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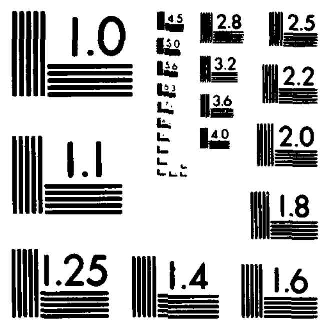

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NEW APPLICATIONS OF GENERALISED CURVES, PARTICULARLY IN PHYSICS

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ABSTRACT

Part I. The traditional assumptions of infinitesimal smoothness, implicitly made in various applied topics, do not agree with what is observed at microscopic and smaller dimensions. Mathematical models closer to what actually occurs, are made possible by the theory of generalised curves, and by the matching up and resonance of the infinitesimal patterns they generate. Phenomena hitherto not fully accounted for, may be partly due to such generalised patterns in stress lines of bridges, streamlines of shock waves, and rays of light.

Part II. In relativistic quantum physics, generalised curves appear on account of the indefiniteness of the metric, and the infinitesimal zigzagging corresponds to successive emission and absorption of radiation. The Nowosad theory, a preview of which is given here, accounts in this way for the elementary particles in a logical manner, and provides at the same time de Broglie's Pilot Wave and the quantum potentials of Bohm and Vigier. Moreover such things as the Pauli exclusion become theorems. The theory makes use of singular integrals and of reduced quaternion-valued analytic functions.

AMS (MOS) Subject Classifications: 81G99

Key Words: Aircraft, de Broglie's pilot wave, calculus of variations, dynamical friction, bridges, Bohm-Vigier potentials, elasticity, elementary particles, existence, fine structure, generalised curves, photons, quantum, quaternions, refraction, relativity, relaxed controls, resonance, shock waves, singular integrals, streamlines, stress and strain lines.

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SIGNIFICANCE AND EXPLANATION

Generalised infinitesimal patterns in stress lines and streamlines, and in light rays, are suggested as possible contributory causes affecting the durability of bridges and aircraft and the refraction of light. This last phenomenon is the basis of a theory of elementary particles in physics, due to Nowosad.

NEW APPLICATIONS OF GENERALISED CURVES, PARTICULARLY IN PHYSICS

L. C. Young

(This is the substance of invited addresses to the Canadian Mathematical Society at Trent University, to McMaster University, to the McLeod Applied Seminar at Oxford University, and to the Ioffe Colloquium at Imperial College. The present version is dedicated to my sister Dr Cecily Tanner on her 80-th birthday, 6 Feb. 1980).

Part I : Generalities, intuitive applications, exact definitions.

1. Introduction. The calculus of variations has long been a most applicable branch of mathematics, and the origin of conceptual advances in convexity, duality, functional analysis, advances that transformed a whole range of subjects. The present applications are conceptual: they link the variational 'fine structure' to the fine structure that science is discovering in the world. What was yesterday abstract acquires a significance more real and more fundamental than our tables and chairs, and the ground beneath our feet. For this, we must overcome subjective judgements tied to the particular evolution of our species and of our minds. To make possible the life from which we spring, we had to find, in the physical world, an average stability, to which our size, our life-span and our senses could adapt, ignoring the chaotic turmoil of underlying micro-phenomena, just as in learning to drive we ignore what goes on under the hood. What we subjectively still call real, many of us, may be little more than a set of buttons to be pushed, on a convenient biological instrument panel mainly consisting of our senses. It is a vastly over-simplified model, an abstraction built into our nature, our outlook, our language, our science.

We speak without reservation of 'drawing a straight line'; we term straight the path of a falling particle, and the bee-line, or crow's line of flight. For Felix Klein, these were more properly thin strips or tubes, and on such lines Hjemslev wrote a 'natural geometry'. This has partly been resurrected with today's 'fuzzy sets', after being discredited in Clifford's Common sense of the exact sciences, where it was stipulated (with suitable conventions as to upper and lower) that the straight line considered to be drawn was to be, not the thin strip traced, but its upper edge. Actually, this edge, when examined closely under a magnifying glass, looks more like a saw-edge, with very fine sharp teeth: it is quite evident that its length is at least half as

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much again as the shortest distance. If we wish to neglect the finite size of the teeth, and yet to retain a mathematical concept resembling what we draw, we must imagine a limiting situation, in which we have, not a straight line, but an infinitely fine saw-edge, or infinitesimal zigzag, that appears to coincide with a straight line. The concept is not even wholly new: books on mechanics have long spoken of a perfectly rough plane, as quite different from an ordinary smooth one. What this means is that the infinitesimal structure is not flat.

