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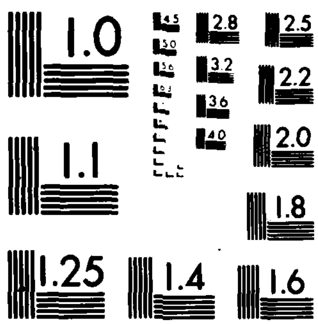

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INFERENCE ON THE STRUCTURE OF INTERACTION IN TWO-WAY CLASSIFICATION MODEL

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1. INTRODUCTION

Under the classical two-way classification model with one observation per cell, the hypotheses of no main effects are tested in practice by using the ratios of the mean squares associated with the main effects to the error mean square. But when the interaction between the main effects is present, these tests are no longer valid. So, there is quite a bit of interest in studying the structure of interaction term and the effect of interaction on the usual tests for main effects. In Section 2 of this chapter, we review Tukey's test for non-additivity (see Tukey (1949)) and certain generalizations of this test by Scheffé (1959, p. 144) and Graybill and Milliken (1970). Some other interesting early developments like the work of Fisher and Mackenzie (1923) and Williams (1952) are also discussed in this section. In Section 3, we discuss the model when the interaction matrix is decomposed by singular value decomposition of a matrix. The work of Gollob (1968), Mandel (1969) as well as the likelihood ratio tests (see Corsten and van Eijnsbergen (1972), Johnson and Graybill (1972), and Yochmowitz and Cornell (1978)) for testing the hypotheses on the structures of interaction term are also reviewed. Krishnaiah and Waikar (1971, 1972) proposed simultaneous test procedures for testing the equality of the eigenvalues of the covariance matrix against certain alternatives. Applications of the above procedures in studying the structure of interaction term are emphasized in Section 3. In Section 4, we discuss the effect of the presence of interaction on the usual tests for the hypotheses of no main effects. Finally, the applications of certain tests for the hypotheses of no interaction are illustrated with some data on monkeys on animal models.

2. SOME EARLY DEVELOPMENTS ON TESTS FOR ADDITIVITY

Consider the model

$$y_{ij} = \mu + \alpha_i + \beta_j + \eta_{ij} + \varepsilon_{ij} \tag{2.1}$$

where $y_{ij}(i=1,\ldots,r;j=1,\ldots,s)$ denotes the observation in i-th row and j-th column and ε_{ij}'s are distributed independently as normal with mean 0 and variance σ^2. Also μ,α_i,β_j and η_{ij} respectively denote the general mean, i-th row effect, j-th column effect, and interaction between i-th row and j-th column. In addition, let $\sum_i \alpha_i = \sum_j \beta_j = \sum_i \eta_{ij} = \sum_j \eta_{ij} = 0$. Tukey (1949) proposed the following procedure for testing the hypothesis H: $\eta=0$ where $\eta=(\eta_{ij})$. The hypothesis H is accepted or rejected according as

$$F_1 \lesseqgtr F_\alpha \tag{2.2}$$

where

$$P\left[F_1 \le F_\alpha | H\right] = (1-\alpha), \tag{2.3}$$

$$F_1 = \frac{s_1^2\{(r-1)(s-1)-1\}}{(s_e^2 - s_1^2)} \tag{2.4}$$

$$s_1^2 = \frac{\left[\sum_i \sum_j (\bar{y}_{i.}-\bar{y}_{..})(\bar{y}_{.j}-\bar{y}_{..})y_{ij}\right]^2}{\{\sum_i(\bar{y}_{i.}-\bar{y}_{..})^2\}\{\sum_j(\bar{y}_{.j}-\bar{y}_{..})^2\}},$$

$$s_e^2 = \sum_i \sum_j (y_{ij}-\bar{y}_{i.}-\bar{y}_{.j}+\bar{y}_{..})^2 ,$$

$$s\bar{y}_{i.} = \sum_j y_{ij},\; r\bar{y}_{.j} = \sum_i y_{ij} \quad \text{and} \quad rs\bar{y}_{..} = \sum_{ij}\sum y_{ij}.$$

When H is true, the statistic F_1 is distributed as the central F distribution with (1,rs-r-s) degrees of freedom. In examining the model (2.1) with $\eta_{ij}=\lambda\ \alpha_i\beta_j$, Ward and Dick (1952) solved the normal equations and arrived at s_1^2 as the sum of squares associated with testing the hypothesis of no interaction. Ghosh and Sharma (1963) showed that the power of Tukey's test for H against the alternative hypothesis $\eta_{ij}=\lambda\ \alpha_i\beta_j$ is high.

Tukey (1955) showed as to how his test can be extended to test for no interaction in the Latin Square. The model equation in this case is given by

$$y_{ijk} = \mu + \alpha_i^A + \beta_j^B + \gamma_k^C + \eta_{ijk} + \varepsilon_{ijk} \qquad (2.5)$$

where α_i^A, β_j^B and γ_k^C (i=1,2,...,r; j=1,2,...,r; k=1,2,...,r) respectively denote the effects of i-th level of A, j-th level of B and k-th level of C. Also, η_{ijk} denotes the interaction of i-th level of A with j-th level of B and k-th level of C. In addition, the errors ε_{ijk} are distributed independently and normally with mean 0 and variance σ^2. If we apply Tukey's test, we accept or reject the hypothesis H of no interaction under the model (2.5) when

$$F_2 \lesseqgtr F_\alpha \qquad (2.6)$$

where

$$P\left[F_2 \leq F_\alpha \mid H\right] = (1-\alpha), \qquad (2.7)$$

$$F_2 = \frac{s_2^2\ (r^2-3r+1}{s_3^2 - s_2^2} \tag{2.8}$$

$$s_2^2 = \left[\sum_i \sum_j e_{ijk}\ u_{ijk}\right]^2 \qquad s_o^2, s_3^2 = \sum_i \sum_j e_{ijk}^2$$

$$s_o^2 = \sum_i \sum_j (u_{ijk} - \bar{u}_{i..} - \bar{u}_{.j.} - \bar{u}_{..k} + 2\ \bar{u}_{...})^2,$$

$$u_{ijk} = (\bar{y}_{i..} + \bar{y}_{.j.} + \bar{y}_{..k} - 3\bar{y}_{...})^2, \ r^2\bar{y}_{...} = \sum_i \sum_j y_{ijk},$$

$e_{ijk} = y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\ \bar{y}_{...}$, $r\bar{y}_{i..} = y_{i..}$, $ry_{.j.} = y_{.j.}$, $r\bar{y}_{..k} = y_{..k}$, $r\bar{u}_{i..} = u_{i..}$, $r\bar{u}_{.j.} = u_{.j.}$, and $r\bar{u}_{..k} = u_{..k}$.

When H is true, F_2 is distributed as the central F distribution with $(1, r^2-3r+1)$ degrees of freedom.

Thus interaction can be tested with only 1 cell replicate in the Latin Square. Mandel (1969) also considered the problem of testing the hypothesis of no interaction under the model (2.5) when $\eta_{ijk} = \lambda u_i v_j$ where u_i and v_j are specified a priori and λ is an unknown constant.

