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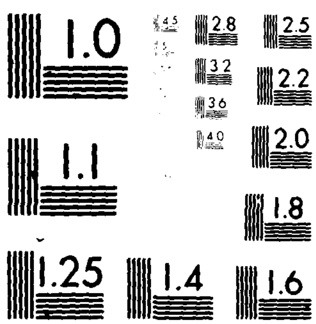

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ENERGY CRITERIA FOR FINITE HYPERELASTICITY

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ABSTRACT

The equations of hyperelasticity have the special feature that their natural entropy is not a globally convex function. Strict convexity of the entropy function is essential in formulating a physically reasonable entropy criterion for shock waves. In this paper we show that the natural entropy of the equations of hyperelasticity is uniformly convex when restricted to the shock curves. This fact enables us to prove the equivalence of the entropy criterion and Lax's shock conditions for existence of weak shocks for problems that are genuinely nonlinear. Furthermore, for problems that are not necessarily genuinely nonlinear we study the (generalized) "E-condition" and show that it is indeed a generalization of the entropy condition. Finally, we consider the viscosity criterion which requires that a motion of a hyper-elastic body is the limit of smooth motions of a family of viscoelastic materials. The relationship between the energy criterion, the E-condition, and the viscosity criterion is then discussed.

AMS(MOS) Subject Classifications: 73C50, 35L65, 35L67

Key Words: elasticity, entropy function, shock waves

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SIGNIFICANCE AND EXPLANATION

The equations of hyperelasticity have many features in common with the equations of gas dynamics. A fundamental property of these equations is that one expects that solutions to the initial value problem develop singularities in derivatives in finite time, that is shock waves develop. Because of this one broadens the definition of solutions and considers generalized solutions. A mathematical problem then arises: Are such solutions unique? The purpose of this paper is to formulate a so-called entropy criterion in order to select a physically reasonable generalized solution. Several such criteria have already been proposed in the theory of nonlinear hyperbolic conservation laws. We study these criteria by combining the rich structure of the equations of elasticity with some known results from the general theory.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

ENERGY CRITERIA FOR FINITE HYPERELASTICITY

Reza Malek-Madani

1. Introduction. Recent studies in continuum mechanics have pointed out special and physically relevant restrictions on the desirable properties of the equations of hyperelasticity. On the other hand, viewed as a system of conservation laws, the dynamic equations exhibit nonuniqueness of solutions of the initial value problem. In order to seek out physically meaningful solutions of these conservations laws additional restrictions, known as entropy criteria, are imposed. The purpose of this work is to study the interrelationship between several proposed entropy criteria in the context of the theory of continuum mechanics.

One of the distinguishing features of material response in more than one space dimension is that the internal energy cannot be a globally convex function of the deformation gradient, without violating the principle of material frame indifference [1, §52]. Yet, as it will become apparent in Section 2, the internal energy is a natural candidate in formulating an entropy criterion for the equations of hyperelasticity. A fundamental result of Lax [2] establishes the equivalence of this entropy criterion and Lax's shock conditions in the context of general hyperbolic systems of conservation laws with a strictly convex entropy. In section 3 this result is proved by observing that the entropy is locally convex along shock curves.

Another feature of the equations of elasticity is that the assumption of "genuine nonlinearity" is not generally satisfied. The entropy criterion is then known to be insufficient to single out a unique solution. Oleinik [3], Leibovich [4], and Liu [5] [6] introduced a strengthened version of Lax's shock conditions, the E-condition, to deal with such problems. Dafermos [7], motivated by the physics of the problem, proposed the entropy rate admissibility criterion, as a generalization of the entropy criterion in order to study nonlinear problems which fail to be genuinely nonlinear. In section 4 it is shown that for the equations of elasticity

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the E-condition is indeed a generalization of the entropy criterion. Finally, section 5 is concerned with the viscosity criterion which requires that a motion of a hyperelastic body is the limit of smooth motions of a family of viscoelastic materials. The relationship between the energy criterion, the E-condition, and the viscosity criterion is then discussed.

2. Preliminaries and notations. Consider a body with reference configuration $\mathcal{B} \subset R^n$, $n = 1, 2,$ or 3. For simplicity assume that the reference configuration is uniform, with uniform density $\rho_0(\underset{\sim}{X}) \equiv 1$, and consider a motion $\underset{\sim}{x}(t,\underset{\sim}{X})$ of $\mathcal{B}$ in R^n. In the absence of external body force the motion satisfies the field equations

$$\ddot{x}_i = T_{i\alpha,\alpha} \qquad i = 1, 2, \text{ or } 3 \tag{2.1}$$

where $,\alpha$ denotes differentiation with respect to X_α, $\underset{\sim}{T}$ is the Piola-Kirchhoff stress tensor [1], and the usual summation convention is used. A material is called elastic if $\underset{\sim}{T} = \underset{\sim}{T}(\underset{\sim}{F})$, where $\underset{\sim}{F}$ is the deformation gradient

$$\underset{\sim}{F} = \frac{\partial \underset{\sim}{x}}{\partial \underset{\sim}{X}} \ , \quad F_{i\alpha} = \frac{\partial x_i}{\partial X_\alpha} \ .$$

Throughout this paper it is assumed that the material is hyperelastic; thus there exists a stored energy function $\sigma = \sigma(\underset{\sim}{F})$ such that

$$T_{i\alpha}(\underset{\sim}{F}) = \frac{\partial \sigma(\underset{\sim}{F})}{\partial F_{i\alpha}} \ . \tag{2.2}$$