In my book (3), the distinction between a straight line, and an infinitesimal zigzag on it, may be seen in the problem of sailing against the wind in a straight river, with a following current which is strongest in midstream. The optimal solution requires tacking, i.e. suitably zigzagging, but the tacks must be infinitesimal, to remain where the current helps most. We cannot actually sail on such an infinitesimal zigzag: we can only approximate to it by a finite zigzag; but then, neither can we sail in a rigorous straight line -- that also we would have to approximate.

2. Elastic shrinkage, dynamical friction, infinitesimal perturbations. Until we get to the exact definitions, we shall use the intuitive language of engineers and physicists, so that infinitesimal will mean, more or less, too small to be seen, even if the object to which this is applied is millions of times larger than our atoms and molecules. If we try to be more exact in this sort of context, we have to bring in micro-phenomena that alter the whole picture.

This being so, a number of familiar objects and phenomena can now illustrate generalised curves. One of the simplest is a thread of an elastic band. Stretched, it can be straight, or it can curve round some object; then, as we reduce the tension, it wrinkles up on an apparently shorter path, and the wrinkles turn it into an infinitesimal zigzag of the original length. Non-elastic materials may wrinkle up too, but do not necessarily smooth out when subsequently stretched. This is how, on a frozen lake, daily temperature changes, and the resulting expansions and contractions, turn a smooth sheet of ice into a triangulated surface.

Another familiar illustration of a generalised curve arises from dynamical friction. (This has been remarked by Nowosad.) When a taxi is stopped by traffic lights, half way up a hill, the driver uses, not his brakes, but his clutch. With the clutch engaged, the engine velocity is transmitted to the taxi's forward motion; with it disengaged, the taxi slips back, accelerated by gravity. Classically, acceleration cannot balance velocity, but the driver achieves the impossible: the vehicle remains apparently stationary. The clutch is neither engaged, nor disengaged, but wobbles from one position to the other, like a country blessed with two political parties. The taxi keeps moving infinitesimally forward and infinitesimally back: the displacement, as function of the time, zigzags infinitesimally on the horizontal. The same dynamical friction can, and must, be used in contexts where there are no reliable brakes, as in that of politics, or in other activities governed by exponential growth: an exponential starts with miserable slowness, but suddenly shoots up; and a second exponential is either too late, or so fast increasing that it poses even greater threats. Dynamical friction can also be used where there is no proper steering, as in space science. (In this subject there are also no brakes, but that is less important.) To stay in orbit, a satellite must balance perturbing gravitational accelerations by bursts of engine power, and therefore by velocity changes, since -- strictly -- the velocity changes when a rocket ejects even one high speed particle. The actual trajectory is thus an infinitesimal zigzag on the desired orbit. This applies equally if the orbit is unstable, for instance for a periodic orbit that is desired to take turns at being close to the Earth and the Moon. We can speak of the motion as subject to infinitesimal balanced perturbations.

More generally, we can argue that almost any motion caused by man is subject to such perturbations, and so follows a generalised, rather than an ordinary, trajectory. This is because it proceeds by infinitesimal spurts: for instance the propulsion produced by revolving engines is uneven during the small time of a revolution. Normally, the spurts never propell quite in the right direction, and they need correcting by steering, so that the resulting motion is infinitesimally perturbed in the balanced manner described above. Natural motion can, of course, sometimes also proceed in this way: the sun's motion is disturbed back and forth by the rocket action of sunspots; and terrestrial objects may have their motion similarly affected by, say, even slight waves of sound.

It almost goes without saying, that an entirely similar phenomenon must affect the rays of light of geometrical optics, and turn them into infinitesimal zigzags, by what Newton called 'fits of easy reflection and refraction'. This is then a proper bridge between geometrical and wave theories of light. Consider, for instance, a ray passing through optical material and emerging on the other side. The refraction law of Harriot, Snell or Descartes -- whichever you may prefer to call it -- fails to explain how light, after slowing down in the material, suddenly manages to emerge at its original speed. Motorists, after passing through a village, press the accelerator: where is the accelerator for light? Have we been deceived? How can we be so sure that light is ever slowed down at all? The slower time through the material can result from a proportionately longer path, that only looks straight, but is really an infinitesimal zigzag, to avoid infinitesimal hurdles set up by matter on the way. If physicists had thought of this possibility, particularly de Broglie, and later Bohm and Vigier, who came close, they might have anticipated the Nowosad theory that I speak of in Part II.