Mandel (1969) has identified many models as special cases of the Factor Analysis of Variance (FANOVA) model given by (3.1) in the next section. These special cases are obtained by assuming very special structures of the interaction term η_{ij} in (2.1) and they are given in the following table:

Special Cases of the FANOVA Model

Structure of η_{ij}	Type of the Model
0	Additive
$\lambda\alpha_i\beta_j$	Concurrent
$R_i\beta_j$	Bundle of Lines - Rows Linear
$C_j\alpha_i$	Bundle of Lines-Columns Linear
$R_i\beta_j+\lambda\alpha_i\beta_j$	Combination of Concurrent and Bundle of Lines
$R_i\beta_j+\alpha_iC_j+\lambda\alpha_i\beta_j$	First Sweep of Tukey's Vacuum Cleaner

The additive model has no interaction. The concurrent model can be tested effectively by using Tukey's test for non-additivity. Mandel (1961) proposed the bundle of lines model with one replication per cell in the fixed two-way layout. The test for no interaction under this model is described below. If we have $\eta_{ij}=R_i\beta_j$, the total sum of squares (s.s) is partitioned as follows.

Source of Variation	d.f.	s.s.
Total	rs	$\sum_i \sum_j y_{ij}^2$
Mean	1	$rs\ \bar{y}_{..}^2$
Rows	r-1	$r \sum_i (\bar{y}_{i.}-\bar{y}_{..})^2$
Columns	s-1	$s \sum_j (\bar{y}_{.j}-\bar{y}_{..})^2$
Slopes	r-1	$\{\sum_i (b_i-1)^2\}\{\sum(\bar{y}_{.j}-\bar{y}_{..})^2\}$
Residual	(r-1)(s-2)	$\sum_i \sum_j \{(y_{ij}-\bar{y}_{i.})-b_i(\bar{y}_{.j}-\bar{y}_{..})\}^2$

where

$$b_i = \frac{\sum_j y_{ij}(\bar{y}_{.j}-\bar{y}_{..})}{\sum_j (\bar{y}_{.j}-\bar{y}_{..})^2} . \tag{2.9}$$

The hypothesis $R_i=0$ is accepted or rejected according as

$$F_3 \lessgtr F_\alpha \tag{2.10}$$

where

$$P\left[F_3 \leq F_\alpha \mid H\right] = (1-\alpha), \tag{2.11}$$

and

$$F_3 = \frac{(s-2)\, s_2^2}{s_3^2} . \tag{2.12}$$

In Eq. (2.12), s_2^2 and s_3^2 are respectively the sums of squares associated with slopes and residual in the preceding table. Also, F_3 has F distribution with r-1 and (r-1)(s-2) degrees of freedom when H is true. When H is rejected, Mandel indicated that the data is represented by a bundle of non-parallel lines with scatter about the lines being measured by the residual mean square. In order to examine whether the multiplicative structure $R_i\beta_j$ is an appropriate descriptor for n_{ij} he partitioned the s.s. (slopes) as follows:

Source	df	ss
Slopes	r-1	$\sum(b_i-1)^2 \sum(\bar{y}_{.j}-\bar{y}_{..})^2$
Concurrence	1	$\dfrac{\left[\sum\sum(\bar{y}_{i.}-\bar{y}_{..})(\bar{y}_{.j}-\bar{y}_{..})y_{ij}\right]^2}{\sum(\bar{y}_{i.}-\bar{y}_{..})^2 \sum(\bar{y}_{.j}-\bar{y}_{..})^2}$
Non-concurrence	r-2	Remainder

The S.S. for concurrence is identical to Tukey's s.s. for 1 df. In the presence of interaction, significant concurrence indicates that the multiplicative model $R_i \beta_j$ will account for most of the interaction. He tests this hypothesis by using F_4 as test statistic where

$$F_4 = \frac{\text{s.s.(concurrence)}(r-2)}{\text{s.s (non-concurrence)}}$$

When there is no concurrence F_4 has F distribution with 1 and r-2 degrees of freedom. Testing for interaction in the bundle of line models is thus a two step procedure.

Step 1 involves testing for no interaction. The second step is to test for the appropriate structure of the interaction if the interaction is present.

We can use simultaneous tests to test both hypotheses simultaneously.

The combination of the concurrent and bundle of lines models can be reparametrized by expressing η_{ij} as $\eta_{ij}=(\lambda\alpha_i+R_i)\beta_j$, and therefore becomes a FANOVA model (see (3.1) below) with a single multiplicative component. The first sweep of Tukey's vacuum cleaner can be reduced to a two component FANOVA model by a similar reparametrization. Future sweeps of Tukey's vacuum cleaner differ from the FANOVA model in that new terms of the vacuum cleaner are functions of the residuals and the preceding sweep. In the FANOVA model, they are functions of the residuals only.

Milliken and Graybill (1970) considered the model

$$\underset{\sim}{y} = X\underset{\sim}{\beta} + Z\underset{\sim}{\lambda} + \underset{\sim}{e} \qquad (2.13)$$

where $\underset{\sim}{e}: n\times 1$ is distributed as a multivariate normal with mean vector $\underset{\sim}{0}$ and covariance matrix $\sigma^2 I_n$, X: n×p is a known matrix of rank q, $Z(z_{ij}(X\underset{\sim}{\beta}))$: n×k is unknown but its elements are known functions of $X\underset{\sim}{\beta}$, $\underset{\sim}{\lambda}$: k×1 and $\underset{\sim}{\beta}$: p×1 are unknown. If Z is known, the usual test statistic used for testing the hypothesis $\underset{\sim}{\lambda} = 0$ is given by F where

$$F = \frac{Q_1(n-r)}{Q_0(r-q)} \quad , \tag{2.14}$$

$$Q_1 = \underset{\sim}{y}' \, [(I-XX^-)Z] \, [(I-XX^-)Z]^- \, \underset{\sim}{y} \tag{2.15}$$

$$Q_0 = \underset{\sim}{y}'[I-XX^-] \, \underset{\sim}{y} - Q_1 \; , \tag{2.16}$$

and r is the rank of $[X,Z]$. Also, q < r < n and A^- denotes the Moore-Penrose generalized inverse of A. Since Z is unknown, we replace Z with $\hat{Z}$ in (2.10) where $\hat{Z}$ is obtained from Z by replacing $X\underset{\sim}{\beta}$ with $X\hat{\underset{\sim}{\beta}}$; here $\hat{\underset{\sim}{\beta}}$ is the least square estimate of $\underset{\sim}{\beta}$ under the model when $\underset{\sim}{\lambda}=\underset{\sim}{0}$. Now let,

$$F^* = \frac{(n-r)\hat{Q}_1}{(r-q)\hat{Q}_0} \tag{2.17}$$

where

$$\hat{Q}_1 = \underset{\sim}{y}'[(I-XX^-)\hat{Z}] \, [(I-XX^-)\hat{Z}]^- \underset{\sim}{y} \tag{2.18}$$

$$\hat{Q}_0 = \underset{\sim}{y}' \, [I-XX^-] \, \underset{\sim}{y} - \hat{Q}_1 \; . \tag{2.19}$$

The hypothesis $\underset{\sim}{\lambda}=\underset{\sim}{0}$ is accepted or rejected according as

$$F^* \lessgtr F_\alpha \tag{2.20}$$

where

$$P[F^* \le F_\alpha | \underset{\sim}{\lambda} = \underset{\sim}{0}] = (1-\alpha). \tag{2.21}$$

When $\underset{\sim}{\lambda} = \underset{\sim}{0}$, the statistic F^* is distributed as central F distribution with (r-q) and (n-r) degrees of freedom. When $\underset{\sim}{\lambda} \neq \underset{\sim}{0}$, the distribution of F^* is not known. The distribution theory given above is essentially contained in Scheffé [(1959); problem 4.9] and the model (2.13) is a slight generalization of the model considered by Scheffé.

