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A ONE-DIMENSIONAL NUMERICAL MODEL OF LASER HEATING OF TARGET SL--ETC(U)

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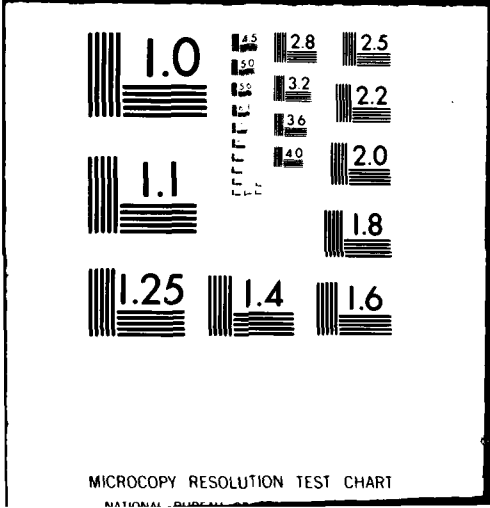
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A ONE-DIMENSIONAL NUMERICAL MODEL
OF LASER HEATING OF TARGET SLABS

J.P. Morency

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(6) A ONE-DIMENSIONAL NUMERICAL MODEL
OF LASER HEATING OF TARGET SLABS

(Un modèle Numérique
Unidimensionnel du Chauffage
d'une Plaque métallique
par Laser) J.P. Morency

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RESUME

Dans ce rapport on développe un schéma numérique de solution de l'équation unidimensionnelle non linéaire de conduction de la chaleur. On applique ensuite ce schéma à l'interaction laser-métaux et on montre l'importance de la variation des propriétés thermophysiques sur la distribution de la température dans une plaque métallique. (NC)

ABSTRACT

↓
This report establishes a numerical scheme to solve the nonlinear one-dimensional heat transfer equation. The scheme is then applied to the case of laser-metals interaction and used to show the importance of thermophysical properties variation on the temperature distribution within a slab. (U)
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1.0 INTRODUCTION

The development of the CO₂ laser with its high efficiency and high output power has made possible a variety of laser-target interaction experiments. However, as the cost of performing exhaustive parametric studies to establish potential applications is prohibitive, computer simulations were developed which can predict laser effects with sufficient accuracy to reduce the number of required experiments to an acceptable level.

Although Hanley (Ref. 1) has developed an operational finite difference code to solve the three-dimensional heat flow equation, his solution does not conserve energy and leads to inaccurate results. For simplicity, in this report, we solve the one-dimensional problem by properly treating the boundary conditions to conserve energy. Exploitation of the Kirchhoff transformation of the temperature scale allows temperature-dependent physical properties to be included relatively simply. Furthermore, by reducing the truncation error to terms of second order in time and fourth order in space, we obtain significantly more accurate results for a specific time or, conversely, we reduce the computation time for a given accuracy. The extension of this numerical scheme to three-dimensional problems is straightforward.

Section 2 deals with the mathematical formulation of the problem, and Section 3 develops the finite-difference approach used to solve the one-dimensional heat diffusion equation. In Section 4, we present some numerical results and compare them with the few existing analytical solutions. In Section 5, we discuss our results and their limitations.

This work was performed at DREV between May 1978 and February 1979 under PCN 33B06, Effects of Laser Beams on Materials.

2.0 FORMULATION OF THE PROBLEM

The mathematical treatment of laser-target interaction involves the solution of the differential equation of heat conduction in bounded media under appropriate initial and boundary conditions. For a stationary, homogeneous, isotropic solid with internal heat generation the differential equation in a Cartesian coordinate is (Refs. 2, 3)

$$\rho C_p(T) \frac{\partial T}{\partial t}(z,t) = \frac{\partial}{\partial z} \left[K(T) \frac{\partial T}{\partial z}(z,t) \right] + g[z,t] \quad [1]$$

In this equation, ρ is the density, C_p , the specific heat at constant pressure, T , the temperature, z , the spatial coordinate, t , the time, K , the thermal conductivity and $g[z,t]$, the internal heat source or sink.

This differential equation of heat conduction has numerous solutions, unless a set of suitable boundary and initial conditions are prescribed. In this study, we consider linear and nonlinear boundary conditions of the first and second kind. Mathematically, they have the following forms:

$$T = T_0 \text{ or } 0 \quad [2a]$$

and/or

$$K[T] \frac{\partial T}{\partial \eta_i} = F \quad [2b]$$

where T_0 is a constant temperature, $\partial/\partial \eta_i$ denotes differentiations along the outward-drawn normal at the boundary surface, S_i , and F can be arbitrary functions of time and surfaces.

When the thermal properties of the solid vary with temperature, the partial differential equation is nonlinear. If it is assumed

that ρ , C_p and K are dependent on temperature but independent of position and time, and that the heat-generation term $g \equiv g(z,t)$ is independent of temperature, then, changing the dependent variable using the Kirchhoff transformation (Refs. 2,4,5) will remove the thermal conductivity from the differential operator.

This transformation is accomplished by defining a new dependent variable, U , as

$$U = \int_0^T \frac{K(T')}{K_0} dT' \quad [3]$$

where $U \equiv U(T)$, $T \equiv T(z,t)$ and K_0 is the value of thermal conductivity for $T = 0$. Since K is a function of T only eq. 1 can be written in the following form

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial z^2} + \frac{\partial K}{\partial z} \cdot \frac{\partial T}{\partial z} + g \quad [4]$$

Expressing $\frac{\partial K}{\partial z}$ in the form

$$\frac{\partial K}{\partial z} = \frac{dK}{dT} \cdot \frac{\partial T}{\partial z} \quad [5]$$

and substituting this into eq. 4 gives

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial z^2} + \frac{dK}{dT} \left(\frac{\partial T}{\partial z} \right)^2 + g \quad [6]$$

From eq. 3 we find

$$\frac{\partial U}{\partial t} = \frac{du}{dT} \cdot \frac{\partial T}{\partial t} = \frac{K}{K_0} \frac{\partial T}{\partial t} \quad [7a]$$

$$\frac{\partial U}{\partial z} = \frac{dU}{dT} \cdot \frac{\partial T}{\partial z} = \frac{K}{K_0} \frac{\partial T}{\partial z} \quad [7b]$$

$$\frac{\partial^2 U}{\partial z^2} = \frac{\partial}{\partial z} \left(\frac{K}{K_0} \frac{\partial T}{\partial z} \right) = \frac{1}{K_0} \frac{\partial K}{\partial z} \cdot \frac{\partial T}{\partial z} + K \frac{\partial^2 T}{\partial z^2} \quad [7c]$$

Substituting eq. 7 in eq. 6 gives

$$\frac{1}{\alpha} \frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial z^2} + \frac{g}{K_0} \quad [8]$$

Since the thermal diffusivity $\alpha = K/\rho C_p$ in eq. 8, is a function of temperature, the equation is still nonlinear, but simpler in form. Through this transformation, the boundary conditions become

$$U = U_0 \quad [9a]$$

and/or

$$K_0 \frac{\partial U}{\partial \eta_1} = F \quad [9b]$$

Even with these simplifications, the solution of the differential equation is very complex and analytical results can only be obtained for a restricted number of particular or specific cases, which we will use without derivation to check the validity of our numerical model. Following Hanley (Ref. 1), we use a standard finite-difference approximation to solve this one-dimensional heat conduction equation. One of

our aims is to obtain a good compromise between computer time and accuracy of the results, particularly those related to a comparison with experimental ones.

3.0 FINITE-DIFFERENCE MODEL

3.1 Finite-Difference Approximation

We use an explicit, central difference scheme to solve the following differential equation:

$$\frac{\partial U}{\partial t}(z,t) = \frac{\alpha \partial^2 U}{\partial z^2}(z,t) + \frac{\alpha g}{K_0} \quad [10]$$

We consider only materials opaque to the incident laser beam (i.e. materials such as metals for which the absorption depth is much less than the wavelength of the laser radiation) and chemically as well as nuclearly stable. Under such circumstances, there is no internal heat source or sink and, therefore, $g(z,t) = 0$.

The boundary-value problem under consideration is that of a slab of finite thickness whose front surface has a uniform and constant thermal load (i.e. $\partial U / \partial z = Cte$ at $z = 0$), whereas the back surface is either insulated (i.e. $\frac{\partial U}{\partial z} = 0$ at $z = L$) or held at the ambient temperature (i.e. $U = U_0$ at $z = L$).

We use a Taylor series expansion of the function as our basic concept in the finite-difference approximation of the differential equation and the related boundary conditions. Since the procedure is

quite straightforward, we will give only the main results here. When a function $U(z,t)$ and its derivatives are finite, continuous, and have a single value, this function can be expanded in the form of the Taylor series about the point z as

$$U(z+\Delta z,t) = U(z,t) + \Delta z \cdot \frac{\partial U}{\partial z}(z,t) + \frac{\Delta z^2}{2!} \frac{\partial^2 U}{\partial z^2}(z,t) + \frac{\Delta z^3}{3!} \frac{\partial^3 U}{\partial z^3}(z,t) + \dots \quad [11]$$

or

$$U(z-\Delta z,t) = U(z,t) - \Delta z \cdot \frac{\partial U}{\partial z}(z,t) + \frac{\Delta z^2}{2!} \frac{\partial^2 U}{\partial z^2}(z,t) - \frac{\Delta z^3}{3!} \frac{\partial^3 U}{\partial z^3}(z,t) + \dots \quad [12]$$

The first-order central-difference approximation is obtained by subtracting eq. 12 from eq. 11

$$\left. \frac{\partial U}{\partial z}(z,t) \right]_z = \frac{U(z+\Delta z,t) - U(z-\Delta z,t)}{2\Delta z} + O(\Delta z^2) \quad [13]$$

The term $O(\Delta z^2)$ on the right-hand side indicates that the error involved in cutting off the infinite series is of the order of (Δz^2) . Similarly, the addition of eqs. 11 and 12 gives the second derivative of the function as

$$\left. \frac{\partial^2 U}{\partial z^2}(z,t) \right]_z = \frac{U(z+\Delta z,t) + U(z-\Delta z,t) - 2U(z,t)}{\Delta z^2} + O(\Delta z^2). \quad [14]$$

where the truncation error is of the order of (Δz^2) .

We also have for the time variable the following finite-difference approximation

$$\left. \frac{\partial U}{\partial t} (z, t) \right]_t = \frac{U(z, t+\Delta t) - U(z, t)}{\Delta t} + 0(\Delta t) \quad [15]$$

We then find, for the finite-difference approximation, the following relation

$$U(R, \ell+1) = U(R, \ell) + \frac{\alpha \Delta t}{[\Delta z]^2} \left[U(R+1, \ell) + U(R-1, \ell) \right] - \frac{2\alpha \Delta t}{[\Delta z]^2} \cdot U(R, \ell) + 0(\Delta t) + 0(\Delta z^2) \quad [16]$$

where Δz is the spatial increment and Δt , the temporal one; R refers to lattice points along the z axis and ℓ , to the integral multiple of the step Δt along the time axis. The coefficient α is the temperature-dependent thermal diffusivity and can be expressed as

$$\alpha(T) = \frac{K(T)}{\rho(T) \cdot C_p(T)} \quad [17]$$

3.2 Boundary Conditions

Using the central-difference approximation, the boundary conditions become

1. Back surface

a) Held to ambient

$$U(R_{\max}, \ell) = U_0 \quad [18a]$$

b) Insulated (i.e., no heat flow)

$$U(R_{\max. + 1}, \ell) = U(R_{\max. - 1}, \ell) \quad [18b]$$

and

2. Front surface

$$U(R_{\min.} - 1, \ell) = U(R_{\min.} + 1, \ell) + \frac{2\Delta z}{K_0} \epsilon(T) \cdot \phi \cdot \cos \theta [19]$$

where $R_{\max.}$ and $R_{\min.}$ indicate respectively the maximum and the minimum values on the z axis, K_0 is the ambient temperature thermal conductivity, $\epsilon(T)$ the temperature-dependent absorption or coupling coefficient, ϕ , the incident flux or power density on the material and θ , the angle of incidence of the laser beam with respect to the normal to the front surface.

