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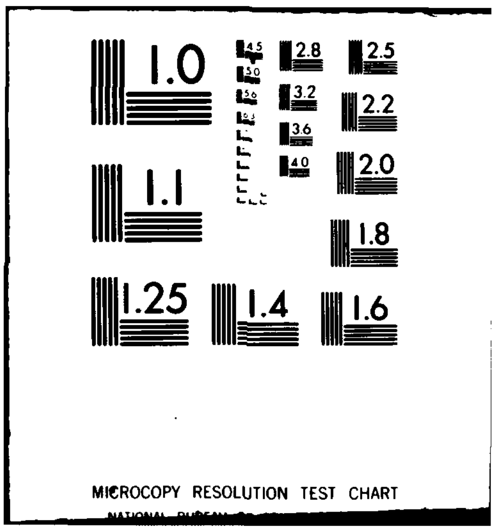
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A FAULT-DIAGNOSIS TECHNIQUE FOR CLOSED FLOW NETWORKS

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Abstract

Closed flow networks represent a mathematical model for transportation networks, multiple resource computer systems and computer communication networks. A fault-diagnosis technique for these networks is presented which can locate all single edge failures in the network. This technique is based on a flow causality relationship developed here. The number of edges that need to be monitored is shown to be (n-1) for an n-node network. These edges constitute the branches of a tree in the network.

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I. Introduction

A flow network[1] can be regarded as a mathematical model of a large class of traffic systems such as computer communication networks. Topologically, they form a network of interconnected branches and nodes.[3] A closed (open) network is a flow network without (with) external inputs/outputs. An open network can be restructured as a closed network with additional nodes(s). Closed networks can be used to model multiple resource computer systems, computer networks, etc.[2]

In this paper, a technique will be developed to locate (diagnose) faults in a closed flow network. The fault model used here assumes that a fault in a communication link results in changing the flow through the link. It will be further assumed that the law of conservation of flow holds at any time. Specifically, it is shown that by monitoring only flow through (n-1) edges for a n-node system one can determine the location of failures that result in the deviation of flow in other links.

This paper has two principal sections. In Section 2, a model is developed for the closed flow network and this forms the basis of the fault-diagnostic technique. Next, is Section 3, a flow causality is developed which describes the relationship between flows in different links when conservation of flow holds. Based on this, a fault-dignostic[4] algorithm is developed.

In this paper the links in a system are classified into two categories: <u>observable</u> links (o-link) and <u>unobservable</u> link (u-link). The flows in the o-links are monitored; by observing the flows through these links, information regarding failure in other links is derived. A pattern of flow deviations in the o-links is called diagnostic pattern. The correspondence between diagnostic patterns and various faults is established. This forms the basis of the fault-diagnostic technique.

II. Flow Model

A flow network can be represented in terms of a labeled, directed graph:

$S = \langle V, E, X(t) \rangle$ where:

$V = \{ v_i \mid i = 1, 2, \ldots n \}$ represents a set of n nodes which connects two or more links

$E = \{ e_j \mid j = 1, 2, \ldots, m \}$ represents a set of edges which connects the nodes in the graph.

$X(t) = \{ x_j(t) \mid j = 1, 2, \ldots m$ and $x_j(t)$ represents the flow through $e_j(t) \}$. This X(t) represents the set of labels for the edges.

A flow vector $\underline{X}(t)$ can be defined as:

$$\underline{X}(t) = \left[x_1(t), x_2(t), \ldots, x_m(t) \right]^t$$

where $[\ ]^t$ represents the transpose operator.

All multiple links between two nodes can be combined into a single link. Thus, S can be represented as a <u>simple</u> labeled graph.

Assuming that S is strongly connected, one has

$n \leq m \leq n^2 - n$.

Let $V_k = \{ v_{k1}, v_{k2}, \ldots, v_{kn} \}$

represent a subset of V.

Let $\bar{V}_k = V - V_k$.

<u>Definition</u>: Let $C(V_k)$ represent a <u>directed</u> <u>cutset</u> defined as:

$C(V_k) = \{ e_k \mid e_k$ is a directed edge from a node in V_k to $\bar{V}_k \}$.

Let the total number of directed cutsets be q. It can be readily seen that:

$$n \leq q \leq 2^n-2.$$

Let $a_{kj} = \begin{cases} 1 \text{ if } e_j \varepsilon C(V_k) \\ 0, \text{ otherwise} \end{cases}$

<u>Definition</u>: Given a directed cutset $C(V_k)$, let a_k represent a <u>directed</u> <u>cut</u> <u>vector</u>, defined as:

$$A_k = [a_{k1}, a_{k2}, \ldots, a_{km}]$$

Let $u_k(t)$ be the total amount of flow through the links contained in $C(V_k)$.

