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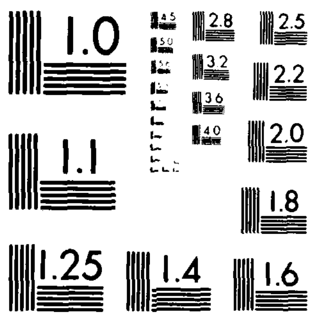

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EXPERIMENTAL RANDOM FATIGUE IN ELASTIC-PLASTIC RANGE - MODELS OF SIGNIFICANT VARIABLES.

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EXPERIMENTAL RANDOM FATIGUE IN ELASTIC-PLASTIC RANGE-
MODELS OF SIGNIFICANT VARIABLES

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ABSTRACT

In the previous study of first order models with 11 variables to predict fatigue life of materials in elastic-plastic range under random vibrations, 5 variables showed significant effects. In this report both first and second order models based on 5 significant variables have been developed. The tables of analysis of variance, and of the predicted lives together with residuals and 95% confidence intervals, are constructed for each model. Based on 24 tests a second order model of 5 significant variables consisting of 10 terms is found to be the best one. The deviations of the lives predicted by this model ranged from -34.3% to 17.4% with an average of 8.32% on the negative side and 6.35% on the positive side. These results contrast with those which are obtained by the principle of linear damage accumulation and cycle counting and involve several hundred percent error as a rule.

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INTRODUCTION

A novel methodology, based on 8 probabilistic parameters and experiment design, and used to develop first-order models which predicted the fatigue life of materials in elastic range under random vibrations, has been established in [1]. In the analysis of the experimental results in [1], the probabilistic parameters which showed significant effects on the fatigue life were then considered in developing second order models in [2]. Very accurate and reliable estimates of fatigue lives were obtained.

The same methodology, based on 11 probabilistic parameters, has been used in developing first order models for random fatigue of materials in the elastic-plastic range in [3]. The variables which showed the significant effects on the fatigue life in [3] are considered in the present report in the development of second order models. Again, very reliable and accurate estimates of fatigue lives are obtained.

I. EXPERIMENTS, PARAMETERS, DESIGNS AND MODELS

Among the 11 variables, 5 variables were found to have significant effects on the fatigue life based on the analysis of first order models for random fatigue in elastic-plastic range [3]. These five significant variables are mean, variance, zero upcrossings, ε_y level upcrossings, and the duration of excursion above ε_f level. In this report 5 more first and second order models also referred to as life predicting equations were obtained using these 5 significant variables for each of the 3 designs.

For the first two designs, the full factorial design with 2 center points and the central composite design with 4 center points, only the first order model of significant variables could be obtained because the second order model required a larger number of tests than are available in either of the two designs. For the third design, the central composite design with 4 center points and 6 replications, both the first and the second order models of significant variables have been obtained. One additional model also was obtained for this design, involving 10 terms which contributed a significant sum of squares to regression in the analysis of variance of the second order model consisting of 20 terms. Thus 5 life predicting equations are given in this report. The tables of analysis of variance, and of the predicted lives together with residuals and 95% confidence intervals were constructed for all life predicting equations of all 3 designs. The confidence intervals are computed using the standard deviation of the predicted life and the t value from the t-table with number of degrees of freedom equal to that of residuals.

II. FULL FACTORIAL DESIGN WITH TWO CENTER POINTS

This design involves the first 10 tests of the experiment in [3]. A first order life predicting equation is obtained by regressing the log of the fatigue life on the 5 coded significant variables. The second order life predicting equation could not be obtained because the number of tests in this design is insufficient.

The first order life predicting equation is given as

$$\hat{y} = 3.41 + 0.0445x_1 - 0.253x_2 - 0.162x_3 - 0.202x_5 + 0.115x_7 \quad (1)$$

The analysis of variance of equation (1) is given in Table 1. The F-ratio for this equation is computed to be 6.183 with 5 and 4 degrees of freedom. This F-ratio is smaller than the corresponding F value of 6.26 from the F table at 95% significance level. This means that the regression is not effective and that the model is not acceptable. It should be noted that the two F-values are fairly close so the model could also be accepted depending upon an individual's judgment. The residual sum of squares is 0.1483 as compared to a total of 1.2951, a 11.5%. The other 88.5% of the total is due to regression. From the analysis of variance it appears that the mean contributes a negligible sum of squares to regression but it will still be considered as a significant variable since it showed significant effect on the basis of analysis in [3].

The predicted lives together with residuals and 95% confidence intervals are given in Table 2. The actual lives of all the tests fall within the predicted confidence intervals. The confidence intervals as such are fairly wide because the t value associated with this model is high.

III. CENTRAL COMPOSITE DESIGN WITH FOUR CENTER POINTS

For this design of 18 tests, only a first order model of the significant variables is obtained. The second order model could not be obtained because the number of tests of this design is less than that required for the second order model. The first order model, obtained by regressing the log of fatigue life on the coded levels of significant variables, is given as

$$\hat{y} = 3.51 - 0.0037x_1 - 0.306x_2 - 0.0863x_3 - 0.0896x_5 + 0.0743x_7 \qquad (2)$$

Table 3 consists of analysis of variance of the above equation. The computed F-ratio of the above model is 11.25 with 5 and 12 degrees of freedom which is greater than the corresponding F value of 3.11 from the F-table at 95% significance level. This implies that the regression is effective and that the model is acceptable. The residual sum of squares is 0.3516 in comparison to a total of 2.0269, a 17.6%. The other 82.4% of the total is due to the regression. The residual sum of squares appears to be relatively high even though the model is acceptable. In this case also the mean contributes the lowest sum of squares to regression.

