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REPORT NO. NADC-80099-60

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WAVE PROPAGATION IN ANISOTROPIC LAYERED MEDIA

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17 June 1980

PHASE REPORT
Task No. ZF61-542-201
Work Unit ZM501

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6 2 4 2 1 1

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) NADC-80099-60	2. GOVT ACCESSION NO. AD-A090 014	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) Wave Propagation in Anisotropic Layered Media		5. TYPE OF REPORT & PERIOD COVERED (9) Phase rept.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) William R. Scott		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS NAVAL AIR DEVELOPMENT CENTER Aircraft and Crew Systems Technology Directorate Warminster, Pennsylvania 18974		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Task No. ZF61-542-201 Work Unit ZM501
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Air Systems Command Department of the Navy Washington, DC 20361 (11)		12. REPORT DATE 17 June 1980
		13. NUMBER OF PAGES 22
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) (12) 24 (16) F61542		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public ~~release~~; distribution unlimited.

(17) ZF61542201

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Anisotropic Media
Wave Propagation
Composite Materials

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A mathematical treatment is presented for the problem of a plane wave obliquely incident upon an attenuating layered anisotropic medium. Detailed treatment is given for cases of interest for laminated composite materials.

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INTRODUCTION

In order to obtain a comprehensive model to describe the interaction of ultrasonic waves with fiber reinforced laminated composite materials , a series of experimental and theoretical studies has been undertaken characterizing the behavior of elastic waves in anisotropic layered media. The principal motivation of these studies has been to develop improved nondestructive evaluation techniques in which measurements are made of properties, such as complex elastic constants, which are critical parameters used in the modeling of failure and fracture in composites.

This report will be devoted entirely to the mathematical treatment of oblique incidence wave propagation in layered anisotropic media. Results will be developed for use in subsequent reports which will detail various experimental studies and will describe the use of a comprehensive computer code for generating accurate numerical results.

No attempt has been made to provide a comprehensive treatment of wave propagation in all anisotropic media, rather consideration has been restricted to symmetries which are found in certain composite materials of interest. Thus, the results apply to materials of orthorhomic symmetry and higher. Within this class of problems a further restriction has been made limiting boundary conditions to treat only those boundary planes coincident with principle planes of symmetry for the material. In so doing we eliminate a few cases of practical interest but obtain the advantage of restricting eigenvalue problems to those for which closed form solutions exist.

The development of wave propagation theory will be carried to the point where total oblique incidence reflection and transmission coefficients can be computed as a function of frequency for rather general laminates consisting of laminae of the treated symmetries.

The mathematical development will proceed by defining the complex secular wave equation for displacement velocity in an anisotropic medium and transforming it to allow its solution in terms of parameters describing wave propagation in the incident medium. The solution to this equation provides the complex phase velocity and displacement velocity eigenvectors which, when coupled with Snell's Law, yield the direction of the propagating waves.

Using this information, impedance matrices are defined which relate the stress and velocity fields for the propagating waves.

Once the stress and velocity fields are characterized for the media on either side of an interface, linear boundary value equations may be written relating the amplitudes for various modes of wave propagation on either side of the interface.

For a system consisting of many interfaces, a method will be developed for evaluating the total reflection and transmission coefficients. This technique is a generalization of one developed by Scott[1] for normal incidence laminate propagation.

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THEORY

The time harmonic equation for a wave in an anisotropic medium may be expressed in the form[2].

$$k^2(l_{iK}\, C_{KL}\, l_{Lj})v_j = \rho\omega^2 v_i \quad . \qquad (1)$$

The quantities in (1) are defined as follows:

k is the complex wave vector for the propagating wave

ω is the angular frequency

ρ is the density

V_i is the vector associated with the i th component of wave displacement velocity

C_{KL} are the components of the complex elastic constant matrix as expressed in contracted engineering notation in which the subscripts run over 6 values, and

l_{iK} is a matrix of the form

$$\begin{bmatrix} l_x & 0 & 0 & 0 & l_z & l_y \\ 0 & l_y & 0 & l_z & 0 & l_x \\ 0 & 0 & l_z & l_y & l_x & 0 \end{bmatrix} \qquad (2)$$

where l_i are complex direction cosine defining the propagation direction of the wave; the transposed matrix, l_{Lj}, is defined analogously.

WAVE EQUATION IN AN ORTHOTROPIC LAMINATE

For the most general case to be considered, an anisotropic solid with orthorhombic symmetry and principal axes along the *x*, *y*, and *z* directions (1) takes the form:

$$\begin{bmatrix} \alpha - \dfrac{\rho\omega^2}{k^2} & \delta & \varepsilon \\ \delta & \beta - \dfrac{\rho\omega^2}{k^2} & \xi \\ \varepsilon & \xi & \gamma - \dfrac{\rho\omega^2}{k^2} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0 \qquad (3)$$

where

$$\alpha = C_{11} \cos^2 \theta \cos^2 \phi + C_{66} \cos^2 \theta \sin^2 \phi + C_{55} \sin^2 \theta$$

$$\beta = C_{66} \cos^2 \theta \cos^2 \phi + C_{22} \cos^2 \theta \sin^2 \phi + C_{44} \sin^2 \theta$$

$$\gamma = C_{55} \cos^2 \theta \cos^2 \phi + C_{44} \cos^2 \theta \sin^2 \phi + C_{33} \sin^2 \theta \qquad (4)$$

$$\delta = (C_{12} + C_{66}) \cos^2 \theta \cos \phi \sin \phi$$

$$\varepsilon = (C_{13} + C_{55}) \cos \theta \sin \theta \cos \phi$$

$$\xi = (C_{23} + C_{44}) \cos \theta \sin \theta \sin \phi$$

θ is chosen as the angle of incidence of the wave with respect to the plane of the laminate (measured up from the plane) and ϕ is the angle between the plane of incidence and the first principal axis in the plane of the laminate.

