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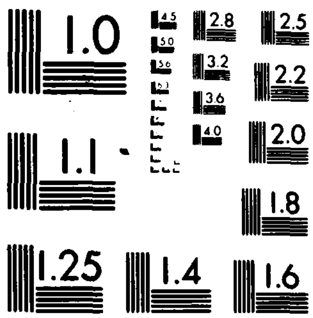

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Effect of Finite Electron Drift Velocity on the Resonant Excitation of a Nonuniform Plasma,

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Effect of Finite Electron Drift Velocity on the Resonant Excitation of a Nonuniform Plasma

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ABSTRACT

An exact analytic solution of the external resonant excitation of a nonuniform plasma in the presence of a zero order finite electron drift is presented. The process of linear mode conversion of the long wavelength external radiation, of frequency ω, into a Langmuir wave limits the peak

omega omega sub p

amplitude of the cold plasma resonance at $\omega = \omega_p$, where ω_p is the local electron plasma frequency. The finite electron drift alters the effective group velocity of the Langmuir wave, and thus it modifies the peak amplitude of the resonance in a significant manner. For drifts toward the overdense side an amplitude enhancement is obtained, while for drifts toward the underdense side a severe quenching takes place. The effect is governed by a single scaled drift parameter $\cancel{u = (2 v_D/3\bar{v})(3\omega_p L/\bar{v})^{1/3}}$, and significant modifications arise when $\cancel{|u| > 1}$. In here, v_D is the small drift velocity, $\bar{v}$ the electron thermal velocity, and L the density scale length.

its absolute value is greater than 1.

I. INTRODUCTION

In recent years the subject of linear mode conversion[1-4] has received considerable attention in regard to the propagation and absorption of electromagnetic radiation in nonuniform plasmas. Some of the areas in which this process plays an important role are: ionosphere modification, laser fusion, RF heating of magnetized plasmas, and the spontaneous radio emissions from astrophysical bodies. The essential feature associated with linear mode conversion is the existence of some point in the plasma where the local dispersion relations of two different modes overlap, e.g., locally $\omega_1 = \omega_2$, $k_1 = k_2$, but asymptotically $k_1 \neq k_2$, for two modes of frequency ω_j and wavenumber k_j. At the mode conversion point energy and momentum can be transferred from mode 1 to mode 2, or vice versa. An important property of the process is that it depends linearly on the wave amplitude, hence the coupling between the two modes depends only on the zero order equilibrium properties of the medium, and it does not exhibit an excitation threshold. This behavior is to be contrasted with parametric instabilities in which the coupling depends explicitly on wave amplitude and a definite threshold exists. Of course, mode conversion also becomes amplitude dependent[4,5] if the wave amplitudes are sufficiently large so that the zero order quantities (density, drift velocity, temperature) are modified by nonlinear interactions (e.g., ponderomotive forces).

The cleanest environment for the study of linear mode conversion consists of the resonant excitation of an unmagnetized nonuniform density plasma by an external capacitor plate[6] field (near field) E_o oscillating at frequency ω. Since the external field has the properties ($k = 0$, ω) it can mode convert into a Langmuir wave which satisfies $\omega^2 = \omega_p^2 + 3k^2 \bar{v}^2$, at a point along the density gradient where $\omega = \omega_p$. In here, ω_p refers to the electron plasma frequency and $\bar{v}$ is the electron thermal velocity. The usage of a capacitor plate pump field for a study of resonant excitation adequately models the more general situation encountered when an electromagnetic wave propagates obliquely at an angle θ relative to the density gradient. The reason is that the electromagnetic wave encounters a cut-off at that location where $\omega_p = \omega\cos\theta$, thus the electric field which reaches the $\omega = \omega_p$ point is evanescent (i.e., $k \simeq 0$). The capacitor plate field is to be identified conceptually with the evanescent field of the electromagnetic wave.

The present investigation is concerned with the modification of the linear mode conversion process by the presence of a small electron drift velocity v_D, with $v_D << \bar{v}$. Such a drift can be found in naturally occuring plasmas, as in the auroral ionosphere, or can be created artificially in the laboratory. Of course, the drift can also be the result of the resonant absorption process itself.

For values of v_D less than are required to trigger the ion acoustic instability, as is assumed in this study, the principal effect consists of the modification of the group velocity v_g of the electron plasma wave at the point $\omega = \omega_p$. For $v_D = 0$, $v_g = 0$ at the resonance point, hence the amplitude of the electric field builds up to a large level given by[4] $E \sim E_o \, (\omega_p L/\bar{v})^{2/3}$ where L represents the scale length of the density profile. In this case the resonant external pumping of the plasma is limited by the leakage of the mode converted wave down the density gradient, an effect which is manifested by the L dependence of the amplitude scaling. However, for $v_D \neq 0$, $v_g = v_D$ at $\omega = \omega_p$. Therefore, the cold plasma resonance is destroyed and a corresponding change in the mode conversion process takes place. In particular, if v_D points in the direction of decreasing density the zero order leakage rate due to the density gradient is enchanced and results in the decrease of the peak amplitude attained by the resonant electric field. However, if v_D points in the direction of increasing density, the leakage down the density gradient is reduced and results in an enhancement of the peak amplitude. In addition to the changes in peak amplitude, the entire Airy-like pattern of the mode converted wave can be blown down (or up) the density gradient, depending on the direction of the drift. This work provides an analytic description of these changes.

Section II presents the physical model underlying the process. In Section III the exact formal solution of the problem is obtained and the asymptotic behavior is extracted. Section IV exhibits the waveforms and parameter dependences of the exact solution. Conclusions are given in Section V.

II. FORMULATION

The essential physics of the problem considered can be treated through a fluid description of the electron response provided that the zero order electron drift velocity v_D remains smaller than the threshold drift required to trigger the ion acoustic instability. It is also understood that the ever increasing electron Landau damping encountered by the mode converted Langmuir wave as it propagates down the density gradient is neglected. This kinetic effect does not significantly alter the peak amplitude of the resonance and its proper inclusion requires an integral equation formulation which is beyond the scope of the present study.

The zero order plasma density profile $n_o(x)$ is taken to be stationary and represented locally by a linear function of position, i.e., $n(x) = n_p (1 + x/L)$, where n_p is the density at the point $x = 0$ defined by the resonance condition $\omega = \omega_p$. The external pump electric field is given by $E_p = E_o \exp(- i\omega t)$ and points along the density gradient. A qualitative sketch of the relevant geometry is shown in Fig. 1.