Table 4 consists of the predicted lives together with residuals and 95% confidence intervals. The confidence intervals are relatively narrower but the actual lives of test numbers 30, 34, 35 and 36 fall out of the predicted confidence intervals. The residuals appear to be large and have a sinusoidal pattern in them. This shows that the model needs some more terms to improve the prediction.

IV. CENTRAL COMPOSITE DESIGN WITH FOUR CENTER POINTS AND SIX REPLICATIONS

This central composite design consists of all 24 tests of the experiment. The design with six replications is shown in Fig. 1. The numbers at different locations represent the test number. One first order and 2 second order models are obtained for this design by regressing the log of fatigue life on the coded levels of significant variables. These models are described below.

1. First Order Model of 5 Significant Variables

The life predicting equation of five significant variables for 24 tests is obtained as

$$\hat{y} = 3.53 + 0.0003x_1 - 0.313x_2 - 0.0808x_3 - 0.0857x_5 + 0.0625x_7 \quad (3)$$

The analysis of variance of equation (3) is given in Table 5. The F-ratio is computed to be 19.45 with 5 and 18 degrees of freedom which is higher than corresponding F value of 2.77 from the F-table at 95% significance level. This shows that the regression is effective and that the model is acceptable. The residual sum of squares is 0.4965 as compared to a total of 3.1801, a 15.6%. The other 84.4% of the total is due to regression. Table 5 also shows that the sum of squares contributed by the mean is comparatively high but the sum of squares contributed due to duration of excursion above ε_f level is low, whereas the reverse is true in the first order models for 10 and 18 tests. These 2 variable factors will be considered for second order models before any conclusion is drawn about their effects.

Table 6 consists of the predicted lives together with residuals and 95% confidence intervals. This table shows that the actual lives of the following 8 tests 27, 30, 34, 35, 36, 43, 44 and 46 do not fall within the predicted confidence intervals. The residuals appear to be large in magnitudes. The analysis of the above model suggests that a higher order model should be tried in order to improve the prediction of the fatigue life and the confidence intervals.

2. Second Order Models of 5 Significant Variables

Two second order models have been obtained for this design. The first model consists of a complete second order polynomial of 20 terms of the 5 significant variables. The second model consists of 10 terms which are considered to have contributed a significant sum of squares to regression in the analysis of first model of this design. The analysis of both models is described below.

a. Twenty Terms The life predicting equation of all 20 terms, a complete second order polynomial of 5 variables is obtained as

$$\begin{aligned}\hat{y} = 3.70 &+ 2.62x_1 - 0.419x_2 - 0.377x_3 - 2.32x_5 - 1.07x_7 \\ &+ 0.917x_1^2 - 0.303x_2^2 - 0.934x_3^2 - 0.509x_5^2 + 0.111x_7^2 \\ &- 0.838x_1x_2 + 4.67x_1x_3 + 5.57x_1x_5 - 0.564x_1x_7 \\ &+ 0.240x_2x_3 - 1.60x_2x_5 + 1.04x_2x_7 + 0.617x_3x_5 \\ &+ 3.13x_3x_7 - 3.16x_5x_7 \end{aligned} \qquad (4)$$

The analysis of variance of equation (4) is given in Table 7. The F-ratio is found to be 7.80 with 20 and 3 degrees of freedom. The F value from the F-table with the same degrees of freedom is 8.66 at 95% significance level. The F-ratio is smaller than the F value from the F-table which means that the regression is not effective and that the model is not acceptable even though the sum of squares due regression is 98.1 percent of the total sum of squares. This is because the number of terms in the equation is large and a good regression cannot be obtained. The residual sum of squares is 0.0601 as compared to a total of 3.1801, a 1.9%. The analysis of variance of this model shows that there are several terms which contribute negligible sum of squares to regression. These terms can be eliminated in order to develop a better regression model because the number of terms is decreased with a negligible effect on the regression sum of squares. This model of 10 significant terms is described in the next section.

Table 8 consists of the predicted lives together with residuals and 95 percent confidence intervals. All the actual lives of the tests fall within the predicted confidence intervals. It should be noted that the confidence intervals are very wide as the value of t used in computing them is high because the number of degrees of freedom associated with this model is only 3.

b. Ten Terms The life predicting equation of 10 significant terms is obtained as

$$\begin{aligned} \hat{y} = 3.58 &+ 0.0008x_1 - 0.312x_2 - 0.0892x_3 - 0.115x_5 \\ &+ 0.110x_7 - 0.0436x_1^2 - 0.066x_3^2 + 0.0363x_7^2 \\ &- 0.119x_2x_5 - 0.015x_5x_7 \end{aligned} \qquad (5)$$

Table 9 consists of the analysis of variance of equation (5). The F-ratio computed for equation (5) is 16.67 with 10 and 13 degrees of freedom which is greater than the corresponding F value of 2.67 from the F-table at 95% significance level. This implies that the regression is effective and that the model is acceptable. The residual sum of squares is 0.2297 in comparison to a total of 3.1801, a 7.2%. The other 92.8% of the total sum of squares is due to regression.

Table 10 gives the predicted lives together with residuals and 95% confidence intervals. The confidence intervals are comparatively very narrow and all the actual lives are included in the intervals except for the test number 44. The actual life of this test is above the upper limit of the confidence interval by 3.2 percent. The residuals are also small in magnitudes and appear to be randomly distributed about zero level. The plot of the residuals is shown in Fig. 2.

