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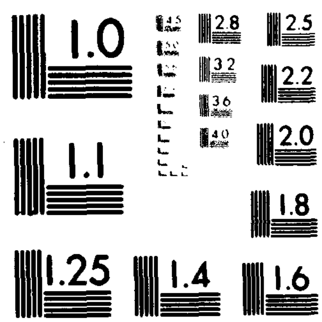

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OPTIMIZATION OF COMBAT DYNAMICS

FINAL REPORT

BY

JAMES G. TAYLOR

NOVEMBER 30, 1979

U. S. ARMY RESEARCH OFFICE

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18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Lanchester Theory of Combat; Combat Modelling; Attrition Modelling; Combat Dynamics; Battle-Outcome Prediction; Zero Points of Differential-Equation Solutions; Military Tactics; Deterministic Combat Attrition; Differential-Equation System Dynamics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report outlines research accomplishments for a two-year project studying the optimization of combat dynamics. Results were obtained for two basic topics: (1) obtaining insights into the dynamics of combat by mathematically analyzing relatively simple Lanchester-type models, and (2) investigating the structure of optimal time-sequential tactical decisions with such simple differential-equation models. However, research efforts were primarily concentrated on the first topic, and new research directions

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were established (e.g. "ground-breaking" work on "simple-approximate" battle-outcome-prediction conditions). A complete list of publications originating from the project is given.

1. Introduction.

The broad objective of the research project reported on by this final report was "to extend the state of the art for the quantitative determination of optimal time-sequential combat strategies," with emphasis placed on extending the state of the art for developing and analyzing analytical solutions to Lanchester-type equations of warfare in order to develop insights into the dynamics of combat. Most of the research accomplished under this particular ARO sponsorship concerned the development so-called of battle-outcome-prediction conditions for Lanchester-type equations (i.e. conditions that analytically relate the outcome of battle to the initial conditions of the differential-equation combat model), although a certain amount of effort was spent on summarizing past research on the structure optimal time-sequential fire-distribution policies. A motivation for investigating battle-outcome-prediction conditions for Lanchester-type equations was that previous research had shown that such conditions are crucial for developing optimal time-sequential combat strategies in dynamic optimization problems in which the system dynamics are provided by such Lanchester-type equations (see TAYLOR [1;2] for further details).

2. Topics Considered.

Research results were obtained for two basic topics:

(1) obtaining insights into the dynamics of combat by mathematically analyzing relatively simple Lanchester-type models,

and (2) investigating the structure of optimal time-sequential tactical decisions with such simple differential-equation models.

However, research efforts were primarily concentrated on the first topic, and new research directions were established (e.g. "ground-breaking" work on "simple-approximate" battle-outcome-prediction conditions). These research efforts have focused on mathematically analyzing the basic paradigms that have been the point of departure for developing combat models currently used by the U.S. Army.

3. Summary of Most Important Results.

The most important results of this research may be summarized as follows:

1. force-annihilation-prediction conditions (based on new mathematical results for the zero of a nonoscillatory [in the strict sense] solution to the second-order linear ordinary differential equation) developed for Lanchester-type equations of modern warfare;

2. occurrence of a zero point of a nonoscillatory (in the strict sense) solution to the second-order linear ordinary differential equation related to the equation's initial conditions;

3. algorithm developed for numerically determining (complete with *a priori* error bounds) the so-called parity-condition parameter for predicting force annihilation in homogeneous-force combat modelled by Lanchester-type equations of modern warfare;

4. "simple-approximate" conditions that are sufficient (but not necessary) to predict battle outcome developed for several Lanchester-type combat models;

5. methodology developed for determining conditions under which it is optimal for the victor to initially commit as many forces as possible to battle in Lanchester-type combat between two homogeneous forces by considering the instantaneous casualty-exchange ratio;

6. approximate solutions (and associated error bounds) developed for several Lanchester-type homogeneous-force combat models.

Further details may be found in Progress Reports No. 1-4 for this project (*see also* papers and interim technical reports listed in Appendixes A and B to this final report).

4. Final Remarks.

The above results allow the quantitative behavior of solutions to variable-coefficient Lanchester-type equations of modern warfare to be analyzed almost as easily as that for solutions to Lanchester's classic constant-coefficient differential-equation combat model. Such results will be useful for analytically investigating optimal time-sequential combat strategies in Lanchester-type differential-game/optimal-control problems. Finally, several new research directions have been established for analytically investigating the quantitative behavior of Lanchester-type combat models.

