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Note on the Energy Release Rate For a Crack Starting from the Apex of a Wedge

by

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Griffith [1] was apparently the first to employ the energy release rate $\mathcal{G}$ as a critical condition of crack extension. In this note we will show, however, that if a crack starts from the apex of a wedge (cf. Figure 1 with n ≠ 1), the initial value of $\mathcal{G}$ is zero, although the stresses at the crack tip are unbounded. This example suggests that you cannot use the initial energy release rate as a critical condition of crack extension unless the opening-angle of the crack faces in the reference is precisely zero. Since the order of the

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We confine our problem to a simple mode III crack; the given solutions are then simple and of closed-form, so that we can examine the precise dependence of G on the crack length a. To the author's knowledge, no such closed-form solutions for an arbitrary crack length a have been presented (see, for example, Khrapkov [2] for Mode I and II).

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Abstract

We show that for a mode III crack starting from the apex of a wedge, the initial value of the energy release rate is zero, although the stresses at the crack tip are unbounded.

Note on the Energy Release Rate for a Crack Starting from the Apex of a Wedge

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Introduction

Griffith [1] was apparently the first to employ the energy release rate $\mathcal{G}$ as a critical condition of crack extension. In this note we will show, however, that if a crack starts from the apex of a wedge (cf. Figure 1 with $n \neq 1$), the initial value of $\mathcal{G}$ is zero, although the stresses at the crack tip are unbounded. This example suggests that you cannot use the initial energy release rate as a critical condition of crack extension unless the opening-angle of the crack faces in the reference is precisely zero. Since the order of the singularity of the strain energy density is less than one, this result may be predicted mathematically but it is not trivial physically.

We confine our problem to a simple mode III crack; the given solutions are then simple and of closed-form, so that we can examine the precise dependence of $\mathcal{G}$ on the crack length a. To the author's knowledge, no such closed-form solutions for an arbitrary crack length a have been presented (see, for example, Khrapkov [2] for Mode I and II).

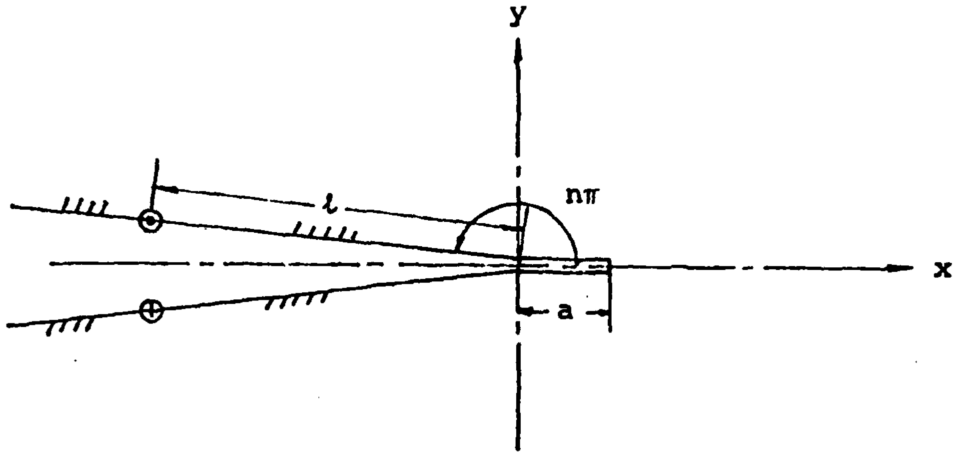

Fig. 1 A crack starting from the apex of a wedge

<u>Brief Methods of Solutions</u>

Following the theory developed by Sih [3], we will analyse Mode III crack of length a which starts from the apex of an infinite wedge subjected to concentrated forces P acting in opposite directions at $z = \ell e^{\pm in\pi}$ (Fig.1).

We employ a function

$$z = w(\zeta) = e^{-n\pi i}(\zeta^2 - a^{1/n})^n, \quad 0 < n \leq 1, \tag{1}$$

which maps the upper side of the crack to $-a^{1/2n} \leq \text{Real}\ (\zeta) \leq 0$, the lower side to $0 \leq \text{Real}\ (\zeta) \leq a^{1/2n}$, and the points at which the forces P are acting to $\beta = \pm (\ell^{1/n} + a^{1/n})^{1/2}$, respectively.

With the aid of equation (16) of Sih [3], the relevant stress function is

$$F(\zeta) = \frac{P}{\pi G} \log \frac{\zeta + \beta}{\zeta - \beta}, \tag{2}$$

where G is the shear modulus. The stresses are then given in the form

$$\sigma_{xz} - i\sigma_{yz} = G\,\frac{F'(\zeta)}{w'(\zeta)}$$

$$= \frac{P}{\pi}\,\frac{2\beta}{\beta^2 - \zeta^2}\,\frac{1}{2ne^{-n\pi i}\zeta(\zeta^2 - a^{1/n})^{n-1}}$$

in the ζ-plane, or

$$= \frac{P}{\pi}\,\frac{(\ell^{1/n} + a^{1/n})^{1/2}}{\ell^{1/n} + z^{1/n}}\,\frac{-1}{n(a^{1/n} - z^{1/n})^{1/2} z^{1-1/n}}\,, \tag{3}$$

$$= \frac{P}{\pi}\,\frac{\ell^{1/2n}}{\ell^{1/n} + z^{1/n}}\,\frac{i}{nz^{1-1/2n}} \quad \text{for} \quad a = 0 \tag{4}$$

in the z-plane.

Equations (3) and (4) show, as is expected, that the stress singularity of order 1/2 has changed discontinuously order to $1 - 1/2n$ at $a = 0^+$. (This is another example of simple closed-form solution exhibiting the singularity transition phenomenon which was studied by Nuismer and Sendeckyj [4].)

Inserting equations (1) and (2) into equation (7) of [3] yields the solution for the stress-intensity factor:

$$K(a) = \frac{P}{\pi}\,\frac{\sqrt{2}}{(\ell^{1/n} + a^{1/n})^{1/2}}\,\frac{a^{1/2n-1/2}}{n^{1/2}}\,.$$

Since $\mathcal{G} = \pi K^2/2G$, the energy release rate is give by

$$\mathcal{G}(a) = \frac{P^2}{\pi G}\,\frac{a^{1/n-1}}{(\ell^{1/n} + a^{1/n})\,n}\,. \tag{5}$$

Equation (5) shows that

$$\mathcal{G}(a) \sim a^{1/n-1} \quad \text{as} \quad a \to 0\,,$$

and the initial value of G(a) at $a = 0^+$ is zero unless $n = 1$, as noted in the introduction.

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