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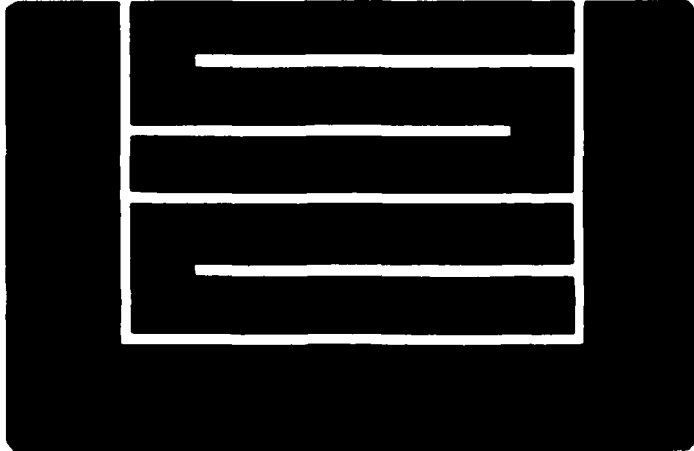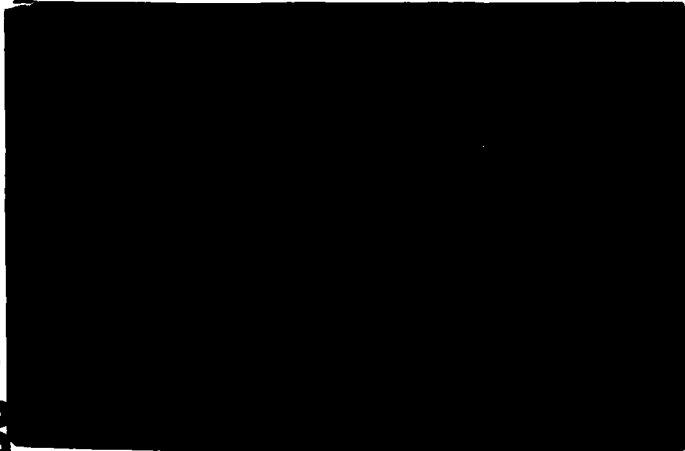
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APPROXIMATE PREDICTION INTERVALS FOR A FUTURE
OBSERVATION FROM THE INVERSE GAUSSIAN DISTRIBUTION*

by

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University of South Carolina
Statistics Technical Report No. 65
62F99-1

Interim

August 1980

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ABSTRACT

The problem of predicting, on the basis of an observed sample from an inverse Gaussian distribution, the mean of a future random sample (or a single future observation) from the same distribution is considered. Approximate prediction intervals are proposed, and their accuracy is investigated via extensive Monte Carlo simulations. The results are useful for predicting the next first passage time for a Brownian motion with positive drift or the failure time of an item having inverse Gaussian life distribution.

1. INTRODUCTION

The inverse Gaussian distribution arises as the first passage time distribution of a Brownian motion process with drift (Cox and Miller, 1965). Tweedie (1957a, 1957b) discussed the statistical properties of this distribution and noted the similarities between the sampling distribution theory of the maximum likelihood estimators of the inverse Gaussian parameters and those of the mean and variance of the normal distribution. Recently, Chhikara and Folks (1977) proposed the inverse Gaussian distribution as a lifetime model in reliability studies which has several advantages over the lognormal model when there is a high occurrence of early failures. Further distribution theory for the inverse Gaussian was given by Chhikara and Folks (1974, 1975), and its use in tests for positive drift in Brownian motion was discussed by Seshadri and Shuster (1974) and Chhikara and Folks (1976), among others.

The form of the probability density function of the two-parameter inverse Gaussian distribution generally used is

$$f(x;\mu,\lambda) = [\lambda/(2\pi x^3)]^{\frac{1}{2}} \exp[-\lambda(x-\mu)^2/(2x\mu^2)], \quad x>0, (\lambda,\mu>0) . \quad (1.1)$$

Other parametric forms are summarized by Johnson and Kotz(1970). In (1.1), μ is the mean and λ is a shape parameter. The variance is μ^3/λ, so that μ is not a location parameter in the usual sense.