The system of equations (2.1) is an example of a system of conservation laws in several space dimensions. This can be seen by letting $v_i = \dot{x}_i$ and observing that (2.1) is equivalent to

$$\begin{aligned} \dot{v}_i - T_{i\alpha,\alpha} &= 0 \qquad i = 1, 2, \text{ or } 3 \\ \dot{F}_{i\alpha} - v_{i,\alpha} &= 0 \qquad \alpha = 1, 2, \text{ or } 3 \end{aligned} \tag{2.3}$$

where a dot denotes differentiation with respect to time; (2.4) is hyperbolic at $\underset{\sim}{F}$ if σ satisfies the strong ellipticity condition:

$$N_\alpha N_\beta \nu_i \nu_j \frac{\partial^2 \sigma(\underset{\sim}{F})}{\partial F_{i\alpha} \partial F_{j\beta}} > 0 \tag{2.4}$$

for arbitrary unit vectors $\underset{\sim}{N}$ and $\underset{\sim}{\nu}$.

Local existence and well posedness of the equations of elastodynamics were obtained by Hughes, Kato, and Marsden [8]. However, the hyperbolic character of (2.3), (2.4) prevents the existence of global smooth solutions for the Cauchy problem and shock waves develop. From experience gained in studying general nonlinear conservation laws (Conway and Smoller [9], Kruzkov [10], DiPerna [11], Glimm [12]) a natural class of solutions of (2.3) (2.4) is the class of functions of bounded variations in the sense of Tonelli-Cesari (cf. Volpert [13]). Typical functions in this class are piecewise smooth and their discontinuities consist of a family of smooth surfaces with simple jump discontinuities. Such functions serve as good models for the mathematical representation of shock waves.

A piecewise smooth pair $(\underset{\sim}{v}(\underset{\sim}{X}, t), \underset{\sim}{F}(\underset{\sim}{X}, t))$ is a weak solution of (2.3), (2.4) if it is a classical solution at points of smoothness and if the Rankine-Hugoniot conditions

$$\begin{aligned} -s[v_i] &= N_\alpha [T_{i\alpha}] \qquad i = 1, 2, \text{ or } 3 \\ -s[F_{i\alpha}] &= N_\alpha [v_i] \qquad \alpha = 1, 2, \text{ or } 3 \end{aligned} \tag{2.5}$$

are satisfied across each shock $\chi = \chi(\underset{\sim}{X}, t)$, where $s = \dot{\chi}$ is the speed of the propagation of the shock, $[u] = u^+ - u^-$ denotes the jump across the shock χ, and $\underset{\sim}{N}$ is a unit normal to the shock at $(\underset{\sim}{X}, t)$ in the direction of propagation.

$S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ denotes the set of states $(\underset{\sim}{v}^1, \underset{\sim}{F}^1)$ which can be connected to $(\underset{\sim}{v}^-, \underset{\sim}{F}^-)$ by a shock with normal $\underset{\sim}{N}$. It is also assumed that the symmetric matrix E

$$E_{ij}(\underset{\sim}{F}) = N_\alpha N_\beta \frac{\partial^2 \sigma(\underset{\sim}{F})}{\partial F_{i\alpha} \partial F_{j\beta}} \tag{2.6}$$

has a simple positive eigenvalue $\lambda(\underset{\sim}{F})$. This assumption is weaker than strong ellipticity and is sufficient for further discussions in this paper. Dafermos [14] proved the following proposition in connection with the local existence of the shock curve $S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ under the additional hypothesis of genuine nonlinearity of σ at $\underset{\sim}{F}^-$, that is,

$$N_\alpha N_\beta N_\gamma \frac{\partial^3 \sigma(\underset{\sim}{F}^-)}{\partial F_{i\alpha} \partial F_{j\beta} \partial F_{k\gamma}} r_i r_j r_k \neq 0 \tag{2.7}$$

where $\underset{\sim}{r} = \underset{\sim}{r}(\underset{\sim}{F}^-)$ is the eigenvector of E associated with λ.

Proposition 2.1. Let λ be a simple eigenvalue of E and assume that σ is genuinely nonlinear at $\underset{\sim}{F}^-$. Then there exists two smooth maps $s_i : (-\varepsilon, \varepsilon) \to R$ and $(\underset{\sim}{v}_i, \underset{\sim}{F}_i) : (-\varepsilon, \varepsilon) \to R^{n^2+n}$ which satisfy (2.5) with $s_i^2(0) = \lambda$ and $(\underset{\sim}{v}_i(0), \underset{\sim}{F}_i(0)) = (\underset{\sim}{v}^-, \underset{\sim}{F}^-)$.

We remark that a weaker hypothesis of

$$N_{\alpha_1} N_{\alpha_2} \dots N_{\alpha_\ell} \frac{\partial^\ell \sigma(\underset{\sim}{F}^-)}{\partial F_{i_1\alpha_1} \partial F_{i_2\alpha_2} \dots \partial F_{i_\ell\alpha_\ell}} r_{i_1} r_{i_2} \dots r_{i_\ell} \neq 0$$

for some $\ell > 2$ is sufficient for the proof of Proposition 2.1 [14].

