ADVANCED TELEPROCESSING SYSTEMS
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SEMIANNUAL TECHNICAL REPORT

MARCH 31, 1980

Principal Investigator: Leonard Kleinrock

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ADVANCED TELEPROCESSING SYSTEMS

Semi-Annual Technical Report

March 31, 1980

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This Semi-Annual Technical Report covers research carried out by the Advanced Teleprocessing Systems Group at UCLA under ARPA Contract No. MDA903-77-C-0272 covering the period October 1, 1979 through March 31, 1980.

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This contract has three primary designated research areas: Multi-Hop Packet Radio Systems, Multi-Access Broadcast on Wires, and Resource Sharing and Allocation.

This report contains the abstracts of the publications which summarize our research results in these areas during this semi-annual period, followed by the main body of the report which is devoted to a treatment of multi-hop packet radio networks. In particular, it consists of the Ph.D. dissertation by John A. Silvester, "On the Spatial Capacity of Packet Radio Networks", conducted under the supervision of Professor Leonard Kleinrock (Principal Investigator for this research). The work presents some results on point-to-point multi-hop networks as regards their capacity, their optimal traffic matrices, and the optimal transmission range. The general behavior of capacity as a function of the number of transmitting terminals is investigated.

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INTRODUCTION

This Semi-Annual Technical Report covers research carried out by the Advanced Teleprocessing Systems Group at UCLA under DARPA Contract No. MDA903-77-C-0272 covering the period October 1, 1979 through March 31, 1980. Under this contract we have three designated tasks as follows:

TASK I. MULTI-HOP PACKET RADIO SYSTEMS

An advanced investigation of the principles of operation, performance evaluation and design of multi-access communications in a local distributed broadcast environment. We will study access schemes suitable for multi-hop systems, power control, hierarchical structures, routing and location procedures, and the asymptotic behavior of all such systems as the number of terminals and nodes gets very large.

TASK II. MULTI-ACCESS BROADCAST ON WIRES

The peculiarities of multi-access broadcast communications in a distributed network connected over terrestrial wires (as opposed to radio communication) will be studied. The issues here involve the investigation of such things as mismatched communication links between different networks, the impact of various topologies, (loops, trees, tandems, stars, etc) stability and control, and the utility of sensing the distance between access ports.

TASK III. RESOURCE SHARING AND ALLOCATION

A single measure of network performance which includes throughput, delay, and blocking will be applied to a number of multi-access computer-communication systems. The behavior of various flow control schemes as evaluated by this definition of power will be conducted. Extended flow control and bandwidth control studies will also be carried out.

The main body of this report is devoted to a treatment of multi-hop packet radio networks. In particular, it consists of the Ph.D. thesis conducted by John A. Silvester under the supervision of Professor Leonard Kleinrock (Principal Investigator for this research). The title of this work is, "On the Spatial Capacity of Packet Radio Networks". The work presents some results on point-to-point multi-hop networks as regards their capacity, their optimal traffic matrices, and the optimal transmission range. The general behavior of capacity as a function of the number of transmitting terminals is investigated.

Below, we give a list of publications which summarize our work during this semi-annual period, followed by the main report. The abstract of each paper is given along with the reference itself.

RESEARCH PUBLICATIONS

1. Kleinrock, L., "On a New Class of Queueing Models for Distributed Environments," *Proceedings of Ninth International Teletraffic Congress*, Torremolinos, Spain, October 1979.

A new class of queueing problems arises when one considers random demands for service which arise in a geographically distributed environment, such as access to communication channels in computer networks. Not only must we suffer the usual consequences of queues and delays due to the randomness in the demand process, but also we must pay a price for organizing these demands into a cooperating queue. It is this second problem which is usually ignored in classical queueing theory.

In this paper, we study these problems associated with geographically distributed access to a common broadcast communication channel in a packet switching environment. We present some solutions to this multi-access broadcast problem, giving the throughput-delay profile both for long-range communication systems (such as satellite packet switching) and for local access in a ground radio packet switching environment. Of interest is the optimum profile one can ever achieve; to this end, we conjecture a lower bound on the mean delay for these systems.

2. Molle, M. and L. Kleinrock, "Analysis of Concentrated ALOHA Satellite Links," *Sixth Data Communications Symposium*, Pacific Grove, California, November 1979, pp. 87-95.

A conventional ALOHA satellite link uses a transponder which blindly echoes all up-channel traffic on the down-channel. An ALOHA channel can never be fully utilized, so an intelligent satellite could statistically multiplex the successful packets from several slotted ALOHA up-channels onto a single down-channel to conserve bandwidth, and hence reduce cost. We refer to this as a concentrated ALOHA system. Throughput, delay and stability effects are considered, varying the number of up-channels per down-channel and the satellite buffer size. Up- and down-channel bandwidths are assigned independent linear costs, and all performance comparisons are between constant cost systems. It is shown that the marginal increase in system performance drops off so quickly that a small number of up-channels maximizes throughput if up-channel bandwidth has a non-zero cost. This small number is a function of the buffer size and the relative cost of up- to down-channel bandwidth. It is also shown that, even if satellite buffer space is free, a small buffer minimizes average delay for some previously studied protocols of this type. A new protocol which improves performance and allows a large buffer to be used effectively is introduced and analyzed. Solving for throughput and delay in concentrated ALOHA systems provides new analytic and numeric results for the $G/D/1$ queue with rest period equal to the service time.

3. Kamoun, F. and L. Kleinrock, "Stochastic Performance Evaluation of Hierarchical Routing for Large Networks," *Computer Networks,* Volume 3, Number 5, November 1979, pp. 337-353.

> In its present form, distributed routing extracts a prohibitive price when used in large networks because of the processing time, nodal storage and line capacity required to update, store and exchange routing information among network nodes. In an earlier paper, we have shown that hierarchical routing schemes with optimally selected clustering structures yield enormous reductions in routing table length and hence in routing cost, at the price of an increase in network path length. That increase was shown to be negligible in the limit of very large networks. In this paper, we evaluate the tradeoff between the reduction in routing table length and the increase in network path length in terms of the more meaningful network performance measures of *delay* and *throughput.* Extended queueing models are developed to exhibit the interrelationships which exist between network variables such as delay, throughput, channel capacity, nodal storage, network path length. routing table length, etc. These models are an extension of the classic model for networks in that they account for line overhead and storage requirements due to routing. The models demonstrate the enormous efficiency of optimized hierarchical routing for a class of large networks.

4. Grnarov, A., L. Kleinrock and M. Gerla, "A New Algorithm for Network Reliability Computation," *Computer Networking Symposium,* Gaithersburg, Maryland, December 1979.

> A new algorithm for *network terminal reliability* computation is presented. The algorithm belongs to the class of path enumeration algorithms and is based on the application of a newly defined operation on the set of all simple paths. Comparisons with existing algorithms on the basis of terms that must be evaluated during the derivation, the number of operations required, and the execution time in several represented benchmarks show that the proposed algorithm is considerably more efficient than currently available schemes.

5. Silvester, J. and L. Kleinrock, "On the Capacity of One-Hop ALOHA Packet Radio Networks with Adjustable Transmission Power," *Pacific Telecommunications Conference Proceedings,* Honolulu, Hawaii, January 1980, pp. 1A-25 to 1A-31.

> In this paper we investigate two possible policies for realizing an arbitrary traffic matrix in a Slotted ALOHA broadcast packet radio netw rk: full connectivity and limited transmission power. The performance fo. he fully connected (point-to-point) network is the same as the known result for a centralized network and allows a maximum throughput of $1/e$. The other approach, wherein we give each node sufficient power to just reach his destination, allows a maximum throughput proportional to the logarithm of the number of (active) nodes in the network. These results, which are derived analytically, are then verified by simulation, showing excellent agreement.

6. Grnarov, A., L. Kleinrock and M. Gerla, "A New Algorithm for Symbolic Reliability Analysis of Computer-Communications Networks," *Pacific Telecommunications Conference Proceedings,* Honolulu, Hawaii, January 1980, pp. 1A-11 to 1A-19.

> A new algorithm for *symbolic network analysis* is presented. The algorithm is based on the application of a newly defined operation on the set of all simple paths. Comparisons with existing algorithms on the basis of terms that must be evaluated during the derivation, the number of operations required, and the execution time in several represented benchmarks show that the proposed algorithm is considerably more efficient than currently available solutions.

7. Kleinrock, L. and P. Kermani, "Static Flow Control in Store and Forward Computer Networks," *IEEE Transactions on Communications,* Vol. COM-28, February 1980, pp. 271-279.

> In this paper we develop an analytic model for end-to-end communication protocols and study the window mechanism for flow control in store-and-forward (in particular message switching) computer-based communication networks. We develop a static flow control model in which the parameters of the system are not dynamically adjusted to the stochastic fluctuation of the system load. Numerical results are presented and it is shown that the throughput-delay performance of a network can be improved by proper selection of the design parameters, such as the window size, the timeout period, etc.

8. Kermani, P. and L. Kleinrock, "Dynamic Flow Control in Store and Forward Computer Networks," *IEEE Transactions on Communications,* Vol. COM-28, February 1980, pp. 263-271.

> In a recent paper we presented an analysis of flow control in store-and-forward computer communication networks, using a token mechanism mechanism. The analysis there assumed equilibrium conditions for a selected set of system parameters which were not dynamically adjusted to stochastic fluctuations in the system load; this mechanism was referred to as "static flow control". In this paper we study a "dynamic flow control" in which parameters of the system are dynamically adjusted to match the availability of resources in the network. Based on Markov decision theory, an optimal policy to dynamically select the number of tokens is formulated. Because an exact solution to the problem is extremely difficult, an effective heuristic solution to the problem is presented. Numerical results are given and it is shown that the throughput-delay performance of a network is better with dynamic control than with static control.

9. Silvester, J., *On the Spatial Capacity of Packet Radio Networks,* Ph.D. Dissertation, Computer Science Department, University of California, Los Angeles, March 1980.

UNIVERSITY OF CALIFORNIA

Los Angeles

On the Spatial Capacity of Packet Radio Networks

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy
in Computer Science

by

John Andrew Silvester

1980

ABSTRACT OF THE DISSERTATION

On the Spatial Capacity of Packet Radio Networks

by

John Andrew Silvester
Doctor of Philosophy in Computer Science
University of California, Los Angeles, 1980
Professor Leonard Kleinrock, Chairman

Previous research in the area of Broadcast Packet Radio Networks has focused on centralized one-hop networks. In this dissertation we present some important results concerning the capacity of *point-to-point multi-hop* networks. We find that in point-to-point networks we can achieve much higher performance levels due to the ability to *spatially reuse* the channel. We derive optimum transmission ranges and retransmission policies (maximizing capacity) for various configurations of ALOHA networks.

For *one-dimensional random* networks (i.e., random topologies) with random traffic matrices, satisfied by exactly adjusted transmission range, allowing communication in one hop, we find that we can obtain a capacity *proportional to the logarithm* of the number of nodes in the network. For *regular* (topology) networks using fixed transmission ranges and multi-hop communication to support uniform traffic matrices, we can obtain a throughput of $2/e$.

For *two-dimensional* networks we find that we can obtain throughput proportional to the *square root* of the number of nodes in the network, for both regular and random topologies. We find that the best *regular* design is a hexagonal tessellation. The optimum average degree for *random* networks in which each node uses the same (fixed) transmission radius is shown to be about 6. Generalizing to networks of n nodes in arbitrary dimensions (k), we find that the throughput is proportional to $n^{\frac{k-1}{k}}$.

When we run simulations of these networks we find that the routing and flow control issues become very important. We, therefore, present several routing algorithms and show (by simulation) that the routing algorithm which balances traffic flow, seems to produce the best performance. With balanced flow, we find performance similar to that predicted by our models.

CHAPTER 1
INTRODUCTION

1.1 Digital Communication Networks

The recent explosive growth in data communications has two main origins: i) As computers and data bases grew larger it was necessary to share the cost of these expensive resources between many users; and ii) with the realization that data communication was feasible and relatively cheap, we saw an expansion of computer technology into many areas hitherto not using computers. Within the first class of applications, we include such items as time-sharing services, centralization of computing facilities of the different branches of a company and so on. Among others, credit checking, automated supermarket checkout and environmental monitoring are examples of new application areas which have become feasible due to the availability of (cheap) digital communication.

The common attribute of the second class of applications mentioned above is that the local function requires very simple machinery (perhaps utilizing microcomputers), and that the central facility can support hundreds, perhaps thousands, of these remote terminals (the computational demands being minimal). We also find that the communication needs can be characterized in a fairly general manner as 'bursty'. This term implies that the demand on the communication resource tends to come in short bursts with long idle periods between them. The average load is fairly low (perhaps on the order of less than ten bits per second), but during a transaction the data rate may be fairly high, possibly several thousand bits per second.

Early data communication services used analog circuits and line switching (through the existing telephone network). Towards the end of the sixties, however, it was found that the telephone network used in a line switching mode was not capable of handling the large volume of traffic that was beginning to arise, in a cost effective manner. One of the main reasons for this is the bursty nature of the traffic generated by interaction with computers [JACK 69, KLEI 74]. With bursty traffic and circuit switching, the (expensive) communication resource is tied up for the duration of the call, even though it is only being used perhaps 1% of the time.

Fortunately, the advent of reasonably cheap minicomputers allowed more sophisticated channel sharing, alleviating the problem. When switching became cheap [ROBE 74], we saw the introduction of *message switching* [BARA 64, KLEI 64] as an alternative to circuit switching. In message switching, the expensive resource (communication bandwidth) is only allocated to the user for the brief duration of time that he actually needs it (i.e., during his burst of activity), thus allowing a reduction in the cost by sharing the channel among many users.

The main technique behind message switching is the use of a communication subnet consisting of switching centers with *store and forward* capability. Messages are passed from one switching center to the next, finding a path to their destination according to the routing algorithm. (This contrasts the line switching scheme in which the whole path is assigned at call setup and not released until termination of the session.) The channels between switching centers are thus only allocated to a particular user when he has a message to send. Packet switching is very similar to message switching except that the messages are split into several packets (often of fixed length for simplicity of buffer allocation and protocols) and then entered into the network. Each packet is independently addressed and may follow a different route to the destination from other packets of the same message. Packet switching tends to reduce the network delay since intermediate nodes on the path do not have to wait for reception of the whole (long) message and packets may also take different paths and be served in parallel. There is, however, an additional delay incurred due to reassembly at the destination since the packets may arrive out of sequence. Packet switching is now used almost exclusively in digital communication networks.

The end of the sixties saw the beginning of the development of generalized computer communication networks, the ARPAnet being a good (if somewhat hackneyed) example [BUTT 74, CARR 70, CROC 72, FRAN 70, FRAN 72, HEAR 70, KLEI 70, ORNS 72, ROBE 70, ROBE 72a]. Following the success of this network, many others sprang up around the world, both private and public (TYMNET, TELENET and DATAPAC for example).

In some applications it is not possible (or economically feasible) to construct conventional communication networks using transmission lines. As a result of this many new technologies have been explored. The ALOHA system at the University of Hawaii [ABRA 70, KUO 73] is such an innovative network using a broadcast radio channel to interconnect the various campuses of the University which are located on different islands. Installing a wire network would have been costly, whereas a radio network was relatively simple and cheap to install. In this scheme any node wishing to use the channel, does so with no regard to other users. If no other node attempts to use the channel at the same time, the packet will be received successfully (ignoring transmission errors due to noise). If, however, two (or more) nodes transmit at the same time, destructive interference will occur, resulting in the loss of one or both of the transmitted packets. (It may be possible to recover one of the overlapped packets by capture effects if the relative power levels of the two signals are sufficiently different.) This situation is handled in the ALOHA system by requiring that the receiving node transmit a positive acknowledgement. If the sender does not receive this acknowledgement within some time-out period, he will retransmit the packet after some (random) delay (to avoid continued collisions).

Another technology gaining in popularity is the use of satellites for data communication. There are two basic modes for using a satellite channel: i) treat it simply as a 'cable in the sky' and assign portions of the available bandwidth to users in a time or frequency multiplexing mode; or ii) use a random access mode such as ALOHA [ABRA 73], so that any user can access the whole channel bandwidth on a contention basis. If the satellite uses a broadcast downlink (as is often the case), the sender will hear his own transmission (delayed by one round-trip propagation delay). He will, thus, be able to determine if the packet was received without error (due to a collision), obviating the need for positive acknowledgements. He can also detect errors caused by noise on the uplink. Downlink noise errors may be localized at the destination and he can, therefore, make no inference about the noise on the downlink.

The current rapid growth in the availability of digital communication facilities has greatly reduced the communication problems for many applications. Certain applications cannot be served with the current facilities, however. In particular, studies of local distribution of communication and computer resources have only just begun. Novel technologies that are currently being explored include fiber optics and coaxial cables, which can also be used in a broadcast mode. This problem will be exaggerated in the near future by the tremendous growth in the home computer market. It is no longer ridiculous to anticipate the day when every home will have a computer of some kind, and it would be shortsighted indeed, to assume that these will not require access to data banks and more powerful computing facilities and thus require communication capability.

Currently, access to computing resources is gained by using the local telephone network to connect to a host or network access point (TIP in ARPAnet terminology). This approach is only feasible since the current telephone charging structure counts these as 'free' local calls. This situation cannot persist. We anticipate new regulations, in the near future, charging for these calls in a realistic manner. When this occurs, other services, possibly using such innovative technologies as packet radio, will become available.

2

Mobile users cannot be supported by wire networks. The use of mobile telephones has grown rapidly in recent years with the reduction in size and cost of electronics. We anticipate that this growth will continue, especially considering the recent introduction of small, portable radio telephones (weighing less than 1 kilogram). For mobile digital communication, random access schemes are more attractive than fixed allocation schemes (such as those used in the mobile telephone network, for example), which were designed to meet speech (i.e., non-bursty) traffic requirements. We thus anticipate the growth of these kinds of facility possibly using ALOHA type networks. We also believe that these *broadcast packet radio* networks may even be a cost effective alternative for fixed (i.e., non-mobile) users, especially where the current telephone system is poor.

1.2 Types of Network

Although many different transmission media are currently in use (wire, microwave radio, satellite, broadcast radio, light pipes etc.), we can separate all communication networks (independent of technology) into two major categories: centralized and point-to-point.

1.2.1 Centralized Networks

Figure 1.1 shows a typical centralized network. In a centralized network, traffic is generated either by terminals (at the leaves of the tree), or by the central node (station). All traffic generated by the terminals is directed to the central node. This station is usually interfaced to a large computer, or could be a gateway into another network. The ALOHAnet is a typical example of this kind of network with the station being the University's main computer center and also a gateway into the ARPAnet.

Routing for these networks is relatively straightforward and is usually accomplished by organizing the nodes into a tree structure as shown in the figure [GITM 76]. The performance of such networks is usually determined by the last level, where most of the traffic is concentrated. (This is especially true for broadcast multi-hop radio networks since all network traffic must eventually contend for the channel over this final 'critical hop' [TOBA 78a, 78b].)

1.2.2 Distributed Networks

Figure 1.2 shows a typical distributed network. In this kind of network, traffic can be generated at any node and be destined to any other node. The classic example of this would be the ARPA network (ARPAnet), which we show in Figure 1.3. The routing problem for these networks is much more complicated [GERL 73, FULT 72]. Most networks employ some form of adaptive routing to compensate for failures or to route around local congestion. Adaptive routing causes significant overhead due to the additional traffic necessary to keep the routing tables current. If the connectivity is rapidly changing, as is the case in a mobile network, these problems are exacerbated. Little work has been done on developing routing algorithms which are capable of handling mobile users (and hence rapid topology changes) in an efficient manner.

