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The finite segment modelling of a long flexible cable is discussed. An equivalent force system representing the fluid and gravitational forces exerted on a typical cable segment is determined. The fluid forces include inertia, drag, and hydrostatic (bouyancy) forces. Application of the results in computer algorithm development is also discussed.

A REPRESENTATION OF FLUID FORCES IN FINITE SEGMENT CABLE MODELS

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ABSTRACT

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INTRODUCTION

The objective of this report is to develop expressions for the fluid forces on a submerged segment of a finite segment cable model. It is expected that these expressions, when used with the computer algorithms of a chain link model, will provide a comprehensive yet efficient simulation of submerged cable dynamics.

There has been an increasing interest recently in the dynamics of submerged cables. This interest is due to the significant, and often deleterious, effects of the fluid forces on cable motion and cable dynamics in a wide range of towing and mooring configurations. This interest, in turn, has stimulated the development of a number of procedures for modelling and studying cable dynamics for a variety of applications. References [1-35]* provide a brief summary of some of these procedures. In a review of these procedures, Choo and Casarella [10] suggest that a promising approach to obtaining an accurate analysis of large displacement, unsteady, three-dimensional motion, is the finite segment (that is, rigid link) modelling of the cable.

Independent of these modelling procedures in cable dynamics, there have also been many recent significant advances in finite segment modelling of other mechanical systems -- particularly "multibody systems." A multibody system or "chain system" (sometimes also called

*Numbers in brackets refer to References at the end of the report.

"open-chain" or "cpen-tree" systems) is defined as a set of rigid bodies arbitrarily assembled such that adjacent bodies share a common point and such that no closed loops are formed. Figure 1. presents a sketch of such a system.

The interest in general chain systems stems from the fact that they are excellent models of many physical systems such as manipulators, cranes, robots, biodynamic systems (for example, human body models), chains, and cables.

The advances in the modelling and analysis of such systems are due to a large extent to corresponding advances in computational technology and in computer hardware. But, beyond this there have also been significant advances in the procedures for developing the governing dynamical equations of motion of these systems. One of the most promising of these procedures is based on using Lagrange's form of d'Alembert's principle to obtain the equations of motion (See References [36-40].). This principle, as exposited by Kane and others [42-44] is similar to Lagrange's equations in that non-working internal constraint forces between the bodies of the system, are automatically eliminated in the analysis. Also, the principle leads directly to the correct number of governing equations (that is, one equation for each degree of freedom). However, unlike Lagrange's equations, the principle does not require the differentiation of lengthy scalar energy functions. Instead, it uses vector derivatives which can be calculated by vector multiplication. Such calculations are ideally

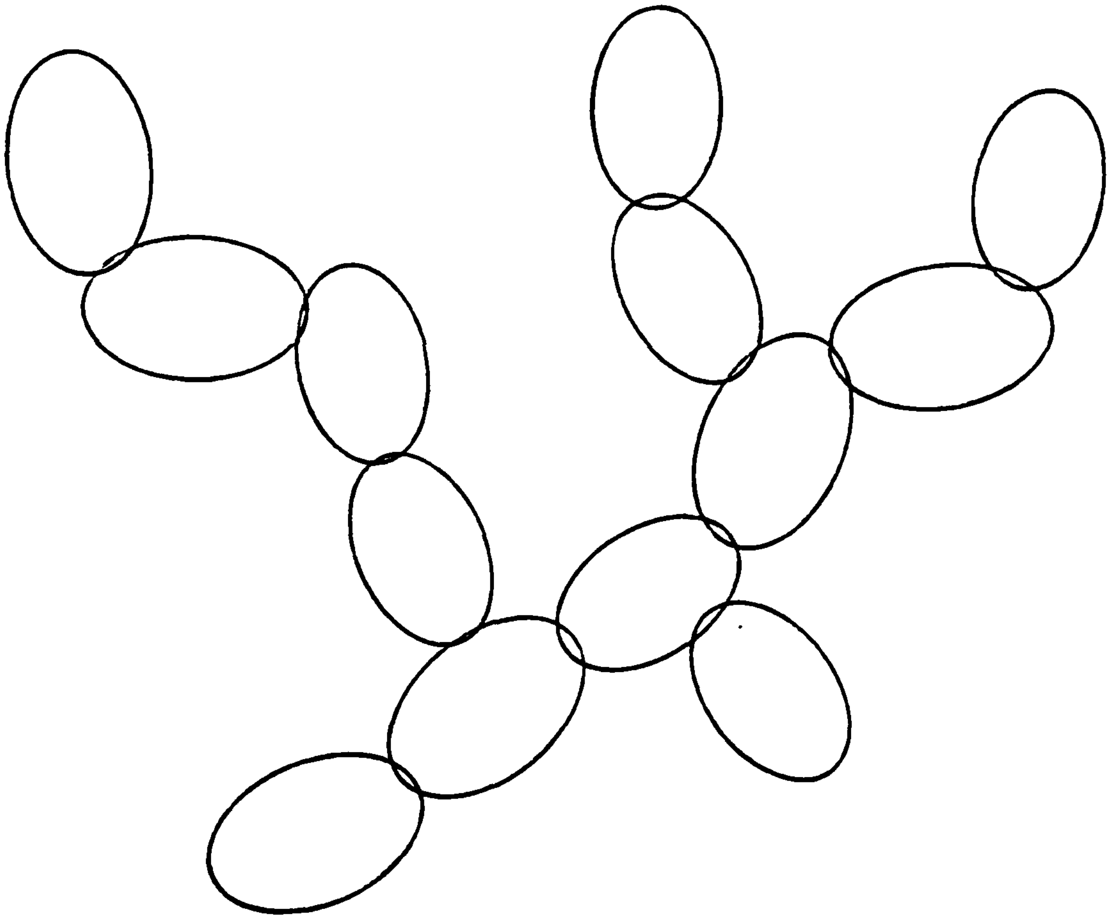

Figure 1. A General Chain System

suited for conversion into computer algorithms. Indeed, in References [36-40],explicit expressions and algorithms for obtaining the coefficients of the governing dynamical equations are presented.

This approach is directly applicable in the modelling and analysis of cable dynamics. However, to develop a cable model, that is, a link or finite-segment cable model, it is necessary to have expressions for the fluid forces acting on the cable segments. Hence, the emphasis of this report is the analysis of these fluid forces and the development of analytical expressions for them -- in a format suitable for their direct incorporation into multibody computer models as described in References [38-40,45,46].

The balance of the report is divided into six sections. The first of these provides the assumptions and the basis of the analysis used in the sequel. The next section describes the fluid inertia forces transmitted to the cable and generated by acceleration of the cable segments. This is followed by two sections describing the normal and tangential drag forces and the weight and bouyancy forces on the segments. The final section contains some concluding remarks.

PRELIMINARY CONSIDERATIONS

NOTATION

a,b,c - Coefficients defined by Equations (32), (33), and (34).

$\underset{\sim}{a}^G$ - Acceleration of the mass center G of a cable segment in an inertial reference frame R.

$\underset{\sim}{a}_{GN}$ - Acceleration of the fluid relative to G in R and normal to the cable segment axis.

$\underset{\sim}{a}_N$ - Normal component of the acceleration of the fluid relative to the cable segment.

$\underset{\sim}{a}^P$ - Acceleration of a typical point P of a cable segment in an inertial reference frame R.

$\underset{\sim}{a}^W$ - Acceleration of the fluid in an inertial reference frame R.

