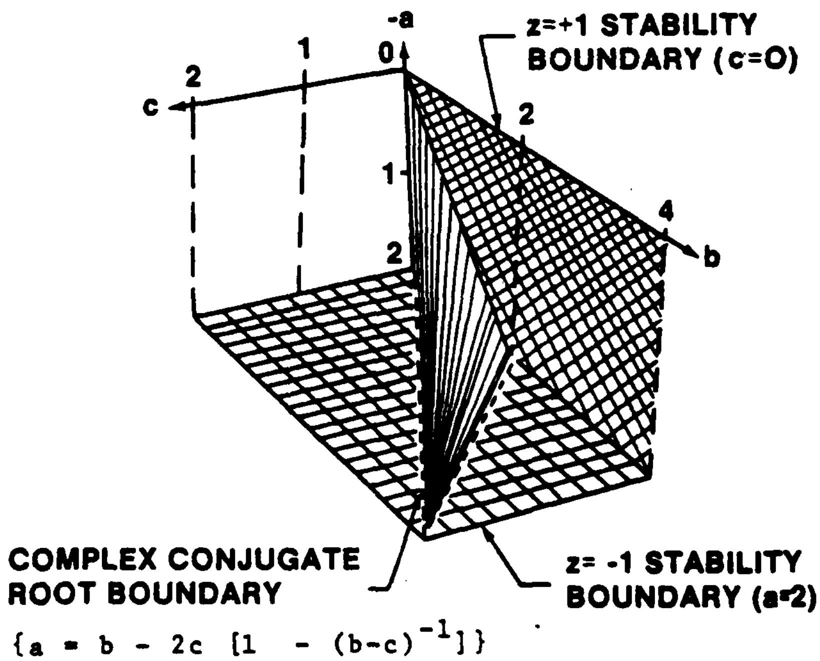


Figure 2. Stability boundaries in 3-dimensions.

associated with double, or complex conjugate, roots. If $J<0$, as in this case, the stable region lies to the right and the right side of the line is double cross-hatched. Single cross-hatching is used on the contours of Equations (35) and (36) associated with single root boundaries. The side of the contour on which to place the cross-hatching is determined by the requirement that cross-hatching be continuous, or on the same side, of the contours as the intersections corresponding to $z=+1$ ($\theta=0$) and $z=-1$ ($\theta=\pi$) are approached along either the complex root or real root stability boundary. Using this criterion, the stable region is found to be bounded by the $c=0$ plane, the $a=2$ plane, and the curve specified by Equations (37) and (38). The latter may be solved for a as a function of b, resulting in the simple expression

$$a=b-2c\left[1-(b-c)^{-1}\right]. \tag{40}$$

The limiting values for a and b may be found by letting θ approach 0 and π, i.e., $z=\pm1$. For the case of $\theta\rightarrow0$, represent B by the series expansion

$$B=\cos\Theta=1-(\Theta^2/2!)+(\Theta^4/4!)-(\Theta^6/6!)+\ldots \tag{41}$$

Then using Equations (37), (38), and (41), one obtains

$$\lim_{\Theta\rightarrow0} b=c \tag{42}$$

$$\lim_{\Theta\rightarrow0} a = \lim_{\Theta\rightarrow0} c\left[(4/\Theta^2)-(2/3)+(\Theta^2/36)-\ldots\right]\rightarrow\infty \tag{43}$$

For $c=0$, Equation (42) still holds (recognize $k_I=0$ is really the case one is interested in). Then, using Equations (34) and (37), one obtains

$$\lim_{\Theta\rightarrow0} a = \lim_{\Theta\rightarrow0} \frac{(k_I\Theta^3/4J_v\omega^3)\left[2-(\Theta^2/2!)+(\Theta^4/4!)-\cdots\right]}{(\Theta^2/2!)-(\Theta^4/4!)+\cdots}$$
$$= \lim (k_I\Theta/2J_v\omega^3)\left[2-(2\Theta^2/6)+(\Theta^4/72)-\cdots\right]=0 \tag{44}$$

For the limiting case of $\theta \rightarrow \pi$:

$$\lim_{\Theta \to \pi} a = \lim_{\Theta \to \pi} \left\{ [(1+B)c/(1-B)]+(j-B) \right\} = 2 \tag{45}$$

and

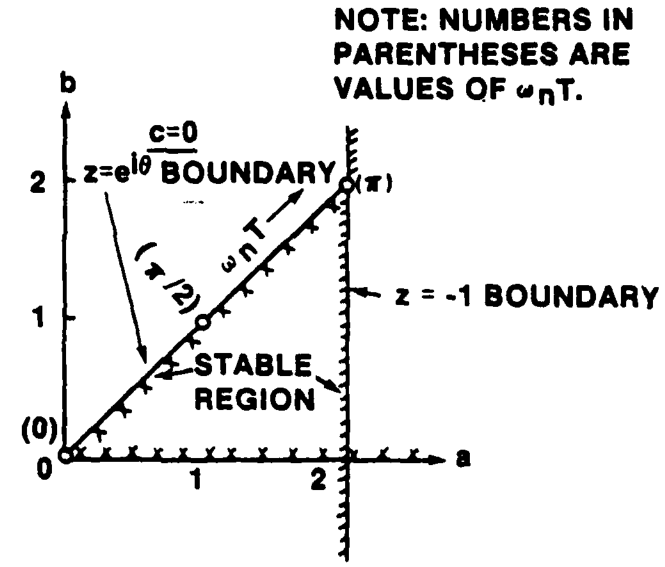
$$\lim_{\Theta \to \pi} b = \lim_{\Theta \to \pi} (c+1-B) = c+2 \tag{46}$$

The results of the foregoing are sketched graphically in two dimensions on *Figure 3*, for the cases where $c = 0$ and when $c \neq 0$.

The design technique employs the pole placement approach. It is based on the premise that the dynamic behavior of the system is related closely to the location of the roots of its associated characteristic equation. The method shows both analytically and graphically the direct correlation between these roots and the control gains and sample period of the controller. The design technique then involves the specification of the characteristic equation root locations and the subsequent determination of the control system gains and sample period needed to attain these locations. The control system designer then must determine the system response resulting from using these numerical values and assess its adequacy. If it is not adequate, he usually relies on his experience to relocate the roots to improve the response in the manner desired for his particular system (i.e., faster settling time, lower peak overshoot, etc.). It is assumed that one wishes the pair of complex conjugate poles of the C.E. to dominate the dynamic response of the system. This response will then be modified by locating the third (real) root.

First consider the pair of complex conjugate roots. One may use Equations (16) - (19), substituting a, b for k_0, k_1, respectively [also using Equations (28) through (31)]. Upon examining the nature of the equations for a, b, one sees that they are functions of r, B, and c, or (finally) functions of ζ, θ, c. For each value of ζ, b versus a may be determined analytically and plotted graphically as a function of θ for a given value of c. This may be repeated for a number of values of c. Now the value of θ may be selected (for selected values of c) that best meets whatever design criteria one chooses. In effect, selecting θ and c will also choose the real root location. This may be shown by designating the real root of the C.E. as z_R, where

$$z_R \equiv \delta = e^{-\sigma_R T} \tag{47}$$



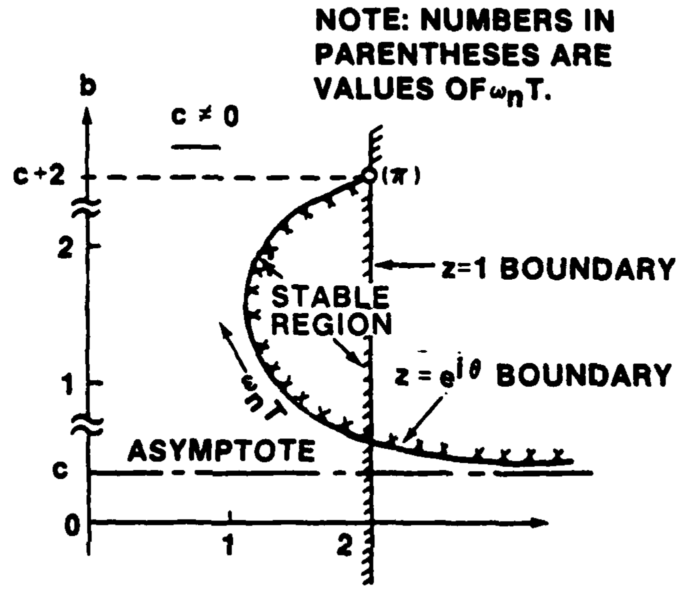


Figure 3. Stability boundaries in 2-dimensions.