When k=1, we obtain

$$\hat{Q}_1 = \frac{[\underset{\sim}{y}'(I-XX^-)\hat{Z}]^2}{\underset{\sim}{\hat{Z}}'(I-XX^-)\hat{Z}} \tag{2.22}$$

$$\hat{Q}_0 = \underset{\sim}{y}'(I-XX^-)\underset{\sim}{y} - \hat{Q}_1 \tag{2.23}$$

$$F^* = \frac{\hat{Q}_1(n-r)}{\hat{Q}_0(r-q)} \quad . \tag{2.24}$$

Graybill and Milliken (1970) discussed some useful special cases of the model (2.13). One of the special cases discussed was the concurrent model

$$y_{ij} = \mu + \alpha_i + \beta_j + \lambda\alpha_i\beta_j + \varepsilon_{ij} \tag{2.25}$$

where λ is unknown and other notations are the same as used in the model (2.1). The hypothesis $\lambda=0$ can be tested by using the test statistic (2.24). In this special case, the test discussed in Graybill and Milliken (1970) is equivalent to Tukey's test for non-additivity.

Fisher and Mackenzie (1923) considered the model when the expected effect is the product of the constants representing the effects of two factors. Williams (1952) considered the following model:

$$y_{ij} = \lambda\alpha_i v_j + \beta_j + \varepsilon_{ij} \tag{2.26}$$

where $\sum\alpha_i = \sum\beta_j = 0$ and $\sum\alpha_i^2 = \sum v_j^2 = 1$. He showed that the least square estimate of λ is the largest root of the matrix $T = (t_{jk})$ where $t_{jk} = \sum_i(y_{ij}-\bar{y}_{.j})(y_{ik}-\bar{y}_{.k})$. Williams (1952) also considered the following model:

$$y_{ij} = c_i d_j \lambda + \alpha_i + \beta_j + \varepsilon_{ij} \tag{2.27}$$

where $\sum\alpha_i = \sum\beta_j = \sum c_i = \sum d_j = 0$ and $\sum c_i^2 = \sum d_j^2 = 1$. He showed that the least square estimate of λ is the largest root of the matrix $V = (v_{jk})$ where $v_{jk} = \sum_i(y_{ij} - \bar{y}_{i.} - \bar{y}_{.j})(y_{ik} - \bar{y}_{i.} - \bar{y}_{.k})$.

3. TESTS FOR THE STRUCTURE OF INTERACTION USING EIGENVALUES OF A RANDOM MATRIX

In the model (2.1), we assume that the rank of $\eta = (\eta_{ij})$ is c. Using the singular value decomposition of a matrix, we know that

$$\eta = \theta_1 \underset{\sim}{u}_1 \underset{\sim}{v}_1' + \ldots + \theta_c \underset{\sim}{u}_c \underset{\sim}{v}_c' \tag{3.1}$$

where $\theta_1^2 \geq \ldots \geq \theta_c^2$ are the eigenvalues of $\eta\eta'$, $\underset{\sim}{u}_i$ is the eigenvector of $\eta\eta'$ corresponding to θ_i^2 and $\underset{\sim}{v}_i$ is the eigenvector of $\eta'\eta$ corresponding to θ_i^2. Gollob (1968) and Mandel (1969) considered the problem of testing the hypothese H_i where H_i: $\theta_i = 0$. Their tests as well as the likelihood ratio tests for testing H_i will be discussed in the later part of this section. We will first discuss as to how the simultaneous tests of Krishnaiah and Waikar (1971, 1972) for sphericity can be applied in the area of testing for the structure of interaction term η_{ij}. Some discussions along these lines were made by Schuurmann, Krishnaiah and Chattopadhyay (1973b) and Krishnaiah and Schuurmann (1974).

It is known (see Gollob (1968)) from a result of Eckert and Young (1936) that the least square estimates of $\theta_i, \underset{\sim}{u}_i$, and $\underset{\sim}{v}_i$ are respectively $\hat{\theta}_i, \hat{\underset{\sim}{u}}_i$ and $\hat{\underset{\sim}{v}}_i$ where $\hat{\theta}_1^2 \geq \ldots \geq \hat{\theta}_{r-1}^2$ are the non-zero roots of DD', $\hat{\underset{\sim}{u}}_i$ is the eigenvector of DD' corresponding to $\hat{\theta}_i^2$, $\hat{\underset{\sim}{v}}_i$ is the eigenvector of $D'D$ corresponding to $\hat{\theta}_i^2$, $D = (d_{ij})$ and $d_{ij} = y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..}$. Now, let I_r: r×r denote the identity matrix and J_r: r×r denote the matrix whose elements are equal to unity.

But

$$DD' = (I_r - \frac{1}{r} J_r)Y(I_s - \frac{1}{s} J_s)Y'(I_r - \frac{1}{r} J_r). \qquad (3.2)$$

We can choose C_r such that $C_r' C_r = I_{r-1}$ and $I_r - \frac{1}{r} J_r = C_r C_r'$. So, it is easily seen (e.g., see Johnson and Graybill (1972a)) that the nonzero roots of DD' are the same as the nonzero eigenvalues of W where $W = C_r'YC_sC_s'Y'C_r$. But the columns of $C_r'Y$ are distributed independently as (r-1)-variate normal with mean vector $C_r'M$ and covariance matrix $C_r'C_r\sigma^2$. So, W is distributed as noncentral Wishart matrix with (s-1) degrees of freedom and noncentrality parameter Ω where $\Omega = C_r'MC_sC_s'M'C_r$, $M = (m_{ij})$ and $m_{ij} = \mu+\alpha_i+\beta_j+\eta_{ij}$. Also, $E(W/(s-1)= \Sigma^*$ where $\Sigma^* = \sigma^2 I + (\Omega/(s-1)$. We can express Ω as

$$\Omega = \sum_{k=1}^{c} \theta_k^2 C_r' \underset{\sim}{u}_k \underset{\sim}{u}_k' C_r . \qquad (3.3)$$

Let $\lambda_1 \geq \ldots \geq \lambda_{r-1}$ be the nonzero roots of Σ^*. Then $\lambda_i = \sigma^2+(\theta_i^2/(s-1))$, $(i=1,2,\ldots,c)$, $\lambda_{c+1}=\ldots=\lambda_{r-1} = \sigma^2$.

It is of interest to test the hypothesis H: $\theta_1=\ldots=\theta_c=0$ and its subhypotheses simultaneously. The hypothesis H is equivalent to testing the hypothesis H* where H*: $\lambda_1=\ldots=\lambda_{c+1}$. So, the problem of testing the hypothesis of no interaction is equivalent to the problem of testing the equality of the eigenvalues of Σ^*. Motivated by this equivalence, we consider the following procedures for testing the hypothesis of no interaction and its subhypotheses in the spirit of the simultaneous tests of Krishnaiah and Waikar (1971).

To fix the ideas, we will first consider the case when $c = r - 2$.