1.3 Broadcast Networks

As previously mentioned, the first implementation of a broadcast packet radio network was the ALOHA system at the University of Hawaii [ABRA 70, KUO 73]. Recent ongoing studies (of which this dissertation is part), supported by the Advanced Research Projects Agency of the Department of Defense (DARPA) have extended these ideas and developed the concept of a packet radio network, (called the PRnet), wherein packets may take multiple hop paths in order to reach their destination [ROBE 72b, KAHN 77, KAHN 78]. Any node in the network can act as a source or destination for traffic in this network. We see, therefore, that the PRnet is topologically similar to the ARPAnet. It is distinguished from the ARPAnet by virtue of the fact that broadcast channels are used, and thus the
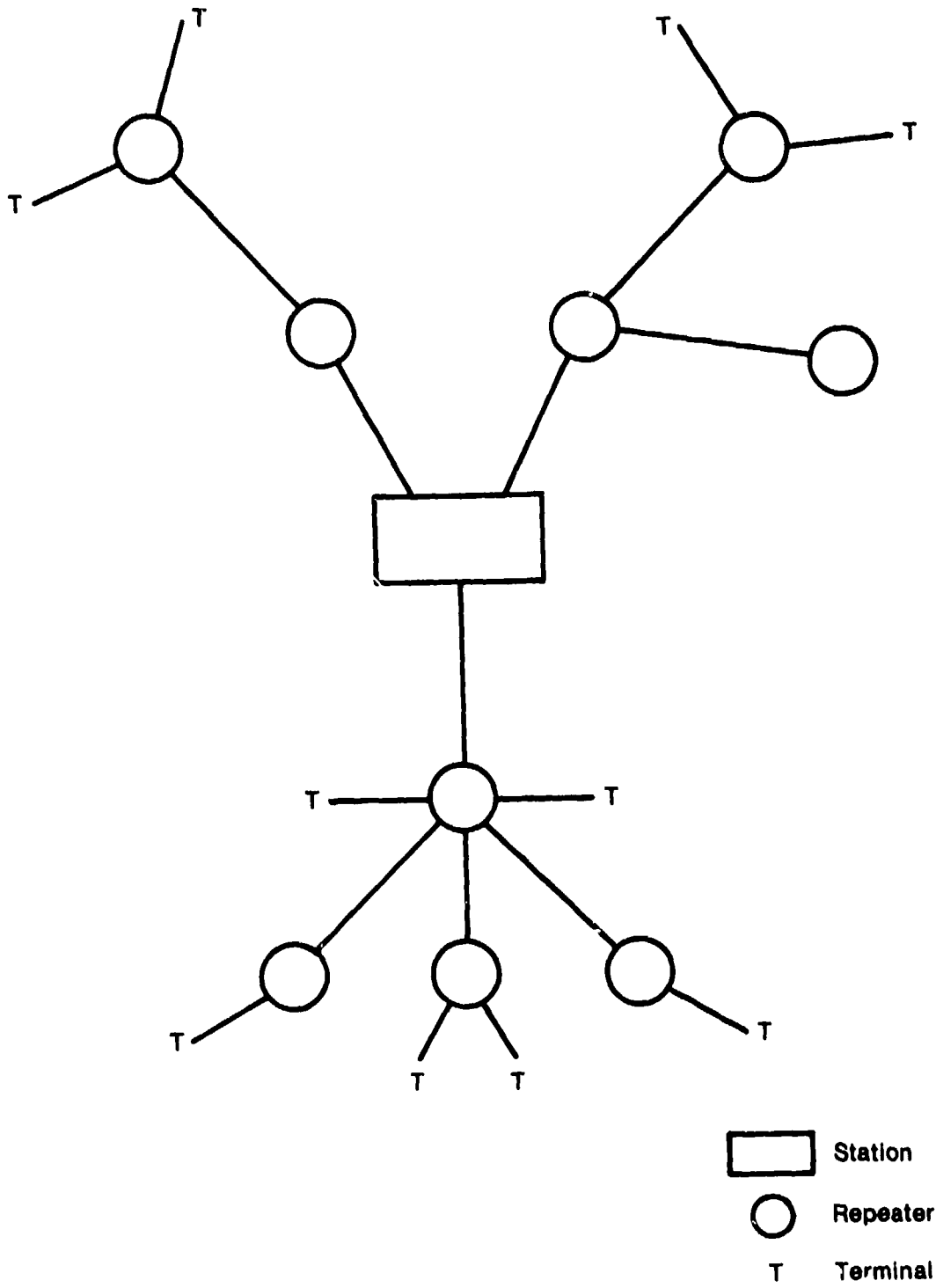
Figure 1.1 Centralized Computer Network
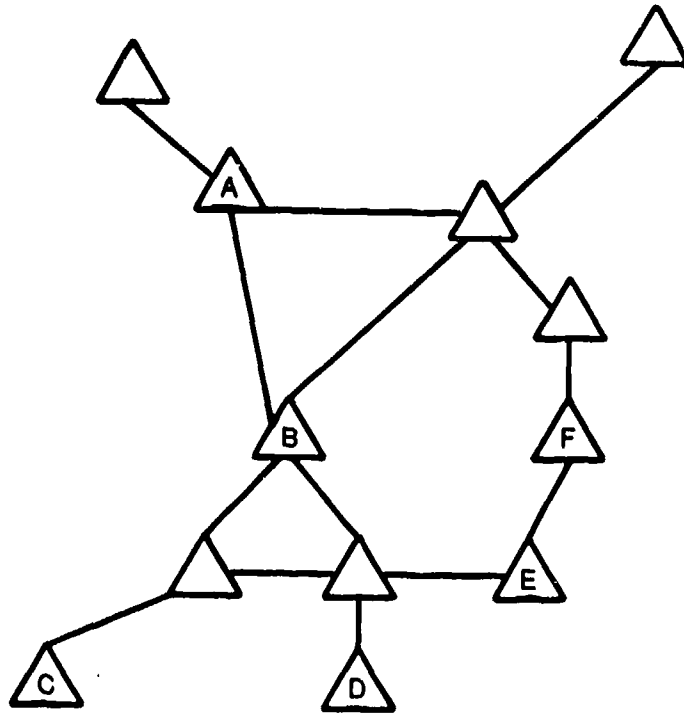
Figure 1.2 A Distributed Network

ARPAnet technology is not directly applicable.

As part of the DARPA project into packet radio networks an experimental network has been built in the San Francisco area. This network is used to run experiments to test the protocols and validate analytical performance studies.

In order to be able to share the same channel with other users (air traffic control in the experimental network) and to reduce the susceptibility to jamming and interception, this network uses Spread Spectrum broadcast radio channels. One additional benefit of utilizing spread spectrum signalling is that Code Division Multiple Access can be used (see section 1.4.4), which allows the system to operate with less interference. The system currently contains about 15 repeaters and two stations. The network provides access to the central stations, (one of which is, in fact, a gateway into the ARPAnet), possibly over multiple hops, for a set of fixed and mobile terminals.

The main advantages of broadcast radio as a communication technology are as follows:

ARPANET GEOGRAPHIC MAP, SEPTEMBER 1979

Legend:
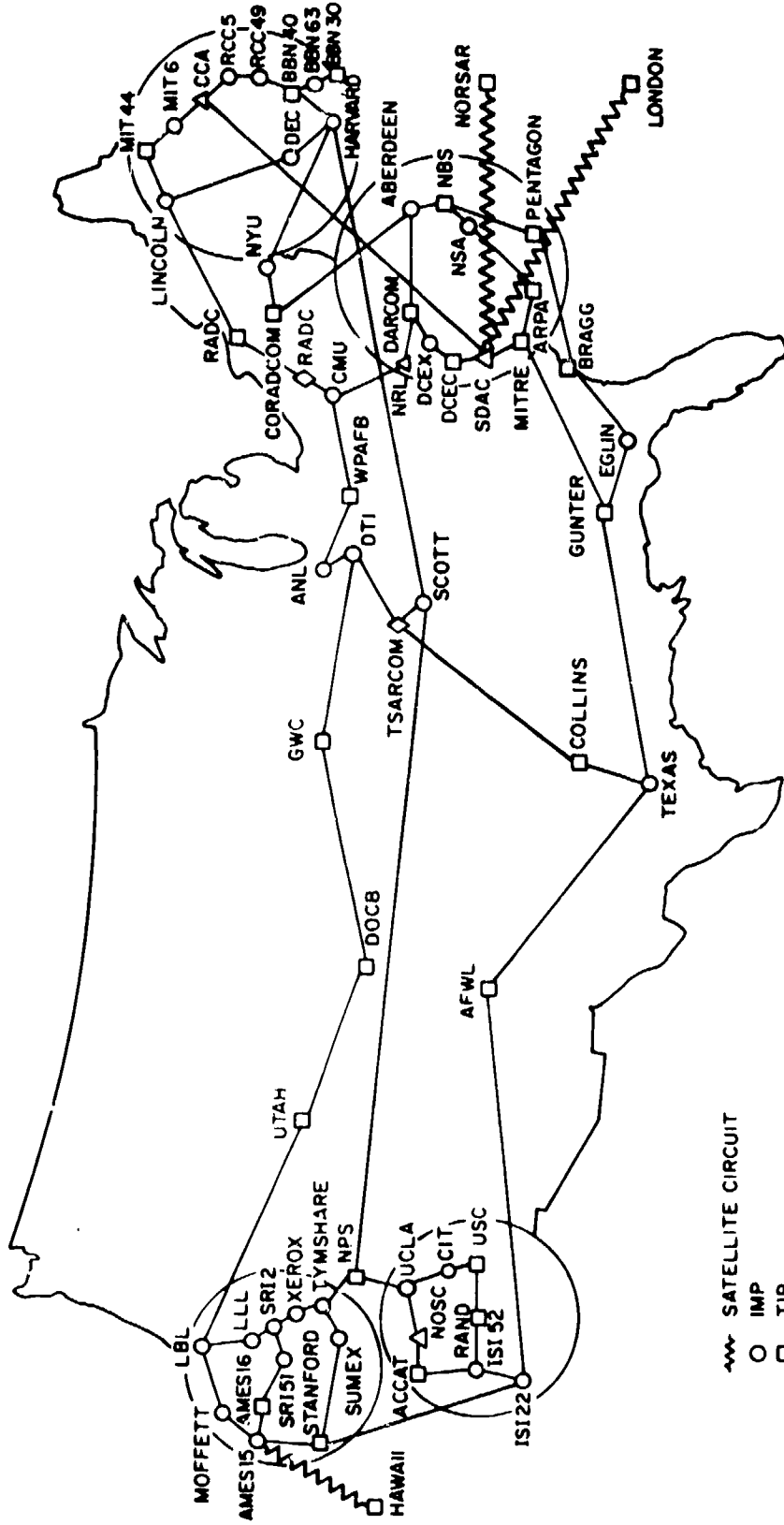
〰 SATELLITE CIRCUIT
◯ IMP
▢ TIP
△ PLURIBUS IMP
◇ PLURIBUS TIP

(NOTE: THIS MAP DOES NOT SHOW ARPA'S EXPERIMENTAL SATELLITE CONNECTIONS)
NAMES SHOWN ARE IMP NAMES, NOT (NECESSARILY) HOST NAMES

i) *Mobility*

In a broadcast radio network, nodes are able to move around the network without losing the capability to communicate. This is very similar to the mobile telephone concept. As a terminal (or any PRU*) moves, it will eventually be out of range of the device with which it was previously communicating. At that time, it will generate special 'search packets' to locate a new repeater to communicate with and thus re-establish contact with the network. Its new location will be noted by the routing center, which will generate new routing tables. If the routing is distributed, the information will slowly permeate through the network until eventually all nodes know the terminal's new location.

ii) *Ease of Deployment*

Such networks are extremely simple to set up. There is no cable to lay, no antennae to align, and most network initialization is a built-in function of the PRU's. This is especially important for establishing communication in areas which are remote or hostile (due to an enemy or the terrain). If necessary, an aeroplane could drop the repeaters by parachute and, once the network initialized itself (establishing topology, routing etc.), communication could begin. An application area where ease of deployment is important, is that of remote sensing (for environmental monitoring or exploration, for example). Being able to drop off the sensors by aeroplane in either of these applications is clearly an attractive proposition.

iii) *Ease of Expansion*

It is extremely easy for the network to expand when new facilities are added. Introducing new terminals (or repeaters to ease traffic congestion) can be accomplished with simple (internally generated) updates to routing tables.

iv) *Cost*

With the current trends in the price of hardware, it is quite reasonable to foresee the day when a radio network such as this will be much cheaper than conventional wire networks. This is clearly true for remote areas (where no facilities currently exist) but we also believe it to be true for urban areas. Less cost benefit would be derived in those areas where good local telephone lines already exist - although the capability of these facilities may soon be exceeded. In some less developed countries, with little or no local telephone service, the cost of building a telephone network (digging up roads and buildings to accommodate wires), is prohibitive and radio using rooftop antennae may be an attractive alternative.

v) *Redundancy and Reliability*

The very nature of broadcast networks allows redundancy. If the connectivity is sufficiently high, the failure of nodes in the network causes little or no performance degradation. The fact that communication is digital and that the PRUs contain microprocessors allows error checking and recovery, thus guaranteeing correct delivery of messages.

---

* All devices in the network (stations, terminals and repeaters) use a common device for channel access called the Packet Radio Unit (PRU).

One major problem of broadcast networks is the issue of security and anti-jam protection, which is of special significance to the military. New coding techniques utilizing Spread Spectrum and programmable decoders allow a certain amount of anti-jam protection and added security. Even so, it seems necessary to end-to-end encode all messages to provide any real security. With current encryption techniques, and the fact that we have microprocessors available in the PRUs to do the encryption, we feel that this problem is not unsurmountable.

1.4 Access Schemes

The main factor that distinguishes a broadcast network from other types of communication facilities is that when one node sends a message (transmits), many other nodes hear the transmission even though it is (usually) only addressed to one of them. (There are instances when a packet will be addressed to many other nodes, in order to establish topology for example). In general, there is no simple way of deciding which node should access the shared resource (channel) at any time. We may therefore encounter collisions at the receiver, either when two nodes transmit to him at the same time, or when two (or more) nodes in his vicinity (hearing range) transmit at the same time. We need therefore some way of controlling these conflicts and guaranteeing safe delivery of messages. Many possibilities for controlling channel access exist, from TDMA and FDMA to pure random access strategies. Due to the bursty nature of the traffic, random access schemes are well suited.

In the sections that follow we give an overview of the various access schemes that are available.

1.4.1 TDMA and FDMA

Time and Frequency division multiplexing are well known techniques for sharing a common channel. They are poorly suited to bursty traffic such as we find in broadcast networks, especially under lightly loaded conditions. We find a good study of these versus random access schemes in [TOBA 74]. Asynchronous time division multiplexing does allow better resource sharing, but requires coordination between the nodes. This coordination is neither cheap nor feasible to implement in a distributed radio network. It may however present a desirable alternative if the number of nodes is small and we are prepared to accept more complex protocols.

1.4.2 ALOHA

The simplest random access scheme is 'pure ALOHA', wherein any node having a packet ready for transmission does so. Of course, collisions occur. These are resolved by the node retransmitting the packet at some (randomly chosen) later time, if no positive acknowledgement is received. It is necessary to randomize the retransmission delays in order to avoid perpetual repetition of the collision. Extensive analysis of this scheme can be found in [ABRA 70, LAM 74], where it is determined that the maximum that the channel can be utilized is 18% (1/2e) of the channel bandwidth. A simple modification to the ALOHA scheme - SLOTTED ALOHA, proposed in [ROBE 75], forces transmissions to commence at the beginning of 'slots' (time divisions of length equal to a packet transmission time). In [ROBE 75, LAM 74] we find analysis of this scheme showing that the capacity is doubled (over unslotted ALOHA), to 36% (1/e).

In real implementations of ALOHA, we can achieve a throughput significantly higher than 1/e, due to the effects of FM capture. This enables a receiver to lock onto the stronger signal, (when a collision occurs), and receive the packet without error. If the capture effect is perfect, we will never waste slots due to interference, as the power levels of different devices will always differ. [ROBE 75] studies this phenomenon.

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Another interesting variation of the ALOHA access scheme is found in [YEMI 79b]. He introduces an 'urn scheme' which dynamically adapts the set of users allowed to transmit in any slot depending on the traffic load. In light traffic, many nodes are given permits to transmit and the system operates like ALOHA. As the traffic load increases, fewer and fewer nodes are given transmission permits, until eventually only one node is allowed to transmit in any slot, which is identical to TDMA. Using this method he shows that the capacity can in fact be much higher than the $1/e$ and even approach 1 (the TDMA performance) for heavily loaded channels.

In [CAPE 78], we find another algorithm which gives ALOHA like performance in low traffic and TDMA like in heavy traffic. In this approach, called the TREE algorithm, conflicts are resolved by continually splitting the population into smaller and smaller groups until the conflict is resolved.

1.4.3 CSMA

Carrier Sense Multiple Access is an obvious modification to the ALOHA scheme in which every node listens to the channel before transmitting. If the node senses a carrier on the channel it remains silent until the channel becomes free (or waits some random time and then resenses the carrier, depending on which of the CSMA protocols is being used). This technique should be familiar to anyone who has listened to police radios or CB. We find extensive analysis of the various CSMA protocols in [TOBA 74]. If i) all nodes are in range of each other and ii) the propagation delay is small compared to the packet transmission time, we can achieve a throughput of about .8 of the channel bandwidth.

In a large network, the propagation delay may become large enough to significantly degrade system performance, eventually reducing to ALOHA levels when the information gained by sensing the channel is so old as to be useless. It is precisely for this reason that CSMA is of no use in satellite networks, where the propagation delay is about .25 seconds. In multi-hop networks the first assumption is violated (lack of range being the motivation for multi-hop), and we must resort to using busy tones which are generated by the receiver as soon as it detects an incoming transmission, which all nodes in his environment hear and realize that they should not transmit. This effectively doubles the propagation delays.

1.4.4 CDMA

Code Division Multiple Access takes advantage of the nature of Spread Spectrum Coding. In a simple implementation of spread spectrum, every node in the network is assigned the same 'CHIP'* This allows the network to coexist on the same channel as other users, who may, for example, be using amplitude modulation. This was the original technique proposed for use in the experimental packet radio network, allowing it to coexist on the same frequencies as the San Francisco airport air traffic control without mutual interference.

If we assign different CHIPS to each node in the network and then require that any node wishing to send a message to node i, use i's code for transmission, we greatly reduce the amount of interference between nodes. There will still be collisions between those packets *addressed* to node i, that are transmitted at about the same time, but packets addressed to other nodes within range will no longer cause interference. It is even possible, by using multiple receivers, to receive more than one concurrent transmission in the same code, provided that they are separated by a sufficient time gap.

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* A CHIP is a code (PN sequence) assigned to a node allowing it receive only those messages sent in this code, all other traffic is seen as noise. Although Spread Spectrum is very wasteful of bandwidth, it allows mutual coexistence, helps prevent jamming and reduces security problems. A good description of spread spectrum techniques can be found in the IEEE special issue on spread spectrum [IEEE 77].

With a single receiver we are able to receive only one of the concurrent packets. We find, therefore, that the interference for Spread Spectrum is much lower than for ALOHA or CSMA and that, consequently, we can achieve higher throughput. However, Spread Spectrum requires the use of much wider bandwidth, which tends to cancel out the effects of the reduced interference. From an information theoretic standpoint it is possible to achieve full utilization of this additional bandwidth by proper coding; these codes and the corresponding receivers may be sophisticated and expensive to build, however.

1.4.5 Reservation Schemes

Many reservation schemes have been proposed [ROBE 73, CROW 73, TOBA 74, BIND 75, KLEI 77, JACO 78]. The basic technique of a reservation scheme is to use some portion of the message channel for reservations which request allocation of the channel for transmission of a message packet in some later slot. The reservation portion of the channel is shared using any of the access schemes mentioned above. Since the reservation packets are much shorter, the reservation channel can be operated at lower throughputs with correspondingly lower delays. Additional reservations (after the first which must be achieved through contention or some form of multiplexing) can be piggy-backed in the header of a data packet.

1.4.6 Overview

In this dissertation we will consider the ALOHA access scheme *exclusively*, for several reasons.

i)   It is simple to implement and analyze.

ii)   In real implementations, when capture and other details are incorporated, the performance is actually not much worse than other access schemes.

iii)   The general phenomenological behavior found for ALOHA will hold true for other access schemes.

1.5 Acknowledgements

In order to resolve the collisions that occur in broadcast networks it is necessary to have positive acknowledgement. In general, two levels of acknowledgement are supported - Hop by Hop acknowledgements (HBH) and End to End acknowledgements (ETE). In the following we describe the motivation for acknowledgements in more detail and give the solutions that were used in the PRnet.

HBH acknowledgements are necessary to support the local protocol, the sender having no way of knowing whether his packet was successfully received or not. Fortunately, this acknowledgement can normally be handled in a *passive* manner, that is we do not have to actually transmit the acknowledgement. The sender listens to the channel until he hears the receiver retransmit the packet (to be passed on to the next repeater). This constitutes an acknowledgement. (This is not the case for CDMA, since the next hop will use a different code for transmission and this code cannot be understood by the previous sender.) Two problems exist here. One is that the sender may not hear the retransmission due to other interference, in which case he will retransmit the packet. It is the responsibility of the final destination to discard duplicate packets. (The intermediate repeaters could also remember the message identification numbers of recently sent messages and not duplicate them.) The other problem is that *destination* nodes do not retransmit the packet. These nodes must therefore, send an *active* acknowledgement (usually consisting of only the header).

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ETE acknowledgements are used to guarantee that the packet did not get lost in a repeater that died, or misrouted due to topology changes. They normally consist of the packet header with a special acknowledgement field set. If after some time out period, the sender has not received an ETE acknowledgement, he sends the packet again. (As before, the destination is responsible for discarding duplicate packets).

1.6 Multi-Hop - Additional Problems

The most difficult problems in multi-hop networks are those of routing and flow control. The routing problem is especially hard if some of the nodes are mobile. We address these issues in a minor way in this dissertation, mainly pointing out the problems and proposing some simple routing algorithms. In Chapter 8 we discuss the issue in greater depth an' present a set of problems for future research.

1.7 Previous Network Capacity Results

In this dissertation we attempt to determine the capacity (maximum achievable throughput) of a Packet Radio Network. In this section we summarize previous results.

Many studies have been conducted to evaluate the capacity of one-hop *centralized* communication networks using broadcast radio as the communication medium. In [ABRA 70] the capacity of fully-connected one-hop centralized pure ALOHA was found to be $1/2e$ and in [ROBE 75] the corresponding result for Slotted ALOHA networks was found to be $1/e$. In [LAM 74] we find an extensive analysis for the fully connected one-hop centralized slotted ALOHA access scheme and in [TOBA 74, KLEI 75a] we find similar results for Carrier Sense Multiple Access (CSMA). Local access networks usually have centralized traffic requirements (the central node often being a gateway to the main network).

In [GITM 75, TOBA 78b] we find some capacity results for two-hop Slotted ALOHA centralized nets and these results are extended to CSMA in [TOBA 78a].

1.8 Outline of this Dissertation

In *Chapter 2* we study the problem in more detail, giving a general formulation and discuss the forms of the models that we will use. We define the key system variables and give some examples of how the capacity of an arbitrary Packet Radio Network can be computed.

Chapter 3 is concerned with realization of arbitrary traffic matrices in a single hop (i.e., not allowing the use of repeaters), for random networks. We first give the performance for a fully connected network (every node has sufficient power to reach every other node), which is the same as for the centralized networks found in [ABRA 70, LAM 74, TOBA 74]. We follow this with a discussion of networks in which each node tunes its transmission power in order to just reach its destination. Our analysis shows that this allows much higher throughput. In fact, we can achieve a throughput which is proportional to the *logarithm* of the number of nodes in the network.