and σ_R represents the damping factor. One may substitute Equation (47) into C.E. (1) and solve for b, obtaining

$$b = [(1-\delta)/1+\delta)]a+[2/(1-\delta)\ c- (1-\delta)^2/(1+\delta)] \tag{48}$$

One sees that Equation (48) yields a straight line contour of b vs. a for a given value of c and a given value of δ. Thus, for a given value of c, where the δ-contour of Equation (48) crosses the ζ-contour of Equations (37) and (38), one obtains a value of θ for the given value of c and those specified values of ζ and δ. From Equation (1), c may be determined as a function of the characteristic equation root locations:

$$c = (\alpha^2 + \beta^2 - 2\alpha + 1)(1 - \delta)/4, \tag{49}$$

where α and β are the real and imaginary parts, respectively, of the pair of complex conjugate roots of characteristic Equation (1). For a specified location (α,β) of the pair of complex conjugate roots, a value for c is established by setting a value for δ (location of the real root).

An additional constraint is imposed by Shannon's Sampling Theorem:

$$\omega < \omega_s/2, \tag{50}$$

where ω is defined in Equation (5) and $\omega_s/2\pi$ is the sampling frequency,

$$\omega_s = 2\pi/T. \tag{51}$$

From Equations (50) and (51), this constraint may be restated as

$$\Theta < \pi\ . \tag{52}$$

The stable region portrayed in *Figures 2* and *3* may be shown for several selected values of c on *Figure 4* using Equations (16), (17) and (48). As c increases in value, the stable region shrinks in size and the ζ contours lie further to the right. When c=0.6, the $\zeta=1/\sqrt{2}$ contour lies entirely to the right (and outside) of the stable region. Finally when $c\geqslant2$, the stable region disappears entirely (for a geometric explanation, see *Figure 2*).

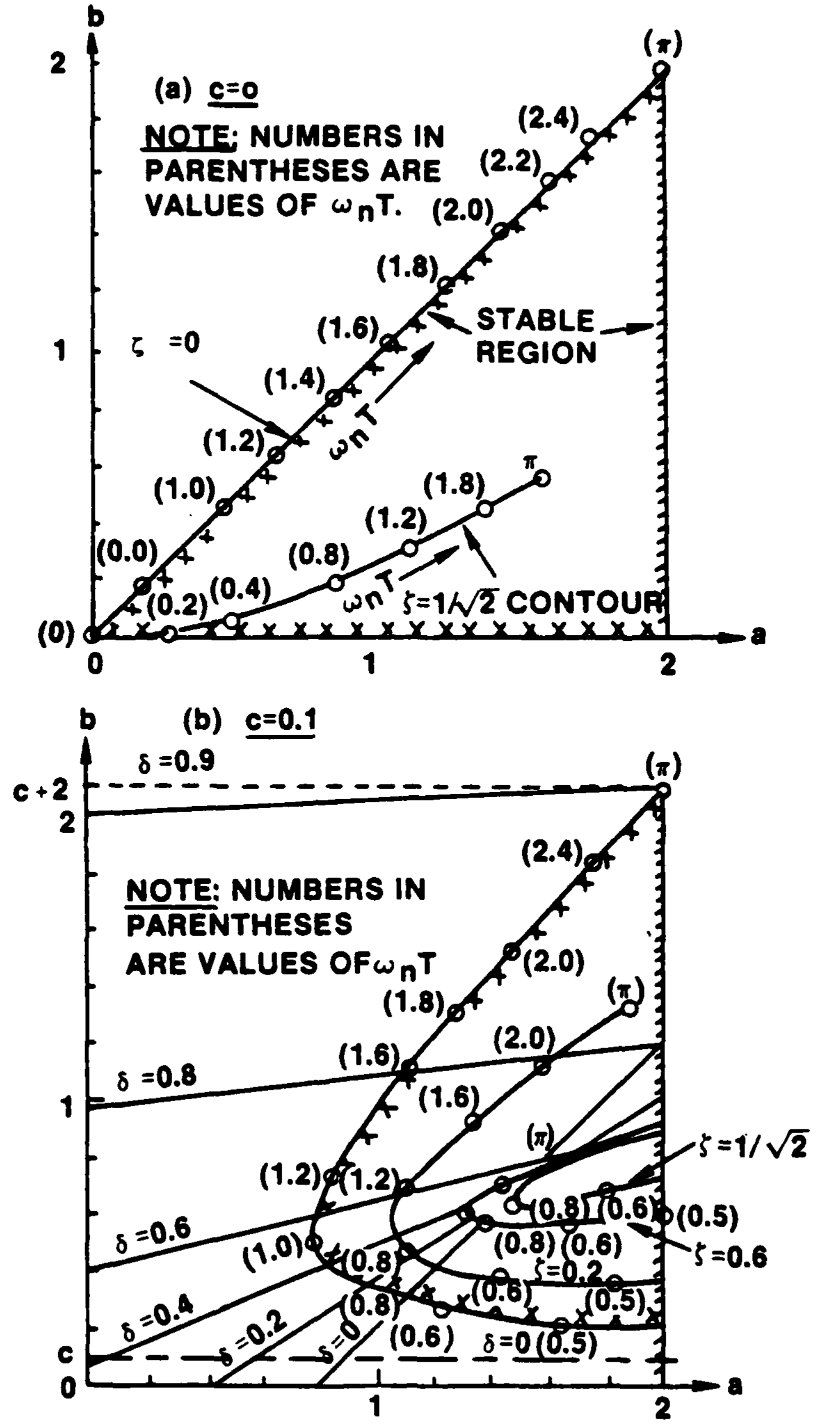


Figure 4. Parameter plane plots portraying root locations (c=0, 0.1, 0.2).

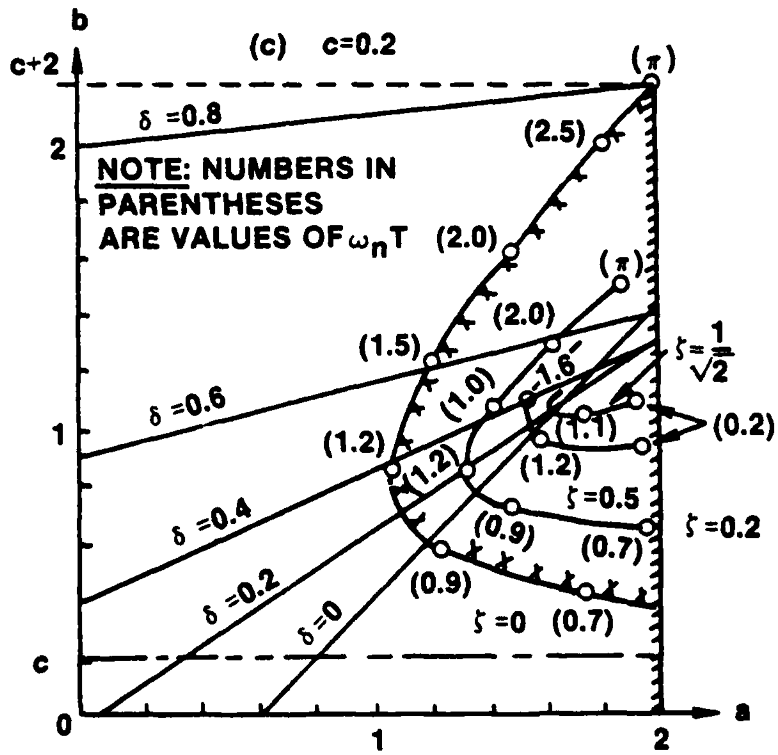


Figure 4. (Concluded).

The application of the foregoing technique will now be summarized.