3. Matching families, and the resonance of stress and strain curves. Generalised curves need completing by a further concept when we come to consider families of curves; and this leads directly to a wave theory. In a family of zigzagging curves, the ups and downs can either get in each other's way, or they can be more or less synchronised. In the latter case we speak of a matched family. This was doubtless at the back of Newton's mind in the quotation above: we have 'fits' of ups, and fits of downs. This takes me to what has been of late a most disturbing topic: the premature collapse of bridges and the breaking up of aircraft. I am concerned with mathematical aspects, hitherto not properly taken into account, and that need completing by an exact mathematical theory. These are things that affect all forms of matter, because the streamlines in fluids, and the lines of stress and strain in solids, can be generalised curves that tend to match. In fluids, this is on account of the fluid pressure; in solids it can arise from a lack of uniformity in the grain of wood, the coarse granular nature of stone, the crystalline patterns of glass and metals, all of which can be considered as regularly or as randomly placed obstacles to lines of stress and strain.

I shall begin with the case of a bridge, and for simplicity I shall treat the latter as if it were in a (vertical) plane, and made up of long filaments stuck together, along which the main stresses and strains are transmitted. To avoid the infinitesimal obstacles I mentioned above, these filaments must be generalised curves, and moreover their infinitesimal zigzag pattern will become more or less pronounced as the material contracts or expands, but since they are not elastic, they tend to break, and to form a gap or an overlap. I pass over factors taken into account in past theory, that accelerate the weakening of the bridge. One important factor, so far ignored, is the matching tendency in the infinitesimal patterns of neighbouring layers. This is induced, and accelerated, by relative lateral sliding of successive filaments, due to traffic vibration, temperature difference, and so on, and it naturally means that cracks tend to match also. Then resonance buids up, until the whole bridge vibrates with the traffic, in larger and larger ups and downs, such as have been photographed just prior to collapse.

4. The generalised streamlines of shock waves. In aircraft, similar causes affect metal fatigue, but past theory has also underestimated the shock waves. If atmospheric streamlines are generalised, the local atmospheric velocity can easily be at least doubled, and this increases even more the pressure, which by Bernoulli's equation varies with the square of the velocity. Experimental evidence indicates, according for instance to Nickel (9), that near the aircraft there are great velocity differences, compared to the basic average streaming. This magnification of local velocities suggest intense infinitesimal zigzagging, but how could the latter be caused? The question takes us back to the zigzagging that I associated earlier with man-induced motion of an object such as an aircraft, and also to the saw-edged nature of the apparently smooth metal. We could hardly expect these infinitesimal patterns not to affect similarly those of nearby streamlines. But in fact the effect is greatly intensified. The velocities concerned, and still more the pressures, are in any case high, so that the air is compressed near the aircraft, and behaves a little more like a liquid or a solid. Through such a medium, we can imagine our finely saw-edged aircraft sawing its way at high speed, much as a mechanical saw slices through wood. From the compressed atmospheric layer, it ejects a spray of high speed atmospheric sawdust that shoots its way through neighbouring layers. The effect is mathematically like dynamical friction.

Intense zigzagging takes place in the infinitesimal elements of the otherwise unexceptionable streamlines that come near the aircraft, as the streamline particles are pushed off their course by the 'sawdust', and pushed back with equal violence by atmospheric pressure. To deal properly with all this wild buffeting by shock waves and turbulence, we need a modern fluid mechanics, with generalised curves as streamlines, and with generalised flows, as in my book (3) and in recent representation theorems of Lewis and Vinter (4).

5. The function-space definitions in terms of duality and probability.
I have not so far distinguished clearly between infinitesimal zigzags and rather fine zigzags, just as I have mainly ignored the fine structure that our world really possesses the quantitised relativistic whirl of micro-particles, and the (already simplified and idealised) statistical wanderings of Brownian molecules. We must be more precise, before we start to apply mathematical concepts in a context where our senses and our intuition no longer guide us.

I shall not in fact need the mathematical description of a matched family of generalised curves C_α, which depend on a parameter α. But since I have used the intuitive concept above, I will go so far as to suggest a definition in terms of the concept of generalised curve itself. If the apparent path of C_α is given by a vector-valued function $x(t,\alpha)$, we simply rewrite the latter $X(t)$, where X denotes a variable continuous function of α, and we consider a generalised curve in the space of the variable X, such that its apparent path is given by $X(t)$. If we then re-express this in terms of the original space, we shall have what I suggest calling a matched family. Thus the concept reduces to that of a generalised curve, and I may now limit myself to the latter.