It should be noted that a number of possible conventions exist for choosing axes in an anisotropic medium. In a unidirectional fiber reinforced lamina the first coordinate index is usually chosen along the fiber axis while the third index is chosen perpendicular to the plane of the laminate. On the other hand, when the wave equations refer to a crystallographic system the third axis is conventionally chosen perpendicular to the plane of hexagonal symmetry which would be the counterpart of the fiber direction. Hence, when values of elastic moduli are chosen for the equations, care must be taken to ensure that they are referenced to the same coordinate system as the equations. In this paper we adhere to former convention.

The wave equation in the form (3) may be solved in closed form whenever the angles defining the propagation direction (θ and ϕ) are known. This is not often the case experimentally.

In most practical problems involving anisotropic media the waves present have risen from refraction or reflection at a given interface and it is desired to solve the equation (3) in terms of the parameters of the incident wave. In order to treat such cases the complex form of Snell's Law

$$k \cos \theta = k' \cos \theta' \qquad (5)$$

is used, where in general θ, θ', k and k' are complex. The complex angles are necessary for separately defining planes of constant phase and attenuation which may not always be parallel in problems involving oblique incidence.

By substituting the relation (5) into (4) which in turn is substituted into (3) a secular equation is derived which expresses the quantity w^2/h^2

in terms of the known quantities θ, C_{ij}, the angle of incidence θ' and the incident wave vector k'.

This is not a secular equation in the classical sense, since the off-diagonal elements involve the variable ω/k; however, the resulting bicubic equation does provide the correct eigenvalues and eigenvectors for the refracted waves. On the other hand, if we were to compute actual numerical values of k and θ from (5) their substitution into (3) and (4) would yield in addition to one correct solution, two spurious solutions corresponding to other eigen modes traveling parallel to the true refracted wave.

As mentioned above, the secular equation (3) is bicubic in ω^2/k^2 and it is easily shown that it is also bicubic in cos θ. Expanding (3) we have

$$\left(\alpha - \frac{\rho\omega^2}{k^2}\right)\left[\left(\beta - \frac{\rho\omega^2}{k^2}\right)\left(\gamma - \frac{\rho\omega^2}{k^2}\right) - \varepsilon\xi\right]$$
$$- \delta\left[\delta\left(\gamma - \frac{\rho\omega^2}{k^2}\right) - \xi\varepsilon\right]$$
$$+ \varepsilon\left[\delta\xi - \varepsilon\left(\beta - \frac{\rho\omega^2}{k^2}\right)\right] = 0 \tag{6}$$

From the relations (4) it is clear that cos θ appears everywhere in even powers since the terms in cos θ sin θ (ε and ξ) always appear in even products and in fact the equation can readily be converted to a cubic equation in $\cos^2\theta$ using the substitution

$$\frac{\omega^2}{k^2} = \frac{\cos^2\theta}{\cos^2\theta'}\,\frac{\omega^2}{k'^2} \tag{7}$$

The fact that the equation can be expressed in terms of even powers of cos θ is a result of (3) being expressed in such a way that incident and reflected waves move with the same speed.

WAVE EQUATION IN A HEXAGONAL MEDIUM

For the case of hexagonal symmetry the secular equation (1) reduces to a biquadratic equation with solutions of the form

$$\left(\frac{k}{\omega}\right)_{PS} = \left(\frac{\rho}{C_{66}\sin^2\alpha + C_{44}\cos^2\alpha}\right)^{\frac{1}{2}} \tag{8}$$

$$\left(\frac{k}{\omega}\right)_{QS} = \left(\frac{2\rho}{C_{11}\sin^2\alpha + C_{33}\cos^2\alpha + C_{44} - \sqrt{\left[(C_{11}-C_{44})\sin^2\alpha + C_{33})\cos^2\alpha\right]^2 + (C_{13}+C_{44})^2\sin^2 2\alpha}}\right)^{\frac{1}{2}} \tag{9}$$

$$\left(\frac{k}{\omega}\right)_{QL} = \left(\frac{2\rho}{C_{11}\sin^2\alpha + C_{33}\cos^2\alpha + C_{44} + \sqrt{\left[(C_{11}-C_{44})\sin^2\alpha - (C_{33}-C_{44})\cos^2\alpha\right]^2 + (C_{44}+C_{13})^2\sin^2 2\alpha}}\right)^{\frac{1}{2}} \tag{10}$$

These are denoted as the pure shear, quasishear and quasilongitudinal solutions respectively, where α is the angle between the unique principal axis and the direction of wave propagation. This solution is a good first approximation for the problem of a unidirectional fiber reinforced laminate, where the fiber direction lies along the principal axis. Clearly this solution yields isotropy in the plane perpendicular to the fiber direction.

For the case in which a wave is obliquely incident upon a hexagonal medium (see figure 1) from a medium of known velocity, equations (6), (7) and (9) can be inverted through the use of Snell's Law giving for the pure shear case,

$$\left(\frac{k}{\omega}\right)^2_{PS} = \frac{(C_{66} - C_{44})\cos^2\theta'\cos^2\phi\,(k'^2/\omega^2) + \rho}{C_{66}} \tag{11}$$

and

$$\cos\theta_{PS} = \left\{\frac{C_{66}}{\rho + (C_{66} - C_{44})\cos^2\phi\cos^2\theta'\,\frac{k'^2}{\omega^2}}\right\}^{\frac{1}{2}} \tag{12}$$

where

- k is the magnitude of the wave vector in the hexagonal medium
- θ is the angle of refraction in the hexagonal medium
- ρ is the density of the hexagonal medium
- θ' is the angle of incidence
- k' is the wave vector in the incident medium
- ϕ is the angle between the x axis (fiber direction) and the plane of incidence (or the plane of refraction), and the
- C_{ij}'s are elements of the elastic constant matrix for the hexagonal material.