For the inverse Gaussian lifetime model, Padgett (1979) obtained approximate confidence bounds for the reliability function, and Padgett and Wei (1979) added a threshold parameter or "guarantee time" and studied the estimation problem for the resulting three-parameter model.

In this paper a prediction interval is proposed for the mean of m independent future observations from (1.1), $Y_1,\ldots,Y_m$, on the basis of a past

random sample of size $n, X_1, \ldots, X_n$, from the same distribution. That is, it is desired to obtain functions of $X_1, \ldots, X_n$, $L(X_1, \ldots, X_n)$ and $U(X_1, \ldots, X_n)$, so that for a specified value γ,

$$P[L(X_1, \ldots, X_n) \leq \bar{Y}_m \leq U(X_1, \ldots, X_n)] = 1-\gamma, \qquad (1.2)$$

where $\bar{Y}_m = \sum\limits_{i=1}^{m} Y_i/m$. If $m=1$, (1.2) provides a prediction interval for a single future observation Y. Note that the correct interpretation of the probability statement (1.2) is that if repeated past samples are taken and used to predict the mean of the future samples by the intervals from $L(X_1, \ldots, X_n)$ to $U(X_1, \ldots, X_n)$, then $\bar{Y}_m$ will be contained in the intervals for a proportion $1-\gamma$ of such pairs of samples. The intervals defined by (1.2) will be useful in predicting the next first passage time of a Brownian motion with positive drift or the future failure time of an item having inverse Gaussian life distribution based on a past sample, for example.

Prediction intervals for the normal and exponential distributions have been studied extensively. For example, among others, Hewett (1968) and Lawless (1970, 1972) considered prediction intervals for the exponential distribution, and Chew (1968), Hahn (1969, 1970), Hall and Prairie (1973), and Mann and Fertig (1977) studied prediction intervals for normal distributions.

It is very difficult, if not impossible, to obtain exact prediction intervals of the form (1.2) for the inverse Gaussian distribution (1.1) when both parameters are unknown. In Section 2, approximate prediction intervals for $\bar{Y}_m$ are proposed based on estimates of μ. The performance of the intervals in terms of coverage probabilities is investigated via extensive Monte Carlo simulations, and the results are reported in Section 3. The proposed approximate prediction intervals seem to perform very well. An example is given in Section 4.

2. THE PROPOSED PREDICTION INTERVALS

Let $X_1, \ldots, X_n$ be a random sample of size n from the inverse Gaussian distribution (1.1) and let $Y_1, \ldots, Y_m$ be a future random sample of size m from the same distribution, independent of the first sample. The maximum likelihood estimators of μ and λ are given by $\hat{\mu} = \bar{X}_n = \sum_{i=1}^{n} X_i/n$ and $\hat{\lambda}^{-1} = \sum_{i=1}^{n} (1/X_i - 1/\bar{X}_n)/n$, respectively (Tweedie, 1957a, 1957b). Also, Tweedie (1957a) showed that $\bar{X}_n$ and $\hat{\lambda}$ are stochastically independent, $\bar{X}_n$ has inverse Gaussian distribution with parameters μ and $n\lambda$, and $n\lambda/\hat{\lambda}$ has chi-square distribution with n-1 degrees of freedom. Also, $\bar{Y}_m$ has inverse Gaussian distribution with parameters μ and $m\lambda$, and by a result of Shuster (1968), $m\lambda(\bar{Y}_m-\mu)^2/\mu^2\bar{Y}_m$ has chi-square distribution with one degree of freedom and is independent of $n\lambda/\hat{\lambda}$. Thus, the random variable $(n-1)m\hat{\lambda}(\bar{Y}_m-\mu)^2/(n\mu^2\bar{Y}_m)$ has F distribution with (1,n-1) degrees of freedom.