In *Chapter 4*, motivated by this increased performance due to range reduction, we proceed to study *local* traffic matrices. Chapter 4 is concerned with 'nearest neighbor' communication, as an attempt to find the 'best traffic matrix'. We start by developing some simple upper and lower bounds on the possible throughput if we were allowed to select the best possible traffic matrix. Since both of these bounds are linear with respect to the number of nodes in the network and the lower bound is achievable we know that the performance of the best traffic matrix will also be linear. We then study some simple schemes which would appear to have low interference patterns. We find, by simulation,

that the performance of some of these schemes exceeds the lower bound that we had previously developed. For one of the schemes we can derive the performance analytically and find that the constant of proportionality is $1/2e$.

Chapters 5,6 and 7 are concerned with 'true' networks i.e., those with multi-hop source-destination paths, using repeaters for store-and-forward packet switching.

Chapter 5 considers networks in which the nodes are regularly placed. First we consider loop networks and determine the optimum average degree of connectivity required (maximizing capacity). We find that we can achieve a capacity of $2/e$. We then look at regular placement on a line finding that the capacity is independent of transmission radius (average degree). We also study two-dimensional networks and find that the network can achieve a throughput proportional to the square root of the number of nodes in the network (provided the average degree is neither i) too large giving a fully connected network; or ii) too small so that the network is not connected). The best performance is when the nodal degree is minimized. By looking at specific tessellations of the plane, we find that the optimum throughput is attained when the nodal degree is 3 (hexagonal tesselation).

Chapter 6 looks at the optimum transmission radius for random networks. We consider two approaches. If every node uses the same transmission radius, we find analytically that we can achieve a throughput proportional to the *square root* of the number of nodes in the network. We find that the transmission radius that maximizes the capacity is such that the average degree is about 6.

Alternately we can make each node have the same degree. This has similar performance (in fact it is superior when we consider flow control in Chapter 7), but is not as simple from an implementation point of view (as the HBH acknowledgements are no longer free). We also present simulation results for both of the above approaches and find, that although the general behavior is similar to that predicted by the model, there is a significant discrepancy due to local overloading in the network. Chapter 7 discusses this issue in greater detail.

In *Chapter 7* we look at the performance of actual networks by means of simulation. In previous chapters we have often made the assumption that the traffic load in the network is homogeneous. We find that the behavior is similar to that predicted by the simple models presented earlier, but that major problems of routing, flow control and the particular topological structure of the random network generated cause bottlenecks and reduce the performance as these links become overloaded. We present and study several routing algorithms which allow a certain amount of load averaging. We find that these allow higher levels of performance that agree more closely with our analytical predictions.

In Chapter 8 we summarize the results and draw conclusions, also giving direction for future research that arises naturally as an extension to this work.

1.9 Contributions of this Research

Previous research in the area of Broadcast Packet Radio Networks has focused on centralized one-hop networks. In this dissertation we present some important results concerning the capacity of *distributed multi-hop* networks. We find that in point-to-point networks we can achieve much higher performance levels due to the ability to *spatially reuse* the channel. We derive optimum transmission ranges and retransmission policies (maximizing capacity) for various configurations of ALOHA networks.

For *one-dimensional random* networks with random traffic matrices, satisfied by exactly adjusted transmission range, allowing communication in one hop, we find that we can obtain a capacity proportional to the *logarithm* of the number of nodes in the network. For *regular* networks supporting uniform traffic matrices and using fixed transmission ranges by multi-hop, we can obtain a throughput of $2/e$.

For *two dimensional random* networks we find that we can obtain a *square root* behavior, for both fixed and variable transmission radii. We find that the best regular design is a hexagonal tesselation and that the optimum average degree for random networks with fixed transmission radii is about 6. Generalizing to networks of n nodes in k-dimensional space we find that the throughput is proportional to $n^{\frac{k-1}{k}}$.

Our simulations of these networks show that the routing, flow control and specific topological structure issues become very important. We, therefore, present several routing algorithms and present simulation data to show that the routing algorithm which balances traffic flow, produces the best performance. With balanced flow, we find the performance is similar to that predicted by our model.

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CHAPTER 2
DEFINITIONS

2.1 General Approach

The general approach that we will use in determining the capacity of a network is to compute the throughput for a typical node and from that generalize to obtain the network capacity. In this approach we assume that the network is homogeneous in terms of topology and traffic load. When we come to run simulations we find that in some cases this homogeneity assumption causes problems and we have to consider a more detailed model incorporating estimates of the flows on the various links.

2.1.1 Basic System Model

The basic system that we are modeling is a SLOTTED ALOHA network operating under heavy traffic with initial retransmission delay. Initial retransmission delay means that the packet will be transmitted with probability p, even when it first arrives to the transmitter (head of the transmit queue). This means that we do not have to distinguish between the initial transmission and later retransmissions caused by collisions or errors. This assumption would affect delay computations, since each message would undergo an initial delay which would not be encountered in a real system where the initial transmission is scheduled immediately. For capacity computations, however, this model is quite satisfactory (especially under heavy traffic, where the queue at each PRU is likely to be non-empty), because we are not interested in the timing of message transmissions (we assume independence between the various hops that a message may take), but rather in the total network load generated.

We consider that the achievable throughput for heavy traffic is the same as the capacity of the network. It is arguable that this is not valid [YEMI 79b]. For example, the maximum achievable throughput for a pair of nodes operating under heavy traffic is 1/2, but if we alter the traffic matrix so that one node dominates, or force each node to alternately transmit and not transmit in each slot, we can attain a higher 'capacity'. We are assuming that this control is not available.

2.1.2 Parameters Affecting Performance

The performance of a Packet Radio Network depends on many system parameters. The following is a list of those that we will vary in this dissertation. We follow that with a discussion of those that we will not vary.

i) *The Number of Nodes in the Network*

Network capacity is always a function of the number of nodes in the network. Even for the fully connected pure ALOHA network, as analyzed in [ABRA 70] the capacity is a function of the number of nodes, and only tends to $1/2e$ in the limit as the number of nodes becomes infinite. We find that when we restrict the transmission range of the nodes, the capacity is an *increasing* function of the number of nodes. This ability to achieve throughput higher than the fully connected ALOHA capability is, in fact, the main idea behind this dissertation.

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ii) *The Traffic Matrix*

The traffic matrix plays a very significant part in the computation of the capacity. Ideally, we should average the capacity over the set of all possible traffic matrices - we take a different approach, however. For each network generated, we compute the capacity for a randomly chosen traffic matrix and then average the capacity over several different random networks. We consider that a random traffic matrix can be used as a representative of the set of all possible traffic matrices. Since there is such a strong dependency of capacity on traffic matrix, in Chapter 4 we look for that traffic matrix which maximizes the capacity.

The specific way in which we generate the random traffic matrix will depend on the kind of topology that is under investigation. When we consider one-hop communication satisfying a random traffic matrix (Chapter 3), we will *pair* nodes in a random manner. That is to say that each node is equally likely to be communicating with any other node in the network. In Chapter 4, we specifically choose a traffic matrix to 'optimize' capacity. In Chapters 5 and 6, we consider regular and random networks capable of satisfying an arbitrary traffic matrix. In the model of capacity for these networks we (implicitly) assume that the traffic generated by a node is split uniformly between all possible destinations.

iii) *The Adjacency Matrix*

The adjacency matrix defines the topology of the network. It specifies which nodes hear when another node transmits (i.e., which nodes are within range of his transmitter). It allows us to determine the interference expected at any node and which nodes can be used to forward packets. Rather than change specific entries in this matrix, we usually modify it globally, by changing the average transmission range of the devices, for example. In fact, we will often couch results in terms of this transmission radius (or equivalently, in terms of the average number of nodes within range).

iv) *The Transmission Probabilities*

These determine the probability with which a node will transmit in any slot and play a very important part in determining the capacity of the network. They allow us to adjust the traffic load in the local environment so that we do not overload the channel and cause degradation of performance caused by constant collisions and excessive contention for the scarce resource (the communication channel).

For a centralized network it is possible to find the optimum transmission probabilities [LAM 74, YEMI 79b]. For the multi-hop network case, however, there is no simple procedure for assigning transmission probabilities. We consider this question in more detail in section 2.4.

v) *The Routing Algorithm*

The routing matrix specifies how packets are passed through the network. The simplest form of routing algorithm is *fixed routing*. In fixed routing, each PRU has a table with entries showing the next node in the path from itself to the ultimate destination of the packet that it is about to transmit. The PRU stores the ID (name) of the next repeater into the header of the packet. When the packet is transmitted, all PRUs that receive it decode the header and only accept the packet if it is addressed to them. In the original routing algorithm for the centralized Packet Radio Network, it was suggested that all routing information should be carried in the header of the packet itself, thus obviating the need for PRU's to store routing tables [GITM 76]. This approach is quite satisfactory for small networks, but causes large overhead (and

waste of the communication channel) for large networks. With current trends for the cost of storage and microprocessors, it seems that storing the routing table in each PRU, may not be a significant cost (although updating them may be a serious problem for mobile devices).

The homogeneity assumption, (i.e., that the environment in all areas is identical and that all nodes carry equal traffic), that we use neglects the effect of the specific routing algorithm being used. We notice, however, that the routing has a profound effect on throughput when we make simulation runs, since some of the links in the system become heavily loaded and thus degrade overall system performance. We introduce three different routing algorithms and show that the one which attempts to balance the traffic load allows the best system performance. The interaction between topology, retransmission assignments and the routing makes the definition of an optimal routing algorithm difficult. An added complexity is that adding flow to a particular path in the network affects the load on neighboring paths (in terms of interference). Even if an optimal algorithm were available, implementation for a mobile distributed network is impossible. The most important reason that this cannot be implemented is that it is impossible for any node to know the 'true' network topology and has to rely on 'old' information. In fact, this is true for any adaptive routing algorithm.

Once we have defined this set of parameters for a particular configuration under consideration, we can compute the throughput for a particular node and then generalize to obtain the network capacity.

Other important system parameters that affect the system performance (but not varied in this dissertation) are:

i) *The Access Scheme*

Throughout this dissertation we consider only the *SLOTTED ALOHA* access scheme (section 1.4 introduced and discussed various different access schemes). As previously mentioned, other schemes allow different capacities to be attained. We feel, however, that the general principles obtained by studying the SLOTTED ALOHA policy apply to other access schemes, at least in a qualitative manner.

ii) *Packet Lengths*

Packet length will have an impact on the capacity of packet radio networks. It has been shown [GAAR 72], that fixed packet lengths always allow higher throughput in ALOHA schemes. We, therefore, study only fixed packet lengths. Capacity results presented in this dissertation will be expressed in terms of packets per slot, where a slot is the time required to transmit a (fixed length) packet.

iii) *Buffer Availability*

Although the number of buffers available in each PRU has an effect on the capacity, the difference is minimal (provided some small (say 5) lower limit is exceeded). Tobagi studies the effect of buffer limitations for centralized networks in [TOBA 78a]. We will assume the existence of infinite buffers and can thus ignore blocking.

iv) *Flow Control Policy*

Little work exists on flow control for packet radio networks. We see some of the results of an uncontrolled network in Chapter 7. In [LAM 74] we find a development of an optimal control policy for fully-connected ALOHA networks, and this is refined in [FAYO 77]. Yemini's 'urn scheme' [YEMI 79b], is a form of flow control mechanism. We make no specific assumption regarding flow control other than that we will attempt to select transmission probabilities that do not allow the local environment to become overloaded. This selection is necessary to maximize throughput in collision type channels.

v) *Processing Capability of The Nodes*

The processing speed of the PRUs will have some impact on the capacity of the network. While the processor is decoding the header, performing buffer queue management and any other tasks, it is unable to use the channel. This may cause a decrease in the capacity of the network. This effect is relatively minor, but should not be ignored, especially if the PRUs are to perform complex tasks such as routing analysis. We assume, however, that the processing time required at a node is zero.

vi) *Acknowledgement Schemes*

Acknowledgement traffic can use a significant portion of the channel and the kind of acknowledgements will therefore impact the network performance. In most studies of these networks the authors choose to ignore this factor; we do also. We consider this traffic to cause an overall scaling down of any performance results that we derive. In order to compare our results with other authors it is necessary to use similar assumptions. In [TOBA 78c] we find a study of the effect of acknowledgement traffic.

2.2 Definitions

In this section we define the important system parameters more precisely and introduce other key variables which constantly crop up in our models. We start out by giving various notational conventions that we use.

2.2.1 Notation

We define the following notation:

i) *Link:* A link between node i and j will be designated (i,j).

ii) *Path:* A path from s to d, usually defined by the routing matrix, will be designated $[s,d]$.

$$[s,d] = (s,i_1)(i_1,i_2) \ldots (i_{k-1},d) \tag{2.1}$$

is a path of length k.

iii) *Summation Convention:* A period (.) used in place of a subscript will imply summation over the range of values of that subscript.

$$p_{i.} = \sum_j p_{ij} \qquad (2.2)$$

2.2.2 Variables

The following variables are used throughout this dissertation and we collect their definitions here.

i) *The Spatial Density of Nodes* λ

λ represents the average number of points (nodes) per unit area (length, volume).

ii) *The Number of Nodes* n

The number of nodes in the network will be denoted by n. In general we generate the network inside a unit area (volume) and thus:

$$\bar{n} = \lambda \qquad (2.3)$$

(where $\bar{x}$ denotes the expected value of x.)

iii) *The Traffic Matrix* τ

The traffic matrix, $\tau = (t_{ij})$, defines the amount of flow between nodes in the network. In this dissertation it will be used to identify the traffic pattern and is assumed to be normalized so that the total network traffic sums to unity (i.e. $t_{..} = \sum\sum t_{ij} = 1$). If $t_{ij} = t$ then the traffic between i and j constitutes a proportion, t, of the total network traffic.

iv) *The Traffic Level* L

This specifies the multiplier to be used on the traffic matrix to find the true network traffic.

v) *The Adjacency Matrix* A

The adjacency matrix, $A = (a_{ij})$, defines the hearing graph, i.e., which nodes can hear which others. This matrix is not necessarily symmetric since the underlying graph is directed (different nodes may be using different transmission radii).

$$a_{ij} = \begin{cases} 1 & \text{if } j \text{ hears } i \\ 0 & \text{otherwise} \end{cases} \qquad (2.4)$$

Note: $a_{ii} = 1$ since a node always hears its own transmission.

vi) *The Hearing Distribution* H_i

The hearing distribution defines the probability that a node can hear i nodes, including himself. H_i is the probability that **A** has i ones in any column.

$$H_i = \text{Pr}\{\text{a node can hear } i \text{ other nodes}\} \qquad i = 1, 2, \ldots, n \qquad (2.5)$$

vii) *The Hitting Distribution* h_i

The hitting distribution represents the probability that i nodes hear a given node's transmission. It corresponds to the probability of **A** having i ones in any row.

$$h_i = \text{Pr}\{ i \text{ nodes hear when a node transmits}\} \qquad i = 1, 2, \ldots, n \qquad (2.6)$$

viii) *The Average Degree* N

We will find that many of the results of this dissertation can be stated in terms of the average degree. This is the average number of nodes that hear when a node transmits.

$$N = \text{E}[\text{number of nodes within range}]$$

$$= \sum_{i=1}^{n} i \, h_i \qquad (2.7)$$

ix) *The Transmission Probability Matrix* **P**

This matrix is an extremely important system parameter, which defines the probability with which a node will transmit in any slot. Since we assume heavy traffic, (every node is always busy), this parameter is sufficient to describe the behavior of a node.

$$p_{ij} = \text{Pr}\{ i \text{ transmits to } j \text{ in any slot}\} \qquad (2.8)$$

The probability that node i transmits to any other node in a slot, p_i, occurs repeatedly in our analysis and, in fact, it is this parameter that we adjust for flow control. We will use the notation p_i for the probability that node i transmits in any slot.

$$p_i = \text{Pr}\{\text{node } i \text{ transmits in any slot}\} \qquad (2.9)$$

$$= p_{i.} \qquad (2.10)$$

x) *The Routing Matrix* **R**

The routing matrix defines how messages are routed through the network. We assume that all messages destined for a particular node, passing through the same intermediate node, will follow the same route from that node onwards, irrespective of the source node. We can state this formally as follows:

Consider the routes (defined by the routing matrix) from the sources s_1 and s_2 to the destination d. Let $\pi_1 = [s_1,d]$ and $\pi_2 = [s_2,d]$, then if these paths have a common node k, we have:

$$\pi_1 = [s_1,k]\ \pi'_1 \quad \text{and} \quad \pi_2 = [s_2,k]\ \pi'_2 \tag{2.11}$$

$$\Rightarrow\ \pi'_1 = \pi'_2 \tag{2.12}$$

This greatly simplifies the routing algorithm, both from an analytical and implementation point of view. We can represent the routing algorithm by the matrix shown below. (This also implies that the PRUs need only store one row of this matrix rather than the whole n by n array.) If packets arriving to node i, destined to node j are passed on to node k, then the routing matrix entry is given by:

$$r_{ij} = k$$

xi) *The Link Flow Matrix* **F**

This represents the load on each (virtual*) link in the net and corresponds to the node to node flow pattern determined from the traffic and routing matrices. It represents the flow requirement for a unit traffic matrix and should be multiplied by the traffic level, L, to find the true flow.

For each source destination pair in the network, we add t_{sd} to the flow for each link on the routing defined path from s to d.

$$f_{ij} = \sum_s \sum_d t_{sd}\ \delta_{ij}^{sd} \tag{2.13}$$

Where, $\delta_{ij}^{sd} = 1 \ <\Rightarrow\ (i,j) \in [s,d]$

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* We do not have physical links between nodes like we do in a conventional network. We can however think of a link existing between any pair of nodes that are able to communicate.

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xii) *The Success Probability Matrix* S

The entries of this matrix represent the probability of a node successfully receiving a packet in any slot.

$$s_{ij} = \Pr\{j \text{ successfully receives from } i\} \qquad (2.14)$$

For convenience we define s to be the probability that a node successfully receives a packet from *any* other node in a slot.

$$s_i = \Pr\{\text{node } i \text{ successfully receives a packet}\} \qquad (2.15)$$

$$= s_{i.} \qquad (2.16)$$

xiii) *The Link Utilization Matrix* U

This defines the utilization of a link in the network and is the ratio of the flow requirement (f_{ij}) to the achievable throughput (s_{ij}). This ratio is multiplied by the traffic level L, to give the actual utilization of this link.

$$u_{ij} = L. \frac{f_{ij}}{s_{ij}} \qquad (2.17)$$

Thus, if u_{ij} is greater than one for any link in the network that link is overloaded and the traffic level must be reduced.

xiv) *The Network Throughput Vector* Γ

In *One Hop* networks we can associate a throughput with every node in the network and we refer to γ_i to mean the maximum rate at which node i can receive messages. In this case the capacity is identical to the heavy traffic success probability.

$$\gamma_i = s_i \qquad (2.18)$$

xv) *The Network Capacity* γ

The network capacity represents the maximum number of end to end messages deliverable in any slot. It is measured in packets per slot, and corresponds to the $1/e$ measure that we know for fully connected slotted ALOHA networks.

For one-hop networks the network capacity is given by:

$$\gamma = \sum_i s_i \qquad (2.19)$$

In *multi-hop* networks the throughput is determined by the utilization of the heaviest loaded link in the network. If we scale the traffic level so that the load on this link is one, then the traffic level L^* that achieves this will be the network capacity.

$$\gamma = L^*$$ (2.20)

Where,

$$L^* = \frac{1}{\min_{\substack{i \\ j}} \{u_{ij}\}}$$ (2.21)

2.3 Computation of Capacity

Below we give an example of how the capacity of a network can be computed for i) a single hop network; and ii) a multi-hop network.

2.3.1 One-Hop Network

For one-hop networks we assume that nodes are paired into communication partners. Thus the success rate for node i, whose partner is j, is given by:

$$s_i = \Pr\{ j \text{ transmits}\}\ \Pr\{ i \text{ does not}\}\ \Pr\{\text{no other node that } i \text{ hears, transmits}\}$$

Assuming heavy traffic,

$$s_i = p_j \prod_{\substack{k=1 \\ k \neq j}}^{n} (1 - a_{ki}\, p_k)$$

$$= \frac{p_j}{1 - p_j} \prod_{k=1}^{n} (1 - a_{ki}\, p_k)$$ (2.22)

Example

Figure 2.1 shows a random network of four nodes, the circles drawn around each node representing the area covered by a transmission of that node. The transmission radii shown are determined by the traffic matrix, which in this example requires that nodes 1 and 2 are a communicating pair and that 3 and 4 are the other pair. The adjacency matrix is:
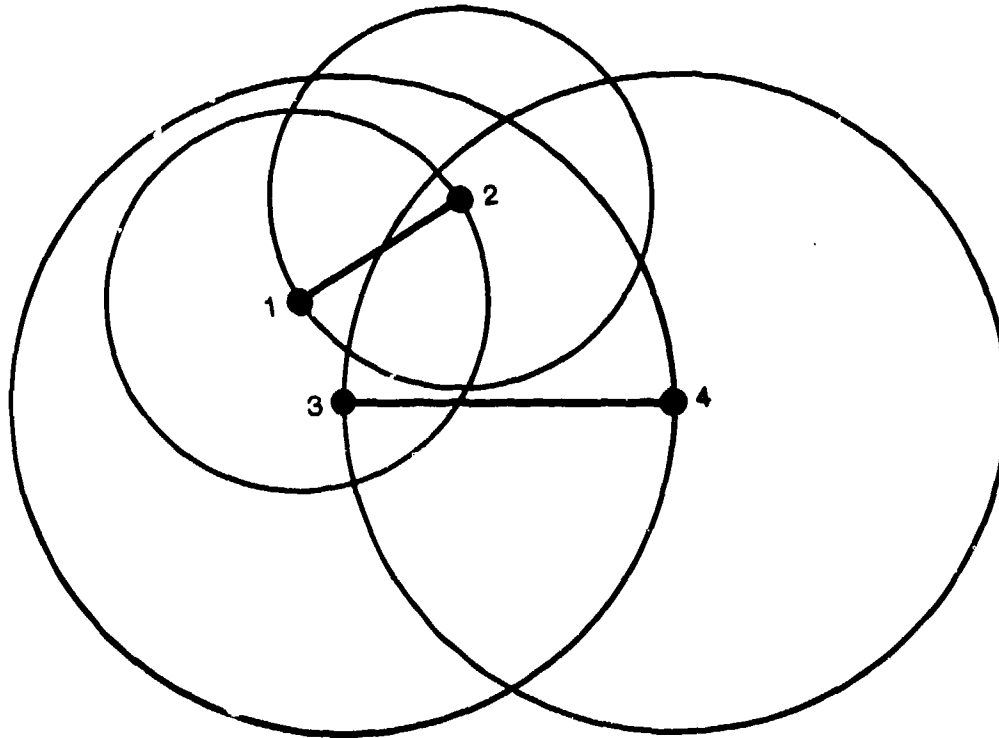
Figure 2.1 A Simple Four Node Network

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

(2.23)

The probability of a successful reception at node 1 is:

$$s_1 = p_2(1-p_1)(1-p_3)$$

(2.24)

For heavy traffic in the Slotted ALOHA mode, the maximum reception rate of node i, γ_i, is equal to the probability of a successful reception, s_i.