Smooth solutions of (2.1) also satisfy the equations of conservation of mechanical energy

$$W_t + \operatorname{div} \underset{\sim}{\phi} = 0 \tag{2.8}$$

where

$$W(\underset{\sim}{v}; \underset{\sim}{F}) = \frac{1}{2} v_i v_i + \sigma(\underset{\sim}{F}) \tag{2.9}$$

is the mechanical energy density and

$$\phi_\alpha(\underset{\sim}{v}, \underset{\sim}{F}) = -v_i T_{i\alpha}(\underset{\sim}{F}) \tag{2.10}$$

is the flux associated with W. A weak solution of (2.1), however, does not necessarily satisfy (2.8). The energy criterion requires that an admissible solution satisfy

$$W_t + \operatorname{Div} \underset{\sim}{\phi} \le 0 \tag{2.11}$$

in the sense of distribution. (2.9), (2.11) are special cases of the entropy function and the entropy criterion defined in Lax [15]. For piecewise smooth weak solutions of (2.1), it is well known [14] that (2.11) is equivalent to

$$s[W(\underset{\sim}{v}, \underset{\sim}{F})] - N_\alpha[\phi_\alpha(\underset{\sim}{v}, \underset{\sim}{F})] \ge 0 \tag{2.12}$$

with $(\underset{\sim}{v}^+, \underset{\sim}{F}^+) \in S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$. It should be noted that (2.11) is the classical Clausius-Duhem inequality of thermodynamics [16], specialized to isothermal materials.

3. <u>The energy criterion and Lax's shock conditions</u>. There are several results pointing to a relationship between the convexity of the entropy function and the entropy criterion [2], [11]. However, due to the geometry of hyperelastic materials, the energy function (2.9) cannot be globally convex. Nevertheless, due to the special structure of the equations of hyperelasticity, we will show in this section that (2.9) is locally convex along the shock curves $S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$. This fact enables us to prove the equivalence of the entropy criterion (2.12) with Lax's shock conditions defined in (3.1).

Assume that conditions of Proposition 2.1 hold. Let $(\underset{\sim}{v}(\varepsilon), \underset{\sim}{F}(\varepsilon)) \in S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$. For ε near zero $E(\underset{\sim}{F}(\varepsilon))$ has a simple eigenvalue $\lambda(\varepsilon)$ near $\lambda(0)$. A shock $(\underset{\sim}{v}^+, \underset{\sim}{F}^+; \underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ satisfies <u>Lax's shock conditions</u> [15] if

$$\begin{aligned} &\lambda(\varepsilon) < s^2(\varepsilon) < \lambda(0) \quad \text{if} \quad s(\varepsilon) > 0 \\ &\lambda(\varepsilon) > s^2(\varepsilon) > \lambda(0) \quad \text{if} \quad s(\varepsilon) < 0 \ . \end{aligned} \tag{3.1}$$

One of the interpretations of (3.1) is that the linearization of (2.1) on either side of the shock is stable.

<u>Proposition</u> 3.1. <u>Let</u> $(\underset{\sim}{v}^+, \underset{\sim}{F}^+) \in S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ <u>and assume</u> (2.12) <u>holds</u>. <u>Then</u>

$$s\{[W(\underset{\sim}{v}, \underset{\sim}{F})] - \frac{\partial W}{\partial v_i}(\underset{\sim}{v}^-, \underset{\sim}{F}^-)[v_i] - \frac{\partial W}{\partial F_{i\alpha}}(\underset{\sim}{v}^-, \underset{\sim}{F}^-)[F_{i\alpha}]\} \geq s[v_i][v_i] \tag{3.2}$$

$$s\{[W(\underset{\sim}{v}, \underset{\sim}{F})] - \frac{\partial W}{\partial v_i}(\underset{\sim}{v}^+, \underset{\sim}{F}^+)[v_i] - \frac{\partial W}{\partial F_{i\alpha}}(v^+, F^+)[F_{i\alpha}]\} \geq -s[v_i][v_i] \tag{3.3}$$

<u>Proof</u>. We prove (3.2). The proof of (3.3) is similar. From (2.5) we have

$$[v_i][v_i] = [T_{i\alpha}][F_{i\alpha}] \ . \tag{3.4}$$

It follows from (2.5) and (3.4) that

$$\begin{aligned} s[W] - N_\alpha[\phi_\alpha] &= s[\sigma] - \frac{s}{2}[v_i][v_i] - s[F_{i\alpha}]T_{i\alpha}^- \\ &= s[\sigma] - \frac{s}{2}[F_{i\alpha}](T_{i\alpha}^+ + T_{i\alpha}^-) \ . \end{aligned} \tag{3.5}$$

On account of (2.12) and (3.5)

$$s[\sigma(\underset{\sim}{F})] \geq \frac{s}{2}[F_{i\alpha}](T_{i\alpha}{}^{+} + T_{i\alpha}{}^{-}) \quad ;$$

this inequality combined with (3.4) yields

$$s\{[W] - \frac{\partial W}{\partial \sigma_i}(\underset{\sim}{v}^{-}, \underset{\sim}{F}^{-})[v_i] - \frac{\partial W}{\partial F_{i\alpha}}(\underset{\sim}{v}^{-}, \underset{\sim}{F}^{-})[F_{i\alpha}]\} \geq s\{\frac{1}{2}[v_i\ v_i] + \frac{1}{2}[F_{i\alpha}](T_{i\alpha}{}^{+} + T_{i\alpha}{}^{-})$$

$$- v_i{}^{-}[v_i] - T_{i\alpha}{}^{-}[F_{i\alpha}]\}$$

$$= s[v_i][v_i] \quad .$$

This completes the proof of Proposition 3.1.

Corollary 3.1. The function $W(\underset{\sim}{v}, \underset{\sim}{F})$ defined by (2.9) is uniformly convex on $S(\underset{\sim}{v}^{-}, \underset{\sim}{F}^{-}; \underset{\sim}{N})$ in a neighborhood of $(\underset{\sim}{v}^{-}, \underset{\sim}{F}^{-})$.