The linearized high frequency electron fluid velocity $\tilde{v}$ oscillating at frequency ω satisfies the force equation

$$\left(-i\omega + v_D \frac{d}{dx}\right)\tilde{v} = -\frac{e}{m}\tilde{E} - \frac{3\bar{v}^2}{n_o}\frac{d}{dx}\tilde{n} \tag{1}$$

where the high frequency electron density $\tilde{n}$ is self-consistently determined by the linearized continuity equation

$$-i\omega\tilde{n} + \frac{d}{dx}\left(v_D\tilde{n} + n_o\tilde{v}\right) = 0 \tag{2}$$

and the total high frequency field $\tilde{E}$ must satisfy Poisson's equation

$$\frac{d}{dx}\left(\tilde{E} - E_o\right) = -4\pi e\,\tilde{n} \tag{3}$$

in which the explicit role of the external charges is retained to account for the pump field.

Inserting Eq (3) into (2) yields

$$-i\omega\frac{d}{dx}\left(\tilde{E} - E_o\right) + \frac{d}{dx}\left[v_D\frac{d}{dx}\left(\tilde{E} - E_o\right) - 4\pi e n_o \tilde{v}\right] = 0 \tag{4}$$

or

$$\frac{d}{dx}\left[D\left(\tilde{E} - E_o\right) - 4\pi e n_o \tilde{v}\right] = 0 \tag{5}$$

where $D = (-i\omega + v_D\, d/dx)$ is the total time derivative operator. Applying D to Eq (5) and using Eq (1) results in

$$D^2\left(\tilde{E} - E_o\right) + \omega_p^2 \tilde{E} - 3\bar{v}^2\frac{d^2}{dx^2}\left(\tilde{E} - E_o\right) = c \tag{6}$$

in which a constant c appears. This constant can be set equal to zero because the correct boundary conditions (vacuum) are automatically satisfied by the explicit inclusion of the external pump in Eq. (3).

Proceeding to expand out D^2 and neglecting the terms $(d/dx)E_o$ and $(d^2/dx^2)E_o$ because the pump amplitude E_o varies slowly over the region of interest (i.e., the resonance scale length) results in

$$\frac{3\bar{v}^2}{\omega^2}\frac{d^2}{dx^2}\tilde{E} + i\frac{2v_D}{\omega}\frac{d}{dx}\tilde{E} + \left(1 - \frac{\omega_p^2}{\omega^2}\right)\tilde{E} = E_o \tag{7}$$

where it is explicitly (and consistently) assumed that $v_D^2 \ll \bar{v}^2$.

The new physics being considered appears in Eq. (7) through the first derivative term which has a purely imaginary coefficient proportional to the electron drift velocity. A physical picture for the role played by the drift can be obtained by analyzing the local dispersion relation for a Langmuir wave having wavenumber k, i.e.,

$$\omega = k v_D + (\omega_p^2 + 3k^2 \bar{v}^2)^{1/2} \tag{8}$$

which is qualitatively sketched in Fig. 2. In this figure the solid curve corresponds to $v_D = 0$ and the dashed curve corresponds to a case $v_D > 0$, i.e., the drift is toward the overdense plasma. It is seen that for $v_D > 0$ the group velocity $v_g > 0$ at k=0. This implies that in this environment the external pump can match k and ω, thus it is still able to excite a Langmuir wave. However, because $v_g > 0$ the excited mode is a backward wave ($k < 0$, $v_g > 0$) which propagates into the overdense plasma up to a point $x = (v_D^2/3\bar{v}^2)L$, where $v_g = 0$. At this point the wave turns around and becomes a forward wave ($k < 0$, $v_g < 0$) which proceeds to propagate down the density gradient. Therefore, for $v_D > 0$ it is expected that the mode conversion pattern is shifted into the overdense plasma and the amplitude of the resonance should be enhanced because the convection due to the drift opposes the natural leakage associated with the density gradient.

For $v_D < 0$ (i.e., drift down the gradient) mode conversion also occurs at $\omega = \omega_p$, but now $v_g < 0$, hence the excited wave is a forward wave from the outset and propagates down the gradient at a rate faster than the leakage rate due to the density gradient alone. Consequently, the mode conversion pattern is expected to be shifted (blown down) to the underdense side and the peak amplitude of the resonance should decrease due to the enhanced convection.

It is useful to introduce the scale amplitude and space variables

$$A = (\tilde{E}/E_o)(\omega_p L/\sqrt{3}\,\bar{v})^{-2/3}$$
$$z = (x/L)(\omega_p L/\sqrt{3}\,\bar{v})^{2/3} \tag{9}$$

to transform Eq. (7) into a single parameter equation

$$\frac{d^2}{dz^2}A + iu\frac{d}{dz}A - zA = 1 \tag{10}$$

in which the new physics associated with the drift is determined by the lumped parameter u defined by

$$u = (2v_D/3\bar{v})(3\omega_p L/\bar{v})^{1/3} \tag{11}$$

Physically, this parameter represents the ratio of the drift velocity to the effective leakage speed associated with the density gradient. It should be noted that although $v_D \ll \bar{v}$, the parameter u can be of order unity or larger because $\omega_p L/\bar{v}$ can attain rather large values both in natural and laboratory plasmas. As is expected from Eq. (10), and shown in Sec. IV, the range of values for which the effect of the drift becomes important is $|u| > 1$.

III. FORMAL SOLUTION

To solve Eq. (10) one proceeds to eliminate the first derivative term by introducing the auxiliary function ψ defined by

$$A(z) = \psi(z)\exp(-iuz/2) \tag{12}$$

which transforms Eq. (10) into

$$\frac{d^2}{dz^2}\psi - (z - u^2/4)\psi = \exp(iuz/2) \tag{13}$$

Defining a shifted coordinate $\xi = z - u^2/4$ yields

$$\frac{d^2}{d\xi^2}\psi - \xi\psi = \exp\left[i(u\xi/2 + u^3/8)\right] \tag{14}$$

which is a generalization of the inhomogeneous Airy equation in which the forcing function now oscillates in space. The reason for this oscillation is that the density gradient destroys the translational invariance in this problem.

The general solution of Eq. (14) is given by the linear superposition of the homogeneous solutions[7] of the Airy equation (Ai, Bi), and the inhomogeneous solution, namely,

$$\psi(\xi) = \alpha\,Ai(\xi) + \beta\,Bi(\xi) - \pi\exp(iu^3/8)\left\{ Ai(\xi)\int_0^\xi dt\,Bi(t)\exp(iut/2) - Bi(\xi)\int_0^\xi dt\,Ai(t)\exp(iut/2) \right\} \tag{15}$$

as can be easily verified by direct substitution.