If we assign a transmission probability of ½ to each node the nodal throughputs, $\Gamma = \gamma_i$, are given by:

$$\Gamma = \begin{Bmatrix} (\frac{1}{2})^3 \\ (\frac{1}{2})^4 \\ (\frac{1}{2})^3 \\ (\frac{1}{2})^2 \end{Bmatrix} = \begin{Bmatrix} \frac{1}{8} \\ \frac{1}{16} \\ \frac{1}{8} \\ \frac{1}{4} \end{Bmatrix} \qquad (2.25)$$

This gives a total throughput, γ $(=\sum\gamma_i)$, of:

$$\gamma = \frac{9}{16} \qquad (2.26)$$
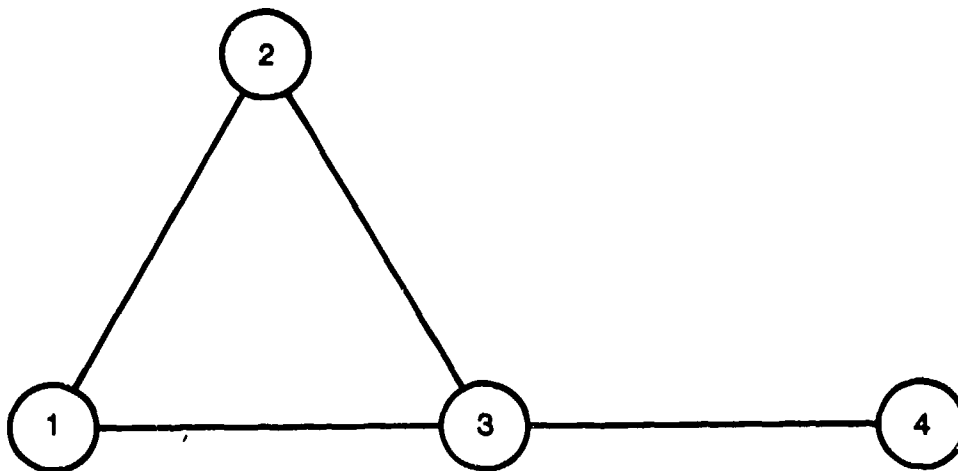
2.3.2 Multi-Hop Network

Figure 2.2 Simple Multi-Hop Network

Figure 2.2 shows a sample multi-hop network. Let us assume a uniform traffic matrix, that is, all nodes communicate with all others on an equal basis. The traffic matrix is:

$$\tau = \frac{1}{12} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \tag{2.27}$$

The adjacency matrix is:

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \tag{2.28}$$

The obvious routing matrix is:

$$R = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \end{bmatrix} \tag{2.29}$$

From this we can determine the flow requirement matrix:

$$F = \frac{1}{12} \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 2 & 0 \\ 2 & 2 & 0 & 3 \\ 0 & 0 & 3 & 0 \end{bmatrix} \tag{2.30}$$

Note that the total network traffic $(f_{..} = \sum\sum f_{ij} = 16/12$, since [1,4], [2,4], [4,1] and [4,2] contain two hops. Using a transmission probability of the reciprocal of the number of nodes that you hit:

$$P = \begin{bmatrix} \dfrac{1}{3} \\[6pt] \dfrac{1}{3} \\[6pt] \dfrac{1}{4} \\[6pt] \dfrac{1}{2} \end{bmatrix} \tag{2.31}$$

With the flow requirements matrix and the transmission probabilities, we determine the probability that node i transmits to node j in any slot, p_{ij} as the probability that node i transmits, p_i, times the fraction of his traffic destined to j, $f_{ij}/f_{i.}$.

$$p_{ij} = p_i \frac{f_{ij}}{\sum_j f_{ij}} \tag{2.32}$$

Evaluating this matrix:

$$\mathbf{P} = \begin{Bmatrix} 0 & \frac{1}{9} & \frac{2}{9} & 0 \\ \frac{1}{9} & 0 & \frac{2}{9} & 0 \\ \frac{1}{14} & \frac{1}{14} & 0 & \frac{3}{28} \\ 0 & 0 & \frac{1}{2} & 0 \end{Bmatrix} \tag{2.33}$$

From this we may determine the success probabilities, a g each node is *always busy*

$$s_{ij} = p_{ij} \prod_{\substack{k=1 \\ k \neq i}}^{n} \left[(1 - p_k)a_{kj} \right] \tag{2.34}$$

Thus,

$$s_{12} = s_{21} = \frac{1}{9} \left(1 - \frac{1}{3} \right) \tag{2.35}$$

$$s_{31} = s_{32} = \frac{1}{14} \left(1 - \frac{1}{3} \right)^2 \tag{2.36}$$

$$s_{13} = s_{23} = \frac{2}{9} \left(1 - \frac{1}{2} \right) \left(1 - \frac{1}{3} \right) \left(1 - \frac{1}{4} \right) \tag{2.37}$$

$$s_{43} = \frac{1}{2} \left(1 - \frac{1}{3} \right)^2 \left(1 - \frac{1}{4} \right) \tag{2.38}$$

$$s_{34} = \frac{3}{28} \left(1 - \frac{1}{2} \right) \tag{2.39}$$

and so,

$$S = \begin{Bmatrix} 0 & \dfrac{1}{18} & \dfrac{1}{18} & 0 \\ \dfrac{1}{18} & 0 & \dfrac{1}{18} & 0 \\ \dfrac{2}{63} & \dfrac{2}{63} & 0 & \dfrac{3}{56} \\ 0 & 0 & \dfrac{1}{6} & 0 \end{Bmatrix} \qquad (2.40)$$

We can no / compute the utilities $u_{ij} = \dfrac{f_{ij}}{s_{ij}}$ (as the initial traffic level $L = 1$), the reciprocal of the maximum of which, determines the network capacity.

$$U = \begin{Bmatrix} 0 & \dfrac{3}{2} & \dfrac{3}{1} & 0 \\ \dfrac{3}{2} & 0 & \dfrac{3}{1} & 0 \\ \dfrac{21}{4} & \dfrac{21}{4} & 0 & \dfrac{28}{9} \\ 0 & 0 & \dfrac{3}{2} & 0 \end{Bmatrix} \qquad (2.41)$$

The largest entry in this table identifies the heaviest loaded link, namely $(1,3)$ or $(2,3)$. The maximum traffic level we can support, L^*, is thus given by:

$$L^* = \frac{1}{u_{23}} = \frac{4}{21} \qquad (2.42)$$

The throughput, γ, is then given by:

$$\gamma = L^* = \frac{4}{21} \qquad (2.43)$$

This is a pessimistic estimate for the capacity because, as mentioned before, we are assuming that the nodes are always busy. In fact, to satisfy this assumption we should (optimally) select the transmission probabilities so that each node is equally busy. As we show below this is not a trivial computation and, in fact, is not even feasible for arbitrary networks, due to the dependence between success rates of neighboring nodes.

2.4 Optimal Retransmission Probabilities

For a fully connected network with each node carrying equal traffic, the optimal retransmission probabilities are known to be $1/n$ where n is the number of nodes in the network (for heavy traffic) [LAM 74]. The general optimality condition (for unequal traffic rates) is that the sum of the offered traffic rates, (which corresponds to the transmission probabilities in our model), should sum to unity. This is no longer true for networks where different nodes are in different environments and the global

optimality condition for this case gives no such simple rule. In [YEMI 79a], we find rules for local optimality which can be used to generate "global optima" by suitable selection of costs to weight the value of transmissions from different nodes. It is not clear how we can apply these costs to satisfy the global flow requirements, however.

In a real network the transmission probabilities should be adjusted as a function of the traffic load. In light traffic, when the probability of a collision is low, a high transmission probability can be used to reduce delay. In heavy traffic the transmission probability must be reduced so that the channel does not become overloaded. Since in this dissertation we are interested only in capacity results we will usually use $p = 1/k$ where k is the number of nodes that hear the transmission. This is not necessarily optimal as we see below. It will, in fact tend to under-estimate the capacity of the network. It will ensure, however, that the local environment is not *overloaded*.
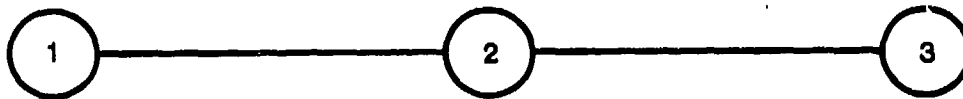

Figure 2.3 Simplest Possible Network

Consider the network shown in Figure 2.3 operating under a uniform traffic pattern; that is τ is given by:

$$\tau = \frac{1}{6} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad (2.44)$$

This generates the flow pattern f_{ij} as given by the matrix $\mathbf{F} = f_{ij}$:

$$\mathbf{F} = \frac{1}{3} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad (2.45)$$

Using the $1/k$ transmission probabilities, we have:

$$\mathbf{P} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{2} \end{pmatrix} \qquad (2.46)$$

We can determine the throughput matrix to be:

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$$S = \begin{pmatrix} 0 & \frac{1}{6} & 0 \\ \frac{1}{12} & 0 & \frac{1}{12} \\ 0 & \frac{1}{6} & 0 \end{pmatrix}$$ (2.47)

Using our simple rule, the maximum throughput that can be supported is therefore:

$$L_{max} = \frac{1}{4}$$ (2.48)

For this case we can easily determine the optimal **P** vector. By symmetry we have that $p_1 = p_3$ and $p_{21} = p_{23} = \frac{1}{2}p_2$. The only interesting throughput terms are s_{12} and s_{21}, which are given by:

$$s_{12} = p_1 (1 - p_2) (1 - p_3) = p_1 (1 - p_2) (1 - p_1)$$ (2.49)

$$s_{21} = p_{21} (1 - p_1) = \frac{1}{2}p_2 (1 - p_1)$$ (2.50)

We must find p_1, p_2 to maximize the ratio of throughput over flow requirement. Since all the flows are equal, and increasing s_{12} decreases s_{21}, the maximum will be attained when $s_{12} = s_{21}$, i.e.:

$$2p_1 (1 - p_2) (1 - p_1) = p_2 (1 - p_1)$$ (2.51)

which gives:

$$p_1 = \frac{p_2}{2(1 - p_2)}$$ (2.52)

and

$$s_{12} = \frac{p_2}{2}\left[1 - \frac{p_2}{2(1 - p_2)} \right]$$ (2.53)

Differentiating with respect to p_2 we have:

$$\frac{ds_{21}}{dp_2} = 1 - \frac{p_2}{(1 - p_2)} - \frac{p_2^2}{2(1 - p_2)^2} = 0$$ (2.54)

$$=> \quad p_2 = 1 \begin{array}{c} + \\ - \end{array} \frac{1}{\sqrt{3}}$$ (2.55)

Only the negative root makes sense, giving:

$$\mathbf{P}_{opt} = \begin{pmatrix} \dfrac{\sqrt{3}-1}{2} \\[2mm] \dfrac{\sqrt{3}-1}{\sqrt{3}} \\[2mm] \dfrac{\sqrt{3}-1}{2} \end{pmatrix} = \begin{pmatrix} .365 \\ .422 \\ .365 \end{pmatrix} \qquad (2.56)$$

and the throughput matrix is given by:

$$\mathbf{S} = \begin{pmatrix} 0 & 0.134 & 0 \\ 0.134 & 0 & 0.134 \\ 0 & 0.134 & 0 \end{pmatrix} \qquad (2.57)$$

Which gives $L_{max}=0.402$ which is significantly better than 1/4. We were only able to carry out this optimization due to the simple network structure. In more realistic cases we are faced with a much more complex problem.

As we shall find later in our simulation runs (Chapter 7), the simple policy of using $p=1/k$ is no good in situations where the traffic loads on nodes are unequal. For these situations we propose the following approach:

$$p_i = \frac{\text{total traffic carried by } i}{\text{total traffic carried by nodes that hear } i} \qquad (2.58)$$

This attempts to set the traffic load in any environment to unity as suggested previously.

For the network of Figure 2.3 we then have:

$$\mathbf{P} = \begin{pmatrix} \dfrac{1}{3} \\[2mm] \dfrac{1}{2} \\[2mm] \dfrac{1}{3} \end{pmatrix} \qquad (2.59)$$

which gives:

$$\mathbf{S} = \begin{pmatrix} 0 & \dfrac{1}{9} & 0 \\[2mm] \dfrac{1}{6} & 0 & \dfrac{1}{6} \\[2mm] 0 & \dfrac{1}{9} & 0 \end{pmatrix} \qquad (2.60)$$

The maximum throughput that can be achieved is therefore, $L^* = 1/3$. This is much closer to the optimum than the simpler scheme outlined above. We will use this approach in our simulations of networks where the traffic is unbalanced.

CHAPTER 3
ONE-HOP COMMUNICATION

The networks that we consider in this chapter consists of a set of nodes randomly located which are able to communicate directly, (i.e., in one-hop). These networks may be thought of as either representatives of the set of all possible networks or as snapshots of a mobile network. In order to model the requirement that the network should be able to handle an arbitrary traffic pattern, we assume a uniform traffic matrix. Our traffic model is, then, of the (instantaneous) communication requirement between some active subset of the total number of nodes in the network (non-active nodes are ignored).

3.1 Network Model

A network is a set of n (active) nodes (with n even to allow pairing) located randomly according to a uniform distribution in a unit hypersphere. These nodes are then randomly paired to represent communicating pairs of nodes. Having generated the network and traffic matrix, we satisfy the communication requirement by suitable choice of transmission power. There are two approaches to satisfying this random communication pairing: i) give every node sufficient power to be able to reach every other node in the network; or ii) give each node sufficient power to just reach his communication partner.

Once the network is established, as above, we have one additional parameter to specify - the probability that a node will transmit in any slot. (This corresponds to the offered channel traffic randomized so that Slotted ALOHA will operate correctly and resolve previous conflicts due to simultaneous transmissions.) In order to compute the throughput we use the 'heavy traffic model', which corresponds to assuming that all (active) nodes are always busy, but which transmit in any given slot depending on this transmission probability. We denote the transmission probability for node i as p_i.

Nodal Throughput: Consider an arbitrary node (say node i) in the network. The probability that this node correctly receives a packet from his partner (say node j) in any slot, is given by:

$$s_i = \Pr\{j \text{ transmits}\} \Pr\{i \text{ does not transmit}\} \Pr\{\text{none of } i\text{'s neighbors transmits}\}$$

$$= p_j (1-p_i) \prod_{k \in N_i} (1 - p_k) \tag{3.1}$$

where N_i is the set of nodes that i can hear (excluding his partner j). The assumption here is that reception is a discrete process, i.e., a node either hears a transmission or does not. Thus another transmission either causes interference or not depending on whether he is more distant than the threshold of reception. In a real network this reception process in not disrete but depends on relative power levels, noise etc.

For the heavy traffic model, s_i corresponds to the (received) throughput γ_i for this node, (recall that we are considering one-hop networks). Thus the total network throughput, γ, is given by:

$$\gamma = \sum_{i=1}^{n} \gamma_i = \sum_{i=1}^{n} s_i \tag{3.2}$$

3.2 Completely Connected Topologies

One approach to satisfying an arbitrary random traffic matrix is to give every node sufficient transmission power so that all the nodes in the network hear when any one transmits. This corresponds to the model of [ROBE 75, ABRA 77] and the total network throughput will therefore approach $1/e$. We proceed to show that our approach gives the same result.

Since the environment for each node is identical, we assume $p_i = p$. The number of nodes that can interfere with a given transmission is $n-2$, so the throughput for each node is:

$$\gamma_i = p \, (1-p) \, (1-p)^{n-2} \tag{3.3}$$

In order to set the offered traffic in any environment to be the optimum value of one packet per slot [ABRA 70, LAM 74, YEMI 79b], we use a transmission probability of $p = 1/n$. (This can easily be seen to be optimal by differentiating with respect to p in Equation 3.3.) We then have:

$$\gamma_i = \frac{1}{n} \left(1 - \frac{1}{n} \right)^{n-1} \tag{3.4}$$

As the throughput for each node is identically distributed, the network throughput, γ , is simply:

$$\gamma = \left(1 - \frac{1}{n} \right)^{n-1} \tag{3.5}$$

which is the exact behavior for finite n and exhibits the expected asymptotic behavior of $1/e$ for large networks.

3.3 Limited Transmitter Power

Another approach for arbitrary traffic matrices is to limit the power of each transmitter so that it exactly reaches its destination (again assuming that reception is a two-state process, either you can or cannot hear a transmission). In Figure 3.1 we show a two-dimensional network of 10 nodes generated in this manner by the simulation program described in section 3.5; the lines joining pairs of nodes represent the traffic matrix and hence the transmission radii (e.g., nodes 3 and 9 are a communicating pair).

3.3.1 General Model

Since the networks we consider are homogeneous, the throughput for all nodes is identically distributed; therefore, we drop the subscripts corresponding to the particular node under investigation. We attempt to set the offered traffic in any environment to unity by selecting the transmission probability to be $1/k$ for a node that interferes (hits) with k others when he transmits, including himself and his transmission partner. Using the notation $\gamma(k)$ to represent the throughput for a node which hits k and making the assumption that both nodes of a partnership hit the same number of nodes*, we obtain the following expression for the throughput:

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* Since both nodes are transmitting at the same range, certainly the *expected* number hit by a transmission will be the same.
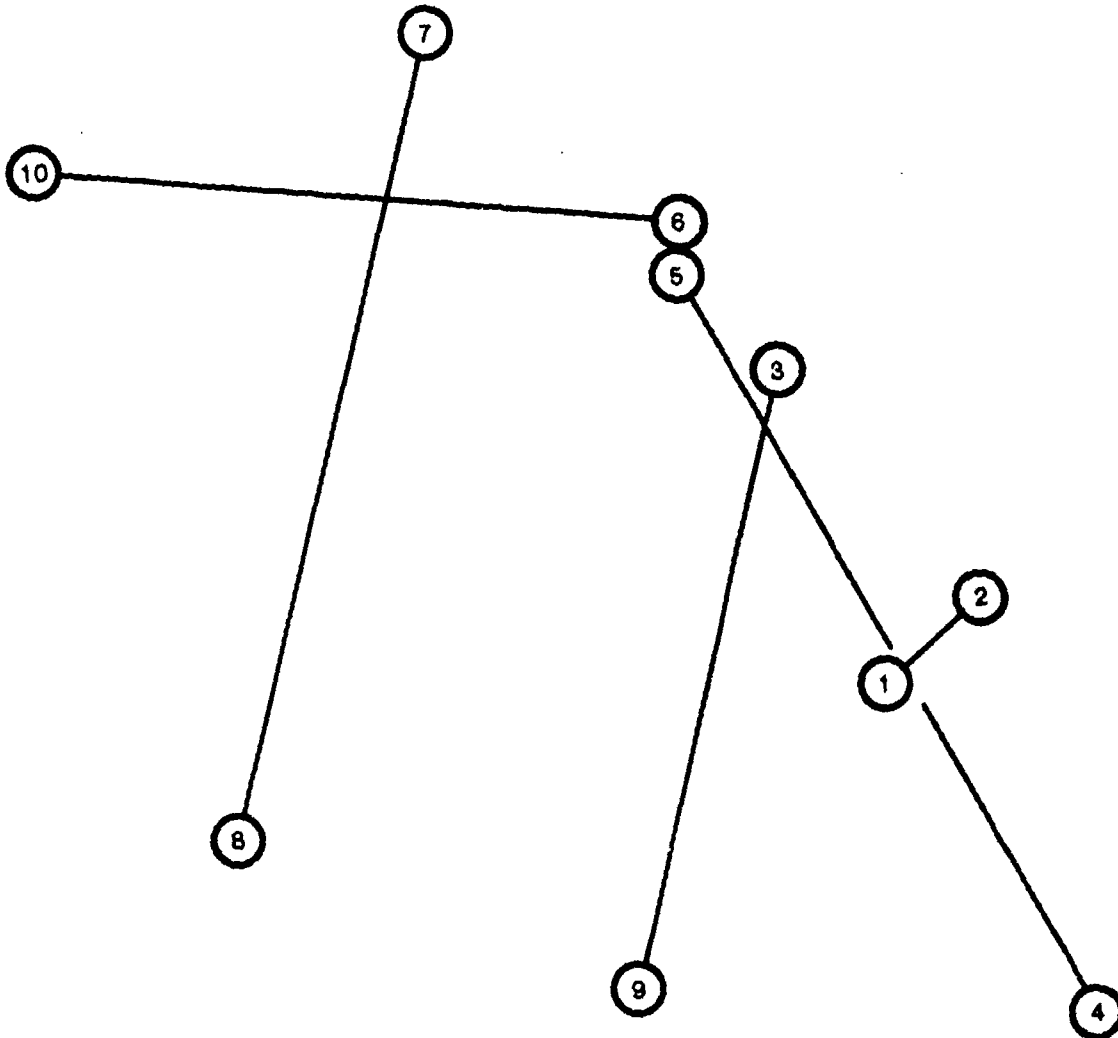
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Figure 3.1 10 Node Limited Power Network

$$\gamma(k) = I \frac{1}{k} \left(1 - \frac{1}{k} \right) \tag{3.6}$$

where I, the interference factor, is the interference contribution from nodes other than the node itself. We can think of this factor as background interference. If we assume that the interference encountered at any node is independent of the degree of that node, then the expected throughput for any node in the network, γ_{node}, is given by:

$$\gamma_{node} = I \sum_{k=2}^{n} h_k \gamma(k) \tag{3.7}$$

where h_k is the probability that a node hits (interferes with) k nodes when he transmits (note that he always hits himself and his partner).