Let $F_\gamma(1,n-1)$ denote the value such that an F random variable with (1,n-1) degrees of freedom satisfies $P[F \leq F_\gamma(1,n-1)] = 1-\gamma$. Then from above

$$P\left[\frac{(\bar{Y}_m-\mu)^2}{\mu^2\bar{Y}_m} \leq \frac{nF_\gamma(1,n-1)}{m(n-1)\hat{\lambda}}\right] = 1-\gamma. \tag{2.1}$$

If the parameter μ is known, then (2.1) can be solved for an exact $(1-\gamma)$ prediction interval for $\bar{Y}_m$. However, if μ is unknown, then an exact prediction interval cannot be obtained from (2.1). An approximate $1-\gamma$ prediction interval may be obtained as follows. Since we have the random sample $X_1, \ldots, X_n$ at hand, suppose that μ^2 in the denominator in (2.1) is approximated by $\bar{X}_n^2$, so that

$$P\left[\frac{(\bar{Y}_m-\mu)^2}{\bar{Y}_m} \leq \frac{n\bar{X}_n^2 F_\gamma(1,n-1)}{m(n-1)\hat{\lambda}}\right] \approx 1-\gamma. \tag{2.2}$$

Also, since μ can be estimated by the future sample mean, the unknown value

of μ in (2.2) can be approximated by $\dfrac{n\bar{X}_n + m\bar{Y}_m}{n+m}$. Then

$$\frac{(\bar{Y}_m - \mu)^2}{\bar{Y}_m} \simeq \left(\frac{n}{n+m}\right)^2 \left(\bar{Y}_m + \frac{\bar{X}_n^2}{\bar{Y}_m} - 2\bar{X}\right).$$

Hence, (2.2) becomes

$$P[\bar{Y}_m + \frac{\bar{X}_n^2}{\bar{Y}_m} \le \frac{(n+m)^2 \bar{X}_n^2 \, F_\gamma(1, n-1)}{nm(n-1)\hat{\lambda}} + 2\bar{X}_n] \approx 1-\gamma. \qquad (2.3)$$

The inequality in (2.3) can be solved for $\bar{Y}_m$ by finding the roots of the

quadratic equation

$$\bar{Y}_m^2 - c(\underline{X})\bar{Y}_m + \bar{X}^2 = 0. \qquad (2.4)$$

where $c(\underline{X}) = \dfrac{(n+m)^2 \bar{X}_n^2 \, F_\gamma(1, n-1)}{nm(n-1)\hat{\lambda}} + 2\bar{X}_n$. It is easy to show that both roots

of (2.4) are always real and positive and that the quantity $\bar{Y}_m + \dfrac{\bar{X}_n^2}{\bar{Y}_m}$

satisfies the inequality in (2.3) when $\bar{Y}_m$ is between these two roots. Denote

the smaller root of (2.4) by $L_1(\underline{X})$ and the larger root by $U_1(\underline{X})$ where

$\underline{X} = (X_1, \ldots, X_n)$. Then the interval $(L_1(\underline{X}), U_1(\underline{X}))$ provides an approximate

$(1-\gamma)$ prediction interval for $\bar{Y}_m$. The closeness of the approximation will

be investigated in Section 3 by Monte Carlo simulations. For large n, since

$\bar{X}_n$ is very near μ with probability one by the strong law of large numbers,

the prediction probability will be close to the nominal value $(1-\gamma)$.

A second approximate prediction interval for $\bar{Y}_m$ may be obtained by using

the first few terms of the Taylor series expansion of $g(\mu) = (\bar{Y}_m - \mu)^2 / (\mu^2 \bar{Y}_m)$

about $\bar{X}_n$. In the Monte Carlo study reported in Section 3, the approximation

using only one or two terms in the series was not very good. However, the approximation using three terms performs fairly well when the variance μ^3/λ of the inverse Gaussian distribution is small and n is relatively large. The three terms are

$$\frac{(\bar{Y}_m - \mu)^2}{\mu^2 \bar{Y}_m} \approx \frac{(\bar{Y}_m - \bar{X}_n)^2}{\bar{X}_n^2 \bar{Y}_m} + 2(\frac{1}{\bar{X}_n^2} - \frac{\bar{Y}_m}{\bar{X}_n^3})(\mu - \bar{X}_n)$$

$$+ (\frac{3\bar{Y}_m}{\bar{X}_n^4} - \frac{2}{\bar{X}_n^3})(\mu - \bar{X}_n)^2$$

$$= \bar{X}_n^{-2} [(6 - 8\mu/\bar{X}_n + 3\mu^2/\bar{X}_n^2) \bar{Y}_m + \bar{X}_n^2 / \bar{Y}_m$$