3.3 Errors Involved in Finite Differences

Since in the process of the numerical solution of differential equations the derivatives are approximated with finite-difference expressions at each nodal point, an analysis of the possible errors involved and of the way to reduce them is paramount. There are two main types of errors: round-off and truncation errors. Furthermore, because of the explicit numerical scheme used, a specific relation between the spatial and the temporal variables must be satisfied. This relation is called the stability criterion. A detailed derivation of the results given below is beyond the scope of the present work. However, interested readers can consult, for more information, anyone of the following publications (Refs. 6-9).

3.3.1 Round-Off Errors

Numerical calculations are carried out only to a finite number of significant figures. At each step, the error introduced by rounding off the numerical calculations is called the round-off error. In linear problems, the effects of these errors superimpose themselves in the

solution. The use of small mesh size, although desirable for a better approximation of the differential equation, increases the cumulative effect of round-off errors. Therefore, one cannot always say that decreasing the mesh size necessarily increases the accuracy in finite-difference calculations. On the other hand, carrying out the numerical calculations at the intermediate stages to two or more additional significant figures will reduce the cumulative effect of round-off errors at the expense of increasing the computation time. However, since the distribution of these errors has many of the features of a random process, it is likely that the effects of these errors will generally cancel each other in part. Therefore, it is very difficult to determine exactly the order of magnitude of the cumulative departure of the solution due to round-off errors.

3.3.2 Stability of Finite-Difference Solutions

At each stage of the calculations, some round-off errors will be present. The solution of finite-difference equations is called stable if the difference between the exact and the numerical solutions tends to zero as Δt and Δz tend to zero and does not increase exponentially. Specifically, the solution will be stable if the following condition is satisfied:

$$2 \alpha \cdot \Delta t \left[\frac{1}{(\Delta z)^2} \right] \leq 1 \quad [20]$$

It should be noted that the form of the difference equations depends on the type of the differentiation scheme used, that of differential equations and the boundary conditions. Therefore, the stability criteria given above cannot be generalized for other systems. In fact, each system must be examined individually for stability.

Unfortunately, there is no general method, for nonlinear problems, that can be used effectively to determine the stability of the resulting finite-difference equation.

3.3.3 Truncation Error

The Taylor series expansion, used in expressing a partial differential equation in finite differences, is truncated after a prescribed number of terms. The error involved in each step of calculation resulting from the truncation of the series is called the truncation error. In our case, that error involves terms of order Δt and $(\Delta z)^2$. As the mesh size is reduced, and accordingly the time step to satisfy the stability criteria, the truncation error is expected to become smaller so that the numerical solution converges faster to the true solution. Of course, this increases the number of nodal points and the computation time. It is interesting to note that the truncation error is reduced to terms of order $(\Delta t)^2$ and $(\Delta z)^4$ by satisfying the following relation:

$$\alpha \cdot \Delta t \left[\frac{1}{(\Delta z)^2} \right] = \frac{1}{6} \quad [21]$$

Under this condition, the finite-difference solution approaches, within a determined residual error, the true solution of the differential equation at a faster rate and it will be used in our numerical scheme. This value is called the "stability constant" and it will be referred to under this name later on.

3.4 Program Summary

A computer program has been written in FORTRAN IV to evaluate the finite-difference approximation described in the previous section. A listing appears in Appendix A along with detailed running instructions.

The program can deal with different types of opaque materials as long as their thermophysical properties and their temperature variation are known. The number of points in the lattice along the z axis is variable to permit the user to satisfy specific needs.

In addition, an analytical solution calculation for constant thermophysical properties has been introduced into the program to check the numerical results. Finally, the user can obtain the temperature distribution as a function of time in tabular and graphical forms.

4.0 RESULTS

4.1 Analytical Solutions

Schriempf (Ref. 10) gives analytical solutions to the heat diffusion equation in the one-dimensional situation. He assumes the laser beam is constant and uniform, the material parameters are temperature independent, the solid is uniform and isotropic. Furthermore, there are no internal heat sources or sinks, and no phase change in the materials is considered.

The solution for the semi-infinite solid is

$$T(z,t) = \frac{2 \cdot \epsilon \cdot \phi \cdot \cos \theta}{K} \cdot \sqrt{at} \cdot \text{ierfc} \left[\frac{z}{2\sqrt{at}} \right] \quad [22]$$

and

$$T(0,t) = \frac{2 \cdot \epsilon \cdot \phi \cdot \cos \theta}{K} \cdot \sqrt{\frac{at}{\pi}} \quad [22a]$$

Similarly, the solution for a slab of finite thickness is

$$T(z,t) = \frac{\epsilon \cdot \phi \cdot (\cos \theta) \cdot L}{K} \left\{ \frac{\alpha \cdot t}{L^2} + \left[\frac{3(L-z)^2 - L^2}{6L^2} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cdot e^{-\alpha n^2 \pi^2 t / L^2} \cdot \left(\cos \frac{n\pi(L-z)}{L} \right) \right] \right\} \quad [23]$$

and

$$T(0,t) = \frac{\epsilon \cdot \phi \cdot (\cos \theta) \cdot L}{K} \left\{ \frac{\alpha \cdot t}{L^2} + \left[\frac{1}{3} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cdot e^{-\alpha n^2 \pi^2 t / L^2} \right] \right\} \quad [23a]$$

where L is the slab thickness, and the condition for an insulated back surface is

$$\left. \frac{\partial T}{\partial z} \right|_{z=L} = 0 \quad [24]$$

Equation 23 has been included into the program appearing in Appendix A, such that the reliability of the numerical results, for similar conditions, can be assessed.

4.2 Numerical Solutions

The numerical results obtained in calculating the function U are identical to those of the temperature T , if we assume that the thermo-physical properties are temperature independent. Otherwise, we have to solve the following equation

$$U(T) = \frac{1}{K_0} \int_0^T K(T') dT' \quad [25]$$

to find the temperature distribution through the slab considered. For metals studied here, the thermal conductivity of the solid phase is

$$K(T') = K_0 (1 + \beta T') \quad [26]$$

Substituting eq. 26 into eq. 25 gives

$$U = T + \frac{\beta}{2} T^2 \quad [27]$$

and finally,

$$T = \frac{1}{\beta} \left[\sqrt{1 + 2\beta U(z,t)} - 1 \right] \quad [28]$$

Furthermore, we assume that the thermal diffusivity is as follows:

$$\alpha(T') = \frac{K(T')}{\rho_0 C_p(T')} = \frac{K_0(1 + \beta T')}{\rho_0 C_{p_0} (1 + \gamma T')} \quad [29]$$

where K_0 is the thermal conductivity, ρ_0 , the density (assumed constant in the temperature range considered) and C_{p_0} , the specific heat at the initial temperature (i.e. room temperature in our case).

4.2.1 Numerical vs Analytical Solutions

This section compares the numerical and the analytical solutions for similar conditions; it also gives an assessment of the accuracy and reliability of our numerical model.

Figure 1 shows the analytical solution (alternating lines) and the numerical results (continuous lines), with temperature independent thermophysical properties, for a 6-node case (i.e. $\Delta z = 0.2$ cm) along the z axis. Each curve represents the temperature distribution at 5 time step increments (i.e. $5\Delta t$). Both models predict the same front surface temperature, within 1%, after only 25 time steps ($25 \Delta t$) or about 0.2 s in the present case. Furthermore, if we calculate, for a specific time, the area under each curve we obtain the same result, which demonstrates that we satisfy the energy conservation principle.

The same results appear in Fig. 2, but for a 21-node case (i.e. $\Delta z = 0.05$ cm) along the z axis. Each curve corresponds to 16 time steps. Except for the first few curves, it is very difficult to differentiate between the results. As a matter of fact, we have, within 0.5%, similar results on the front surface after only 16 time steps or about 0.006 s in this case. However, this accuracy was obtained at the expense of increasing the computation time by a factor of 64. This is because Δz is 4 times smaller and Δt is 16 times smaller since $\Delta t \propto (\Delta z)^2$.

This demonstrates that the numerical solution can give, with some compromise, results comparable to those of the analytical one. In the next section, the numerical solution is applied to cases where no analytical solutions exist, but where there are a few approximate ones.

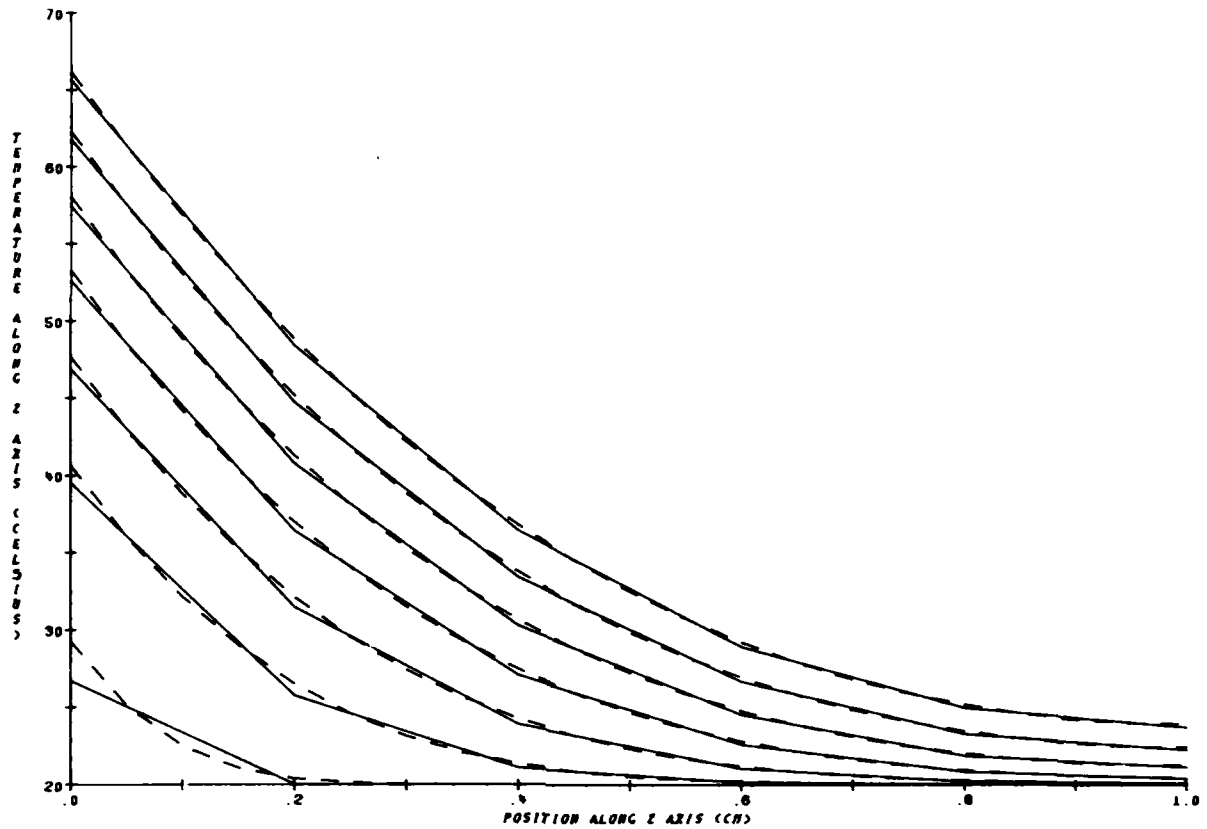


FIGURE 1 - Temperature distribution within a 1 cm-thick Cu target for an incident intensity of $2,000 \text{ W/cm}^2$ and a coupling coefficient of 0.02. Each curve corresponds to 5 time increments ($5 \Delta t$) of the numerical solution. The continuous lines represent the temperature independent thermophysical properties numerical case (6 nodes along z axis) whereas the alternating lines show the analytical solution.

