AD-A090 022 SPIRE CORP BEDFORD MA F/G 20/9
TECHNIQUES FOR THE SIMULATION OF THE SPACE PLASMA ENVIRONMENT.(U)
JUN 80 W D HALVERSON F19628-80-C-0055
UNCLASSIFIED SCIENTIFIC-1 AFGL-TR-80-0203 NL

1 OF 1
AD A
090022

END
DATE
FILMED
11-80
DTIC

LEVEL II

AFGL-TR-80-0203

TECHNIQUES FOR THE SIMULATION OF THE SPACE PLASMA ENVIRONMENT

AD A090022

Ward D. Halverson

Spire Corporation
Patriots Park
Bedford, Massachusetts 01730

Scientific Report No. 1

June 1980

AIR FORCE GEOPHYSICS LABORATORY
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HANSCOM AFB, MASSACHUSETTS 01731

80 10 7 044

Qualified requestors may obtain additional copies from the Defense Documentation Center. All others should apply to the National Technical Information Service.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFGL-TR-80-0203	2. GOVT ACCESSION NO.: AD-A090 022
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): TECHNIQUES FOR THE SIMULATION OF THE SPACE PLASMA ENVIRONMENT	5. TYPE OF REPORT & PERIOD COVERED: Scientific Report No. 1
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Ward D. Halverson	8. CONTRACT OR GRANT NUMBER(s): F19628-80-C-0055
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Spire Corporation, Patriots Park, Bedford, MA 01730	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 62101F, 766108AE
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Geophysics Laboratory, Hanscom Air Force Base, Massachusetts 01731, Monitor/A.H. Wendel/PHG	12. REPORT DATE: June 1980
	13. NUMBER OF PAGES: 22
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): SCIENTIFIC-1	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Spacecraft charging, laboratory simulation, electron and ion beams

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The effects of space plasmas with distributed spectra of particle energies can be simulated in the laboratory by combinations of monoenergetic electron and ion beams. The energy and current density of beams are calculated to match various moments of the velocity distribution function of distributed spectra. Analytical solutions have been found for one- and two-beam matching, respectively, of two- and four-velocity moments of arbitrary distribution functions. A three-energy solution which matches six moments of a Maxwellian

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. ABSTRACT (Concluded)

distribution is presented, as well as multiple-beam solutions for matching two-Maxwellian distributions.

FOREWORD

The author wishes to acknowledge the contributions of Stephen N. Bunker in computing the three-energy solution presented in Section 2.2.3 and of Steven H. Face in finding the solutions for beams with arbitrarily assigned energies, presented in Section 2.4.

TABLE OF CONTENTS

SECTION 1
INTRODUCTION

Over the past several years the interactions between spacecraft and the vacuum, electromagnetic and particle radiation environment in space have been of considerable interest to the scientific and engineering community. Electrostatic charging of high altitude satellites by the effects of the ambient plasma and solar radiation is of serious importance, since it can affect the operation of several on-board systems in scientific, military, and commercial satellites.

Some laboratory facilities are presently available (see Refs. 1-3, for example) and others are projected[4] which can provide a degree of simulation of the radiation and plasma environment of space. A simulation facility is necessarily limited by engineering and economic considerations to providing only a partial re-creation of the actual environment encountered by a space vehicle. For example, only a few monoenergetic beams, incident on a target from a limited number of directions, can be envisioned to simulate geomagnetic substorm plasmas which generally have distributed energy spectra and are fairly isotropic. The question as to what constitutes a "good" simulation of a given plasma environment has often been posed, but there are presently no consistent answers.

The purpose of the present investigation is to analyze specific techniques for choosing the energies and current densities of monoenergetic electron and ion beams to simulate given space plasmas with distributed particle energy spectra. The plasmas are characterized by various averages of their velocity distribution functions, and the energies and particle densities of the monoenergetic beams are computed so that there is a good match between the characteristics of the distributed spectra and those of the beams. We hope that this analysis will help provide a concrete link between the measurements of the space environment and the engineering specifications for charged particle beams in laboratory simulation facilities.