$$- 6\bar{X}_n + 6\mu - 2\mu^2/\bar{X}_n]. \tag{2.5}$$

From (2.1) and (2.5),

$$P[(6 - 8\mu/\bar{X}_n + 3\mu^2/\bar{X}_n^2)\bar{Y}_m + \bar{X}_n^2/\bar{Y}_m$$

$$\leq \frac{nF_\gamma(1, n-1)\bar{X}_n^2}{m(n-1)\hat{\lambda}} + 6\bar{X}_n - 6\mu + 2\mu^2/\bar{X}_n]$$

$$\approx 1 - \gamma. \tag{2.6}$$

The probability (2.6) still depends upon μ, and as before, an approximation must be used. A lower confidence bound on μ was stated in Padgett (1979). Using $\bar{X}_n$ for μ on the left side of the inequality in (2.6) and the lower confidence bound $[1/\bar{X}_n + (F_\gamma(1, n-1)/((n-1)\bar{X}_n\hat{\lambda}))]^{-1}$ on the right-hand side gives a rough approximation

$$P[\bar{Y}_m + \frac{\bar{X}_n^2}{\bar{Y}_m} \leq c_2(\underline{X})] \approx 1 - \gamma, \tag{2.7}$$

where

$$c_2(\underline{X}) = \frac{nF_\gamma(1,n-1)\bar{X}_n^2}{m(n-1)\hat{\lambda}} + 2\bar{X}Q(\underline{X}),$$

with

$$Q(\underline{X}) = 3[\bar{X}_n F_\gamma(1,n-1)]^{\frac{1}{2}}/[((n-1)\hat{\lambda})^{\frac{1}{2}} + (\bar{X}_n F_\gamma(1,n-1))^{\frac{1}{2}}]$$

$$+ (n-1)\hat{\lambda}/[((n-1)\hat{\lambda})^{\frac{1}{2}} + (\bar{X}_n F_\gamma(1,n-1))^{\frac{1}{2}}]^2.$$

As before, (2.7) has a unique solution of the form

$P[L_2(\underline{X}) \le \bar{Y}_m \le U_2(\underline{X})] \approx 1-\gamma$, where $L_2(\underline{X})$ and $U_2(\underline{X})$ are the real positive roots

of the equation $\bar{Y}_m^2 - c_2(X)\bar{Y}_m + \bar{X}_n^2 = 0$.

Other approximations for prediction intervals for $\bar{Y}_m$ are possible, but

the two given here seem to behave better than any of the many others tried

in the Monte Carlo study. The Monte Carlo simulation study is described and

some of the results are reported in the next section.

3. MONTE CARLO SIMULATIONS

Before describing the Monte Carlo simulation results, a brief description

of the procedure for generating a random number from the inverse Gaussian

distribution is given. Let X denote a random variable with inverse Gaussian

distribution (1.1). Then as before $Y = \lambda(X-\mu)^2/(X\mu^2)$ has chi-square dis-

tribution with one degree of freedom. Hence, a value y of Y is generated,

and the solution for x in terms of y from above is

$$x = [\mu(2\lambda + y\mu) \pm \mu(y\mu(y\mu + 4\lambda))^{\frac{1}{2}}]/2\lambda. \tag{3.1}$$

It is obvious that both solutions given by (3.1) are positive and that the

plus sign gives $x>\mu$ and the minus sign gives $x<\mu$, with $x=\mu$ only if $y=0$.

Therefore, a value u of a uniform random variable U on (0,1) is generated. Let x_1 denote the solution from (3.1) given by

$$x_1 = [\mu(2\lambda + y\mu) + \mu(y\mu(y\mu + 4\lambda))^{\frac{1}{2}}]/2\lambda$$

and let

$$x_2 = [\mu(2\lambda + y\mu) - \mu(y\mu(y\mu + 4\lambda))^{\frac{1}{2}}]/2\lambda.$$

Let $p(y) = \dfrac{\mu}{x_1 + \mu}$. Then choose the value of the inverse Gaussian random variable X to be x_1 if $u \leq p(y)$ and x_2 otherwise, since the pdf of X is given by the mixture $f_1(x)p(y) + f_2(x)[1-p(y)]$ for a given y, where $f_1(x_1)$ denotes the pdf of X_1 and $f_2(x_2)$ is the pdf of X_2. Repeating this procedure for n independent values of Y and U yields a (pseudo) random sample of size n from (1.1).