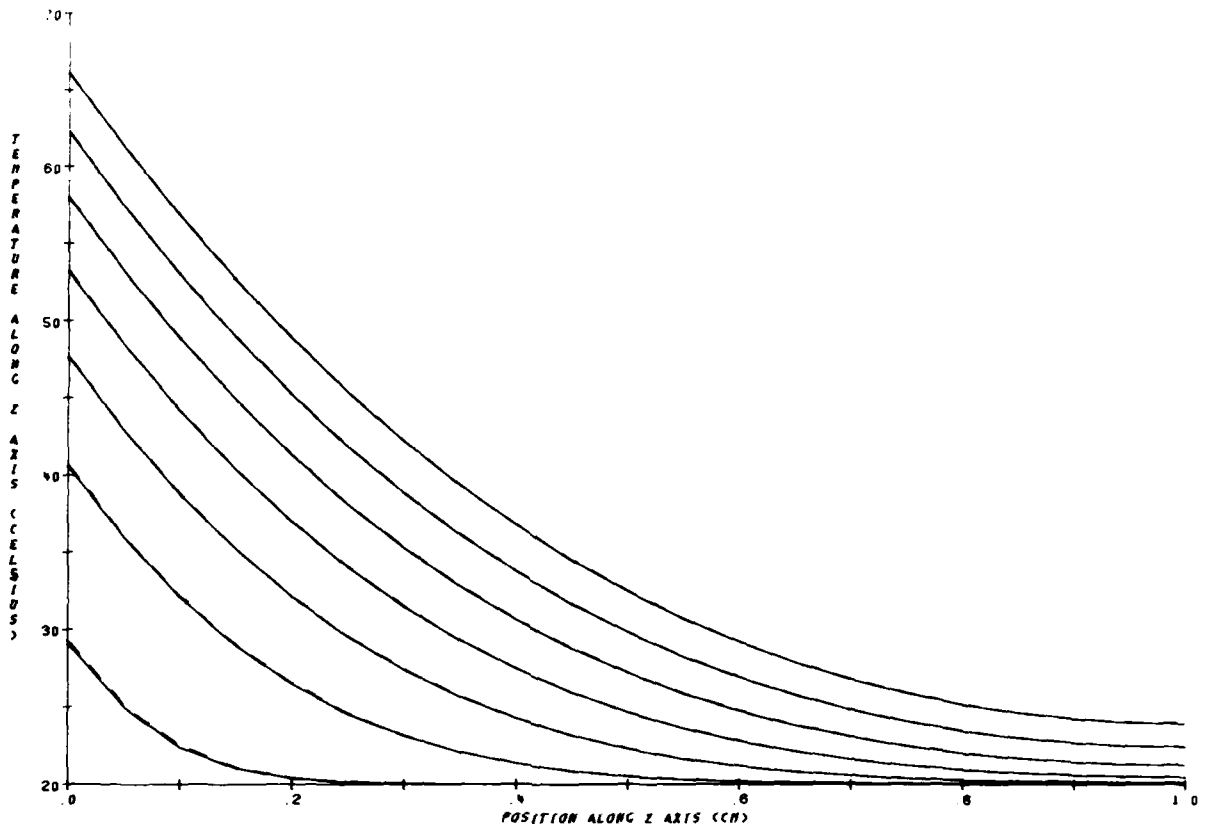


FIGURE 2 - Similar to Fig. 1 but for 21 node numerical case. Each curve corresponds to 16 time increments ($16 \Delta t$).

4.2.2 Temperature-Dependent Thermophysical Properties

The temperature dependence of thermophysical properties is an important factor in dealing with laser-matter interactions. Figure 3 shows some results for copper with 6 nodes along the z axis. We have assumed that the thermophysical properties vary linearly with temperature. The continuous curves represent the temperature-dependent results while the alternating curves are the results for the constant room-temperature case. The time interval between curves is 0.25 s. Both temperature distributions are similar at 0.25 s but quite different after 4.5 s. We calculate an error of 10% or about 0.5 s of the total time to reach melting on the front surface between the two models. In other words, it takes 0.5 s longer to reach melting on the front surface, for the specified conditions, if you consider temperature-dependent properties as in the physical world.

Figure 4 shows the absolute error in temperature for the preceding conditions. The error increases from the front to the back surface. These results indicate a decrease in the thermal diffusivity of copper with temperature as expected (Ref. 11). Figure 5 presents the relative error in percentage for the same conditions.

Figure 6 illustrates the results obtained for stainless steel # 304. This case demonstrates the drastic influence of the temperature-dependent thermophysical properties. Under the present conditions, there is a twofold increase in the time required to reach melting on the front surface between the two models. In the variable model, melting on the front surface can be reached in 3 s, whereas in the constant model, this can be achieved within 1.5 s.

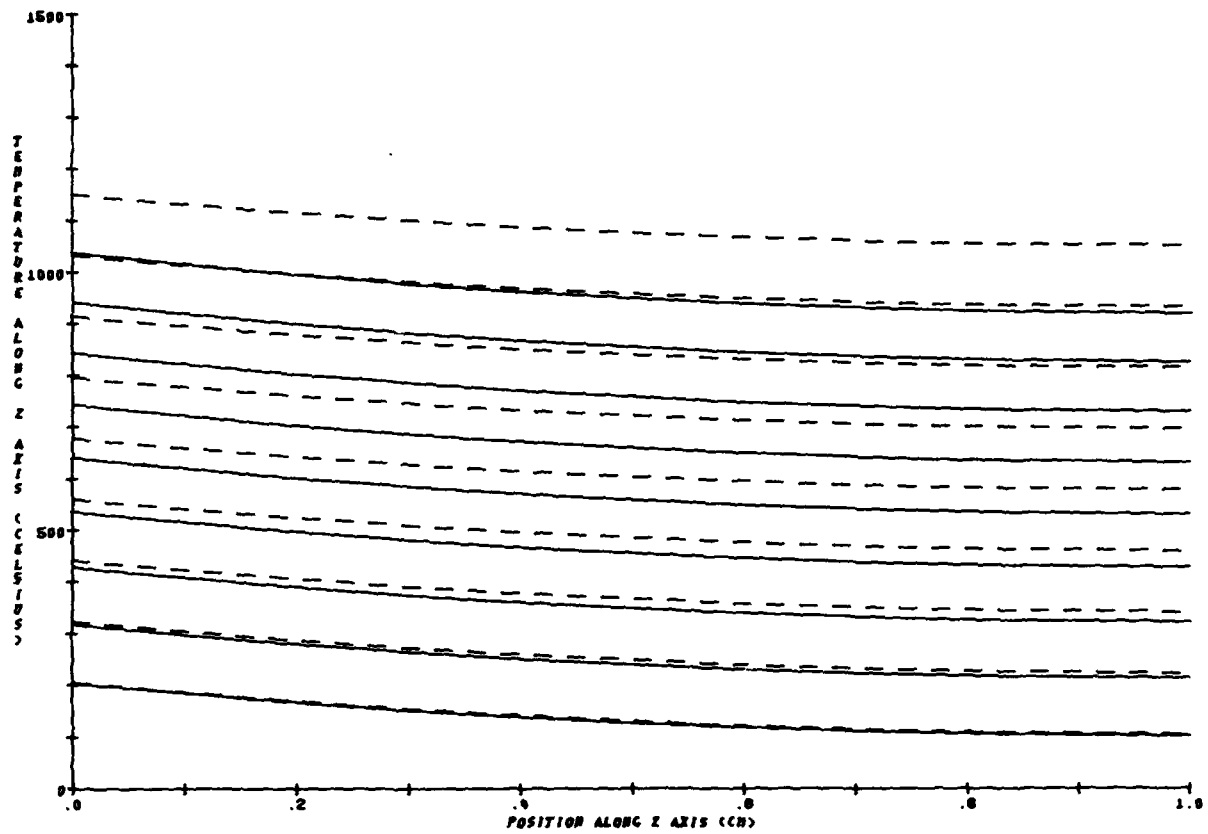


FIGURE 3 - Temperature distribution within a 1 cm-thick Cu target (6 nodes along z axis) for an incident intensity of $20,000 \text{ W/cm}^2$ and a coupling coefficient of 0.04. The time interval between each curve is 0.25 s for the temperature-dependent case (continuous lines) and for the constant room-temperature case (alternating lines).