Based on desired system response characteristics, tentatively select locations for the three roots of the characteristic equation. Although the zeros of the system characteristic equation also affect the dynamics, placement of the poles will dominantly affect the dynamics.

- A root location criteria might be to select numerical values for ζ and ω_n.

- Determine computer design constraints. One might be to select as large a value of T as possible to minimize on-board memory capacity requirements.

- Then qualitatively determine how much integral gain is desired for the types of disturbance inputs expected. This gives an indication of the value of c to use.

Using the value of c tentatively established in the above paragraph, plot the stability boundaries in the b vs a parameter plane, using Equations (36) through (38). Then plot the desired ζ-contour (as a function of argument $\omega_n T$) on the b vs. a parameter plane, using Equations (16) and (17).

Using various values of δ, plot δ-contours on the same plot, using Equation (48).

Find an intersection of a δ-contour which gives a desired value for $\omega_n T$ (and hence T, since ω_n has already been specified) and δ. In general, if $\delta \to 0$, its effects on the system dynamics is small compared to the effect of the pair of complex conjugate poles (placed by $\omega_n T$, ζ).

Check the response of the system using the values of a, b, c, and $\omega_n T$ (and hence K_p, K_I, K_D, and T) associated with the selected intersection. If it is unsatisfactory, reiterate the above procedure, selecting another value of c.

If the design procedure were merely to specify the three root locations (such as in terms of α, β, δ, $\omega_n T$, δ), iteration would not be required. Unfortunately, in design practice one must exercise engineering judgement in selecting the three root locations, giving rise to iterative design procedures such as the one outlined above. However, another possibility does exist. If the real root is placed near the origin of the z-plane ($\delta \to 0$), and if the pair of complex conjugate roots are located near the unit circle in the z-plane (i.e., specifying ζ, and $\omega_n T$), the latter two roots will dominate the system response. Then the already developed tools for specifying the system response (such as determining explicitly the maximum overshoot and the settling time) of second-order systems may be applied handily. The foregoing procedure may be amplified through the application of realistic numerical values for the system parameters.

Assume a digital on-board controller, and further assume that it is desired to have a controller natural frequency (ω_n) of six rad/s, and that the moment of inertia (J_v) about the single axis is 4.3×10^4 kg.m^2. Assume that integral control is desired to drive to zero the effect of constant input disturbances. This means a non-zero value of c is desired. If a value of ζ of $1/\sqrt{2}$ is selected, and if the value of δ is desired to be kept as close to zero as possible, it is implied from *Figure 4c* that, for $\delta > 0$, no intersections of the ζ-contour occur when $c > 0.2$. Hence, a value of $c = 0.1$ is chosen. The smallest value of δ whose corresponding δ-line intersects the ζ-contour, with a reasonable factor of safety, is $\delta = 0.4$. The value of $\omega_n T = 1.2$ is on the ζ-contour at the intersection, yielding a value of 0.20s for T. Corresponding values of a and b are 1.41 and 0.68, respectively. This gives a set of gain values of 1.14×10^7 N·m/rad, 1.67 $\times 10^8$ N·m/rad·s, and 2.35×10^6 N·m·s/rad for K_P, K_I, and K_D, respectively. If response studies show that a larger value of integral gain is deemed necessary, a larger value of c (and consequently a smaller value of ζ) would be selected.

6. CONCLUSIONS

An analytical method for portraying stability regions in a selected parameter space has been shown for a digital system. The method requires that the system characteristic equation be available and expressed in the complex z-domain. It also is possible to apply pole placement to obtain desired dynamic characteristics using this modified parameter space technique. The advantage of the technique over existing classical sampled-data methods is that the stability and dynamic response characteristics are expressed in terms of several (rather than merely one) selected parameters. Also, the sampling period, T, need not be expressed numerically before the design technique begins, giving the system designer one more degree of freedom.

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APPENDIX
DERIVATION OF RECURSIVE RELATIONS

It is possible to obtain algebraic recursive relations for the real and imaginary parts of the complex variable, z, when it is raised to the j^{th} power (j is a possible integer). From Equation (3), z is defined as the vector (or complex variable),

$$z = e^{Ts} = r\, e^{i\theta} = r \cos\theta + i\, r \sin\theta. \tag{A1}$$

Using Equation (5) in Equation (A1), i.e., letting $B = \cos\theta$, leads to

$$z = rB + i\, r\sqrt{1-B^2}. \tag{A2}$$

The real and imaginary parts of z^j may be defined as R_j and I_j, respectively, i.e.,

$$z^j = R_j + i\, I_j\ , \tag{A3}$$

If j=1, Equation (A3) becomes identical with Equations (A1) and (A2), leading to

$$R_1 = rB \tag{A4R}$$

and

$$I_1 = r\sqrt{1-B^2}\ . \tag{A4I}$$

If j=0, then Equation (A3) becomes

$$z^0 = 1 = R_0 + i\, I_0\ , \tag{A5}$$

or

$$R_0 = 1 \tag{A6R}$$

and

$$I_0 = 0. \tag{A6I}$$

If one squares the left and right hand sides of Equation (A1), one may obtain

$$
\begin{aligned}
z^2 &= r^2\, e^{i2\theta} = r^2\,(\cos 2\theta + i \sin 2\theta) \\
&= r^2\,(2\cos^2\theta - 1) + i2\, r^2 \sin\theta\cos\theta \\
&= 2\, r^2 \cos\theta\,(\cos\theta + i\sin\theta) - r^2 \\
&= 2rBz - r^2. \qquad \text{(A7)}
\end{aligned}
$$

One may now multiply each term of Equation (A7) by z^{k-1} to yield

$$z^{k+1} = 2rBz^k - r^2 z^{k-1} \,. \qquad \text{(A8)}$$

Substitution of Equation (A3) into Equation (A8), letting j = k-1, k, and k+1, leads to

$$R_{k+1} + i\, I_{k+1} = 2rB(R_k + i\, I_k) - r^2(R_{k-1} + iI_{k-1}) \,. \qquad \text{(A9)}$$

Separately equating the real and imaginary parts, respectively, of Equation (A9) leads to the two equations,

$$R_{k+1} = 2rBR_k - r^2 R_{k-1} \qquad \text{(A10R)}$$

and

$$I_{k+1} = 2rBI_k - r^2\, I_{k-1} \,. \qquad \text{(A10I)}$$

One may observe the identical forms of Equations (A10) and write them both as a single equation,

$$X_{k+1} = 2rBX_k - r^2 X_{k-1} \,, \qquad \text{(A11)}$$

where X_K may be used to represent either R_k or I_k. Of course the dummy index k may be exchanged with j, yielding Equation (8) of the text. If one knows the two values X_k and X_{k-1} of Equation (A11), one may determine the third value X_{k+1}. Since two sets of values of X_k are known, i.e., for k = 0 and k = 1 [see Equations (A4) and (A6)], as many values of X_k as are needed for the particular problem at hand can be determined recursively from Equation (A11).