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[1] J. G. Taylor, "On the Isbell and Marlow Fire Programming Problem," Naval Res. Log. Quart. 19, 539-556 (1972).

[2] J. G. Taylor, "On the Treatment of Force-Level Constraints in Time-Sequential Combat Problems," Naval Res. Log. Quart. 22, 617-650 (1975).

APPENDIX A: List of All Publications

1. J. G. Taylor, "Approximate Solution (with Error Bounds) to a Nonlinear, Nonautonomous Second-Order Differential Equation," Journal of the Franklin Institute 306, 195-208 (1978).

2. J. G. Taylor and G. G. Brown, "Numerical Determination of the Parity-Condition Parameter for Lanchester-Type Equations of Modern Warfare," Computers and Operations Research 5, 227-242 (1978).

3. J. G. Taylor, "Overview of a Lanchester-Type Aggregated-Force Model of Conventional Large-Scale Ground Combat," pp. 551-562 in Proceedings of the Seventeenth U.S. Army Operations Research Symposium, Fort Lee, VA (1978).

4. J. G. Taylor, "Recent Developments in the Lanchester Theory of Combat," pp. 773-806 in Operational Research '78, Proceedings of the Eighth IFORS International Conference on Operational Research, K. B. Haley (Editor), North-Holland, Amsterdam (1979).

5. J. G. Taylor, "Attrition Modelling," pp. 139-189 in Operationsanalytische Spiele für die Verteidigung, R. K. Huber, K. Niemeyer, and H. W. Hofmann (Editors), Oldenbourg, München (1979).

6. J. G. Taylor, "Optimal Commitment of Forces in Some Lanchester-Type Combat Models," Opns. Res. 27, 96-114 (1979).

7. J. G. Taylor, "Prediction of Zero Points of Solutions to Lanchester-Type Differential Combat Equations for Modern Warfare," SIAM J. Appl. Math. 36, 438-456 (1979).

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9. J. G. Taylor, Force-on-Force Attrition Modelling, Military Applications Section of the Operations Research Society of America, Arlington, VA (1980).

10. J. G. Taylor, "Dependence of the Parity-Condition Parameter on the Combat-Intensity Parameter for Lanchester-Type Equations of Modern Warfare," OR Spektrum 1, 199-205 (1980).

11. J. G. Taylor, "Theoretical Analysis of Lanchester-Type Combat Between Two Homogeneous Forces with Supporting Fires," revised manuscript submitted to Naval Res. Log. Quart. 27, 109-121 (1980).

12. J. G. Taylor, "Using the Gronwall-Bellman Inequality to Develop Error Bounds for the Liouville-Green (or WKB) Approximation," submitted to J. Math. Anal. Appl.

13. J. G. Taylor, "A Lanchester-Type Aggregated-Force Model of Conventional Ground Combat," submitted to Naval Res. Log. Quart.

14. J. G. Taylor, "'Simple Approximate' Battle-Outcome-Prediction Conditions for Variable-Coefficient Lanchester-Type Equations of Modern Warfare," submitted to Naval Res. Log. Quart.

15. J. G. Taylor, "Battle-Outcome-Prediction Conditions for Variable-Coefficient Lanchester-Type Equations for Area Fire," submitted to Journal of the Franklin Institute.

16. J. G. Taylor and G. G. Brown, "Anniholation Prediction for Lanchester-Type Models of Modern Warfare," revised manuscript submitted to Opns. Res.

17. J. G. Taylor, "Sufficient Conditions for Battle-Outcome Prediction by Prüfer-Type Analysis of Lanchester-Type Differential Equations for Modern Warfare," submitted to SIAM J. Appl. Math.

APPENDIX B: List of Interim Technical Reports Published

1. J. G. Taylor and G. G. Brown, "A Table of Lanchester-Clifford-Schläfli Functions," NPS55-77-39, Naval Postgraduate School, Monterey, CA, October 1977 (AD A050 248).

2. J. G. Taylor and G. G. Brown, "A Short Table of Lanchester-Clifford-Schläfli Functions," NPS55-77-42, Naval Postgraduate School, Monterey, CA, October 1977 (AD A049 863).