V. DISCUSSIONS AND CONCLUSIONS

Five life predicting equations based on 5 significant variables have been obtained in this report. These five variables are mean, variance, zero upcrossings, ε_y level upcrossings and duration of excursion above ε_f level. They were identified to be significant in the investigation of the first order models in the previous report [3].

The percent deviations of the predicted lives from the actual lives and the precent residual sum of squares of the total of all the models investigated in this report and the best first order model of the previous report [3] are given in Table 11. Among all the first order models in the previous [3] and the present reports, the best model is found to be the best first order model of the previous report [3] which consists of all 11 variables based on 24 tests. However the best first order model is inferior to the second order models represented by equations (4) and (5) as evidenced from Table 11 by comparing the percent deviations of predicted lives and percent residual sum of squares.

The model of equation (4) gives the minimum percent residuals and lower percent deviations of predicted lives as shown in Table 13.

On the contrary this model gives a very wide confidence interval and does not qualify the F-test. The percent residual sum of squares and the percent deviations of predicted lives for the model of equation (5) are a little higher than the ones obtained for equation (4). But the number of terms in equation (5) are 10 less than in equation (4) and the width of the confidence intervals of the model of equation (5) on the average is only 44% of the width of the confidence intervals of equation (4) as shown in Table 12.

Considering the adequacy, accuracy of life predictions, width of the confidence intervals, residual sum of squares and the number of terms in the model, the model of equation (5) is considered to be the statistically best one. The deviations of the predicted lives from the actual lives for this model are within a range from -34.3% to 17.4% with average deviations of 8.3% on the negative side and 6.3% on the positive side.

VI. SUMMARY

(1) On the basis of analysis of first order models presented in a previous study [3], 5 variables showed significant effects on the fatigue life under random vibrations. These variables have been considered for further developing first and second order models in the present study.

(2) The first and second order models of significant variables have been developed for 3 designs. For each model the analysis of variance and predicted lives together with 95% confidence intervals are obtained.

(3) From the analysis of variance F-ratio is computed to check whether the regression is effective and the model is acceptable.

(4) The best first order model is found to be the one which consists of all 11 variables based on 24 tests as it should be.

(5) Among all the second order models investigated and the best first order model, a second order model of 10 terms based on 24 tests is found to be the statistically best one.

(6) The percent deviations of the predicted lives of the statistically best model (second order) range from -34.3% to 17.4% with average deviations of 8.3% on the negative side and 6.4% on the positive side.

ACKNOWLEDGMENTS

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2. Huang, T. C. and Nagpal, Vinod K., "Experimental Random Fatigue in Elastic Range - Models of Significant Variables," ONR Technical Report No. UW/RF-4, University of Wisconsin-Madison.
3. Huang, T. C. and Nagpal, Vinod K., "Experimental Random Fatigue in Elastic-Plastic Range - First Order Models," ONR Technical Report No. UW/FR-5, University of Wisconsin-Madison.

Table 1 Analysis of Variance of 10 Tests
First Order Model of 5 Significant Variables
Life predicting equation:

$$\hat{y} = 3.41 + 0.0445x_1 - 0.253x_2 - 0.162x_3 - 0.202x_5 + 0.115x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.0100	1	0.0100	
Due to Variance	0.8493	1	0.8493	
Due to Zero Upcrossings	0.0264	1	0.0264	
Due to ε_y Level Upcrossings	0.1718	1	0.1718	
Due to Duration of Excursion Above ε_f Level	0.0894	1	0.0894	
Due to Regression	1.1468	5	0.2294	
Residuals	0.1483	4	0.0371	6.183
Total	1.2951	9		

F-ratio is smaller than the table value of 6.26 with 5 and 4 degrees of freedom at 95% significance level. So regression is not effective and the model is rejected.

Table 2 Results of 10 Tests, First Order Model of 5 Significant Variables
Life predicting equation:

$$\hat{y} = 3.41 + 0.0445x_1 - 0.253x_2 - 0.162x_3 - 0.202x_5 + 0.115x_7$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
25	49.58	3.904	4.004	54.82	-0.100	3.629	4.379	37.68	79.74
26	50.75	3.927	3.961	52.51	-0.034	3.522	4.400	33.87	81.42
27	18.41	2.913	2.977	19.63	-0.064	2.483	3.471	11.98	32.17
28	52.50	3.957	3.800	44.70	0.157	3.439	4.161	31.16	64.13
29	22.33	3.106	3.126	22.78	-0.020	2.715	3.537	15.11	34.36
30	30.67	3.423	3.581	35.91	-0.158	3.140	4.022	23.09	55.83
31	24.00	3.178	3.125	22.76	0.053	2.706	3.544	14.97	34.61
32	30.00	3.401	3.511	33.48	-0.110	3.083	3.939	21.83	51.34
33	40.92	3.712	3.697	40.33	0.015	3.219	4.174	25.02	65.00
34	36.92	3.794	3.533	34.23	0.261	3.305	3.761	27.26	42.96

Table 3 Analysis of Variance of 18 Tests
First Order Model of 5 Significant Variables
Life predicting equation:

$$\hat{y} = 3.51 - 0.0037x_1 - 0.306x_2 - 0.0863x_3 - 0.0896x_5 + 0.0743x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio
Due to Mean	0.0108	1	0.0108	
Due to Variance	1.4728	1	1.4728	
Due to Zero Upcrossings	0.0572	1	0.0572	
Due to ε_y Level Upcrossings	0.0744	1	0.0744	
Due to Duration of Excursion Above ε_f Level	0.0557	1	0.0557	
Due to Regression	1.6708	5	0.3342	
Residuals	0.3561	12	0.0297	11.25
Total	2.0269	17		