In my book (3), there are two definitions of generalised curves: one involves duality, and corresponds, in the history of functional analysis, to Hadamard's representation of a linear functional on the space of continuous functions, as the limit of a certain type of expression; the other, which similarly corresponds to the Riesz representation of the functional considered by Hadamard, is in terms of probability measures in the space of directions. First it is necessary to identify by definition an ordinary curve with the corresponding curvilinear integral, i.e. with a linear functional of a special form on the space of integrands. For the purpose of our applications in Part II, the ordinary curve in question will be supposed, in the parametric case, rectifiable and given in terms of the arc length by a Lipschitzian function. In the non-parametric case, the ordinary curve will be supposed Lipschitzian, with a derivative whose absolute value does not exceed the velocity of light. Further, all curves concerned are to be situated in a compact set A in the space of x, and we denote by B a suitable ball in a similar space of the variable $\dot{x}$, or else (in the parametric case) the unit sphere.

We denote by F the space of continuous functions $f(x,\dot{x})$ on $A \times B$. Instead of defining our ordinary curve by the function $x(t)$, we identify it with the linear functional

$$L(f) = \int f(x(t), dx(t)/dt)\, dt \qquad f \in F \quad ,$$

i.e. with a linear functional L possessing at least one such representation in terms of a corresponding Lipschitzian $x(t)$. Thus our ordinary parametric or non-parametric curve becomes an element L of the dual of the space F. We term generalised curve a weak * limit of such elements, i.e. an expression

$$\lim_n L_n(f) \quad ,$$

where the L_n are linear functionals of the kind just defined above in terms of corresponding Lipschitzian functions $x_n(t)$, and where the limit exists for each $f \in F$. This is the definition in terms of duality. The alternative (Riesz type) definition involves a unit measure μ on B, which is associated with a Lipschitzian $x(t)$ and which depends itself on t. We term μ a tangential probability at t, attached to $x(t)$. For fixed t, we term mean value of f at the time t, or at the point of parameter t, the integral in μ of the function of $\dot{x}$ given by $f(x(t),\dot{x})$; we shall denote this mean value, or expectation, by $M_t(f)$, and we extend the notation temporarily to the case where f is vector-valued. The tangential probability will be required to satisfy the condition that $M_t(f)$ reduces to $dx(t)/dt$ almost everywhere when we choose for f the identity function $f(x,\dot{x}) = f(\dot{x}) = \dot{x}$. This being so, we term, in our second definition, generalised curve a linear functional $L(f)$, if and only if there exist such a tangential probability and an associated Lipschitzian $x(t)$, such that

$$L(f) = \int M_t(f)\, dt \qquad \text{for} \quad f \in F \quad .$$

We note that in the parametric case t represents a generalised arc length, which in general increases faster than the arc length of the curve defined by $x(t)$; and that, in the non-parametric case, we can identify one of the components of x with the variable t, if we wish the underlying space to have this additional dimension.

A local tangential probability, to replace a tangent or derivative, is the first sign of a relationship with the fine structure of modern physics. This substitution is immaterial for integrands $f(x,\dot{x})$ linear in $\dot{x}$. Thus generalised curves are a non-linear theory, whereas Schwartz distributions are a linear theory. Similarly probability does not enter into the linear summation processes of Cesaro, Abel and so on, in the theory of divergent series and sequences. Thus we find no probability in books on divergent series. Yet the series $1 - 1 + 1 - 1 + \ldots$, or equivalently the sequence $1, 0, 1, 0, \ldots$, corresponds to a sequence of alternate castings of 'heads' and 'tails' for a coin: the Cesaro sum $\frac{1}{2}$ could only represent a fictitious mean position, with the coin on its rim. A more natural description of a limiting state of affairs is obtained by assigning equal probabilities to the values 0 and 1. In such matters, the use of probability comes from number theory, a subject reputed to contribute less than its share to human progress, and I think wrongly so reputed. Probability appears, for instance, in the classical Kronecker-Weyl theorem, and to disregard it in plausible arguments can have unfortunate results, as in the main lemma of (5).

Duality also is connected with modern physics, but in its basic philosophy. Physical quantities are known to us only by experiments whose outcome is expressed by the values of integrals, i.e. by elements of a dual space. In mathematics, things are much the same: mathematical entities are important by their use, and this again is in evaluating certain integrals, such as the curvilinear integral, considered above. In either case, what really matters is some element of a dual space. Geometers had duality long ago, but, as stressed by Dieudonné (6), they spoilt it by insisting that the dual space be identified with the original one. It was only when this identification was given up, that it became possible to use duality to define new elements, such as generalised curves or Schwartz distributions, which are needed in existence theorems. Modern duality thus begins with Minkowski, with Banach, and with Dieudonné and Schwartz (7).