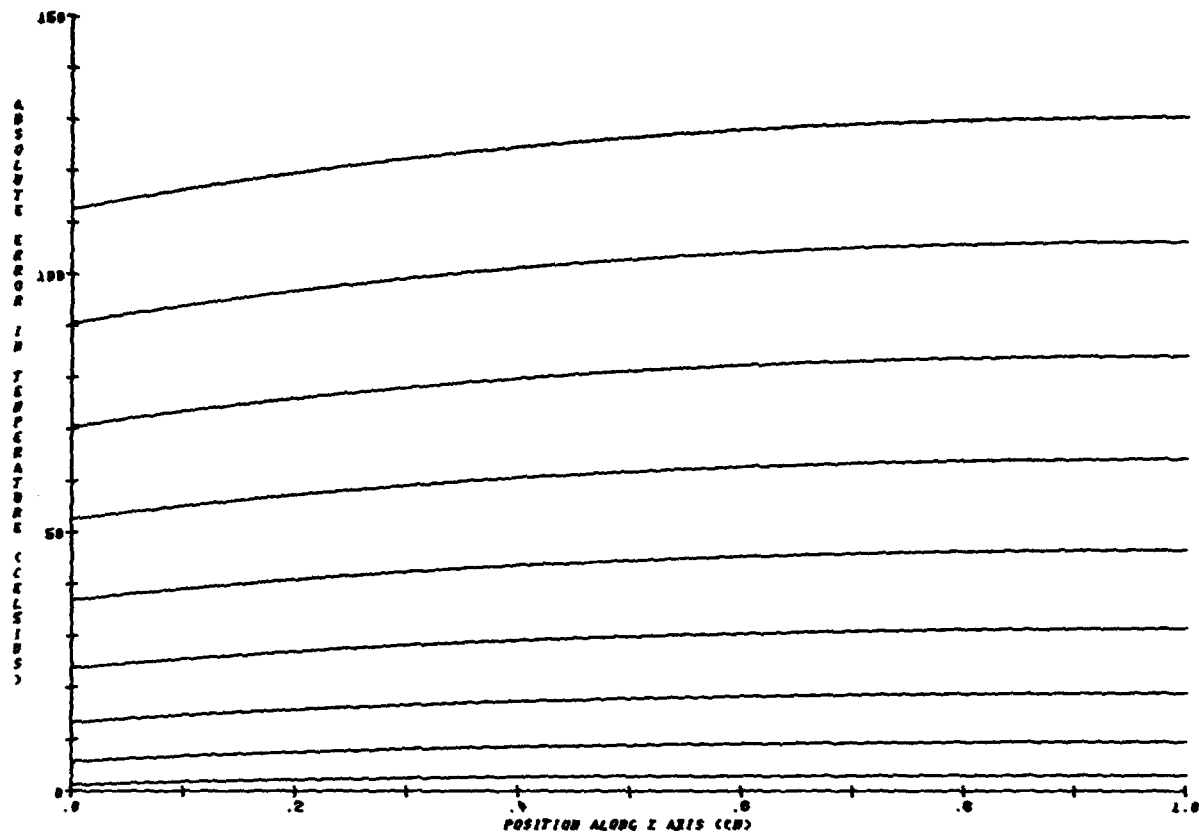


FIGURE 4 - Absolute error in temperature distribution between the two models of Fig. 3.

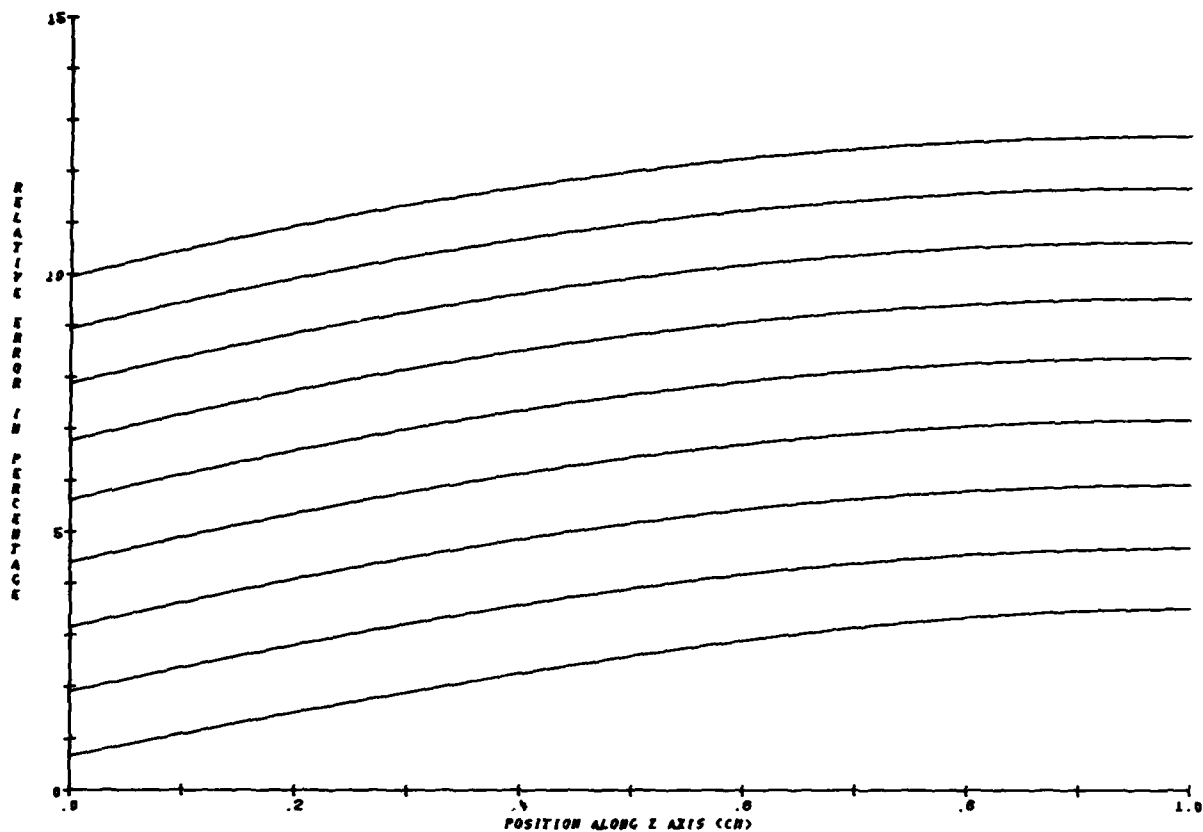


FIGURE 5 - Percentage of error in temperature distribution between the two models of Fig. 3.

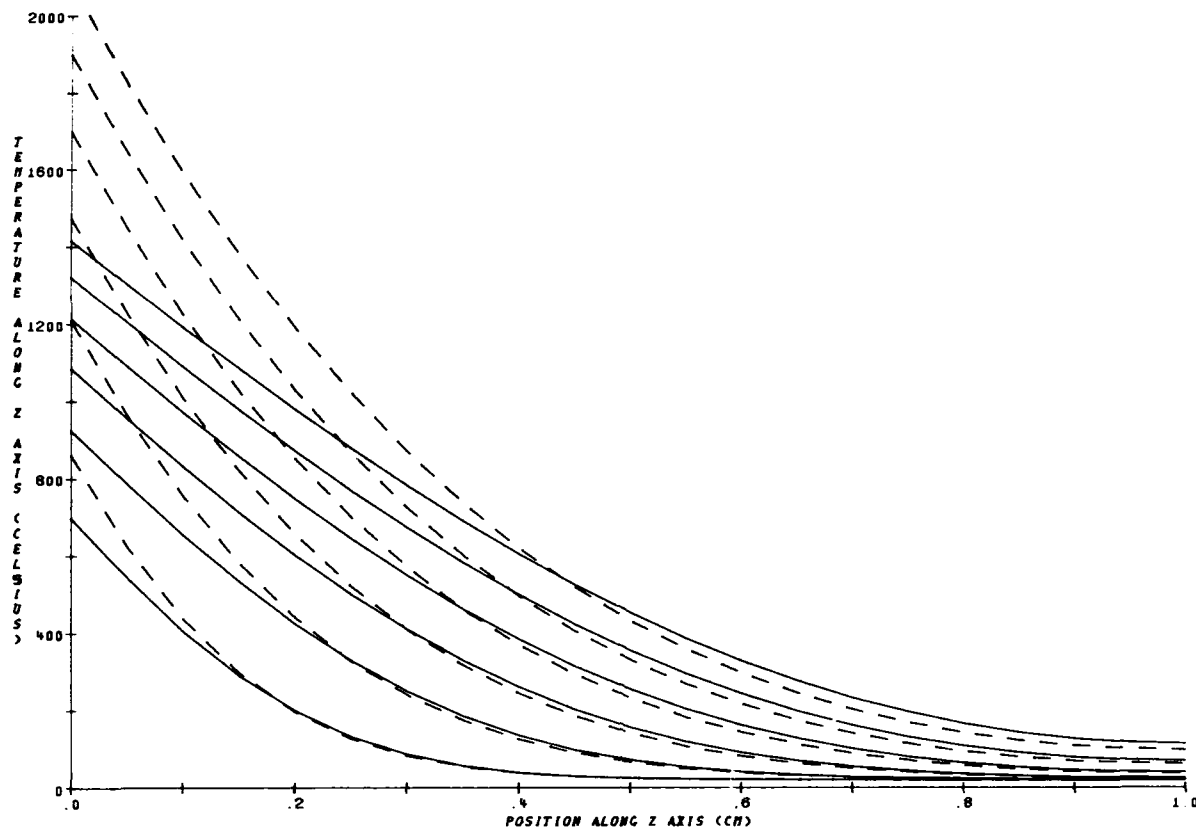


FIGURE 6 - Temperature distribution within a 1 cm-thick stainless steel #304 target (21 nodes along z axis) for an incident intensity of $20,000 \text{ W/cm}^2$ and a coupling coefficient of 0.04. The time interval between each curve is 0.25 s for the temperature-dependent case (continuous lines) and for the constant room-temperature case (alternating lines).

Furthermore, the curves cross one another and we find a higher value of the temperature on the back surface in the variable case. Figure 7 shows the absolute error in temperature along the z axis for the same time interval.

4.2.3 Averaging of the Thermophysical Properties

We define the average value of the thermophysical properties in the following way:

$$K_{av.} = \frac{K(T_i) + K(T_m)}{2} \quad [30]$$

and

$$C_{pav.} = \frac{C_p(T_i) + C_p(T_m)}{2} \quad [31]$$

where T_i is the initial temperature and T_m the melting temperature. If we use these values in the constant properties model instead of those of the room temperature, we obtain the results shown in Fig. 8 for copper and those in Fig. 9 for stainless steel #304. We expect similar results because the thermophysical properties of these two materials vary linearly with temperature so that these properties are overestimated at low temperature and underestimated at high temperature by the average properties values. However, although the discrepancy in time to reach melting on the front surface is removed, the temperature distribution through the slab, specially in stainless steel #304 is quite different. The distribution behaves as if the coupling coefficient was varying with temperature and, depending on the position selected along the z axis, it seems to increase or to decrease with time.

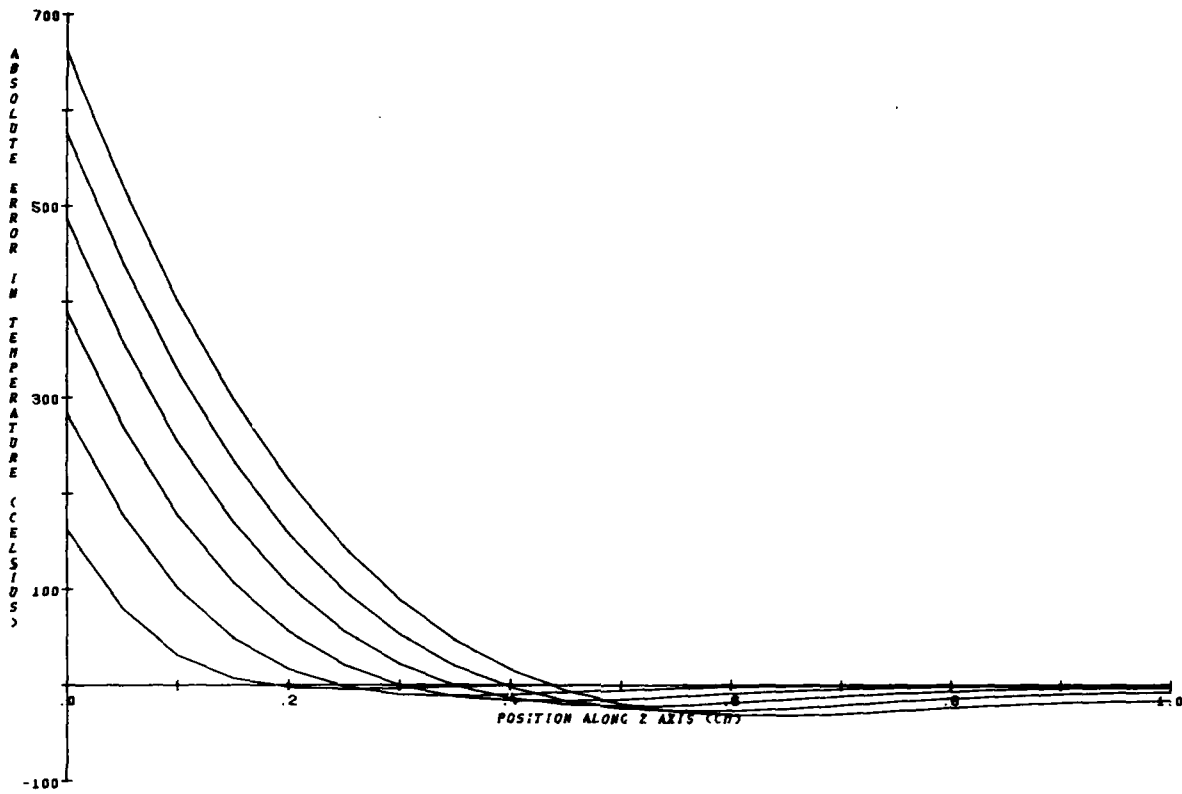


FIGURE 7 - Absolute error in temperature distribution for conditions of Fig. 6.