Proof. The proof follows immediately from (3.2) and (3.3) by considering shocks with positive and negative speeds.

Theorem 3.1. Assume the genuine nonlinearity condition (2.7) holds and that ε is near 0. Then the energy criterion (2.11) is equivalent to Lax's shock conditions (3.1).

Proof. Let $(\underset{\sim}{v}(\varepsilon), \underset{\sim}{F}(\varepsilon)) \in S(\underset{\sim}{v}^{-}, \underset{\sim}{F}^{-}; \underset{\sim}{N})$ with $(\underset{\sim}{v}(0), \underset{\sim}{F}(0)) = (\underset{\sim}{v}^{-}, \underset{\sim}{F}^{-})$. We follow an argument of Lax [2]. Differentiating (2.7) with respect to ε and evaluating at $\varepsilon = 0$ we have

$$s^2(0)v_i'(0) = E_{ik}(\underset{\sim}{F}^{-})v_k' \quad , \quad -sF_{i\alpha}'(0) = N_\alpha\ v_i'(0) \quad , \tag{3.6}$$

thus $s^2(0) = \lambda(\underset{\sim}{F}^{-})$, $v'(0) = \underset{\sim}{\gamma}(\underset{\sim}{F}^{-})$. Without loss of generality assume that $s(0)$ is positive. Differentiating (2.5) twice and using (3.6) we deduce

$$\begin{aligned} 4s^2\ s'\ v_i' + s^3\ v_i'' &= -N_\alpha\ N_\beta\ N_\gamma \frac{\partial^2\ T_{i\alpha}(\underset{\sim}{F}^{-})}{\partial F_{k\beta}\ \partial F_{p\gamma}} v_k'\ v_p' + sE_{ik}(\underset{\sim}{F}^{-})v_k'' \\ - 2s'\ F_{i\alpha}' - sF_{i\alpha}'' &= N_\alpha\ v_i'' \quad . \end{aligned} \tag{3.7}$$

Since E is symmetric $\underset{\sim}{v}'(0)$ is also a left eigenvector for λ. It follows from (3.7) that

$$4s^2\, s'\, v_i'\, v_i' = -N_\alpha\, N_\beta\, N_\gamma \frac{\partial^2\, T_{i\alpha}}{\partial F_{k\beta}\, \partial F_{p\gamma}}\, v_k'\, v_p'\, v_i' \quad . \tag{3.8}$$

Thus $s'(0)$ is nonzero by the genuine nonlinearity assumption.

Differentiating

$$E_{ik}(\underset{\sim}{F}(\varepsilon))r_k(\varepsilon) = \lambda(\varepsilon)r_i(\varepsilon)$$

and comparing the result with (3.8) yields

$$4ss' = \lambda' \quad .$$

We normalize (3.8) so that $s(0)s'(0)$ is positive, or equivalently $s'(0)$ is positive. In that case (3.1) is satisfied if and only if ε is negative. Let

$$h(\varepsilon) = s(\varepsilon)[W(\underset{\sim}{v}, \underset{\sim}{F})] - N_\alpha[\Phi_\alpha(\underset{\sim}{v}, \underset{\sim}{F})] \quad .$$

then *on account of* (2.10) we have

$$h'(\varepsilon) = s'[W] + sv_i\, v_i + sT_{i\alpha}\, F_{i\alpha}' + N_\alpha\, T_{i\alpha}\, v_i' + N_\alpha\, v_i \frac{\partial T_{i\alpha}}{\partial F_{j\beta}} F_{j\beta}' \quad .$$

Since

$$-s'[v_i] - sv_i' = N_\alpha \frac{\partial T_{i\alpha}}{\partial F_{j\beta}} F_{j\beta}'$$

$$-s'[F_{i\alpha}] - sF_{i\alpha}' = N_\alpha[v_i]$$

$h'(\varepsilon)$ reduces to

$$h'(\varepsilon) = s'\{[W] - \frac{\partial W(\varepsilon)}{\partial F_{i\alpha}} [F_{i\alpha}] - \frac{\partial W}{\partial v_i} [v_i]\} \quad .$$

By corollary 3.1 the term in the brackets is negative for ε near 0. Thus, the energy criterion is satisfied if and only if ε is negative. This completes the proof.

4. The E-condition and the energy criterion. Although the genuine nonlinearity assumption (2.7) is reasonable for a local analysis, it turns out that for many problems (2.7) does not hold globally (In Section 5 we will give an example of a constitutive relation for which (2.7) fails). In the works of Oleinik [3], Wendroff [17], Leibovich [4], and Liu [5] a generalization of Lax's shock conditions (3.1), called the (generalized) E-condition, has been introduced in order to study the solutions of such problems. In this section we outline Liu's abstraction of the E-condition and show that it is a generalization of the energy criterion (2.11).

A shock $(s; \underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{v}^+, \underset{\sim}{F}^+; \underset{\sim}{N})$ is said to satisfy the (generalized) E-condition if for all $(\underset{\sim}{v}, \underset{\sim}{F}) \in S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ between $(\underset{\sim}{v}^-, \underset{\sim}{F}^-)$ and $(\underset{\sim}{v}^+, \underset{\sim}{F}^+)$

$$s(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{v}^+, \underset{\sim}{F}^+; N) \le s(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{v}, \underset{\sim}{F}; \underset{\sim}{N}) \quad . \tag{4.1}$$

As we will show below, a shock that satisfies the energy criterion (2.11) also satisfies the E-condition "on the average." To be more precise, the line integral of (4.1) along $S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ turns out to be (2.12).

