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A THEORETICAL ANALYSIS OF THE RADAR CROSS SECTION OF THE BICONI--ETC(U)
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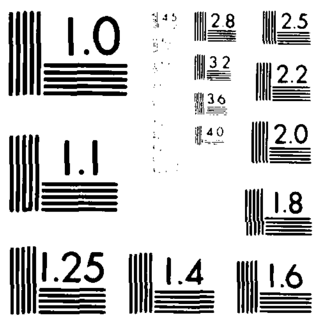

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TECHNICAL REPORT

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J.L. WHITROW

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S U M M A R Y

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The biconical corner reflector is a useful device where moderate enhancement of the radar cross section is required. This report uses the principles of physical optics to develop a formula for the radar cross section of an arbitrary biconical corner reflector illuminated at an arbitrary angle of incidence within the confines of the faces of the reflector. Previously published results containing arithmetic and typographical errors for the special case of normal incidence on a symmetrical reflector are corrected.

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TABLE OF CONTENTS

LIST OF APPENDICES

LIST OF FIGURES

1. INTRODUCTION

The biconical corner reflector has received mention in several texts and papers(refs.1,2,3) as a useful device for the passive enhancement of the radar cross section when invariance of the cross section with azimuth angle is a system requirement. It consists of two cones, one inverted on the top of the other, with apex angles such that the included angle between the surfaces of the cones is 90°. Most commonly the cone apex angles are both equal to 45°, and the cone apices do not coincide for structural reasons. The reflector functions by directing the electromagnetic radiation incident on each cone surface onto the other from whence it is rescattered. Using the concept of rays described by Deschamps(ref.4), it can be seen that for those rays in the plane containing the axis of the reflector this rescattering is such as to direct the rays back towards the illuminating radar (figure 1). However any other rays are dispersed away from the radar,and hence the enhancement of the radar cross section is not as great as, say, that of the trihedral corner reflector. In practical terms the biconical corner reflector can be regarded as occupying a useful position between those scatterers which, in the optical limit, have no dispersion, such as flat plates, Luneburg lenses and trihedral reflectors, and those which cause dispersion in both planes, such as spheres and, more generally, ellipsoids.

It is the purpose of this work to determine the radar cross section of the generalised corner reflector of arbitrary size and apex angle. This does not appear to have previously been the subject of research. Of the references cited above on the biconical corner reflector, only the second provides details of the computations necessary to calculate the radar cross section. It is of limited value, though, for it only treats incidence normal to the axis of a reflector constructed from cones each with an apex angle of 45°. The technique of calculation is not particularly illuminating of the physical process involved, and the derivation given contains arithmetic errors. These errors have been perpetuated in reference 1 and an additional typographical error included.

In this report the geometrical optics techniques developed by Deschamps (ref.4) are used to determine the magnetic field and hence the induced surface current on the lower cone resulting from the reflection of the incident electromagnetic field by the upper cone. From this, the field radiated by the surface current back towards the radar may be determined by using the techniques of physical optics. It is shown that the reflection of the incident field from the lower cone to the upper cone, and thence back to the radar produces exactly the same field strength. The total radar cross section of the biconical corner reflector then follows from simple calculations. It will be appreciated that the field directly incident on each face of the reflector will induce currents which will reradiate back towards the radar as well as towards the other face of the reflector. Provided, however, that incidence is not near the normal of either cone, and each face of the reflector is at least several wavelengths long, this reradiation is not significant and can safely be neglected in comparison with the doubly reflected field. Since the normal design objective is to use the biconical reflector at angles of incidence midway between the faces of the reflector, this contribution to the radar cross section has been neglected in this report. Furthermore diffraction by the edges of the reflector has been ignored. The results therefore should only be considered valid when all the critical dimensions of the reflector exceed several wavelengths.

The final result that is obtained is in the form of an integral of a fairly well behaved real function, valid for arbitrary cone apex angles and angles of incidence within the region subtended by the cones. As the general result, in the form of an integral, is amenable only to numerical evaluation, it does not highlight the effect of the various dimensions of the reflector on the total radar cross section. In three special cases though,

(i) when the cones touch at their apices,
(ii) when the apex angles are 45^0 and the angle of incidence is 0^0, and
(iii) when the lower cone has an apex angle of 90^0,

the integral can be evaluated analytically, yielding simple expressions for the radar cross section. In these cases the effect of a change in the dimensions of the cones is immediately apparent.

In this work it is assumed that the cone is illuminated by a plane electromagnetic wave, of angular frequency ω, polarised with the magnetic field parallel to the y axis of the xyz coordinate system used to describe the problem, with the direction of propagation of the incident wave subtending an angle β to the x axis. A positive time harmonic dependency, $\exp(i\omega t)$ of all the fields will be assumed throughout the analysis.

2. THEORETICAL ANALYSIS

2.1 The physical optics approximation

The simplest method of determining the radar cross section of an object is to apply the techniques of geometrical optics, but in situations where either or both principal radii of curvature of the reflecting surface are infinite (gaussian curvature is zero) this method predicts an infinite cross section in the specular direction, and zero elsewhere. The essential feature that geometrical optics omits is that for a body of characteristic dimension h, diffraction will spread the scattered field over an angular interval (λ/h) radians, where λ is the wavelength of the radiation. It is then necessary to have recourse to the techniques of physical optics(ref.1,p51) to determine the radar cross section. In this method an approximation to the field, and hence the current, on the surface of the scatterer obtained by geometrical optics techniques is used in the Stratton-Chu integral for the radiated field(ref.1,p54). Since the field point in this integral lies on the surface of the scatterer, the above objection to geometrical optics no longer applies provided the relevant dimensions of the scatterer are large in terms of the wavelength, and physical optics yields an accurate estimate of the main beam and adjacent sidelobes of the scattered field. Thus, if $\underset{\sim}{H}$ is the total magnetic field on the surface of the scatterer, the scattered magnetic field $\underset{\sim}{H}_s$ is(ref.2,p238) (note the difference in sign of the exponential term resulting from the different time convention in this reference)

$$\underset{\sim}{H}_s = \frac{1}{4\pi}\int_{\xi'} (\hat{n} \times \underset{\sim}{H}) \times \nabla \frac{\exp(-ikR)}{R}\, d\xi' \qquad (1)$$

where the various parameters are as defined in figure 2 and $\hat{n}$ is the unit normal to ξ'. For the field point at a large distance from the body

$$\underset{\sim}{H}_s = -\frac{ik}{4\pi}\frac{\exp(-iks)}{s}\int_{\xi'} (\hat{n} \times \underset{\sim}{H}) \times \hat{s}\ \exp(ik\hat{s}\cdot\underset{\sim}{s}')\, d\xi' \qquad (2)$$

where $\hat{s}$ is a unit vector in the direction of $\underset{\sim}{s}$, and s is the magnitude of $\underset{\sim}{s}$. $\underset{\sim}{s}'$ is a vector describing the position of the element of area $d\xi'$ of the scatterer. If the incident magnetic field is equal to H_0, the radar cross section then becomes

$$\sigma = \frac{\pi}{H_0^2\lambda^2}\left|\int_{\xi'}(\hat{n} \times \underset{\sim}{H}) \times \hat{s} \;\; \exp(ik\hat{s}\cdot\underset{\sim}{s}')\; d\xi'\right|^2 \qquad (3)$$