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APPENDIX F

```
10   REM  ***************************************************************
20   REM  *                                                             *
30   REM  *        THIS PROGRAM IS STORED UNDER MICPP                   *
40   REM  *                                                             *
50   REM  *        THE PROGRAM CALCULATES THE VALUES OF P1 AND P2 IN THE *
60   REM  *        PARAMETER PLANE METHOD.  THE STABILITY BOUDARIES ARE *
70   REM  *        CALCULATED IF ZETA = 0.  IN ADDITION THE PROGRAM WILL *
80   REM  *        CALCULATE THE VALUES OF P1 AND P2 FOR A LINEAR SET OF *
90   REM  *        EQUATIONS.                                           *
100  REM  *                                                             *
110  REM  *                                                             *
120  REM  *                                                             *
130  REM  *                                                             *
140  REM  *                                                             *
150  REM  *                                                             *
160  REM  *                                                             *
170  REM  ***************************************************************
180  OPTION BASE 0
190  DEFAULT ON
200  FLOAT 4
210  DIM C(20),S(20),A(20),B(20),F(20),Dd(20),Mm(10,3),Eta(20),The(20)
220  DIM Tem1(20),Tem2(20),Tem3(20),Tem(20),Rr(20),Ir(20),R(20),Dl(20)
230  DIM Zb(20),Wnb(20),Phi(20),Alpha(20)
240  !
250  !
260  !    THE USER IS GIVEN THE OPTION OF HAVING DATA PLOTTED AND/OR
270  !    HAVING THE RESULTS PRINTED. THE PRINTER IS DESIGNATED BY EITHER
280  !    "7,6" FOR THE THERMAL PRINTER OR "16" FOR THE CRT"
290  !
300  !
310  DISP " This is the MICPP program. Please press CONTINUE"
320  PAUSE
330  LINPUT "IS DATA TO BE PLOTTED? Y OR N ?",Qp$
340  Ltyp=-1                                     !  SETS COUNTER FOR PLOTTER
350  IF (Qp$="Y") OR (Qp$="y") THEN GOSUB 2790
360  LINPUT "DO YOU WISH A HARD COPY OF THE PROGRAM RESULTS? Y/N",Test$
370  PRINTER IS 7,6
380  IF (Test$="N") OR (Test$="n") THEN PRINTER IS 16
390  PRINT LIN(3)
400  LINPUT "S,Z, OR W TRANSFORM?",Tl$
410  !    THE USER HERE HAS THE OPTION OF SELECTING EITHER Z ,W OR S TRANSFORM
420  !
430  !
440  IF Tl$="S" THEN PRINT "S TRANSFORM METHOD"
450  IF Tl$="Z" THEN PRINT "Z TRANSFORM METHOD"
460  IF Tl$="Z" THEN 530
470  IF Tl$="S" THEN 540
480  BEEP
490  DISP " ONLY Z OR S ALLOWED,  ";Tl$;", WAS INPUT, PRESS CONTINUE TO GO
 ON"
500  PAUSE
510  GOTO 400
520  !
530  !
540  INPUT "WHAT IS THE ORDER OF THE CHARACTERISTIC EQUATION?",N
550  PRINT USING 560;N
560  IMAGE "THE SYSTEM ORDER IS =",DD
570  PRINT LIN(1)
```

```
580  !
590  !
600  !
610  !
620  !
630  REDIM C(N),S(N),A(N),B(N),F(N),Dd(N),Rr(N),Ir(N)
640  INPUT "INPUT ELEMENTS OF A MATRIX, A(0),A(1),...",A(*)
650  PRINT "ELEMENTS OF A MATRIX  A(0),A(1),A(2),...",A(*);

660  IF (Q1p$="Y") OR (Q1p$="y") THEN 680
670  INPUT "INPUT ELEMENTS OF B MATRIX, B(0),B(1),...",B(*)
680  PRINT "ELEMENTS OF B MATRIX  B(0),B(1),B(2),...",B(*);
690  IF (Q1p$="Y") OR (Q1p$="y") THEN 710
700  INPUT "INPUT ELEMENTS OF F MATRIX, F(0),F(1),...",F(*)
710  PRINT "ELEMENTS OF F MATRIX  F(0),F(1),F(2),...",F(*);
720  ! ****************************************************************
730  !
740  !
750  ! ****************************************************************
760  !   THE VALUE OF ZETA (DAMPING RATIO) IS INPUT HERE FOR USE IN THE
770  !   PARAMETER SPACE METHOD.
780  !
790  !
800  INPUT "FOR THE PARAMETER PLANE CALCULATIONS, WHAT IS VALUE OF ZETA?",Zeta
810  IF Zeta>1 THEN GOTO 1460
820  PRINT LIN(4)
830  PRINT "ZETA =";Zeta
840  Ltyp=Ltyp+1
850  Pass=0             ! SET VALUES
860  IF Zeta>1 THEN GOTO 1460
870                                           ! IF ZETA IS EQUAL TO ZERO THE
880  IF (T1$="Z") AND (Zeta=0) THEN GOSUB 2620 ! PROGRAM CALCULATES THE
890                                           ! STABILITY BOUNDARIES FOR Z=+-1
900  !
910  !
920  ! INPUT THE VALUES OF INITIAL AND FINAL OMEGA TO BE USED.  IN ADDITION
930  ! IF THE NUMBER OF STEPS TO BE USED IS EQUAL TO ZERO, THEN THE PROGRAM
940  ! ASSUMES THAT THE USER WISHES TO SKIP THE SECTION ON THE PARAMETER-
950  ! SPACE AND SKIPS DIRECTLY TO THE OPTION TO EITHER CALCULATE THE ROOTS
960  !
970  !
980  INPUT "INPUT INITIAL OMEGA, MAXIMUM OMEGA, NO OF STEPS",Wint,Maxw,Nwnt
990  PRINT "INITIAL OMEGA =";Wint;TAB(35);"FINAL OMEGA =";Maxw
1000 IMAGE , "NUMBER OF STEPS=",MDDD,
1010 PRINT USING 1000;Nwnt
1020 IF Wint=0 THEN Wint=1E-25
1030 IF Nwnt<=0 THEN 1460
1040 !
1050 ! *****************************************************************
1060 !
1070 ! THE CALCULATIONS ASSOCIATED WITH THE PARAMETER-SAPCE DESIGN FOLLOW
1080 !
1090 !
1100 !
```

```
1110 IF (T1$="S") OR (T1$="s") THEN Theta=PI-ASN(SQR(1-Zeta^2))
1120 FOR I=0 TO Nwnt
1130 Wnt=Wint+I*(Maxw-Wint)/Nwnt
1140 GOSUB 2250
1150 GOSUB 2110
1160 IF X=0 THEN GOTO 1200
1170 IF T1$="Z" THEN PRINT "WnT=";Wnt;TAB(20);"DET=";X;TAB(40);"P1=";P(0);TAB(60
);"P2=";P(1)
1180 IF T1$="S" THEN PRINT "Wn=";Wnt;TAB(20);"DET=";X;TAB(40);"P1=";P(0);TAB(60)
;"P2=";P(1)
1190 GOTO 1280
1200 IF T1$="S" THEN PRINT "Wn =";Wnt
1210 IF T1$="Z" THEN PRINT "WnT =";Wnt
1220 PRINT "THE DETERMINANT OF THE COEFFICIENT MATRIX IS EQUAL TO ZERO"
1230 PRINT "MATRIX AB ="                 ! IF SO THEN SKIP THE NORMAL ROUTINE
1240 MAT PRINT Ab
1250 PRINT
1260 PRINT "MATRIX Fp ="
1270 MAT PRINT Fp
1280 IF (Qp$="Y") OR (Qp$="y") THEN GOSUB 2820
1290 NEXT I
1300 !
1310 ! ******************************************************************
1320 !
1330 ! A SERIES OF CONTROL STATEMENTS FOLLOW THAT GIVE THE USER VARIOUS OPTIONS
1340 ! TO CHANGE CERTAIN VARIBLES.
1350 !
1360 !
1370 IF (Qp$="Y") OR (Qp$="y") THEN GOSUB 2930
1380 LINPUT "CHANGE OMEGA RANGE?,ZETA?,OR F MATRIX? THEN INPUT W,Z,F OR NO",Q1$
1390 IF (Q1$="NO") OR (Q1$="N") OR (Q1$="no") OR (Q1$="n") THEN 1460