Theorem 4.1. Let $(\underset{\sim}{v}^+, \underset{\sim}{F}^+) \in S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; N)$ be such that (4.1) holds. Then

$$s[W] - N_\alpha[\phi_\alpha] \ge 0 \quad .$$

Proof. Let $(s(\tau), \underset{\sim}{v}(\tau), \underset{\sim}{F}(\tau))$ be a parametrization of $S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$, $\tau \in [\tau_1, \tau_2]$ such that

$$\begin{aligned} \underset{\sim}{v}(\tau_1) &= \underset{\sim}{v}^- & \underset{\sim}{v}(\tau_2) &= \underset{\sim}{v}^+ \\ \underset{\sim}{F}(\tau_1) &= \underset{\sim}{F}^- & \underset{\sim}{F}(\tau_2) &= \underset{\sim}{F}^+ \quad . \end{aligned} \tag{4.2}$$

Without loss of generality assume $s(\tau_1) > 0$. On account of (2.5) we have

$$s^2(\tau)(F_{i\beta}(\tau) - F_{i\beta}^-)\dot{F}_{i\beta} = N_\alpha N_\beta(T_{i\alpha}(F(\tau)) - T_{i\alpha}(F^-))\dot{F}_{i\beta}$$

where a dot denotes differentiation with respect to τ. Since by the definition of parametrization the coefficient of $s^2(\tau)$ is positive, we can use (4.1) to obtain

$$\int_{\tau_1}^{\tau_2} N_\alpha N_\beta T_{i\alpha}(\underset{\sim}{F}(\tau))\dot{F}_{i\beta}\, d\tau - \int_{\tau_1}^{\tau_2} N_\alpha N_\beta T_{i\alpha}(\underset{\sim}{F}^-)\dot{F}_{i\beta}\, d\tau$$

$$\geq s^2(\tau_2) \int_{\tau_1}^{\tau_2} (F_{i\beta}(\tau) - F_{i\beta}^-)\dot{F}_{i\beta}\, d\tau \quad . \tag{4.3}$$

Since we perform a line integral along $S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ we note that if different parametrications are needed for separate segments of S, we repeat the remainder of the proof for each segment and sum the resulting line integrals. On account of (2.5)

$$\begin{aligned} N_\alpha N_\beta [F_{i\alpha}] &= [F_{i\beta}] \\ N_\alpha N_\beta \dot{F}_{i\alpha} &= \dot{F}_{i\beta} \quad . \end{aligned} \tag{4.4}$$

Thus (4.3) reduces to

$$\int_{\tau_1}^{\tau_2} T_{i\alpha}(\underset{\sim}{F}(\tau))\dot{F}_{i\alpha}\, d\tau - T_{i\alpha}(\underset{\sim}{F}^-)[F_{i\alpha}] \geq \frac{1}{2} s^2(\tau_2)[F_{i\alpha} F_{i\alpha}] - s^2(\tau_2)F_{i\alpha}^-[F_{i\alpha}] \quad . \tag{4.5}$$

Since by (2.5)

$$s^2(\tau_2)[F_{i\alpha}] = N_\alpha N_\beta [T_{i\beta}(\underset{\sim}{F})]$$

(4.5) becomes

$$\int_{\tau_1}^{\tau_2} T_{i\alpha} \dot{F}_{i\alpha}\, d\tau - \frac{1}{2}(T_{i\alpha}^+ + T_{i\alpha}^-)[F_{i\alpha}] \geq 0 \quad . \tag{4.6}$$

Finally, we deduce from the definition of a hyperelastic material that the integral term in (4.6) is $[\sigma(\underset{\sim}{F})]$ and that (4.6) is equivalent to the energy criterion (see (3.5)).

5. <u>The viscosity criterion</u>. This criterion views an admissible solution of (2.1) as the limit of smooth solutions of the equations of a family of viscoelastic materials defined in (5.1) below. The perturbed equations generally arise by introducing an artificial viscosity into the problem. In conjunction with (2.1) consider a one-parameter family of linearly viscous materials [1] with the constitutive relation

$$\underset{\sim}{t}_\varepsilon(\underset{\sim}{F}, \dot{\underset{\sim}{F}}) = \underset{\sim}{t}_1(\underset{\sim}{F}) + \varepsilon \underset{\sim}{t}_2(\underset{\sim}{D}) \tag{5.1}$$

where $\underset{\sim}{t}_\varepsilon$ is the Cauchy stress and $\underset{\sim}{D}$ is the stretching tensor, i.e., the symmetric part of the velocity gradient

$$L_{km} = \partial \dot{x}_k / \partial x_m \quad . \tag{5.2}$$

Further assume that $\underset{\sim}{T}_2$ satisfies a positive definiteness condition

$$T_{2,ik} D_{ik} > 0 \quad . \tag{5.3}$$

That solutions of (2.1) obtained via (5.1) satisfy the energy criterion is the subject of the following theorem.

<u>Theorem</u> 5.1. <u>Let inequality</u> (5.3) <u>hold</u>. <u>Let</u> $\underset{\sim}{x}^\varepsilon$ <u>be a solution of</u> (2.1), (5.1) <u>such that</u> $(\underset{\sim}{x}^\varepsilon, \dot{\underset{\sim}{x}}^\varepsilon)$ <u>converges almost everywhere and boundedly to</u> $(\underset{\sim}{x}, \dot{\underset{\sim}{x}})$. <u>Then</u> $(\underset{\sim}{x}, \dot{\underset{\sim}{x}})$ <u>satisfies</u> (2.1), (5.1) <u>with</u> $\varepsilon = 0$, <u>and the energy criterion</u>

$$W_t + \text{Div}\, \phi \leq 0$$

<u>holds in the sense of distributions</u>.