The essential problem in this work is to evaluate equation (2) on a conical surface illuminated by a system of rays reflected by the adjacent conical surface. Consider the situation shown in figures 1 and 3 in which the incident rays propagate parallel to the xz plane, making an angle β to the x axis. It will be assumed that the incident magnetic field is polarised in the $\hat{y}$ direction and has a magnitude H_0. The lower cone has an apex angle α and the upper one an apex angle $\pi/2 - \alpha$.

From figure 3 it is immediately apparent that the total ray path length from the radar back to the radar for rays in the xz plane is independent of the position at which the ray is incident on the upper cone, but for rays in adjacent planes there is an increase in the total path length. Mathematically, the path length from the radar to point B and thence to A is contained within the phase term of $\underset{\sim}{H}$ in equation (2) and the propagation back to the radar is contained within the term $\hat{s}\cdot\underset{\sim}{s}' - s$.

If r is the distance from an arbitrary point on the lower cone to the junction between the cones, illustrated for point A in figure 3, then an element of area on the lower cone is

$$d\xi' = (a + r \sin\alpha)\; dr\; d\phi \qquad (4)$$

where ϕ measures the angular displacement around the cone from the point A in the xz plane. Equation (2) can thus be reduced to the form

$$H_s = H_0\iint g(r,\phi)\; \exp(ikh(r,\phi))\; dr\; d\phi \qquad (5)$$

in which

$$\left.\frac{\partial h(r,\phi)}{\partial\phi}\right|_{\phi=0} = 0 \qquad (6)$$

The integration with respect to ϕ can therefore be carried out using the method of stationary phase (ref.5,p274), yielding the result

$$H_s \sim H_0\int\left[\frac{2\pi}{k|h''(r,0)|}\right]^{\frac{1}{2}} g(r,0)\; \exp(ikh(r,0) - i\frac{\pi}{4})\; dr \qquad (7)$$

where " denotes the second derivative of $h(r,\phi)$ with respect to ϕ. The sign of the term $i\pi/4$ is negative since $h''(r,o)$ is negative in this problem. It is interesting to note that, in the stationary phase approximation of equation (2), the amplitude of the magnetic field is only required in the plane $\phi = 0$, thus to some extent simplifying the calculations that would otherwise be required in evaluating equation (2). However it is still necessary to know the behaviour of the phase as ϕ varies about 0 in order that its second derivative may be calculated. The evaluation of these parameters now follows.

2.2 The tracing of the rays

Consider an arbitrary point A on the lower cone at which an incident ray reflected at point B on the upper cone is specularly reflected back towards the source radar (figure 3). Since the rays from the source radar are incident at an angle β to the x axis and are parallel to the xz plane, the points A and B must of necessity lie in the xz plane, ie the plane $\phi = 0$. To determine the radar cross section of the biconical reflector, the analysis of the previous section indicates that it is only necessary to know the amplitude of the magnetic field at A and the behaviour of the phase of the field on the surface of the cone in the vicinity of A. On letting

$$AC = r \tag{8}$$

it follows that

$$BC = \frac{r}{\tan(\alpha-\beta)} \tag{9}$$

and

$$l = \frac{r}{\sin(\alpha-\beta)} \tag{10}$$

The point of incidence on the upper cone, and the ray path length are thus specified.

The next step in the procedure is to determine the effect of the curvature of the upper cone at B on the wave reflected from B. From inspection the intersection of the cone with the plane $\phi = 0$ is a straight line, which has zero curvature. It follows immediately that this plane is one of the principle planes of the surface at B, and the other principal plane is orthogonal to it and contains the normal to the cone at B. The curvature of the intersection of this plane and the cone is the other principal curvature at B, and follows quite simply from a general theorem on the normal curvature vector. Since the intersection of the plane $z = z_B$ is a circle of radius $a + z_B \cot \alpha$, the normal curvature at B, and hence the other principle curvature, is

$$K_B = \frac{\sin \alpha}{(a + z_B \cot \alpha)}$$

$$= \frac{\sin \alpha \tan(\alpha-\beta)}{a \tan(\alpha-\beta) + r \cos \alpha} \tag{11}$$

Following the techniques developed in Appendix II for determining the effect of the curvature of the surface on the reflected wavefront it is now necessary to construct coordinate systems u_i, v_i, w_i, $i = 1, 2, 3$ at B to describe the incident wavefront, the surface, and the reflected wavefront. The vectors $\hat{u}_2$, $\hat{v}_2$, and $\hat{w}_2$ are chosen to be coincident, with $\hat{u}_1$ directed to make u_i a left hand coordinate system. Since the incident wave is a plane wave its curvature matrix is

$$Q_B^i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{12}$$

Furthermore, following the notation of Appendix II

$$P^i = P^r = \begin{bmatrix} \sin(\alpha-\beta) & 0 \\ 0 & 1 \end{bmatrix} \tag{13}$$

$$p_{33}^i = - \sin(\alpha-\beta) \tag{14}$$

and

$$p_{33}^r = \sin(\alpha-\beta) \tag{15}$$

It then follows from equation (II.16) that the curvature matrix of the reflected wave at B is

$$Q_B^r = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{2 \sin\alpha \sin^2(\alpha-\beta)}{a \sin(\alpha-\beta) + r \cos\alpha \cos(\alpha-\beta)} \end{bmatrix} \tag{16}$$

Equation (I.6) of Appendix I can then be used to determine the curvature matrix of the ray incident on A. Thus

$$Q_A^i = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{2 \sin\alpha \sin^2(\alpha-\beta)}{a \sin(\alpha-\beta) + r \cos\beta + r \sin\alpha \sin(\alpha-\beta)} \end{bmatrix} \tag{17}$$

The amplitude of the incident magnetic field at A due to the ray reflected from B is therefore (from equation (I.8))

$$|H_i| = H_0 \left(\frac{a \sin(\alpha-\beta) + r \cos\alpha \cos(\alpha-\beta)}{a \sin(\alpha-\beta) + r \cos\beta + r \sin\alpha \sin(\alpha-\beta)} \right)^{\frac{1}{2}} \tag{18}$$

where H_0 is the amplitude of the plane wave incident upon the biconical reflector at B. (Note that the boundary conditions on the surface at B require that the amplitude of the reflected wave at B also be equal to H_0).