$\underset{\sim}{a}^{W/G}$ - Acceleration of the fluid relative to the cable segment at G, the mass center of the segment.

$\underset{\sim}{a}^{W/P}$ - Acceleration of the fluid relative to the cable segment at a typical point P.

A,B,C - Fluid force coefficients of Equations (1), (2), (3), and (4).

$\underset{\sim}{B}$ - Bouyancy force on a submerged right circular cylinder.

$\underset{\sim}{B}_N$ - Component of B normal to the cable segment axis.

$\underset{\sim}{B}_T$ - Component of B parallel to the cable segment axis.

C_M - Added mass coefficient.

C_N - Normal drag coefficient.

C_T - Tangential drag coefficient.

d - Diameter of the cylindrical cable segment.

$\underset{\sim}{f}$ - Fluid force per unit length exerted on a cable segment.

$\underset{\sim}{F}_M$ - Resultant added mass force passing through the mass center of a cable segment.

$\underset{\sim}{F}$ - Resultant fluid and gravitational force passing through the mass center G of the cable segment.

$\underset{\sim}{F}_N$ - Resultant normal drag force passing through the mass center G of the cable segment.

$\underset{\sim}{F}_T$ - Resultant tangential drag force passing through the mass center G of the cable segment.

g - Gravity constant.

G - Mass center of a typical cable segment.

$\underset{\sim}{k}$ - Vertical unit vector.

L - Length of a typical cable segment.

$\underset{\sim}{n}$ - A unit vector parallel to the cable segment axis.

P - A typical point on the cable segment (See Figure 3.).

R - An inertial reference frame.

R_e - Normal Reynolds number (See Equation (8)).

R_{eT} - Tangential Reynolds number (See Equation (9)).

$\underset{\sim}{T}$ - Torque of the couple of the equivalent fluid and gravitational force system.

$\underset{\sim}{T}_M$ - Torque of the couple of the equivalent added mass force system.

$\underset{\sim}{T}_N$ - Torque of the couple of the equivalent normal drag force system.

$\underset{\sim}{T}_T$ - Torque of the couple of the equivalent tangential drag force system.

$\underset{\sim}{V}$ - Cable segment volume.

$\underset{\sim}{V}_G$ - Velocity of the mass center G of a cable segment in an inertial reference frame R.

$\underset{\sim}{V}_{GN}$ - Velocity of the fluid relative to G in R and normal to the cable segment axis.

$\underset{\sim}{V}_N$ - Normal component of the velocity of the fluid relative to the cable segment.

$\underset{\sim}{V}^P$ - Velocity of a typical point P of the cable segment in an inertial reference frame R.

$\mathbf{V}^{P/G}$ - Velocity of the fluid relative to the cable segment at G, the mass center of the segment.

$\mathbf{V}_T$ - Tangential component of the velocity of the fluid relative to the cable.

$\mathbf{V}^W$ - Velocity of the fluid in an inertial reference frame R.

$\mathbf{W}$ - Resultant weight force on the cable segment.

x - Coordinate of P relative to G.

X - Defined by Equation (40).

$\boldsymbol{\alpha}$ - Angular acceleration of a typical cable segment.

μ - Viscosity of the fluid.

ρ - Mass density of the fluid.

ρ_c - Mass density per unit length of the cable.

$\boldsymbol{\omega}$ - Angular velocity of a typical cable segment.

CABLE MODELLING

Consider a finite segment model of a cable as shown in Figure 2. The segments are rigid cylinders connected at their ends with spherical pins. The model is thus a chain simulation of the cable. As mentioned earlier, recent developments in the analysis of multibody system dynamics has led to computer formulations of the governing equations of motion of such chain systems. These formulations have the option of accomodating models with segments of different and varying lengths. The shorter segments can then be used to model the cable in those regions where the cable has a small radius of curvature. This can be accomplished by computing the angle between the cylinder axis of the segment and the direction of the resultant force vector transmitted across the segment joint. If this angle exceeds an arbitrarily selected value, the segment length can be reduced. (Recall that for a light flexible cable, the resultant force at a cross section is nearly tangent to the cable.) This scheme can also be used to optimize the number of segments in the model.

FLUID FORCES ON THE CYLINDRICAL SEGMENTS

Consider a typical segment of the cable model as shown in Figure 3. Using results of Webster [35], the fluid force $\underset{\sim}{f}$ per unit length at a typical point P may be expressed as:

$$\underset{\sim}{f} = A\, \underset{\sim}{a}_N + B|\underset{\sim}{V}_N|\underset{\sim}{V}_N + C|\underset{\sim}{V}_T|\underset{\sim}{V}_T \qquad (1)$$

