





A COMPENSATION METHOD FOR QUADRATURE MODULATION

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BY

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1. INTRODUCTION

Notice the near-term implementation of clutter or extended targets. A test of quadrature modulation has been made with fairly good results (44 dB suppression for single-sided Gaussian spectrum), but manual adjustments were required which are operationally impractical for the 32 modulators of the system. Amplitude-phase modulation has not yet been tested; however, some form of compensation will be required, and it is likely to be complicated, based on the experience with the array pointing attenuators and phase shifters. Tests of amplitude-phase modulation are required to determine the complexity of the compensation.

Because of the encouraging results with quadrature modulation, it would be desirable to pursue the investigation further with this modulation scheme. Towards this end we present a relatively simple compensation method for quadrature modulation which, based on a postulated error model, improves the sidelobe suppression, removes the necessity of frequent manual adjustments, and is amendable to non-real time automatic calibration. A test of the compensation method is required to determine the actual degree of improvement and the complexity of the compensation. We show that such a test can be carried out using presently available hardware. It is recommended that such a test be conducted.

Section 2 presents the error models, Section 3 the compensation methods, and Section 4 a test method and an algorithm for determining the compensation tables. Recommendations are summarized in Section 5.

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2. ERROR MODELS

A general and an approximate error model for a double balanced mixer is postulated. The resultant error model using two mixers is derived.

2.1 MIXER ERROR MODELS

Figure 1 shows the schematic diagram of a double balanced mixer. We postulate that an input x on the I port produces an attenuation and a phase shift of the L port signal. Thus the output is

$$z = A(x)\cos[\omega_t + \phi(x)]$$

=
$$A(x)\cos[\phi(x)]\cos\omega_t - A(x)\sin[\phi(x)]\sin\omega_t$$
 (1)

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where A(x), $\phi(x)$ are functions determined by the error characteristics of the mixer.

Let $f(x) = A(x)\cos[\phi(x)]$ and $g(x) = A(x)\sin[\phi(x)]$. Then

$$z = f(x) \cos \omega_{t} - g(x) \sin \omega_{t} . \qquad (2)$$

If only major sources of error are considered, the above equation can be simplified. The quantity $\phi(x)$ is small and A(x) is practically linear (for small signal input on the I port); hence (2) simplifies to

$$z = x \cos \omega_{t} t - g(x) \sin \omega_{t} t .$$
 (3)



Figure 1. Double Balanced Mixer.

2.2 QUADRATURE MODULATION ERROR MODELS

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Figure 2 shows the schematic diagram of a quadrature modulator.



Figure 2. Quadrature Modulator.

Based on the general mixer error model of Equation (2) the output is

$$z = f_{x}(x) \cos \omega_{c} t - g_{x}(x) \sin \omega_{c} t - f_{y}(y) \sin \omega_{c} t - g_{y}(y) \cos \omega_{c} t$$
$$= [f_{x}(x) - g_{y}(y)] \cos \omega_{c} t - [f_{y}(y) + g_{x}(x)] \sin \omega_{c} t . \qquad (4)$$

Based on the simplified mixer error model of Equation (3) this reduces to

$$z = [x-g_y(y)] \cos \omega_c t - [y+g_x(x)] \sin \omega_c t .$$
 (5)

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3. COMPENSATION

General compensation is shown to be impractical, and a practical simplified compensation is presented. It is shown that this simplified compensation incorporates the current manual adjustments of offset, gain and relative carrier phase, and is amenable to computer controlled calibration (non-real time). This simplified compensation is capable of more than just replacing the current manual adjustment and hence should result in better sidelobe suppression than is currently achieved. A test will have to be performed to determine the degree of improvement. Section 4 discusses a test method using currently available hardware.

3.1 GENERAL COMPENSATION

From Equation (4) we wish to solve for x and y as functions of x^{D} and y^{D} when

$$x^{D} = f_{x}(x) - g_{y}(y) , y^{D} = f_{y}(y) + g_{x}(x) .$$
 (6)

Unfortunately these equations for general compensation with arbitrary error functions cannot be easily solved. Two methods, a full table lookup and an iteration procedure are possible. The full table would require 64K words of memory. The number of iterations required would depend on the error functions. Either method is very similar to what would be required for general amp-phase compensation and is not very practical.

3.2 SIMPLIFIED COMPENSATION

The simplified compensation model is shown in Figure 3. It requires four 256-word tables and 2 adders (which could be multiplexed over a number of modulators).