We now proceed to find the hitting distribution, h_k. Consider an arbitrary node, P, in the network and rank the $n-1$ other nodes in order of their distance from P. If P is paired with a node in the $k+1$st. position in this list (i.e., his kth neighbor), he will interfere with (hit) exactly $k+1$ nodes when he transmits. As P is equally likely to be paired with any of the nodes, the hitting distribution is given by:

$$h_k = \frac{1}{n-1} \qquad k = 2,3, \cdots ,n \tag{3.8}$$

Substituting this into Equation 3.7 we have:

$$\gamma_{node} = \frac{I}{n-1} \left[\sum_{k=2}^{n} \frac{1}{k} \left(1 - \frac{1}{k} \right) \right] \tag{3.9}$$

3.3.2 The Interference Factor

I is the product of terms corresponding to the interference generated by each node that is in range. We can group these terms depending on the number of nodes that the source of the interference hits when he transmits. We define I_k to be the total interference contribution of nodes that hit k when they transmit. The total interference will then be the product of these factors.

$$I = \prod_{k=2}^{n} I_k \tag{3.10}$$

Let us call a node that hits k others when he transmits a 'k-hitter'. Then,

$$I_k = \sum_{j=0}^{n-2} \Pr\{\text{a node hears } j \ k\text{-hitters}\} (1 - f_k)^j \tag{3.11}$$

where f_k is the transmission probability of a node that hits k others (equal to $1/k$ for this example).

We wish to evaluate the probability that an arbitrary node in the network will hear another node. Let us call the probability of hearing a particular k-hitter, α_k. Assuming that the 'hits' of this node are uniformly distributed over the set of nodes, we can evaluate α_k.

$$\alpha_k = \frac{\begin{pmatrix} n-3 \\ k-3 \end{pmatrix}}{\begin{pmatrix} n-2 \\ k-2 \end{pmatrix}} = \frac{k-2}{n-2} \qquad (3.12)$$

By unconditioning on k, we can evaluate the probability, α, that we hear any particular other node.

$$\alpha = \sum_{k=2}^{n} h_k \frac{k-2}{n-2}$$

$$= \sum_{k=2}^{n} \frac{1}{n-1} \frac{k-2}{n-2}$$

$$= \frac{(n-2)(n-1)}{2} \frac{1}{(n-1)(n-2)} = \frac{1}{2} \qquad (3.13)$$

Thus the probability of hearing j other nodes, H_j (the hearing distribution), is the binomial distribution given below. (Note the subscript for H is $j+2$ since a node always hears two others - himself and his partner.)

$$H_{j+2} = \begin{pmatrix} n-2 \\ j \end{pmatrix} \left(\frac{1}{2} \right)^j \left(\frac{1}{2} \right)^{n-2-j} \qquad j=0,1,\ldots,n-2 \qquad (3.14)$$

In the determination of the interference factor, however, we need to evaluate the probability that the node hears j k-hitters, H_j^k.

$$H_j^k = \sum_{l=j}^{n-2} \Pr\{\text{Total of } l \ k-\text{hitters}\} \Pr\{\text{hear } j \text{ of } l\}$$

$$= \sum_{l=j}^{n-2} \begin{pmatrix} n-2 \\ l \end{pmatrix} (h_k)^l (1-h_k)^{n-2-l} \begin{pmatrix} l \\ j \end{pmatrix} \left(\frac{k-2}{n-2} \right)^j \left(1 - \frac{k-2}{n-2} \right)^{l-j} \qquad (3.15)$$

We thus have:

$$I_k = \sum_{j=0}^{n-2} \sum_{l=j}^{n-2} \begin{pmatrix} n-2 \\ l \end{pmatrix} (h_k)^l (1-h_k)^{n-2-l} \begin{pmatrix} l \\ j \end{pmatrix} \left(\frac{k-2}{n-2} \right)^j \left(1 - \frac{k-2}{n-2} \right)^{l-j} (1-f_k)^j \qquad (3.16)$$

We can switch the order of summation to get:

$$I_k = \sum_{l=0}^{n-2} \binom{n-2}{l} (h_k)^l (1-h_k)^{n-2-l} \sum_{j=0}^{l} \binom{l}{j} \left[\frac{k-2}{n-2}\right]^j \left[1 - \frac{k-2}{n-2}\right]^{l-j} (1-f_k)^j$$

$$= \sum_{l=0}^{n-2} \binom{n-2}{l} (h_k)^l (1-h_k)^{n-2-l} \left[\frac{k-2}{n-2}(1-f_k) + \left(1 - \frac{k-2}{n-2}\right)\right]^l$$

$$= \sum_{l=0}^{n-2} \binom{n-2}{l} (h_k)^l \left(1 - \frac{k-2}{n-2} f_k\right)^l (1-h_k)^{n-2-l}$$

$$= \left[h_k \left(1 - \frac{k-2}{n-2} f_k\right) + (1-h_k)\right]^{n-2}$$

$$= \left[1 - h_k \left(\frac{k-2}{n-2}\right) f_k\right]^{n-2} \tag{3.17}$$

Thus the interference factor, I, is given by:

$$I = \prod_{k=3}^{n} \left(1 - \frac{k-2}{n-2} h_k f_k\right)^{n-2} \tag{3.18}$$

For large n, we can use the exponential approximation to find:

$$\lim_{n \to \infty} I = \prod_{k=3}^{n} e^{-(k-2)h_k f_k}$$

$$= e^{-\sum_{k=3}^{n} (k-2)h_k f_k} \tag{3.19}$$

For $f_k = 1/k$ and $h_k = 1/n-1$, the exponent of Equation 3.19 will be:

$$\lim_{n \to \infty} \sum_{k=3}^{n} \frac{k-2}{k} \frac{1}{n-1} = 1$$

and the interference factor will be:

$$I = \frac{1}{e} \qquad \text{(for large networks)} \tag{3.20}$$

3.3.3 Throughput

We are now in a position to evaluate the throughput. We first substitute for f_k in Equation 3.18 to obtain:

$$I - \prod_{k=3}^{n} \left(1 - \frac{k-2}{n-2} \frac{h_k}{k} \right)^{n-2} \tag{3.21}$$

We can now evaluate Equation 3.9 to obtain the total network throughput.

$$\gamma_{node} = \frac{1}{n-1} \left[\prod_{k=3}^{n} \left(1 - \frac{k-2}{n-2} \frac{1}{k} \frac{1}{n-1} \right)^{n-2} \right] \left[\sum_{k=2}^{n} \frac{1}{k} \left(1 - \frac{1}{k} \right) \right] \tag{3.22}$$

Since the throughput for each node is identically distributed, the total network throughput, γ, will be given by $n\gamma_{node}$.

$$\gamma = \frac{n}{n-1} \left[\prod_{k=3}^{n} \left(1 - \frac{k-2}{n-2} \frac{1}{k} \frac{1}{n-1} \right)^{n-2} \right] \left[\sum_{k=2}^{n} \frac{1}{k} - \frac{1}{k^2} \right] \tag{3.23}$$

With some manipulation, we find that the asymptotic behavior for large networks is given by:

$$\lim_{n \to \infty} \gamma = \frac{\log(n) + C - \frac{\pi^2}{6}}{e} \tag{3.24}$$

Where C is Euler's constant. This can be approximated by:

$$\gamma \approx \frac{\log(n) - 1}{e} \tag{3.25}$$

The above results were derived with no reference to the dimensionality of the network. We can therefore achieve a throughput *logarithmically* proportional to the network size for all networks satisfying an arbitrary traffic pattern by exact adjustment of transmission range. It must be pointed out, however, that the throughput for all pairs of nodes in the network is not the same. Nodes that are close together (and thus have high transmission probabilities since they do not interfere with many other nodes) will achieve higher throughputs than those that are far apart (recall that the background interference is uniform for all nodes in the network). Even the node with the smallest throughput (in the worst case this node will hit $n-2$ other nodes) will have a throughput of $1/ne$ for large networks, which is the same as that for the fully connected case (in which *every* node achieves a throughput of $1/ne$). Thus the node experiencing the worst performance will be doing no worse than for the fully connected case, whereas nodes close together will far exceed this throughput.

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3.4 Simulation

In order to check the validity of this model, we developed a simulation program to compute the throughputs for these networks. This program operates as follows (described for a two-dimensional network):

A uniformly distributed random network is generated and located inside the unit circle. Pairs are then randomly assigned, in fact, we pair node 1 to node 2, node 3 to node 4 and so on, (this being a perfectly random pairing). With this pairing, the transmission radii are determined so that communication can take place, and the adjacency matrix is computed. We then determine the transmission probabilities, based on the number of nodes within range of the node. From this we can compute the success probabilities for each node and hence the network throughput. These data are then averaged over several runs.

In Figure 3.2 we plot the model and simulation results for one-dimensional networks averaged over 50 networks. We also show the throughput for a fully connected network for reference purposes. We see excellent agreement between model and simulation. The retransmission policy used for this run was $p=1/k$, where k is the number of nodes hit by your transmission (partners not necessarily using the same k).

Figure 3.3 shows similar results for the two dimensional case. We noti   od agreement between the model and the simulation results for large n. In two dimensions the mo... requires larger networks before the agreement is good, due to the higher proportion of nodes on the edge of the area which suffer less interference. It is for this reason that the simulation results exceed that predicted by the model for small networks.

3.5 Other transmission Policies

In addition to using the transmission policy of $p=1/k$, we have also investigated several others. We note (from the form of Equation 3.22) that if p is any other function of k, then either the interference will increase and dominate (reducing throughput) or the success term will become smaller and dominate. We expect therefore, that using $p=1/k$ will give the best performance. For the schemes that we have tried this is indeed the case. The performance of the following schemes is shown in Figures 3.4 through 3.7 (pages 44-47).

i) *Fixed Transmission Probability* If we use a fixed transmission probability (independent of the hitting degree and the network size), the throughput as a function of network size rapidly falls to zero since too much interference is generated. Figure 3.4 shows the behavior for $p=\frac{1}{2}$. Note that if $p=1/n$ we expect a throughput of $1/e$.

ii) *Hearing Degree* We also tried using the hearing degree rather than the hitting degree for determining the transmission probability. Figure 3.5 shows the performance for the case where a node uses its own hearing degree to determine the transmission probability. We see that the throughput is independent of the network size and appears to be constant at $2/e$. A justification of this is that the average number heard (from the binomial distribution above) is $n/2$ and in fact, for large n, this distribution will have a sharp peak. Each node, therefore, hears $n/2$ nodes each of which transmits with probability $2/n$. The nodal throughput is thus:

$$Y_{node} = \frac{2}{n}\left(1 - \frac{2}{n}\right)^{\frac{n}{2}} \approx \frac{2}{ne} \qquad (3.26)$$
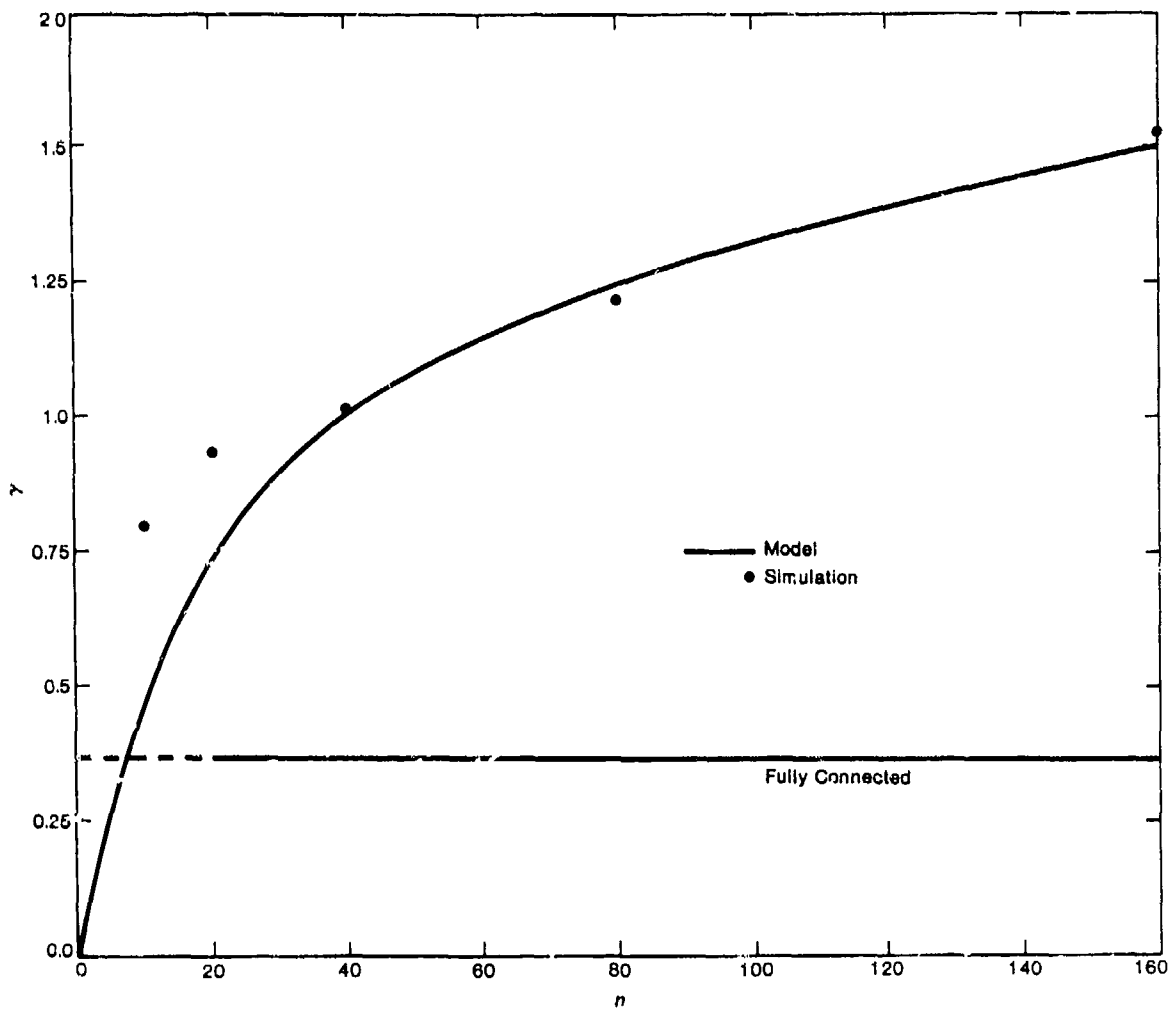
40

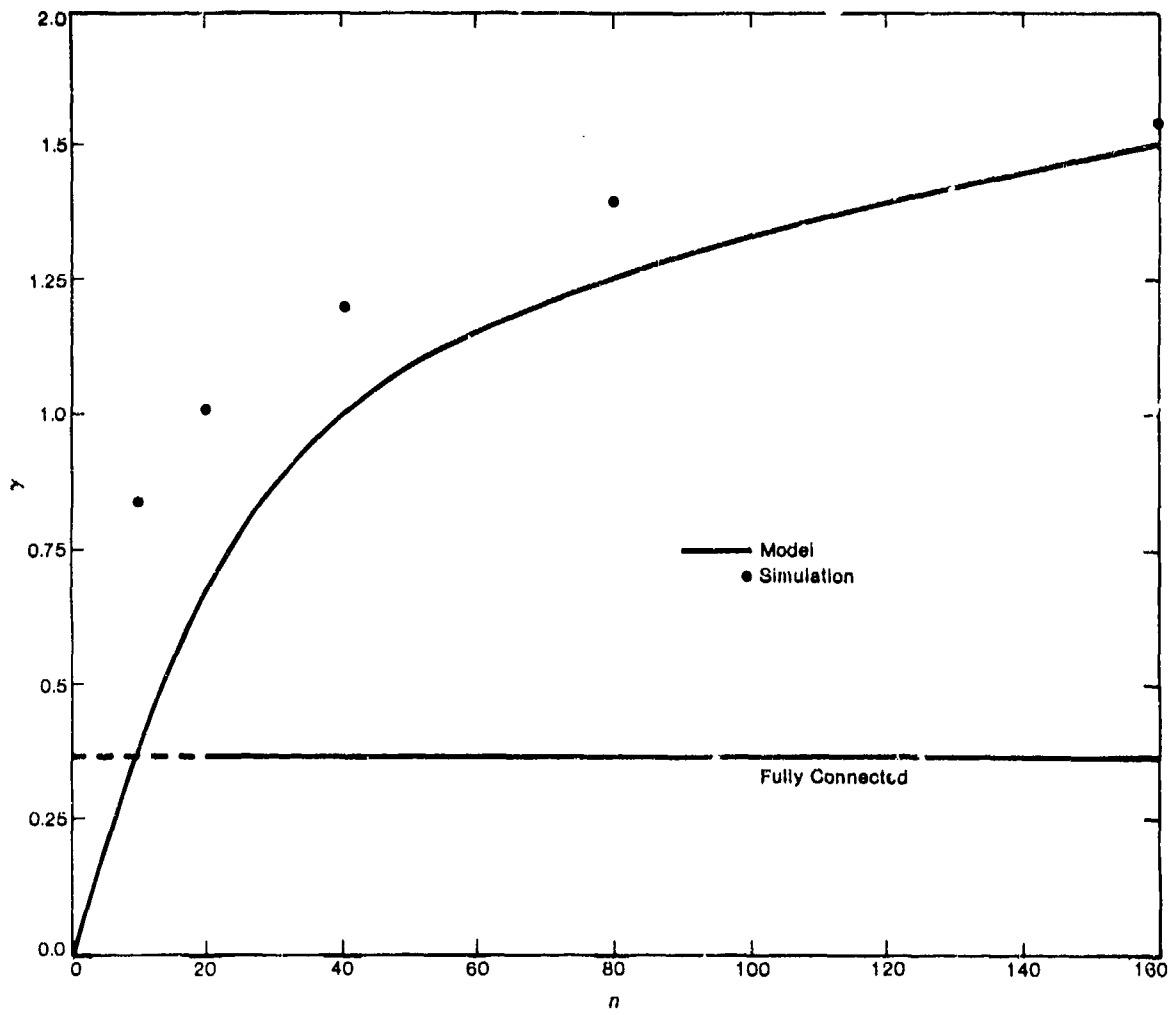Figure 3.2 1-d Random Network - Throughput

Figure 3.3 2-d Random Network - Throughput

and thus the total network throughput is:

$$\gamma = n\gamma_{node} = \frac{2}{e}$$
(3.27)

iii) *Partner's Hearing Degree* Instead of using a transmission probability based on the hearing degree of the node itself, we tried using the hearing degree of the node's communication partner. We show the simulation results for this case in Figure 3.6. We see that this has very similar performance to that found when the hearing degree of the node itself was used.

iv) *Estimated Degree* From a practical (implementation) point of view, it may be difficult for a node to determine exactly how many nodes hear when it transmits. We tried using an estimate of the hitting degree, equal to the number expected to be within range based on the transmission power and density of nodes (both quantities would probably be available to a node in a real network). Figure 3.7 shows the performance of this scheme and we find, as expected, that the throughput grows logarithmically with the network size (note that for this case both nodes of a partnership will use the same transmission probability). We note that the performance is not quite as good for this scheme as when we used the actual hitting degree. This is probably mainly due to edge effects where the nodes actually have lower degrees than would be expected (and also suffer from less interference).