In the calculus of variations, as I show in my book (3), generalised curves provide almost automatic existence theorems. However, in the applications to physics, it is important to note that generalised curves are needed also as the solutions of a large class of problems, typical examples of which are given in (3). These are the problems in which the integrand $f(x,\dot{x})$ fails to be convex in $\dot{x}$. This is precisely what happens to the problem of geodesics in relativistic space-time, and this accounts for the need of generalised curves in physics, as we shall shortly see.

Part II : Elementary particle physics, the Nowosad theory.

6. Preliminary remarks. This is a preview of what I can only call a breakthrough in micro-physics, and except for my having introduced long ago the notion of generalised curve, my part in it is expository. I hope the present brief account will make it easier for Dr Nowosad to publish soon the complete details of his researches to date, rather than wait until every single one of the known elementary particles has been fitted into his scheme. We have given joint public lectures in this manner, I giving the first lecture, and he the second. I have also, of course, had the advantage of long oral discussions with him, and of seeing written notes and summaries that he used in his lectures, the latest being a lecture to the international physics institute at Trieste in Sept. 1979, the summary of which is reproduced at the end of the present paper.

The path followed by Nowosad is extremely logical: there are no mysterious 'q-numbers', no a priori indeterminacy rules nor exclusion principles; any such matters are now theorems. Everything must follow from the local relativistic metric, an indefinite quadratic differential form (Lagrangian). We are actually concerned only with what happens in a minute portion of space-time, and the local metric represents in a sense the total effect on this portion, exerted by the outside world. We can therefore ignore the outside, and expand our minute portion to a whole 4-dimensional relativistic manifold with a local metric. On this manifold, our first task is to seek for primitive, or intrinsic, objects. The latter include, of course, the geodesics determined by the metric, and among them there will be generalised curves. These will be the paths of the particles of light, the photons. These are then the primitive particles, from which all others must be derived. Indeed, in the Nowosad mathematical model which simulates what goes on in our tiny portion of space time, matter, or the illusion of matter, is derived by infinitesimal zigzagging of photons. In this way nothing extraneous has been introduced, since the zigzags result directly from the indefinite metric. The key to the Nowosad theory is thus the concept of generalised curve.

It is hard to believe: in our tiny portion of space-time, matter behaves like flickering light. Yet, if we are to achieve unity at all, out of the host of long or short lived elementary particles that our remarkable experiments have uncovered, this is the one solution that stares us in the face. It will seem even more logical as we get to the details, and we shall see that so far it agrees very well with experiment. Of course we must remember that it applies to a limited context, a tiny portion of space-time; moreover, physics has seen a number of nice theories, each in turn leading to remarkable experiments that put it out of date. The Quantum Mechanics of the Dirac era was in its way quite beautiful, prior to the discovery of a host of micro-particles. I would be sorry to see the present limited theory made the excuse of some grandiose program, some new gospel, in which philosophers, theologians and popular scientists would vie with one another in their biblical pronouncements "In the beginning there was light", "As it was in the beginning, ..." -- and then to see it all upset by yet more beautiful experiments and by a more perfect theory, as physics progresses along its own zigzag path. Nevertheless, I cannot pass by this opportunity to quote the pioneer of the present theory, de Broglie (8):

> Light has just revealed itself as capable of condensing into matter, whilst matter is capable of dissipating into light. Giving free scope to our imagination, we could suppose that at the beginning of time, on the morrow of some divine 'Fiat Lux', light, at first alone in the universe, has little by little produced by progressive condensation the material universe such as, thanks to light itself, we can contemplate it today. And perhaps one day, when time will have ended, the universe, recovering its original purity, will again dissolve into light.

7. Connection with the Dirac and wave theory. Particles will here be associated with waves, and we must begin by studying the latter. The primitive object that corresponds to our metric, is then the dual, or Hamiltonian, quadratic form $q^*(y)$, whose coefficients, like those of the original (Lagrangian) quadratic, depend in general on the point x; here x varies, as explained, on a relativistic 4-dimensional manifold. From either form, we derive a corresponding volume magnification, which is then a scalar, and I shall assume that the latter is properly taken into account in all formulae. As further primitive objects, we associate with a twice differentiable function $v(x)$, the squared norm of its gradient, and the Laplacian. To define these, we set $q^*_y = \partial q^*/\partial y$ and take for y the gradient $\nabla v = v_x = \partial v/\partial x$, and we write

$$|\nabla v|^2 = q^*(y) \quad , \quad \Delta v = \gamma^{-1}(\partial/\partial x, \gamma q^*_y) \quad ,$$

where γ is the appropriate scalar.