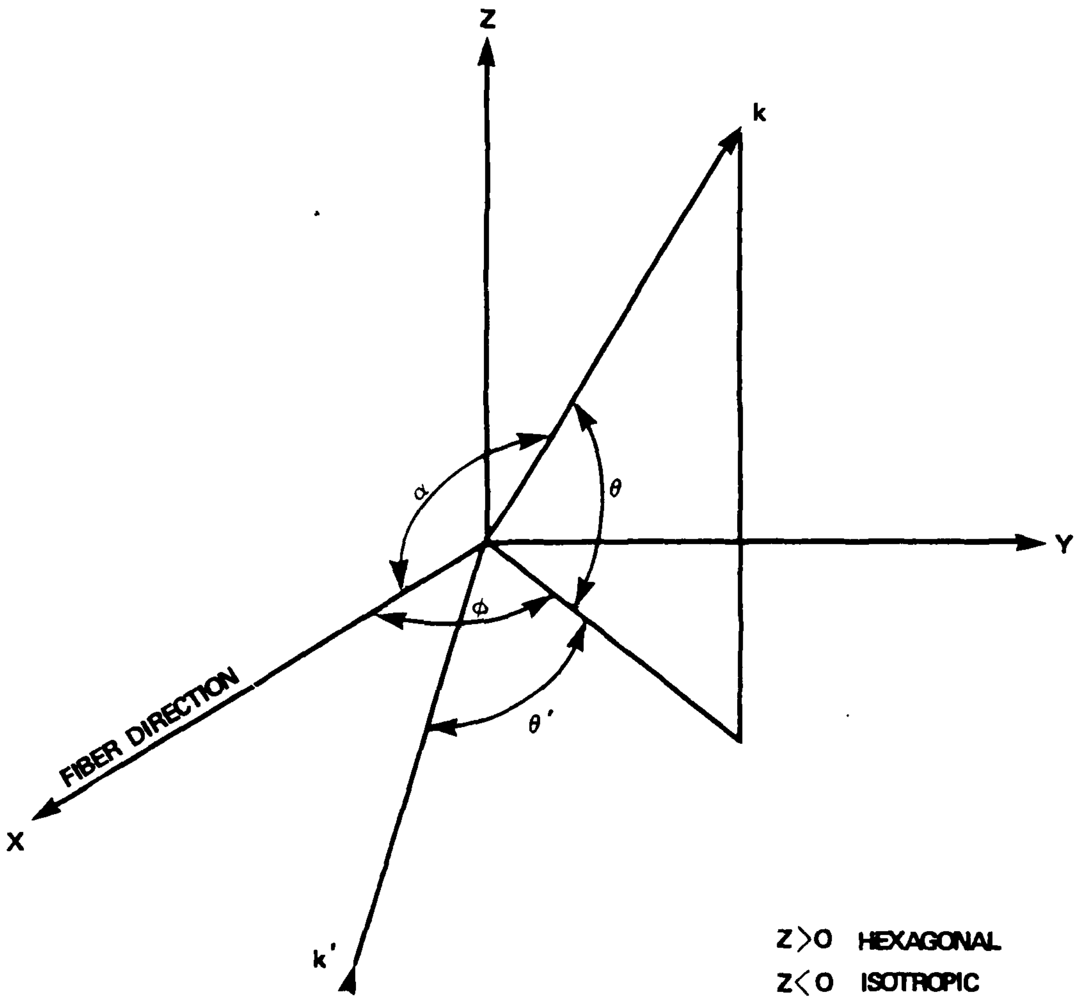

Figure 1. Plane Wave Incident Obliquely Upon a Hexagonal Medium

For the cases of quasishear and quasilongitudinal waves the expressions are somewhat more cumbersome:

$$\cos^2 \theta_{QS,QL} = \frac{-B}{A} \pm \left(\frac{B^2}{A^2} - \frac{4\, C_{11}\, C_{44}}{A} \right) \tag{13}$$

where

$$B = \left[(C_{11} - C_{44})\,(C_{11} - 2C_{44} + C_{33}) - 2(C_{13} + C_{44})^2 \right] \cos^2 \phi$$
$$-2 \left[C_{44} + C_{11} \right] \left[\frac{\omega^2 \rho}{k'^2 \cos^2 \theta'} + (C_{11} - C_{33}) \cos^2 \phi \right] \tag{14}$$

and

$$A = \left[\frac{\omega^2 \rho}{k'^2 \cos^2 \theta} + (C_{11} - C_{33}) \cos^2 \phi \right]^2$$
$$- \left[(C_{11} - 2\, C_{44} + C_{33})^2 - 4(C_{13} + C_{44})^2 \right] \cos^4 \phi \tag{15}$$

IMPEDANCE MATRICES AND THE BOUNDARY VALUE PROBLEM

In order to solve the complete boundary value problem for waves traveling between two anisotropic media it is also necessary to compute the normal component of the stress fields, or traction forces associated with waves on either side of the interface. This relation is given by[2]

$$-T_{in} = \frac{n_{iK}\, C_{KL}\, K_{Lj}}{\omega}\, Vj \tag{16}$$

where

$$n_{iK} = \begin{bmatrix} n_x & 0 & 0 & 0 & n_z & n_y \\ 0 & n_y & 0 & n_z & 0 & n_x \\ 0 & 0 & n_z & n_y & n_x & 0 \end{bmatrix} \tag{17}$$

n_i are normals to the interface

$$K_{Lj} = \begin{bmatrix} K_x & o & o \\ 0 & K_y & 0 \\ 0 & 0 & K_z \\ 0 & K_z & K_y \\ K_z & 0 & K_y \\ K_y & K_x & 0 \end{bmatrix} \tag{18}$$

K_1 are wave vectors for the wave within the given medium and T_{in} (the traction force) consists of three components transforming as a vector.