3.6 Conclusions

We have shown that restricting the transmission power of the nodes in a Packet Radio Network increases the capacity of the network by taking advantage of spatial separation. By reducing the transmission rights of those nodes which interfere with many other nodes, we are able to obtain a throughput proportional to the logarithm of the number of nodes in the network. We anticipate that additional reduction of the transmission range (thus decreasing interference and increasing spatial reuse), may allow even higher throughputs to be attained. We were unable to do this due to two constraints: i) the traffic matrix - this assumption will be relaxed in Chapter 4 when we attempt to find the 'best' traffic matrix; and, more importantly, ii) no multi-hop capability - this assumption will be relaxed in Chapters 5 and 6 when we consider store-and-forward networks.
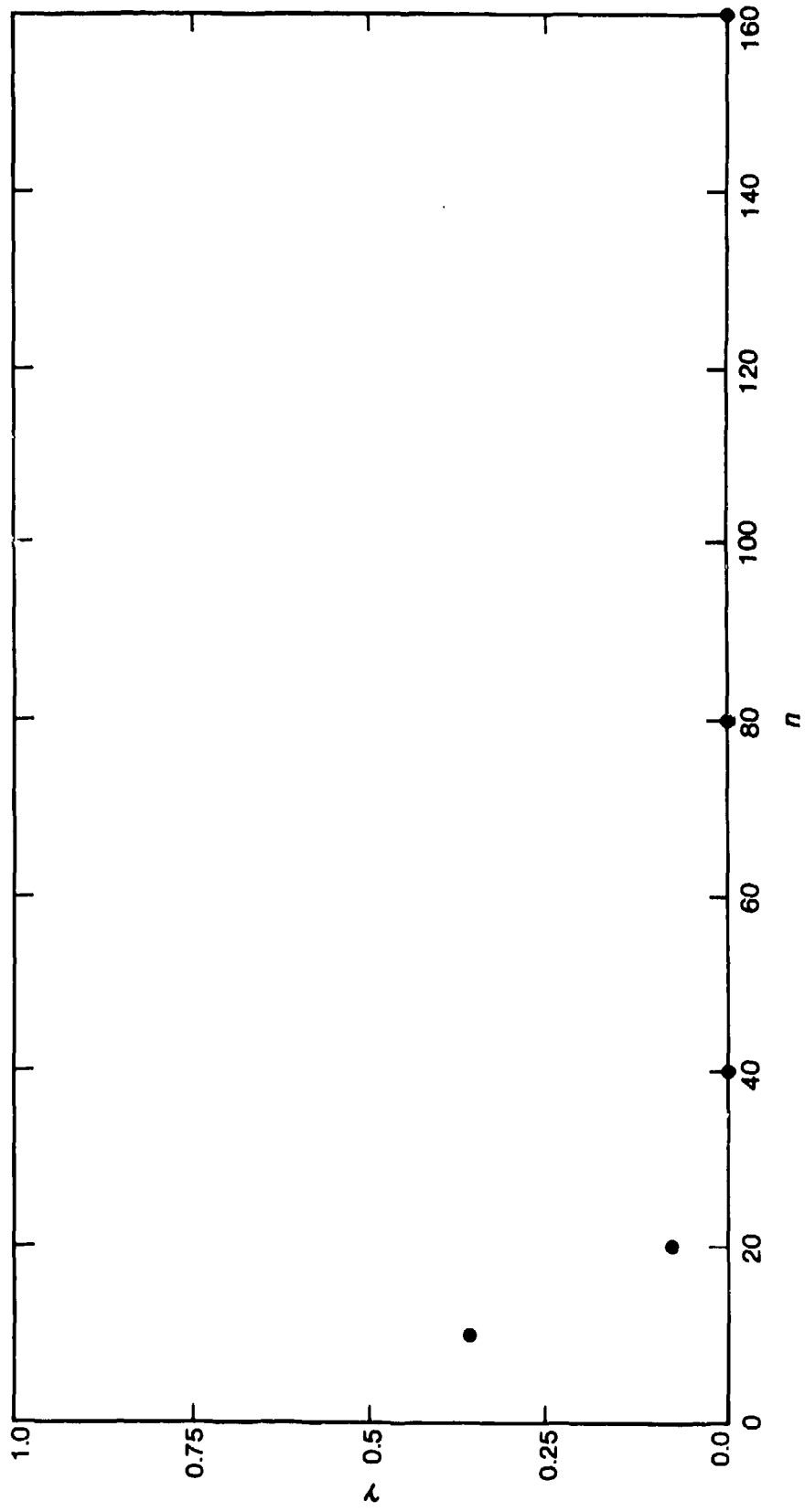
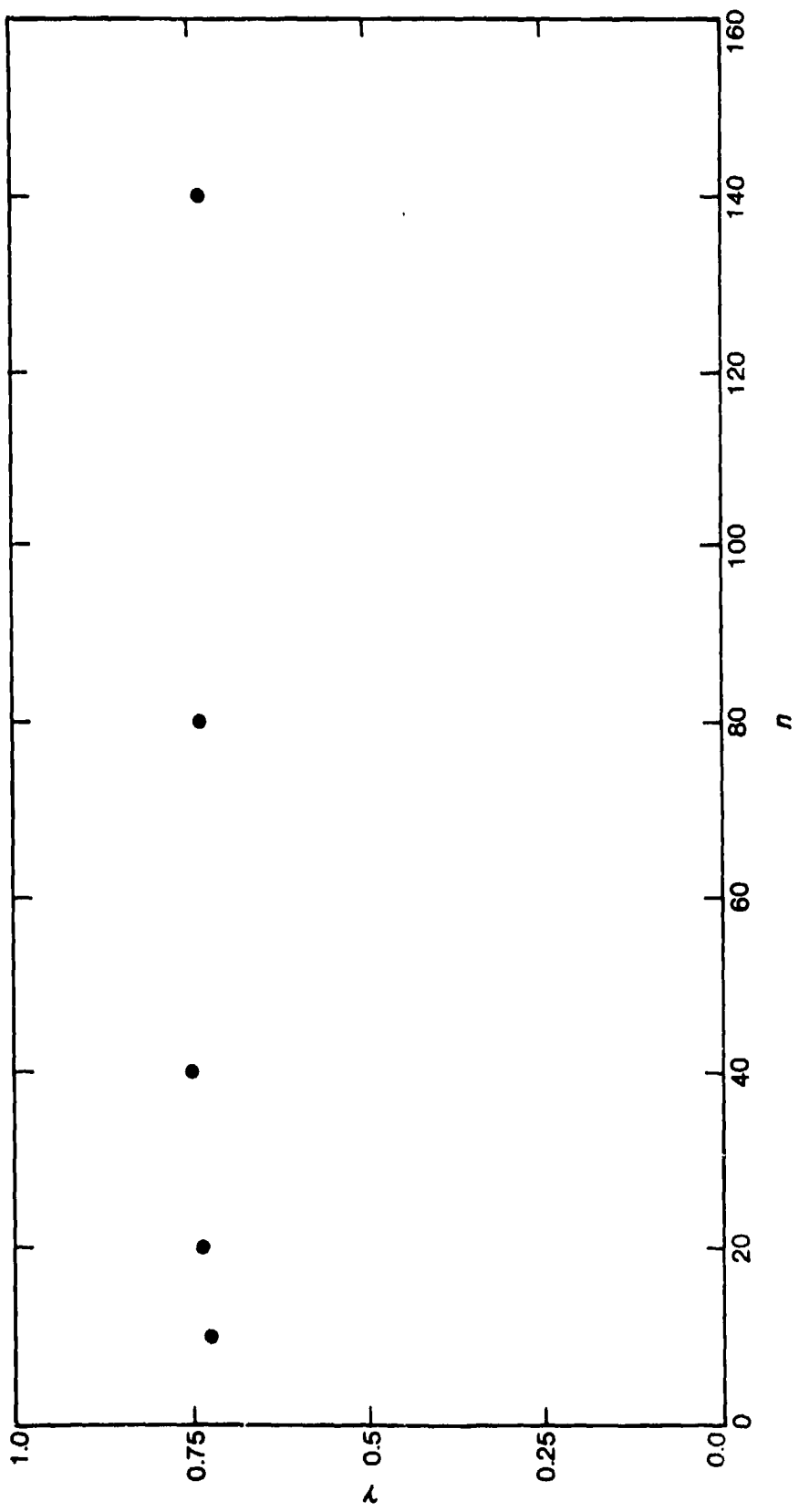Figure 3.4 Fixed Transmission Probability (2-d)
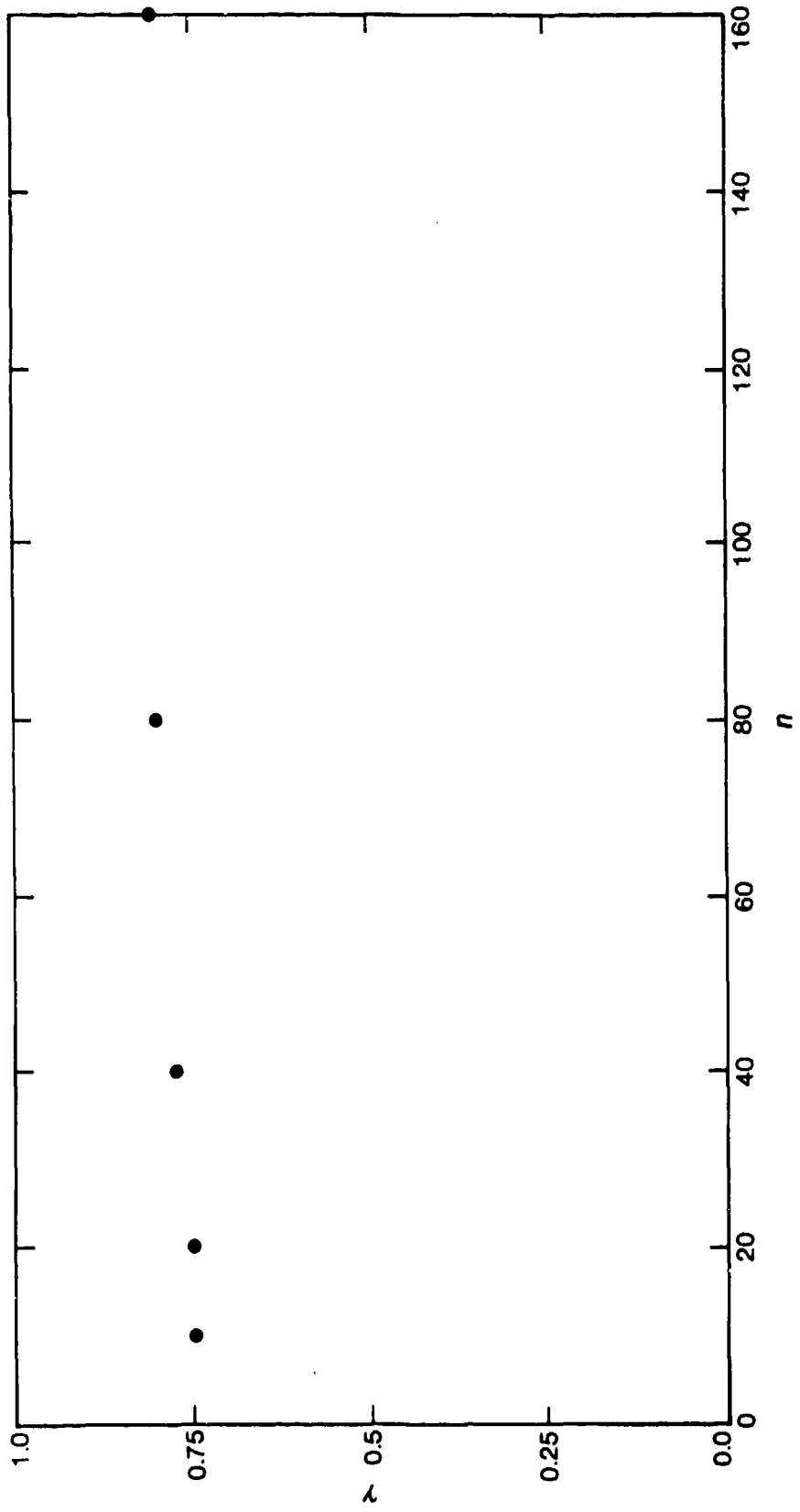
44

Figure 3.5 Hearing Degree (2-d)

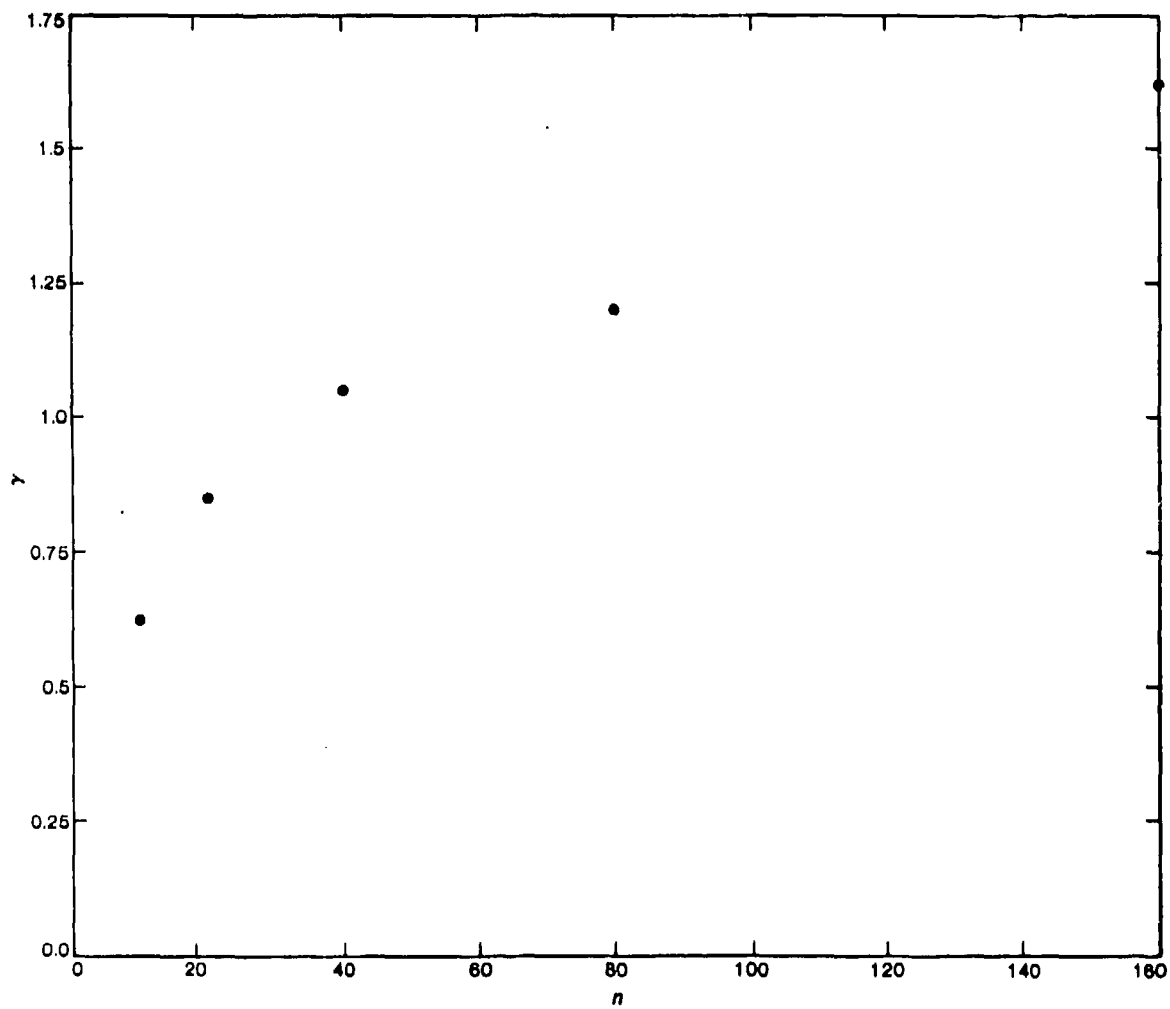Figure 3.6 Partner's Hearing Degree (2-d)

Figure 3.7 Estimate Hitting Degree (2-d)

CHAPTER 4
LOCAL TRAFFIC MATRICES

In Chapter 3 we found that having the ability to perfectly adjust the transmission range (and the resulting reduction in interference) allowed significant increases in throughput. We anticipate that further reduction in range would allow even greater spatial separation and thus higher capacities. In the previous discussion the transmission range was determined by the traffic matrix. Since we did not allow multi-hop paths, we required that the transmission power of a node be exactly sufficient to reach his communication partner. By changing the traffic matrix we can therefore further reduce the transmission ranges. We study this problem in this chapter, attempting to answer the following question: *For a random placement of nodes, what traffic matrix allows the highest traffic levels to be supported?* We note here that we only need consider one-hop communication, since we could improve any multi-hop configuration (to achieve a higher throughput in terms of end-to-end messages) by considering each hop of the message to be a separate message. In Chapters 5 and 6, we re-impose the arbitrary traffic requirement but allow multi-hop traffic. We hope that we can reap the benefits of low interference without paying too much of a penalty for the multiple hops.

In this chapter, then, we are concerned with attempting to find the 'best' traffic (to maximize throughput for a given topology). In the first section we develop some simple upper and lower bounds on the maximum throughput that can be attained under any traffic matrix. It is clear that the performance of the 'best' traffic matrix will lie between these bounds. The determination of the true 'best' traffic matrix is hard. We proceed, therefore, by trying various schemes that appear to have low interference. We find that some of these are not bad at all and can support throughputs in excess of the lower bound that we obtain in the first section.

The networks that we consider consist of a set of nodes randomly located in the unit hypersphere. We show a typical two-dimensional network in Figure 4.1

4.1 Simple Bounds on Performance

In this section we give simple upper and lower bounds on the performance for the best possible traffic matrix (BTM).

4.1.1 Upper Bound

If there were *no* interference between pairs of nodes, we would be able to achieve a performance equal to that obtainable by $\frac{n}{2}$ independent pairs. One independent pair is able to support a throughput of $\frac{1}{2}$ (which is achieved for a transmission probability of $\frac{1}{2}$) [ABRA 70]. Thus,

$$\gamma_{BTM} \leq \frac{n}{4} \tag{4.1}$$

4.1.2 Lower Bound

As a lower bound we consider how many nodes (of the n total) can be paired up without any of them causing interference to any other pairs (clean pairs). The unpaired nodes are assumed to generate no traffic. Consider a pair of nodes in the network, P and Q. If these are to communicate without causing any interference, Q must be P's nearest neighbor and P must also be Q's nearest neighbor.
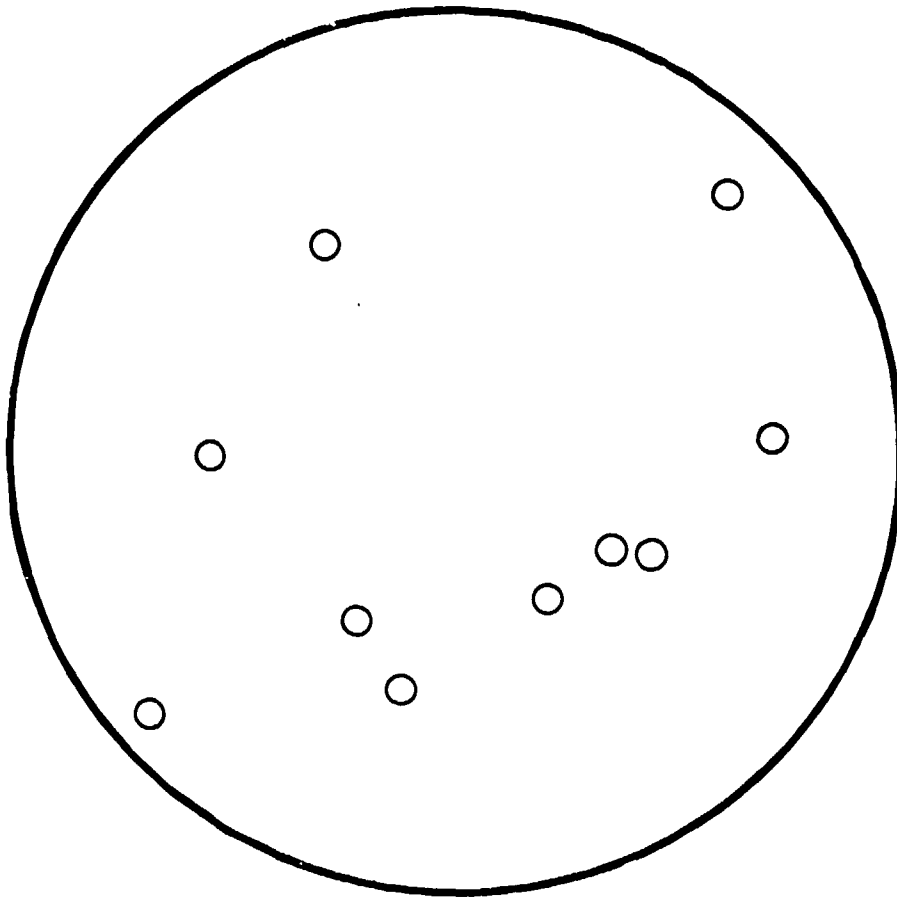
49

Figure 4.1 A 2-Dimensional Random Network

4.1.2.1 One Dimension

There are n nodes randomly located on the unit line. For simplicity we approximate this by a Poisson process of density λ $(=n)$. (Note that this approximation is only good for $n >> 1$.) Figure 4.2 shows two points P and Q in this random network. Suppose Q is P's nearest neighbor. The distribution, $F_X(x)$, of X $(\overline{PQ})$ can be found as follows (see also [KEND 63, ROAC 68]):

$$F_X(x) = \Pr\{X \leqslant x\}$$
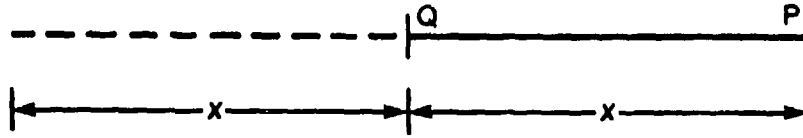
$$= 1 - \Pr\{X > x\}$$

Figure 4.2 No Interference in 1-Dimensional Network

$$
= \begin{cases} 1 - e^{-2\lambda x} & \text{1-dimension} \\ 1 - e^{-\lambda \pi x^2} & \text{2-dimension} \end{cases}
$$

which gives the density:

$$
f(x)\,dx = \begin{cases} 2\lambda e^{-2\lambda x}\,dx & \text{1-D} \\ 2\lambda \pi x e^{-\lambda \pi x^2} & \text{2-D} \end{cases} \tag{4.2}
$$

For no interference we require that there be no point closer to Q than P. That is, there is to be no point on the dashed line. The length of this line is clearly equal to x, and the probability of finding no point there, f, is:

$$
f = e^{-\lambda x} \tag{4.3}
$$

So the probability that a point is a member of a clean pair, g, is:

$$
g = \int_0^\infty 2\lambda e^{-2\lambda x} e^{-\lambda x}\,dx
$$

$$
= \int_0^\infty 2\lambda e^{-3\lambda x} = \frac{2}{3} \tag{4.4}
$$

We see, then, that we can find a traffic matrix which can support $\dfrac{.67n}{2}$ clean pairs, which will allow a throughput of $\dfrac{.67n}{4}$. Since this is readily achievable, it is clearly a lower bound on the performance of the 'best' traffic matrix. Figure 4.3 shows a comparison of simulation results for the number of no-interfering pairs of nodes that can be supported. We see excellent agreement with the predictions of the analytical model.