It can be readily seen that

$$u_k(t) = A_k X(t) \qquad k = 1, 2, \ldots, q. \tag{1}$$

The amount, $u_k(t)$ is referred to as the <u>cut</u> <u>flow</u>.

Using the conservation law of flow, one has

$$\frac{du_k(t)}{dt} = 0, \; k = 1, 2, \ldots, q. \tag{2}$$

III. <u>Flow Causality in Closed Networks</u>

We now develop some results to characterize the flow vector, $X(t)$, by using the fundamental equation described in (2).

From (1) and (2), one has:

$$a_k \frac{dx(t)}{dt} = 0 \quad k = 1, 2, \ldots, q \tag{3}$$

Let $\Delta X(t) = \left[\delta x_1(t), \; \delta x_2(t), \ldots, \; \delta x_m(t)\right]$, where $\delta x_i(t) = x_i(t+\Delta t)-x_i(t)$ represents a small change of flow in the i-th link between a small time interval, Δt.

Using the notation, one can rewrite (3) as:

$$A_k \cdot \Delta X(t)=0 \quad k = 1, 2, \ldots, q. \tag{4}$$

Now consider the solutions of the simultaneous equations described in (4).

Let S be a complete graph.# Thus $m = n^2 - n$ and $q = 2^n - 2$. But the total number of linearly independent equations in (4) can be shown to be equal to

$$s = \frac{(n-1)\ (n+1)}{2}$$

Thus there exists:

$$w = m - s = \frac{(n-1)\ (n-2)}{2} \qquad (5)$$

degrees of freedom for the system of simultaneous equations given in (4).

Let the set of independent simulataneous equation in (4) be:

$$A_k . \Delta x(t) = 0 \quad k = 1, 2, \ldots, s \qquad (6)$$

Let the solution of (6) be given as:

$$\Delta x(t)^* = \left[\delta \overset{*}{x}_1\ (t)\ \ \delta \overset{*}{x}_2\ (t), \ldots, \delta \overset{*}{x}_m\ (t)\right].$$

This solution in general can be expressed as a linear function of s independent variables given as:

$$\delta x_j^*(t) = \sum_{i=1}^{s} \psi_{ij} \delta x_i^*\ (t) \quad j = 1, 2, \ldots, m \qquad (7)$$

where $\psi_{ij} = 1, 0, -1$.

<u>Example 1</u>: Consider a closed network consisting of 3 nodes and 6 links, as shown in Fig. 1. The relationship described in (6) can be derived as:

#As will be seen later, the results of the paper apply to <u>incomplete</u> graphs, as well.

$$
\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \delta x_1(t) \\ \delta x_2(t) \\ \delta x_3(t) \\ \delta x_4(t) \\ \delta x_5(t) \\ \delta x_6(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

For n=3, the number of independent variables is w=1. Let $\delta x^*_3(t) = z$. The soultion, $\Delta x^*(z)$, can be expressed in terms of z as:

$$\Delta x^*(z) = [z, z, z, -z, -z, -z] \tag{8}$$

It may be observed from the above solution that any deviation in flow in one link affects all the other links according to (8). We will call this type of relationship a <u>flow causality</u> of the network.

This flow causality can be also described in terms of a node-to-node matrix shown below:

$$D_3(z) = \begin{bmatrix} 0 & z & -z \\ -z & 0 & z \\ z & -z & 0 \end{bmatrix}$$

The matrix $D_n(z)$, for any n-node graph, will be referred to as a flow <u>causality matrix</u>.

Let $Z = \left\{ z_i \quad z_i = \delta x^*_i(t),\ i \quad 1, 2, \ldots, w \right\}$ be the set of w independent variables in the solution of equation (6). The entries in the matrix, $D_n(z)$, can be expressed in terms of the variables in Z.

In the following, an algorithm is described to construct the matrix $D_n(z)$, for any general n. Let d_{ij} be the element in the i-th row j-th column of $D_n(z)$.

Algorithm to construct $D_n(z)$

A1: Generate an undirected graph, $\tilde{G} = (V, \tilde{E})$, by excluding the direction of each link in S.

A2: Choose a tree, $\tilde{T}$, from $\tilde{G}$ so as to form a path. Assign the number i to the i-th node in the path.

A3: Assign a parameter, z_k to each link, e_{ij}, in the cotree.

A4: Set $d_{ii} = 0$ for all i.

A5: For any d_{ij}, where $i \leq j-2$ and which has not been assigned in

A3: Assign $d_{ij} = 0$.