In Eq. (15) α and β are constants which depend on the parameter u, and their values are determined by the asymptotic boundary conditions as

$\xi \to \pm \infty$. For $\xi \to +\infty$ (overdense region) the electric field amplitude (and hence ψ) must remain bounded. Since in the limit $\xi \to \infty$

$$\mathrm{Bi}(\xi) \sim \exp\left(\tfrac{2}{3}\xi^{3/2}\right)/(\pi^2 \xi)^{1/4}$$

$$\mathrm{Ai}(\xi) \sim \exp\left(-\tfrac{2}{3}\xi^{3/2}\right)/(\pi^2 \xi)^{1/4} \tag{16}$$

Eq. (15) requires that β must satisfy

$$\beta(u) = -\pi \exp(iu^3/8)\,\Lambda(u)$$

$$\Lambda(u) \equiv \int_0^\infty dt\,\mathrm{Ai}(t)\exp(iut/2) \tag{17}$$

To evaluate α one demands that as $\xi \to -\infty$ (underdense side) ψ must asymptotically approach a wave travelling down the density gradient. Since to evaluate α explicitly one needs to know Λ as well as the asymptotic behavior of the other terms in Eq. (15), we proceed to examine $\Lambda(u)$.

In calculating $\Lambda(u)$ it should be noted that the quantity $\Lambda(0)$ can be easily obtained by applying Cauchy's theorem in the complex t plane and using the integral definition of Ai(t). However, it is nontrivial to extend this technique for $u \neq 0$. Consequently, we proceed to find $\Lambda(u)$ by direct manipulation in the real t plane. Integrating by parts twice in Eq. (17) yields

$$\Lambda(u) = (2i/u)\mathrm{Ai}(0) + (2i/u)^2\left[\mathrm{Ai}'(0) + \int_0^\infty dt\,\mathrm{Ai}''(t)\exp(iut/2)\right] \tag{18}$$

where the prime notation represents the derivative operator. Recognizing that $Ai'' = tAi$ and using the definition of Λ in Eq. (17) yields a differential equation in u

$$\Lambda(u) = (2i/u)Ai(0) - (2/u)^2 Ai'(0) + (2i)(2/u)^2 \frac{d}{du}\Lambda(u) \tag{19}$$

which can be integrated over u to yield

$$\Lambda(u) = \exp(-iu^3/24)\left\{\begin{array}{l}\Lambda(0) \\ -\frac{1}{2}\int_0^u du \exp(iu^3/24)\left[\frac{u}{2}Ai(0) + iAi'(0)\right]\end{array}\right\} \tag{20}$$

The remaining indefinite integrals in Eq. (20) can be expressed in terms of the incomplete γ function[8]

$$\begin{aligned}\int_0^u du \exp(iu^3/24) &= \frac{2\exp(i\pi/6)}{(3)^{2/3}}\gamma(1/3; -iu^3/24) \\ \int_0^u du\, u \exp(iu^3/24) &= \frac{4\exp(i\pi/3)}{(3)^{1/3}}\gamma(2/3; -iu^3/24)\end{aligned} \tag{21}$$

to find

$$\Lambda(u) = \exp(-iu^3/24)\left\{\begin{array}{l}\frac{1}{3} - \frac{Ai(0)\exp(i\pi/3)\gamma(2/3; -iu^3/24)}{(3)^{1/3}} \\ - \frac{iAi'(0)\exp(i\pi/6)\gamma(1/3; -iu^3/24)}{(3)^{2/3}}\end{array}\right. \tag{22}$$

and where $Ai(0) = (3)^{-2/3}/\Gamma(2/3)$, $Ai'(0) = -(3)^{-4/3}/\Gamma(4/3)$. In the small u limit Eq. (22) results in

$$\Lambda(u) \approx \frac{1}{3} - \left(\frac{u^2}{8}\right)Ai(0) - i\left(\frac{u}{2}\right)Ai'(0) \tag{23}$$

Using the integral representation for the functions Ai, Bi, it is straightforward to apply the saddle point method to extract the asymptotic behavior of the terms appearing in Eq. (15). The results for $\xi \to \infty$ are

$$\int_0^{\xi} dt\, Ai(t) \exp(iut/2) \sim \Lambda(u) - \frac{1}{2\sqrt{\pi}} \frac{\exp[i(u\xi/2) - \eta]}{[(\xi)^{1/2} - iu/2]} \tag{24}$$

$$\int_0^{\xi} dt\, Bi(t) \exp(iut/2) \sim \frac{1}{\sqrt{\pi}} \frac{\exp[i(u\xi/2) + \eta]}{[(\xi)^{1/2} + iu/2]} \tag{25}$$

where, $\eta = (2/3)\ (\xi)^{3/2}$. In the limit $\xi \to -\infty$ these terms take the form

$$\int_0^{\xi} dt\, Ai(t) \exp(iut/2) \sim \Lambda(u) - \exp(-iu^3/24)$$
$$+ \frac{\exp(-iu|\xi|/2)}{2\sqrt{\pi}\, |\xi|^{1/4}} \left\{ \frac{\exp[-i(\eta + \pi/4)]}{|\xi|^{1/2} + u/2} + \frac{\exp[i(\eta + \pi/4)]}{|\xi|^{1/2} - u/2} \right\} \tag{26}$$

$$\int_0^{\xi} dt\, Bi(t) \exp(iut/2) \sim \Gamma(u)$$
$$- \frac{i \exp(-iu|\xi|/2)}{2\sqrt{\pi}\, |\xi|^{1/4}} \left\{ \frac{\exp[-i(\eta + \pi/4)]}{|\xi|^{1/2} + u/2} - \frac{\exp[i(\eta + \pi/4)]}{|\xi|^{1/2} - u/2} \right\} \tag{27}$$

The spatially independent term $\Gamma(u)$ in Eq. (27) can be obtained in a manner analogous to the calculation of $\Lambda(u)$. The result is

$$\Gamma(u) = -\exp(-iu^3/24) \left\{ \frac{Bi(0) \exp(i\pi/3)\, \gamma(2/3; -iu^3/24)}{(3)^{1/3}} + \frac{i\, Bi'(0) \exp(i\pi/6)\, \gamma(1/3; -iu^3/24)}{(3)^{2/3}} \right\} \tag{28}$$

and in the small u limit

$$\Gamma(u) \approx -i\,\frac{u}{2}\,Bi'(0) - \frac{u^2}{8}\,Bi(0) \tag{29}$$

Taking into consideration that as $\xi \to -\infty$

$$\begin{aligned} Ai(\xi) &\sim \frac{1}{\sqrt{\pi}\,|\xi|^{1/4}} \sin(\eta + \pi/4) \\ Bi(\xi) &\sim \frac{1}{\sqrt{\pi}\,|\xi|^{1/4}} \cos(\eta + \pi/4) \end{aligned} \tag{30}$$

and using the asymptotic forms in Eqs. (26) and (27) yields the leading behavior in the underdense side of the profile

$$\psi(\xi) \sim \frac{1}{\sqrt{\pi}\,|\xi|^{1/4}} \left\{ \begin{array}{l} \left[\alpha - \pi \exp(iu^3/8)\,\Gamma(u)\right] \sin(\eta + \pi/4) \\ - \pi \exp(iu^3/12) \cos(\eta + \pi/4) \end{array} \right\} \tag{31}$$

which indicates that the choice

$$\alpha = \pi \exp(iu^3/8)\,\Gamma(u) - i\pi \exp(iu^3/12) \tag{32}$$

results in an asymptotically travelling wave (i.e., the mode converted wave).