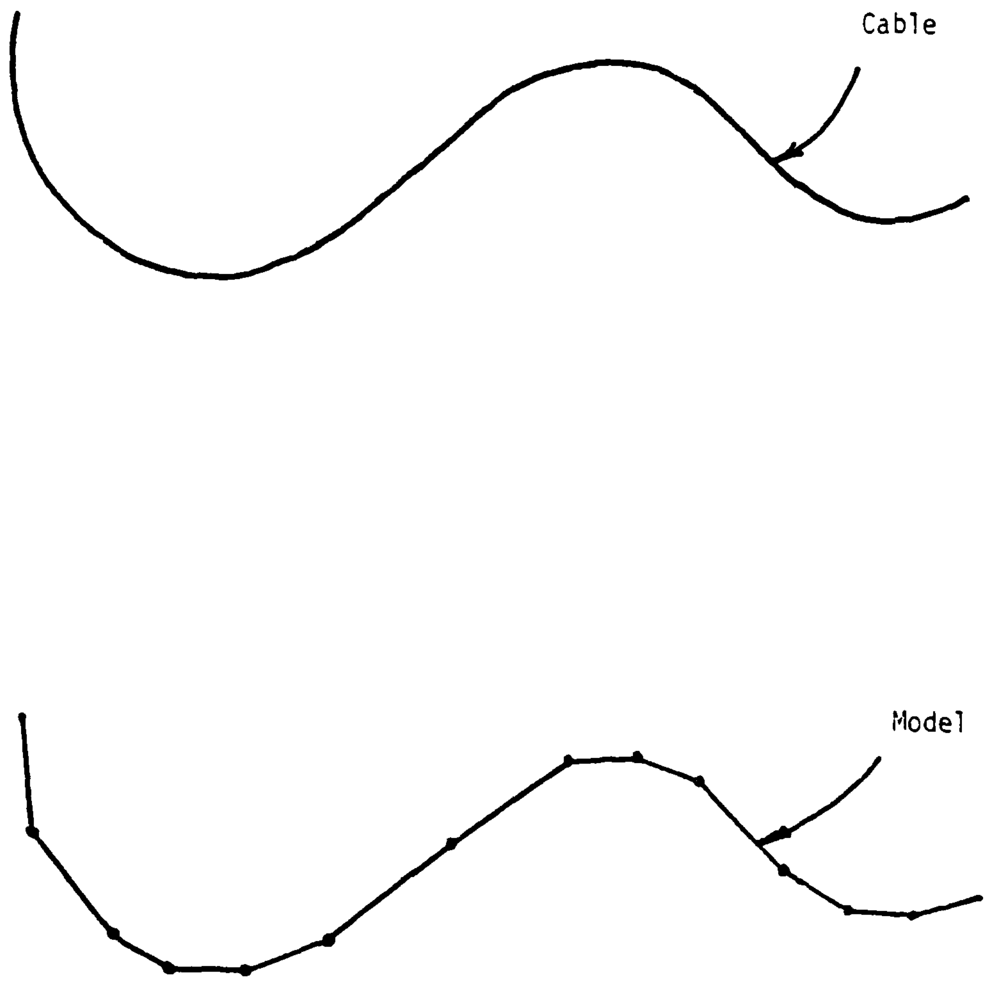

Figure 2. A Finite Segment Cable Model

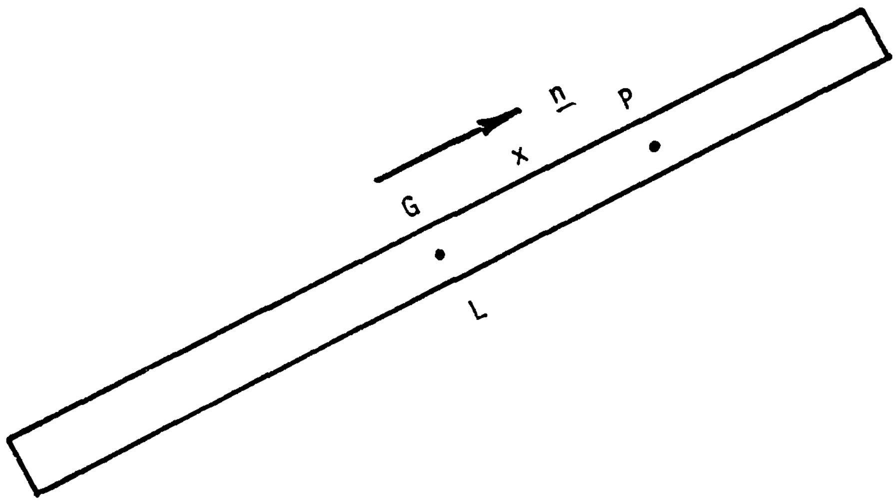

Figure 3. A Typical Cylindrical Cable Model Segment

where $\underset{\sim}{V}_N$ is the normal component of the fluid velocity relative to the cable segment at P, $\underset{\sim}{a}_N$ is the normal component of the fluid acceleration relative to the cable segment at P, $\underset{\sim}{V}_T$ is the tangential component of the fluid velocity relative to the cable segment at P, and the coefficients A, B, and C are:

$$A = C_M \rho(\pi/4)d^2 \quad (2)$$

$$B = C_N \rho(d/2) \quad (3)$$

and

$$C = C_T \rho_W (d/2) \quad (4)$$

where ρ is the fluid mass density and d is the diameter of the cylindrical segment. C_M, C_N, and C_T are coefficients dependent upon the Reynolds number of the fluid flow past the cable segment. These are usually determined experimentally and the reported results may vary slightly. Webster [35], for example, records them as:

$$C_M = 1.0 \quad (5)$$

$$C_N = \begin{cases} 0.0 \text{ for } R_e \le 0.1 \\ 0.45 + 5.93/(R_e)^{0.33} \text{ for } 0.1 < R_e \le 400. \\ 1.27 \text{ for } 400. < R_e \le 10^5 \\ 0.3 \text{ for } R_e > 10^5 \end{cases} \quad (6)$$

and

$$C_T = \begin{cases} 1.88/(R_{eT})^{0.74} \text{ for } 0.1 < R_{eT} \leq 100.55 \\ 0.062 \text{ for } R_{eT} > 100.55 \end{cases} \tag{7}$$

where the Reynolds numbers R_{eN} and R_{eT} are defined as

$$R_e = \rho d|\underset{\sim}{V}_N|/\mu \tag{8}$$

and

$$R_{eT} = \rho d|\underset{\sim}{V}_T|/\mu \tag{9}$$

where μ is the viscosity of the fluid.

Of these coefficients C_N, the normal drag coefficient is undoubtedly the most significant in affecting cable motion. It is also the most widely studied in the literature (See for example, References [47-54].) The variation of C_N with R_{eN} is often depicted graphically as in Figure 4. A linear curve fit of this graph leads to the expressions:

$$C_N = \begin{cases} 1.45 + 8.55\,(R_e)^{-0.40} \text{ for } 1 < R_e < 30 \\ 1.0 + 4(R_e)^{-1/2} \text{ for } 30 \leq R_e < 100 \\ 2.25 - 0.45\log(R_e) \text{ for } 100 \leq R_e < 1000 \\ 0.90 \text{ for } 1000 \leq R_e < 4000 \\ 1.05 + 0.54\log(R_e) \text{ for } 4000 \leq R_e < 15000 \\ 1.21 \text{ for } 15{,}000 \leq R_e < 150{,}000 \\ 0.3 \text{ for } 150{,}000 \leq R_e \end{cases} \tag{10}$$

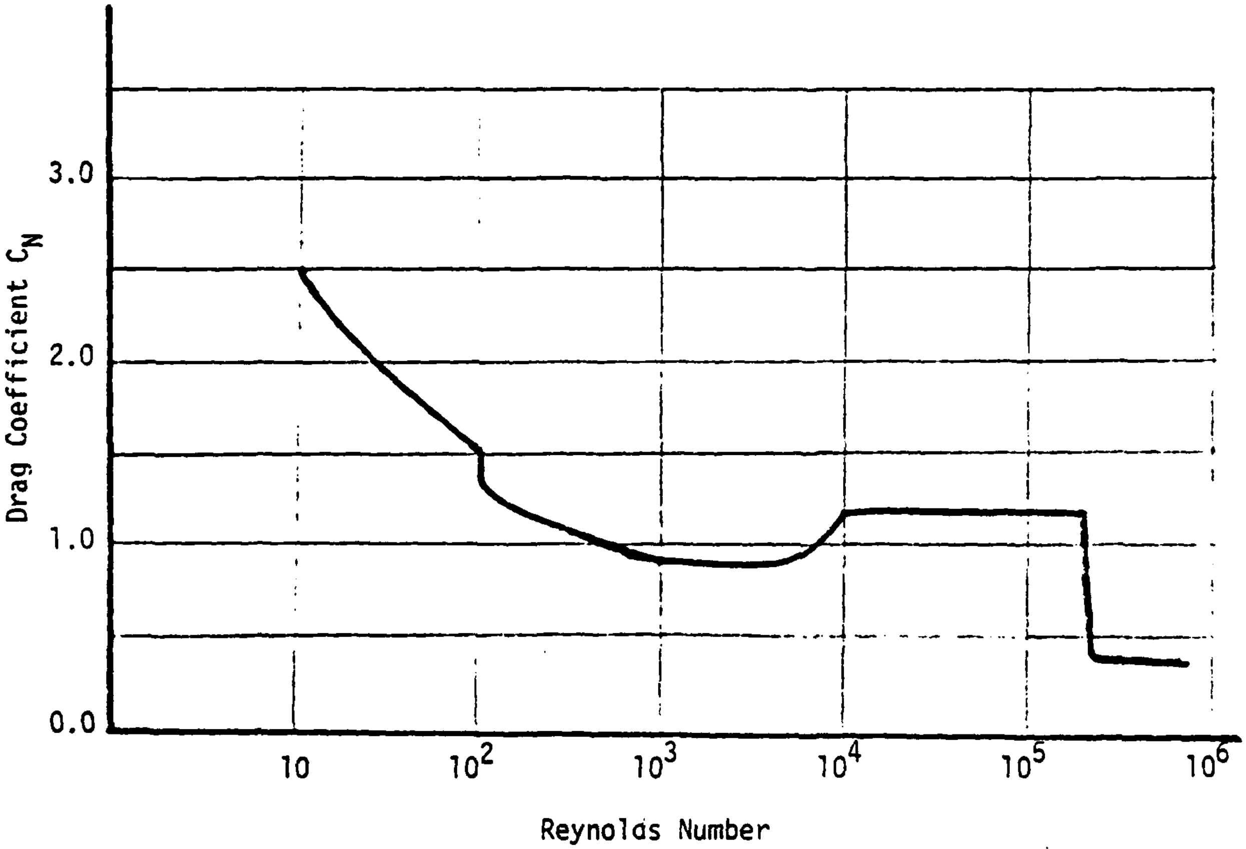

Figure 4. Normal Drag Coefficient (References [47-54]).