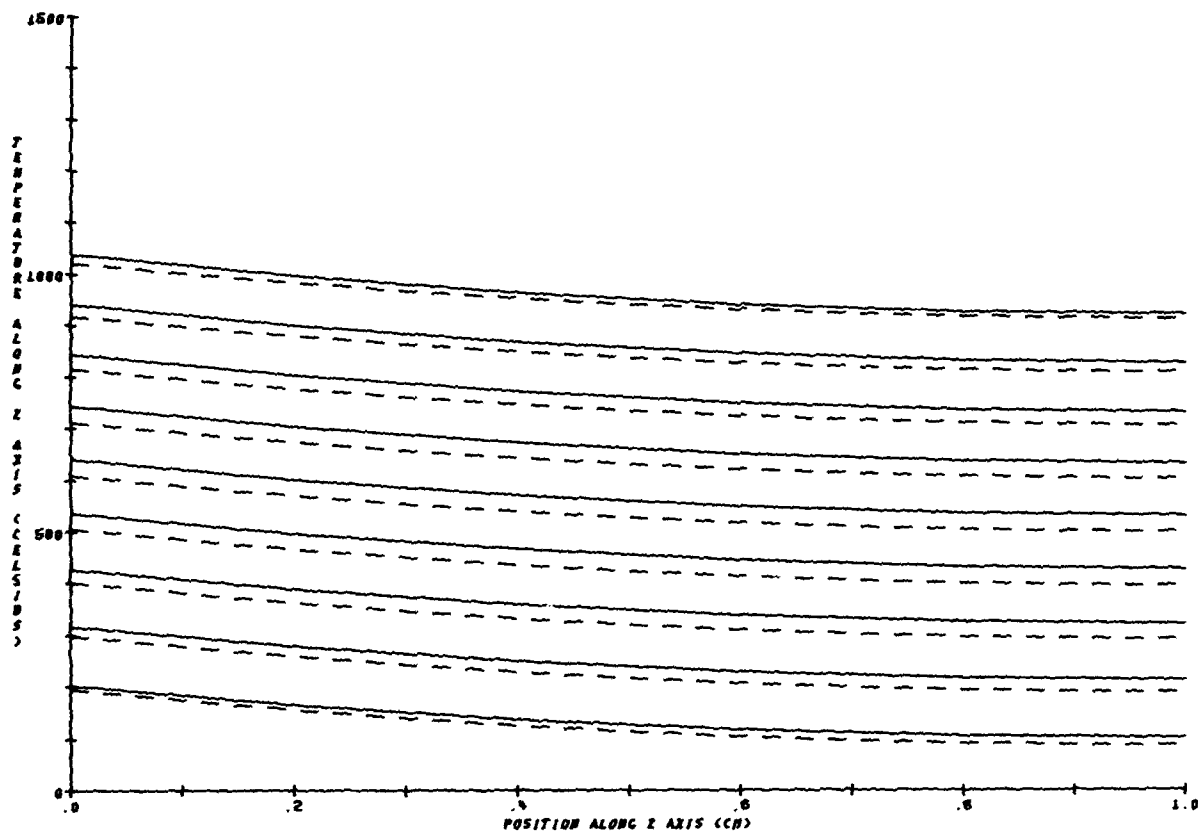


FIGURE 8 - Temperature distribution within a 1 cm-thick Cu target (6 nodes along z axis) for an incident intensity of $20,000 \text{ W/cm}^2$ and a coupling coefficient of 0.04. The time interval between each curve is 0.25 s for the temperature-dependent case (continuous lines) and the average-value case (alternating lines).

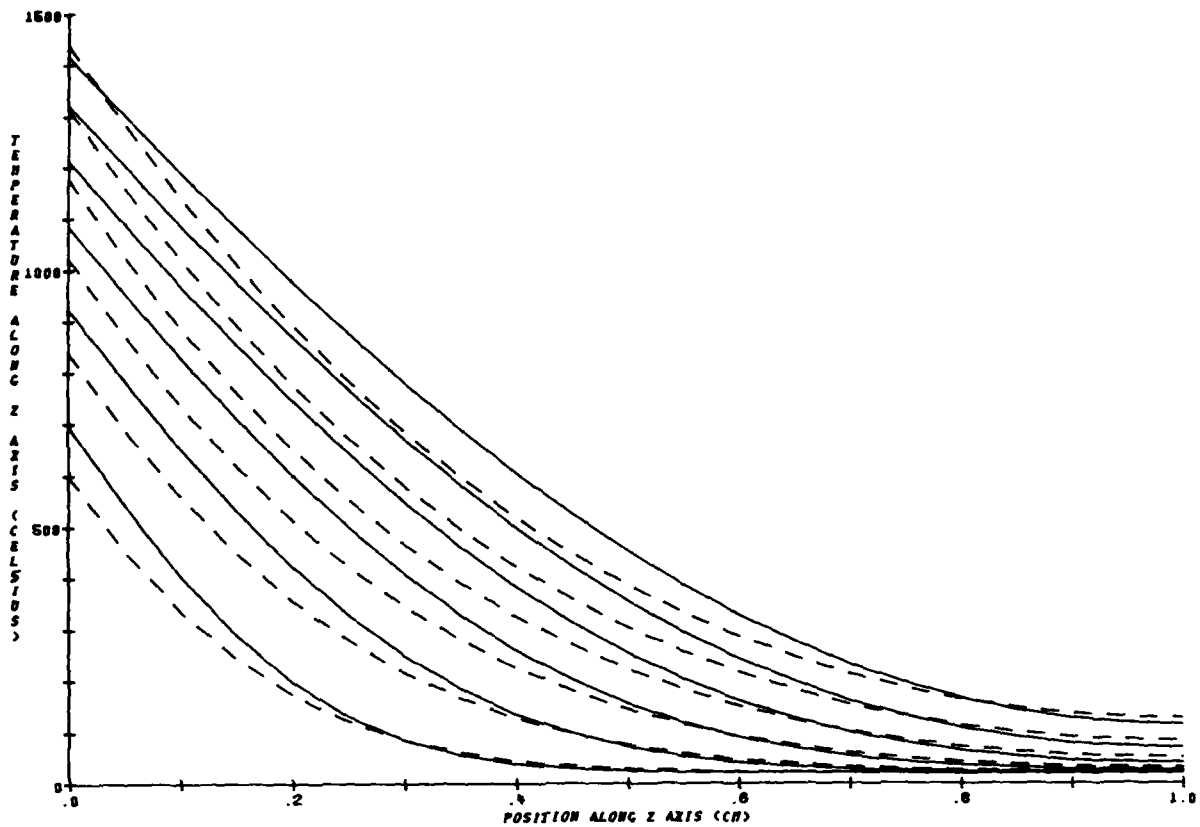


FIGURE 9 - Temperature distribution within a 1 cm-thick stainless steel #304 target (21 nodes along z axis) for an incident intensity of $20,000 \text{ W/cm}^2$ and a coupling coefficient of 0.04. The time interval between each curve is 0.25 s for the temperature-dependent case (continuous lines) and the average-value case (alternating lines).

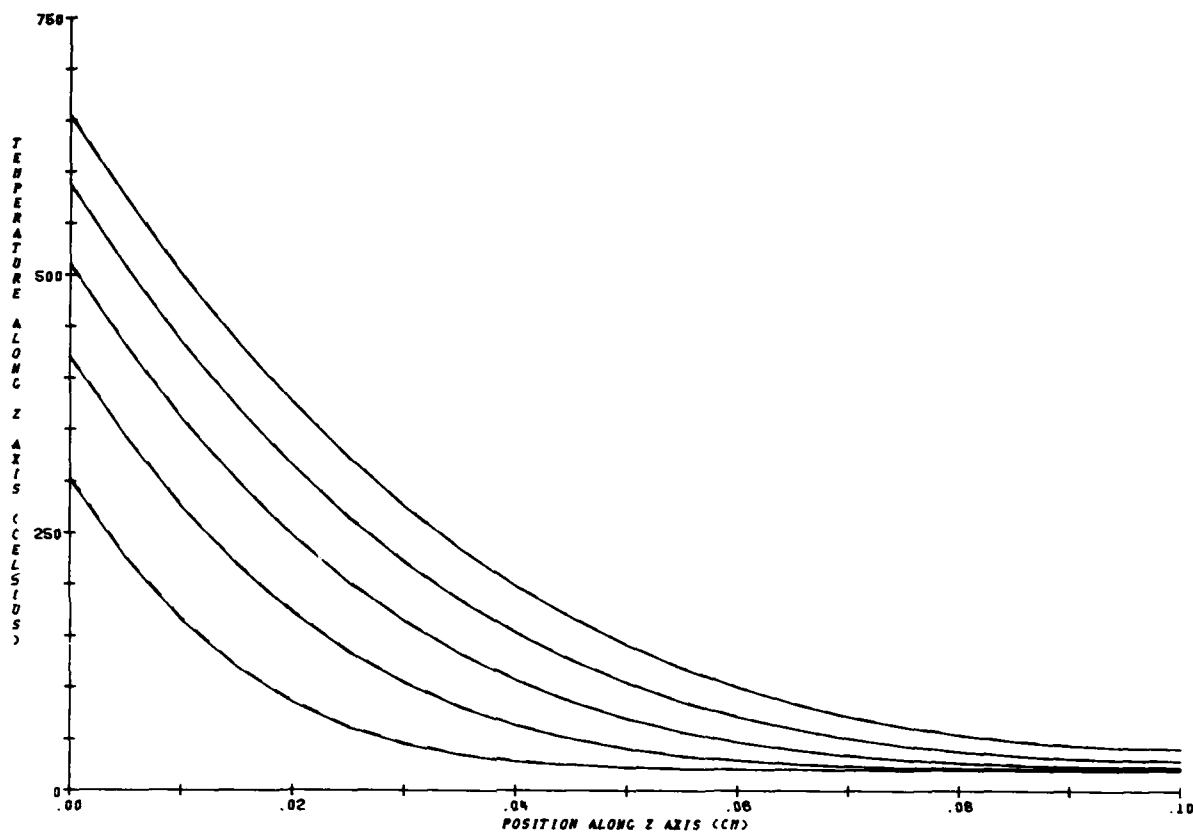


FIGURE 10 - Temperature distribution within a 0.1 cm-thick Al target (21 nodes along z axis) for an incident intensity of 2×10^6 W/cm² and a coupling coefficient of 0.02. The time interval between each curve is 134×10^{-6} s for the numerical case (continuous lines) and the analytical average value case (alternating lines).