In order to investigate the behavior of the approximate prediction intervals proposed in Section 2, extensive Monte Carlo simulations were performed. For various fixed values of γ, n, m, μ, and λ, estimates of the coverage probabilities and mean widths of the prediction intervals for $\bar{Y}_m$ discussed in Section 2 were computed based on 1000 pairs of samples $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$. The results were essentially the same when 2000 pairs of samples were used, so most of the simulations were performed with 1000 samples in order to reduce computing time. For each fixed set of values γ, n, m, λ, and μ, 1000 pairs of samples $x_1, \ldots, x_n$ and $y_1, \ldots, y_m$ were generated from the inverse Gaussian distribution (1.1). For each such pair, the approximate $(1-\gamma)$ prediction interval, say $L_1(\underline{x})$ to $U_1(\underline{x})$, was computed and checked to see whether or not $\bar{y}_m$ was contained in the computed interval. The width of the interval was also computed and stored. Then the proportion of the 1000

such pairs of samples which gave prediction intervals containing the corresponding $\bar{y}_m$ was used as the estimate of the actual coverage probability. Also the average interval width was computed for the 1000 pairs of samples.

Tables 1-3 give some of the Monte Carlo results for $1-\gamma = 0.99$, 0.95, and 0.90 and various sample sizes n and m and values of μ and λ. The symbol I_1 refers to the prediction interval $(L_1(\underline{X}), U_1(\underline{X}))$ in Section 2 and I_2 refers to $(L_2(\underline{X}), U_2(\underline{X}))$. Based on the simulation results, I_1 has a better overall performance as an approximate $(1-\gamma)$ prediction interval for $\bar{Y}_m$ than any other approximation which was tried. The prediction interval I_2 is somewhat conservative when the variance of the underlying inverse Gaussian distribution, $\sigma^2 = \mu^3/\lambda$, is small and does not give a good approximation when the variance is large and n is small, for example $\mu=3$, $\lambda=\frac{1}{2}$, and n=5. As expected, the approximation becomes closer and the average interval width decreases as n increases. Also, as the variance $\sigma^2 = \mu^3/\lambda$ decreases, the prediction improves and the prediction intervals tend to be conservative. Both I_1 and I_2 perform better for m=1 than for other values of m.

4. EXAMPLE

Chhikara and Folks (1977) showed that the maintenance data reported by Von Alven (1964) on active repair times (hours) for an airborne communications transceiver fit an inverse Gaussian distribution. The n=46 observed repair times were:

.2,.3,.5,.5,.5,.5,.6,.6,.7,.7,.7,.8,.8,1.0,1.0,1.0,1.0,1.1,

1.3,1.5,1.5,1.5,1.5,2.0,2.0,2.2,2.5,2.7,3.0,3.0,3.3,3.3,

4.0,4.0,4.5,4.7,5.0,5.4,5.4,7.0,7.5,8.8,9.0,10.3,22.0,24.5.

The respective maximum likelihood estimates of μ and λ^{-1} were $\bar{X}_{46} = 3.61$ and

Table 1. Simulation Results for $1-\gamma = 0.99$

μ	λ	n	m	I_1 Ave.Width	Cov.Prob.	I_2 Ave.Width	Cov.Prob.
1	0.25	5	1	208.308	0.983	148.0918	0.980
		5	5	118.512	0.995	33.834	0.967
		5	10	135.319	0.990	19.600	0.940
1	0.25	30	1	38.365	0.989	37.566	0.989
		30	15	6.640	0.996	5.313	0.995
3	0.25	5	1	3430.014	0.977	2393.380	0.967
		5	5	1629.692	0.967	420.363	0.920
		5	10	2285.851	0.962	268.782	0.891
3	0.25	30	1	368.060	0.990	351.081	0.990
		30	15	61.311	0.981	36.551	0.968
1	1	15	1	12.354	0.993	12.304	0.994
		15	10	4.114	0.994	3.641	0.994
		15	20	3.976	0.999	3.063	1.000
1	4	5	1	9.157	0.988	8.266	0.995
		5	5	5.886	0.997	3.993	1.000
1	4	15	1	4.084	0.994	4.520	0.998
		15	10	1.671	0.998	2.114	1.000
1	4	30	1	3.531	0.991	3.933	0.996
		30	15	1.120	0.998	1.690	1.000
5	1	15	1	332.821	0.990	303.860	0.988
		15	20	94.316	0.992	34.503	0.970
5	4	5	1	220.023	0.989	166.451	0.987
		5	5	138.620	0.996	52.584	0.990
5	4	30	1	62.778	0.992	64.342	0.994
		30	15	14.571	0.994	15.493	1.000

