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by

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1. Introduction and Summary.

Trees in a forest, stars in a galaxy and bacteria on a petri dish are a few examples wherein there is interest in the spatial distributions of individuals over regions. In pursuit of better understanding of these distributions, attempts have been made to use probabilistic and statistical tools [Eberhardt (1967), Pielou (1969), Pollard (1971), Patil and Stiteler (1974), Cox (1976), Cox and Lewis (1976)]. Very little effort has been made to develop coherent statistical structures for the locations of individuals in a region. The main purpose of this paper is to propose such structures in order to avoid inconsistencies that can be found in the literature, to provide a sound probabilistic basis for a variety of inference procedures that are used, and to permit the development of better and consistent statistical methodology in the area.

We show that, under certain conditions, the collection of random variables representing the numbers of individuals in subsets of a region can be regarded as a stochastic process. The models presented have a stationarity property. In Section 2 we present simple random models which have, in addition to stationarity, a property of independent increments. We prove that simple models lead to a compound Poisson process. In a number of the references cited above, Poisson and negative binomial distributions have been used to describe the numbers of individuals in subsets of a region. Our formulation shows that these two distributions arise as marginal distributions in special cases of simple random models.

In Section 3 we introduce models that are stationary, but do not necessarily have the property of independent increments. We show that a negative binomial distribution proposed by Eberhardt (1967) to characterize the random behavior of the numbers of individuals in subsets of a region is a marginal

distribution resulting from two different stochastic models that do not have independent increments. Section 4 is devoted to the development of several different stochastic models when the region under consideration has finite Lebesgue measure. It is shown that the binomial distribution suggested by Eberhardt (1967) can be derived from all of these models.

2. Simple Random Models.

Some basic notation is needed to define spatial stochastic models. Throughout the paper, R will denote a Euclidean space, λ will be the Lebesgue measure on R , $X(A)$ will be the number of individuals in a subset $A \subseteq R$, and Σ will represent the set of all Borel subsets of R with finite Lebesgue measures.

Definition 2.1. Individuals are said to be located in R in a simple random way if the following conditions hold:

- (i) Stationarity: For every set $A \in \Sigma$, the distribution of the random variable $X(A)$ depends only on $\lambda(A)$.
- (ii) Independent increments: If A_1, \dots, A_m ($m > 1$) represent any m disjoint sets in Σ , the random variables, $X(A_1), \dots, X(A_m)$, are independent.

The objective of this section is to show that, under conditions (i) and (ii), the collection, $\{X(A), A \in \Sigma\}$, is a stochastic process determined by a positive number and a sequence of numbers in $[0, 1]$ summing to unity.

For any two sets A_1, A_2 in Σ and nonnegative integers k_1, k_2 , we have clearly

$$P[X(A_1) = k_1, X(A_2) = k_2] = \sum_{\substack{j+r=k_1 \\ j+l=k_2}} P[X(A_1 \cap A_2) = j, X(A_1 \cap \bar{A}_2) = r, X(\bar{A}_1 \cap A_2) = l]. \quad (2.1)$$

Now let q be an arbitrary point in R and $Y_q(t)$, the number of individuals in a sphere centered at q having Lebesgue measure $t \geq 0$, $Y_q(0) \equiv 0$. Then, using (i), (ii), we can write

$$P[X(A_1) = k_1, X(A_2) = k_2] = \sum_{\substack{j+r=k_1 \\ j+l=k_2}} P[Y_q(t_1) = j]P[Y_q(t_2) = r]P[Y_q(t_3) = l], \quad (2.2)$$

where t_1 , t_2 and t_3 are $\lambda(A_1 \cap A_2)$, $\lambda(A_1 \cap \bar{A}_2)$ and $\lambda(\bar{A}_1 \cap A_2)$ respectively. Without difficulty, equations (2.1) and (2.2) can be extended to any finite number of sets in Σ . The general result is stated in the following theorem.

Theorem 2.1. For individuals located in R in a simple random way, the joint distribution of any finite collection of random variables from $\{X(A), A \in \Sigma\}$ is determined uniquely by a joint distribution of a finite collection of random variables from $\{Y_q(t), t \geq 0\}$.

The proof is omitted. It remains to show that, under (i) and (ii), the collection $\{Y_q(t), t \geq 0\}$ is a stochastic process determined by a positive number and a probability sequence.