As mentioned above, these expressions are of slightly different form than those in Equation (6).

In Equation (1), the first term is due to the acceleration of the fluid relative to the cable segment. It is sometimes called the "added mass force" (See References [55-57]). The second term is the "drag force." The third term, the "tangential drag force," is tangent to the cable segment and it is generally smaller than the other fluid forces. The effects of each of these forces is examined in more detail in the following sections of the report. It is assumed throughout the analysis that the coefficients A, B, and C of Equation (1) are constant, or at least "slowly varying" along the length of the cable segment. Also, the fluid is assumed to be incompressible.

ADDED MASS FORCES

RELATIVE ACCELERATION OF THE FLUID NORMAL TO THE CABLE

To determine the force system exerted on a cable segment due to the added mass forces, it is necessary to develop a description of $\underset{\sim}{a}_N$, the normal component of the fluid acceleration relative to the cable segment in terms of the ambient motion of the fluid and in terms of the motion of the cable segment. To do this, let G be the mass center of a typical cable segment as shown in Figure 3. Let x locate a typical point P relative to G and let L be the length of the cable segment. Finally, let $\underset{\sim}{n}$ be a unit vector parallel to the axis of the segment as shown. Then x is positive if the vector $\underset{\sim}{GP}$ has the same sense as $\underset{\sim}{n}$.

Let $\underset{\sim}{a}_W$ represent the acceleration of the fluid in an inertial reference frame R. (Usually $\underset{\sim}{a}_W$ will be zero.) Then, at point P the acceleration of the fluid relative to the cable segment may be written as:

$$\underset{\sim}{a}^{W/P} = \underset{\sim}{a}^{W} - \underset{\sim}{a}^{P} \tag{11}$$

where $\underset{\sim}{a}^P$ is the acceleration of P in R. $\underset{\sim}{a}_P$ may be expressed in terms of $\underset{\sim}{a}_G$, the acceleration of G in R, as:

$$\underset{\sim}{a}^{P} = \underset{\sim}{a}^{G} + \underset{\sim}{a}^{P/G} \tag{12}$$

where $\underset{\sim}{a}^{P/G}$ is the acceleration of P relative to G in R. However, since both P and G are fixed on the cable segment, $\underset{\sim}{a}^{P/G}$ may in turn be written as

$$a^{P/G} = \alpha X(xn) + \omega x[\omega x(xn)] \quad (13)$$

where α and ω are the angular velocity and the angular acceleration of the segment in R. Since a_N, in Equation (1), is the component of $a^{W/P}$ perpendicular to n, it may be written as:

$$a_N = a^{W/P} - (a^{W/P} \cdot n)n \quad (14)$$

By substituting Equation (13) into (12), (12) into (11), and finally (11) into (14), a_N becomes

$$a_N = a^{W/G} - x\alpha Xn - x\omega X(\omega Xn) - (a^{W/G} \cdot n)n + x(\omega \cdot n)^2 n - x\omega^2 n \quad (15)$$

where $a^{W/G}$ is $a^W - a^G$, the acceleration of the fluid relative to the cable at G in R, and ω^2 is $\omega \cdot \omega$. Let a_{GN} be the acceleration in R of this fluid relative to G perpendicular to the segment axis. Then, a_{GN} is

$$a_{GN} = a^{W/G} - (a^{W/G} \cdot n)n \quad (16)$$

Hence, from Equations (15) and (16), a_N may be written as

$$a_N = a_{GN} - x\alpha Xn - x\omega X(\omega Xn) + x(\omega \cdot n)^2 - x\omega^2 n \quad (17)$$

RESULTANT ADDED MASS FORCE AND MOMENT

The added mass force at a typical point P of the cable segment is $A\underset{\sim}{a}_N$. Let the set of added mass forces acting on the entire cable segment be represented by an equivalent force system consisting of a single force $\underset{\sim}{F}_M$ passing through G together with a couple with torque $\underset{\sim}{T}_M$. Then $\underset{\sim}{F}_M$ and $\underset{\sim}{T}_M$ are given by

$$\underset{\sim}{F}_M = \int_{-L/2}^{L/2} A\underset{\sim}{a}_N dx \tag{18}$$

and

$$\underset{\sim}{T}_M = \int_{-L/2}^{L/2} x\underset{\sim}{n} x A\underset{\sim}{a}_N dx \tag{19}$$

By assuming that A is independent of position along the cable segment and by substituting from Equation (17) into Equations (18) and (19), $\underset{\sim}{F}_M$ and $\underset{\sim}{T}_M$ become, upon integration

$$\underset{\sim}{F}_M = AL\underset{\sim}{a}_{GN} \tag{20}$$

and

$$\underset{\sim}{T}_M = -A(L^3/12)[\underset{\sim}{\alpha} - (\underset{\sim}{\alpha} \cdot \underset{\sim}{n})\underset{\sim}{n} + (\underset{\sim}{\omega} \cdot \underset{\sim}{n})\underset{\sim}{n} X \underset{\sim}{\omega}] \tag{21}$$

DRAG FORCES

RELATIVE VELOCITY OF THE FLUID NORMAL AND TANGENT TO THE CABLE SEGMENT

To determine the drag forces exerted by the fluid on the model segments, it is necessary, from Equation (1), to have expressions for $\underset{\sim}{V}_N$ and $\underset{\sim}{V}_T$, the normal and tangential components of the fluid velocity relative to the segment at a typical point P. These velocities may be obtained by using the same procedures outlined in the foregoing section of the report. Specifically, let $\underset{\sim}{V}^W$ represent the fluid velocity in R. Then, at P the velocity of the fluid relative to the segment may be expressed as

$$\underset{\sim}{V}^{W/P} = \underset{\sim}{V}^W - \underset{\sim}{V}^P \qquad (22)$$

where $\underset{\sim}{V}^P$ is the velocity of P in R. $\underset{\sim}{V}_P$ may be expressed in terms of $\underset{\sim}{V}^G$, the velocity of G in R as

$$\underset{\sim}{V}^P = \underset{\sim}{V}^G + \underset{\sim}{V}^{P/G} \qquad (23)$$

where $\underset{\sim}{V}^{P/G}$ is the velocity of P relative to G in R. However, since P and G are both fixed on the cable segment, $\underset{\sim}{V}^{P/G}$ may be written as

$$\underset{\sim}{V}^{P/G} = \underset{\sim}{\omega} X x \underset{\sim}{n} \qquad (24)$$

where, as before, $\underset{\sim}{\omega}$ is the angular velocity of the cable segment in R. Since $\underset{\sim}{V}_N$ is the component of $\underset{\sim}{V}^{W/P}$ perpendicular to $\underset{\sim}{n}$, it may be written as

$$\underset{\sim}{V}_N = \underset{\sim}{V}^{W/P} - (\underset{\sim}{V}^{W/P} \cdot \underset{\sim}{n})\underset{\sim}{n} \tag{25}$$