The quantity

$$\frac{n_{1K}\ C_{KL}\ K_{Lj}}{\omega} = Z_{ij} \tag{19}$$

in (16) is defined as the normal acoustic impedance matrix and is useful in solving the boundary value equations for a given interface.

Boundary value equations may be written by applying the conditions that the displacement velocities and traction forces at the interface between two media must match. This implies:

$$\sum_u V_u = \sum_v V'_v \tag{20}$$

$$\sum_u Z_u V_u = \sum_v Z'_v V'_v \tag{21}$$

where the unprimed variables refer to the incident medium. In general, for orthotropic symmetry, these equations will contain six unknowns corresponding to three possible modes for the reflected and refracted waves (2 quasishear, 1 quasilongitudinal).

Each vector V_i may be written in the form $V_i = A_i\ v_i$ where v_i is an eigenvector of (3) and A_i is the displacement velocity amplitude. Since the components of each v_i are known within a constant factor (10) and (11) each may be broken down into 3 component equations allowing solutions for the six unknowns to be obtained.

Care must be taken in (10) and (11) to insure that identical components are being matched for the incident and refracted media. For media in which principal axes are not aligned, this is most easily done by rotating the T's and v's on the right side of the equation after initially computing them in a system for which the coordinate axes and principal axes are coincident.

COMPUTATION OF THE Z MATRIX

In order to compute the Z matrix of expression (9) it is necessary to know the wave vector k for the particular mode in question. For refracted waves this information comes directly from the solution to equation (3). When reflected waves are considered one can, by the use of symmetry arguments, obtain the k vectors for the refracted waves directly from those of the reflected waves by changing the sign of the component of k normal to the interface. If all three modes of the incident wave are present this will permit immediate computation of the reflected Z matrices; however, if only one mode is present this may be used to determine the corresponding reflected mode which may then be used in place of the incident k' and θ' for the solution of equation (3).

Having computed the oblique incidence reflection and transmission coefficients for waves incident upon interfaces between anisotropic media of interest, it becomes possible to compute frequency dependent reflection and transmission coefficients for arbitrary laminates consisting of such media.

PLANE WAVES IN LAYERED MEDIA

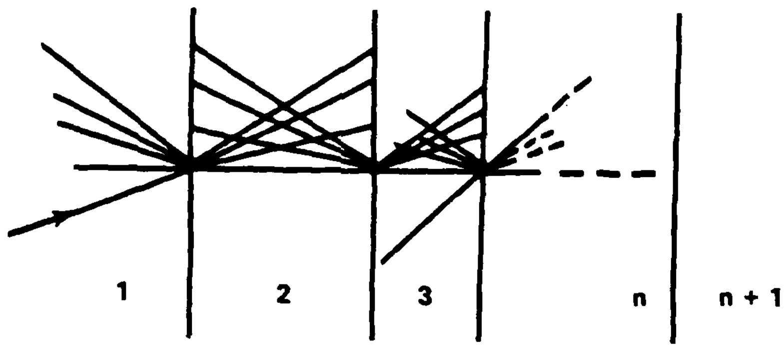

Figure 2. Plane Waves being Reflected and Mode Converted at the Interfaces of a Layered Medium

Figure 2 depicts a laminate for which a plane wave is obliquely incident upon an interface from the lower left. The wave is monochromatic, and by definition infinite in width and may, therefore, be equally well represented by any parallel ray. Upon striking the interface the wave produces, in the most general case, three reflected and three refracted modes whose amplitudes may be denoted by the vector

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \qquad (22)$$

For the sake of convention the components may be ordered according to corresponding velocity for orthorhombic symmetry. For hexagonal symmetry the ordering quasilongitudinal, quasishear, pure shear is less ambiguous since modes do not change at points where velocities cross over. It is clear from Snell's Law and elementary geometry that for plane parallel media, even numbered multiple reflections in a given direction will always be scattered into one of the three modes present in the transmitted wave. In other words, for the situation depicted in figure 2, a single vector a as in (12) is adequate for describing all of the waves propagating to the right within any given medium. This implies that, since we are dealing with a linear system of plane waves, any reflection, propagation, transmission, etc. of such waves can be treated as a matrix multiplication operation. For example, a vector a whose components represent the three possible modes of propagation with a given $(k' \sin \theta')$ for any given medium will have components of the form