This interim report outlines the approach which we have followed to define beam energies and densities to simulate distributed plasma spectra. It does not consider the equally important problems of the effects of discrete angular directions of the charged particle beams or the interactions of the incident beams with the target. We have, however, made considerable progress in both these areas, which will be reported in other publications and in the final report of the program.

SECTION 2
MOMENT-MATCHING TECHNIQUES

2.1 BACKGROUND

A plasma can be characterized by various averages of the velocity distributions of its constituent particles. In general, the "velocity moments" of a given distributon function, f(v), are defined by

$$M_k = 4\pi \int_0^\infty v^k \; f(v) \; v^2 dv \tag{1}$$

$$k = 0,1,2,----.$$

where the $4\pi v^2$ dv term represents an infinitesimal element in (isotropic) velocity space.

The velocity moments, M_k, can be related to physical averages for several values of k. For example, M_0, M_1, M_2, and M_3 are related, respectively, to the average number density <N>, particle flux, <NF>, pressure, <P>, and energy flux, <EF>, of the given particle type in the plasma.

$$M_0 = \langle N \rangle = n \tag{2}$$

$$M_1 = 4\pi \langle NF \rangle = n \langle v \rangle = n \left(\frac{8}{\pi} \frac{kT}{m} \right)^{1/2} \tag{3}$$

$$M_2 = \frac{3}{m} \langle P \rangle = \frac{3\pi}{8} n \left(\frac{8}{\pi} \frac{kT}{m} \right)^{1} \tag{4}$$

$$M_3 = \frac{8\pi}{m} \langle EF \rangle = \frac{\pi}{2} n \left(\frac{8}{\pi} \frac{kT}{m} \right)^{3/2} \tag{5}$$

$$M_4 = \frac{15}{64} \pi^2 n \left(\frac{8}{\pi} \frac{kT}{m} \right)^{2} \tag{6}$$

$$M_5 = \frac{3}{8} \pi^2 n \left(\frac{8}{\pi} \frac{kT}{m} \right)^{5/2} \tag{7}$$

The average speed, $\langle v \rangle$, in Equation (3) is defined by

$$\langle v \rangle = \frac{M_1}{M_0} \tag{8}$$

The expressions on the right-hand side of Equations (2)-(7) are given for the case of a Maxwellian velocity distribution,

$$f(v) = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} \tag{9}$$

where n, m, and T are respectively the number density, mass, and temperature of the particles and k is Boltzmann's contant.

Average and RMS "Temperatures"

A useful method for characterizing a non-Maxwellian plasma is to define effective temperatures which are related to ratios of the velocity moments.[5] The average and RMS temperatures are given by

$$T_{AV} = \frac{1}{k}\frac{\langle P \rangle}{\langle N \rangle} = \frac{m}{3k}\frac{M_2}{M_0} \tag{10}$$

$$T_{RMS} = \frac{1}{k}\frac{\langle EF \rangle}{2\langle NF \rangle} = \frac{m}{4k}\frac{M_3}{M_1} \tag{11}$$

The two temperatures are equal when the velocity distribution is Maxwellian.

2.2 MONOENERGETIC BEAMS TO MATCH VELOCITY MOMENTS

A technique to simulate a plasma with a distributed velocity distribution is to choose the velocities and particle densities of mononergetic beams so that their velocity moments match those of the plasma. Under these conditions, the average parameters of the beams, such as number density, pressure, or energy flux, are equal to those of the plasma component under simulation.

In general, a single beam can match two moments of the distributed spectrum, so that two beams can match four moments, three beams, six moments, etc. As discussed in Section 2.4, it is also possible to overspecify the problem and use more than the minimum number of beams to match a given number of velocity moments.