Conditions (i) and (ii) respectively insure that the process $Y_q(t)$ has stationary and independent increments. That is, for real numbers $0 \leq t_1 \leq t_2 \leq \dots \leq t_m$ ($m \geq 2$), the random variables $Y_q(t_1)$, $Y_q(t_2) - Y_q(t_1)$, \dots , $Y_q(t_m) - Y_q(t_{m-1})$ are independent and, moreover, $Y_q(t_l) - Y_q(t_{l-1})$ has a distribution depending only on $t_l - t_{l-1}$, $l = 1, \dots, m$ ($t_0 = 0$). Upon application of a well known characterization of such processes, it follows [see, for example, Khintchine (1960), p. 36] that $Y_q(t)$ is the compound Poisson process represented as

$$Y_q(t) = Z_1 + Z_2 + \dots + Z_{N(t)},$$

where $\{N(t), t \geq 0\}$ is a simple Poisson process and $\{Z_i, i \geq 1\}$ is a sequence of i.i.d. random variables on the positive integers, independent of $N(t)$. If $Z_i \equiv 1$, then $Y_q(t) \equiv N(t)$, whereas if Z_i has a logarithmic distribution, $Y_q(t)$ has a negative binomial distribution, as is well known. These two marginal distributions of $X(A)$ have been frequently used in the literature.

3. A Generalization of the Simple Random Models.

In this section we present two methods of developing a general stationary stochastic structure for the random quantities $\{X(A)\}$. This is done in two steps. Firstly, for $m \geq 1$, we present the joint distribution of $X(A_1), \dots, X(A_m)$, where A_1, \dots, A_m are any m disjoint sets of Σ . We then show that this set of finite dimensional distributions determines uniquely the stochastic structure of $\{X(A)\}$.

For this purpose we introduce some notation. Let $\{M(t), t \geq 0\}$ be a stochastic process on the nonnegative integers, stochastically increasing in t , that is, $M(t) \leq M(s)$ for $0 \leq t \leq s$, and $\{W_i, i \geq 1\}$, a sequence of random variables with positive integer values, independent of $\{M(t)\}$. We assume that $\{M(t), t \geq 0\}$ and $\{W_i, i \geq 1\}$ depend on the parameters θ_1 and θ_2 respectively, ranging in parameter spaces P_1 and P_2 , where P_2 is a collection of infinite sequences, P_1 and P_2 being probability spaces with F and G the respective probability measures.

Definition 3.1. Let A_1, \dots, A_m be disjoint sets in Σ with respective Lebesgue measures t_1, \dots, t_m , and let k_1, \dots, k_m be nonnegative integers. We define the joint distribution of $\{X(A_i), i = 1, \dots, m\}$ as

$$P\{X(A_i) = k_i, i = 1, \dots, m\} = \int_{P_1} \int_{P_2} \prod_{i=1}^m P\left[\sum_{j=1}^{M(t_i)} W_j = k_i\right] dF(\theta_1) dG(\theta_2). \quad (3.1)$$

Definition 3.2. With the same notation as above, we define

$$\int_{P_1 \times P_2} \int \prod_{j=r_{i-1}+1}^{r_i} W_j = k_i, \quad i = 1, \dots, m \int dF(\theta_1) dG(\theta_2), \quad (3.2)$$

where $r_i = \sum_{q=1}^i M(t_q)$, $i = 1, \dots, m$, $r_0 = 0$.

The consistency of the set of joint distributions of finite collections of random variables from $\{X(A), A \in \mathcal{E}\}$, generated by disjoint sets and defined by (3.1) or (3.2), follows from the stochastic structure imposed on $\{M(t), t \geq 0, W_i, i \geq 1\}$. The stationarity condition (i) is clearly satisfied, since definitions (3.1) and (3.2) vary only with the Lebesgue measures of the respective sets. The generalization of equation (2.1) to any finite number of sets in \mathcal{E} , not necessarily disjoint, provides a way to extend definitions (3.1) and (3.2) to any such sets. These extensions preserve the required consistency and stationarity of $\{X(A), A \in \mathcal{E}\}$. We have proved the following:

Theorem 3.1. If the joint distributions of every finite number of random variables in $\{X(A), A \in \mathcal{E}\}$ are given by the extensions of (3.1) or (3.2), then $\{X(A), A \in \mathcal{E}\}$ is a stationary stochastic process.

It is important to note that (3.1) and (3.2) yield the same marginal distributions for $X(A), A \in \mathcal{E}$, but they define different processes. Statistical methodologies based on the two models may be quite different and this has been ignored in the literature.

In Section 2, we proved that simple random models depend on a positive number, say μ , and a probabilistic sequence, say $\{P_i, i \geq 1\}$. Now we show that those models are particular cases of the stochastic processes presented by (3.2). To do so, let $\{M(t), t \geq 0\}$ be a Poisson process with parameter μ , and $\{W_i, i \geq 1\}$ be an i.i.d. sequence of random variables given by $P(W_i = i) = P_i$ ($i \geq 1$). In addition we take P_1 and P_2 to be sets containing only μ and the sequence $\{P_i\}$ respectively. Now the extension of (3.2) reduces directly to a simple random model.