Having evaluated the coefficients $\alpha(u)$ and $\beta(u)$ the solution is completely determined

$$A(z) = \psi(z - u^2/4) \exp(-iuz/2) \tag{33}$$

with

$$\psi(\xi) = \pi \exp(iu^3/8) \begin{cases} Ai(\xi)\left[\Gamma(u) - i\exp(-iu^3/24) - \int_0^\xi dt\, Bi(t)\exp(iut/2)\right] \\ + Bi(\xi)\left[-\Lambda(u) + \int_0^\xi dt\, Ai(t)\exp(iut/2)\right] \end{cases} \tag{34}$$

The asymptotic behavior for the field amplitude can be obtained with the help of the expressions in Eqs. (24) - (27). In the overdense side, $z \to +\infty$, the quantity ψ behaves as

$$\psi(\xi) \sim -\frac{\exp\left[i(u^3/8 + u\xi/2)\right]}{(\xi + u^2/4)} \tag{35}$$

which together with Eq. (31) results in the leading term

$$A(z) \sim -\frac{1}{z} \tag{36}$$

showing that deep into the plasma the behavior is insensitive to the value of u, i.e., the response is entirely determined by the cold fluid response $E \sim E_o/\varepsilon$ with $\varepsilon = 1 - \omega_p^2/\omega^2$.

In the underdense region $z \to -\infty$ the leading behavior consists of

$$A(z) \sim -\frac{1}{z} + \frac{\sqrt{\pi}\exp\left[i(u^3/12 - \pi/4)\right]\exp\left\{i\left[(2/3)|z - u^2/4|^{3/2} - uz/2\right]\right\}}{|z - u^2/4|^{1/4}} \tag{37}$$

which explicitly shows the coexistence of the mode converted wave and the pump. It is seen that asymptotically the effective origin of the Langmuir wave is shifted up the gradient by an amount $u^2/4$. However, the asymptotic amplitude of the mode converted wave does not exhibit any u dependent changes.

Near the resonance region ($z \simeq 0$), where the peak electric field is generated, there is no convenient simple representation of the formal solution. Therefore, we investigate this interesting behavior of the system by directly plotting Eq. (33), as shown in the next section. For the sake of completeness, the response of the system to a delta function $\delta(x)$ source in the presence of finite electron drift is included in the appendix.

IV. SPATIAL BEHAVIOR

The exact spatial behavior of the scaled electric field $A(z)$ is obtained by evaluating the formal solution, given by Eq. (33), with the help of a digital computer. The resulting waveforms are shown in Fig. 3 for $u = 0, \pm 5.0$. In this figure the solid curves represent the real part of A, and the dashed curves the imaginary part. The corresponding modulus (i.e., $|A|^2$) is exhibited in Fig. 4. In this figure the slopping straight line shown for the $u = 0$ case represents the effective cold plasma diaelectric, $\varepsilon' = \varepsilon(\omega_p L/\sqrt{3}\,\bar{v})^{2/3}$, or equivalently, the scaled density profile. It should be noted that in Fig. 4 different amplitude scales are used for different values of u in order to appropriately display the waveforms.

It is observed from Figs. 3 and 4 that the qualititative behavior obtained from the simple consideration of Eq. (8), and discussed in Sec. II, is indeed reproduced by the formal solution. For $u < 0$, i.e., when the drift is directed toward the underdense side, the Airy-like pattern is blown down the gradient. Since in this case the effective group velocity is increased, the peak amplitude of the resonance is reduced. For $u > 0$, i.e., when the drift points toward the overdense side, the opposite behavior is encountered. In this case the Airy-like pattern penetrates beyond the resonance point ($z = 0$), and since the drift opposes the convection, the peak amplitude of the resonace is enhanced above the $u = 0$ level. It is also found from Figs. 3 and 4 that for $|z| \gg 1$ the field amplitude is independent of u, as predicted by the asymptotic analysis given in Eq. (37). In addition to these effects, it is found that the wavelength of the mode converted wave is shortened as $|u|$ increases, as expected from Eq. (8).

The dependence of the location Z_m of the peak electric field on the drift parameter is exhibited in Fig. 5. It is found that for $u > 0$, Z_m increases as

u^2. The corresponding u dependence of the square of the maximum value of the resonant electric field, $|A_m|^2$, is shown in Fig. 6. It is observed that for $0 \leq u \leq 4.0$ the peak value increases monotonically with u, eventually attaining a maximum enhancement factor $|A_m(u)|^2/|A_m(0)|^2 \simeq 3.6$. However, the enhancement saturates as u increases further because of the decreasing scale length of the pattern as u increases. For $u \gg 1$ the enhancement becomes nearly independent of u and approaches a value $|A_m(u)|^2/|A_m(0)|^2 \simeq 3.2$ asymptotically. For $u < 0$ the peak amplitude of the resonance decreases monotonically with u due to the enhanced wave convection.

V. CONCLUSIONS

An exact analytic solution of the resonant excitation of a nonuniform plasma in the presence of a zero order finite electron drift velocity has been obtained. The process of linear mode conversion of the external long wavelength radiation into a short wavelength Langmuir wave is found to limit the peak amplitude of the cold plasma resonance occurring at $\omega = \omega_p(x)$. The inclusion of a finite electron drift alters the effective group velocity of the Langmuir wave, and thus it modifies the peak amplitude of the resonance. For drifts that point up the density gradient an enhancement is obtained, while for drifts down the gradient a severe quenching can take place. The effect is shown to be governed by the scaled drift parameter $u = (2v_D/3\bar{v})(3\omega_p L/\bar{v})^{1/3}$, and significant modifications are obtained when $|u| > 1$. This level of u may be attainable in some practical situations at very low drift velocities v_D because of the large density scale lengths L that can be encountered in the laboratory or the ionosphere. Consequently, the various effects found in this study should be considered when interpreting and or planning experiments concerned with the resonant excitation of nonuniform plasmas in these environments.