The hypothesis H^* can be expressed as $H^* = \bigcap_{i=1}^{r-2} H^*_{i,r-1}$, $H^* = \bigcap_{i=1}^{r-1} H^*_{i,i+1}$, and $H^* = \bigcap_{i=1}^{r-2} H^*_i$ where $H^*_{ij}: \lambda_i = \lambda_j, (i<j), H^*_i: \lambda_i = \bar{\lambda}$ and $(r-1)\,\bar{\lambda} = \lambda_1 + \ldots + \lambda_{r-1}$. Also, let $A^*_1 = \bigcup_{i=1}^{r-2} A^*_{i,r-1}$, $A^*_2 = \bigcup_{i=1}^{r-1} A^*_{i,i+1}$ and $A^*_3 = \bigcup_{i=1}^{r-2} A^*_i$ where $A^*_{ij}: \lambda_i > \lambda_j, (i<j)$, $A^*_i: \lambda_i > \bar{\lambda}$, and $(r-1)\,\bar{\lambda} = (\lambda_1 + \ldots + \lambda_{r-1})$. The hypothesis H^* when tested against A^*_1 is accepted or rejected according as

$$\frac{\ell_1}{\ell_{r-1}} \lesseqgtr c_{1\alpha} \tag{3.4}$$

where

$$P\left[\frac{\ell_1}{\ell_{r-1}} < c_{1\alpha} \middle| H^*\right] = (1-\alpha). \tag{3.5}$$

If H^* is rejected, we accept or reject $H^*_{i,r-1}$ against $A^*_{i,r-1}$ according as

$$\frac{\ell_i}{\ell_{r-1}} \lesseqgtr c_{1\alpha}. \tag{3.6}$$

Here we note that $H^*_{i,r-1}$ is equivalent to the hypothesis that $\theta_i = 0$.

Next, consider the problem of testing H^* against A^*_2. In this case, we accept H^* if

$$\frac{\ell_i}{\ell_{i+1}} \leq c_{2\alpha} \tag{3.7}$$

for $i = 1,2,\ldots,r-1$ and reject it otherwise where

$$P\left[\frac{\ell_i}{\ell_{i+1}} \le c_{2\alpha};\ i = 1,2,\ldots,r-2 \middle| H^*\right] = (1-\alpha). \tag{3.8}$$

If H^* is rejected, we accept or reject $H^*_{i,i+1}$ according as

$$\frac{\ell_i}{\ell_{i+1}} \lesseqgtr c_{2\alpha} \tag{3.9}$$

The hypothesis $H^*_{i,i+1}$ is equivalent to the hypothesis that $\theta_i = \theta_{i+1}$.

If we test H^* against A^*_3, we accept or reject H^* according as

$$\frac{\ell_1}{\ell_1+\ldots+\ell_{r-1}} \lesseqgtr c_{3\alpha} \tag{3.10}$$

where $c_{3\alpha}$ is chosen such that

$$P\left[\frac{\ell_1}{\ell_1+\ldots+\ell_{r-1}} \le c_{3\alpha} \middle| H^*\right] = (1-\alpha). \tag{3.11}$$

If H^* is rejected, we accept or reject H^*_i against A^*_i according as

$$\frac{\ell_i}{\ell_1+\ldots+\ell_{r-1}} \lesseqgtr c_{3\alpha} \tag{3.12}$$

Here we note that H^*_i is equivalent to the hypothesis that $\theta_i = \bar{\theta}$ where $(r-1)\,\bar{\theta} = \theta_1+\ldots+\theta_{r-1}$.

It is known that W is distributed as the central Wishart matrix with (s-1) degrees of freedom and $E(W/(s-1)) = \sigma^2 I$. When H^* is true, Schuurmann, Krishnaiah and Chattopadhyay (1973a,b) investigated the exact distribution of $\ell_1/(\ell_1+\ldots+\ell_{r-1})$ whereas Krishnaiah and Schuurmann (1974) investigated the distribution of ℓ_1/ℓ_{r-1}. Percentage points of the above statistics are reproduced in Chapter 24 of Krishnaiah in this volume for some values of the parameters. The exact distribution of $\max(\ell_1/\ell_2, \ell_2/\ell_3, \ldots, \ell_{r-2}/\ell_{r-1})$ is not known. But, we know that

$$P\left[\frac{\ell_1}{\ell_p} \leq c_{2\alpha} \middle| H^*\right] \leq P\left[\max_i (\ell_i/\ell_{i+1}) \leq c_{2\alpha} \middle| H^*\right]. \tag{3.13}$$

Using the inequality (3. 13) and the results on the distribution of ℓ_1/ℓ_p, we can obtain upper bounds on the values of $c_{2\alpha}$ where $c_{2\alpha}$ is given by (3. 8). Computer programs are also available for computing percentage points of various ratios like ℓ_1/ℓ_p, $\ell_1/(\ell_1+\ldots+\ell_p)$ and $\max_i(\ell_i/\ell_{i+1})$ by using Monte Carlo methods.

We will now discuss simultaneous test procedures to test H^* when $c \leq r-2$. In this case, we can express H^* as $H^* = \bigcap_{i=1}^{c} H^*_{i,c+1}$. Motivated by this decomposition, we propose the following procedure. We accept or reject H^* against $\bigcup_{i=1}^{c} \{\lambda_i > \lambda_{c+1}\}$ when

$$\frac{\ell_1}{\ell_{c+1}} \lesseqgtr c_{4\alpha} \tag{3.14}$$

where

$$P\left[\frac{\ell_1}{\ell_{c+1}} \leq c_{4\alpha} \middle| H^*\right] = (1-\alpha). \tag{3.15}$$

When H^* is rejected, we accept or reject $H^*_{1,c+1}$ according as

$$\frac{\ell_1}{\ell_{c+1}} \lessgtr c_{4\alpha} \tag{3.16}$$

where

$$P\left[\frac{\ell_1}{\ell_{c+1}} \leq c_{4\alpha} \middle| H^*\right] = (1-\alpha). \tag{3.17}$$

The above test for H^* is equivalent to testing $H_{1,c+1},\ldots,H_{c,c+1}$ simultaneously against appropriate alternatives and accepting H^* if and only if all the subhypotheses $H_{i,c+1}(i=1,\ldots,c)$ are accepted. The hypothesis $H_{i,c+1}$ is equivalent to the hypothesis that $\theta_i = 0$. In proposing the test discussed above, $\ell_{c+1}/(s-1)$ is used as an estimate of λ_{c+1}. One may use any of the eigenvalues $\ell_{c+2}/(s-1),\ldots,\ell_{r-1}/(s-1)$ also as estimates of λ_{c+1}. Alternatively, one may use $(\ell_{c+1}+\ldots+\ell_{r-1})/(r-c-1)$ as an estimate of λ_{c+1}. So, procedures can be proposed to test H^* and $H^*_{i,c+1}(i=1,\ldots,c)$ simultaneously by replacing ℓ_{c+1} with ℓ_{c+i} $(i=2,3,\ldots,r-1)$ or $(\ell_{c+1}+\ldots+\ell_{r-1})/(r-c-1)$. Computer programs are available for computing the percentage points of the test statistics ℓ_1/ℓ_{c+i}, $(i=1,2,\ldots,r-c-1)$, and $\ell_1/(\ell_{c+1}+\ldots+\ell_{r-1})$. Also,

$$P\left[\frac{\ell_1}{\ell_{c+1}} \leq c_{4\alpha} \middle| H^*\right] \geq P\left[\frac{\ell_1}{\ell_{r-1}} \leq c_{4\alpha} \middle| H^*\right], \tag{3.18}$$

$$P\left[\frac{\ell_1}{\ell_{c+1}+\ldots+\ell_{r-1}} \leq c_{4\alpha} \middle| H^*\right] \geq P\left[\frac{\ell_1}{\ell_1+\ldots+\ell_{r-1}} \leq c_{4\alpha} \middle| H^*\right]. \tag{3.19}$$

When H^* is true, we can use inequalities (3.18) and (3.19) and the known results on the distributions of ℓ_1/ℓ_{r-1} and $\ell_1/(\ell_1+\ldots+\ell_{r-1})$

to obtain bounds on the critical values associated with the procedures discussed above for testing H^* and $H^*_{i,c+1}(i=1,2,\ldots,c)$.