$$a_i = a_i \exp i(\omega t - k_i \cdot r) \tag{23}$$

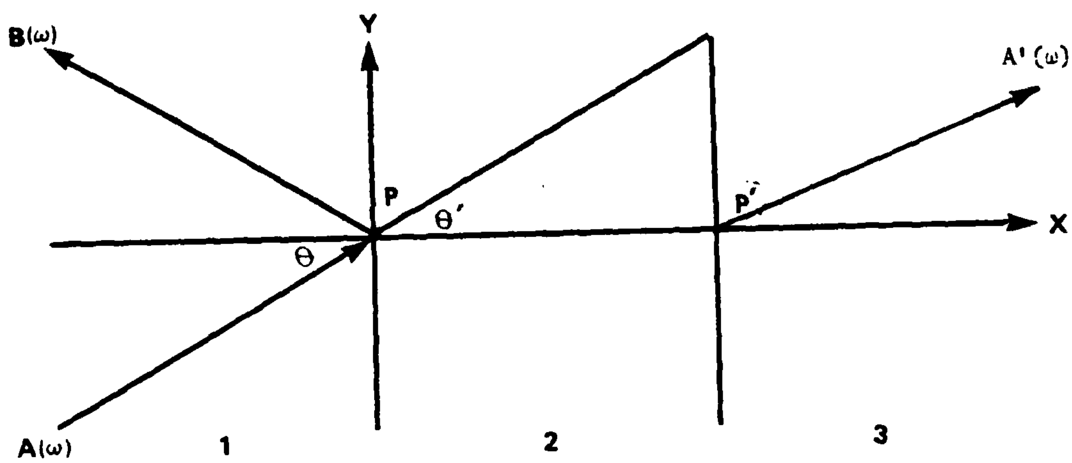

Figure 3. Waves Referenced to the Origin

For simplicity the phase of wave (23) will be referenced to point P(r=0) (see figure 3). Thus in passing from medium 1 to medium 2 the vector a will undergo the component transformation

$$a'_j = \sum_i T_{ij}\, a_i \exp i(\omega t - k'_j \cdot r), \tag{24}$$

where

- a is a mode vector which represents waves with the same "k sin θ" incident at P
- a_i is the complex amplitude of the i th component of such waves at r=0 and t=0
- k_i is the wave vector for the i th component and
- T_{ij} is the amplitude transmission coefficient from the i th incident mode to the i th refracted mode

The variables a', k', are defined analogously for medium 2 which implies the matrix equation

$$a'_j = \sum_i T_{ij}\, a_i \text{ or } a' = \overline{T}\, a \tag{25}$$

It should be noted that the vanishing of the phase component of (15) results from the assumption that the wave front travels an infinitesimal distance in going from medium 1 to medium 2 in the neighborhood of P and is independent of the fact that r = 0. Hence, any phase shift experienced would be entirely the result of transmission phenomena described in equations (10) and (11) from which T_{ij} is computed.

In computing phase shift matrices corresponding to propagation delays, it is convenient to describe phase shifts seen at a given point in terms of frequency domain spectra. Thus a wave traveling from point P to point P' in medium 2 will undergo a phase shift such that

$$\begin{aligned} a'_i\,(P') &= a'_i\,(P)\exp i(-k'\,D\cos\theta'_i) \\ &= a_i\,(P)\exp - i(\omega D\cos\theta'_i/C_i) \end{aligned} \tag{26}$$

which is described by a diagonal matrix of the form

$$\Phi_{ij} = \delta_{ij}\exp -i(\omega D\cos\theta'_i/C_i) \tag{27}$$

$$a'_i\,(P') = \overline{\Phi}_{ij}\, a_i\,(P) \tag{28}$$

$$a\,(P') = \overline{\Phi}\, a\,(P) \tag{29}$$

The other matrix of interest is that associated with a reflection. Again no propagation delay is associated with this phenomenon and reflection matrices produce the transformation in medium 1

$$b_i\,(P) = \sum R_{ij}\, a_j\,(P) \tag{30}$$

or

$$b = \overline{R}\, a \quad \text{where} \quad b_i = b_i \exp i(\omega t + k_i\cdot r) \tag{31}$$

where R_{ij} is the amplitude reflection coefficient for i mode incident waves going to j mode reflected waves.

Again the coefficients R_{ij} are computed by solving the equations (10) and (11).

MULTIPLE REFLECTIONS IN A THICK PLATE

In calculating the reflection and transmission coefficients of a thick plate it is convenient to compute the total amplitude of waves at the points P and P' and to consider wave fronts as being reflected between those points (figure 3). This is permissible since it is easily shown that the phase shift of the wavefront is independent of the point from which the reflection is assumed to occur. Then for an incident wave A (ω) the total reflected wave at the point P is of the form

$$B(\omega) = (\bar{R}_{12} + \bar{T}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}\,\bar{T}_{12} + \bar{T}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2\,\bar{R}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2\,\bar{T}_{12} + $$
$$+ \bar{T}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2\,\bar{R}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2\,\bar{R}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2\,\bar{T}_{12} + \ldots)\,A(\omega)$$

$$B(\omega) = (\bar{R}_{12} + \bar{T}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2 \sum_{n=0} (\bar{R}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2)^n\,\bar{T}_{12})\,A(\omega) \tag{32}$$

$$B(\omega) = \left[\bar{R}_{12} + \bar{T}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2\,(\bar{\mathbf{1}} - \bar{R}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi})^{-1}\,\bar{T}_{12}\right] A(\omega) \tag{33}$$

where $B(\omega)$ is the amplitude vector for waves of frequency ω traveling in medium 1 with a negative y component of velocity and matrices of the form $\bar{T}_{ij}$ or $\bar{R}_{ij}$ are incident from medium i onto medium j.

A composite reflection coefficient may now be defined

$$\bar{R}_{13} = [\bar{R}_{12} + \bar{T}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2\,(\bar{\mathbf{1}} - \bar{R}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2)^{-1}\,\bar{T}_{12}] \tag{34}$$

which describes the total contribution in medium 1 from interface reflections occurring to the right of medium 1 (see figure 2).