F-ratio is greater than the table value of 3.11 with 5 and 12 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 4 Results of 18 Tests, First Order Model of 5 Significant Variables
Life predicting equation:

$$\hat{y} = 3.51 - 0.0037x_1 - 0.306x_2 - 0.0863x_3 - 0.0896x_5 + 0.0743x_7$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
25	49.58	3.904	3.974	53.20	-0.070	3.778	4.170	43.72	64.72
26	50.75	3.927	3.993	54.22	-0.066	3.697	4.289	40.31	72.92
27	18.41	2.913	3.108	22.38	-0.195	2.857	3.359	17.42	28.75
28	52.50	3.961	3.881	48.47	0.080	3.705	4.057	40.63	57.83
29	22.33	3.106	3.145	23.22	-0.039	2.875	3.415	17.72	30.42
30	30.67	3.423	3.777	43.68	-0.354	3.563	3.991	35.29	54.08
31	24.00	3.178	3.201	24.56	-0.023	2.998	3.404	20.05	30.07
32	30.00	3.401	3.387	29.58	0.014	3.187	3.587	24.20	36.14
33	40.92	3.712	3.625	37.52	0.087	3.364	3.886	28.89	48.74
34	44.42	3.794	3.566	35.37	0.228	3.451	3.681	31.52	39.71
35	38.17	3.642	3.530	34.12	0.112	3.436	3.624	31.07	37.48
36	36.92	3.609	3.488	32.72	0.121	3.379	3.597	29.34	36.49
37	33.08	3.499	3.442	31.25	0.057	3.209	3.675	24.75	39.45
38	23.33	3.150	3.045	21.01	0.105	2.838	3.252	17.08	25.84
39	30.83	3.428	3.466	32.01	-0.038	3.294	3.638	26.95	38.02
40	32.17	3.471	3.545	34.64	-0.074	3.281	3.809	26.61	45.09
41	40.42	3.699	3.846	46.81	-0.147	3.578	4.114	35.80	61.19
42	71.33	4.267	4.056	57.74	0.211	3.825	4.287	45.83	72.75

Table 5 Analysis of Variance of 24 Tests
First Order Model of 5 Significant Variables
Life predicting equation:

$$\hat{y} = 3.53 + 0.0003x_1 - 0.313x_2 - 0.0808x_3 - 0.0857x_5 + 0.0625x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio
Due to Mean	0.1223	1	0.1223	
Due to Variance	2.3905	1	2.3905	
Due to Zero Upcrossings	0.0482	1	0.0482	
Due to ε_y Level Upcrossings	0.0516	1	0.0516	
Due to Duration of Excursion Above ε_f Level	0.0262	1	0.0262	
Due to Regression	2.6836	5	0.5367	
Residual	0.4965	18	0.0276	19.45
Total	3.1801	23		

F-ratio is greater than the table value of 2.77 with 5 and 18 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 6 Results of 24 Tests, First Order Model of 5 Significant Variables
Life predicting equation:

$$\hat{y} = 3.53 + 0.0003x_1 - 0.313x_2 - 0.0808x_3 - 0.0857x_5 + 0.0625x_7$$

Test No.	Actual Life T	Actual Life y	Predicted Life $\hat{y}$	Predicted Life $\hat{T}$	Residuals $y-\hat{y}$	95% Confidence Interval $\hat{y}$ Lower	95% Confidence Interval $\hat{y}$ Upper	95% Confidence Interval $\hat{T}$ Lower	95% Confidence Interval $\hat{T}$ Upper
25	49.58	3.904	3.990	54.05	-0.086	3.822	4.158	45.69	63.95
26	50.75	3.927	4.005	54.87	-0.078	3.736	4.274	41.93	71.80
27	18.41	2.913	3.130	22.87	-0.217	2.928	3.332	18.70	27.99
28	52.50	3.961	3.905	49.65	0.056	3.752	4.058	42.59	57.88
29	22.33	3.106	3.180	24.05	-0.074	2.938	3.422	18.89	30.62
30	30.67	3.423	3.819	45.56	-0.396	3.659	3.979	38.84	53.45
31	24.00	3.178	3.221	25.05	-0.043	3.051	3.391	21.13	29.70
32	30.00	3.401	3.389	29.64	0.012	3.221	3.557	25.05	35.06
33	40.92	3.712	3.649	38.44	0.063	3.416	3.882	30.44	48.53
34	44.42	3.794	3.588	36.16	0.206	3.491	3.685	32.83	39.83
35	38.17	3.642	3.552	34.88	0.090	3.470	3.634	32.14	37.86
36	36.92	3.609	3.516	33.65	0.093	3.430	3.602	30.87	36.68
37	33.08	3.499	3.475	32.30	0.024	3.278	3.672	26.51	39.35
38	23.33	3.150	3.053	21.18	0.097	2.895	3.211	18.09	24.79
39	30.83	3.428	3.499	33.08	-0.071	3.348	3.650	28.44	38.49
40	32.17	3.471	3.554	34.95	-0.083	3.333	3.775	28.03	43.58
41	40.42	3.699	3.857	47.32	-0.158	3.611	4.103	37.01	60.51
42	71.33	4.267	4.100	60.34	0.167	3.928	4.272	50.79	71.69
43	19.75	2.983	3.170	23.81	-0.187	3.027	3.313	20.64	27.46
44	29.92	3.399	3.206	24.68	0.193	3.076	3.336	21.67	28.11
45	48.17	3.875	3.852	47.09	0.023	3.694	4.010	40.22	55.12
46	44.50	3.795	3.596	36.45	0.199	3.508	3.684	33.37	39.81
47	21.00	3.045	2.964	19.38	0.081	2.796	3.132	16.38	22.92
48	64.92	4.173	4.073	58.73	0.100	3.924	4.222	50.59	68.18