The final step is to determine the behaviour of the phase of the magnetic field in the vicinity of A, in particular on the intersection of the lower conical surface and the plane $z = z_A$. In this work it is convenient to refer all the phases to the origin of the xyz coordinate system. The path length S(B) of the incident wave at B, from which the phase follows by multiplication by k, is

$$S(B) = -a \cos \beta - r \cot(\alpha-\beta) \cos(\alpha-\beta) \tag{19}$$

and the path length at A is

$$\begin{aligned} S(A) &= S(B) + l \\ &= - a \cos \beta + r \sin(\alpha-\beta) \end{aligned} \tag{20}$$

For an arbitrary point in the vicinity of A, described by the vector $\underset{\sim}{U} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$, the path length is

$$S^i(\underset{\sim}{U}) = S(A) + u_3 + \frac{1}{2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \cdot Q_A^i \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{21}$$

where u_i is a coordinate system constructed at A such that $\hat{u}_3$ is a unit vector in the direction of propagation of the ray incident on A and $\hat{u}_1$, $\hat{u}_2$ coincide with the principal directions of the wavefront incident at A; these are parallel to $\hat{w}_1$, $\hat{w}_2$ erected at B, and are indicated in figure 4.

Since the intersection of the plane $z = z_A$ and the lower conical surface is a circle, any arbitrary point on the circle can be referred to by the angular coordinate ϕ of the spherical polar coordinate system, with $\phi = 0$ on the circle corresponding to the point A. Then erecting the coordinate system t_1, t_2, at A, as indicated in figure 4, in the vicinity of $\phi = 0$, the coordinates of a point on the circle are, to second order,

$$t_1 = -\frac{1}{2}(a + r \sin \alpha)\, \phi^2 \tag{22}$$

$$t_2 = - (a + r \sin \alpha)\, \phi \tag{23}$$

Thus in equation (21) the parameters are

$$u_3 = -t_1 \cos(2\alpha - \beta) \tag{24}$$

$$u_1 = t_1 \sin(2\alpha - \beta) \tag{25}$$

$$u_2 = t_2 \tag{26}$$

The final factor to be accounted for in the exponential term in equation (2) is the path length from the point on the lower cone back to the illuminating radar. Referred to the origin, this is

$$S^r(\underset{\sim}{U}) = - a \cos \beta - r \sin(\alpha-\beta) + \frac{1}{2} (a + r \sin \alpha) \cos \beta \; \phi^2 \tag{27}$$

The parameter $h(r,\phi)$ appearing in equation (5) is the negative of the sum of equations (27) and (21), ie

$$h(r,\phi) = 2 a \cos \beta - \frac{(a + r \sin \alpha)(a \sin(\alpha-\beta) + r \cos \beta)\cos \beta}{a \sin(\alpha-\beta) + r \cos \beta + r \sin \alpha \sin(\alpha-\beta)} \phi^2 \tag{28}$$

As noted in the introduction, the phase of ray returning to the radar is independent of r in the plane $\phi = 0$.

2.3 Evaluation of the radiation integral

It now only remains to substitute the various terms into equation (2) and carry out the integration to complete the problem. An element of area $d\xi'$ on the cone may be expressed as

$$d\xi' = (a + r \sin \alpha) \, d\phi \, dr \tag{29}$$

The limits of integration of the ϕ variable are strictly $-\pi$ to $+\pi$, but since the method of stationary phase is to be used, only the region near $\phi = 0$ is important. The integration with respect to r is carried out over that region of the lower cone that is illuminated by the field reflected by the upper cone, or the total length of the lower cone, whichever is the smaller. Mathematically, the limits of r are 0 and R, the minimum of R_1 and $R_2 \tan(\alpha-\beta)$ (figure 1).

So far in the development of the theory, little has been said about the polarisation of the field. In the opening discussion it was arbitrarily assumed for convenience that the incident magnetic field is polarised parallel to the Y axis, although precisely the same result for the radar cross section would ultimately be obtained if the electric field were polarised in this direction. The factor $(\underset{\sim}{n} \times \underset{\sim}{H}) \times \underset{\sim}{r}$ in equation (2) at the arbitrary point A on the surface of the cone therefore has the magnitude

$$2 \hat{y} \cos(\alpha-\beta) \, |H_i| \tag{30}$$

where $|H_i|$ is given by equation (18). The additional factor 2, above, accounts for the fact that in equation (2), $\underset{\sim}{H}$ is the total magnetic field on the surface of the cone, which, since it is the sum of the rays incident and reflected from the surface, is twice the incident field on the surface.

Thus on substituting the expressions for the amplitude and phase of the magnetic field into equation (2) and performing the integration with respect to ϕ, as detailed in equation (7), the following result is obtained

$$\underset{\sim}{H}_s = -\frac{i}{2}\left(\frac{k}{\pi}\right)^{\frac{1}{2}} H_0\, \hat{y}\, \frac{\exp(-ikS)}{S} \exp(i2ka \cos\beta - i\frac{\pi}{4})$$

$$\frac{\cos(\alpha-\beta)}{(\cos\beta)^{\frac{1}{2}}} \int_0^R \frac{\left[a \sin(\alpha-\beta) + r \cos\alpha \cos(\alpha-\beta)\right]^{\frac{1}{2}} \left[a + r \sin\alpha\right]^{\frac{1}{2}}}{\left[a \sin(\alpha-\beta) + r \cos\beta\right]^{\frac{1}{2}}}\, dr \quad (31)$$

The alternative path whereby a ray can return to the radar, viz, reflection of the incident ray by the lower cone onto the upper cone, thence back to the radar yields precisely the same result and hence the total magnetic field reflected back towards the radar by the biconical corner reflector is twice that of equation (31). That the contribution of the two ray paths are equal may easily be proven by replacing α by $\pi/2 - \alpha$, β by $-\beta$ and the upper limit of integration R by R $\cot(\alpha-\beta)$ in equation (31).

From the formal definition of radar cross section

$$\sigma = \lim_{r\to\infty} 4\pi r^2 \left|\frac{H_s}{H_0}\right|^2 \quad (32)$$

it therefore follows that the radar cross section of the biconical corner reflector is

$$\sigma = 4k\, \frac{\cos^2(\alpha-\beta)}{\cos\beta}$$

$$\cdot\left[\int_0^R \frac{\left[a \sin(\alpha-\beta) + r \cos\alpha \cos(\alpha-\beta)\right]^{\frac{1}{2}} \left[a + r \sin\alpha\right]^{\frac{1}{2}}}{\left[a \sin(\alpha-\beta) + r \cos\beta\right]^{\frac{1}{2}}}\, dr\right]^2 \quad (33)$$

In general this integral does not yield to algebraic manipulation, but it is extremely well behaved, and hence simple to integrate numerically on a digital computer. However such a technique, while of use for specific examples, does not throw any light on the physical significance of the various parameters affecting the radar cross section. Fortunately there are a few specific examples, illustrated in figure 7, in which the integral can be evaluated algebraically to assist in this regard.