We will now consider the problem of testing H^* against the alternatives $\bigcup_{i=1}^{c} [\lambda_i > \lambda_{i+1}]$. In this case, the hypothesis H^* is decomposed as $H^* = \bigcap_{i=1}^{c} H^*_{i,i+1}$ and the following procedure may be used. We accept H^* if

$$\ell_i/\ell_{i+1} \leq c_{5\alpha} \tag{3.20}$$

for $i = 1,2,\ldots,c$ and reject it otherwise where

$$P\left[\frac{\ell_i}{\ell_{i+1}} \leq c_{5\alpha};\ i = 1,2,\ldots,c \,\middle|\, H^*\right] = (1-\alpha) \tag{3.21}$$

When H^* is rejected, $H_{i,i+1}$ $(i = 1,2,\ldots,c)$ is accepted or rejected according as $(\ell_i/\ell_{i+1}) \lessgtr c_{5\alpha}$. As before we can replace ℓ_{c+1} with $\ell_{c+i}(i=2,\ldots,r-c-1)$ or $(\ell_{c+1}+\ldots+\ell_{r-1})/(r-c-1)$ in the above procedure.

Next, consider the problem of testing H^* against $\bigcup_{i=1}^{c} [\lambda_i > \bar{\lambda}]$. In this case, we accept or reject H^* according as

$$\frac{\ell_1}{\ell_1+\ldots+\ell_{r-1}} \lessgtr c_{6\alpha} \tag{3.22}$$

where

$$P\left[\frac{\ell_1}{\ell_1+\ldots+\ell_{r-1}} \leq c_{6\alpha} \,\middle|\, H^*\right] = (1-\alpha). \tag{3.23}$$

When H^* is rejected, we accept or reject the hypothesis $\lambda_i = \bar{\lambda}$ $(i=1,\ldots,c)$ against $\bigcup_{i=1}^{c} \left[\lambda_i > \bar{\lambda}\right]$ according as

$$\frac{\ell_1}{\ell_1+\ldots+\ell_{r-1}} \lesseqgtr c_{6\alpha}. \tag{3.24}$$

Here we note that the hypothesis $\lambda_i = \bar{\lambda}$ is equivalent to the hypothesis that $\theta_i^2 = (\theta_1^2+\ldots+\theta_c^2)/(r-1)$. We may decompose H^* as $H^* = \bigcap_{i=1}^{c} \{\lambda_i = \bar{\lambda}^*\}$ where $(c+1)\bar{\lambda}^* = \lambda_1+\ldots+\lambda_{c+1}$. In view of this decomposition, we propose the following procedure for testing H^* against $\bigcup_{i=1}^{c} [\lambda_i > \bar{\lambda}^*]$. We accept or reject H^* according as

$$\frac{\ell_1}{\ell_1+\ldots+\ell_{c+1}} \lesseqgtr c_{7\alpha} \tag{3.25}$$

where

$$P\left[\frac{\ell_1}{\ell_1+\ldots+\ell_{c+1}} \leq c_{7\alpha} \middle| H^*\right] = (1-\alpha). \tag{3.26}$$

When H^* is rejected, the hypothesis $\lambda_i = \bar{\lambda}^*$ is accepted or rejected according as

$$\frac{\ell_i}{\ell_1+\ldots+\ell_{c+1}} \lesseqgtr c_{7\alpha}. \tag{3.27}$$

In the above procedure, we may replace ℓ_{c+1} with $(\ell_{c+1}+\ldots+\ell_{r-1})/(r-c-1)$ and apply the test.

Next, consider the problem of testing the hypothesis $(r-c-1)(\lambda_1+\ldots+\lambda_c) = c(\lambda_{c+1}+\ldots+\lambda_{r-1})$ against the alternative that $(r-c-1)(\lambda_1+\ldots+\lambda_c) > c(\lambda_{c+1}+\ldots+\lambda_{r-1})$. In this case, the hypothesis is accepted if

$$\frac{\ell_1+\ldots+\ell_c}{\ell_{c+1}+\ldots+\ell_{r-1}} \leq c_{8\alpha} \tag{3.28}$$

and rejected otherwise where

$$P\left[\frac{\ell_1+\ldots+\ell_c}{\ell_{c+1}+\ldots+\ell_{r-1}} \leq c_{8\alpha} \,\middle|\, H^*\right] = (1-\alpha). \tag{3.29}$$

Here, we note that the hypothesis $(r-c-1)(\lambda_1+\ldots+\lambda_c) = c(\lambda_{c+1}+\ldots+\lambda_{r-1})$ is equivalent to the hypothesis that $\theta_1 = \ldots = \theta_c = 0$.

Next, consider the problem of testing the hypothesis $H_{(a)}$ where $H^*_{(a)}: \lambda_a = \lambda_{a+1} = \ldots = \lambda_c = \lambda_{c+1}$. We can express $H^*_{(a)}$ as $\bigcap_{i=1}^{c} H^*_{(a)i}$ where $H^*_{(a)i}$: $(r-a)\,\lambda_i = (\lambda_a+\ldots+\lambda_{r-1})$. Also, let $A^*_{(a)i}$: $(r-a)\,\lambda_i > (\lambda_a+\ldots+\lambda_{r-1})$. Then, the hypothesis $H^*_{(a)}$ is accepted if

$$\frac{\ell_a}{\ell_a+\ldots+\ell_{r-1}} \leq c_{9\alpha} \tag{3.30}$$

and rejected otherwise where

$$P\left[\frac{\ell_a}{\ell_a+\ldots+\ell_{r-1}} \leq c_{9\alpha} \,\middle|\, H^*_{(a)}\right] = (1-\alpha). \tag{3.31}$$

But the distribution of $\ell_a/(\ell_a+\ldots+\ell_{r-1})$ involves $\theta_1,\ldots,\theta_{a-1}$ as nuisance parameters even when $H^*_{(a)}$ is true. So, the above test cannot be applied unless bounds (free from nuisance parameters) are obtained on the distribution of the above test statistics. Here, we note that the hypothesis $H^*_{(a)}$ is equivalent to the hypothesis that $\theta_a=\ldots=\theta_c=0$, and $A^*_{(a)i}$ is equivalent to the hypothesis that $\theta_i^2 > (\theta_a^2+\ldots+\theta_c^2)/(r-a)$. Procedures similar to the above can be proposed for testing $H^*_{(a)}$ against alternatives $\bigcup_{i=a}^{c} [\lambda_i > \lambda_{c+1}]$ and $\bigcup_{i=a}^{c} [\lambda_i > \lambda_{i+1}]$.

Next, consider the problem of testing the hypothesis $H_{oab}(r-a-b)(\lambda_a+\ldots+\lambda_c) = (c-a+1)(\lambda_{a+b}+\ldots+\lambda_{r-1})$ against the alternative that $(r-a-b)(\lambda_a+\ldots+\lambda_c) > (c-a+1)(\lambda_{a+b}+\ldots+\lambda_{r-1})$. In this case we accept or reject the null hypothesis according as

$$\frac{(\ell_a+\ldots+\ell_c)}{(\ell_{a+b}+\ldots+\ell_{r-1})} \lessgtr c_{10\alpha} \tag{3.32}$$

where

$$P\left[\frac{\ell_a+\ldots+\ell_c}{\ell_{a+b}+\ldots+\ell_{r-1}} \le c_{10\alpha} \middle| H_{oab}\right] = (1-\alpha). \tag{3.33}$$

The distribution of the test statistics in (3.32) involves nuisance parameters even when H_{oab} is true and so bounds free from nuisance parameters should be obtained to apply this procedure. Here we note that H_{oab} is equivalent to the hypothesis that $(r-a-b)\sum_{i=1}^{c} \theta_i^2 = (c-a+1)\sum_{a+b}^{c} \theta_i^2$, that is $\sum_{i=a+b}^{c} \theta_i^2\,(r-b-c-1)$ $+ (r-b-c-1)\sum_{i=a+b}^{c} \theta_i^2 + (r-a-b)\sum_{i=a}^{a+b-1} \theta_i^2 = 0$ and so $\theta_1\ldots=\theta_c=0$.