Table 7 Analysis of Variance of 24 Tests
Second Order Model of 5 Significant Variables
Life predicting equation:

$$\hat{y} = 3.70 + 2.62x_1 - 0.419x_2 - 0.377x_3 - 2.32x_5 - 1.07x_7 + 0.917x_1^2 - 0.303x_2^2 - 0.934x_3^2 - 0.509x_5^2 + 0.111x_7^2 - 0.838x_1x_2 + 4.67x_1x_3 + 5.57x_1x_5 - 0.564x_1x_7 + 0.240x_2x_3 - 1.60x_2x_5 + 1.04x_2x_7 + 0.671x_3x_5 - 3.13x_3x_7 - 3.16x_5x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio
Due to Mean	0.1223	1	0.1223	
Due to Variance	2.3905	1	2.3905	
Due to Zero Upcrossings	0.0482	1	0.0482	
Due to ε_y Level Upcrossings	0.0812	1	0.0812	
Due to Duration of Excursion Above ε_f Level	0.0414	1	0.0414	
Due to Mean Square	0.0887	1	0.0887	
Due to Variance Square	0.0004	1	0.0004	
Due to Zero Upcrossings Square	0.1073	1	0.1073	
Due to ε_y Level Upscrossings Square	0.0071	1	0.0071	
Due to Duration of Excursion Above ε_f Level Square	0.0299	1	0.0299	
Due to Mean * Variance	0.0115	1	0.0115	
Due to Mean * Zero Upcrossings	0.0034	1	0.0034	
Due to Mean * ε_y Level Upcrossings	0.0076	1	0.0076	
Due to Mean * Duration of Excursion Above ε_f Level	0.0003	1	0.0003	
Due to Variance * Zero Upcrossings	0.0029	1	0.0029	
Due to Variance * ε_y Level Upcrossings	0.0880	1	0.0880	
Due to Variance * Duration of Excursion Above ε_f Level	0.0187	1	0.0187	
Due to Zero Upcrossings * ε_y Level Upcrossings	0.0064	1	0.0064	

Table 7 (Continued)

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio
Due to Zero Upcrossings * Duration of Excursion Above ε_f Level	0.0235	1	0.0235	
Due to ε_y Level Upcrossings * Duration of Excursion Above ε_f Level	0.0405	1	0.0405	
Due to Regression	3.1199	20	0.1560	
Residual	0.0601	3	0.0200	7.80
Total	3.1801	23		

F-ratio is smaller than the table value of 8.66 with 20 and 3 degrees of freedom at 95% significance level. So regression is not effective and the model is not accepted.

Table 8 Results of 24 Tests, Second Order Model of 5 Significant Variables

Life predicting equation:

$$\hat{y} = 3.70 + 2.62x_1 - 0.419x_2 - 0.377x_3 - 2.32x_5 - 1.07x_7 + 0.917x_1^2 - 0.303x_2^2$$
$$- 0.934x_3^2 - 0.509x_5^2 + 0.111x_7^2 - 0.838x_1x_2 + 4.67x_1x_3 + 5.57x_1x_5 - 0.554x_1x_7$$
$$+ 0.240x_2x_3 - 1.60x_2x_5 + 1.04x_2x_7 + 0.671x_3x_5 - 3.13x_3x_7 - 3.16x_5x_7$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
25	49.58	3.904	3.938	51.32	-0.034	3.502	4.374	33.18	79.36
26	50.75	3.927	3.931	50.96	-0.004	3.479	4.383	32.43	80.07
27	18.41	2.913	2.933	18.78	-0.020	2.488	3.378	12.03	29.33
28	52.50	3.961	3.936	51.21	0.025	3.494	4.378	32.91	79.70
29	22.33	3.106	3.121	22.67	-0.015	2.672	3.570	14.47	35.50
30	30.67	3.423	3.519	33.72	-0.095	3.108	3.928	22.37	50.83
31	24.00	3.173	3.133	22.94	0.045	2.713	3.553	15.07	34.92
32	30.00	3.401	3.389	29.64	0.012	2.953	3.825	19.16	45.83
33	40.92	3.712	3.713	40.98	-0.001	3.264	4.162	26.16	64.18
34	44.42	3.794	3.808	45.06	-0.014	3.385	4.231	29.51	68.80
35	33.17	3.642	3.662	38.94	-0.020	3.277	4.047	26.50	57.23
36	36.92	3.609	3.502	33.18	0.107	3.171	3.833	23.83	46.20
37	33.03	3.499	3.514	33.58	-0.015	3.069	3.959	21.51	52.43
38	23.33	3.150	3.240	25.53	-0.090	2.871	3.609	17.65	36.93
39	30.83	3.428	3.438	31.12	-0.010	2.989	3.887	19.87	48.75
40	32.17	3.471	3.475	32.30	-0.004	3.026	3.924	20.62	50.59
41	40.42	3.699	3.681	39.69	0.018	3.232	4.130	25.34	62.16
42	71.33	4.267	4.247	69.90	0.020	3.833	4.661	46.22	105.71
43	19.75	2.983	3.024	20.57	-0.041	2.598	3.458	13.43	31.51
44	29.92	3.399	3.305	27.25	0.094	2.895	3.715	18.07	41.08
45	48.17	3.875	3.787	44.12	0.088	3.373	4.201	29.18	66.73
46	44.50	3.795	3.870	47.94	-0.075	3.507	4.233	33.36	68.91
47	21.00	3.045	3.004	20.17	0.041	2.578	3.430	13.17	30.89
48	64.92	4.173	4.175	65.04	-0.002	3.784	4.566	43.97	96.20