2.2.1 Single Beam Energy

A monoenergetic beam can match two moments according to the simultaneous equations,

$$\left.\begin{aligned} n_b v_b^{\,j} &= M_j \\ n_b v_b^{\,k} &= M_k \end{aligned}\right. \qquad (j \neq k) \tag{12}$$

where n_b and v_b are the density and velocity of the beam particles.

For example, when the zeroth (number density) and second (pressure) moments are chosen,

$$\begin{aligned} n_b &= n \\ v_b &= \left(\frac{M_2}{M_0}\right)^{1/2} = \left(\frac{3\ k\ T_{AV}}{m}\right)^{1/2} \end{aligned} \tag{13}$$

or, in terms, of beam energy, E_b,

$$E_b = \frac{3}{2}\,k\ T_{AV} \tag{14}$$

If the first (number flux) and third (energy flux) moments are used,

$$\begin{aligned} n_b &= n \\ v_b &= \left(\frac{4\ k\ T_{RMS}}{m}\right)^{1/2} \end{aligned} \tag{15}$$

or

$$E_b = 2\ k\ T_{RMS} \tag{16}$$

2.2.2 Two Beam Energies

The densities and velocities of two monoenergetic beams can be found to match the zeroth through third velocity moments of the distributed spectrum by solving four simultaneous equations:

$$\begin{aligned} n_1 + n_2 &= n \\ n_1 v_1 + n_2 v_2 &= n <v> \\ n_1 v_1^{\ 2} + n_2 v_2^{\ 2} &= \frac{3}{m}\, n\ T_{AV} \\ n_1 v_1^{\ 3} + n_2 v_2^{\ 3} &= \frac{4}{m}\, n <v>\ T_{RMS} \end{aligned} \tag{17}$$

where n_1, n_2, v_1, and v_2 are the densities and velocities of the beams, and the velocity moments of the distributed spectrum have been replaced by relations (8), (10), and (11). Boltzmann's constant, k, has been taken to be unity.

It is shown in Appendix A that the velocities and densities of the monoenergetic beams can be found analytically.

$$v_{1,2} = \frac{<v>}{6\ T_{AV} - 2\ m <v>^{2}} \cdot \left\{ 4\ T_{RMS} - 3\ T_{AV} \pm \left[8\ T_{RMS}\ (2\ T_{RMS} - 9\ T_{AV} + 2\ m <v>^{2}) - 27\ T_{AV}^{2} \left(1 - \frac{4\ T_{AV}}{m <v>^{2}}\right) \right]^{1/2} \right\} \tag{18}$$

$$n_1 = n\left(\frac{v_2 - \langle v\rangle}{v_1 - v_2}\right)$$

$$n_2 = n - n_1 \tag{19}$$

For a Maxwellian plasma, where $T_{AV} = T_{RMS} = T$, Eq. (18) simplifies somewhat,

$$v_{1,2} = \frac{\pi \langle v\rangle}{6\pi - 16}\left\{1 \pm \left(\frac{27}{2}\pi + \frac{128}{\pi} - 83\right)^{1/2}\right\} \tag{20}$$

The beam densities and energies are then found to be

$$n_1 = 0.382\ n$$
$$n_2 = 0.618\ n \tag{21}$$

$$E_1 = 3.007\ T$$
$$E_2 = 0.568\ T \tag{22}$$

2.2.3 Three Beam Energies

Six moments of the distributed spectrum can be used to compute the densities and velocities of three monoenergetic beams. No analytical solutions have been found for this case, but iterative techniques can be used to find solutions of the set of six simultaneous, nonlinear equations.

As discussed in Appendix B, the beam velocities and densities can be found in terms of the average speed and density of the plasma particles. For the case of a Maxwellian plasma with temperature, T, the beam densities and energies are

$$n_{1,2,3} = [0.087,\ 0.588,\ 0.325]\ n$$
$$E_{1,2,3} = [4.931,\ 1.657,\ 0.303]\ T \tag{23}$$

Different values will be found for other types of velocity distribution functions, but the method used to compute the Maxwellian results is general for all realistic spectral shapes.