Case (a) ; a = 0.

In this case the cones touch at their apices. Whilst it is acknowledged that the physical optics technique is not valid for radii of curvature of the order of, or less than one wavelength ie near the apices of the cones, the effect of this on the total radar cross section of a large reflector is not expected to be significant. Thus

$$\sigma = \frac{8k}{9} R^3 \frac{\cos^3(\alpha-\beta)\sin(2\alpha)}{\cos^2\beta} \quad (34)$$

Case (b) ; $\alpha = \frac{\pi}{4}$, $\beta = 0$

This case corresponds to normal incidence on a symmetrical biconical corner reflector. Equation (33) reduces to

$$\sigma = k\left[\int_0^R \frac{(\sqrt{2}\ a + r)}{(a + \sqrt{2}\ r)^{\frac{1}{2}}}\, dr\right]^2$$

$$= \frac{k}{9}\left[(a + \sqrt{2}\ R)^{\frac{1}{2}}(4a + \sqrt{2}\ R) - 4a^{\frac{3}{2}}\right]^2 \qquad (35)$$

If instead, this expression for the radar cross section is written in terms of the maximum radius of the cone, b, that is illuminated, rather than the face length, R, ie

$$b = a + \frac{R}{\sqrt{2}} \qquad (36)$$

this result becomes

$$\sigma = \frac{8\pi}{9\lambda}\left[(2b - a)^{\frac{1}{2}}(b + a) - 2a^{\frac{3}{2}}\right]^2 \qquad (37)$$

This result should be compared with that of reference 2, p262 which is incorrect due to mathematical errors in the evaluation of an integral, and reference 1, p596 which further introduces a typographical error to the result in reference 2.

Case (c) ; $\alpha = \frac{\pi}{2}$

The third special case of interest that can be treated by the general result is the degenerate case $\alpha = \pi/2$, in which the lower cone opens out to become a disc of radius R + a and the upper cone is a circular cylinder of radius a. In this case

$$\sigma = 4kaR^2 \tan^2\beta \cos\beta \qquad (38)$$

Then if this expression is written in terms of the length L of the cylinder that is illuminated by the ray reflected by the disc, ie

$$L = R \tan\beta \qquad (39)$$

this result becomes the familiar expression(ref.1, p312)

$$\sigma = 4kaL^2 \cos\beta \qquad (40)$$

for the bistatic radar cross section of a circular cylinder of overall length 2L.

2.4 Comparison with the truncated cone

It is interesting to compare the above formulae for the radar cross section of the biconical corner reflector with that for the truncated cone illuminated at normal incidence to the cone surface(ref.2,p99)

$$\sigma = \frac{8\pi}{9\lambda}\left[b^{\frac{3}{2}} - a^{\frac{3}{2}}\right]^2 \frac{\sin\alpha}{\cos^4\alpha} \tag{41}$$

The parameters are defined in figure 8. It is precisely this result which would be obtained if the biconical corner reflector were illuminated normal to one of its faces. The transition from equation (41) to equation (37), say, as incidence is varied from the normal depends upon the relative phasing of the two signals, a factor which is not included in either of the expressions. As the biconical corner reflector is not intended to be used with incidence near the normal to either of its faces, the added complication in including the direct backscattered signal from each cone in the analysis was not considered worthwhile.

3. DISCUSSION

A general formula suitable for digital computer calculation of the radar cross section of an arbitrary biconical corner reflector illuminated at an arbitrary angle of incidence has been developed from the principles of geometrical and physical optics. In certain special cases of cone apex angles and angles of incidence which are significant in the practical application of the reflector the integral has been evaluated analytically, yielding insight on the effect on the radar cross section of changes in the dimensions of the reflector and the angle of incidence. It is considered that the results are valid provided that all dimensions of the reflector are at least several times the wavelength of the incident radiation. The results are not valid if the angle of incidence approaches the normal to either of the conical surfaces which comprise the biconical corner reflector.

NOTATION

The following table lists the main symbols used throughout this report. Other symbols appearing in the text have meanings requiring elaborate explanation; they are fully detailed where they first appear in the text.

a	radius of circle at the junction of the upper and lower conical surfaces.
b	radius of lower conical surface at edge of illumination
e_1, e_2, e_3	coordinate system describing an arbitrary ray
i	$\sqrt{-1}$
k	wavenumber of incident radiation ($= 2\pi/\lambda$)
l	distance from point A to point B on conical surface
r	distance of point on lower conical surface to junction of cross
s	distance from origin of x, y, z coordinate system to radar.
t_1, t_2	coordinate system describing circle on lower conical surface
u_1, u_2, u_3	coordinate system at point of reflection describing an incident ray
v_1, v_2, v_3	coordinate system at point of reflection describing reflecting surface
w_1, w_2, w_3	coordinate system at point of reflection describing a reflected ray
x, y, z	rectangular cartesian coordinate system
A	point of reflection on lower conical surface
B	point of reflection on upper conical surface
C	curvature matrix of surface
H	magnetic field strength
H_0	magnetic field strength of rays illuminating the biconical reflector
Q	curvature matrix of ray
R	minimum of, length of lower conical surface illuminated by reflection, and R_1
R_1	length of lower conical surface
R_2	length of upper conical surface
S	ray path length
α	apex angle of lower conical surface

β	angle of incidence of incident radiation to x axis
λ	wavelength of incident radiation
σ	radar cross section
ϕ	angular coordinate describing point on lower conical surface
ω	angular frequency of incident radiation

In the text the following notation is used, for example,

s	distance from origin of x, y, z coordinate system to radar
$\underset{\sim}{s}$	vector from origin to radar
$\hat{s}$	unit vector in direction of $\underset{\sim}{s}$

REFERENCES

No	Author	Title
1	Ruck, G.T. et alia	"Radar Cross Section Handbook". Volumes I and II, Plenum Press, New York, 1970
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3	Robertson, S.D.	"Targets for Microwave Radar Navigation". Bell Systems Technical Journal Vol 26 pp852-869 (1947)
4	Deschamps, G.A.	"Ray Techniques in Electromagnetics". Proceedings of the IEEE, Vol 60, no 9, pp1022-1035, September 1972
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APPENDIX I

PROPERTIES OF A GEOMETRICAL OPTICS FIELD

The basic concept of geometrical optics is that the field from an arbitrary source can be expressed as an amplitude and a phase term, thus

$$U = A(S)\exp(-ikS) \qquad (I.1)$$

in which the path length S can be determined independently of the amplitude term, and leads to the specification of a system of rays along which the energy propagates. The surfaces on which S is constant are called wavefronts and the associated rays are everywhere normal to the wavefronts. The amplitude of the field along a ray follows from simple energy considerations, independently of the behaviour of the amplitude along any other rays describing the total field.