<u>Proof</u>. Let

$$\underset{\sim}{T}(\underset{\sim}{F}) = \det \underset{\sim}{F} \underset{\sim}{t}_1(\underset{\sim}{F}) \underset{\sim}{F}^{-T} \tag{5.4}$$

be the Piola-Kirchoff stress associated with $T_1(\underset{\sim}{F})$. Then $\underset{\sim}{x}^\varepsilon$ satisfies

$$\ddot{x}_i^\varepsilon - T_{i\alpha}(\underset{\sim}{F}^\varepsilon)_{,\alpha} = \varepsilon(\det \underset{\sim}{F}^\varepsilon\, t_{2,ik}\, G_{k\alpha}^\varepsilon)_{,\alpha} \tag{5.5}$$

where $\underset{\sim}{G} = \underset{\sim}{F}^{-T}$. Multiplying (5.5) by $\dot{\underset{\sim}{x}}$ and using (2.9) and (2.10) we obtain

$$\frac{dW}{dt} + \Phi_{\alpha,\alpha} = \varepsilon(\det \underset{\sim}{F}^{\varepsilon}\ t_{2,ik}\ G^{\varepsilon}_{k\alpha}\ \dot{x}^{\varepsilon}_{i})_{,\alpha} - \varepsilon \det \underset{\sim}{F}^{\varepsilon}\ t_{2,ik}\ G^{\varepsilon}_{k\alpha}\ \dot{x}^{\varepsilon}_{i,\alpha} \quad . \tag{5.6}$$

Since $\underset{\sim}{t}_2$ is symmetric and $\underset{\sim}{L} = \dot{\underset{\sim}{F}}\ \underset{\sim}{F}^{-1}$ it follows

$$t_{2,ik}\ G^{\varepsilon}_{k\alpha}\ \dot{x}^{\varepsilon}_{i,\alpha} = t_{2,ik}\ D^{\varepsilon}_{ik} \quad .$$

Therefore, on account of (5.3), (5.6) implies that

$$\frac{dW}{dt} + \Phi_{\alpha,\alpha} \leq \varepsilon(\det \underset{\sim}{F}^{\varepsilon}\ t_{2,ik}\ G^{\varepsilon}_{k\alpha}\ \dot{x}^{\varepsilon}_{i})_{,\alpha} \quad . \tag{5.7}$$

Since $(\underset{\sim}{x}^{\varepsilon}, \dot{\underset{\sim}{x}}^{\varepsilon})$ converges almost everywhere and boundedly to $(\underset{\sim}{x}, \dot{\underset{\sim}{x}})$, the right hand side of (5.7) approaches zero in the sense of distributions. This completes the proof

One does not expect that the converse of theorem 5.1 holds true. In the case $n = 1$ the viscosity criterion was shown to be equivalent to the E-condition by Wendroff [15]. Since for weak shocks the E-condition and the energy criterion are equivalent one may conjecture that in this case the energy criterion is sufficient to guarantee that the viscosity method will choose the proper shock. We carry this program for a special and simple class of linearly viscous material, namely, we assume

$$\underset{\sim}{t}_2(\underset{\sim}{D}) = \underset{\sim}{D} \quad . \tag{5.8}$$

The following theorem draws heavily on the qualitative theory of connecting orbits developed by Conley and Smoller [18], [19].

Theorem 5.2. Let $(\underset{\sim}{v}^+, \underset{\sim}{F}^+) \in S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ be a weak shock, i.e., $|(\underset{\sim}{v}^+, \underset{\sim}{F}^+) - (\underset{\sim}{v}^-, \underset{\sim}{F}^-)|$ is small. Assume $\det F^{\pm}$ are positive and $N_\alpha A_{i\alpha}{}^{\pm}$ are nonzero, where

$$\underset{\sim}{A} = \frac{\partial \det \underset{\sim}{F}}{\partial \underset{\sim}{F}} \quad . \tag{5.9}$$

Assume the energy criterion holds. Then there exists a one-parameter family of travelling wave solutions of (2.1), (5.1), (5.8) which converge to the weak shock $(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{v}^+, \underset{\sim}{F}^+; \underset{\sim}{N})$ almost everywhere and boundedly.

Proof. (2.1), (5.1), (5.8) take the form

$$\ddot{x}_i - T_{i\alpha,\alpha} = \varepsilon(D_{ik}\ A_{k\alpha})_{,\alpha} \quad ,$$

which reduce to

$$\dot{v}_i - T_{i\alpha,\alpha} = \varepsilon((v_{i,\beta} X_{\beta,k} + v_{k,\beta} X_{\beta,i})A_{k\alpha})_{,\alpha}$$
$$\dot{F}_{i\alpha} = v_{i,\alpha} \tag{5.10}$$

since $D_{ik} = \frac{1}{2}(\dot{x}_{i,k} + \dot{x}_{k,i})$. We consider solutions $(\underset{\sim}{v}(\xi), \underset{\sim}{F}(\xi))$ of (5.10) with $\xi = \frac{N_\alpha X_\alpha - st}{\varepsilon}$ which satisfy the boundary condition

$$(\underset{\sim}{v}(\xi), \underset{\sim}{F}(\xi)) = \begin{cases} (\underset{\sim}{v}^-, \underset{\sim}{F}^-) & \text{as } \xi \to -\infty \\ (\underset{\sim}{v}^+, \underset{\sim}{F}^+) & \text{as } \xi \to +\infty \ . \end{cases} \tag{5.11}$$