1400 IF Q1$="W" THEN 980
1410 IF Q1$="Z" THEN 800
1420 IF (Q1$="F") AND (Q1p$="Y") THEN 1380
1430 IF Q1$="F" THEN 700
1440 IF (Q1$<>"W") OR (Q1$<>"Z") OR (Q1$<>"F") THEN BEEP
1450 IF (Q1$<>"W") OR (Q1$<>"Z") OR (Q1$<>"F") THEN 1380
1460 LINPUT "DO WISH TO CALCULATE THE CONTOURS FOR REAL ROOTS?",Rr$
1470 IF Rr$="Y" THEN GOTO 3180
1480 !
1490 !
1500 ! THE USER IS GIVEN THE OPTION OF CALCULATING THE COEFFICIENTS OF THE
1510 ! CHARACTERISTIC EQUATION.
1520 !
1530 !
1540 LINPUT "CALCULATE COEFFICIENTS OF THE CHAR. EQU. OR ROOTS? Y/N?",T$
1550 IF (T$="N") OR (T$="n") THEN 1950
1560 INPUT "INPUT P1,P2",Cp1,Cp2       ! P1 IS ASSOCIATED WITH A AND P2 WITH B
1570 MAT Tem1=A*(Cp1)
1580 MAT Tem2=B*(Cp2)
1590 MAT Tem3=Tem1+Tem2
1600 MAT Tem=Tem3+F
1610 PRINT "COEFFICIENTS OF THE CHARACTERISTIC EQUATION WITH"
1620 PRINT "P1=";Cp1;TAB(30);"P2=";Cp2
1630 PRINT
```

```
1640 PRINT "C(0) + C(1)*S + C(2)*S^2 + C(3)*3^3 + ..."
1650 FOR I=0 TO N
1660 PRINT USING 1670;I,Tem(I)
1670 IMAGE "C(",DD,")=",MZ.6DE
1680 Tem1(I)=0
1690 NEXT I
1700 DISP "WAIT FOR SILJAK"
1710 !
1720 ! Call statement for SILJAK
1730 !
1740 CALL Siljak(N,Tem(*),Tem1(*),.0000001,.0000001,50,Rr(*),Ir(*))
1750 IF TI$="S" THEN PRINT LIN(2),"ROOTS:",LIN(1),SPA(8),"REAL","   IMAGINARY","
DAMPING RATIO","      Wn",LIN(2)
1760 IF TI$="Z" THEN PRINT LIN(2),"ROOTS:",LIN(1),SPA(8),"REAL","   IMAGINARY","
DAMPING RATIO","      WnT",LIN(2)
1770 FOR I=1 TO N
1780 IF ABS(Ir(I))<=1E-11 THEN Ir(I)=0
1790 Rtem=(Rr(I)^2+Ir(I)^2)^.5
1800 Stem=Rr(I)/Rtem
1810 IF Ir(I)=0 THEN Stem=1
1820 IF TI$="S" THEN GOTO 1880
1830 Sgmt=LOG(Rtem)
1840 Omgt=ACS(Stem)
1850 Omegant=SQR(Sgmt^2+Omgt^2)
1860 Zetant=-Sgmt/Omegant
1870 GOTO 1900
1880 Omegant=Rtem
1890 Zetant=-Stem
1900    PRINT USING 1930;Rr(I),Ir(I),Zetant,Omegant
1910 NEXT I
1920 PRINT LIN(5)
1930 IMAGE 9X,MZ.6DE,5X,MZ.6DE,6X,MZ.6DE,6X,MZ.6DE
1940 GOTO 1380
1950 IF (Qp$="Y") OR (Qp$="y") THEN GOSUB 3000
1960 GOTO 200
1970 !
1980 !
1990 !
2000 !
2010 !
2020 !
2030 !
2040 !
2050 !
2060 !
2070 !
2080 !
2090 ! CALCULATE A ,B AND F MATRICES AT EACH OMEGA
2100 !
2110   Ab(0,0)=DOT(A,C)
2120   Ab(0,1)=DOT(B,C)
2130   Ab(1,0)=DOT(A,S)
2140   Ab(1,1)=DOT(B,S)
2150   Fp(0)=-DOT(F,C)
2160   Fp(1)=-DOT(F,S)
2170   DIM D(1,1),Ab(1,1),P(1),Fp(1)
2180   MAT D=INV(Ab)
2190   MAT P=D*Fp
2200   X=DET
2210   RETURN
```

```
2220    ! ____________________________________________
2230    !
2240    !
2250      FOR K=0 TO N
2260      IF (T1$="Z") OR (T1$="z") THEN 2300
2270      Rk=Wnt^K
2280      Argk=Theta*K
2290      GOTO 2320
2300      Rk=EXP(-Zeta*Wnt*K)                    ! THIS SUBPROGRAM WILL SET THE
2310      Argk=K*Wnt*(1-Zeta^2)^.5               ! VALUES OF THE C AND S MATRICES
2320      C(K)=Rk*COS(Argk)                      ! FOR K = 1 TO N, GIVEN VALUES OF
2330      S(K)=Rk*SIN(Argk)                      ! ZETA AND WN*T
2340      NEXT K
2350      RETURN
2360       !
2370       !
2380       !
2390       ! ---------------------------------------------
2400       !
2410       !
2420 Elm:    ! THE SUBROUTINE TO CALCULATE CONSTANT ZETA CONTOURS
2430         !
2440         !
2450           FOR I=0 TO N
2460           Dd(I)=Dk^I
2470           NEXT I
2480        E=DOT(A,Dd)
2490        L=DOT(B,Dd)
2500        M=DOT(F,Dd)
2510        IF L<>0 THEN PRINT "P2=";-E/L;"*P1 + ";-M/L
2520        IF (L=0) AND (E<>0) THEN PRINT "P1 =";-M/E
2530        IF (L=0) AND (E=0) THEN PRINT "COEFFICIENTS OF P1 & P2 EQUAL TO ZERO"
2540        PRINT
2550      PRINT "THE AXES INTERCEPTS ARE AS FOLLOWS:"
2560      IF L<>0 THEN PRINT "AT P1=0 , P2=";-M/L
2570      IF E<>0 THEN PRINT "AND AT P2=0 , P1=";-M/E
2580      IF L=0 THEN PRINT "P1 = CONSTANT =";-M/E
2590      IF E=0 THEN PRINT "P2 = CONSTANT =";-M/L
2600      PRINT LIN(2)
2610        RETURN
2620           PRINT "THE STABILITY BOUNDARIES FOR Z=+1 AND -1 ARE AS FOLLOWS",LI
N(1)
2630           Dk=-1
2640           PRINT "FOR Z=-1"
2650           GOSUB Elm
2660           PRINT LIN(2)
2670           Dk=1
2680           PRINT "FOR Z=+1"
2690           GOSUB Elm
2700           PRINT LIN(2)
2710           RETURN
2720           !
2730           !
2740           !
2750           ! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
2760           ! THE FOLLOWING SECTIONS BETWEEN THE "!!!!!!" ARE FOR PLOTTING !
2770           !                                                              !
2780           ! !                                                            !
```

```
2790     ASSIGN #1 TO "PDATA"
2800     READ #1,4
2810     RETURN
2820            IF Rr$<>"Y" THEN PRINT #1;P(0),P(1),0,Zeta,END
2830            IF Rr$="Y" THEN PRINT #1;P(0),P(1),0,1+Dk,END
2840                 IF Rr$="Y" THEN GOTO 2920
2850                 IF Pass=0 THEN Mix=Max=P(0)
2860                 IF Pass=0 THEN Miy=May=P(1)
2870                 Pass=1
2880                 IF P(0)<=Mix THEN Mix=P(0)
2890                 IF P(0)>=Max THEN Max=P(0)
2900                 IF P(1)<=Miy THEN Miy=P(1)
2910                 IF P(1)>=May THEN May=P(1)
2920                 RETURN
2930         PRINT #1;P(0),P(1),1,Ltyp,END
2940         REDIM Mm(Ltyp,3)
2950         Mm(Ltyp,0)=Mix
2960         Mm(Ltyp,1)=Max
2970         Mm(Ltyp,2)=Miy
2980         Mm(Ltyp,3)=May
2990         RETURN
3000   PRINTER IS 16
3010   PRINT "The program is now over and the data is on the tape."
3020   PRINT
3030   PRINT "If you would like to plot the data"
3040   PRINT "then you must load the plot routine entitled PLOTER."
3050   IF (Test$="y") OR (Test$="Y") THEN PRINTER IS 7,6
3060                        REDIM Mm(Ltyp,3)
3070                        PRINT #1,1;Ltyp,Mm(*)
3080                        RETURN
3090     !
3100     !
3110     ! !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
3120     !
3130     !
3140  !
3150  ! THIS PART OF THE PROGRAM CALCULATES THE CONSTANT ZETA CONTOURS
3160  !
3170  !
3180  ! CALCULATE THE BOUNDARIES AT ZETA >1.0
3190  PRINT "FOR REAL ROOTS OF THE CHARACTERISTIC EQUATION",LIN(1)