A6: Let $d_{k(k+1)}$ that value which satisfies the following:

$$\sum_{i=k+1}^{n} \sum_{j=1}^{k} d_{ij} = 0 \qquad k = 1, 2, .., n-1$$

A7: Set $d_{ij} = -d_{ji}$ for $i \geq j$.

It may be noted that the above algorithm is general and is not restricted to complete graphs only.

Example 2: Consider a closed network consisting of eight nodes and seventeen links below:

Generate an undirected graph corresponding to Figure 2 shown in Figure 3. Choose a tree (Gothic path shown in Figure 3) and assign the parameters z_k ($k = 1, 2, \ldots, 10$) to the links in the cotree. We then have the following matrix based on the above algorithm.

In the following, y variables correspond to the branches of The chosen tree and z variables correspond to the links.

$$D_8(Z)=\begin{bmatrix} 0 & -y_1 & z_1 & z_2 & 0 & z_3 & 0 & z_4 \\ y_1 & 0 & -y_2 & 0 & z_5 & 0 & z_6 & 0 \\ -z_1 & y_2 & 0 & -y_3 & 0 & 0 & z_8 & z_7 \\ -z_2 & 0 & y_3 & 0 & -y_4 & 0 & z_9 & 0 \\ 0 & -z_5 & 0 & y_4 & 0 & -y_5 & 0 & z_{10} \\ -z_3 & 0 & 0 & 0 & y_5 & 0 & -y_6 & 0 \\ 0 & -z_6 & -z_8 & -z_9 & 0 & y_6 & 0 & -y_7 \\ -z_4 & 0 & -z_7 & 0 & -z_{10} & 0 & y_7 & 0 \end{bmatrix}$$

The relationships between y and z variables are given below:

$$y_1 = z_1 + z_2 + z_3 + z_4$$

$$y_2 = z_1 + z_2 + z_3 + z_4 + z_5 + z_6$$

$$y_3 = z_2 + z_3 + z_4 + z_5 + z_6 + z + z_8$$

$$y_4 = z_3 + z_4 + z_5 + z_6 + z_7 + z_8 + z_9$$

$$y_5 = z_3 + z_4 + z_6 + z_7 + z_8 + z_9 + z_{10}$$

$$y_6 = z_4 + z_6 + z_7 + z_8 + z_9 + z_{10}$$

$$y_7 = z_4 + z_7 + z_{10}$$

IV. Fault diagnosis

It is assumed here that a fault in the network results in an abnormal change of flow through one or more edges. If all the edges are monitored then detection of such change is a trivial problem. However monitoring all the edges may be impractical since there are $O(n^2)$ possible edges for an n-node network. In this section, we provide a solution to this problem. A technique is presented to locate faulty edges by observing the flow on a small subset of edges. The number of edges to be monitored need not exceed (n-1) for an n-node graph. Furthermore, these edges constitute the branches of a tree and therefore monitoring these edges may be easy to implement.

First we present an example to provide motivation for the result presented later in the section.

Example 3: Consider the network shown in Figure 3. The following table will be hereafter referred to as a flow causality table and can be derived from the D matix shown in Example 2. This table describes a relationship between the flows in different edges.

Table 1 Flow causality table for Example 2

Link	e_{13}	e_{14}	e_{16}	e_{81}	e_{25}	e_{27}	e_{38}	e_{37}	e_{47}	e_{58}
e_{12}	*	*	*	*						
e_{23}	*	*	*	*	*	*				
e_{34}		*	*	*	*	*	*	*		
e_{45}			*	*	*	*	*	*	*	
e_{56}			*	*		*	*	*	*	*
e_{67}				*		*	*	*	*	*
e_{78}				*			*			*

This table provides complete information regarding how a flow deviation in one edge results in compensating changes in other edges. For example, change in flow e_{13} results in compensating changes in e_{12} and e_{23}.

By monitoring the flows through the branches of the tree, one can obtain information regarding any changes of flow in other edges. In fact, there exists a unique correspondence between the change in flow in any edge and the change of flow through the branches of the tree. This fact can be used to design a syndrome table; this table is shown below. Here, a 1(0) represents a change (no change) in the flow. It may be seen from the different syndrome patterns that a faulty link which produces a change of flow through itself will result in a change of flow in a combination of branches of the tree. This combination is distinct for two different edges. Thus, a faulty edge can be located by using the syndrome table.