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BIBLIOGRAPHY

1. T. H. Stix, Phys. Rev. Lett. 15, 878 (1965).
2. A. D. Piliya, Zh. Tekh. Fiz. 36, 819 (1966) [Sov. Phys. - Tech. Phys. 11, 609 (1966).]
3. D. L. Kelly and A. Baños, Jr., Plasma Physics Group, University of California, Los Angeles Report PPG-170 (1974).
4. G. J. Morales and Y. C. Lee, Phys. Fluids 20, 1135 (1977).
5. G. J. Morales, Phys. Fluids, 20 1164 (1977).
6. H. C. Kim, R. Stenzel, and A. Y. Wong, Phys. Rev. Lett., 33, 866 (1974).
7. H. A. Antosiewicz, in Handbook of Mathematical Functions, edited by M. Abromowitz and I. A. Stegun (Dover, New York, 1970), p. 444.
8. P. J. Davis, in Handbook of Mathematical Functions, edited by M. Abromowitz and I. A. Stegun (Dover, New York, 1970), p. 260.

APPENDIX

For completeness, the response of the plasma to a delta function source in the presence of finite electron drift is presented. The relevant scaled equation, equivalent of Eq. (10), is

$$\frac{d^2}{dz^2} A + iu \frac{d}{dz} A - zA = \delta(z - z_0) \tag{A1}$$

where z_o is the location of the source. Transforming as in Eq. (12) and shifting coordinates leads to

$$\frac{d^2}{d\xi^2}\psi - \xi\psi = \exp(iuz_0/2)\, \delta\left(\xi + \frac{u^2}{4} - z_0\right) \tag{A2}$$

Applying the outgoing wave boundary condition for $\xi \to -\infty$ results in the two separate expressions

$$\psi(\xi) = -\pi \, Ai(z_0 - u^2/4) \exp(iuz_0/2) \left[Bi(\xi) + i\,Ai(\xi)\right] \tag{A3}$$

for $\xi < z_0 - u^2/4$, and

$$\psi(\xi) = -\pi \left[Bi(z_0 - u^2/4) + i\,Ai(z_0 - u^2/4)\right] \exp(iuz_0/2)\, Ai(\xi) \tag{A4}$$

for $\xi > z_0 - u^2/4$.

Transforming back to obtain the electric field A in terms of the scaled coordinate z yields

$$A(z) = -\pi\, Ai(z_+ - u^2/4) \exp\left[-iu(z - z_0)/2\right] \left[Bi(z_- - u^2/4) + i\,Ai(z_- - u^2/4)\right] \tag{A5}$$

where z_+ is the greater of (z, z_o) and z_- the lesser of (z, z_o). The result of (A5) has been plotted for several (~10) uniformly spaced sources and it is found that the behavior predicted by Eq. (33) is recovered. Of course, some additional sharp edges are obtained which are not present for the uniform pump case.

FIGURE CAPTIONS

Fig. 1. Qualitative sketch of the geometry of the problem. The density n(x) corresponds to the slopping line and the small electron drift v_D can point up or down the density gradient.

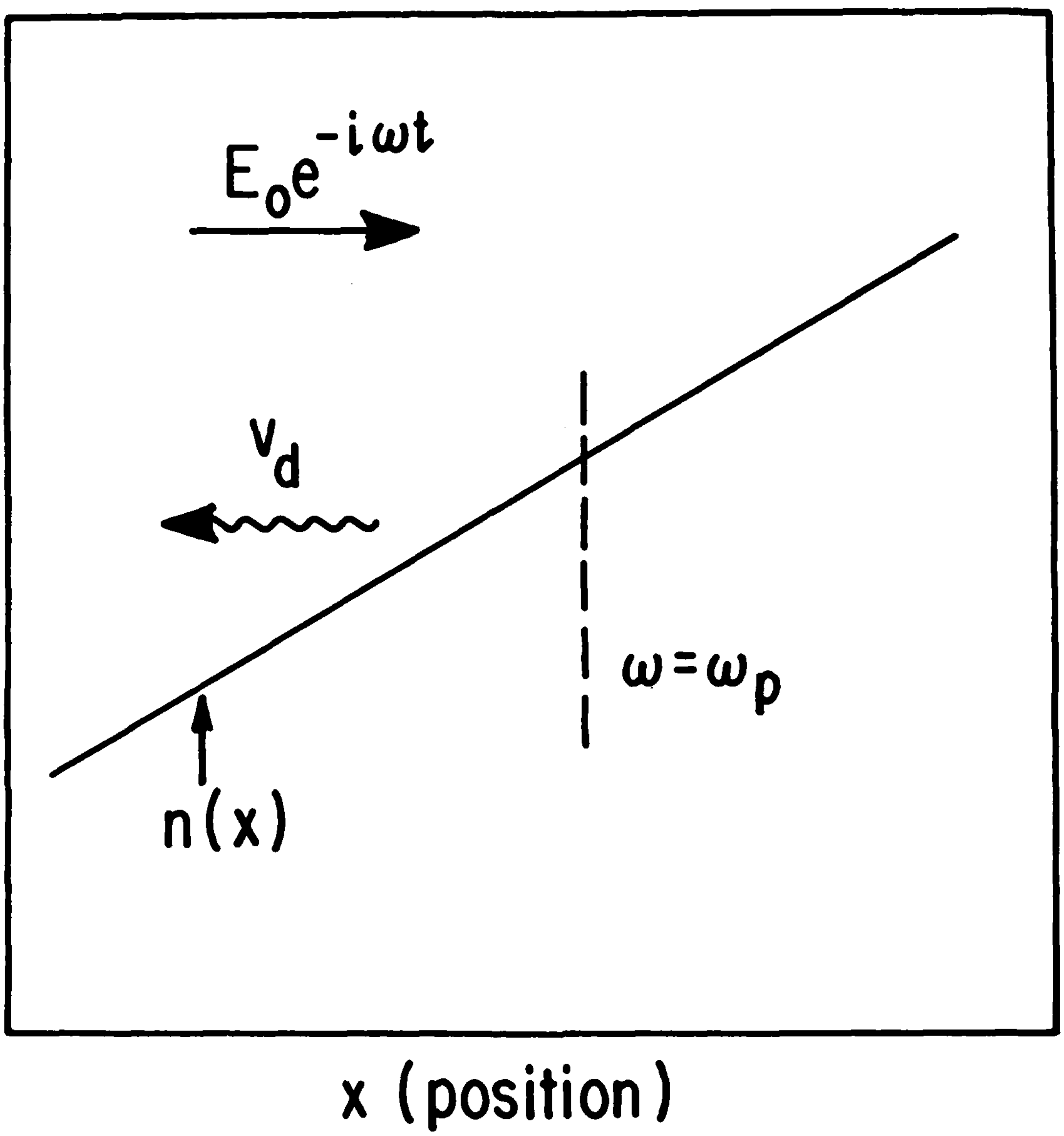
Fig. 2. Qualitative sketch of the effect of finite electron drift velocity on the local dispersion relation for Langmuir waves.

Fig. 3. Spatial dependence of the scaled electric field A for values of the scaled drift parameter $u = 0, \pm 5.0$. The solid curves correspond to the real part and the dashed curves to the imaginary part.