Table 2. Simulation Results for $1-\gamma = 0.95$

μ	λ	n	m	I_1 Ave.Width	Cov.Prob.	I_2 Ave.Width	Cov.Prob.
1	0.25	5	1	73.393	0.942	53.844	0.933
		5	5	39.481	0.952	13.439	0.921
1	0.25	30	1	21.850	0.962	21.815	0.963
		30	15	4.421	0.974	4.053	0.976
3	0.25	5	1	1441.764	0.948	1012.451	0.935
		5	5	503.665	0.925	137.920	0.858
3	0.25	30	1	226.021	0.957	217.445	0.956
1	1	15	1	7.161	0.956	7.466	0.966
		15	10	2.653	0.983	2.787	0.989
1	1	50	1	5.964	0.960	6.359	0.966
		50	40	1.194	0.990	1.724	1.000
1	4	5	1	4.420	0.959	4.564	0.981
		5	5	2.888	0.974	2.704	0.997
1	4	30	1	2.387	0.944	2.757	0.964
		30	15	0.810	0.983	1.388	1.000
5	1	15	1	198.000	0.973	184.395	0.966
		15	10	48.958	0.965	31.178	0.953
5	1	30	1	138.490	0.948	136.992	0.950
		30	15	24.509	0.969	21.461	0.967
5	4	5	1	100.599	0.956	81.492	0.959
		5	5	56.788	0.973	28.971	0.974
5	4	30	1	37.529	0.942	39.581	0.955
		30	15	9.941	0.980	12.077	0.997

Table 3. Simulation Results for $1-\gamma = 0.90$

μ	λ	n	m	Ave.Width I_1	Cov.Prob.	Ave.Width I_2	Cov.Prob.
		5	1	46.122	0.897	34.753	0.888
1	0.25	5	5	26.807	0.931	10.095	0.894
		5	10	30.468	0.921	7.079	0.868
		5	1	739.880	0.895	523.699	0.875
3	0.25	5	5	353.523	0.880	99.665	0.815
		5	10	494.554	0.885	67.965	0.789
1	4	15	1	2.027	0.893	2.461	0.949
		15	10	0.948	0.966	1.518	0.999
5	4	15	1	31.853	0.887	33.583	0.905
		15	10	12.130	0.950	13.221	0.975

$\hat{\lambda}^{-1} = 0.587$ (or $\hat{\lambda} = 1.70$). For this data and $m=1$, a 95% prediction interval

for the next repair time is obtained from $(L_1(\underline{X}), U_1(\underline{X}))$ in Section 2 as

(0.3185, 40.8393). Similarly, $(L_2(\underline{X}), U_2(\underline{X}))$ yields a 95% prediction interval

of (0.3074, 42.3139), slightly wider than $(L_1(\underline{X}), U_1(\underline{X}))$. For $m=10$, a 95%

prediction interval for the mean of the next ten repair times is found from

$(L_1(\underline{X}), U_1(\underline{X}))$ to be (1.2009, 10.8311) and $(L_2(\underline{X}), U_2(\underline{X}))$ yields (1.0575,12.2997).

5. CONCLUSION

Prediction intervals for the mean $\bar{Y}_m$ of m future observations (or a

single future observation) based on a current independent random sample of

size n from the same inverse Gaussian distribution (1.1) have been investigated.

When the mean μ of the inverse Gaussian distribution is known, exact $(1-\gamma)$

prediction intervals are easily obtained. If μ is unknown, some approximate

$(1-\gamma)$ prediction intervals for $\bar{Y}_m$ have been proposed. Based on the results

of Monte Carlo simulations, the approximate prediction interval I_1 given by

$(L_1(\underline{X}), U_1(\underline{X}))$ in Section 2 is relatively simple to compute and performs

best overall of those considered in the study.