e_{12}	e_{23}	e_{34}	e_{45}	e_{56}	e_{67}	e_{78}	Faulty edge
1	0	0	0	0	0	0	e_{12}
0	1	0	0	0	0	0	e_{23}
0	0	1	0	0	0	0	e_{34}
0	0	0	1	0	0	0	e_{45}
0	0	0	0	1	0	0	e_{56}
0	0	0	0	0	1	0	e_{67}
0	0	0	0	0	0	1	e_{78}
1	1	0	0	0	0	0	e_{13}
1	1	1	0	0	0	0	e_{14}
1	1	1	1	1	0	0	e_{16}
1	1	1	1	1	1	1	e_{81}
0	1	1	1	0	0	0	e_{25}
0	1	1	1	1	1	0	e_{27}
0	0	1	1	1	1	1	e_{38}
0	0	1	1	1	1	0	e_{37}
0	0	0	1	1	1	0	e_{47}
0	0	0	0	1	1	1	e_{58}

Table 2: Syndrome Table

In the following, we prove the general result:

Theorem 1: A failure in any edge in a closed flow network S, can be diagnosed (located) by observing the flows through the branches of any tree $\tilde{T}$ in S.

Proof: Let the branches of a tree, T, correspond to o-links and the remaining edges correspond to u-links.

An elementary cycle can be constructed by using only one o-link and two or more u-links. These sets of elementary cycles form a linearly independent set.

Suppose a failure arises in the o-link i in S. Let i be contained in the elementary cycle L_i. There are no other o-links included in L_i. Thus the symptom of the failure will appear in o-link i only and hence is diagnosed.

On the other hand consider a failure that arises in u-link i. Let i be included in the elementary cycles $L_{i1}, L_{i2}, \ldots, L_{ir}$. It may be seen that : (a) $k \geq 2$; (b) the failure in i will result in a change of flow in exactly k, o-links contained in $L_{i1}, L_{i2}, \ldots, L_{ik}$ and (c) the combination $L_{i1}, L_{i2}, \ldots, L_{ik}$ is unique for each link i. Hence this failure is also diagnosable. Q.E.D.

Corollary: The observable points in closed network S for single failure diagnosis need not exceed (n-1).

It may be seen that certain multiple failures may also be diagnosable. For example a double failure can be diagnosed if they occur in two disjoint elementary cycles.

V. Conclusion:

This paper presents a fault-diagnosis technique for a closed flow network. A flow causality is derived from the topology of a flow network. Based on this, a technique is described that locates all single edge failures by observing flow through (n-1) edges. These edges constitute the branches of a tree of an n-node network.

Other potential applications of the results of the paper to other areas such as system fault-diagnosis,[4] distributed computing need to be investigated.

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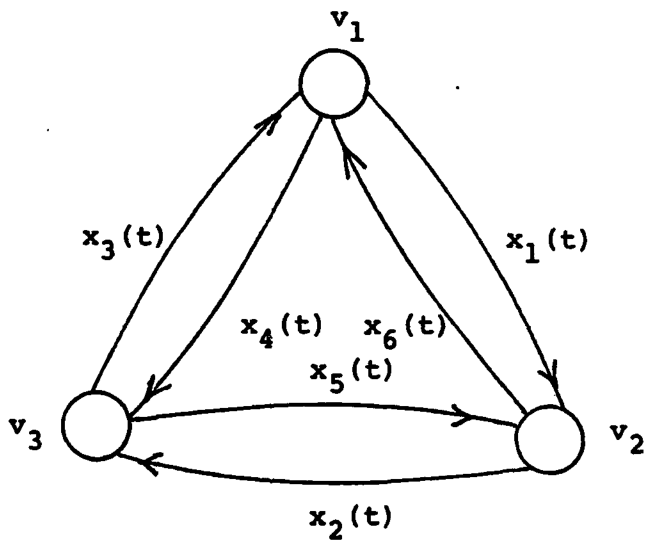