By substituting Equation (24) into (23), (23) into (22), and finally (22) into (25), $\underset{\sim}{V}_N$ becomes

$$\underset{\sim}{V}_N = \underset{\sim}{V}^{W/G} - (\underset{\sim}{V}^{W/G} \cdot \underset{\sim}{n})\underset{\sim}{n} - \underset{\sim}{\omega} X \underset{\sim}{n} \tag{26}$$

where $\underset{\sim}{V}^{W/G}$ is $\underset{\sim}{V}^{W} - \underset{\sim}{V}^{G}$, the velocity of the fluid relative to G in R. Let $\underset{\sim}{V}_{GN}$ be the velocity of the fluid relative to G in R perpendicular to the cable segment axis. Then $\underset{\sim}{V}_{GN}$ is

$$\underset{\sim}{V}_{GN} = \underset{\sim}{V}^{W/G} - (\underset{\sim}{V}^{W/G} \cdot \underset{\sim}{n})\underset{\sim}{n} \tag{27}$$

Hence, from Equations (26) and (27), $\underset{\sim}{V}_N$ finally becomes

$$\underset{\sim}{V}_N = \underset{\sim}{V}_{GN} - \underset{\sim}{\omega} X \underset{\sim}{n} \tag{28}$$

Similarly, since $\underset{\sim}{V}_T$ is the component of $\underset{\sim}{V}^{W/P}$ parallel to $\underset{\sim}{n}$, it may be written as:

$$\underset{\sim}{V}_T = (\underset{\sim}{V}^{W/P} \cdot \underset{\sim}{n})\underset{\sim}{n} \tag{29}$$

Using Equations (22), (23), and (24), this may be written as

$$\underset{\sim}{V}_T = (\underset{\sim}{V}^{W/G} \cdot \underset{\sim}{n})\underset{\sim}{n} = \underset{\sim}{V}_{GT} \tag{30}$$

where $\underset{\sim}{V}_{GT}$ is the velocity of the fluid relative to G in R parallel to the cable segment axis.

Finally, in Equation (1), it is necessary to have expressions for the magnitudes of $\underset{\sim}{V}_N$ and $\underset{\sim}{V}_T$. From Equation (28) the magnitude of $\underset{\sim}{V}_N$ may be written in the form

$$|\underset{\sim}{V}_N| = (a + bx + cx^2)^{1/2} \tag{31}$$

where a, b, and c are

$$a = \underset{\sim}{V}_{GN} \cdot \underset{\sim}{V}_{GN} = (\underset{\sim}{V}_{GN})^2 \tag{32}$$

$$b = -2\underset{\sim}{V}_{GN} \cdot \underset{\sim}{\omega} x \underset{\sim}{n} \tag{33}$$

and

$$c = (\underset{\sim}{\omega} x \underset{\sim}{n}) \cdot (\underset{\sim}{\omega} x \underset{\sim}{n}) = (\underset{\sim}{\omega} x \underset{\sim}{n})^2 \tag{34}$$

Similarly, from Equation (30), the magnitude of $\underset{\sim}{V}_T$ may be written as

$$|\underset{\sim}{V}_T| = |\underset{\sim}{V}_{GT}| \tag{35}$$

NORMAL DRAG FORCE AND MOMENT

The normal drag force at a typical point P of the cable segment is $B|\underset{\sim}{V}_N|\underset{\sim}{V}_N$. Let the set of normal drag forces acting on the entire cable segment be represented by an equivalent force system consisting of a single force $\underset{\sim}{F}_N$ passing through G together with a couple with torque $\underset{\sim}{T}_N$. Then, $\underset{\sim}{F}_N$ and $\underset{\sim}{T}_N$ are given by:

$$\underset{\sim}{F}_N = B \int_{-L/2}^{L/2} |\underset{\sim}{V}_N|\underset{\sim}{V}_N \, dx \tag{36}$$

and

$$\underset{\sim}{T}_N = B \int_{-L/2}^{L/2} |\underset{\sim}{V}_N| x\underset{\sim}{n} X \underset{\sim}{V}_N \, dx \tag{37}$$

Consider first $\underset{\sim}{F}_N$. From Equations (28) and (31), $\underset{\sim}{F}_N$ may be written as:

$$\underset{\sim}{F}_N = B \int_{-L/2}^{L/2} (a + bx + cx^2)^{\frac{1}{2}} (\underset{\sim}{V}_{GN} - \underset{\sim}{\omega} X x\underset{\sim}{n}) dx \tag{38}$$

or

$$\underset{\sim}{F}_N = B\underset{\sim}{V}_{GN} \int_{-L/2}^{L/2} (a + bx + cx^2)^{\frac{1}{2}} dx - B\underset{\sim}{\omega} x \underset{\sim}{n} \int_{-L/2}^{L/2} x(a + bx + cx^2)^{\frac{1}{2}} dx \tag{39}$$

The integrals in Equation (39) may be evaluated in closed form. In [57] it is found that by letting X be defined as

$$X = a + bx + cx^2 \tag{40}$$

the first integral may be evaluated as

$$\int_{-L/2}^{L/2} X^{\frac{1}{2}}dx = [(cL + b)/4c]X^{\frac{1}{2}}\Big|_{L/2} + [(cL - b)/4c]X^{\frac{1}{2}}\Big|_{-L/2}$$

$$+ [(4ac - b^2)/8c^{3/2}]\log\left(\frac{X^{\frac{1}{2}}\big|_{L/2} + \frac{Lc^{\frac{1}{2}}}{2} + \frac{b}{2c^{\frac{1}{2}}}}{X^{\frac{1}{2}}\big|_{-L/2} - \frac{Lc^{\frac{1}{2}}}{2} + \frac{b}{2c^{\frac{1}{2}}}}\right) \tag{41}$$

Similarly, the second integral in Equation (39) may be evaluated as:

$$\int_{-L/2}^{L/2} xX^{\frac{1}{2}}dx = (1/3c)X^{3/2}\Big|_{L/2} - (1/3c)X^{3/2}\Big|_{-L/2} - [b(cL + b)/8c^2]X^{\frac{1}{2}}\Big|_{L/2}$$

$$+ [b(b - cL)/8c^2]X^{\frac{1}{2}}\Big|_{-L/2}$$

$$- [b(4ac - b^2)/16c^{5/2}]\log\left(\frac{X^{\frac{1}{2}}\big|_{L/2} + \frac{Lc^{\frac{1}{2}}}{2} + \frac{b}{2c^{\frac{1}{2}}}}{X^{\frac{1}{2}}\big|_{-L/2} - \frac{Lc^{\frac{1}{2}}}{2} + \frac{b}{2c^{\frac{1}{2}}}}\right) \tag{42}$$

Finally, for the purpose of evaluating $\underset{\sim}{T}_N$, it is useful to also record the result:

$$\int_{-L/2}^{L/2} x^2X^{\frac{1}{2}}dx = (1/4c)[(L/2) - (5b/6c)]X^{3/2}\Big|_{L/2} + (1/4c)[(L/2) + (5b/6c)]X^{3/2}\Big|_{-L/2}$$

$$+ [(5b^2 - 4ac)/16c^2]\{[(cL + b)/4c]X^{\frac{1}{2}}\Big|_{L/2} + [(cL - b)/4c]X^{\frac{1}{2}}\Big|_{-L/2}$$

$$+ [(4ac - b^2)/8c^{3/2}]\log\left(\frac{X^{\frac{1}{2}}\big|_{L/2} + \frac{Lc^{\frac{1}{2}}}{2} + \frac{b}{2c^{\frac{1}{2}}}}{X^{\frac{1}{2}}\big|_{-L/2} - \frac{Lc}{2} + \frac{b}{2c^{\frac{1}{2}}}}\right)\} \tag{43}$$