2.3 TWO-MAXWELLIAN PLASMAS

Garrett showed that a two-Maxwellian fit is often a good representation of plasma distribution functions measured during geomagnetic substorms.[6] The density and temperature of each Maxwellian component can be found from four velocity moments of the measured spectrum. It is possible, in principle, to find three-Maxwellian fits which match six moments, although the effects of errors in measurement of the plasma spectrum become increasingly exaggerated when computing the high-order moments. It should also be possible to find multiple-Maxwellian least-square fits directly from the measured distribution functions without computing the velocity moments of the data.

2.3.1 Single Beam Energy

A two-Maxwellian distribution has average and RMS temperatures given by

$$T_{AV} = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2} \tag{24}$$

$$T_{RMS} = \frac{n_1 T_1^{3/2} + n_2 T_2^{3/2}}{n_1 T_1^{1/2} + n_2 T_2^{1/2}} \tag{25}$$

where n_1, n_2, T_1, and T_2 are the respective densities and temperatures of the two components of the spectrum.

A single monoenergetic beam can match two velocity moments of the distributed spectrum if its density and energy are chosen according to Eqs. (13)-(16) above. For example, if the beam density is equal to the total plasma density, $n_1 + n_2$, and its energy is 3/2 T_{AV}, then the zeroth and second velocity moments of the two-Maxwellian plasma and the monoenergetic beam are equal.

2.3.2 Two Beam Energies

Two methods exist for matching the velocity moments of a two-Maxwellian distribution by two monoenergetic beams. First, the energy and density of each beam can be chosen individually to match two moments of each of the Maxwellian components of the spectrum. In this case, Eqs. (13)-(16) would be employed along with the densities and temperatures of the two-Maxwellian fit.

The second approach is to use the average and RMS temperatures of the two-Maxwellian fit, Eqs. (24) and (25) and to calculate the beam velocities and densities from Eqs. (18) and (19). In both cases, as many as four moments of the two-Maxwellian distribution function can be matched by two monoenergetic beams. In practical situations, physical considerations would be required to make a choice between the two methods of matching velocity moments.

2.3.3 Three or More Beam Energies

The moments of a two-Maxwellian distribution function can be matched in several different combinations with multiple monoenergetic beams. As in the two-beam case, each Maxwellian component of the plasma can have one or more beams assigned to it which individually match velocity moments. For six-moment matching, three beam energies and densities could be selected using Eq. (23) for each component, and a total of six beam energies would be required to simulate the two-Maxwellian plasma. As mentioned in Section 2.2.3, the computed values of the zeroth through fifth moment of the full spectrum can also be used directly to find three beam energies and densities through the iterative minimization procedure described in Appendix B.

2.4 ARBITRARILY ASSIGNED BEAM ENERGIES

The velocity moments of a measured distribution function can also be matched by monoenergetic beams whose velocities are chosen arbitrarily. As an example, four monoenergetic beams can match four velocity moments:

$$\begin{aligned}
n_1 + n_2 + n_3 + n_4 &= M_0 \\
v_1 n_1 + v_2 n_2 + v_3 n_3 + v_4 n_4 &= M_1 \\
v_1^2 n_1 + v_2^2 n_2 + v_3^2 n_3 + v_4^2 n_4 &= M_2 \\
v_1^3 n_1 + v_2^3 n_2 + v_3^3 n_3 + v_4^3 n_4 &= M_3
\end{aligned} \tag{26}$$

When the four beam velocities are fixed, then it is only a matter of solving a set of linear simultaneous equations for the beam densities, n_1 through n_4. It should be pointed out that not all combinations of beam velocity may be chosen for v_1 through v_4, because negative, and therefore unphysical, solutions for the beam densities can be obtained in some cases.

Table 1 gives the densities calculated for three, four, and five beams as a function of preassigned beam energies. The beam energies and densities are normalized to the temperature and density of a Maxwellian distribution, and the velocity moments used for the calculations are given by the right-hand side of Eqs. (2)-(6). The first three-beam solution in Table 1 is a check of the six-moment solution, Eq. (23), found by the iterative procedure discussed in Section 2.2.3.