With the help of these quantities, we obtain as dual primitive objects the extremal functions for the Dirichlet or energy integral $\int |\nabla v|^2$, and therefore the solutions of Laplace's equation $\Delta v = 0$, which is here a wave equation, whose additive system of solutions can termed harmonic or wave functions. To derive 'states' in physics, we must pass to a multiplicative system, by writing $\varphi = e^{iv}$, where the 'phase' v is harmonic. We then have the fundamental identity

$$\varphi^{-1} \Delta \varphi = -|\nabla v|^2 ,$$

which follows from the general identity

$$\Delta f(w) = f' \Delta w + f'' |\nabla w|^2$$

for a twice differentiable function f of a twice differentiable function w . The fundamental identity means that, for the state φ whose phase is the harmonic, or critical, object v , the Dirichlet or energy integral becomes the singular integral

$$- \int \varphi^{-1} \Delta \varphi ,$$

involving the Laplacian. Instead of φ^{-1} , we would write here, in the Quantum Mechanics of the Dirac era, the complex conjugate of φ .

8. The quantum jumps. In the singular integral of the preceding section, mathematicians will see rather an analogy with the complex contour integral $\int f^{-1} df$, whose value is an integer multiple of $2i\pi$: if we vary the contour or the analytic function f , the integral can alter only by multiples of what we may call a 'quantum jump' $2i\pi$. Nowosad's doctor thesis shows that our singular integral behaves similarly. More generally, if ℓ is a linear functional and A a linear operation, he found quantum jumps for the functional $\ell(\varphi^{-1} A \varphi)$ in corresponding commutative B* algebras. Quantum jumps thus arise automatically, because of the logarithmic singularity introduced by the multiplicative nature of the superposition of states in physics.

Nowosad gives a simple example to illustrate how this comes about. A is the Laplacian, and the underlying function-algebra is that of continuous almost periodic functions in space-time; the functional $\ell(f)$ denotes the limit of the mean value of f in a ball whose radius tends to infinity. In the basic exponentials e^{ikx} , which generate the algebra, kx denotes an ordinary Euclidean scalar product of vectors, while for coefficients k, k_o, etc, the expressions $|k|^2$, (kk_o) etc, indicate squared norms and scalar products in the dual metric of that of special relativity. The state e^{ikx} is a so-called 'plane wave', and is termed 'isotropic' if $|k|^2 = 0$; it satisfies in any case the relation $\Delta\varphi = -|k|^2\varphi$, so that $\ell(\varphi^{-1}\Delta\varphi) = -|k|^2$. Evidently, if φ is isotropic, the phase kx must reduce to a scalar multiple of $\xi - ct$, where c is the velocity of light, and where ξ, t are orthogonal projections of x in 3-space and on the time axis, so that we may regard φ as a plane wave travelling with the velocity of light along a direction in 3-space. Consider now two plane waves φ_o, φ_1, where $\varphi_1 = \varphi\varphi_o$ and φ is isotropic: the corresponding coefficients are related by $k_1 = k + k_o$, and we shall suppose k, k_o not orthogonal in the metric, to ensure that the difference $|k_1|^2 - |k_o|^2 = 2(kk_o) = 2(kk_1)$ does not vanish. If we set $\varphi_t = t\varphi_1 + (1-t)\varphi_o$ and $\varepsilon = t/(1-t)$, we find that, on the portion $0 < \varepsilon < 1$, i.e. $0 < t < \frac{1}{2}$, of the φ-space segment joining φ_o, φ_1,

$$\varphi_t^{-1}\Delta\varphi_t = \frac{-|k_o|^2\varphi_o - \varepsilon|k_1|^2\varphi_t}{\varphi_o + \varepsilon\varphi_1} = \frac{-|k_o|^2 - \varepsilon|k_1|^2\varphi}{1 + \varepsilon\varphi}$$

$$= (-|k_o|^2 - \varepsilon|k|^2)(1 - \varepsilon\varphi + \varepsilon^2\varphi^2 - \dots) .$$

The series converges uniformly: we multiply through and integrate term by term in the ball of radius r, and we then make r tend to infinity. The mean values still converge uniformly, and since $\ell(\varphi^n) = 0$ for $n=1,2,\dots$, we see that $\ell(\varphi_t^{-1}\Delta\varphi_t)$ is the constant $-|k_o|^2$ for $0 < t < \frac{1}{2}$. Similarly it is the different constant $-|k_1|^2$ for $\frac{1}{2} < t < 1$. At $t = \frac{1}{2}$ we get a 'quantum jump', and not without reason: the denominator becomes $1 + \varphi$, which vanishes when kx/π is an odd integer, i.e. on a family of parallel planes of 3-space which move with the velocity of light.