Hence, from Equations (39), (41), and (42), $\underset{\sim}{F}_N$ may be written as

$$F_N = B\{\omega \times n(1/3c)(X^{3/2}|_{-L/2} - X^{3/2}|_{L/2})[(b/2c)\omega \times n$$

$$+ V_{GN}][(\frac{cL + b}{4c})X^{1/2}|_{L/2} + (\frac{cL - b}{4c})X^{1/2}|_{-L/2}$$

$$+ (\frac{4ac - b^2}{8c^{3/2}})\log(\frac{X^{1/2}|_{L/2} + \frac{Lc^{1/2}}{2} + \frac{b}{2c^{1/2}}}{X^{1/2}|_{L/2} - \frac{Lc^{1/2}}{2} + \frac{b}{2c^{1/2}}})]\} \qquad (44)$$

Next, consider T_N. From Equations (37), (28), (31), and (40), T_N may be written as:

$$T_N = B\int_{-L/2}^{L/2} xX^{1/2}\, nX(V_{GN} - \omega X x n)dx \qquad (45)$$

By expanding the vector product T_N becomes:

$$T_N = Bn x V_{GN}\int_{-L/2}^{L/2} xX^{1/2}dx - B[\omega - (\omega \cdot n)n]\int_{-L/2}^{L/2} x^2X^{1/2}dx \qquad (46)$$

By using Equations (42) and (43), T_N takes the form

$$T_N = Bn x V_{GN}[(1/3c)X^{3/2}|_{L/2} - (1/3c)X^{3/2}|_{-L/2}$$

$$- \frac{b(cL + b)}{8c^2} X^{1/2}|_{L/2} + \frac{b(-cL + b)}{8c^2} X^{1/2}|_{-L/2}$$

$$- \frac{b(4ac - b^2)}{16c^{5/2}} \log(\frac{X|_{L/2} + \frac{Lc^{1/2}}{2} + \frac{b}{2c^{1/2}}}{X|_{L/2} - \frac{Lc^{1/2}}{2} + \frac{b}{2c^{1/2}}})]$$

$$+ B[(\omega \cdot n)n - \omega]\{(1/4c)[(L/2) - (5b/6c)]X^{3/2}|_{L/2} \qquad (47)$$

$$+ (1/4c)[(L/2) + (5b/4c)]X^{3/2}|_{-L/2} + [(5b^2 - 4ac)/16c^2][\frac{cL+b}{4c} X^{1/2}|_{L/2}$$

$$+ \frac{cL-b}{4c} X^{1/2}|_{-L/2} + (\frac{4ac-b^2}{8c^{3/2}}) \log(\frac{X^{1/2}|_{L/2} + \frac{Lc^{1/2}}{2} + \frac{b}{2c^{1/2}}}{X^{1/2}|_{L/2} - \frac{Lc^{1/2}}{2} + \frac{b}{2c^{1/2}}})]\}$$

TANGENTIAL DRAG FORCE AND MOMENT

The tangential drag force at a typical point P of the cable segment is given by the third term in Equation (1) as: $C|\mathbf{V}_T|\mathbf{V}_T$. Hence, let the set of tangential drag forces acting on the entire cable segment be replaced by an equivalent force system consisting of a single force $\mathbf{F}_T$ passing through G together with a couple with torque $\mathbf{T}_T$. Then $\mathbf{F}_T$ and $\mathbf{T}_T$ are given by

$$\mathbf{F}_T = C \int_{-L/2}^{L/2} |\mathbf{V}_T| \mathbf{V}_T \, dx \tag{48}$$

and

$$\mathbf{T}_T = C \int_{-L/2}^{L/2} |\mathbf{V}_T| x \mathbf{n} \times \mathbf{V}_T \, dx \tag{49}$$

Consider first $\mathbf{F}_T$. By substituting for $\mathbf{V}_T$ from Equation (30), $\mathbf{F}_T$ becomes

$$\mathbf{F}_T = CL|\mathbf{V}_{GT}|\mathbf{V}_{GT} \tag{50}$$

Consider next $\mathbf{T}_T$. From Equation (30) it is seen that $\mathbf{V}_T$ is parallel to $\mathbf{n}$. Hence, the vector product is zero and therefore T_T becomes:

$$\mathbf{T}_T = 0 \tag{51}$$

WEIGHT AND BOUYANCY FORCES

Consider finally the weight and bouyancy forces acting on the cable segment. The weight forces* may be represented simply by the single vertical (downward) force $\underset{\sim}{W}$ passing through the mass center G as shown in Figure 5. If ρ_c is the mass density per unit length of the cable segment, then $\underset{\sim}{W}$ may be expressed as

$$\underset{\sim}{W} = -\rho_c g L \underset{\sim}{k} \tag{52}$$

where $\underset{\sim}{k}$ is a vertical unit vector and g is the gravity constant.

The bouyancy forces are not quite as simple as the weight forces. The bouyancy forces are due to the hydrodynamic pressure forces exerted on the cable segment. These forces are normal to the surface of the segment and they increase in magnitude linearly with the depth. For a submerged cylinder, the bouyancy forces may be represented by the single vertical force $\underset{\sim}{B}$ passing through the geometric center G of the cylinder as shown in Figure 5. However, since the cable segment represents a finite segment modelling of a continuous cable, the ends 1 and 2 of the segment are not exposed to the fluid. That is, they are "shielded" from the fluid forces by the adjacent joining cable segments. Therefore, the hydrostatic forces on the cable segment are all normal to the segment axis. If the force $\underset{\sim}{B}$ is represented by components $\underset{\sim}{B}_N$ and $\underset{\sim}{B}_T$ perpendicular

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*These are the only non-fluid forces considered in the analysis. They are included in the analysis because of their close association with bouyancy forces.