Finally, Fig. 10 presents some results for aluminum, using the average properties values as defined previously. We used 21 nodes for this 0.1 cm-thick slab because we were interested in a high-input intensity ($\phi = 2 \times 10^6 \text{ W/cm}^2$) for a short time duration, as would be encountered with pulsed lasers. The time to reach melting on the front surface is within 1% of the one obtained by Gautier (Ref. 12).

5.0 DISCUSSION

Our aim of developing a reliable numerical procedure for the non-linear one-dimensional heat conduction equation has been achieved. The boundary conditions are calculated through a numerical procedure different from the one used by Hanley (Ref. 1); therefore, we satisfy the energy conservation law whereas his scheme does not. Furthermore, our scheme is simpler, easier to understand and to work with, and through proper selection of the "stability constant", converges more rapidly to the true solution for an optimal number of steps. The time-independent or continuous-intensity laser beam has been considered in the present work.

Our present solution is applicable to cases where lateral conduction of heat is negligible. These situations correspond to short interaction time consideration or to experiments in which the laser beam dimensions are similar or larger than those of the target. However, extension of our numerical model to 3 dimensions is easy and straightforward.

Similarly, the model does not include any explicit terms for radiative and convective heat losses but these could be considered by modifying the appropriate boundary conditions.

6.0 CONCLUSIONS

A numerical method of solving the nonlinear partial differential equations of heat conduction with a computer has been developed. The reliability of the numerical results has been demonstrated by comparison with the analytical solution in one dimension. By proper selection of the "stability constant" the numerical procedure converges more rapidly to the results of the analytical solution by minimizing the truncation error. Agreement is within 1% after only 25 time steps ($25 \Delta t$).

We have shown the importance of taking into account the effect of temperature-dependent thermophysical properties when dealing with laser-matter interactions. For example, in the case of stainless steel #304, the calculated time required to reach melting on the front surface in the case of variable properties is twice the time necessary when room-temperature values are used. Although this discrepancy disappears when average-temperature values are used, the calculated temperature distributions through the material are significantly different.

7.0 ACKNOWLEDGMENTS

The author wishes to thank Drs. M. Gravel and R.W. MacPherson for their valuable suggestions and discussions. He also recognizes the valuable participation of Mr. J.C. Anctil in the preliminary stage of posing the problem. The full collaboration of Mr. A. Blanchard, R. Gouge and R. Tremblay from Data Systems was a great asset in the use of the computer.

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APPENDIX AA.1 Program Description and Listing

The program written in Fortran IV may be used under the CP-V Version E01 operating system on DREV Xerox 560 MP computer. The listing shows the materials, with their corresponding thermophysical properties, available to the user. Having selected the material, the user inputs the thickness of his target, the number of nodes through it, the ambient temperature, the incident flux, its angle of incidence from normal, the coupling coefficient, the total time of run, the print out interval and the type of boundary conditions. From there on, the program starts its calculation by establishing the initial conditions throughout the slab. Then, it determines the temperature for the next time increment and checks if it has to print out those results. In the event it has, we obtain tables of values for the front and back surface temperature and its distribution through the slab. Furthermore, it calculates, for this time, the corresponding analytical solution and outputs it. Then, the program checks if the total run time is exhausted and if not, it goes back through the loop and calculates the temperature distribution for the next time step. Once the allotted time is passed the program calls subroutines to create APL files with the complete set of data. These files are used to produce graphical representations similar to the ones appearing in this report.

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1.000 •JOB 66043,JPBMORENCY,7. (TERMINAL JOB BFALACOCA).
2.000 •LIMIT (TIME,20),(CORE,48)
3.000 •ASSIGN F:105,(FILE,HTINPUT1),(IN)
5.500 •ASSIGN M:SI,(FILE,FALACOCA1),(IN)
6.000 •FORTRAN LS,SI,GO,BC
7.000 •LOAD (GO),(LMN,FALACOCALM),(PERM),(EF,(APLFNS,LPR))
8.000 •RUN

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ONE-DIMENSION HEAT CONDUCTION LISTING

```
1.      DIMENSION UO(0:21,2),TO(21,2),TGRA(21,150),TO(21),TM(21,100)
2.      DOUBLE PRECISION F8
3.      1 FORMAT('1')
4.      10  FORMAT(' 1 ...ALUMINUM, 2 ...COPPER, 3 ...MOLYBDENUM,')
5.      20  FORMAT(' 4 ...IRON, 5 ...NICKEL, 6 ...TITANIUM, 7 ...AL(2024)')
6.      25  FORMAT(' 8 ...STAINLESS STEEL(304)')
7.      30  FORMAT(' INPUT # FOR TARGET MATERIAL')
8.      40  FORMAT(I4)
9.      50  FORMAT(' MATERIAL THICKNESS (CM)')
10.     51  FORMAT('NUMBER OF NODES ALONG Z-AXIS')
11.     55  FORMAT(2I5,F20.5)
12.     60  FORMAT(F20.5)
13.     90  FORMAT(' AMBIENT TEMPERATURE (C)')
14.    100  FORMAT(' INCIDENT INTENSITY (WATTS/CM*CM) ')
15.    120  FORMAT(' ANGLE OF INCIDENCE (DEG) FROM NORMAL')
16.    123  FORMAT(' PREMELT ABSORPTION COEFFICIENT')
17.    130  FORMAT(' TIME OF RUN (SEC)')
18.    135  FORMAT(' PRINT OUT INTERVAL (SEC)')
19.    150  FORMAT(' 1 ...BACK SURFACE INSJLATED ')
20.    160  FORMAT(' 2 ...BACK SURFACE HEAT SINKED TO AMBIENT ')
21.    185  FORMAT('FRONT SURFACE STARTS TO MELT AT POSITION (1)
22.     1AND AT TIME T= ',F10.5,' SECONDS')
23.    190  FORMAT('1TIME IN SECONDS',F11.7,/)
24.    191  FORMAT('0FRONT SURFACE')
25.    192  FORMAT(6F15.8)
26.    193  FORMAT('0BACK SURFACE')
27.    194  FORMAT('0TEMPERATURE ALONG Z-AXIS')
28.    195  FORMAT(' RUN COMPLETED')
29.    196  FORMAT('0ANALYTICAL SOLUTION ALONG Z-AXIS')
30.     WRITE (108,1)
31.     C      SELECT..... MATERIAL CODE
32.     WRITE (108,10)
33.     WRITE (108,20)
34.     WRITE (108,25)
35.     WRITE (108,30)
36.     READ (105,40) I11
37.     OUTPUT I11
38.     GO TO (200,210,220,230,240,241,242,245),I11
39.     200  CONTINUE
40.     C      I11=1 ...ALUMINUM
41.     A0=.660.
42.     F0=.25
43.     F2=.57361
44.     A3=2.7
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```
45.          A4=94.
46.          F1=0.
47.          F3=0.
48.          F4=0.
49.          F5=30.
50.          F9=2.43
51.          GO TO 300
52.    210    CONTINUE
53.    C      I11=2 ...COPPER
54.          A0=1083.
55.          F0=.091
56.          F2=.956
57.          A3=4.89
58.          A4=42.
59.          F1=2.456E-5
60.          F3=-1.567E-4
61.          F4=0.
62.          F5=0.0
63.          F9=8.217
64.          GO TO 300
65.    220    CONTINUE
66.    C      I11=3 ...MOLYBDENUM
67.          A0=2610.
68.          F0=.06162
69.          F2=.3460
70.          A3=10.2
71.          A4=131.
72.          F1=2.2E-5
73.          F3=-3.46E-5
74.          F4=0.
75.          F5=10.
76.          F9=8.1
77.          GO TO 300
78.    230    CONTINUE
79.    C      I11=4 ...IRON
80.          A0=1535.
81.          F0=.1060
82.          F2=.1080
83.          A3=7.85
84.          A4=65.
85.          F1=9.6E-5
86.          F3=-1.08E-5
87.          F4=0.
88.          F5=18.
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89. F9=6.88
90. GO TO 300
91. 2+0 CONTINUE
92. C I11=5 ...NICKEL
93. A0=1453.
94. F0=.1095
95. F2=.1425
96. A3=8.75
97. A4=73.
98. F1=5.49E-5
99. F3=-4.56E-5
100. F4=0.
101. F5=0.
102. F9=7.9
103. GO TO 300
104. 2+1 CONTINUE
105. C I11=6 ...TITANIUM
106. A0=1690.
107. F0=.139
108. F2=.0372
109. A3=4.54
110. A4=103.9
111. F1=0.
112. F3=-4.E-6
113. F4=0.
114. F5=21.
115. F9=4.09
116. GO TO 300
117. 2+2 CONTINUE
118. C I11=7 ...AL(2024)
119. A0=630.
120. F0=.215
121. F2=.334
122. A3=2.77
123. A4=95.6
124. F1=7.7E-5
125. F3=9.0E-4
126. F4=0.
127. F5=0.
128. F9=2.43
129. GO TO 300
130. 2+5 CONTINUE
131. C I11=8 ...ACIER(304)
132. A0=1450.