Let us assume that the sequence of discrete random variables $\{W_i\}$ is degenerate at 1, P_1 is $[0, \infty)$, P_2 is only the sequence $(1, 0, 0, \dots)$, $\{M(t)\}$ is Poisson, and F is given by

$$F(x) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^x e^{-\beta y} y^{\alpha-1} dy & x > 0 \\ 0 & x \leq 0 \end{cases} \quad \alpha, \beta > 0. \quad (3.3)$$

Then the marginal distribution of $X(A)$, $A \in \Sigma$, computed by (3.1) or (3.2), is negative binomial with $a = \alpha$ and $b = \frac{\alpha}{\beta} \lambda(A)$. This marginal distribution for $X(A)$ was assumed by Eberhardt (1967) and by Patil and Stiteler (1974).

4. Models for Finite Regions

In the previous two sections, we presented stochastic models when individuals were randomly located in an infinite region. In this section, five different models are developed for situations wherein individuals are located in a Borel set $R_f \subseteq R$ of finite Lebesgue measure.

Let Σ_f be the set of all Borel sets contained in R_f and $X_f(A)$, the number of individuals in set A , $A \in \Sigma_f$. Our objective is to develop a stochastic structure for the collection $\{X_f(A), A \in \Sigma_f\}$. One way to achieve this objective is to select a stochastic process developed in Section 3 for the infinite region R , say $\{X(A), A \in \Sigma\}$, and condition it by an event related to the random variable $X(R_f)$. Specifically, let A_1, \dots, A_m be sets in Σ_f and let k_1, \dots, k_m be nonnegative integers. We define the desired probability as

$$P[X_f(A_i) = k_i, i = 1, \dots, m] = P[X(A_i) = k_i, i = 1, \dots, m | X(R_f)]. \quad (4.1)$$

The consistency and stationarity of the process $\{X_f(A), A \in \Sigma_f\}$ defined in (4.1) are self-evident.

For the four remaining alternative ways of developing a stochastic structure for $\{X_f(A)\}$, we refer to the process $\{M(t)\}$, the sequence $\{W_i\}$, the sets P_1 and P_2 , and the probability measures F and G , which were introduced in Section 3. We define first the joint distribution of $X_f(A_1), \dots, X_f(A_m)$, where A_1, \dots, A_m are disjoint sets in Σ_f and $m \geq 1$, then extend these definitions to any finite collection of sets in Σ_f . Since the extension technique has been used twice, it is omitted from our present discussion. Now let A_1, \dots, A_m be disjoint subsets of R_f , $\lambda(R_f) = t_0$, $\lambda(A_i) = t_i$, $i = 1, \dots, m$, and let k_1, \dots, k_m be nonnegative integers. We define the desired probability in the following four ways:

$$P[X_f(A_i) = k_i, i = 1, \dots, m] = \int_{P_1} \int_{P_2} \prod_{i=1}^m P\left(\sum_{j=1}^i W_j = k_i \mid \sum_{j=1}^0 W_j = M(t_0)\right) dF(\theta_1) dG(\theta_2). \quad (4.2)$$

$$P[X_f(A_i) = k_i, i = 1, \dots, m] = \int_{P_1} \int_{P_2} P\left(\sum_{j=r_{i-1}+1}^{r_i} W_j = k_i, i = 1, \dots, m \mid \sum_{j=1}^{r_0} W_j = M(t_0)\right) dF(\theta_1) dG(\theta_2), \quad (4.3)$$

$$P[X_f(A_i) = k_i, i = 1, \dots, m] = \int_{P_1} \int_{P_2} \prod_{i=1}^m P\left(\sum_{j=1}^i W_j = k_i \mid M(t_0)\right) dF(\theta_1) dG(\theta_2). \quad (4.4)$$

$$P[X_f(A_i) = k_i, i = 1, \dots, m] = \int_{P_1} \int_{P_2} P\left(\sum_{j=r_{i-1}+1}^{r_i} W_j = k_i, i = 1, \dots, m \mid M(t_0)\right) dF(\theta_1) dG(\theta_2). \quad (4.5)$$

where r_0, \dots, r_m are as in definition 3.2.

The stochastic processes determined by (4.2)-(4.5) have been constructed to be both consistent and stationary. Definitions (4.2) and (4.3) or (4.4) and (4.5), yield identical marginal distributions for $X_f(A)$, $A \in \Sigma_f$, but define different processes.

If we let $M(t)$ be a Poisson process, $\{W_i\}$, a sequence of random variables degenerate at 1, P_1 and P_2 singleton sets, then the marginal distribution of $X_f(A)$, $A \in \Sigma_f$, according to each of the definitions (4.1) - (4.5) is binomial with $X(R_f)$ corresponding to the number of Bernoulli trials and $\lambda(A)/\lambda(R_f)$ corresponding to the probability of success in a single trial.

5. Concluding Remarks.

The selection of models for stochastic processes for particular applications and the development of pertinent statistical methodologies have not been addressed in this paper. We have demonstrated that models proposed may be used to yield marginal distributions assumed in the literature; they may also be used to avoid unwarranted assumptions and inconsistencies that arise. We propose in subsequent work to use the general models of this paper to devise improved statistical methodologies for problems involving the location of individuals in a habitat.

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