In free space the path length S can be determined from

$$(\nabla S)^2 = 1 \qquad (I.2)$$

and hence the ray paths are straight lines. In the neighbourhood of an axial ray Oe_3 (figure 5) the wavefront through O can be represented by a second degree equation

$$e_3 = -\frac{1}{2}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \cdot Q \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \qquad (I.3)$$

where $\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$ is the transverse position vector in the orthonormal frame of reference e_1, e_2, e_3 and Q is a 2 x 2 symmetric matrix called the curvature matrix of the surface. Because the matrix Q is symmetric it has two orthogonal eigenvectors $\hat{E}_1$ and $\hat{E}_2$ such that

$$Q\,\hat{E}_i = \frac{1}{R_i}\,\hat{E}_i \qquad i=1,2 \qquad (I.4)$$

The R_i are called the principal radii of curvature and the planes $\hat{e}_3\hat{E}_i$ are the principal planes. In these planes the radii of curvature of the wavefront assume the maximum and minimum values possible. In one plane the rays appear to converge at a point f_1 and in the other at a point f_2 (figure 5). If r is an arbitrary point with coordinates e_1, e_2, e_3, adjacent to the axial ray then

$$S(r) = S(0) + e_3 + \frac{1}{2}\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \cdot Q(e_3) \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \qquad (I.5)$$

Note that the variation of Q with e_1 and e_2 is ignored. It will always be at least one order higher than the curvature and therefore is not significant in these calculations. Under appropriate circumstances the variation with e_3 can also be ignored. For e_3 large, however, the variation of Q can be determined from

$$Q^{-1}(e_3) = Q^{-1}(0) + e_3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (I.6)$$

In the geometrical optics field the energy propagates in the direction of the rays. The variation in amplitude along a ray can therefore be determined by tracing the variation in cross section of a tube of paraxial rays adjacent to the ray of interest. If the cross section of the tube as a function of e_3 is $\Sigma(e_3)$ it can be shown that

$$\frac{\Sigma(e_3)}{\Sigma(0)} = \frac{(R_1 + e_3)\ (R_2 + e_3)}{R_1\ R_2} \quad (I.7)$$

and hence

$$U(e_3) = U(0) \left[\frac{R_1\ R_2}{(R_1 + e_3)\ (R_2 + e_3)}\right]^{\frac{1}{2}} \exp(-ikS(e_3)) \quad (I.8)$$

It is possible to show that the amplitude law can be expressed in the more general form

$$U(e_3) = U(0) \left[\frac{\det\ Q(e_3)}{\det\ Q(0)}\right]^{\frac{1}{2}} \exp(-ikS(e_3)) \quad (I.9)$$

This completes the discussion of the properties of the geometrical optics field that are required in this work. They are discussed at greater length in reference 4.

APPENDIX II

REFLECTION OF A PENCIL OF RAYS BY A CURVED SURFACE

Consider now an arbitrary axial ray undergoing reflection at a point O on a curved surface (figure 6). The behaviour of the reflected ray follows directly from the condition that the phase of the incident and reflected waves must be equal everywhere on the surface in every neighbourhood of the point of reflection O. Now construct orthonormal coordinate systems $\hat{u}_i$, $\hat{w}_i$, i=1,2,3 , to describe the incident and reflected rays with $\hat{u}_3$ and $\hat{w}_3$ directed along the direction of propagation of the incident and reflected rays, and the origins coincident at O. Also at O construct the system $\hat{v}_i$, i=1,2,3 with $\hat{v}_3$ normal to the surface to provide a frame of reference for the surface. Then an arbitrary point on the surface in the neighbourhood of O may be described by a vector $\underset{\sim}{V}$ given by

$$\underset{\sim}{V} = \underset{\sim}{v} - \frac{1}{2}(\underset{\sim}{v}\cdot C\underset{\sim}{v})\,\hat{v}_3 \tag{II.1}$$

where C is the curvature matrix describing the surface at O and $\underset{\sim}{v}$ is an arbitrary vector in the $\hat{v}_1\hat{v}_2$ plane. In the incident pencil of rays the path length at an arbitrary point described by the vector $\underset{\sim}{U} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ is

$$S^i(\underset{\sim}{U}) = S(O) + u_3 + \frac{1}{2}\,\underset{\sim}{u}\cdot Q^i\underset{\sim}{u} \tag{II.2}$$

where $\underset{\sim}{u}$ is the projection of $\underset{\sim}{U}$ on the $\hat{u}_1\hat{u}_2$ plane. It is now necessary to describe $\underset{\sim}{U}$ in terms of $\underset{\sim}{v}$. To this end, introduce

$$p^i_{mn} = \hat{u}_m\cdot\hat{v}_n \qquad m,n=1,2,3 \tag{II.3}$$

which are the projections of the incident ray base vectors on the base vectors of the surface, and

$$P^i = \begin{bmatrix} p^i_{12} & p^i_{12} \\ p^i_{21} & p^i_{22} \end{bmatrix} \tag{II.4}$$

It then follows that the path length of the incident wave at an arbitrary point $\underset{\sim}{V}$ on the surface is

$$\begin{aligned} S^i(V) &= S(O) + \underset{\sim}{v}\cdot\hat{u}_3 - \frac{1}{2}\,\underset{\sim}{v}\cdot C\underset{\sim}{v}\;p^i_{33} \\ &\quad + \frac{1}{2}(P^i\underset{\sim}{v})\cdot Q^i(P^i\underset{\sim}{v}) + \text{higher order terms} \\ &= S(O) + \underset{\sim}{v}\cdot\hat{u}_3 + \frac{1}{2}\,\underset{\sim}{v}\cdot\Gamma^i\underset{\sim}{v} \end{aligned} \tag{II.5}$$

where

$$\Gamma^i = (P^i)^T\,Q^i\,P^i - C\,p^i_{33} \tag{II.6}$$

The superscript T denotes the transpose of the matrix.