9. The photon and its pilot wave. In the preceding example, the quantum jump in the energy integral can be regarded as radiation from planes. To obtain a photon, we would have to have radiation from a point, moving with the velocity of light, i.e. from a 1-dimensional set in space-time. In place of $1 + \varphi$, we would need a function which vanishes on such a 1-dimensional set. We must generalise the apparatus, and use a more general form of Nowosad's quantum jumps. Instead of an isotopic plane wave, we can introduce a so-called monochromatic wave $\varphi = e^{iv}$, in which the phase v is subject to $\Delta v = |\nabla v|^2 = 0$. Alternatively, we can take a complex-valued phase $u + iv$, in which case the same conditions become,

for the real and imaginary parts, $0 = \Delta u = \Delta v = (\nabla u . \nabla v) = |\nabla u|^2 - |\nabla v|^2$. By the general identity of section 7, these conditions mean that an arbitrary twice differentiable f(v) in the real case, or analytic f(u+iv) in the complex case, satisfies, as composite function, Laplace's equation. In both cases, however, the zeros of such a function do not occupy one-dimensional sets. Can we increase the number of components of f by a generalisation of analytic function, such as is provided, say, by minimal surfaces? Since we require a function-algebra, there is little choice: we have to go over to quaternions, which are not commutative. Fortunately we can obtain the desirable commutativity with 'reduced quaternions' u+iv+jw , whose fourth component vanishes. Since the square of iv+jw is $-v^2-w^2$, it follows successively that the square and n-th power of a reduced quaternion, and generally the sum of any convergent series of its powers with real coefficients, are likewise reduced quaternions. Such series are the quaternion-valued analytic functions of <u>Fueter</u> (1931): nobody imagined that they might some day acquire practical significance. A Fueter function f is derived by

$$f = \left(\frac{\partial}{\partial u} + i \frac{\partial}{\partial v} + j \frac{\partial}{\partial w} \right) P$$

from the real part P(u,v,w) of an ordinary analytic function F(u+ir) with real coefficients, where $r^2 = v^2 + w^2$

I shall depart slightly from the Nowosad definitions, by now substituting for u,v,w functions subject to $0 = \Delta(u+iv+jw) = (\nabla u, \nabla v) = (\nabla u, \nabla w)$, $|\nabla u|^2 = |\nabla v|^2 + |\nabla w|^2$, and to the ratio $w/v = \theta$ satisfying $|\nabla\theta|^2 = 0$. This ensures that every such P then satisfies Laplace's equation. These matters, and the precise place at which exponentials are introduced to provide the quantum jumps, as explained earlier in this note, will doubtless be cleared up when Nowosad's work appears -- here I merely give a preview. In this way we obtain what may be termed a monochromatic reduced-quaternion valued function algebra, generated by the functions u,v,w . What is important is that, for an f in this Fueter algebra, the zeros are obtained, either from 3 , or from 2 equations: the two possibilities arise because the 'imaginary part' of f is the product of iv + jw by a real factor $r^{-1}P_r$. However only the first possibility provides zeros on loci of the right dimension, on which our quantum jump, our photon, then moves with the velocity of light. Elaborate though this all may seem, it does then provide, for the photon, the simplest of all particles, a mathematical model associating it with a reduced quaternion-valued f , as its 'pilot wave'.

10. The zigzagging mechanism of material particles. Only the simplest generalised curves, other than ordinary curves, appear in the Nowosad theory, the ones in which, at each point, there are no more than two associated directions. This is because, for a particle other than a photon, there is both emission and absorption of radiation, and in general the outgoing and incoming direction of light quanta will be different. To each of the two directions, we associate a corresponding model of a photon, i.e. a reduced quaternion-valued algebra of monochromatic waves. To have a generalised curve, we must associate further, with the two directions, their respective probabilities p_1, p_2. Evidently, if V_1, V_2 are velocity vectors for light in the two directions, the mean velocity

$$V = p_1 V_1 + p_2 V_2$$

will be that of the particle on its apparent path. Similarly, if f_1, f_2 are the functions in our two algebras whose singularities are associated with the local emission and absorption, the mean

$$f = p_1 f_1 + p_2 f_2$$

will be a fully quaternion-valued function, also associated with the apparent path. In this way, not only is the particle slowed down apparently to the mean velocity vector, but there is an apparently associated, fully quaternion-valued f, for which we find by combination a complicated wave equation with a right-hand side: this right-hand side turns out to be a certain multiple of fQ, where Q is a potential occurring in the work of Bohm and Vigier. Fully quaternion-valued and even complex quaternion-valued states are not uncommon in the literature. (See, for instance, Edmonds 10.)