Table 9 Analysis of Variance of 24 Tests
Second Order Model of 5 Significant Variables (10 terms)
Life predicting equation:

$$\hat{y} = 3.58 + 0.0008x_1 - 0.312x_2 - 0.0892x_3 - 0.115x_5 + 0.110x_7 - 0.0436x_1^2 - 0.066x_3^2 + 0.0363x_7^2 - 0.119x_2x_5 - 0.015x_5x_7$$

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Due to Mean	0.1223	1	0.1223	
Due to Variance	2.3905	1	2.3905	
Due to Zero Upcrossings	0.0482	1	0.0482	
Due to ε_y Level Upcrossings	0.0812	1	0.0812	
Due to Duration of Excursion Above ε_f Level	0.0414	1	0.0414	
Due to Mean Square	0.0887	1	0.0887	
Due to Zero Upcrossings Square	0.1071	1	0.1071	
Due to Duration of Excursion Above ε_f Level Square	0.0288	1	0.0288	
Due to Variance of ε_y Level Upcrossings	0.0406	1	0.0406	
Due to ε_y Level Upcrossing * Duration of Excursion Above ε_y Level	0.0016	1	0.0016	
Due to Regression	2.9504	10	0.2950	
Residuals	0.2297	13	0.0177	16.67
Total	3.1801	23		

F-ratio is greater than the table value of 2.67 with 10 and 13 degrees of freedom at 95% significance level. So the regression is effective and the model is accepted.

Table 10 Results of 24 Tests, Second Order Model of 5 Significant variables (10 terms) Life Predicting equation is

$$\hat{y} = 3.58 + 0.0008x_1 - 0.312x_2 - 0.0892x_3 - 0.115x_5 + 0.110x_7 - 0.0436x_1^2 - 0.066x_3^2 + 0.0363x_7^2 - 0.119x_2x_5 - 0.015x_5x_7$$

Test No.	Actual Life		Predicted Life		Residuals	95% Confidence Interval			
						$\hat{y}$		$\hat{T}$	
	T	y	$\hat{y}$	$\hat{T}$	$y-\hat{y}$	Lower	Upper	Lower	Upper
25	49.58	3.904	4.021	55.76	-0.117	3.836	4.206	46.32	67.12
26	50.75	3.927	3.960	52.46	-0.033	3.684	4.236	39.81	69.13
27	18.41	2.913	2.875	17.73	0.038	2.599	3.151	13.45	23.36
28	52.50	3.961	3.960	52.46	0.001	3.786	4.134	44.07	62.44
29	22.33	3.106	3.129	22.85	-0.023	2.869	3.389	17.62	29.64
30	30.67	3.423	3.718	41.18	-0.295	3.562	3.874	35.23	48.14
31	24.00	3.178	3.223	25.10	-0.045	3.076	3.370	21.67	29.08
32	30.00	3.401	3.388	29.61	0.013	3.214	3.562	24.87	35.24
33	40.92	3.712	3.691	40.08	0.021	3.433	3.949	30.97	51.88
34	44.42	3.794	3.818	45.51	-0.024	3.553	4.083	34.93	59.30
35	38.17	3.642	3.590	36.23	0.052	3.454	3.726	31.64	41.50
36	36.92	3.609	3.561	35.20	0.048	3.430	3.692	30.87	40.13
37	33.08	3.499	3.370	29,08	0.129	3.132	3.608	22.93	36.87
38	23.33	3.150	3.160	23.57	-0.010	2.981	3.339	19.71	28.18
39	30.83	3.428	3.513	33.55	-0.085	3.361	3.665	28.83	39.04
40	32.17	3.471	3.487	32.69	-0.016	3.209	3.765	24.75	43.17
41	40.42	3.699	3.643	38.21	0.056	3.372	3.914	29.12	50.12
42	71.33	4.267	4.133	62.36	0.134	3.961	4.305	52.51	74.06
43	19.75	2.983	3.119	22.62	-0.136	2.906	3.332	18.29	27.98
44	29.92	3.399	3.207	24.70	0.192	3.049	3.365	21.09	28.94
45	48.17	3.875	3.786	44.08	0.089	3.641	3.931	38.14	50.95
46	44.50	3.795	3.715	41.06	0.080	3.604	3.826	36.75	45.87
47	21.00	3.045	3.106	22.33	-0.061	2.900	3.314	18.18	27.44
48	64.92	4.173	4.170	64.72	0.003	4.023	4.317	55.87	74.97

Table 11 Comparison of Percent Deviations of Predicted Lives and Residual Sum of Squares for Six Models