Now introduce the projections of the reflected wave base vectors onto those of the surface, thus

$$p^r_{mn} = \hat{w}_m \cdot \hat{v}_n \qquad m,n=1,2,3 \tag{II.7}$$

and

$$P^r = \begin{bmatrix} p^r_{12} & p^r_{12} \\ p^r_{21} & p^r_{22} \end{bmatrix} \tag{II.8}$$

If Q^r is the curvature matrix of the reflected wave, then following the previous analysis, the path length of the reflected ray at an arbitrary point $\underset{\sim}{V}$ of the surface is

$$\begin{aligned} S^i(\underset{\sim}{V}) &= S(0) + \underset{\sim}{v}\cdot\hat{w}_3 - \frac{1}{2}\,\underset{\sim}{v}\cdot C\underset{\sim}{v}\; p^r_{33} \\ &+ \frac{1}{2}\,(P^r\underset{\sim}{v})\cdot Q^r(P^r\underset{\sim}{v}) + \text{higher order terms} \end{aligned} \tag{II.9}$$

The condition of phase matching on the surface in some neighbourhood of 0 is satisfied if the first and second order terms are equated. Thus

$$\underset{\sim}{v} \cdot \hat{u}_3 = \underset{\sim}{v} \cdot \hat{w}_3 \tag{II.10}$$

or alternatively

$$\hat{v}_i \cdot \hat{u}_3 = \hat{v}_i \cdot \hat{w}_3 \qquad i=1,2 \tag{II.11}$$

which asserts the equality of the angles of incidence and reflection. The matching of the second order terms requires that

$$(P^r\underset{\sim}{v})\cdot Q^r(P^r\underset{\sim}{v}) = (P^i\underset{\sim}{v})\cdot Q^i(P^i\underset{\sim}{v}) - 2\,\underset{\sim}{v}\cdot C\underset{\sim}{v}\; p^i_{33} \tag{II.12}$$

since

$$p^i_{33} = -\,p^r_{33} \tag{II.13}$$

It follows, since this holds for all $\underset{\sim}{v}$, that

$$(P^r)^T\, Q^r\, P^r = (P^i)^T\, Q^i\, P^i - 2\,C\,p^i_{33} \tag{II.14}$$

It is then straightforward to deduce the value of Q^r which describes the curvature of the reflected ray.

In the special case in which the axes $\hat{u}_2$, $\hat{v}_2$ and $\hat{w}_2$ coincide and $\hat{u}_1$ is directed such as to make $\hat{u}_i$, i=1,2,3 a left handed coordinate system,

$$P^i = P^r = \begin{bmatrix} \cos\theta & 0 \\ 0 & 1 \end{bmatrix} \tag{II.15}$$

and equation (II.14) reduces to the simple result.

$$Q^r = Q^i + 2\,(P^r)^{-1}\,C\,(P^r)^{-1}\cos\theta \tag{II.16}$$

where θ is defined in figure 6.

This appendix parallels the work in reference 4 for refraction by an arbitrary curved surface.