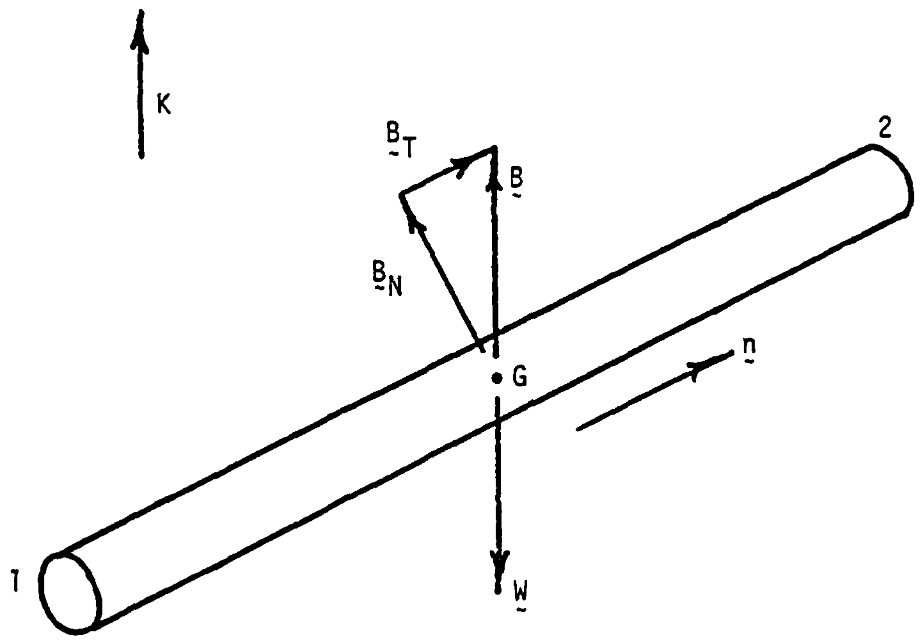

Figure 5. Weight and Bouyancy Forces
Acting on the Cable Segment

and parallel to the cable segment, then the resultant, equivalent hydrostatic force on the cable segment may be represented by the single force $\underset{\sim}{B}_N$ passing through G. (This is also seen by noting that the hydrostatic forces on the ends of a submerged right circular cylinder are parallel to the axis of the cylinder and, hence, may be represented by the component $\underset{\sim}{B}_T$.) The vertical force $\underset{\sim}{B}$ of Figure 5. may be expressed in the form (See, for example, Reference [49].)

$$\underset{\sim}{B} = -\rho g V \underset{\sim}{k} \tag{53}$$

where ρ is the mass density of the fluid and V is the volume of the submerged segment. Hence, since $\underset{\sim}{B}_N$ is the component of $\underset{\sim}{B}$ perpendicular to the segment axis, it may be represented as:

$$\underset{\sim}{B}_N = \underset{\sim}{B} - (\underset{\sim}{B} \cdot \underset{\sim}{n})\underset{\sim}{n} = \underset{\sim}{n} x (\underset{\sim}{B} x \underset{\sim}{n}) \tag{54}$$

or

$$\underset{\sim}{B}_N = -\rho g \underset{\sim}{n} x (\underset{\sim}{k} x \underset{\sim}{n}) \tag{55}$$

CONCLUSIONS

To summarize the foregoing results, the force system exerted on a segment of a finite segment cable model by the fluid inertia, drag, and hydrostatic forces, together with the gravitational forces, may be represented by a single force $\underset{\sim}{F}$ passing through the mass center of the cable segment together with a couple with torque $\underset{\sim}{T}_T$. $\underset{\sim}{F}$ and $\underset{\sim}{T}$ are given by the expressions

$$\underset{\sim}{F} = \underset{\sim}{F}_M + \underset{\sim}{F}_N + \underset{\sim}{F}_T + \underset{\sim}{B}_N + \underset{\sim}{W} \tag{56}$$

and

$$\underset{\sim}{T}_T = \underset{\sim}{T}_M + \underset{\sim}{T}_N \tag{57}$$

where $\underset{\sim}{F}_M$, $\underset{\sim}{F}_N$, $\underset{\sim}{F}_T$, $\underset{\sim}{B}_N$, $\underset{\sim}{W}$, $\underset{\sim}{T}_M$, and $\underset{\sim}{T}_N$ are given by Equations (20), (44), (50), (55), (52), (21), and (47) respectively.

Equations (56) and (57) are in a format which may be directly converted into subroutine algorithms for use in finite segment computer codes (such as UCIN-SUPER [46]) written specifically for multibody dynamics analyses. That is, given suitable initial conditions for the kinematics of the cable segments together with the segment physical and geometrical parameters, and the fluid properties, the Reynolds numbers and drag coefficients can be determined. This, in turn, determines $\underset{\sim}{F}$ and $\underset{\sim}{T}$ which may then be used in the governing equation algorithm. Upon numerical integration, the kinematics at the end of a short time interval is determined. The above process can then be successively repeated until the time history of the cable motion and dynamics is known.

Reports on the development, validation, and application of such computer algorithms and codes are currently being planned and prepared.

REFERENCES

1. Abel, J. M., "Cable Interactions in a Depth Controlled Submersible," Journal of Hydronautics, Vol. 6., No. 2., pp. 83-89.

2. Breslin, J. P., "Dynamic Forces Exerted by Oscillating Cables," Journal of Hydronautics, Vol. 8., No. 1., January, 1974, pp. 19-31.

3. Calkins, D. E., "Faired Towline Hydrodynamics," Journal of hydronautics, Vol. 4., No. 3., July, 1970, pp. 113-119.

4. Cannon, T. C., and Genin, J., "Three-Dimensional Dynamical Behavior of a Flexible Towline," Aeronautical Quarterly, Vol. 23., Part 3., August 1972, pp. 201-210.

5. Cannon, T. C., and Genin, J., "Dynamic Behavior of Materially Damped Flexible Towed Cable," Aeronautical Quarterly, Vol. 23., Part 2., May 1972, pp. 109-120.

6. Carson, W. W., and Emery, A. F., "An Energy Method Determination of Large Cable Dynamics," Journal of Applied Mechanics, June 1976, pp. 330-334.

7. Casarella, M. J., and Parsons, M., "A Survey of the Configuration and Motion of Cable Systems Under Hydrodynamic Loading," Marine Technology Society Journal, Vol. 4., No. 4., July-August 1970, pp. 27-44.

8. Choo, Y., and Casarella, M. J., "Hydrodynamic Resistance of Towed Cables," Journal of Hydronautics, Vol. 5., No. 4., October 1971, pp. 126-131.

9. Choo, Y., and Casarella, M. J., "Configuration of a Towline Attached to a Vehicle Moving in a Circular Path," Journal of Hydronautics, Vol. 6., No. 1., January 1972, pp. 51-57.

10. Choo, Y., and Casarella, M. J., "A Survey of Analytical Methods for Dynamic Simulation of Cable-Body Systems," Journal of Hydronautics, Vol. 7., No. 4., October 1973, pp. 137-144.

11. Collier, L., "Investigation of Hyrdynamic Loading Functions on Two Faired Towlines," Report 142, Hydrospace Research Corporation, November 1966, Defense Technical Information Center AD 804913.

12. Dawson, T. H., "Dynamics of Shallow Towing Cables," Ocean Engineering, Vol. 4., 1977, pp. 187-196.

13. Dominguez, R. F., "Dynamic Analysis of Cable Systems," Journal of the Structural Division, ASCE, ST8, 1972, pp. 1817-1834.

14. Etter, R. J., and Huang, Thomas T., "An Experimental Investigation of the Static Hydrodynamic Characteristics of Several Simulated Faired Cables Having Symmetrical NACA Airfail Sections," Hydronautics, Inc., Report 530-1, July 1967.

15. Genin, J., and Cannon, T. C., "Equilibrium Configuration and Tensions of a Flexible Cable in a Uniform Flow Field," Journal of Aircraft, Vol. 4., May-June, 1967, pp. 200-202.

16. Genin, J., Citron, S. J., and Huffman, R. R., "Coupling of Longitudinal and Transverse Motions of a Flexible Cable in a Uniform Flow Field," Journal of the Acoustical Society of America, Vol. 52., No. 1., Part 2., July 1972, pp. 438-440.

17. Goeller, J. E. and Laura, P. A., "Analytical and Experimental Study of the Dynamic Response of Segmented Cable Systems," Journal of Sound and Vibration, Vol. 18., No. 3., 1971, pp. 311-324.

18. Goodman, T. R., and Breslin, J. P., "Statics and Dynamics of Anchoring Cables in Waves," Journal of Hydronautics, Vol. 10, No. 4., 1976, pp. 113-120.