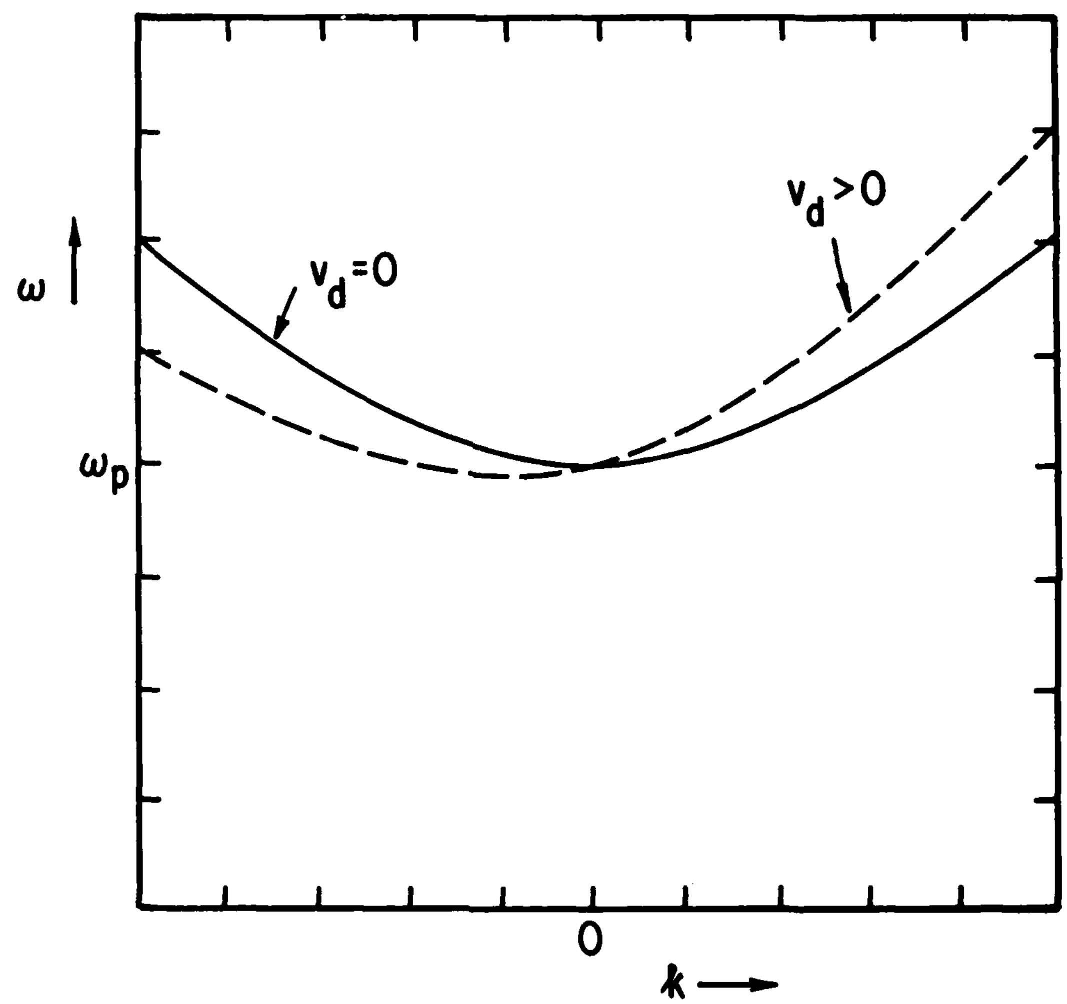
Fig. 4. Spatial dependence of the square of the modulus corresponding to the waveforms of Fig. 3. Note that different amplitude scales are used. The slopping line shown for $u = 0$ represents ϵ' the effective dielectric, or equivalently, the scaled density profile.

Fig. 5. Dependence of the position z_m of the peak amplitude of the electric field on the drift parameter. The cold plasma resonance corresponds to $z_m = 0$.

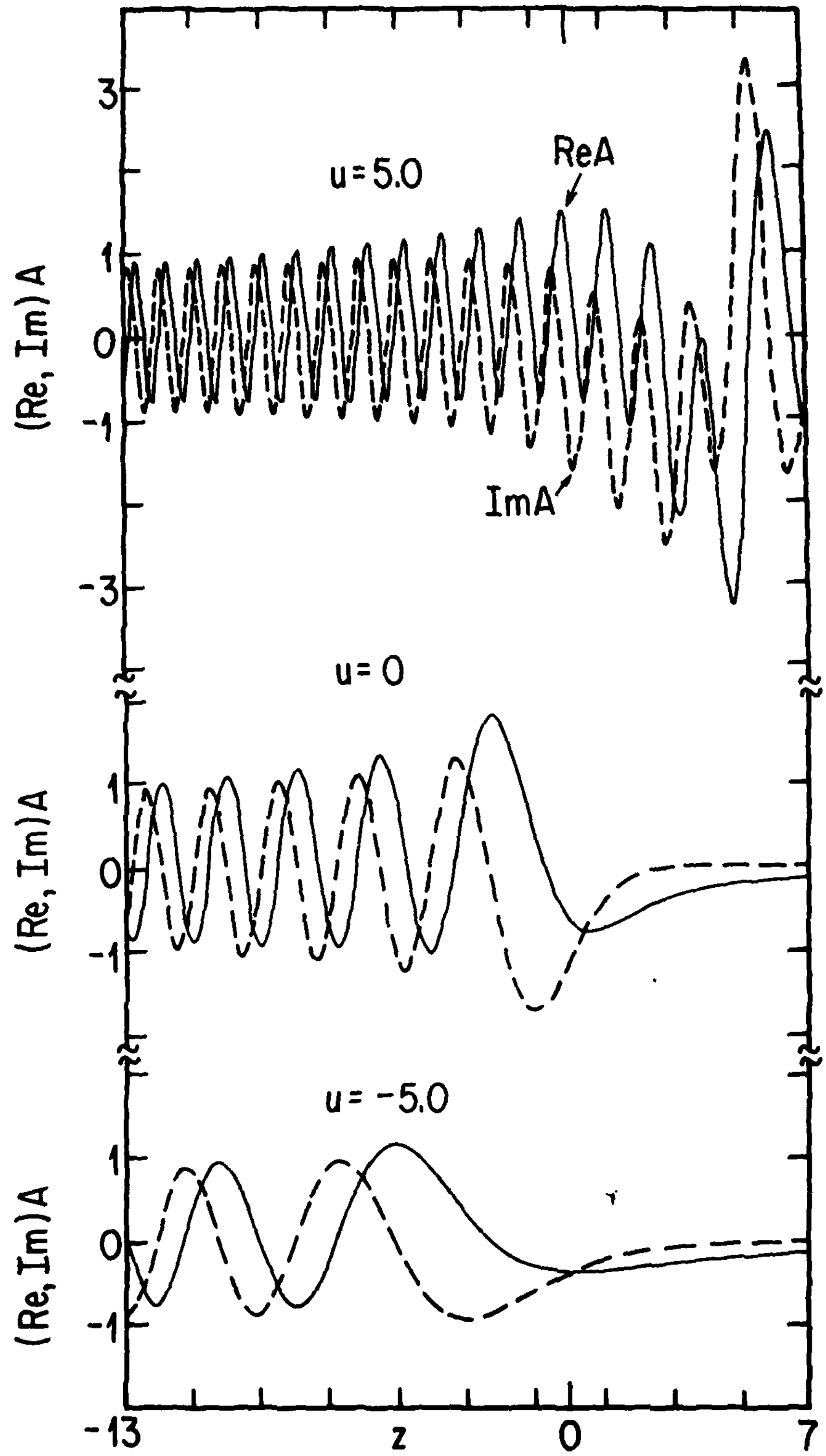
Fig. 6. Dependence of the peak amplitude squared $|A_m|^2$ on the drift parameter. $u > 0$ corresponds to drifts toward the overdense side.