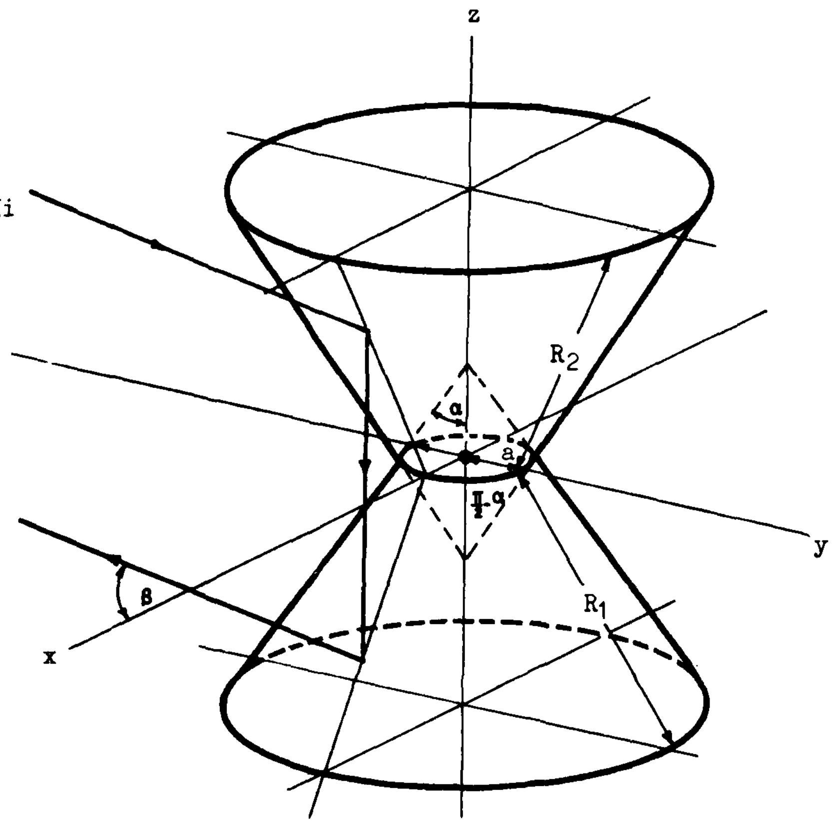


Figure 1. The biconical corner reflector

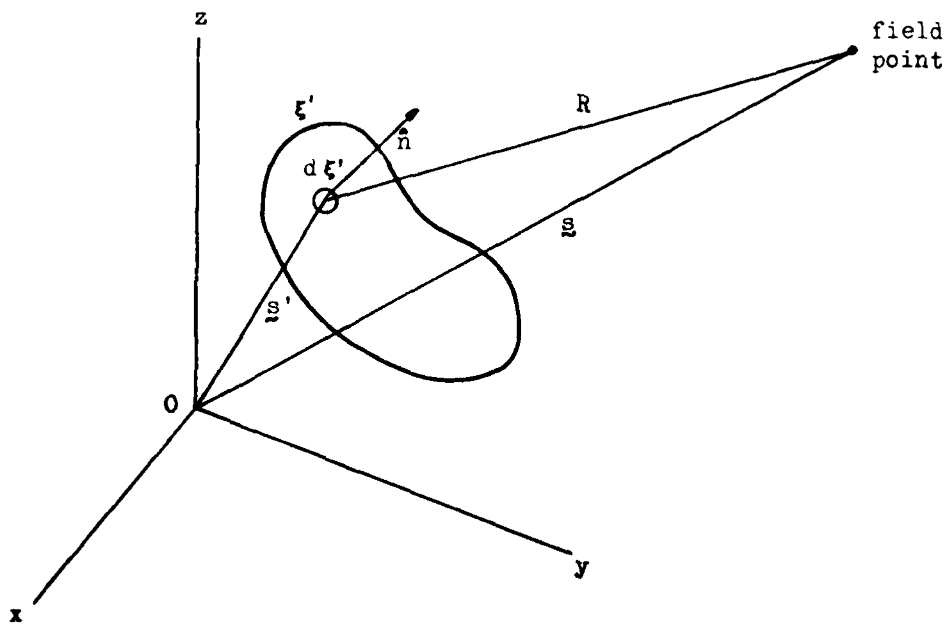


Figure 2. Coordinates of the radiation integral

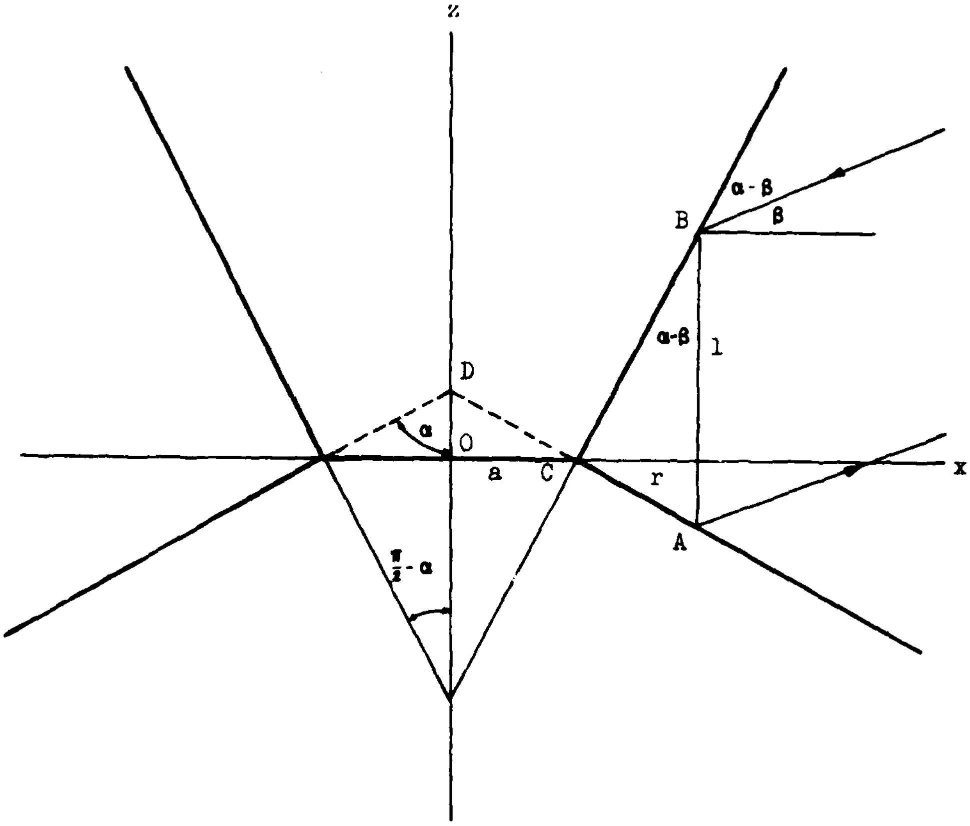


Figure 3. Coordinates of the ray path

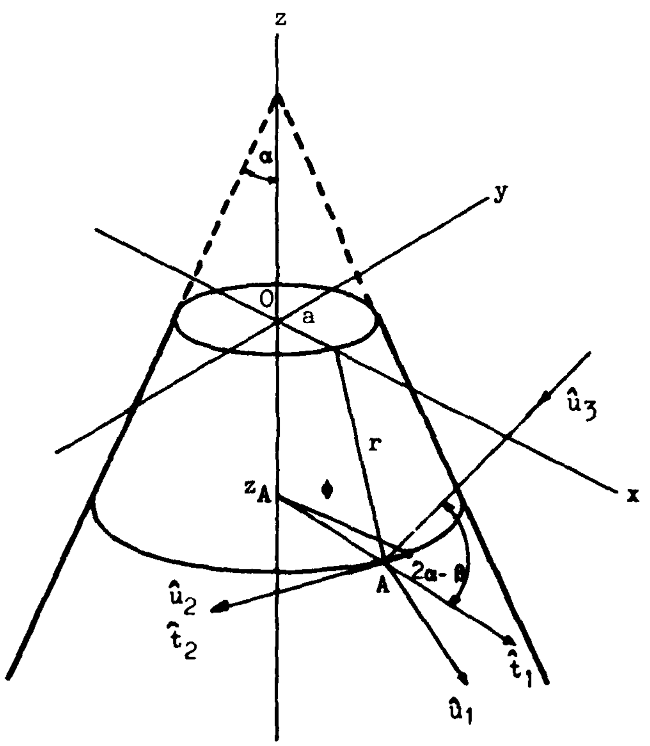


Figure 4. Coordinate systems for the lower conical surface

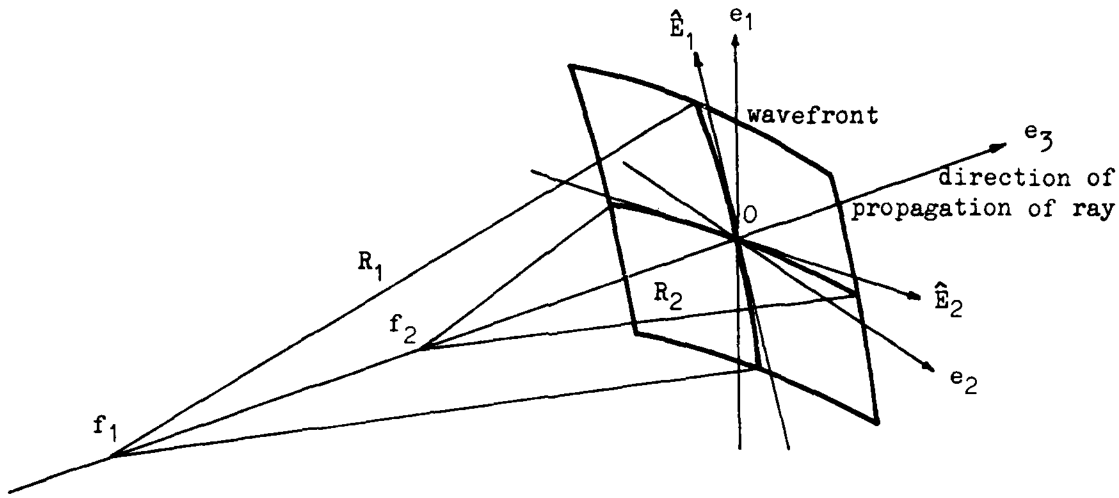


Figure 5. Wavefront associated with an axial ray