We now will discuss the likelihood ratio test statistics for testing the hypotheses $\theta_i=0$ and observe the relationship of these procedures with the procedures discussed above. Corsten and van Eijnbergen (1972) derived the likelihood ratio test statistics for testing the hypothesis that $H: \theta_1=\cdots=\theta_c=0$. The test procedure in this case is to accept or reject H according as

$$L_1 \lesseqgtr c_{11\alpha} \tag{3.34}$$

where $c_{11\alpha}$ is chosen such that

$$P[L_1 \leq c_{11\alpha}|H] = (1-\alpha) \tag{3.35}$$

where $L_1 = (\ell_1+\ldots+\ell_c)/(\ell_1+\ell_2+\ldots+\ell_{r-1})$.
When c=1, Johnson and Graybill (1972) derived the likelihood ratio test independently. The distribution of L_1 for $c > 1$ is not known but a program is available to compute the percentage points of L_1, by using Monte Carlo methods. Here we note that the likelihood ratio test statistic described above is equivalent to the test statistic for testing the hypothesis that $(r-c-1)\sum_{i=1}^{c}\lambda_i = c(\sum_{i=c+1}^{r-1}\lambda_i)$ against the alternative $(r-c-1)\sum_{i=1}^{c}\lambda_i > \sum_{i=c+1}^{r-1}\lambda_i$.

When c=1, the likelihood ratio statistic L_1, is equivalent to the test statistic given in (3.22) for testing H^* against $\lambda_1 > \bar{\lambda}$. Yochmowitz(1974a,b) and Yochmowitz and Cornell (1978) discussed the likelihood ratio statistic for testing the hypothesis $\theta_j=0$ against the alternative $\theta_j\neq0$ and $\theta_{j+1}=0$. The test procedure in this case is to accept or reject the null hypothesis according as

$$T_j \lessgtr c_{12\alpha} \tag{3.36}$$

where

$$P\,[T_j \leq c_{12\alpha} | \theta_j=0] = (1-\alpha) \tag{3.37}$$

and

$$T_j = \ell_j/(\ell_j+\ell_{j+1}+\cdots+\ell_{r-1}). \tag{3.38}$$

But the distribution of T_j even in the null case involves $\theta_1,\ldots,\theta_{j-1}$ as nuisance parameters. When c=2, Hegemann and Johnson (1976) have independently discussed the likelihood ratio test for $\theta_2=0$. Krishnaiah (1978) discussed the likelihood ratio test for $\theta_j=0$ against the alternative that $\theta_j\neq0, \theta_{j+1}\neq0,\ldots,\theta_{j+a}\neq0,\ \theta_{j+a+1}=0$.

Yochmowitz and Cornell (1978) discussed a step-wise procedure to test θ_j's by making use of the distribution of $\ell_1/(\ell_1+\ldots+\ell_{r-1})$ considered by Schuurmann, Krishnaiah and Chattopadhyay (1973). At the first stage, the hypothesis $\theta_1=0$ is accepted or rejected according as

$$T_1 \lessgtr c_{13\alpha} \tag{3.39}$$

where

$$P[T_1 \leq c_{13\alpha} | \theta_1=0] = (1-\alpha). \tag{3.40}$$

If the hypothesis of $\theta_1=0$ is accepted and T_j was defined by (3.41), we do not proceed further. If $\theta_1=0$ is rejected, we proceed further and accept or reject $\theta_2=0$ according as

$$T_2 \lesseqgtr c_{14\alpha} \tag{3.41}$$

where

$$P[T_2 \leq c_{14\alpha} | \theta_2=0] = (1-\alpha) . \tag{3.42}$$

If the hypothesis of $\theta_2=0$ is accepted, we do not proceed further. Otherwise, we proceed and test the hypothesis of $\theta_3=0$ by using T_3 as test statistic. This procedure is continued until $\theta_j=0$ is accepted for any j or $\theta_c=0$ is rejected. At the first stage, the test can be implemented since the null distribution of T_1 is free from nuisance parameters. But the distribution of T_j $(j=2,\ldots,c)$ involves $\theta_1,\ldots,\theta_{j-1}$ as nuisance parameters. As an <u>ad hoc</u> procedure, Yochmowitz and Cornell assumed that the joint distribution of $\ell_j \geq \ldots \geq \ell_{r-1}$ is approximately equivalent to the joint density of the roots of the central Wishart matrix W_j of order $(r-j)x(r-j)$ with $(s-1)$ degrees of freedom and $E(s_j/s-1) = \sigma^2 I_{r-j}$. Johnson and Graybill (1972) and

Yochmowitz and Cornell (1978) suggested approximations of T_1 with central F distribution.

Gollob (1968) and Mandel (1969) considered the problem of testing the hypotheses on θ_j's. The tests of Gollob were motivated by the assumption that the eigenvalues ℓ_j are distributed independently as chi-square variables. But these eigenvalues are neither distributed independently nor as chi-square variables. Mandel (1969) computed $\nu_j = E(\ell_j)$ by using Monte Carlo methods. Using these values of ν_j, he suggested heuristically to examine the magnitude of $\ell_j/\nu_j\hat{\sigma}^2$ to determine as to which of the θ_j's are significant; here $\hat{\sigma}^2 = (\ell_{c+1}+\ldots+\ell_{r-1})/(\nu_{c+1}+\ldots+\nu_{r-1})$. But Mandel did not consider the evaluation of the distribution of $\ell_j/\nu_j\hat{\sigma}^2$.

For discussions on tests for the structure of interaction term in two-way classification with replications, the reader is referred to Gollob (1968) and Krishnaiah (1979).

4. TESTS FOR THE MAIN EFFECTS

In this section, we discuss the problem of testing the main effects in presence of interaction. Let H_{01} denote the hypothesis of no block effect and let H_{02} denote the hypothesis of no treatment effect. The sum of squares associated with variation between blocks is given by s_3^2 where $s_3^2 = s \sum_{i=1}^{r} (\bar{x}_{i.} - \bar{x}_{..})^2$. Similarly, the sum of squares associated with variation between treatments is denoted by s_4^2 where $s_4^2 = r \sum_{j=1}^{s} (\bar{x}_{.j} - \bar{x}_{..})^2$. We know that

$$E(s_3^2/r-1) = \sigma^2 + (\sum \alpha_i^2/b-1) \tag{4.1}$$

$$E(s_4^2/s-1) = \sigma^2 + (\sum \beta_j^2/t-1) \ . \tag{4.2}$$

Now let,

$$F_{01} = \frac{s_3^2}{\hat{\sigma}^2 \ (r-1)} \ , \tag{4.3}$$

$$F_{02} = \frac{s_4^2}{(s-1) \ \hat{\sigma}^2} \ . \tag{4.4}$$

where $\hat{\sigma}^2$ is an estimate of σ^2. We may divide the data into two sets and use one set to estimate σ^2 and the other set to test H_{01} and H_{02}. Another possibility is to use some previous set of data to estimate σ^2. Of course, we can use the maximum likelihood estimate of σ^2. Also the maximum likelihood estimate of σ^2 is known (e.g., see Johnson and Graybill (1972)) to be $(\ell_{c+1}+\ldots+\ell_{b-1})/bt$. If we are testing H_{0i} individually, we accept or reject H_{0i} according as