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The compensating equations are:

$$y = f_{1}(x^{D}) + f_{2}(y^{D})$$
(7)
$$y = f_{3}(x^{D}) + f_{4}(y^{D}) .$$

3.3 INCORPORATION OF PRESENT MANUAL ADJUSTMENTS

Currently compensation is achieved by manually adjusting the offset and gain of the two modulation signals, and the rewire carrier phase. It is shown that the simplified computation model incorporates these adjustments under the assumption that the phase error is small enough so that

$$\cos\phi = 1 \text{ and } \sin\phi = \phi$$
 . (8)

The manual adjustments are equivalent to assuming that

$$A(\mathbf{x}) = a_{\mathbf{x}} + \beta_{\mathbf{x}} \mathbf{x} \quad A(\mathbf{y}) = a_{\mathbf{y}} + \beta_{\mathbf{y}} \mathbf{y}$$
(9)
$$\phi(\mathbf{x}) = \gamma_{\mathbf{x}} \qquad \phi(\mathbf{y}) = \gamma_{\mathbf{y}} \quad .$$

These equations are linear in x and y; hence we can solve for x and y in the form

$$x = a_{x}x^{D} + b_{x}y^{D} + c_{x}$$
(11)
$$y = a_{y}x^{D} + b_{y}y^{D} + c_{y}$$

which can be written as

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$$x = f_{1}(x^{D}) + f_{2}(y^{D})$$
(12)
$$y = f_{3}(x^{D}) + f_{4}(y^{D})$$

where

 $f_1(x^D) = a_x x^D + c_x$ $f_2(y^D) = b_x y^D$ $f_3(x^D) = a_y y^D + c_y$ $f_{4}(y^{D}) = b_{y}y^{D}$

This shows that the simplified compensation with linear tables incorporates the manual adjustments. Since the tables need not be restricted to linear functions the simplified compensation is capable of additional sidelobe suppression.

3.4 AUTO-CALIBRATION

If we can measure, under computer control, the amplitude and phase of the modulator output, then an automatic, non-real time procedure for determining the table entries for the simplified compensation can be implemented. The procedure consists of measuring the output amplitude and phase for a number of input x^{D} and y^{D} values. The program then inverts these equations and thereby determines the table entries. The details of this algorithm are described in Section 4.

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4. TEST METHOD

The test method consists of two phases, a calibration phase and a compensation phase. The calibration phase consists of measuring the output amplitude and phase for a set of input quadrature modulation signals and an algorithm to determine the compensation table values. The compensation phase consists of applying the simplified compensation to our current test signal (say a single-sided Gaussian) and measuring the resultant spectrum.

4.1 CALIBRATION PHASE

4.1.1 Hardware

Figure 4 shows the hardware setup for the calibration phase. The offsets and gains are set to nominal values. This can be accomplished by an adjustment with the single tone test signal and subsequent spectral analysis.





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4.1.2 Software

The software for calibration consists of two parts. Part 1 (which can be programmed on Mini 6, or on the Datacraft with input values transferred to Mini 6) consists of the determination of the modulator transfer function, i.e., tables A(x,y), $\phi(x,y)$ for selected x^D , y^D values. It would be desirable to print out these tables. Part 2 consists of deriving the compensation table values from this transfer function. This program could also be implemented on Mini 6, but to allow the use of a Fortran program it would be desirable to do the programming on the Datacraft. An algorithm is presented in Section 4.3.

4.2 COMPENSATION PHASE

The compensation phase consists of modifying the orignal test signal (say single-sided Gaussian) according to Equation (7). This consists of 4 lookups and 2 adds and can be implemented either real-time on Mini 6 or offline on Mini 6, the Datacraft or CDC.

4.3 COMPENSATION ALGORITHM

4.3.1 General Algorithm

During the calibration phase tables of output amplitude and phase are obtained for input quadrature components. These tables may readily be converted to output quadrature components

$$x^{D} = g_{x}^{\dagger}(x,y)$$
 (14)
 $y^{D} = g_{y}^{\dagger}(x,y)$.

These tables need to be inverted, i.e., x,y as a function of x^D , y^D and then values for the simplified compensation tables must be found that are some best fit to the inverted function. Assuming 8 bit x and y values, the inverted function consists of 64K data points while simplified compensation has only $4 \times 256 = 1K$ table values. Measuring the modulator transfer function at 64K points is time consuming and is not really required since the restricted nature of simplified compensation forces its own interpolation for 63K of those points. Furthermore the determination of the optimum simplified

compensation table values from the modulator transfer function is a rather complicated algorithm. Instead of a general optimum algorithm we propose a simplified algorithm which has the characteristic that when the modulator transfer function has the form of the simplified compensation then this algorithm will provide the correct table values. How well this simplified algorithm approximates the optimum solution when the modulator transfer function is more complex is yet to be determined.