In a similar fashion, the wave at point P' emerging into medium 3 can be written

$$A'(\omega) = \Big\{\bar{T}_{23}\,\bar{\Phi}_2\,\bar{T}_{12} + \bar{T}_{23}\,\bar{\Phi}_2\,\bar{R}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}\,\bar{T}_{12}$$
$$+ \bar{T}_{23}\,\bar{\Phi}\,\bar{R}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}_2\,\bar{R}_{21}\,\bar{\Phi}_2\,\bar{R}_{23}\,\bar{\Phi}\,\bar{T}_{12}\ \ldots\ldots\ A(\omega)\Big\} \tag{35}$$

$$A'(\omega) = \left\{\bar{T}_{23}\ \bar{\Phi} \sum_{n=0} (\bar{R}_{21}\ \bar{\Phi}_2\ \bar{R}_{23}\ \bar{\Phi}_2)^n\ \bar{T}_{12}\right\} A(\omega)$$

$$= \left\{\bar{T}_{23}\ \phi \left\{\bar{\mathbb{1}} - R_{21}\ \phi_2\ R_{23}\ \phi\right\}^{-1} T_{12}\right\} A(\omega) \qquad (36)$$

The total transmission coefficient may now be defined as,

$$T_{13} = \left\{ T_{23}\ \phi_2 \left\{ \bar{\mathbb{1}} - R_{21}\ \phi_2\ R_{23}\ \phi\right\}^{-1} T_{12} \right\}. \qquad (37)$$

REFLECTION AND TRANSMISSION COEFFICIENTS FOR LAYERED MEDIA

In the frequency domain, expressions (34) and (37) may be applied iteratively to construct reflection and transmission coefficients for multilayered media. In figure 2 a wave is incident from the left upon a stratified medium having n + 1 layers, the n + 1 layer being the incident layer and the n th interface being the first interface.

If the media beyond medium n and in front of medium 1 (with respect to the direction of wave motion) are considered as a single virtual interface having reflection and transmission matrices $\bar{R}_{n-1}$, $\bar{T}_{n-1}$; then the problem becomes equivalent to that of a thick plate with reflection and transmission matrices,

$$\bar{R}_n = [\bar{r}_{nf} + \bar{t}_{nb}\ \bar{\Phi}_n\ \bar{R}_{n-1}\ \bar{\Phi}_n\ (\bar{\mathbb{1}} - \bar{r}_{nb}\ \bar{\Phi}_n\ \bar{R}_{n-1}\ \phi_n)^{-1}\ \bar{t}_{nf}] \qquad (38)$$

$$\bar{T}_n = [\bar{T}_{n-1}\ \bar{\Phi}_n\ (\bar{\mathbb{1}} - \bar{r}_{nb}\ \bar{\Phi}_n\ \bar{R}_{n-1}\ \bar{\Phi}_n)^{-1}\ t_{nf}] \qquad (39)$$

where r_{nf} (r_{nb}) and t_{nf} (t_{nb}) are the forward (backward) reflection and transmission coefficients respectively.

Clearly equations (38) and (39) form recursion relations which allow the computation of $\bar{R}_n$ and $\bar{T}_n$ in terms of the phase shift matrices $\bar{\Phi}_i$ and the reflection and transmission matrices $\bar{t}_{if}$, $\bar{t}_{ib}$, $\bar{r}_{if}$, $\bar{r}_{ib}$.

This set of recursion relations can be put in matrix form using the method of Scott[1] by employing the definitions

$$R_n = a_n\ b_n^{-1}\ , \qquad (40)$$

$$a_n = \left(r_{nf}\ t_{nf}^{-1} - r_{nf}\ t_{nf}^{-1}\ r_{nb}\ \phi_n\ R_{n-1}\ \phi_n + t_{nb}\ \phi_n\ R_{n-1}\ \phi_n\right) \qquad (41)$$

and

$$b_n = (t_{nf}^{-1} - t_{nf}^{-1}\ r_{nb}\ \phi_n\ R_{n-1}\ \phi_n) \qquad (42)$$

where the bar matrix notation is suppressed. Expression (40) may be shown to follow from (38), (41) and (42) by direct substitution. By use of definition (40) we may write

$$R_n = a_n \cdot b_n^{-1} = \left[(r_{nf}\, t_{nf}^{-1} - r_{nf}\, t_{nf}^{-1}\, r_{nb}\, \phi_n\, R_{n-1}\, \phi_n)\phi_n^{-1}\, b_{n-1} + t_{nb}\, \phi_n\, a_{n-1} \right]$$

$$\times \left[t_{nf}^{-1}\, \phi_n^{-1}\, b_{-1} - t^{-1}\, r_{nb}\, \phi_n\, a_{n-1} \right]^{-1}$$

$$= a_n\, \phi_n^{-1}\, b_{n-1}\, b_{n-1}^{-1}\, \phi_n\, b_n^{-1} \quad . \tag{43}$$

Thus for each pair of matrices a_n, b_n a matrix relationship may be established of the form:

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} (t_{nb} - r_{nf}\, t_{nf}^{-1}\, r_{nb})\, \phi_n & r_{nf}\, t_{nf}\, \phi_n^{-1} \\ -t_{nf}^{-1}\, r_{nb}\, \phi_n & t_{nf}^{-1}\, \phi_n^{-1} \end{pmatrix} \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} \tag{44}$$

Extending the relationship through all of the layers of the laminate yields:

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \prod_{m=n,1} \begin{pmatrix} (t_{mb} - r_{mf}\, t_{mf}^{-1}\, r_{mb})\, \phi_m & r_{mf}\, t_{mf}\, \phi_m^{-1} \\ -t_{mf}^{-1}\, r_{mb}\, \phi_m & t_{mf}^{-1}\, \phi_m^{-1} \end{pmatrix} \begin{pmatrix} r_{1f} \\ \mathbf{1} \end{pmatrix} \tag{45}$$

The expressions (44) and (45) above could be expanded into a form containing 6x6 and 6x3 matrices; however, the above forms are much easier to work with and are expressed in terms of the 3x3 matrices desired in the result.