19. Goodman, T. R., Kaplan, P., Sargent, T. P., and Bentson, J., "Statis and Dynamic Analysis of a Moored Buoy System," National Oceanic and Atmospheric Administration Report 10829 under Contract NAS 8-26879, April 1972.

20. Huffman, R. R. and Genin, J., "The Dynamical Behavior of Flexible Cable in a Uniform Flow Field," Aeronautical Quarterly, Vol. 22., Part 2., May 1971, pp. 183-195.

21. Landweber, L. and Protter, M. H., "The Shape and Tension of a Light Flexible Cable in a Uniform Current," Journal of Applied Mechanics, Vol. 14, June, 1947.

22. Lehner, J. R. and Batterman, S. C., "Static and Dynamic Finite Deformations of Cables Using Rate Equations," Computer Methods in Applied Mechanics and Engineering, Vol. 2., 1973, pp. 349-366.

23. Leonard, J. W. and Recker, W. W., "Nonlinear Dynamics of Cables with Low Initial Tension," Journal of the Engineering Mechanics Division, ASCE, EM2, 1972, pp. 293-309.

24. Migliore, H. J. and Webster, R. L., "Current Methods for Analyzing Dynamic Cable Response," Journal of Shock and Vibration, June, 1979, pp. 3-16.

25. Y. Narkis, "Approximate Solution for the Shape of Flexible Towing Cables," Journal of Aircraft, Vol. 14, No. 9., September 1977, pp. 923-925.

26. Paidoussis, M. P., "Stability of Towed, Totally Submerged Flexible Cylinders," Journal of Fluid Mechanics, Vol. 34., Part 2., pp. 273-297, 1968.

27. Pode, L., "Tables for Computing the Equilibrium Configuration of a Flexible Cable in a Uniform Stream," Report 687, David Taylor Model Basin, 1951.

28. Rogge, T. R., "Equations of Motion for Flexible Cables," Journal of Aircraft, Vol. 9., No. 11., 1972, pp. 799-800.

29. Roussel, P., "Numerical Solution of Static and Dynamic Equations of Cables," Computer Methods in Applied Mechanics and Engineering, Vol. 9., No. 1., 1976, pp. 65-74.

30. Schram, J. W., and Reyle, S. P., "A Three Dimensional Analysis of a Towed System," Journal of Hydronautics, Vol. 2., No. 4., October 1968, pp. 213-220.

31. Skop, R. A. and Choo, Y., "The Configuration of a Cable Towed in a Circular Path," Journal of Aircraft, Vol. 8., No. 11., 1971, pp. 856-863.

32. Skop, R. A., and Griffin, O. M., "On a Theory for the Vortex-Excited Oscillations of Flexible Cylindrical Structures," Journal of Sound and Vibration, Vol. 41., 1975, pp. 263-274.

33. Springston, G. B., Jr., "Generalized Hydrodynamic Loading Functions for Bare and Faired Cables in Two-Dimensional Steady-State Cable Configurations," Naval Ship Research and Development Report 242A S-F00603 02 Task 7462, 1967.

34. Wang, H. T., "A FORTRAN IV Program for the Three-Dimensional Steady-State Configuration of Extensible Flexible Cable Systems," Naval Ship Research and Development Center, September 1974, NTIS AD/A-000 954.

35. Webster, R. L., "An Application of the Finite Element Method to the Determination of Nonlinear Static and Dynamic Responses of Underwater Cable Structures," General Electric Technical Information Series Report No. R76EMH2, Syracuse, N. Y., January 1976.

36. Huston, R. L. and Passerello, C. E., "On the Dynamics of Chain Systems," ASME Paper No. 74-WA/Aut 11, 1974.

37. Passerello, C. E. and Huston, R. L., "Dynamics of General Chain Systems," NTIS Report PB 257184, August, 1976.

38. Huston, R. L., Passerello, C. E., and Harlow, M. W., "Dynamics of Multirigid-Body Systems," Journal of Applied Mechanics, Vol. 45, No. 4., 1978, pp. 889-894.

39. Huston, R. L. and Passerello, C. E., "On Multi-Rigid-Body System Dynamics," Computers and Structures, Vol. 10., 1979, pp. 439-446.

40. Huston, R. L., and Passerello, C. E., "Multibody Structural Dynamics Including Translation Between the Bodies," Computers and Structures, Vol. 11., 1980.

41. Kane, T. R., "Dynamics of Nonholonomic Systems," Journal of Applied Mechanics, Vol 28., 1961, pp. 574-578.

42. Kane, T. R. and Wang, C. E., "On the Derivation of Equations of Motion," Journal of the Society for Industrial and Applied Mathematics, Vol. 13., 1965, pp. 487-492.

43. Kane, T. R., Dynamics, Holt, Rinehart, and Winston, New York, 1968.

44. Huston, R. L. and Passerello, C. E., "On Lagrange's Form of d'Alembert's Principle," The Matrix and Tensor Quarterly, Vol. 23., 1973, pp. 109-112.

45. Huston, R. L., Passerello, C. E., and Harlow, M. W., "User's Manual for UCIN Vehicle-Occupant Crash Study Model-Version II," University of Cincinnati, Engineering Analysis Report No. ONR-UC-EA-120174-3.

46. Huston, R. L., Passerello, C. E., Harlow, M. W., and Winget, J. M., User's Manual for UCIN-SUPER: Chain Systems," University of Cincinnati, Engineering Science Report No. ONR-UC-ES-080177-4.

47. Schlichting, H., Boundary Layer Theory, McGraw Hill, New York, 1960, p. 16.

48. Dryden, H. L., Murnaghan, F. P., and Bateman, H., Hydrodynamics, Dover, 1956, New York, pp. 11-12.

49. Streeter, V. L. Fluid Mechanics, McGraw Hill, New York, 1966, pp. 345-346.

50. Hoerner, S. F., Fluid Dynamic Drag, published by the author, Brick Town, N.J., 1965, pp. 3-9.

51. Bootle, W. J., "Forces on an Inclined Circular Cylinder in Supercritical Flow," AIAA Journal, Vol. 9., No. 3., 1971, pp. 514-516.

52. Danton, W. L., "A Survey of Available Data on the Normal Drag Coefficient of Cables Subjected to Cross Flow," Civil Engineering Laboratory, Naval Construction Battalion Center, Port Hueneme, CA, August 1977.

53. Delaney, N. K., and Sorensen, N. E., "Low Speed Drag of Cylinders of Various Shapes," NACA TN-3038, November 1953.

54. Folb, R., "Experimental Determination of Hydrodynamic Loading for Ten Cable Fairing Models," Naval Ship Research and Development Center Report 4619 AD-A021 588, November 1975.

55. Garrison, C. J., "Added Mass of a Circular Cylinder in Contrast with a Rigid Boundary," Journal of Hydronautics, Vol. 6., No. 1., January 1972, pp. 59-60.

56. Imloy, F. H., "The Complete Expressions for 'Added Mass' of a Rigid Body Moving in an Ideal Fluid," Hydromechanics Laboratory Research and Development Report 1528, July 1961.

57. Isaacson, M. de St. Q., "Nonlinear Inertia Forces on Bodies," ASCE Journal of the Waterway Port Coastal and Ocean Divisions, Paper no. WW3, August 1979, pp. 213-227.

58. CRC Standard Mathematical Tables, The Chemical Rubber Co., Cleveland, Ohio, 14th Ed., 1965, pp. 321-322.

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