4.3.2 Simplified Algorithm

We assume that the compensation tables (f_1, f_2, f_3, f_4) of Equation (7)) consist of 256 words. This gives us 1K of freedom. Thus we will compute the calbriation functions g'_x, g'_y for only 1K points specifically on a rectangular grid of 32×32 values of x and y. The table values will be computed according to the following formula:

$$f_{1}(x_{i}^{D}) = \frac{1}{32} \sum_{y_{k}^{D}} h_{x}(x_{i}^{D}, y_{\ell}^{D}) - \frac{1}{2} \frac{1}{(32)^{2}} \sum_{x_{k}^{D}} \sum_{y_{\ell}^{D}} h_{x}(x_{k}^{D}, y_{\ell}^{D})$$

$$f_{2}(y_{j}^{D}) = \frac{1}{32} \sum_{\substack{x_{k}^{D} \\ x_{k}}} h_{x}(x_{k}^{D}, y_{j}^{D}) = \frac{1}{2} \frac{1}{(32)^{2}} \sum_{\substack{x_{k}^{D} \\ x_{k}}} \sum_{\substack{y_{\ell} \\ y_{\ell}}} h_{x}(x_{k}^{D}, y_{\ell}^{D})$$

$$f_{3}(x_{1}^{D}) = \frac{1}{32} \sum_{y_{k}^{D}} h_{y}(x_{1}^{D}, y_{\ell}^{D}) - \frac{1}{2} \frac{1}{(32)^{2}} \sum_{x_{k}^{D}} \sum_{y_{\ell}^{D}} h_{y}(x_{k}^{D}, y_{\ell}^{D})$$

$$f_{4}(y_{j}^{D}) = \frac{1}{32} \sum_{\mathbf{x}_{k}^{D}} h_{y}(\mathbf{x}_{k}^{D}, y_{j}^{D}) - \frac{1}{2} \frac{1}{(32)^{2}} \sum_{\mathbf{x}_{k}^{D}} \sum_{\mathbf{y}_{k}^{D}} h_{y}(\mathbf{x}_{k}^{D}, y_{k}^{D})$$

where

 x_{i}^{D}, y_{j}^{D} are the possible (each 256) values of I&Q x_{k}^{D}, y_{i}^{D} are the I,Q values on the coarser grid of 32×32 $x=h_{x}(x^{D}, y^{D})$ $y=h_{y}(x^{D}, y^{D})$ are the inverse functions of the measured calibration p_{j}^{D} 1(

$$x = g_x(x,y)$$
$$y^{D} = g_y^{+}(x,y) .$$

The desired values of h_x, h_y are obtained by linear interpolation on the functions g_x^{\dagger} and g_y^{\dagger} .

Approximations of Simplified Algorithms

The determination of the optimum table values would require both the measurement of the modulation transfer function at many points and a complex iterative algorithm. The simplified algorithm assumes that the modulation transfer function is reasonably approximated by a piecewise linear (twodimensional) function specified at a fraction of the total grid (32×32 instead of 256×256). Since the table values admit only 1K degrees of freedom, the improvement obtainable by measuring the function at more than 1K points is most likely to be insignificant. The simplified algorithm has an important property. If the modulation transfer function is of the same form as the compensation then it will yield the correct solution. This characteristic is shown below.

If

 $h_{x}(x^{D}, y^{D}) = g_{1}(x^{D}) + g_{2}(y^{D})$

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then

$$f_{1}(x_{1}^{D}) = \frac{1}{N} \sum_{y^{D}} (g_{1}(x^{D}) + g_{2}(y^{D})) - \frac{1}{2N^{2}} \sum_{x^{D}} \sum_{y^{D}} (g_{1}(x^{D}) + g_{2}(y^{D}))$$

= $g_{1}(x^{D}) + \frac{1}{N} \sum_{y^{D}} g_{2}(y^{D}) - \frac{1}{2N} \sum_{x^{D}} g_{1}(x^{D}) - \frac{1}{2N} \sum_{y^{D}} g_{2}(y^{D})$

$$f_{2}(y_{j}^{D}) = g_{2}(y^{D}) + \frac{1}{N} \sum_{x^{D}} g_{1}(x^{D}) - \frac{1}{2N} \sum_{x^{D}} g_{1}(x^{D}) - \frac{1}{2N} \sum_{y^{D}} g_{2}(y^{D})$$

Hence

$$f_1(x_1^D) + f_2(y_j^D) = g_1(x^D) + g_2(y_D)$$
.

The same argument applies for $h_y(x^D, y^D)$ and f_3 and f_4 .

If the modulation transfer function is not of the same form as the compensation then the simplified algorithm yields only an approximation to the optimum solution. How well the optimum is approximated, in this case, has not yet been determined. N. S. Sec.

5. RECOMMENDATIONS

Tests of quadrature modulation without compensation achieved fairly good results (44 dB suppression for single-sided Gaussian spectrum). This memorandum presents a relatively simple compensation method which should improve the performance. The degree of improvement actually obtainable will have to be determined by a test. Compared to the amplitude-phase compensation this quadrature compensation requires less hardware because it does not require the additional step of conversion to amplitude-phase. It is recommended that a test of this quadrature compensation be conducted to determine the degree of improvement.