$$F_{0i} \lessgtr F_{1\alpha} \tag{4.5}$$

where

$$P[F_{0i} \leq F_{1\alpha} | H_{0i}] = (1-\alpha), \tag{4.6}$$

and

$$F_{01} = \frac{s_3^2}{(b-1)\ \hat{\sigma}^2}\ , \tag{4.7}$$

$$F_{02} = \frac{s_4^2}{(t-1)\ \hat{\sigma}^2}\ . \tag{4.8}$$

When the interaction is present, the distribution of $(\ell_{c+1}+\ldots+\ell_{r-1})$ is not only complicated but also involves nuisance parameters. If we are testing H_{01} and H_{02} simultaneously, we accept or reject H_{0i} according as

$$F_{0i} \lessgtr F_{\alpha} \tag{4.9}$$

where

$$P[F_{0i} \leq F_{\alpha};\ i=1,2 | H_{01} \cap H_{02}] = (1-\alpha) \tag{4.10}$$

The critical values F_α can be obtained by using Monte Carlo methods. The statistics F_{01} and F_{02} are the like-

lihood ratio statistics (see Yochmowitz (1974))
for testing H_{01} and H_{02} respectively, if $\hat{\sigma}^2$ is the maximum likelihood estimate of σ^2. When c=1, this was pointed out in Johnson and Graybill (1972).
Next, let

$$F_1 = (b-1)(t-1)\, s_3^2/(b-1)\, s_e^2 \;, \quad F_2 = (b-1)(t-1)s_4^2/(t-1)s_e^2$$

where s_e^2 was defined by (2. 5). The statistics F_1 and F_2 have been used extensively to test the hypotheses of no block effect and no treatment effect respectively, under two-way classification additive model with one observation per cell. But if the true model is (2.1), then the statistics F_1 and F_2 are distributed as doubly noncentral F distribution with nuisnace paramenters even in the null cases. So, the usual F tests are no longer valid. Approximations to doubly noncentral F distribution were discussed in Mudholkar, Chaubey and Lin (1976).

5. ILLUSTRATIVE EXAMPLES

In this section, we illustrate the methods described before with real data sets. Table 1 gives data from an experiment* involving the effects of doses A, B, C, D of benactyzine upon the performance of trained rhesus monkeys where A = 0.54 mg/kg, B = 0.17 mg/kg, C = 0.054 mg/kg and D = 1.7 mg/kg.

The subjects were trained to control the position of a primate equilibrium platform (see Yochmowitz, Patrick, Jaeger and Barnes (1977a)) and to press fire and alert buttons on an instrument panel upon their illumination. The platform was perturbed by a random signal and the alert light was triggered at random. The alert light caused one of four fire buttons to light at random. Data were collected at three minute intervals and included the adjusted RMS (i.e., the root mean square position of the platform adjusted about its mean position (see Yochmowitz, Patrick, Jaeger and Barnes (1977b) and the reaction times necessary to extinguish the alert and fire lights. Animal training costs prevented extensive testing and the experiment was limited to 4 subjects. The treatments were administered in the following counter-balanced design:

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*The animals involved in this study were procured, maintained, and used in accordance with the Animal Welfare Act of 1970 and the "Guide for the Care and Use of Laboratory Animals" prepared by the Institute of Laboratory Animal Resources - National Research Council.

	Trial			
Subject	1	2	3	4
1	A	B	C	D
2	B	C	D	A
3	C	D	A	B
4	D	A	B	C

Trials were preceded by a diluent run which served as a standard against which succeeding treatments were compared. For a detailed description of the experiment, the reader is referred to Farrer et al (1979). Z-scores were computed for each variable as follows:

$$Z = \frac{X - Y_p}{s}$$

X is the mean 3 minute score over a 30 minute test period. Y_p is the corresponding predicted level of performance from a linear least squares fit to the preceding diluent run and s is the root mean square error from the linear fit. Z-scores less than -3 represent unusually good performance relative to the preceding diluent run. Conversely, z-scores in excess of 3 represent unusually poor performance relative to the preceding diluent run.

TABLE 1

Mean Adjusted RMS Z-Scores

Trial

Subject	1	2	3	4
1	A 7.26	B 0.27	C -0.80	D 1.91
2	B -0.61	C -0.24	D -0.55	A -1.29
3	C 0.65	D 3.83	A -0.75	B 1.3
4	D 1.99	A 0.02	B -0.63	C -0.8

ANOVA table for the data in TABLE 1 is given below:

Source	S.S.	D.F.	M.S
Subjects	18.539	3	6.180
Trials	19.156	3	6.385
Doses	11.769	3	3.923
Residual	24.002	6	4.000

The sum of squares due to non-additivity is 19.76. The test statistic associated with Tukey's test for non-additivity is 23.35. The critical value from F tables with (1,5) degrees of freedom at 5% level is 6.61. So, we reject the hypothesis of additivity.

In other studies (see Boster (1978)), biochemical measurements* are taken on male and female rhesus monkeys in a long term chronic study. Cholesterol measurements in milligrams per deciliter (MG/DL) on 19 males serving as controls are provided in the following table. 771, 772 and 773 respectively represent the first, second and third test periods in 1977. Similarly, 781, 782 and 783 are the first, second and third test periods in 1978.

Subjects	771	772	773	781	782	783
1	125	105	106	107	130	158
2	122	106	93	97	126	126
3	116	84	89	118	129	130
4	111	149	73	101	130	148
5	120	88	104	116	124	173
6	127	231	139	109	138	164
7	135	94	142	98	119	148
8	130	103	127	124	132	149
9	170	120	125	173	160	196
10	132	105	132	117	136	158
11	121	149	104	107	94	120
12	108	76	108	112	116	132
13	134	75	112	107	113	148
14	105	128	141	108	135	143
15	143	119	114	118	153	145
16	110	86	99	102	100	117
17	119	91	105	123	121	149
18	124	98	118	103	102	127
19	107	99	98	77	110	125

*The animals involved in this study were procured, maintained, and used in accordance with the Animal Welfare Act of 1970 and the "Guide for the Care and Use of Laboratory Animals" prepared by the Institute of Laboratory Animal Resources - National Research Council.

We assume the model (2.1) with interaction term given by (3.1). We assume that y_{ij} represents the observation made on j-th subject (male monkey) at i-th time period. In the notation of the model (2.1), we have $r = 6$ and $s = 19$. We also assume that $c = 1$. The non-zero eigenvalues of DD' in this case are $\ell_1 = 19{,}519.2$, $\ell_2 = 5263.3$, $\ell_3 = 2184.8$, $\ell_4 = 1{,}667.7$ and $\ell_5 = 1255.7$. In this case, we have $\ell_1/\mathrm{tr}DD' = 0.653$. We apply the procedure given by (3.10) - (3.12) to test $\theta_1 = 0$. Upper 5% point of the distribution of $\ell_1/\mathrm{tr}DD'$ is given by the entry corresponding to $\alpha = 0.05$, $j = 1$, $p = 5$, $r = 6$ in Table 19 of Chapter 24 in this volume; this percentage point is 0.4531. But $\ell_1/\mathrm{tr}DD'$ calculated from the data is greater than 0.4531 and so the hypothesis $\theta_1 = 0$ is rejected. Here $\theta_1 = 0$ is the hypothesis of no interaction between subjects and time periods.