3200     INPUT "WHAT IS THE VALUE OF DELTA?",Dk
3210     PRINT "DELTA =";Dk
3220     GOSUB Elm
3230     PRINT
3240     IF (Qp$="Y") OR (Qp$="y") THEN GOSUB 3320
3250     GOTO 1460
3260     !
3270     !
3280     ! Subroutine to prepare values for plotting of the constant
3290     ! zeta contours for real roots.
3300     !
3310     !
3320        IF L=0 THEN GOTO 3420
3330        INPUT "INPUT MINIMUM & MAXIMUM VALUES OF P1",P1mi,P1ma
3340        P(0)=P1mi
3350        P(1)=-(E*P(0)+M)/L
3360        GOSUB 2820
3370        P(0)=P1ma
```

```
3380        P(1)=-(E*P(0)+M)/L
3390        GOSUB 2820
3400        GOSUB 2930
3410        GOTO 3490
3420        INPUT "THE CONTOUR IS VERTICAL,THUS INPUT MAX,MIN VALUES OF P2",G,H
3430        P(0)=-M/E
3440        P(1)=G
3450        GOSUB 2820
3460        P(1)=H
3470        GOSUB 2820
3480        GOSUB 2930
3490        RETURN
3500  !
3510  !
3520  !
3530  END
3540  !
3550  !
3560  !
3570  !
3580  !
3590  !
3600  !
3610  !
3620  !
3630 SUB Siljak(N,Rcoef(*),Icoef(*),Tola,Tolf,Itmax,Rroot(*),Iroot(*))
3640 DIM Xsiljak(0:N),Ysiljak(0:N)
3650 !
3660 !
3670 !
3680 ! **********************************************
3690 ! *** POLYNOMIAL ROOTFINDER USING SILJAK'S METHOD.
3700 ! **********************************************
3710 !
3720 !
3730 ! *** BAD DATA CHECK.
3740 Baddta=(N<=0) OR (Tola<=0) OR (Tolf<=0) OR (Itmax<=0)
3750 IF Baddta=0 THEN 3890
3760 ! *** PRINT ERROR MESSAGE AND PAUSE.
3770 ! *** USER MAY CORRECT DATA AND CONTINUE.
3780 PRINT LIN(2),"ERROR IN SUBPROGRAM Siljak."
3790 PRINT "N=";N,"Tola=";Tola
3800 PRINT "Tolf=";Tolf,"Itmax=";Itmax,LIN(2)
3810 PAUSE
3820 GOTO 3740
3830 !
3840 !
3850 ! *** BEGIN SUBPROGRAM.
3860 ! *** INITIALIZE LOCAL VARIABLES.
3870 ! *** Rroot(*) AND Iroot(*) HOLD THE ROOT APPROXIMATIONS.
3880 ! *** INITIALIZE Rroot(*) AND Iroot(*) TO 9.999999E99.
3890 MAT Rroot=(9.999999E99)
3900 MAT Iroot=(9.999999E99)
3910 !
3920 !
3930 Nn=N
3940 ! *** SPECIAL CHECK FOR N=1.
3950 IF N=1 THEN 5060
3960 !
3970 !
```

```
3980 ! *** Xsiljak(*) AND Ysiljak(*) HOLD THE SILJAK COEFFICIENTS.
3990 Y=Ysiljak(1)=Xsiljak(0)=1
4000 X=Xsiljak(1)=.1
4010 Ysiljak(0)=L=0
4020 !
4030 !
4040 ! *** BRANCH TO COMPUTATION OF SILJAK COEFFICIENTS.
4050 GOSUB Siljak
4060 ! *** G GETS FORMER VALUE OF F, INITIALIZE AND INCREMENT
4070 ! *** ITERATION COUNTER.
4080 G=F
4090 M=Q=P=0
4100 L=L+1
4110 ! *** Z=(DU/DX)^2+(DV/DX)^2.
4120 FOR K=1 TO N
4130    P=P+K*(Rcoef(K)*Xsiljak(K-1)-Icoef(K)*Ysiljak(K-1))
4140    Q=Q+K*(Rcoef(K)*Ysiljak(K-1)+Icoef(K)*Xsiljak(K-1))
4150 NEXT K
4160 Z=P*P+Q*Q
4170 ! *** Deltax AND Deltay ARE THE RESPECTIVE CHANGES IN X AND CHANGES IN Y.
4180 Deltax=-(U*P+V*Q)/Z
4190 Deltay=(U*Q-V*P)/Z
4200 ! *** INCREMENT SUCCESSIVE QUARTERING COUNTER.
4210 M=M+1
4220 !
4230 !
4240 ! *** NEW ROOT APPROXIMATIONS X,Y LOADED INTO Xsiljak(1) AND Ysiljak(1).
4250 Xsiljak(1)=X+Deltax
4260 Ysiljak(1)=Y+Deltay
4270 ! *** RECOMPUTE SILJAK COEFFICIENTS.
4280 GOSUB Siljak
4290 !
4300 !
4310 ! *** IF NEW ERROR ESTIMATE GREATER THAN OLD, QUARTER THE SIZE
4320 ! *** OF Deltax AND Deltay.
4330 IF F>=G THEN 4530
4340 ! *** CHECK IF INCREMENTS ARE SMALL ENOUGH TO SATISFY THE
4350 ! *** STOPPING CONDITIONS.
4360 IF (ABS(Deltax)<Tola) AND (ABS(Deltay)<Tola) THEN 4790
4370 ! *** CHECK IF MAXIMUM NUMBER OF ITERATIONS HAS BEEN EXCEEDED.
4380 ! *** IF SO, PRINT ERROR MESSAGE AND PAUSE.
4390 IF L>Itmax THEN 4700
4400 !
4410 !
4420 ! *** ITERATE AGAIN.
4430 X=Xsiljak(1)
4440 Y=Ysiljak(1)
4450 GOTO 4080
4460 !
4470 !
4480 ! *** QUARTER THE INTERVAL SIZE AND TRY AGAIN.
4490 ! *** MAXIMUM # OF QUARTERINGS HAS BEEN SET AT 20.
4500 ! *** IF THIS IS EXCEEDED, THEN A FINAL TEST IS MADE ON U AND V TO
4510 ! *** SEE IF THE FUNCTIONAL TOLERANCE IS SATISFIED.  IF NOT, AN
4520 ! *** ERROR MESSAGE IS PRINTED AND THE PROGRAM EXITS.
4530 IF M>20 THEN 4570
4540 Deltax=Deltax/4
4550 Deltay=Deltay/4
4560 GOTO 4210
```

```
4570 IF (ABS(U)<=Tolf) AND (ABS(V)<=Tolf) THEN 4790
4580 !
4590 !
4600 ! *** PRINT ERROR MESSAGE AND PAUSE.
4610 PRINT LIN(2),"ERROR IN SUBPROGRAM Siljak."
4620 PRINT "THE INTERVAL SIZE HAS BEEN QUARTERED 20 TIMES AND "
4630 PRINT "THE TOLERANCE FOR FUNCTIONAL EVALUATIONS IS STILL NOT MET."
4640 PRINT "Tolf=";Tolf,"U=";U,"V=";V,LIN(2)
4650 PAUSE
4660 !
4670 !
4680 ! *** MAXIMUM # OF ITERATIONS HAS BEEN EXCEEDED.
4690 ! *** PRINT ERROR MESSAGE AND PAUSE.
4700 PRINT LIN(2),"ERROR IN SUBROUTINE Siljak."
4710 PRINT "MAXIMUM # OF ITERATIONS HAS BEEN EXCEEDED."
4720 PRINT "L=";L,"Itmax=";Itmax,LIN(2)
4730 PAUSE
4740 GOTO 4390
4750 !
4760 !
4770 ! *** ROOT FOUND.  STORE COMPUTED ROOT IN Rroot(*) AND Iroot(*)
4780 ! *** ARRAY ELEMENT.
4790 Rroot(N)=Xsiljak(1)
4800 Iroot(N)=Ysiljak(1)
4810 !
4820 !
4830 ! *** INTIALIZE VARIABLES FOR SYNTHETIC DIVISION.
4840 A=Rcoef(N)
4850 B=Icoef(N)
4860 Rcoef(N)=Icoef(N)=0
4870 X=Xsiljak(1)
4880 Y=Ysiljak(1)
4890 ! *** SYNTHETIC DIVISION TO CALCULATE NEW COEFFICENTS Rcoef(*)
4900 ! *** AND Icoef(*).
4910 FOR K=N-1 TO 0 STEP -1
4920    C=Rcoef(K)
4930    D=Icoef(K)
4940    U=Rcoef(K+1)
4950    V=Icoef(K+1)
4960    Rcoef(K)=A+X*U-Y*V
4970    Icoef(K)=B+X*V+Y*U
4980    A=C
4990    B=D
5000 NEXT K
5010 N=N-1
5020 ! *** REDUCE NUMBER OF COEFFICIENTS AND BEGIN AGAIN.
5030 IF N<>1 THEN 3990
5040 ! *** SINCE DEGREE OF RESULTANT POLYNOMIAL IS ONE,
5050 ! *** COMPUTE FINAL ROOT ALGEBRAICALLY.
5060 A=Rcoef(0)
5070 U=Rcoef(1)
5080 B=Icoef(0)
5090 V=Icoef(1)
5100 T=U*U+V*V
5110 Rroot(1)=-(A*U+B*V)/T
5120 Iroot(1)=(A*V-U*B)/T
5130 N=Nn
5140 SUBEXIT
```

```
5150 !
5160 !
5170 ! *** SUBROUTINE TO COMPUTE SILJAK COEFFICIENTS.
5180 Siljak: Z=Xsiljak(1)*Xsiljak(1)+Ysiljak(1)*Ysiljak(1)
5190 T=2*Xsiljak(1)
5200 FOR K=0 TO N-2
5210    Xsiljak(K+2)=T*Xsiljak(K+1)-Z*Xsiljak(K)
5220    Ysiljak(K+2)=T*Ysiljak(K+1)-Z*Ysiljak(K)
5230 NEXT K
5240 U=V=0
5250 FOR K=0 TO N
5260    U=U+Rcoef(K)*Xsiljak(K)-Icoef(K)*Ysiljak(K)
5270    V=V+Rcoef(K)*Ysiljak(K)+Icoef(K)*Xsiljak(K)
5280 NEXT K
5290 F=U*U+V*V
5300 RETURN
5310 SUBEND
```