Figure 1. **A closed** network **for Example 1**

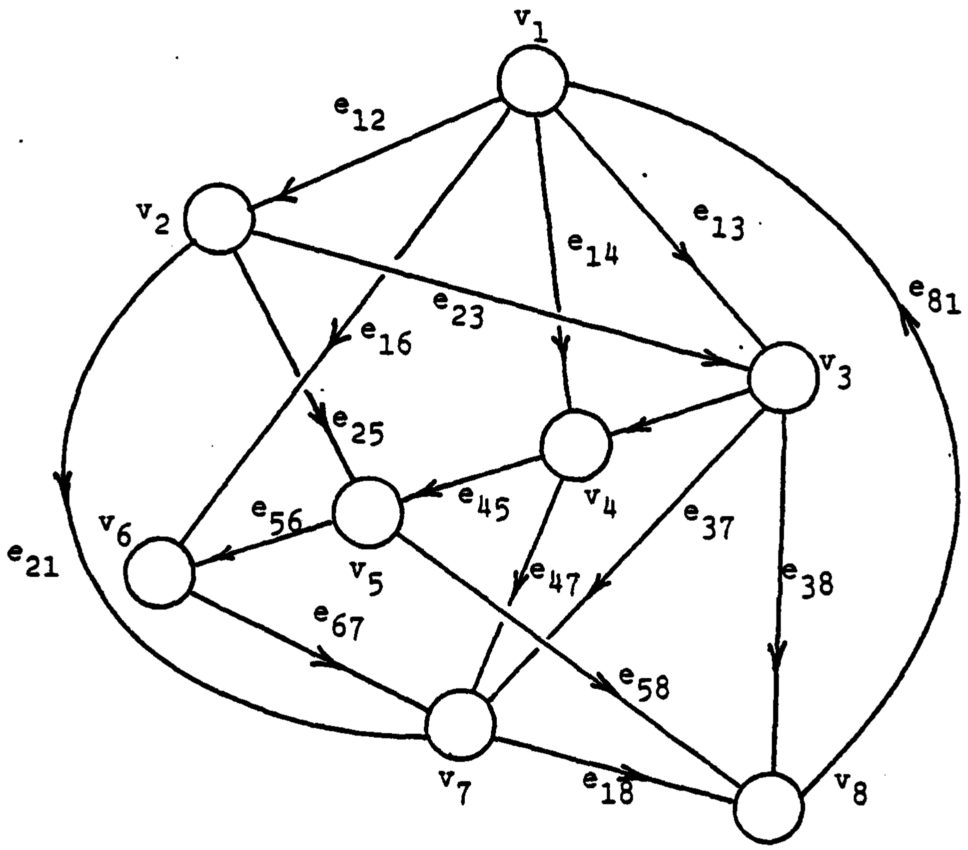

Figure 2. A closed network for Example 2

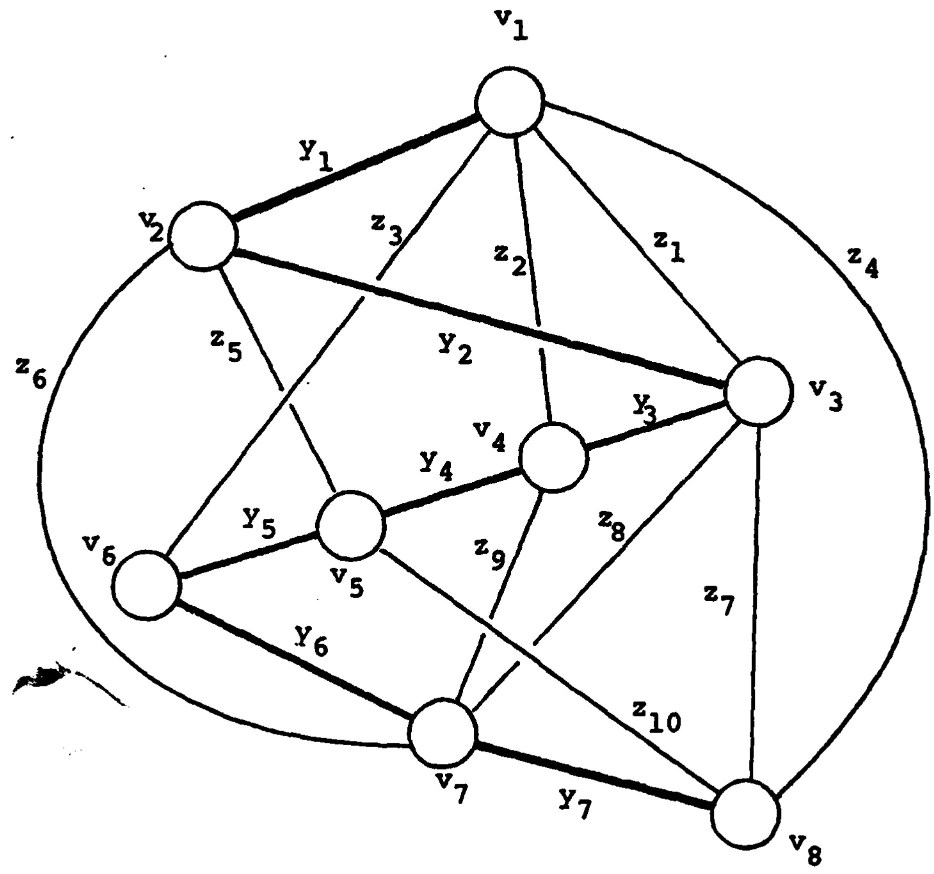

Figure 3. Associated graph G