11. Summary of further results. We see that we have come close to things that have been tried: the main differences are conceptual. But this is precisely what makes it possible to put the theory to practical use and practical test, by solving a number

of theoretical and quantitative problems in our mathematical model of micro-physics. For instance, Dr Nowosad has confirmed orally that in his theory the Pauli exclusion principle and the existence of the most ellusive quark are now among the theorems; moreover some quantitative results are mentioned in the abstract, reproduced below, of his Trieste September 1979 lecture. It is almost inconceivable that this agreement with experiment and with accepted principles could be merely by chance. Neither is there room in Nowosad's logically constructed set up for any tinkering: Quantum theory is reduced to a study of specific integrals and of their discontinuities, the form of the integrals being in the main dictated by that of the relativistic line element, which in its turn incorporates the effect of electro-magnetic theory and so forth on the minute corner of space-time, an effect produced by the outside world or by our own experimental laboratory.

12. The official abstract of Dr Nowosad's Trieste lecture "Quanta and geometry -- a constructive approach." A light quantum is defined in a V_4 by means of a real commutative algebra with values on the reduced quaternions, having three real generators, all its elements being monochromatic waves. This is done in order that solutions of the wave equation with a singularity on a single characteristic curve may be constructed. A particle is characterised by two such algebras, one representing emission (outgoing) and the other absorption (incoming) of light quanta. Massive quanta, moving on time-like trajectories, are represented through the concept of generalised curve, constructed out of two families of light rays, one for each algebra. The generators of the above algebras satisfy a given set of partial differential equations, and it is shown that the metric must then have the form

$$ds^2 = a(x_1,x_4)(dx_1^2 - dx_4^2) + b(x_3,x_4)(dx_2^2 + dx_3^2) \quad .$$

It is shown that the elements in this algebra generated by the given six generators (of which only four are functionally independent) are analytic functions of $x_2 + ix_3$, hence the wave equation for them reduces to the string equation. This gives the correct quantization of the internal states of the particle. By requiring that the momentum-energy tensor be that of the electro-magnetic field, that the background manifold be R^4 and that the singularities in the space sections be bounded, an explicit expression is obtained for ds^2 , involving three arbitrary constants, and allowing for both stable and unstable particles.

Using the values of the electric charge and mass of the proton and the mass of its quantum (pion), one obtains, disregarding integrations inside the singularity, the values

$r_o \cong 10^{-14}$ cm (radius),

$\lambda \cong 10^{23}$ cm^{-2} (cosmological constant),

$K \cong 10^{-36}$ (coupling factor between the gravitational and the electromagnetic fields).

The area of the Euclidean unit sphere is twice as small in the given metric; this accounts for the gyromagnetic factor.

Finally the need for generalised curves brings in and clarifies the issue of indeterminism and hidden variables. It is shown that also the pseudo-Riemannian metric must be taken in the generalised sense, in order to accomodate more than one particle in the given manifold.

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Part I. The traditional assumptions of infinitesimal smoothness, implicitly made in various applied topics, do not agree with what is observed at microscopic and smaller dimensions. Mathematical models closer to what actually occurs, are made possible by the theory of generalised curves, and by the matching up and resonance of the infinitesimal patterns they generate. Phenomena hitherto not fully accounted for, may be partly due to such generalised patterns in stress lines of bridges, streamlines of shock waves, and rays of light.

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ABSTRACT (continued)

Part II. In relativistic quantum physics, generalised curves appear on account of the indefiniteness of the metric, and the infinitesimal zigzagging corresponds to successive emission and absorption of radiation. The Nowosad theory, a preview of which is given here, accounts in this way for the elementary particles in a logical manner, and provides at the same time de Broglie's Pilot Wave and the quantum potentials of Bohm and Vigier. Moreover such things as the Pauli exclusion becomes theorems. The theory makes use of singular integrals and of reduced quaternion-valued analytic functions.