APPENDIX G

```
10   REM *****************************************************************
20   REM *                                                               *
30   REM *      THIS PROGRAM CALCULATES THE RESPONSE USING THE          *
40   REM *      CROSS MULTIPLICATION TECHNIQUE                           *
50   REM *                                                               *
60   REM *                                                               *
70   REM *      IT IS STORED UNDER "CROSS"                               *
80   REM *                                                               *
90   REM *****************************************************************
100  OPTION BASE 0
110  DIM B(25),A(25),R(200),Cc(25),Rr(25),C(200),Mm(3,10)
120  L=-1
130  LINPUT "DO YOU WISH A HARD COPY OF DATA AND RESULTS?",Qhc$
140  PRINTER IS 16
150  IF Qhc$="Y" THEN PRINTER IS 7,6
160  LINPUT "DO YOU WISH TO HAVE THE RESULTS PLOTTED? (Y/N)",Qp$
170  IF Qp$="N" THEN GOTO 200
180  ASSIGN #1 TO "PDATA"
190  READ #1,3
200  INPUT "INPUT THE ORDER OF THE NUMERATOR, DENOMINATOR",N,D
210       REDIM A(N),B(D)
220  PRINT USING 270;N,D
230  IMAGE "NUMBER OF STEPS FOR RESPONSE =",DDD
240  INPUT "INPUT SAMPLE INTERVAL",T
250  PRINT "SAMPLE INTERVAL =";T
260  GOSUB Input
270  IMAGE "ORDER OF NUMERATOR IS ",DD,10X,"ORDER OF DENOMINATOR IS ",DD
280  GOTO 540
290 Inp: ! Subroutine to calculate input function
300  LINPUT "DO YOU WISH  UNIT STEP(U),SIN(S),RAMP(R),OR USER DEFINED(D) FUNCTIO
N AS INPUT?",Qr$
310  IF Qr$="U" THEN GOSUB Step
320  IF Qr$="S" THEN GOSUB Sin
330  IF Qr$="R" THEN GOSUB Ramp
340  IF Qr$="D" THEN GOSUB User
350  IF (Qr$<>"U") AND (Qr$<>"S") AND (Qr$<>"R") AND (Qr$<>"D") THEN GOTO 370
360  GOTO 410
370       BEEP
380       DISP "ONLY  U, S, R,  OR  D ALLOWED"
390       WAIT 3500
400       GOTO 300
410  RETURN
420  SUBEND
430  !
440  !
450  !
460 Input:PRINT LIN(2),"CALCULATIONS MADE ACCORDING TO FOLLOWING EQUATION DEFINI
TION"
470  PRINT LIN(1),"             C(Z)/R(Z) = NUMERATOR/DENOMINATOR"
480  PRINT "          NUMERATOR = A(0)*Z^0 + A(1)*Z^1 + A(2)*Z^2 . . . A(N)*Z^N"
490  PRINT "        DENOMINATOR = B(0)*Z^0 + B(1)*Z^1 + B(2)*Z^2 . . . B(D)*Z^D"
500  RETURN
510  SUBEND
520  !
530  !
540  DISP "TO N=";N;" INPUT NUMERATOR COEFFICIENTS   A(0),A(1),A(2),...A(N)";
550  INPUT A(*)
560  PRINT LIN(3),"NUMERATOR COEFFICIENTS  A(0),A(1),A(2),...A(N)",A(*);
570  LINPUT "ANY CHANGES? (Y/N)",Qcha$
580  IF Qcha$="N" THEN GOTO 630
590  INPUT "COEFFICIENT NUMBER  I,A(I)",I,A(I)
600  PRINT USING 610;I,A(I)
610  IMAGE "A(",K,") = ",K
620  GOTO 570
630  DISP "TO D=";D;" INPUT DENOMINATOR COEFFICIENTS   B(0),B(1),B(2),...B(D)";
640  INPUT B(*)
```

```
650  PRINT "DENOMINATOR COEFFICIENTS   B(0),B(1),B(2),...B(N)",B(*);
660  LINPUT "ANY CHANGES? (Y/N)",Qcha$
670  IF Qcha$="N" THEN GOTO 720
680  INPUT "COEFFICIENT NUMBER  I,B(I)",I,B(I)
690  PRINT USING 610;I,B(I)
700  IMAGE "B(",K,") = ",K
710  GOTO 660
720  Bd=B(D)
730  REDIM B(D-1),Rr(N),Cc(D-1)
740  L=L+1
750  INPUT "INPUT NUMBER OF STEPS FOR RESPONSE",Nstep
760  PRINT USING 230;Nstep
770  REDIM R(Nstep)
780  GOSUB Inp
790  FOR K=0 TO Nstep
800  FOR J=0 TO N
810  IF K+J-D<0 THEN Rr(J)=0
820  IF K+J-D<0 THEN GOTO 840
830  Rr(J)=R(K+J-D)
840  NEXT J
850  FOR J=0 TO D-1
860  IF K+J-D<=0 THEN Cc(J)=0
870  IF K+J-D<=0 THEN GOTO 890
880  Cc(J)=C(K+J-D)
890  NEXT J
900  C(K)=1/Bd*(DOT(A,Rr)-DOT(B,Cc))
910  IF K=0 THEN Miy=May=C(0)
920  IF Miy>C(K) THEN Miy=C(K)
930  IF May<C(K) THEN May=C(K)
940  IF Qp$="Y" THEN GOSUB Tape
950  NEXT K
960  IF Qp$="N" THEN GOTO 1020
970  PRINT #1;Nstep*T,R(Nstep),C(Nstep),1,L+1,END
980  Mm(0,L)=0
990  Mm(1,L)=Nstep*T
1000 Mm(2,L)=Miy
1010 Mm(3,L)=May
1020 GOSUB Output
1030 LINPUT "CHANGE THE INPUT DATA(I)? NUMBER OF SAMPLES(N)? OR STOP(S)?",Qip$
1040 IF Qip$="N" THEN GOTO 750
1050 IF Qip$="I" THEN GOTO 200
1060 IF Qip$<>"S" THEN BEEP
1070 IF Qip$<>"S" THEN GOTO 1030
1080 IF Qp$="Y" THEN GOSUB Final
1090 END
1100 Step: ! Routine to set values for R when input is unit step
1110          PRINT "THE INPUT IS A UNIT STEP FUNCTION"
1120           MAT R=CON
1130           RETURN
1140           SUBEND
1150 !
1160 !
1170 !
1180 Sin:! Routine to set values fo R when input is SIN function
1190        PRINT "INPUT FUNCTION IS SIN WAVE"
1200        INPUT "INPUT FREQUENCY IN RAD/SEC,AMPLITUDE",W,X
1210        PRINT "FREQUENCY (RAD/SEC) IS =";W
1220        PRINT "AMPLITUDE OF SINE WAVE =";X
1230        FOR K=0 TO Nstep
1240        R(K)=X*SIN(W*K*T)
1250        NEXT K
1260        RETURN
1270        SUBEND
1280 !
1290 !
1300 !
```

```
1310 Ramp:!         Routine to set values of R when input is ramp
1320                PRINT "INPUT IS RAMP"
1330                INPUT "INPUT SLOPE OF RAMP (PER SECOND)",W
1340                PRINT "SLOPE IS =";W
1350                FOR K=0 TO Nstep
1360                R(K)=W*K*T
1370                NEXT K
1380                RETURN
1390                SUBEND
1400 !
1410 !
1420 !
1430 User:! Routine to set values of R for user defined input
1440        PRINT "THE INPUT IS A USER DEFINED FUNCTION"
1450        FOR K=0 TO Nstep
1460        FIXED 0
1470        DISP "INPUT R(";K;")";
1480        INPUT R(K)
1490        NEXT K
1500        PRINT "INPUT FOR USER DEFINED FUNCTION R(0),R(1),R(2),. . . R(Nstep)"
1510        STANDARD
1520        PRINT R(*)
1530        RETURN
1540        SUBEND
1550 Output:! Routine to print the data results
1560 PRINT LIN(3)
1570 PRINT "  K";SPA(8);"TIME";SPA(16);"INPUT";SPA(13);"RESPONSE"
1580 FOR K=0 TO Nstep
1590 PRINT USING 1600;K,K*T,R(K),C(K)
1600 IMAGE ,DDD5X,MD.DDDE,10X,MD.DDDE,10X,MD.DDDE
1610 NEXT K
1620 RETURN
1630 SUBEND
1640 Tape: ! PLACE THE DATA ON TAPE FOR PLOTTING
1650 PRINT #1;K*T,R(K),C(K),0,L+1,END
1660 RETURN
1670 SUBEND
1680 !
1690 !
1700 !
1710 Final:! FINAL DATA ON TAPE ROUTINE
1720 PRINT #1,1;L
1730 FOR J=0 TO L
1740 PRINT #1;Mm(0,J),Mm(1,J),Mm(2,J),Mm(3,J),END
1750 NEXT J
1760 RETURN
1770 SUBEND
```