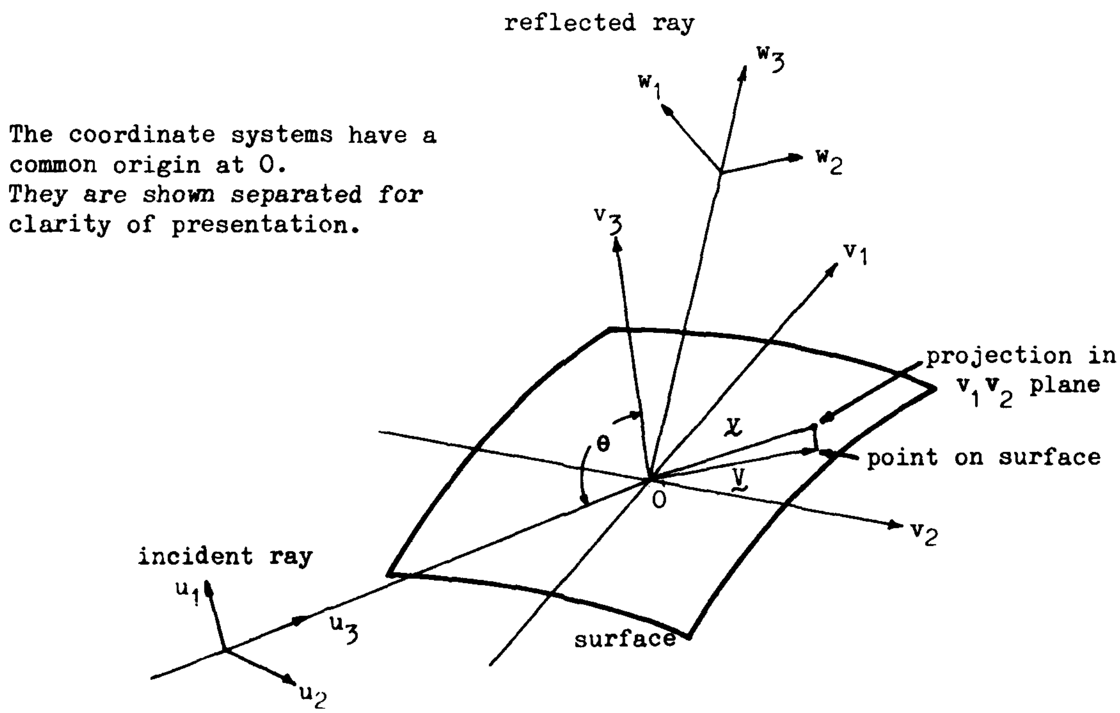
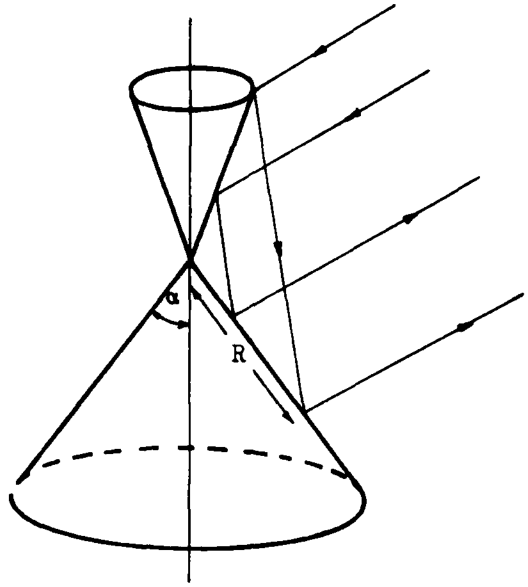
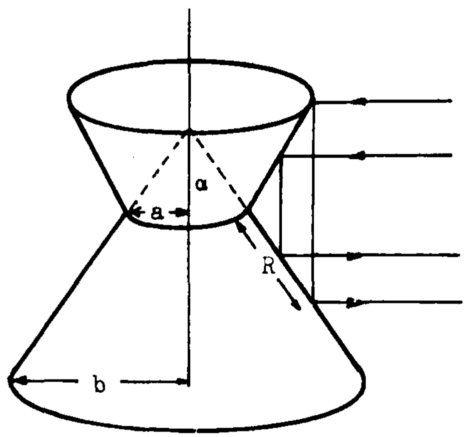


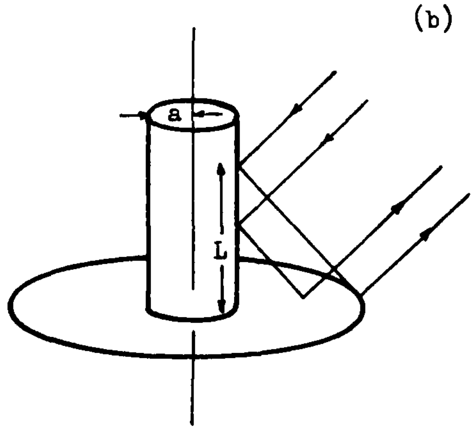
Figure 6. Reflection by a curved surface



(a) $a = o$



(b) $\alpha = \frac{\pi}{4}, \ \beta = o$



(c) $\alpha = \frac{\pi}{2}$

Figure 7. Specific forms of the biconical corner reflector

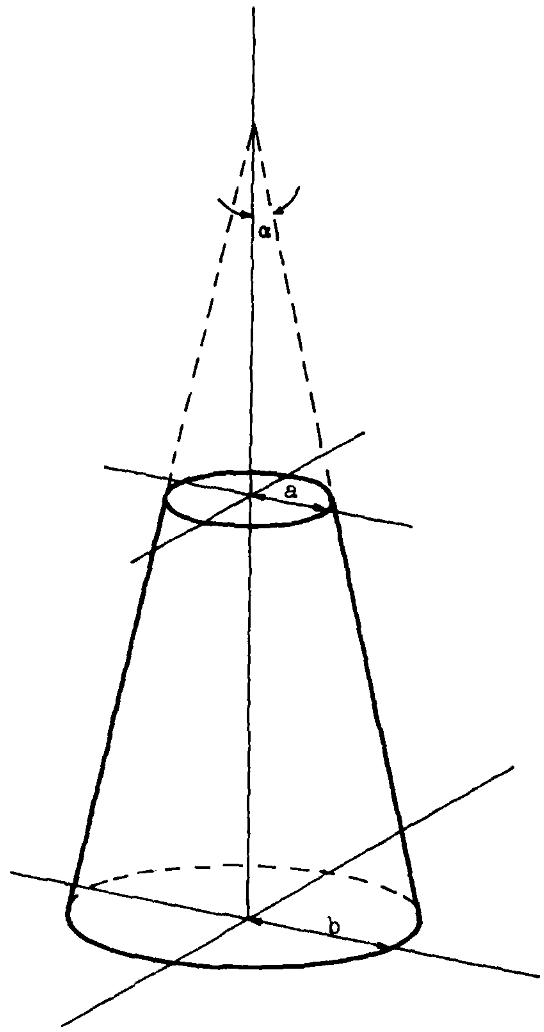


Figure 8. The truncated cone

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