TABLE 1. PARTICLE DENSITIES FOR PREASSIGNED BEAM ENERGIES

No. of Beams	E_i/kT	n_i/n
	0.303	0.325
3	1.657	0.588
	4.931	0.087
	0.5	0.521
3	2.0	0.339
	4.0	0.140
	0.4	0.282
4	0.8	0.333
	1.6	0.069
	3.2	0.316
	0.5	0.518
	1.5	0.156
4	2.5	0.134
	3.5	0.192
	0.2	0.198
	0.6	0.061
5	1.3	0.548
	3.0	0.127
	5.0	0.066
	0.2	0.179
	0.6	0.162
5	1.5	0.502
	3.0	0.086
	5.0	0.071

SECTION 3

CONCLUSIONS AND RECOMMENDATIONS

It has been shown in Section 2 that the low order velocity moments of plasmas with arbitrary distributed energy spectra can be matched by combinations of monoenergetic beams. The beams simulate, in a mathematically precise manner, certain properties of an undisturbed space plasma. However, interactions between the plasma and a complicated object such as a satellite were not considered in this analysis. Spacecraft charging, on the other hand, is a phenomenon which depends not only on the properties of the undisturbed plasma, but also on the detailed interactions of the plasma particles with the surface and the magnitude and shape of the equipotentials around the spacecraft.

Charging models of various degrees of complexity have been developed[7] in an attempt to account for the observed interactions between the environment of space and instrumented spacecraft. An important task of the present program is to use such a model to study the consequences of substituting monoenergetic beams for plasmas with distributed spectra. We believe that comparisons of this kind will provide insight into the selection of a few combinations of beam parameters which will provide good simulation of spacecraft charging phenomena among the many, mathematically equivalent, techniques for matching averaged properties of plasmas and monoenergetic beams.

REFERENCES

1. N. H. Stone and W. K. Rehmann, "The Simulation of Ionospheric Conditions for Space Vehicles", Report No. NASA TN D-5894, NASA Marshall Space Flight Center, Huntsville, Alabama, 1970.

2. F. D. Berkopec, N. John Stevens, and J. C. Sturman, "The Lewis Research Center Geomagnetic Substorm Simulation Facility", Report No. NASA TM X-73602, NASA Lewis Research Center, Cleveland, Ohio, 1976.

3. O. L. Pearson, "Modification of a Very Large Thermal-Vacuum Test Chamber for Ionosphere and Plasmasphere Simulation", American Institute of Aeronautics and Astronautics, Inc., New York, Paper 78-1625, 1978.

4. W. Halverson, "Modifications of Ionospheric Simulation Capability. Volume 1: Study", Report No. FR-60021, Spire Corporation, Bedford, Massachusetts, 1979.

5. H. B. Garrett, E. G. Mullen, E. Ziemba, and S. E. DeForest, "Modeling of the Geosynchronous Orbit Plasma Environment - Part 2", Report No. AFGL-TR-78-0304, Air Force Geophysics Laboratory, Hanscom AFB, Massachusetts, 1978.

6. H. B. Garrett, "Modeling of the Geosynchronous Orbit Plasma Environment - Part 1", Report No. AFGL-TR-77-0288, Air Force Geophysics Laboratory, Hanscom AFB, Massachusetts, 1977.

7. E. C. Whipple, Jr., "Modeling of Spacecraft Charging" (in Proc. Spacecraft Charging Conf., C. P. Pike and R. R. Lovell, ed.), Report Nos. AFGL-TR-77-0051 and NASA TMX-73537, Air Force Geophysics Laboratory, Hanscom AFB, Massachusetts, 1977.

8. P. R. Bevington, Data Reduction and Error Analysis for the Physical Sciences (McGraw-Hill Book Company, New York, 1969), Ch. 11.