APPENDIX H

CONTROL DYNAMICS COMPANY

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HUNTSVILLE, ALABAMA 35802

TELEPHONE
205 / 881-1342

April 2, 1980

Mr. James McLean
US Army Missile Command
Redstone Arsenal, AL 35809

ATTN: DRSMI-RGN

Dear Jim:

In accordance with your instructions I participated in the PERSHING II Autopilot Design Review at The Martin Company in Orlando on 31 March 1980. At the conclusion of the review, I made a number of comments on my observations. The Martin Company representatives requested that Mr. Jesse Armstrong (PERSHING Project Office) make all such comments available in writing. Mr. Armstrong agreed. My comments are summarized in this letter. I have not included the comments made by Mr. Ed. Herbert.

1. General.

The autopilot work that was reported upon was excellent, as were the presentations. The participants appeared extremely capable. The results of the linear analyses conducted appeared quite satisfactory, although examples of associated analyses were not presented. The review was presented at a very fast pace. It is recommended that the next such review be conducted at a pace more conducive to technical interplay between the presenters and the reviewers. This would be enhanced by providing all reviewers with copies of the material to be reviewed -- perhaps a week before the review.

2. Nonlinearities.

My recollection is that only intentionally included nonlinearities (e.g. the saturation-type nonlinearity following each digital "integrator") were recognized as existing. It was assumed that the various system states would lie within these saturation bounds, hence precluding the application of a linear analysis. I am concerned with the possibility of other nonlinearities existing within the controlled missile plant, actuators, and/or sensors and their dynamic effect. I do not believe this possibility has been given much attention. The analysis of nonlinear effects on a digitally-controlled plant is difficult, time-consuming, and tedious. Still, it has been used in the past to uncover unexplained simulation dynamics and proven its worth. On other programs I have always found rewarding, if difficult.

3. Quantization.

Quantization is a special nonlinearity associated with digital controllers. It's dynamic and accuracy effects apparently have not been investigated.

4. Modeling.

During the presentation, it was brought to light that the dynamics of all elements of the system might not be known (e.g. engine nozzle characteristics). One technique to include their effects would be to incorporate the actual hardware in the loop.

5. Tolerance analysis.

In the planned tolerance analysis, significant attention should be devoted to the variation of system parameters. Two areas in particular should receive attention:

(1) Bending characteristics. During the presentation, the statement was made that the NASTRAN program is producing bending parameters with an accuracy of 5%. My experience with NASTRAN leads me to question this statement. The significance of accurate bending data becomes particularly crucial when one attempts to use "notch filters" to compensate for body bending dynamics.

(2) Pole-zero cancellation. Several times during the presentation, pole-zero cancellation techniques were alluded to. Pole-zero cancellation is very sensitive to not knowing precise numerical values of system parameters. I would be wary of such an approach.

6. Computer capacity.

During the presentation it was stated that the computer is already filled to 70% capacity. Typically, at this stage of development, computer requirements would be expected to grow more than 30%. Consequently, efforts should be implemented to keep this growth to a minimum and perhaps to investigate the possibility of decreasing (through innovation) computer requirements now.

As a result of the foregoing observations, the following recommendations are offered for consideration.

(1) Provide copies of the proposed presentation material to all reviewers approximately one week before the next technical review.

(2) Investigate the possible existence of dynamically significant nonlinearities. Predict their dynamic effect by mathematical analysis to be verified by computer simulation. Where this is deemed impractical, include in the hybrid simulation the actual hardware that gives rise to the nonlinearity.

(3) Investigate the sensitivity of the control system to variations in numerical values of bending parameters. View the 5% accuracy figure associated with the NASTRAN program with skepticism. This should have a strong effect on the planned "notch filters."

(4) Investigate analytically the dynamic (and accuracy) effects of digital computer quantization.

(5) Investigate the sensitivity of the control system to realistic variations in numerical values of system parameters involved in any pole-zero cancellation techniques.

(6) Initiate action to decrease computer capacity requirements and/or strongly limit future additional demands upon the onboard computer.

I would appreciate your reviewing this letter and forwarding it to Mr. Jesse Arsmtrong. Thank you.

Sincerely yours,

Sherman M. Seltzer

Sherman M. Seltzer

cc -- Dr. H. Pastrick, DRSMI-RGN
Mr. E. Herbert, DRSMI-RGN

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This technical report covers the Control Dynamics Company's efforts performed in completing the requirements set forth in Technical Requirement No. T-0208, entitled, "Application of Digital Technology to Guidance and Control Theory." Described is a simple analytical set of tools developed to help the practising control system design engineer. These tools are the Parameter Space technique and the Cross-Multiplication technique. Also described is a comprehensive Digital Design seminar conducted for selected members of the Guidance and Control Directorate. Application of advanced digital theory to the PERSHING II digital

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autopilot is described, as well as the beginning efforts to evaluate the consequences of microprocessor implementation of digital filters.