Test No.	Actual Life T	Percent Deviations of Predicted Lives Eq(1)	Eq(2)	Eq(3)	Eq(4)	Eq(5)	Eq(5) of [1]
25	49.58	-10.6	-7.3	-9.1	-3.5	-12.5	-9.0
26	50.75	-3.5	-6.8	-8.1	-0.4	-3.4	-0.7
27	18.41	-6.6	-21.6	-24.2	-2.0	3.7	-9.6
28	52.50	14.9	7.7	5.4	2.5	0.1	5.5
29	22.33	-2.0	-4.0	-7.7	-1.5	-2.3	-10.6
30	30.67	-17.1	-42.4	-48.5	-9.9	-34.3	-22.2
31	24.00	5.2	-2.3	-4.4	4.4	4.6	-5.8
32	30.00	-11.6	1.4	1.2	1.2	1.3	6.0
33	40.92	1.4	8.3	6.1	-0.1	2.0	3.6
34	44.42	7.3	20.4	18.6	-1.4	-2.5	23.2
35	38.17		10.6	8.6	-17.4	5.8	5.3
36	36.92		11.4	8.8	10.1	4.7	2.4
37	33.08		5.5	2.4	-1.5	12.9	-7.9
38	23.33		9.9	9.2	-9.4	-0.7	4.4
39	30.83		-3.8	-7.3	-0.9	-8.8	-12.7
40	32.17		-7.7	-8.6	-0.4	-1.6	-26.0
41	40.42		-15.8	-17.1	1.8	5.5	-17.9
42	71.33		19.1	15.4	2.0	12.6	10.2
43	19.75			-20.6	-4.2	14.5	-2.4
44	29.92			17.5	8.9	17.4	19.8
45	48.17			2.2	8.4	8.5	3.1
46	44.50			18.1	-7.7	7.7	14.2
47	21.00			7.7	3.9	-6.3	2.2
48	64.92			9.5	-0.2	0.3	8.8
		Average Deviations					
Negative side		8.6	12.4	15.6	4.0	8.3	13.4
Positive side		7.2	10.5	9.3	4.8	6.3	8.4
		Residuals					
Percent residual sum of squares of the total		11.5	17.6	15.6	1.9	7.2	11.0

Table 12 Comparison of Width of Confidence Intervals for Two Models

Test No.	Actual Life T	Equation (4)			Equation (5)			Percent Ratio $\frac{(U-L)_4}{(U-L)_5} \times 100$
		Predicted Life $\hat{T}$	Lower Limit L	Upper Limit U	Predicted Life $\hat{T}$	Lower Limit L	Upper Limit U	
25	49.58	51.32	33.18	79.36	55.76	46.32	67.12	45.0
26	50.75	50.96	32.43	80.87	52.46	39.81	69.13	60.5
27	18.41	18.78	12.93	29.33	17.73	13.45	23.36	60.4
28	52.50	51.21	32.91	79.70	52.46	44.07	62.44	39.3
29	22.33	22.67	14.47	35.50	22.85	17.62	29.64	57.2
30	30.67	33.72	22.37	50.83	41.18	35.23	48.14	45.4
31	24.00	22.94	15.87	34.92	25.10	21.67	29.08	38.9
32	30.00	29.64	19.16	45.83	29.61	24.87	35.24	38.9
33	40.92	40.98	24.16	64.18	40.08	30.97	51.88	52.2
34	44.42	45.06	29.51	68.85	45.51	34.93	59.30	61.9
35	33.17	38.94	26.50	57.23	36.23	31.64	41.50	32.1
36	36.92	33.18	23.83	46.20	35.20	30.87	40.13	41.4
37	33.03	33.58	21.51	52.43	29.08	22.93	36.87	45.1
38	23.33	25.53	17.65	36.93	23.57	19.71	28.18	43.9
39	30.83	31.12	18.87	48.75	33.55	28.83	39.04	34.2
40	32.17	32.30	20.62	50.59	32.69	24.75	43.17	61.5
41	40.42	39.69	25.34	62.18	38.21	29.12	50.12	57.0
42	71.33	69.90	46.32	105.71	62.36	52.51	74.06	36.3
43	19.75	20.57	13.43	31.51	22.62	18.29	27.98	53.6
44	29.92	27.25	18.07	41.08	24.70	21.09	28.94	34.1
45	48.17	44.12	29.18	66.73	44.08	38.14	50.95	34.1
46	44.50	47.94	33.36	68.91	41.06	36.75	45.87	25.7
47	21.00	20.17	13.17	30.89	22.33	18.18	27.44	52.3
48	64.92	65.04	43.97	96.20	64.72	55.87	74.97	36.6
Average:								43.7%