APPENDIX A
TWO-BEAM SOLUTION TO MATCH FOUR VELOCITY MOMENTS

Garrett[6] has shown for a similar problem that the set of nonlinear simultaneous equations, Eq. (17), can be reduced to a single quadratic equation of the form,

$$AX^2 + BX + C = 0 \tag{A-1}$$

where A, B, and C are functions of the right-hand side of Eqs. (17). For the problem of calculating the velocities and densities of monoenergetic beams, we have set

$$X = \frac{v_{1,2}}{<v>}$$

$$A = 1 - \frac{3}{m}\frac{T_{AV}}{<v>^2}$$

$$B = \frac{4}{m}\frac{T_{RMS}}{<v>^2} - \frac{3}{m}\frac{T_{AV}}{<v>^2} \tag{A-2}$$

$$C = \frac{9}{m^2}\left(\frac{T_{AV}}{<v>}\right)^2 - \frac{4}{m}\frac{T_{RMS}}{<v>^2}$$

The beam velocities are found to be

$$v_{1,2} = \frac{<v>}{6\ T_{AV} - 2\ m<v>^2} \cdot \left\{ 4\ T_{RMS} - 3\ T_{AV} \pm \left[8\ T_{RMS}\left(2\ T_{RMS} - 9\ T_{AV} + 2\ m<v>^2\right) - 27\ T_{AV}^2\left(1 - \frac{4\ T_{AV}}{m<v>^2}\right)\right]^{1/2} \right\} \tag{A-3}$$

where v_1 (v_2) corresponds to the + (-) sign of Eq. (A-3).

The beam densities corresponding to the velocities are found by substitution,

$$n_1 = n\left(\frac{v_2 - \langle v \rangle}{v_1 - v_2}\right)$$

$$n_2 = n - n_1 \tag{A-4}$$

APPENDIX B

THREE-BEAM SOLUTION TO MATCH SIX VELOCITY MOMENTS

To find a three-beam solution to match six velocity moments of a distributed spectrum, six nonlinear simultaneous equations must be solved. These equations are of the form

$$\sum_{k=1}^{3} \sum_{j=0}^{5} n_k v_k^{\,j} = M_j \tag{B-1}$$

where n_k and v_k are the densities and velocities of the beams, and M_j represents the j^{th} velocity moment of the distributed spectrum.

Although any physically reasonable distribution function can be utilized, we used the six velocity moments of a Maxwellian distribution, Eqs. (2)-(7), to find solutions of Eq. (B-1). The beam densities and velocities were normalized by changing variables,

$$\begin{aligned} a,b,c &= \frac{n_{1,2,3}}{n} \\ x,y,z &= \frac{v_{1,2,3}}{\langle v \rangle} \end{aligned} \tag{B-2}$$

so that the moment equations for a Maxwellian distribution become

$$\begin{aligned} a + b + c &= 1 \\ ax + by + cz &= 1 \\ ax^2 + by^2 + cz^2 &= \frac{3\pi}{8} \\ ax^3 + by^3 + cz^3 &= \frac{\pi}{2} \\ ax^4 + by^4 + cz^4 &= \frac{15}{64}\pi^2 \\ ax^5 + by^5 + cz^5 &= \frac{3}{8}\pi^2 \end{aligned} \tag{B-3}$$

Eqs. (B-3) can be solved by an iterative procedure that minimizes the expression represented by the absolute sum of Eqs. B-3. Trial solutions are substituted into a computer program which then converges on the best solution through an iterative process.[8]

The only solutions of Eqs. (B-3) found by the iterative routine were permutations of the following:

$$(x,y,z) = (1.9679,\ 1.14085,\ 0.4881)$$
$$(a,b,c) = (0.0866,\ 0.5879,\ 0.3255) \qquad \text{(B-4)}$$

The energies of the beams were found from

$$E_1 = \frac{4}{\pi} T x^2 \qquad \text{(B-5)}$$

and similar expressions for E_2 and E_3.