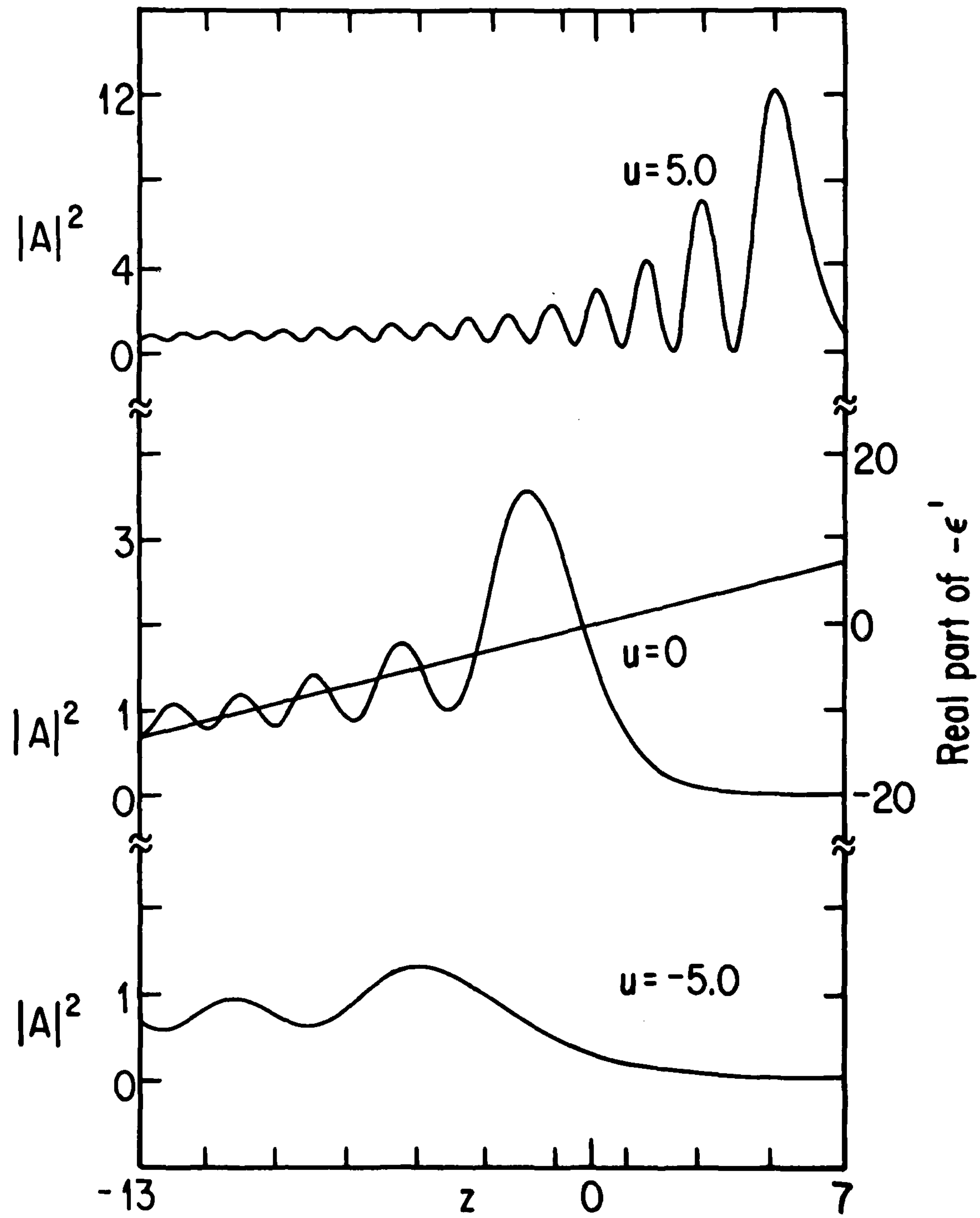
$E_o e^{-i\omega t}$
v_d
$\omega = \omega_p$
n(x)
x (position)