Letting γ^1_{BTM} represent the throughput of the best possible configuration for a 1-dimensional network, we have:
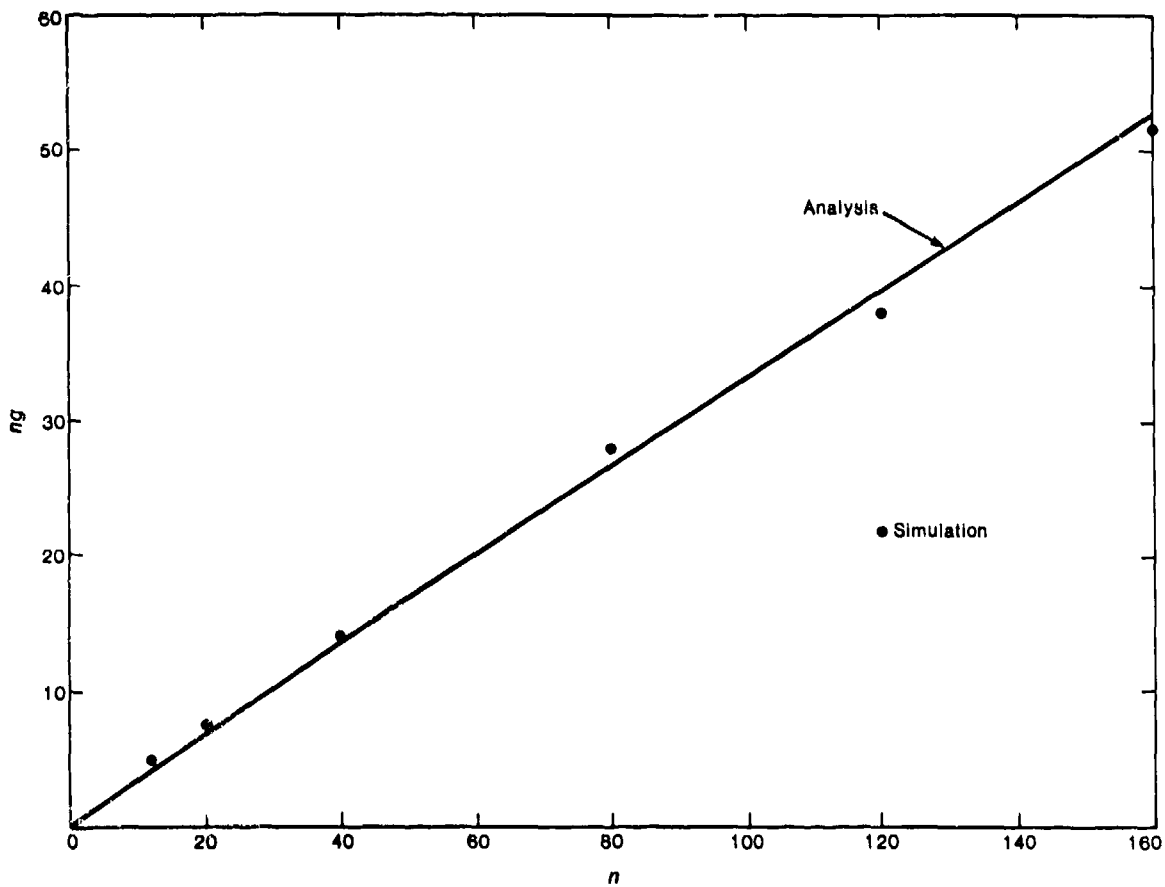
Figure 4.3 Clean Pairs in 1-Dimension

$$\frac{.67n}{4} \leqslant \gamma^1_{BTM} \leqslant \frac{n}{4} \tag{4.5}$$

4.1.2.2 Two Dimensions

We now consider the two dimensional analog. Consider Figure 4.4, let Q be P's nearest neighbor, we assume that P and Q are randomly located in the unit circle by a Poisson process of parameter λ. (This model is not exact, as in fact we place *precisely* n points in a unit circle but for large n it is a good approximation.) The distribution, $f(x)$, of $\overline{PQ}$ is given by (Equation 4.2):
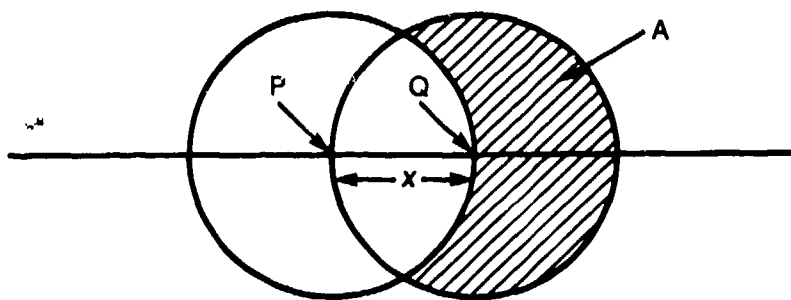
Figure 4.4 No Interference in Two Dimensions

$$f(x)\,dx = 2\lambda\pi x e^{-\lambda\pi x^2}dx \tag{4.6}$$

For no interference we require that there be no point closer to Q than P. That is, there is to be no point in the shaded area, A, encircling Q (there is no point in the circle around P since Q is the nearest neighbor). This area can be found to be:

$$A = x^2\left[\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right]$$

$$= 1.913x^2 \tag{4.7}$$

The probability of finding no point in this area is $e^{-\lambda A}$. So the probability that a point is a member of a clean pair, g, is:

$$g = \int 2\lambda\pi x e^{-\lambda A} e^{-\lambda\pi x^2}dx$$

$$= \frac{\pi}{\pi + 1.913}$$

$$= 0.622 \tag{4.8}$$

This result can also be found in [DEWI 77].

Thus we can find a traffic matrix allowing a throughput of $\frac{.62n}{4}$, which is therefore a lower bound. In Figure 4.5 we show simulation results for two-dimensional networks and also plot the bounds of Equations 4.1 and 4.9, again noting excellent agreement. Combining these equations we have the following relationship:
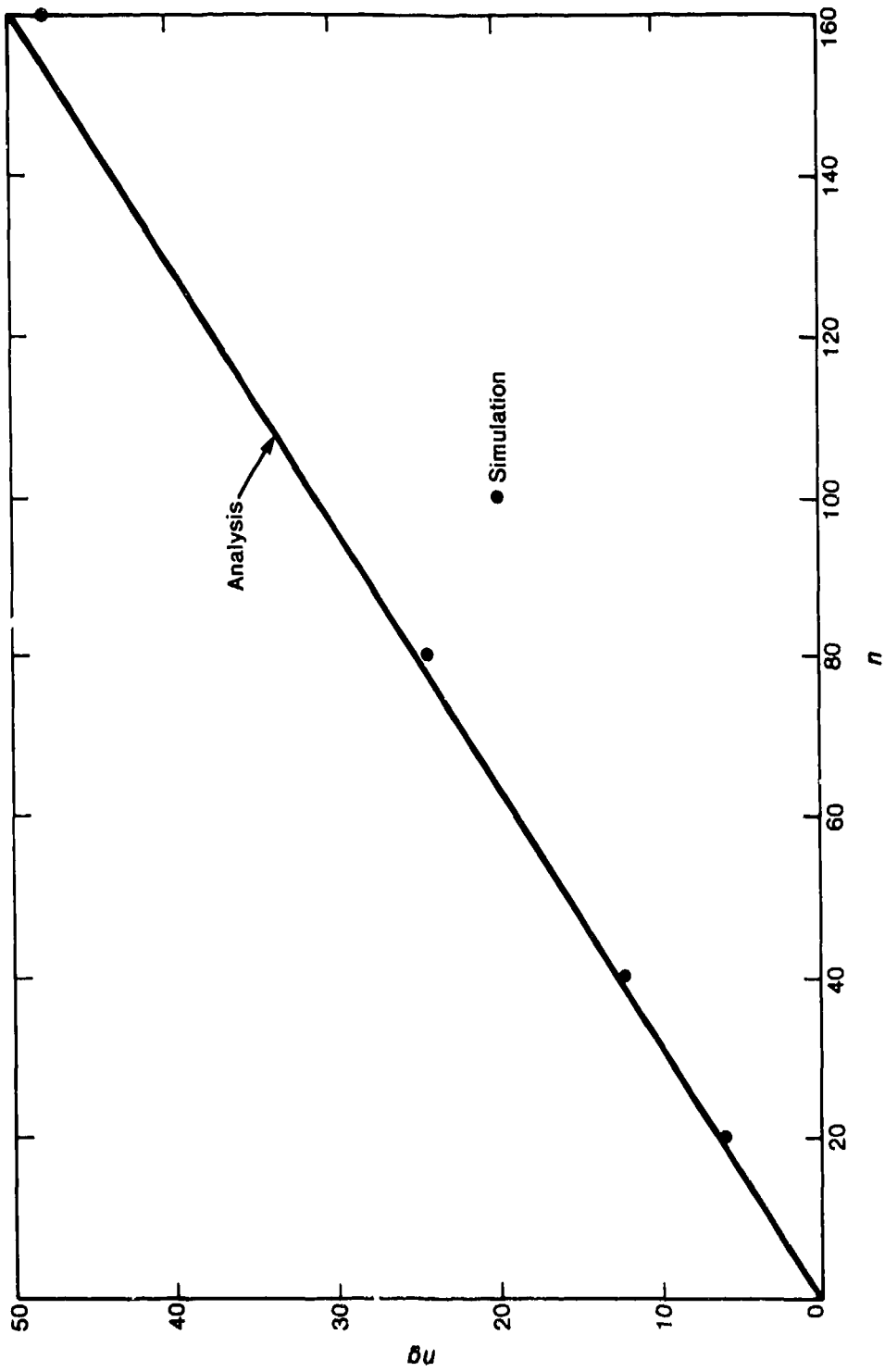
53

Figure 4.5 Clean Pairs in 2-Dimensional Networks

$$\frac{.62n}{4} \leqslant \gamma_{BTM}^2 \leqslant \frac{n}{4} \qquad (4.9)$$

4.2 Case Studies

Determination of the optimal traffic matrix is a hard problem. We choose, therefore, to look at some specific connection strategies allowing us to achieve high throughputs. For some of these cases we can proceed with the analysis outlined earlier, but in all cases we give simulation results.

4.2.1 Nearest Unpaired Neighbor (NUN) in 2-D

In this section we consider random two-dimensional networks and give a low interference connection strategy, the behavior of which exceeds the lower bound given in section 4.1.

Algorithm 4.1, below, describes the policy for pairing nodes in the Nearest Unpaired Neighbor scheme.

Algorithm 4.1 - Nearest Unpaired Neighbor (NUN)

1)   Generate the random network,
        consisting of an even number of nodes.

2)   Mark all nodes as unpaired.

3)   Find the two closest unpaired nodes and connect them; mark them
        as paired.

4)   If all nodes are paired, we have finished; otherwise, return
        to step 3.

The traffic pattern generated by this algorithm is satisfied by giving each node sufficient power to exactly reach his destination. In Figure 4.6 we show the network of Figure 4.1 with connections defined by this algorithm.

As in Chapter 3 we tried using several different transmission policies.

i) *Fixed:* Each node uses a transmission probability of ½. We show the performance of this scheme in Figure 4.7a.

ii) *Hitting Degree:* Each node uses a transmission probability equal to $1/k$, where k is the number of nodes that hear his transmission (including himself and his partner). The results are shown in Figure 4.7b.

iii) *Hearing Degree:* We again use a transmission probability of $1/k$, but k is now the number of nodes that you hear. The results for this scheme are shown in Figure 4.7c.
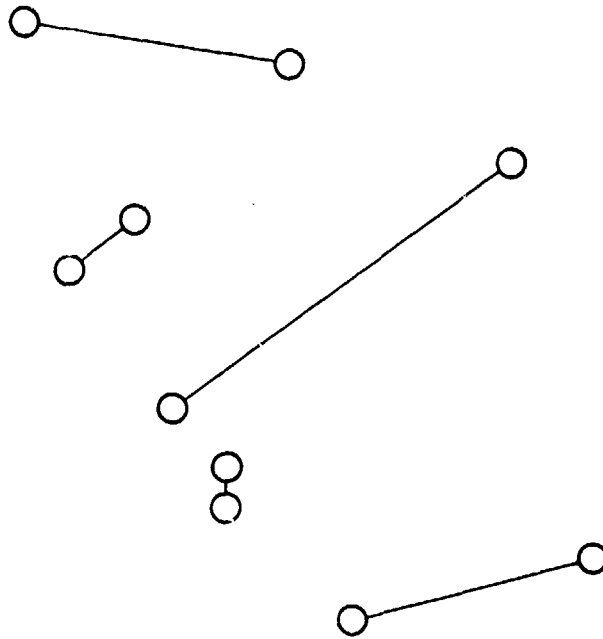
Figure 4.6 A NUN Network

iv) *Partner's Hearing Degree:* In this scheme the hearing degree of your transmission partner is used rather than your own degree. The results are shown in Figure 4.7d.

v) *Estimate:* Since in a real network it would be difficult to determine the number of nodes that actually hear your transmission (or indeed, the number that you can potentially hear), we tried running a simulation estimating the number of nodes that would hear your transmission. This estimate is based on the transmission power used and the average density of nodes within the region of interest. If the transmission radius is r, then the estimate is $\lambda \pi r^2$. The results for this approach are shown in Figure 4.7e. This scheme is the one most closely represented by the model of section 4.4.

We see that all of the schemes corresponding to using a transmission probability based on hitting degrees have similar performance. These schemes appear to achieve a performance which is linear with respect to the size of the network. The other schemes, using either a fixed transmission probability or one based on the hearing degree of either the node or its partner increase at a rate less than linearly.

In studying the networks produced by this algorithm we found that at the end of the pairing process nodes that are far apart become connected. These long links are the main source of interference in the network. We therefore tried the following more drastic approach to reduce the interference caused by these long links, which we call *expurgation.*

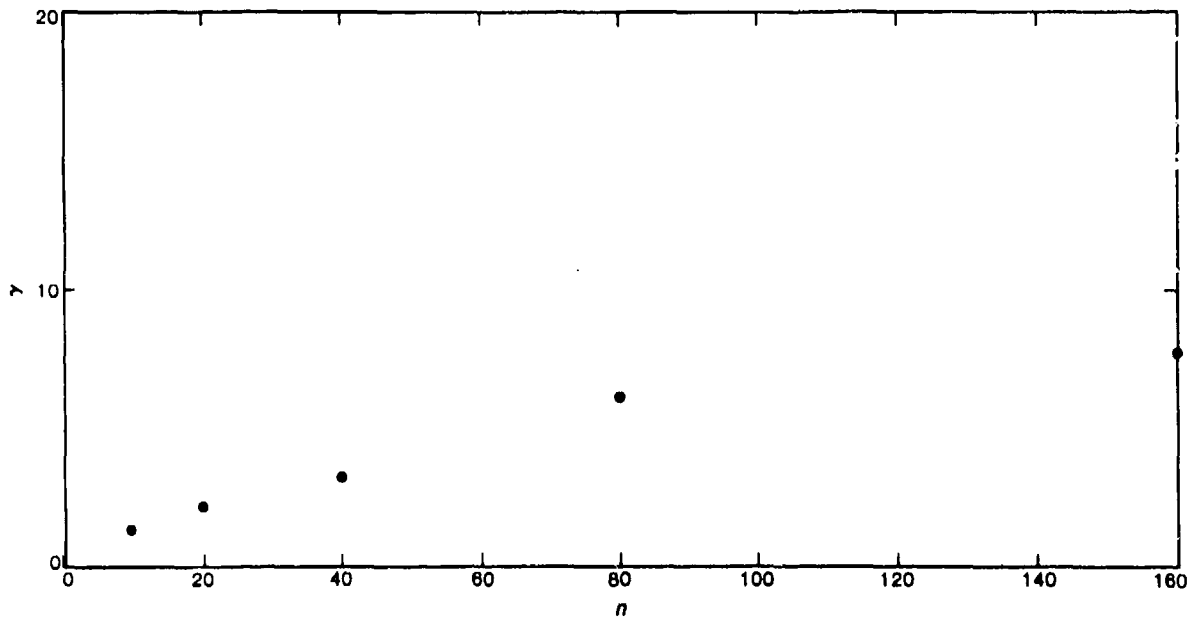*Algorithm 4.2* Expurgation

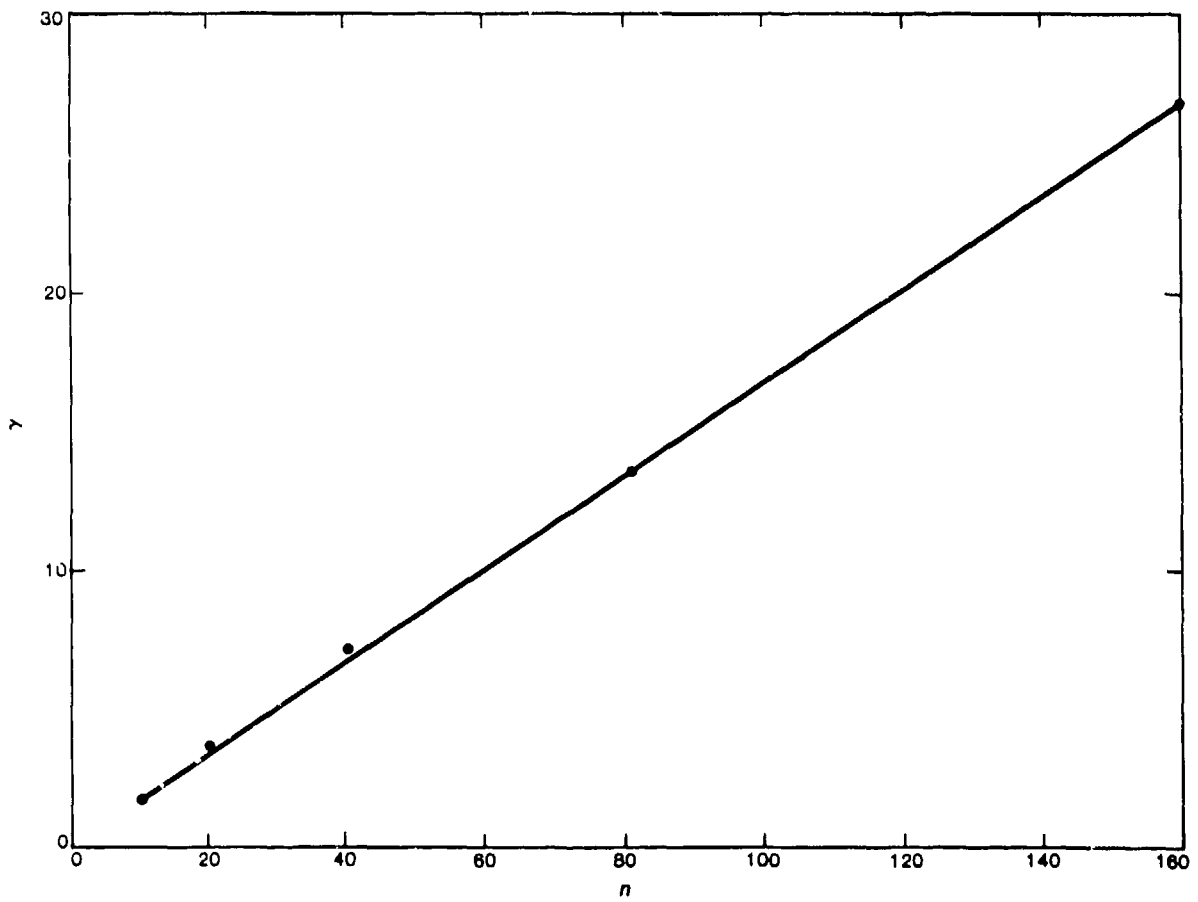Figure 4.7a 2-D NUN - Fixed $p=\frac{1}{2}$