Let prime denote differentiation with respect to ξ. Then (5.10) is transformed into

$$-sv_i' - N_\alpha T_{i\alpha}' = \frac{1}{2}\{N_\alpha(N_\beta X_{\beta,k} v_i' + N_\beta X_{\beta,i} v_k')A_{k\alpha}\}'$$
$$-sF_{i\alpha}' = N_\alpha v_i' \tag{5.12}$$

Equations (5.12) can be integrated once to yield

$$-s(v_i - v_i^-) - N_\alpha(T_{i\alpha} - T_{i\alpha}^-) = \frac{1}{2} N_\alpha N_\beta A_{k\alpha} X_{\beta,k} v_i'$$
$$+ \frac{1}{2} N_\alpha N_\beta A_{k\alpha} X_{\beta,i} v_k' \tag{5.13}$$
$$-s(F_{i\alpha} - F_{i\alpha}^-) = N_\alpha(v_i - v_i^-)$$

where the constants of integration are chosen so that $(\underset{\sim}{v}^-, \underset{\sim}{F}^-)$ is an equilibrium point of (5.13). The Rankine-Hugoniot conditions (2.5) then imply that $(\underset{\sim}{v}^+, \underset{\sim}{F}^+)$ also is an equilibrium point. Let

$$a_i = N_\alpha A_{i\alpha} \tag{5.14}$$

and observe that (5.13b) implies that a_i is constant when $n = 2$ and is a linear function in $\underset{\sim}{v}$ when $n = 3$. Similarly using the definition of $\underset{\sim}{A}$ and $\underset{\sim}{F}^{-1}$,

$$N_\beta X_{\beta,k} = \frac{1}{\det \underset{\sim}{F}} a_k$$

which reduces (5.13) to

$$-s(v_i - v_i^-) - N_\alpha(T_{i\alpha} - T_{i\alpha}^-) = \frac{1}{2 \det \underset{\sim}{F}} \{a_k a_k v_i' + a_k a_i v_k'\} \quad . \tag{5.15}$$

Let $\underset{\sim}{P} = \mathrm{tr}\, \underset{\sim}{a} \otimes \underset{\sim}{a} + \underset{\sim}{a} \otimes \underset{\sim}{a}$, $\psi_i(\underset{\sim}{v}) = -s(v_i - v_i^-) - N_\alpha(T_{i\alpha} - T_{i\alpha}^-)$. Then (5.15) can be written in the familiar form

$$\underset{\sim}{P}\underset{\sim}{v}' = 2 \det \underset{\sim}{F}\psi(\underset{\sim}{v}) \quad . \tag{5.16}$$

A simple calculation shows that P is symmetric and positive definite. Also (5.13) implies that $\det \underset{\sim}{F}(\underset{\sim}{v})$ is a polynomial of degree $n - 1$ in $\underset{\sim}{v}$, in particular, for $n = 2$

$$\det \underset{\sim}{F} = \det \underset{\sim}{F}^- - \frac{1}{s} \mathrm{tr}(\underset{\sim}{F}^- \underset{\sim}{N} \otimes (\underset{\sim}{v} - \underset{\sim}{v}^-)) \quad . \tag{5.17}$$

Since $\det \underset{\sim}{F}(\underset{\sim}{v}^-) = \det \underset{\sim}{F}^-$ and $\underset{\sim}{F}^+$ is near $\underset{\sim}{F}^-$ it follows that $\det \underset{\sim}{F}$ is nonzero for $v_i \in [v_i^-, v_i^+]$. For the same reason $\underset{\sim}{v}^-, \underset{\sim}{v}^+$ are the unique critical points of (5.16) in a small neighborhood of $\underset{\sim}{v}^-$. In turn Theorem 3.1 guarantees the nondegeneracy of these equilibrium points. Finally we note that $\underset{\sim}{\psi}$ is a gradient function as the material is hyperelastic. Thus all assumptions of the lemma on p. 297 in [19] are satisfied (in particular see the discussion on p. 299 of [19] where $\frac{1}{\det \underset{\sim}{F}} \underset{\sim}{P}$ plays the role of the viscosity matrix) which implies the existence of an orbit of (5.16) connecting the critical points $(\underset{\sim}{v}^-, \underset{\sim}{F}^-)$, $(\underset{\sim}{v}^+, \underset{\sim}{F}^+)$. This completes the proof.

The above theorem depends heavily on the fact that $(\underset{\sim}{v}^+, \underset{\sim}{F}^+)$ is in a small neighborhood of $(\underset{\sim}{v}^-, \underset{\sim}{F}^-)$. To obtain global results, that is connecting orbits for strong shocks, one needs additional hypotheses on the stress function to insure that the unstable manifold of $(\underset{\sim}{v}^-, \underset{\sim}{F}^-)$ reaches the region of attraction of the node $(\underset{\sim}{v}^+, \underset{\sim}{F}^+)$. The main tools in implementing the above is intrinsically the same as in Theorem 5.2. We carry this out for a particular constitutive relation and for the case $n = 2$.