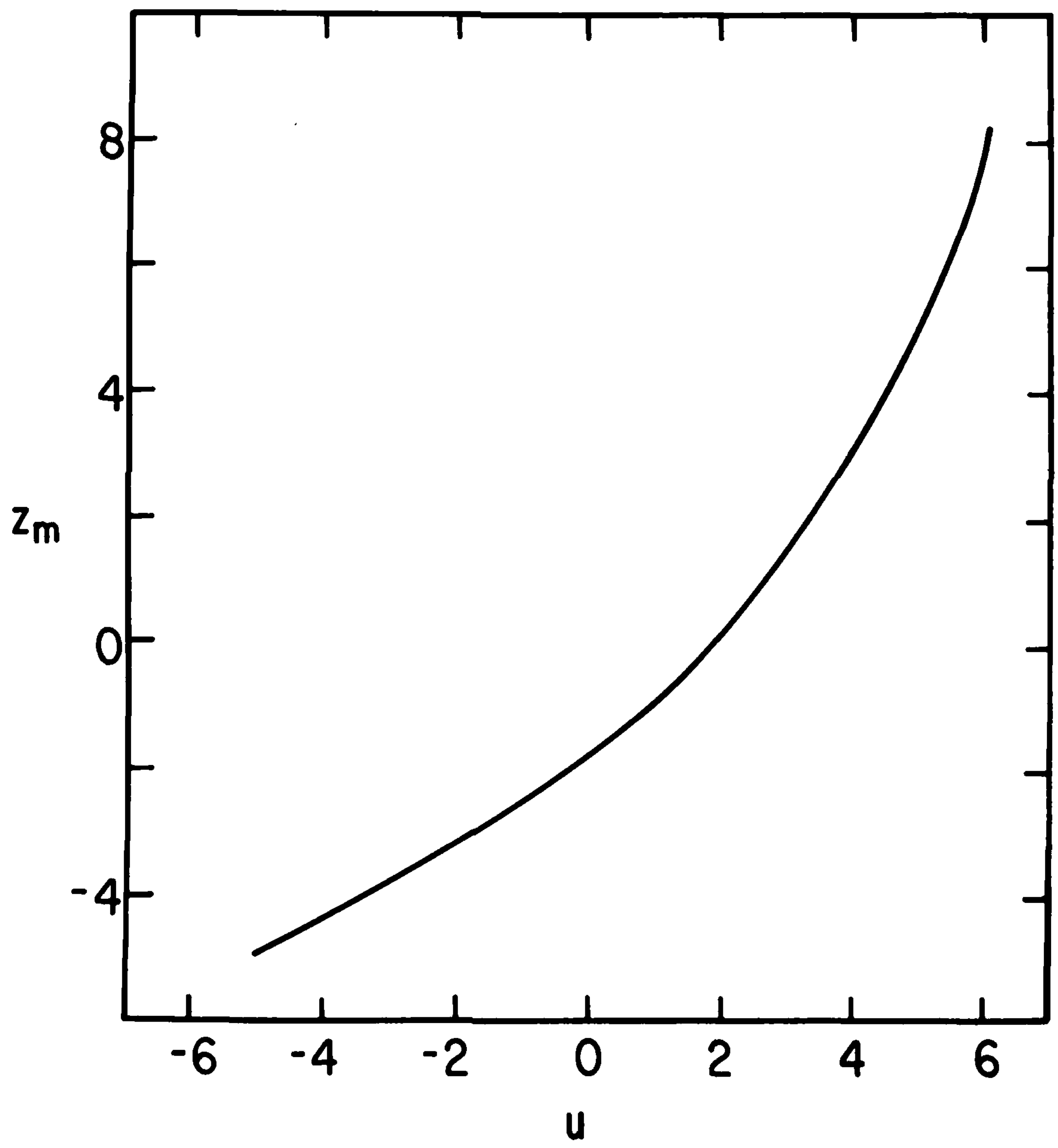
ω
$v_d=0$
$v_d>0$
ω_p
0
k



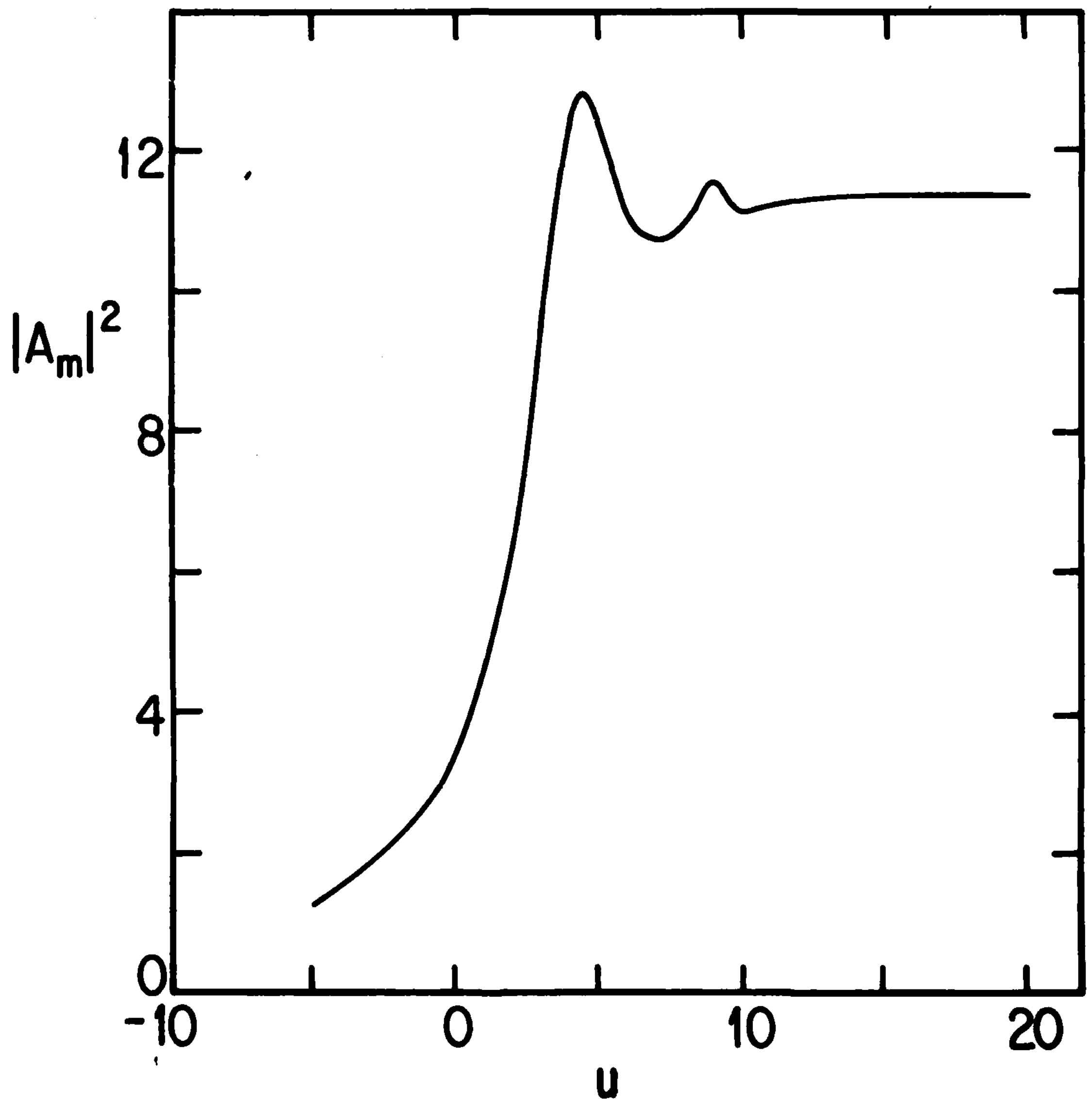
u=5.0
ReA
ImA
3
1
0
-1
-3
(Re, Im) A
u= 0
1
0
-1
(Re, Im) A
u= -5.0
1
0
-1
(Re, Im) A
-13
z
0
7



|A|²
12
4
0
u=5.0
3
1
0
|A|²
u=0
20
0
-20
Real part of -ε'
|A|²
1
0
u=-5.0
-13
z
0
7



8
4
0
-4
z_m
-6
-4
-2
0
2
4
6
u



12
8
4
0
-10
0
10
20
u
|Am|2

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