```
133.      F0=.11712
134.      F2=.03615
135.      A3=7.9
136.      A4=65.
137.      F1=3.74E-5
138.      F3=3.32E-5
139.      F4=0.
140.      F5=20.
141.      F9=7.
142.      300 CONTINUE
143.      C      INPUT..... MATERIAL THICKNESS (CM)
144.      WRITE (108,50)
145.      READ (105,60) A6
146.      OUTPUT A6
147.      C      INPUT.....NUMBER OF NODES ALONG Z-AXIS
148.      WRITE(108,51)
149.      READ(105,60) KK
150.      OUTPUT KK
151.      NK=KK-1
152.      C      INPUT..... AMBIENT TEMPERATURE (C)
153.      WRITE (108,90)
154.      READ (105,60) A9
155.      OUTPUT A9
156.      C      INPUT..... INCIDENT INTENSITY (WATTS/CM*CM)
157.      WRITE (108,100)
158.      READ (105,60) B0
159.      OUTPUT B0
160.      B0=B0/4.184
161.      C      INPUT..... ANGLE OF INCIDENCE (DEG) FROM NORMAL
162.      WRITE (108,120)
163.      READ (105,60) B2
164.      OUTPUT B2
165.      C      INPUT..... PREMELT ABSORPTION COEFFICIENT
166.      WRITE (108,123)
167.      READ (105,60) A5
168.      OUTPUT A5
169.      C      INPUT..... TIME OF RUN (SEC)
170.      WRITE (108,130)
171.      READ (105,60) B3
172.      OUTPUT B3
173.      C      INPUT..... PRINT OUT INTERVAL (SEC)
174.      WRITE (108,135)
175.      READ (105,60) D0
176.      OUTPUT D0
```

```

177.      C9=0.
178.      C      SELECT.....BOUNDARY CONDITIONS: IB5
179.      WRITE (108,150)
180.      WRITE (108,160)
181.      C.....IB5=1 *) BACK SURFACE INSULATED
182.      C.....IB5=2 *) BACK SURFACE HEAT=SINKED TO AMBIENT
183.      READ (105,40) IB5
184.      OUTPUT IB5
185.      C      FRONT SURFACE.....INCIDENT FLUX
186.      P=B0
187.      D1=D0
188.      INDI=0.
189.      C      INITIAL CONDITIONS
190.      DO 344 K=1, KK
191.      TO(K,1)=A9
192.      IF(F3.EQ.0.) GO TO 345
193.      UO(K,1)=A9-F5-F3*F5+A9/F2+.5*F3*F5*F5/F2+.5*F3*A9+A9/F2
194.      GO TO 344
195.      345  UO(K,1)=A9
196.      344  CONTINUE
197.      CTE=1./6.
198.      C2=CTE*(F0+F1*A9)*A3*A6*A6/(F2+F3*(A9-F5))/NK/NK
199.      C0=C2*NK*NK/A6/A6
200.      C4=2.*C2*NK*NK/A6/A6
201.      THET=3.141592*82/180.
202.      G3=COS(THET)*A5*A6*2./NK
203.      500  IF(TO(1,1).LE.A0)      GO TO 505
204.      WRITE(108,185),C9
205.      GO TO 1000
206.      505  CONTINUE
207.      F8=A3
208.      DO 560 K=1, KK
209.      UO(0,1)=UO(2,1)+G3*P/F2
210.      GO TO (530,535), IB5
211.      530  UO(KK+1,1)=UO(KK=1,1)
212.      GO TO 550
213.      535  UO(KK+1,1)=2.*UO(KK,1)-UO(KK-1,1)
214.      550  CONTINUE
215.      A1=(F2+F3*(TO(K,1)=F5))/FB/(F0+F1*TO(K,1))
216.      E1=A1*C0*(UO(K+1,1)-UO(K=1,1))
217.      UO(K,2)=E1+(1.-A1*C4)*UO(K,1)
218.      IF(F3.EQ.0.0) GO TO 555
219.      BETA=F3/F2
220.      TO(K,2)=(((1.+2.*BETA*UO(K,2))*5)/BETA)+F5-1./BETA)

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221.          GO TO 560
222.    555    TO(K,2)=U0(K,2)
223.    560    CONTINUE
224.          C9=C9+C2
225.    580    IF(C9.LT.D1) GO TO 900
226.          D1=D1+D0
227.          INDI=INDI+1
228.          DO 600 K=1, KK
229.            TGRA(K, INDI)=TO(K,1)
230.    600    CONTINUE
231.          TDIM=C9-C2
232.          WRITE(108,190) TDIM
233.          WRITE(108,191)
234.          WRITE(108,192) (TO(1,1))
235.          WRITE(108,193)
236.          WRITE(108,192) (TO(KK,1))
237.          WRITE(108,194)
238.          WRITE(108,192) (TO(L,1),L=1, KK)
239.    C.....ANALYTICAL SOLUTION
240.          LL=INDI
241.          PI=3.14159265
242.          DIFF=(F2+F3*(A0=F5)/2.)/A3/(F0+F1*(A0=F5)/2.)
243.    700    Z=0.0
244.          J=1
245.    725    W1=EXP(-DIFF*PI*PI*TDIM/A6/A6)
246.          N=1
247.          C=0.0
248.    750    W=(-1)**N/N/N*EXP(-DIFF*N*N*PI*PI*TDIM/A6/A6)
249.          W2=W*COS(N*PI*(A6-Z)/A6)
250.          VI=ABS(W/W1)
251.          IF(VI.LE.0.000002) GO TO 775
252.          C=C+W2
253.          N=N+1
254.          GO TO 750
255.    775    C=C+W2
256.          TO(J)=A5*P*TDIM*DIFF/(F2+F3*(A0=F5)/2.)/A6+A5*P*A6/(F2+F3*(
257.    1A0=F5)/2.)*(13*(A6-Z)**2-A6*A6)/6./A6/A6-2.*C/PI/PI)+A9
258.          Z=Z+A6/20.
259.          J=J+1
260.          IF(Z.GT.A6) GO TO 800
261.          GO TO 725
262.    800    WRITE(108,196)
263.          WRITE(108,192) (TO(J),J=1,21)
264.          DO 850 I=1,21

```

```
265.          TM(I,LL)=TO(I)
266.      850  CONTINUE
267.      C.....END OF ANALYTICAL SOLUTION
268.      900  IF(C9.GT.B3.OR.C9.EQ.B3) GO TO 950
269.          DO 910 K=1,KK
270.          UO(K,1)=UO(K,2)
271.          TO(K,1)=TO(K,2)
272.      910  CONTINUE
273.          GO TO 500
274.      950  WRITE(108,195)
275.      1000 CONTINUE
276.          CALL GRAPH(TM,INDI,21,1)
277.          CALL GRAPH(TGRA,INDI,K,5)
278.          CALL EXIT
279.          END
```

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```
1.      SUBROUTINE GRAPH(TEMP,INDI,KK,IAN)
2.      DIMENSION TEMP(KK,INDI)
3.      DIMENSION ITYPE(2),ISIZE(3)
4.      ISIZE(1)=2
5.      ISIZE(2)=21
6.      ISIZE(3)=INDI
7.      ITYPE(1)=3
8.      ITYPE(2)=4
9.      CALL FTIE(1,'JCAOUT')
10.     CALL FREPLACE(1,IAN,TEMP,ISIZE,ITYPE)
11.     ITYPE(1)=2
12.     ITYPE(2)=2
13.     ISIZE(1)=0
14.     CALL FREPLACE(1,100,KK,ISIZE,ITYPE)
15.     CALL FREPLACE(1,101,INDI,ISIZE,ITYPE)
16.     CALL FUNTIE(1)
17.     RETURN
18.     END
```

A.2 Input and Output Examples

The program is run from a terminal in two possible modes. In the ON-LINE interactive mode the program asks for the specific input file considered and then starts the calculation. The output is displayed on the terminal used and is similar to that obtained in the BATCH mode. In the BATCH mode, a job file, which contains a set of controls, the program and data input and output files, are run. The output is printed on the computer line printer as shown in the typical example shown hereafter.

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INPUT DATA FILE

1 ...ALUMINUM, 2 ...COPPER, 3 ...MOLYBDENUM,
4 ...IRON, 5 ...NICKEL, 6 ...TITANIUM, 7 ...AL(2024)
8 ...STAINLESS STEEL(304)
INPUT # FOR TARGET MATERIAL
I11 = 8
MATERIAL THICKNESS (CM)
A6 = 1.00000
NUMBER OF NODES ALONG Z-AXIS
KK = 21
AMBIENT TEMPERATURE (C)
A9 = 20.0000
INCIDENT INTENSITY (WATTS/CM*CM)
B0 = 20000.0
ANGLE OF INCIDENCE (DEG) FROM NORMAL
B2 = .000000
PREMELT ABSORPTION COEFFICIENT
A5 = 4.000000E-02
TIME OF RUN (SEC)
B3 = 3.00000
PRINT OUT INTERVAL (SEC)
D0 = .250000
1 ...BACK SURFACE INSULATED
2 ...BACK SURFACE HEAT SINKED TO AMBIENT
IBS = 1

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OUTPUT DATA FILE

TIME IN SECONDS .2468463

FRONT SURFACE
517.17797852

BACK SURFACE
20.00000000

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|--------------|--------------|--------------|--------------|-------------|-------------|
| 517.17797852 | 349.60815430 | 217.49487305 | 125.55175781 | 70.00808594 | 41.05175781 |
| 27.87646484 | 22.62304688 | 20.77685547 | 20.20263672 | 20.04687500 | 20.00952148 |
| 20.00122070 | 20.00000000 | 20.00000000 | 20.00000000 | 20.00000000 | 20.00000000 |
| 20.00000000 | 20.00000000 | 20.00000000 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|--------------|--------------|--------------|--------------|-------------|-------------|
| 430.88012695 | 290.82983398 | 188.14064026 | 117.87338257 | 73.20011902 | 46.90753174 |
| 32.62438965 | 25.48023987 | 22.19616699 | 20.81092834 | 20.27543640 | 20.08557129 |
| 20.02426147 | 20.00599670 | 20.00085449 | 19.99990845 | 19.99990845 | 19.99990845 |
| 19.99971008 | 19.99990845 | 20.00009155 | | | |

TIME IN SECONDS .4936965

FRONT SURFACE
693.62304688

BACK SURFACE
20.00000000

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|--------------|--------------|--------------|--------------|--------------|--------------|
| 693.62304688 | 537.38012695 | 399.22753906 | 283.46118164 | 192.53442383 | 126.16528320 |
| 81.33718820 | 53.30810547 | 37.02905273 | 28.20971680 | 23.73413086 | 21.60327148 |
| 20.64916992 | 20.24731445 | 20.08837891 | 20.02905273 | 20.00952148 | 20.00219727 |
| 20.00000000 | 20.00000000 | 20.00000000 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|--------------|--------------|--------------|--------------|--------------|--------------|
| 601.07008836 | 455.33862305 | 336.76147461 | 243.46356201 | 172.60578918 | 120.73976135 |
| 84.19859314 | 59.44902039 | 43.34881592 | 33.29827881 | 27.28207397 | 23.83041342 |
| 21.93449402 | 20.93702698 | 20.43501282 | 20.19314575 | 20.08219910 | 20.03311157 |
| 20.01274109 | 20.00474548 | 20.00303650 | | | |

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TIME IN SECONDS .705447

FRONT SURFACE
820.0000000

BACK SURFACE
20.01562500

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|--------------|--------------|--------------|--------------|--------------|--------------|
| 820.0000000 | 672.27124023 | 536.35839844 | 415.24316406 | 311.39135742 | 226.25805664 |
| 159.88891602 | 110.84252930 | 76.50537109 | 53.70288086 | 39.30004883 | 30.62207031 |
| 25.62304688 | 22.86401367 | 21.40307617 | 20.66162109 | 20.29907227 | 20.13085938 |
| 20.05517578 | 20.02490234 | 20.01562500 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|--------------|--------------|--------------|--------------|--------------|--------------|
| 731.66186523 | 583.39404297 | 457.50317383 | 352.94433594 | 268.08398438 | 200.84373474 |
| 148.88979675 | 109.70841980 | 80.96261597 | 60.41841125 | 46.13006592 | 36.46318054 |
| 30.10462952 | 26.03887939 | 23.51322937 | 21.98948669 | 21.09774780 | 20.59326172 |
| 20.32151794 | 20.19039917 | 20.15176392 | | | |