REFERENCES

Chew, Victor (1968). Simultaneous prediction intervals. Technometrics 10, 323-330.

Chhikara, R. S. and Folks, J. L. (1974). Estimation of the inverse Gaussian distribution function. J. Amer. Statist. Assoc. 69, 250-254.

Chhikara, R. S. and Folks, J. L. (1975). Statistical distributions related to the inverse Gaussian. Commun. Statist. 4, 1081-1091.

Chhikara, Raj S. and Folks, J. Leroy (1976). Optimum test procedures for the mean of first passage time distribution in Brownian motion with positive drift. Technometrics 18, 189-193.

Chhikara, R. S. and Folks, J. L. (1977). The inverse Gaussian distribution as a lifetime model. Technometrics 19, 461-468.

Cox, D. R. and Miller, H. D. (1965). The Theory of Stochastic Processes. Methuen and Co., London.

Hahn, G. J. (1969). Factors for calculating two-sided prediction intervals for samples from a normal distribution. J. Amer. Statist. Assoc. 64, 878-888.

Hahn, G. J. (1970). Additional factors for calculating prediction intervals for sample from a normal distribution. J. Amer. Statist. Assoc. 65, 1668-1676.

Hall, I. J. and Prairie, R. R. (1973). One-sided prediction intervals to contain at least m out of k future observations. Technometrics 15, 897-914.

Hewett, J. E. (1968). A note on prediction intervals based on partial observations in certain life test experiments. Technometrics 10, 850-853.

Johnson, N. and Kotz, S. (1970). Distributions in Statistics. Continuous Univariate Distributions-I. Houghton-Mifflin Co., Boston.

Lawless, J. F. (1970). A prediction problem concerning samples from the exponential distribution, with application in life testing. Technometrics 13, 725-730.

Lawless, J. F. (1972). On prediction intervals for samples from the exponential distribution and prediction limits for system survival. Sankhyā, Series B 34, 1-14.

Mann, N. R. and Fertig, K. W. (1977). One-sided prediction intervals for at least p out of m future observations from a normal population. Technometrics 19, 167-177.

Padgett, W. J. (1979). Confidence bounds on reliability for the inverse
    Gaussian model. IEEE Transactions on Reliability R-28, 165-168.

Padgett, W. J. and Wei, L. J. (1979). Estimation for the three-parameter
    inverse Gaussian distribution. Commun. Statist. - Theor. Meth. A8,
    129-137.

Seshadri, V. and Shuster, J. J. (1974). Exact tests for zero drift based
    on first passage times in Brownian motion. Technometrics 16, 133-134.

Shuster, J. (1968). On the inverse Gaussian distribution function.
    J. Amer. Statist. Assoc. 63, 1514-1516.

Tweedie, M.C.K. (1957a). Statistical properties of inverse Gaussian
    distributions. I. Ann. Math. Statist. 28, 362-377.

Tweedie, M.C.K. (1957b). Statistical properties of inverse Gaussian
    distributions. II. Ann. Math. Statist. 28, 696-705.

Von Alven, W. H., Ed. (1964). Reliability Engineering by ARINC.
    Prentice-Hall, Inc., New Jersey.

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7. AUTHOR(s)	8 CONTRACT OR GRANT NUMBER(s)
W. J. Padgett	F49620-79-C-0140

9 PERFORMING ORGANIZATION NAME AND ADDRESS	10 PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
University of South Carolina Department of Mathematics and Statistics Columbia, South Carolina 29208	61102F 2304/A5

11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Air Force Office of Scientific Research/NM Bolling AFB, Washington, DC 20332	August, 1980
	13 NUMBER OF PAGES 17

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15 SECURITY CLASS. (of this report)
	UNCLASSIFIED
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Prediction; First passage time distribution; Life testing; Reliability; Monte Carlo simulation.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The problem of predicting, on the basis of an observed sample from an inverse Gaussian distribution, the mean of a future random sample (or a single future observation) from the same distribution is considered. Approximate prediction intervals are proposed, and their accuracy is investigated via extensive Monte Carlo simulations. The results are useful for predicting the next first passage time for a Brownian motion with positive drift or the failure time of an item having inverse Gaussian life distribution.

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