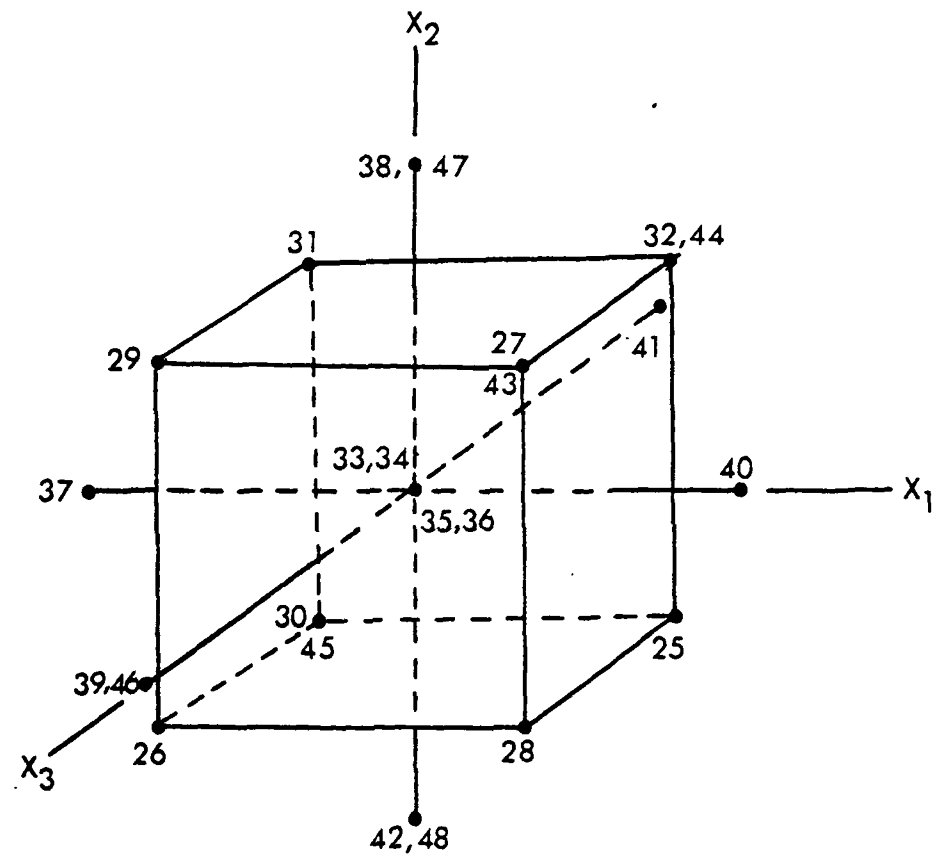

Fig. 1 Test Numbers and Test Locations for the Central Composite Design with Four Center Points and Six Replications

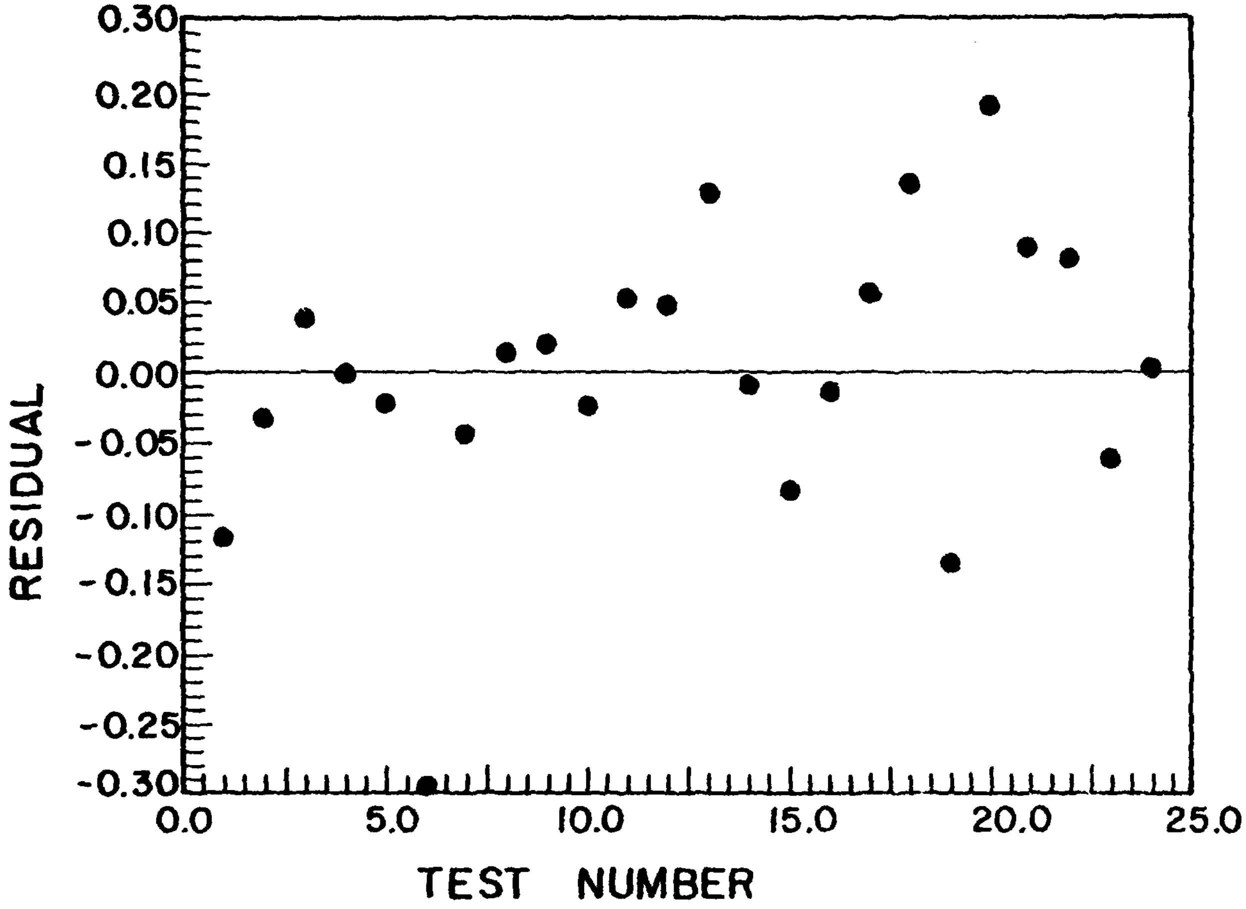

Fig. 2 Distribution of Residuals for the Best Second Order Model, Eq. (5)

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In the previous study of first order models with 11 variables to predict fatigue life of materials in elastic-plastic range under random vibrations, 5 variables showed significant effects. In this part of the research both first and second order models based on 5 significant variables have been developed. The tables of analysis of variance, and of the predicted lives together with residuals and 95% confidence intervals, are constructed for each of the first and second order models.

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