REFERENCES

Anscombe, F. and Tukey, J. (1963). The examination and analysis of residuals. Technometrics 5, 141-160.

Cornelius, P. L., Templeton, W. C. and Taylor, T. H. (1979). Curve fitting by regression on smoothed singular vectors. Biometrics 35, 849-859.

Corsten, L. C. A. and van Eijnsbergen, A. C. (1972). Multiplicative effects in two-way and analysis of variance. Statistica Neerlandica 26, 61-68.

Davis, A. W. (1972). On the ratios of the individual latent roots to the trace of a Wishart matrix. J. Multivariate Anal., 2, 440-443.

Eckart, C. and Young, G. (1936). The approximation of one matrix by another of lower rank. Psychometrika 1, 211-218.

Elston, R. C. (1961). On additivity in the analysis of variances. Biometrics 17, 209-219.

Farrer, D., Yochmowitz, M., Mattsson, J., Barnes, D., Lof, N., Bachman, J. and Bennett, C. (1979). Behavioral effects of benactyzine maintenance and a multiple response task. USAF School of Aerospace Medicine, SAM-TR-79-19, Brooks AFB, Texas.

Fisher, R. A. and Mackenzie, W. A. (1923). Studies in crop variation. II. The manurial response of different potato varieties. J. Agric. Sci., 13, 311- .

Ghosh, M. N. and Sharma, D. (1963). Power of Tukey's test for non-additivity. J. Roy. Statist. Soc. Ser. B., 25, 213-219.

Gollob, H. F. (1968). A statistical model which combines features of factor analytic and analysis of variance techniques. Psychometrika 33, 73-116.

Hegemann, V. and Johnson, D. E. (1976a). On analyzing two-way AoV data with interaction. Technometrics 18, 273-281.

Hegemann, V. and Johnson, D. E. (1976b). The power of two tests for non-additivity. J. Amer. Statist. Assoc., 71, 945-948.

Johnson, D. E. and Graybill, F. A. (1972a). An analysis of a two-way model with interaction and no replication. J. Amer. Statist. Assoc., 67, 862-868.

Johnson, D. E. and Graybill, F. A. (1972b). Estimation of σ^2 in a two-way classification model with interaction. J. Amer. Statist. Assoc., 67, 388-394

Krishnaiah, P. R. and Waikar, V. B. (1971). Simultaneous tests for equality of latent roots against certain alternatives -I. Ann. Inst. Statist. Math., 23, 451-468.

Krishnaiah, P. R. and Waikar, V. B. (1972). Simultaneous tests for equality of latent roots against certain alternatives -II. Ann. Inst. Statist. Math., 24, 81-85.

Krishnaiah, P. R. and Schuurmann, F. J. (1974). On the evaluation of some distributions that arise in simultaneous tests for the equality of the latent roots of the covariance matrix. J. Multivariate Anal., 4, 265-282.

Krishnaiah, P. R. (1978). Some recent developments on real multivariate distributions. In Developments in Statistics, Vol. 1 (P. R. Krishnaiah, editor). Academic Press, Inc.

Krishnaiah, P. R. (1979). Computations of some multivariate distributions. In Handbook of Statistics, Vol. 1 (P. R. Krishnaiah, editor). North-Holland Publishing Company.

Krishnaiah, P. R. (1979). Some developments on simultaneous test procedures. In Developments in Statistics, Vol. 2 (P. R. Krishnaiah, editor). Academic Press, Inc.

Mandel, J. (1961). Non-additivity in two-way analysis of variance. J. Amer. Statist. Assoc., 56, 878-888.

Mandel, J. (1969). Partitioning the interaction in analysis of variance. J. Res. Nat. Bur. Standards Sect. B., 73B, 309-328.

Mandel, J. (1971). A new analysis of variance for non-additive data. Technometrics 13, 1-18.

Milliken, G.A. and Graybill, F. A. (1970). Extensions of the general linear hypothesis. J. Amer. Statist. Assoc., 65, 797-807.

Moore, P. and Tukey, J. (1954). Answer to query 112. Biometrics 10, 562-568.

Mudholkar, G. S., Chaubey, Y. P. and Lin, C. C. (1976). Approximations for the doubly noncentral F distribution. Comm. Statist. Theory Methods A 5, 49-63.

Osborne, N. S., McKelvy, E. C. and Bearce, H. W. (1913). Density and thermal expansion of ethyl alcohol and its mixtures with water. Bulletin of the Bureau of Standards 9, 327-474.

Scheffé, H. <u>The Analysis of Variance</u>. (Sixth Printing) Wiley, New York.

Schuurmann, F. J., Krishnaiah, P. R. and Chattopadhyay, A. K. (1973a). Tables for the distributions of the ratios of the extreme roots of the trace of Wishart matrix. Aerospace Research Laboratories, TR No. 73-0010, Wright-Patterson AFB, Ohio.

Schuurmann, F. J., Krishnaiah, P. R. and Chattopadhyay, A. K. (1973b). On the distributions of the ratios of the extreme roots to the trace of the Wishart matrix. <u>J. Multivariate Anal.</u>, **3**, 445-453.

Slater, P. B. (1973). Spatial and temporal effects in residential sales prices. <u>J. Amer. Statist. Assoc.</u>, **68**, 554-561.

Snee, R. D. (1972). On the analysis of response curve data. <u>Technometrics</u> **14**, 47-62.

Snee, R. D., Acuff, S. F. and Gibson, J. R. (1979). A useful method for the analysis of growth studies. <u>Biometrics</u> **35**, 835-848.

Tukey, J. W. (1949). One degree of freedom for non-additivity. <u>Biometrics</u> **5**, 232-242.

Tukey, J. W. (1955). Answer to Query 113. <u>Biometrics</u> **11**, 111-113.

Tukey, J. W. (1962). The future of data analysis. <u>Ann. Math. Statist.</u>, **33**, 1-67.

Ward, G. C. and Dick, I. D. (1952). Non-additivity in randomized block designs and balanced incomplete block designs. <u>New Zealand Journal of Science and Technology</u> **33**, 430-436.

Williams, E. J. (1952). The interpretation of interactions in factorial experiments. <u>Biometrika</u> **39**, 65-81.

Yochmowitz, M. G. (1974). A note on partitioning the interaction in a two-way model with no replication. (Abstract). <u>IMS Bulletin</u> **80**.

Yochmowitz, M. G. (1974). Testing for multiplicative components of interaction in some fixed models. Ph.D. dissertation. University of Michigan.

Yochmowitz, M. G., Patrick, R., Jaeger, R. and Barnes, D. (1977a). Protracted radiation-stressed primate performance. <u>Aviat. Space Environ. Med.</u> **48**(7): 598-606.

Yochmowitz, M. G., Patrick, R., Jaeger, R. and Barnes, D. (1977b) New metrics for the primate equilibrium platform. Perceptual and Motor Skills **45**, 227-234.

Yochmowitz, M. G. and Cornell, R. G. (1978). Stepwise tests for multiplicative components of interaction. Technometrics **20**, 79-84.

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In this report, the authors give a review of the literature on tests for studying the structure of interaction in two-way classification with one observation per cell. Special emphasis is made on simultaneous test procedures.

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