TIME IN SECONDS .9981252

FRONT SURFACE
925.62231445

BACK SURFACE
20.25854492

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|--------------|--------------|--------------|--------------|--------------|--------------|
| 925.62231445 | 784.70214844 | 652.32104492 | 530.60986328 | 421.55859375 | 326.76611328 |
| 247.17163086 | 182.84936523 | 132.94091797 | 95.79077148 | 69.24682617 | 51.01342773 |
| 38.94702148 | 31.23779297 | 26.47460938 | 23.62622070 | 21.97729492 | 21.05395508 |
| 20.56298628 | 20.32714844 | 20.25854492 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|--------------|--------------|--------------|--------------|--------------|--------------|
| 848.21044922 | 696.37353516 | 565.90307617 | 454.12353516 | 359.96093780 | 282.01391602 |
| 218.64448547 | 168.07295227 | 128.47384644 | 98.06254578 | 75.16455078 | 58.26657104 |
| 46.04951477 | 37.39862061 | 31.40240479 | 27.33912659 | 24.63455627 | 22.93768311 |
| 21.89921570 | 21.34771729 | 21.17568970 | | | |

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44

TIME IN SECONDS 1.2449541

FRONT SURFACE
1011.34643555

BACK SURFACE
21.30419922

| | | | | | |
|--------------------------|--------------|--------------|--------------|--------------|--------------|
| TEMPERATURE ALONG Z-AXIS | | | | | |
| 1011.34643555 | 875.65283203 | 746.64135742 | 625.93310547 | 515.12182617 | 415.64184570 |
| 322.59887695 | 254.60688945 | 193.66650391 | 145.12255859 | 107.74902344 | 79.93212891 |
| 55.89379883 | 45.90429688 | 36.42260742 | 30.17968750 | 26.18798828 | 23.72265625 |
| 21.27929888 | 21.53271464 | 21.30419922 | | | |

| | | | | | |
|----------------------------------|--------------|--------------|--------------|--------------|--------------|
| ANALYTICAL SOLUTION ALONG Z-AXIS | | | | | |
| 942.73048875 | 791.87182617 | 658.40185547 | 541.83129883 | 441.37695313 | 356.00170898 |
| 284.47045898 | 225.41101074 | 177.37993188 | 138.89701843 | 108.55567932 | 85.00825500 |
| 67.05033447 | 53.53417969 | 43.58168030 | 36.38682556 | 31.31039429 | 27.89150146 |
| 25.63603210 | 24.40707397 | 24.01371765 | | | |

TIME IN SECONDS 1.4917831

FRONT SURFACE
1086.60424805

BACK SURFACE
24.01245117

| | | | | | |
|--------------------------|--------------|--------------|--------------|--------------|--------------|
| TEMPERATURE ALONG Z-AXIS | | | | | |
| 1086.60424805 | 955.26074219 | 829.37280273 | 710.21801758 | 599.09204102 | 497.22680664 |
| 405.69165039 | 325.27392578 | 256.37207031 | 198.91552734 | 152.35009766 | 115.69091797 |
| 87.65380859 | 66.80859375 | 51.73193359 | 41.12133789 | 33.86914063 | 29.09033203 |
| 26.12451172 | 24.51928711 | 24.01245117 | | | |

| | | | | | |
|----------------------------------|--------------|--------------|--------------|--------------|--------------|
| ANALYTICAL SOLUTION ALONG Z-AXIS | | | | | |
| 1030.08884766 | 878.45361328 | 742.75244141 | 622.59082031 | 517.36499023 | 426.26879884 |
| 348.32910156 | 282.44848833 | 227.44612122 | 182.10473633 | 145.20967102 | 115.58514404 |
| 92.12615967 | 73.81950378 | 59.76214600 | 49.16986084 | 41.38488770 | 35.87284851 |
| 32.22424316 | 30.14958191 | 29.47726440 | | | |

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45

TIME IN SECONDS 1.7493439

FRONT SURFACE
1156.86865234

BACK SURFACE
29.54296875

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|---------------|---------------|--------------|--------------|--------------|--------------|
| 1156.86865234 | 1029.38769531 | 906.46044922 | 789.12060547 | 678.42895508 | 575.43090820 |
| 481.09497070 | 396.23071289 | 321.40991211 | 256.88818359 | 202.55200195 | 157.91894531 |
| 122.17993164 | 94.28808594 | 73.07543945 | 57.36938477 | 46.08593750 | 38.29370117 |
| 33.25634766 | 30.44555664 | 29.54296875 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|---------------|--------------|--------------|--------------|--------------|--------------|
| 1113.79589044 | 961.56835937 | 824.05737305 | 700.96704102 | 581.81660352 | 495.96777344 |
| 412.63110352 | 340.91357422 | 279.84057617 | 228.39118958 | 185.52926636 | 150.23272705 |
| 121.52125549 | 98.47676086 | 80.26164246 | 66.13415527 | 55.45605469 | 47.70117186 |
| 42.45678711 | 39.42846680 | 38.43872070 | | | |

TIME IN SECONDS 1.9961729

FRONT SURFACE
1218.06225586

BACK SURFACE
38.14111328

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|---------------|---------------|--------------|--------------|--------------|--------------|
| 1218.06225586 | 1093.79028320 | 973.43701172 | 857.86254883 | 747.96118164 | 644.62866211 |
| 548.73120117 | 461.04028320 | 382.17993164 | 312.55859375 | 252.32128906 | 201.31542969 |
| 155.09350586 | 124.95654297 | 98.02758789 | 77.34228516 | 61.94262695 | 50.95410156 |
| 43.65087891 | 39.49096680 | 38.14111328 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|---------------|---------------|--------------|--------------|--------------|--------------|
| 1188.42285156 | 1035.72387695 | 896.81689453 | 771.45874023 | 659.25537109 | 559.67358398 |
| 472.06201172 | 395.66967773 | 329.67211914 | 273.19702148 | 225.34904480 | 185.23338318 |
| 151.98117065 | 124.76715088 | 102.82881165 | 85.48120117 | 72.12719727 | 62.26831055 |
| 55.51147461 | 51.57177734 | 50.27758789 | | | |

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TIME IN SECONDS 2.2430019

FRONT SURFACE
1274.43041992BACK SURFACE
50.40991211

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|---------------|---------------|---------------|--------------|--------------|--------------|
| 1274.43041992 | 1152.99023438 | 1034.98266602 | 921.13524391 | 812.21191406 | 708.99047852 |
| 612.23535156 | 522.66796875 | 440.90844727 | 367.44184453 | 302.55639648 | 246.32324219 |
| 198.56884766 | 155.98574219 | 126.67724609 | 101.22167969 | 81.74414063 | 67.49609375 |
| 57.82836914 | 52.23754883 | 50.40991211 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|---------------|---------------|--------------|--------------|--------------|--------------|
| 1258.57446289 | 1105.48559570 | 965.42089844 | 838.17744023 | 723.42456055 | 620.71313477 |
| 525.49145508 | 449.12158203 | 378.89770508 | 318.06787109 | 265.85400391 | 221.47293091 |
| 184.15600486 | 153.16758728 | 127.82128906 | 107.49467891 | 91.64184570 | 79.80322266 |
| 71.61254283 | 66.80566406 | 65.22119141 | | | |

TIME IN SECONDS 2.4898310

FRONT SURFACE
1326.81542969BACK SURFACE
66.54907227

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|---------------|---------------|---------------|--------------|--------------|--------------|
| 1326.81542969 | 1207.90551758 | 1092.04345703 | 979.85742188 | 872.00585937 | 769.17065430 |
| 672.03344727 | 581.25219727 | 497.42944336 | 421.07379047 | 355.56225586 | 292.10693359 |
| 235.73242188 | 195.26977539 | 158.36865234 | 128.54541016 | 105.23315430 | 87.84887695 |
| 75.86474609 | 66.85424805 | 66.54907227 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|---------------|---------------|---------------|--------------|--------------|--------------|
| 1324.98828125 | 1171.57031250 | 1030.52929688 | 901.69262695 | 784.77856445 | 679.40356445 |
| 585.09643555 | 501.30810547 | 427.43017578 | 362.81004859 | 306.76983125 | 258.62133789 |
| 217.68812561 | 183.31578064 | 154.89184570 | 131.85742188 | 113.71899414 | 100.05981445 |
| 90.54614258 | 84.93652344 | 83.08276367 | | | |

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TIME IN SECONDS 2.7473917

FRONT SURFACE
1377.90942383

BACK SURFACE
87.51660156

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|---------------|---------------|---------------|---------------|--------------|--------------|
| 1377.90942383 | 1261.38085938 | 1147.57592773 | 1037.04199219 | 930.35327148 | 828.11010742 |
| 730.94772461 | 639.37939453 | 554.06347656 | 475.48242188 | 404.08250000 | 340.11499023 |
| 283.81103016 | 235.16967773 | 194.06103516 | 160.22531836 | 133.31372070 | 112.93090820 |
| 96.69506836 | 90.29345703 | 87.51660156 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|---------------|---------------|---------------|--------------|--------------|--------------|
| 1390.91577148 | 1237.20581055 | 1095.29736328 | 965.04244047 | 840.20117188 | 739.44531250 |
| 641.37011719 | 554.50390625 | 477.32128906 | 409.25781250 | 343.72338867 | 294.11840820 |
| 253.84878540 | 216.33911133 | 185.04760742 | 159.47874977 | 133.19287109 | 123.81689453 |
| 113.05249023 | 106.58237305 | 104.57397461 | | | |

TIME IN SECONDS 2.9942207

FRONT SURFACE
1443.96826172

BACK SURFACE
111.40527344

TEMPERATURE ALONG Z-AXIS

| | | | | | |
|---------------|---------------|---------------|---------------|--------------|--------------|
| 1443.96826172 | 1309.50903320 | 1197.52455820 | 1088.49833789 | 984.93530273 | 881.37646444 |
| 784.36791992 | 692.46393750 | 606.19482422 | 526.07397461 | 454.05004883 | 385.99414063 |
| 324.67675781 | 274.75927734 | 230.28198242 | 193.17114258 | 163.26684570 | 140.35009766 |
| 124.19238261 | 114.59008789 | 111.40527344 | | | |

ANALYTICAL SOLUTION ALONG Z-AXIS

| | | | | | |
|---------------|---------------|---------------|---------------|--------------|--------------|
| 1451.33032227 | 1297.37488281 | 1154.75659180 | 1023.33325195 | 902.90087891 | 793.17602533 |
| 693.80639648 | 604.38248008 | 524.44506836 | 453.50170898 | 391.03588867 | 336.52001953 |
| 265.43139648 | 249.26220703 | 215.53198242 | 187.40078125 | 160.67797852 | 144.83056641 |
| 136.99218750 | 129.96826172 | 127.64013672 | | | |

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DREV, P.O. Box 880, Courcellette, Que. GOA IRO

"A One-Dimensional Numerical Model of Laser Heating of Target Slabs"
by J.P. Morency

This report establishes a numerical scheme to solve the nonlinear one-dimensional heat transfer equation. The scheme is then applied to the case of laser-metals interaction and used to show the importance of thermo-physical properties variation on the temperature distribution within a slab. (U)

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CRDV, C.P. 880, Courcellette, Qué. GOA 1R0

"Un modèle numérique unidimensionnel du chauffage d'une plaque métallique par laser" par J.P. Morency

Dans ce rapport on développe un schéma numérique de solution de l'équation unidimensionnelle non linéaire de conduction de la chaleur. On applique ensuite ce schéma à l'interaction laser-métaux et on montre l'importance de la variation des propriétés thermophysiques sur la distribution de la température dans une plaque métallique. (NC)

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