From expression (39) a similar definition can be made for transmission matrices. Thus, letting

$$c_n = t_{n-1}\, \phi_n \tag{46}$$

and as before

$$b_n = (t_{nf}^{-1} - t_{nf}^{-1}\, r_{nb}\, \phi_n\, R_{n-1}\, \phi_n) \tag{47}$$

the expression (39) becomes

$$T_n = c_n \cdot b_n^{-1} \quad . \tag{48}$$

This leads to the recursion relation

$$T_n = [c_{n-1} \quad b_{n-1}^{-1} \phi_n] \, [t_{nf}^{-1} - t_{nf}^{-1} r_{nb} \phi_n a_{n-1} b_{n-1}^{-1} \phi_n]^{-1}$$

$$= c_{n-1} \cdot [t_{nf}^{-1} \phi_n^{-1} b_{n-1} - t_{nf}^{-1} r_{nb} \phi_n a_{n-1}]^{-1} \tag{49}$$

from which is derived the matrix relation

$$\begin{pmatrix} c_n \\ a_n \\ b_n \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & (t_{nb} - r_{nf} t_{nf}^{-1} r_{nb})\phi_n & r_{nf} t_{nf} \phi_n^{-1} \\ 0 & -t_{nf}^{-1} r_{nb} \phi_n & t_{nf}^{-1} \phi_n^{-1} \end{pmatrix} \begin{pmatrix} c_{n-1} \\ a_{n-1} \\ b_{n-1} \end{pmatrix} . \tag{50}$$

A matrix product of the type in expression (45) could readily be constructed, however it is clear that there is no mixing of the c_ns with other matrix elements and the immediate result is

$$T_n = t_{1f} \cdot b_n^{-1} \tag{51}$$

where b_n is obtained directly from expression (45).

RESULTS

The above analytical technique has been utilized for calculating reflection and transmission coefficients for a number of layered anisotropic and isotropic materials both lossless and attenuating. Although the author has not as yet found independent results in the literature for checking the most general cases which can be treated by this technique, verification has been performed for a number of results for reflection and transmission between crystalline media and crystalline and amorphous media given by Auld.[2] In addition, formulas given by Ewing Jardetzky and Press[3] for liquid to solid and solid to liquid interfaces have also been checked and give agreement of the type shown in figures 4 and 5. Excellent agreement was also found with results given by Brekhowskikh[4] for transmission through a thick aluminum plate in water as a function of angle. Furthermore all solutions generated for very general laminate systems have satisfied energy conservation laws.

CONCLUSIONS

The above formalism is believed to provide an accurate and efficient technique for treating rather general problems involving the oblique indicence transmission and reflection of stress waves through anisotropic, layered media. The technique has proved adequate to handle problems involving evanescent waves and material attenuation and should be useful in treating practical problems involving composite materials.