Consider the isotropic compressible hyperelastic material whose stored energy function is given by

$$\sigma(F) = \gamma I + g(III) \tag{5.18}$$

with

$$g(III) = \int^{III} \frac{P(s)}{\sqrt{s}}\, ds \tag{5.19}$$

where $p' < 0$, and I and III are the principal invariants of the left Cauchy-Green tensor $\underset{\sim}{B} = \underset{\sim}{F}\,\underset{\sim}{F}^T$. γ is a material constant and is positive. (5.18) is a two-dimensional compressible model for the classical Mooney-Riolin material for rubber [1]. The Piola-Kirchoff stress tensor has the form

$$\underset{\sim}{T} = -p(III)\underset{\sim}{A} + \gamma\underset{\sim}{F} \tag{5.20}$$

where A is defined by (5.9). We note that this material is strongly elliptic while it is not necessarily genuinely nonlinear since convexity of P is not assumed.

Theorem 5.3. *Let* $(\underset{\sim}{v}^+, \underset{\sim}{F}^+) \in S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ *with the stress function given by* (5.20). *Assume* $\det F^{\pm}$ *are positive and* $N_\alpha A_{i\alpha}^{\pm}$ *are nonzero. If the* (strict) *E-condition holds, i.e.,*

$$s(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{v}^+, \underset{\sim}{F}^+; \underset{\sim}{N}) < s(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{v}, \underset{\sim}{F}; \underset{\sim}{N}) \tag{5.21}$$

for all $(\underset{\sim}{v}, \underset{\sim}{F}) \in S(\underset{\sim}{v}^-, \underset{\sim}{F}^-; \underset{\sim}{N})$ *between* $(\underset{\sim}{v}^-, \underset{\sim}{F}^-)$ *and* $(\underset{\sim}{v}^+, \underset{\sim}{F}^+)$, *then there exists a one-parameter family of solutions of* (2.1), (5.1), (5.8), (5.20) *which connects* $(\underset{\sim}{v}^-, \underset{\sim}{F}^-)$ to (v^+, F^+).

Proof. The following proof relies on the concept of the isolating block developed in [19]. Associated with the system (5.16) we consider the vector field

$$\underset{\sim}{v}' = 2 \det \underset{\sim}{F}\underset{\sim}{\psi}(\underset{\sim}{v}) \quad . \tag{5.22}$$

First we construct a region $\mathcal{D}$ in (v_1, v_2) plane which contains the equilibrium points $(\underset{\sim}{v}^-, \underset{\sim}{F}^-)$, $(\underset{\sim}{v}^+, \underset{\sim}{F}^+)$, and such that the vector field (5.22) is tangent to the boundary of $\mathcal{D}$ in exactly two points. A simple calculation shows that

$$\psi_{1,v_2} = \psi_{2,v_1} = -\frac{2}{s} \det \underset{\sim}{F}\, N_\alpha A_{1\alpha}^-\, N_\beta A_{2\beta}^-\, p'(III) \tag{5.23}$$

which is never zero by the hypothesis. Therefore, the two curves $\psi_1(\underset{\sim}{v}) = 0$ and $\psi_2(\underset{\sim}{v}) = 0$ are one-to-one in the (v_1, v_2) plane for $v_i \in [v_i^-, v_i^+]$. Since $\det \underset{\sim}{F}(\underset{\sim}{v})$ is not zero in that region we can construct $\mathcal{D}$ with the above specifications as a rectangle obtained from the intersection of the lines $v_i = v_i^{\pm} \pm \epsilon_i^{\pm}$. $\epsilon_i^{\pm}$ are chosen near zero and with the appropriate sign so that $\underset{\sim}{v}^{\pm}$ are in the interior of $\mathcal{D}$. (5.23) then implies that the vector field (5.22) is tangent to the boundary of $\mathcal{D}$ in exactly two points. We also note that (5.22) is a gradient-like system, that is, for

$$F(v_1, v_2) \equiv \frac{s}{2}(v_i - v_i^-)(v_i - v_i^-) + s\sigma(F(\underset{\sim}{v})) + N_\alpha T_{i\alpha}^-(v_i - v_i^-)$$

we have

$$2 \det \underset{\sim}{F}\psi(\underset{\sim}{v}) \cdot \text{grad } F \le 0 \quad .$$

Hypothesis (5.21), in turn, implies that (v_1^-, v_2^-), (v_1^+, v_2^+) are the only equilibrium points of (5.22) in $\mathcal{D}$. We can now apply Theorem 6.1 of [18] to insure the existence of a connecting orbit for (5.22). To see that this orbit persists for the system (5.16) we note that since $\underset{\sim}{P}$ is positive definite and symmetric and the matrix of linearization of (5.22) at the node is symmetric, the critical point (v_1^+, v_2^+) remains an attractive node for (5.16) (cf. Theorem 6.2 [18]). This completes the proof.

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The equations of hyperelasticity have the special feature that their natural entropy is not a globally convex function. Strict convexity of the entropy function is essential in formulating a physically reasonable entropy criterion for shock waves. In this paper we show that the natural entropy of the equations of hyperelasticity is uniformly convex when restricted to the shock curves. This fact enables us to prove the equivalence of the entropy criterion and Lax's shock conditions for existence of weak shocks for problems that are genuinely nonlinear. Furthermore, for problems that are not

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20. ABSTRACT (Cont'd.)

necessarily genuinely nonlinear we study the (generalized) "E-condition" and show that it is indeed a generalization of the entropy condition. Finally, we consider the viscosity criterion which requires that a motion of a hyperelastic body is the limit of smooth motions of a family of viscoelastic materials. The relationship between the energy criterion, the E-condition, and the viscosity criterion is then discussed.