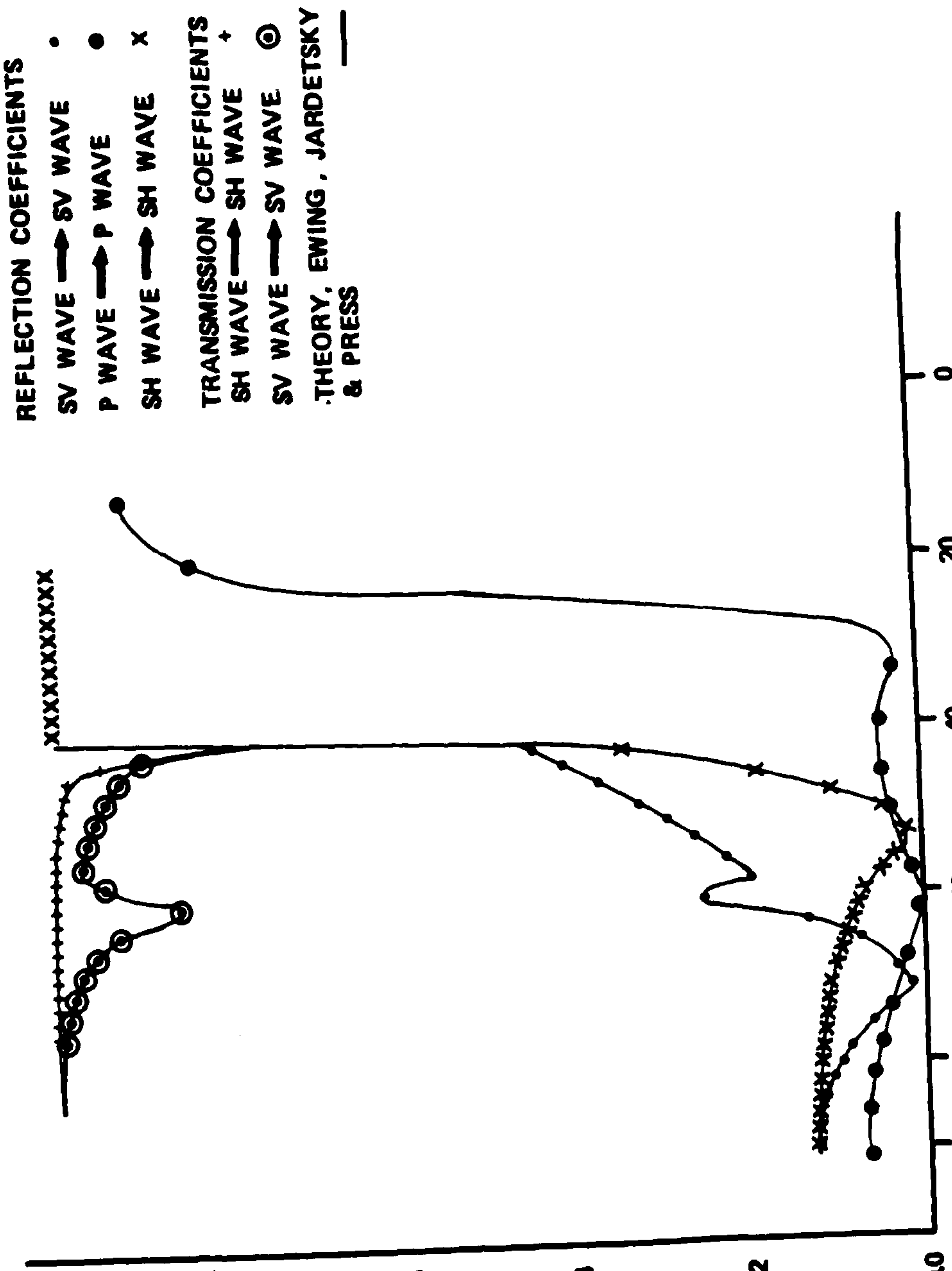

Figure 4 - Reflection and Transmission Coefficients for Interfaces Between Two Solid Media

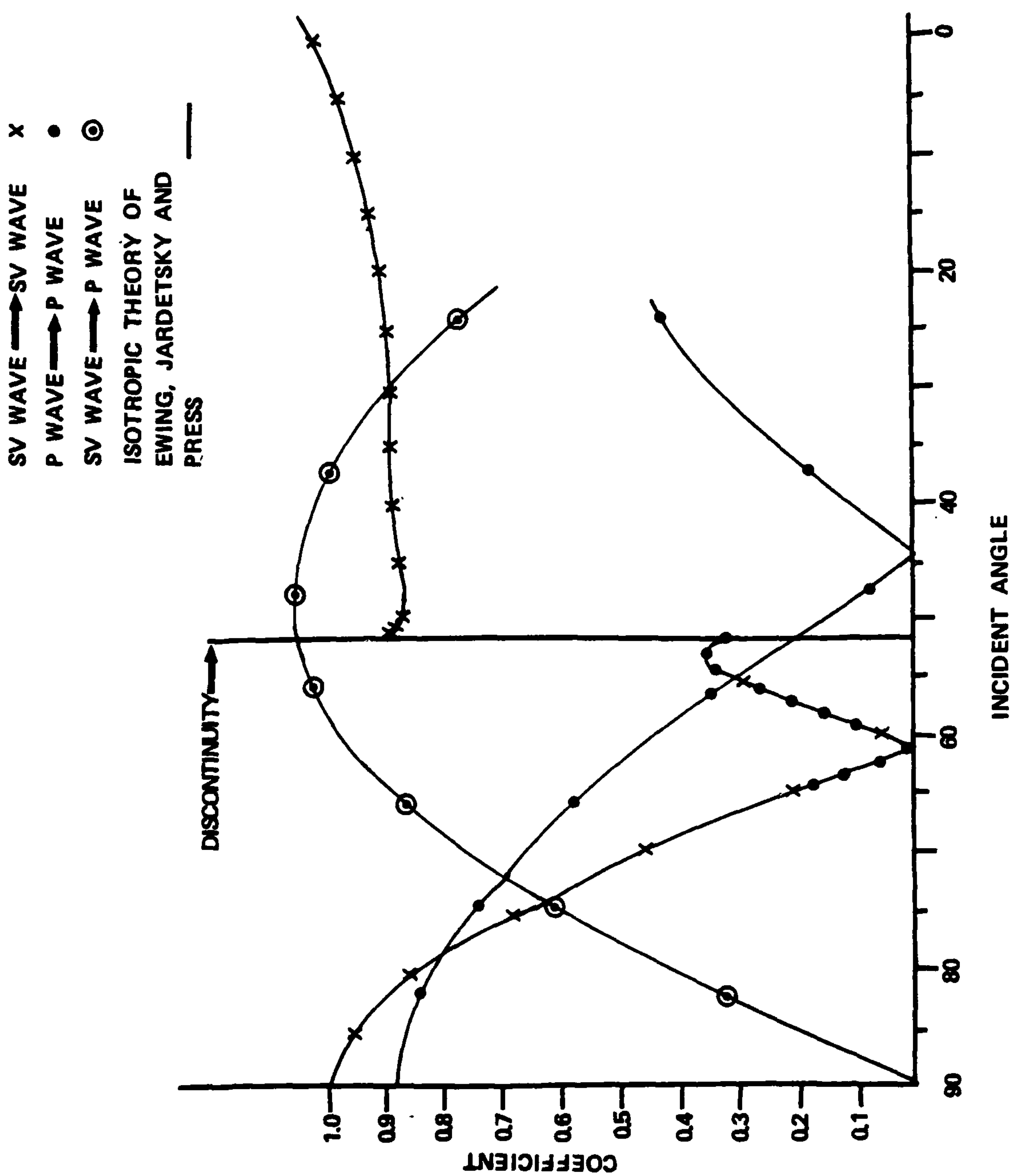

Figure 5 - Reflection Coefficients for Waves Traveling from a Solid into a Liquid

A C K N O W L E D G M E N T S

The author greatfully acknowledges the efforts of Dr. Ed Corwin and in particular Dr. Carl Schmeidekamp for work on computer programs to verify the analysis presented here.

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