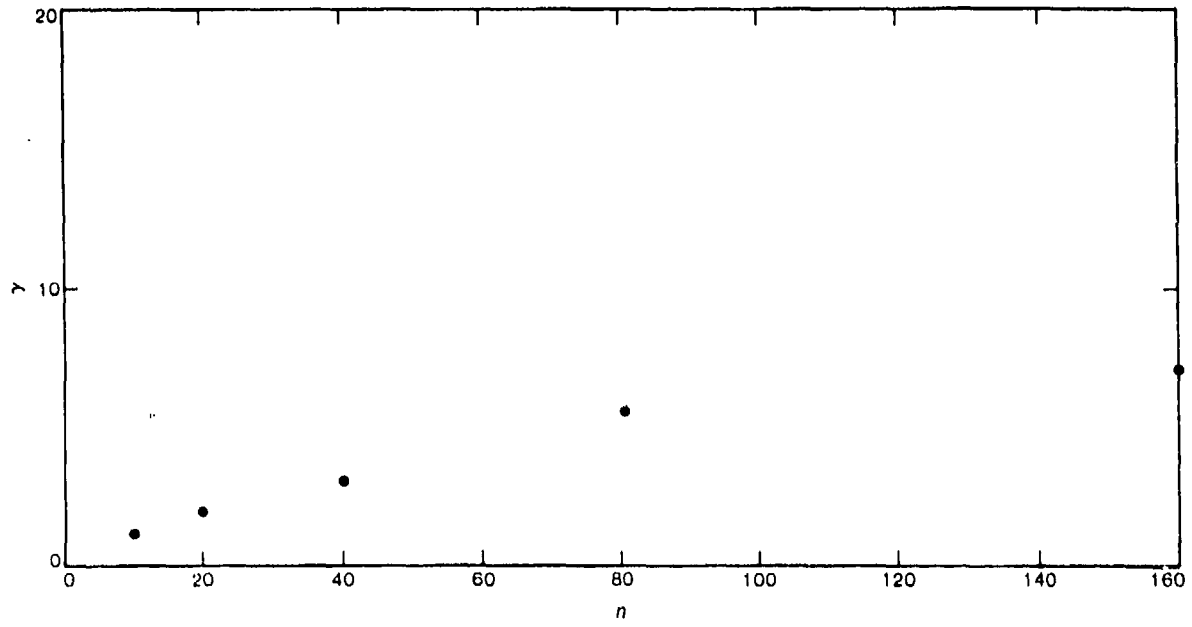Figure 4.7b 2-D NUN - Hitting Degree

Figure 4.7c 2-D NUN - Hearing Degree
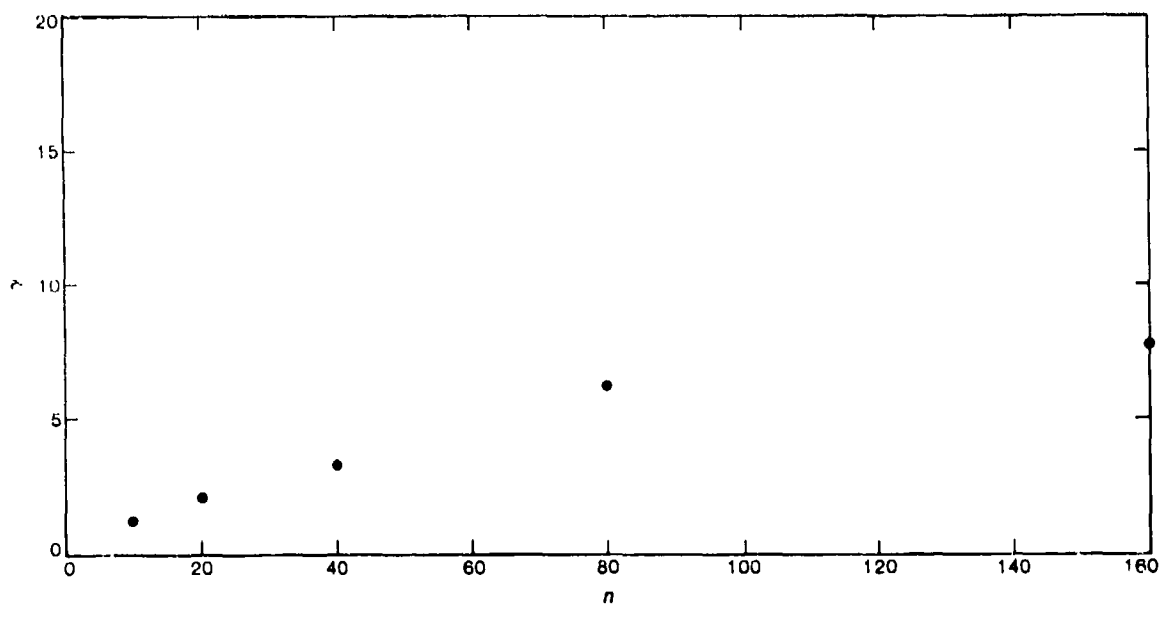

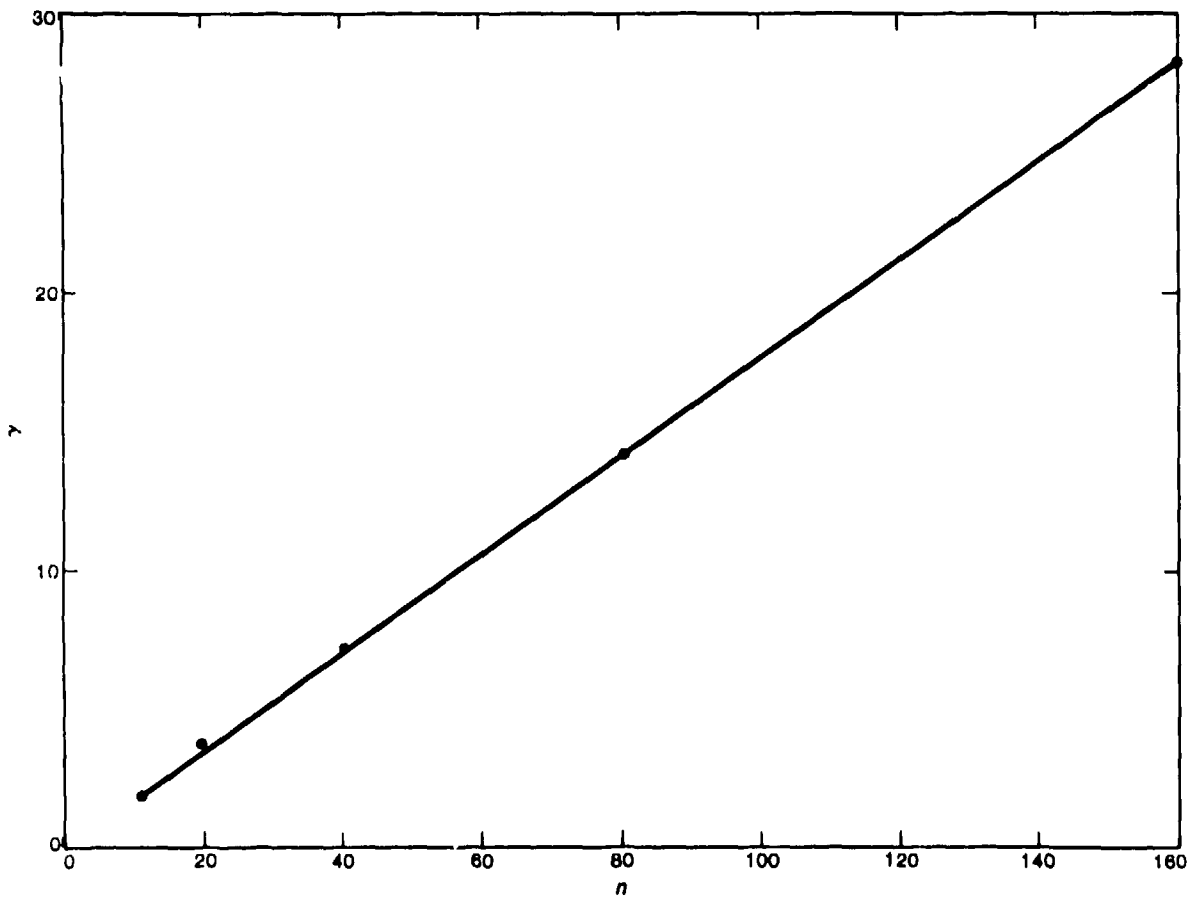Figure 4.7d 2-D NUN - Partner's Hearing Degree

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Figure 4.7e 2-D Estimate of Hitting Degree

1)   Connect all nodes as in algorithm 4.1.

2)   Assign $p=\frac{1}{2}$.

3)   Compute the throughput.

4)   Delete the longest link (i.e., one pair).

5)   If there are still links in the network return to step 3; else stop.

We show the effect of expurgation in Figure 4.8, plotting the net~ork throughput (for various network sizes) against the proportion of links expurgated. In all cases the throughput increases until about 16% of the nodes are no longer communicating. As we delete additional pairs of nodes, the throughput decreases linearly to zero since the pairs being expurgated are, in fact, not causing interference. Each expurgated link now causes a drop of $\frac{1}{2}$ in throughput.

If we stop the expurgation process when the peak is reached, we have the *optimally expurgated* scheme. We show the performance of this in Figure 4.9 and we again notice linear behavior similar to that of the schemes using a transmission probability based on the hitting degree.

The slopes of these graphs (Figure 4.7b, 4.7c, 4.7e and 4.9) are about 0.18 (which is $\approx 1/2e$, unfortunately we cannot derive an expression for this slope from the analysis in the 2-dimensional case).

In the following section we look at the same approach for one-dimensional networks and find identical performance. We then look at a slightly modified one-dimensional scheme (ADJ) which once again performs similarly. The advantage of this particular scheme is that it lends itself very well to analysis. We analyze this and conjecture that all of the schemes presented here in fact obey this model.

4.2.2 Nearest Unpaired Neighbor (NUN) in 1-D

This is the one-dimensional equivalent of the two-dimensional scheme outlined above. Figure 4.10 shows the simulation results for networks using transmission probabilities based on the node's hitting degree and Figure 4.11 shows similar performance for the optimally expurgated case.

In the following section we consider a simpler version of this, in which every node is connected (adjoined) to his left (or right) neighbor.

4.2.3 Adjoining in One Dimension (ADJ)

For this scheme we randomly locate n points on the unit line and then connect adjacent pairs starting from one end. In Figure 4.12 we show the performance for this scheme. We notice that the performance is very similar to the NUN scheme outlined above. We will find in section 4.4 that this (ADJ) scheme lends itself well to analysis.

4.3 An Overview of These Schemes

All of the schemes of the previous sections have very similar performance. In fact, we ran the simulations for highei dimensional networks and again found similar performance. In Figures 4.13 and 4.14 we show the performance of the NUN schemes in one- and two-dimensions respectively, in relation to the bounds developed earlier. We see that we have exceeded the simple lower bound developed in that section and that the slopes of these curves are about .18. We feel that these schemes are in fact
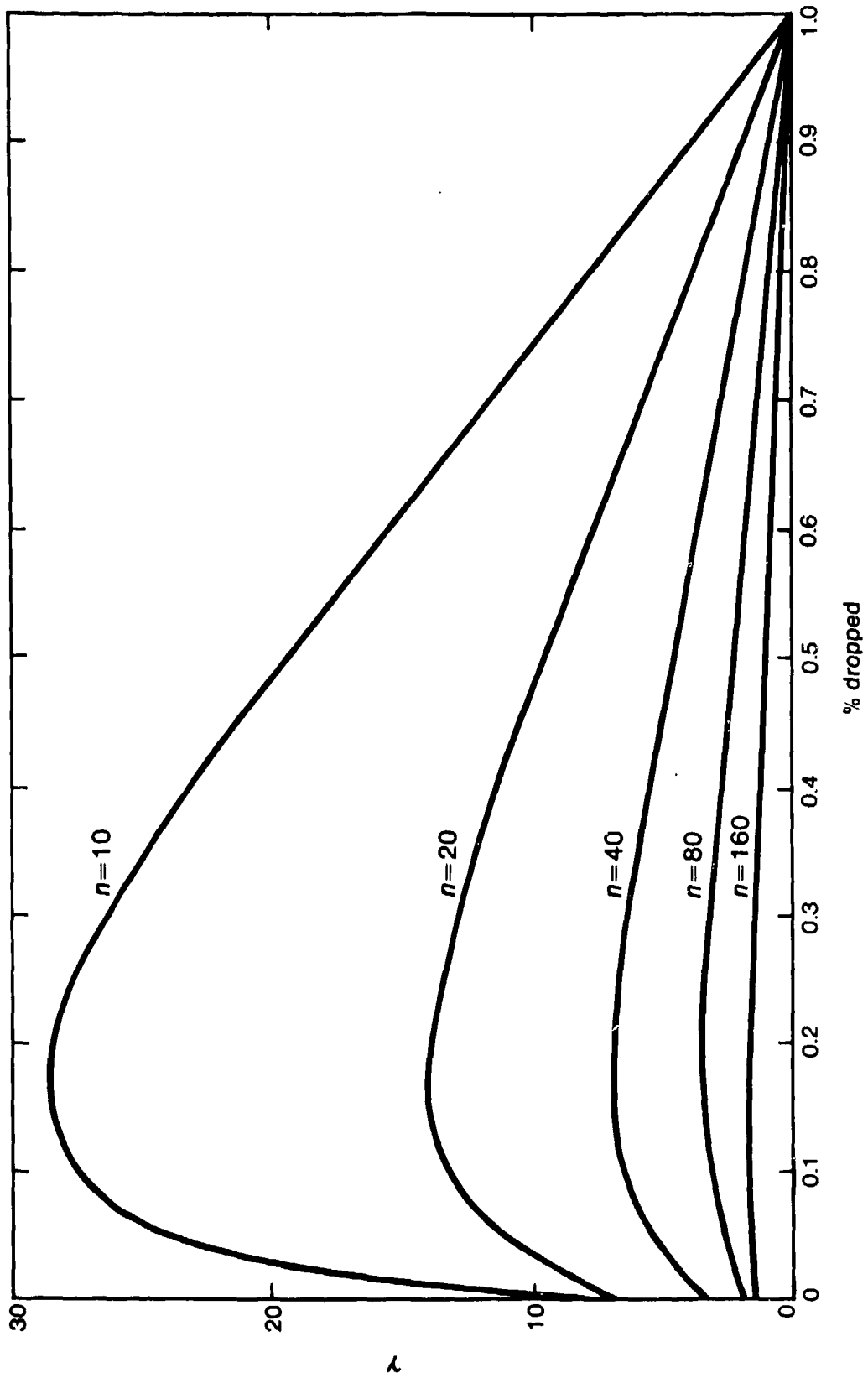
Figure 4.8 2-D NUN - Expurgation
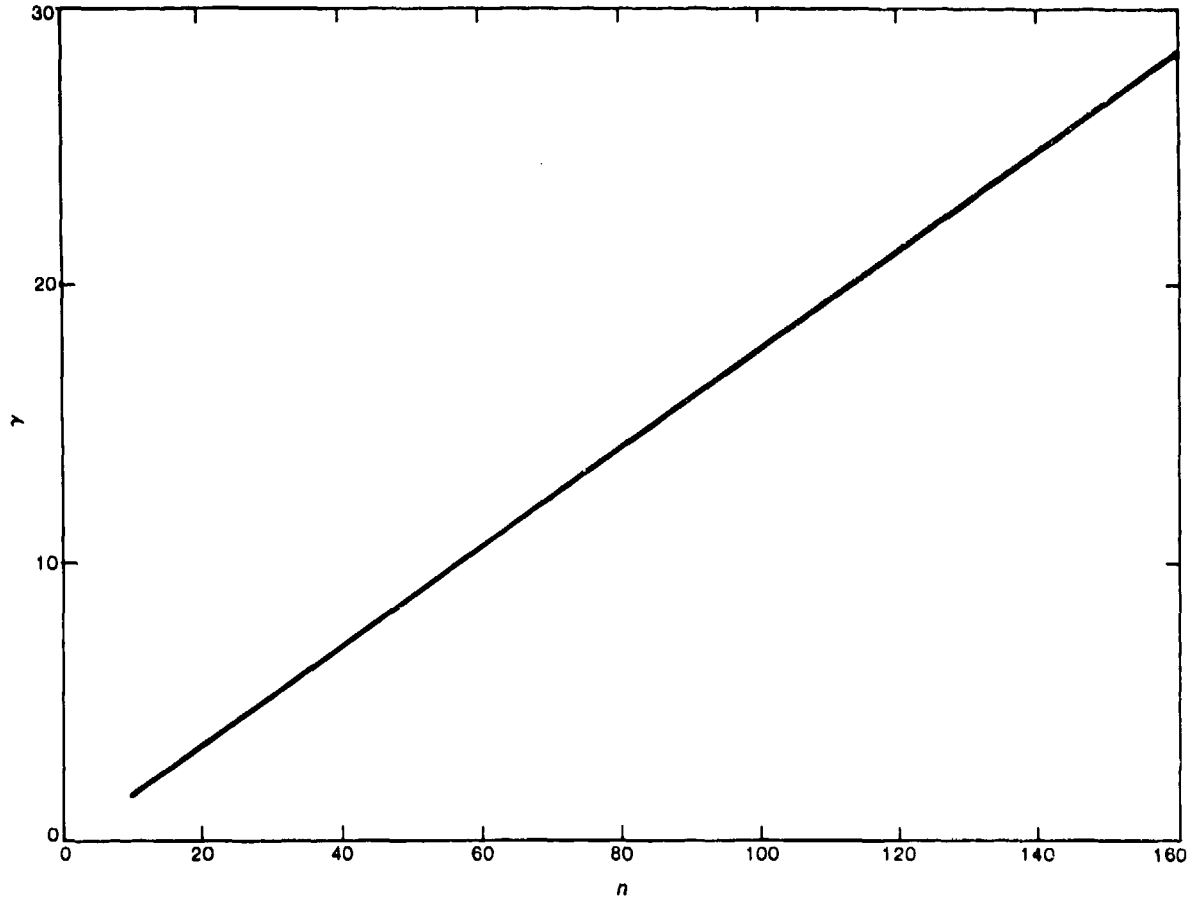
Figure 4.9 2-D NUN - Optimally Expurgated

very close to the best possible traffic matrix, although we do not have any concrete justification for this statement at this time.

4.4 Analytical Model for ADJ

For the one-dimensional ADJ scheme we can develop an analytical model similar to that used in Chapter 3, since we are able to derive the hitting and hearing distributions.

Suppose that i and j are a pair of communicating nodes, i.e., $t_{ij} = 1$.

Figure 4.10 1-D NUN - Hitting Degree

Figure 4.11 1-D NUN - Optimally Expurgated

Figure 4.12 1-D ADJ - Hitting Degree

Figure 4.13 1-D NUN - Comparison to Bounds

Figure 4.14 2-D NUN - Comparison to Bounds

$s_i = \Pr\{j \text{ transmits}\} \Pr\{i \text{ does not transmit}\} \Pr\{\text{none of } i\text{'s neighbors transmits}\}$

$$= p_j \, (1-p_i) \, I \tag{4.10}$$

where

$$I = \Pr\{\text{no interference}\} \tag{4.11}$$

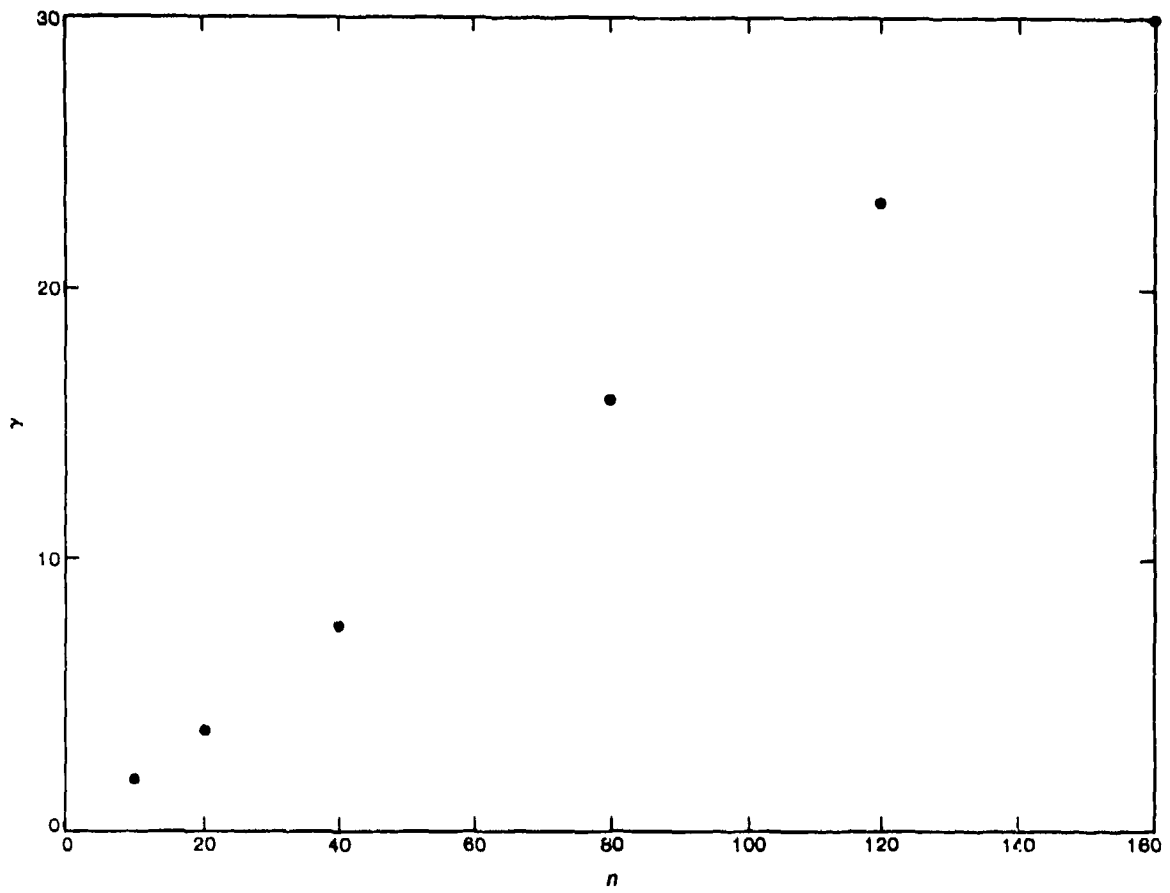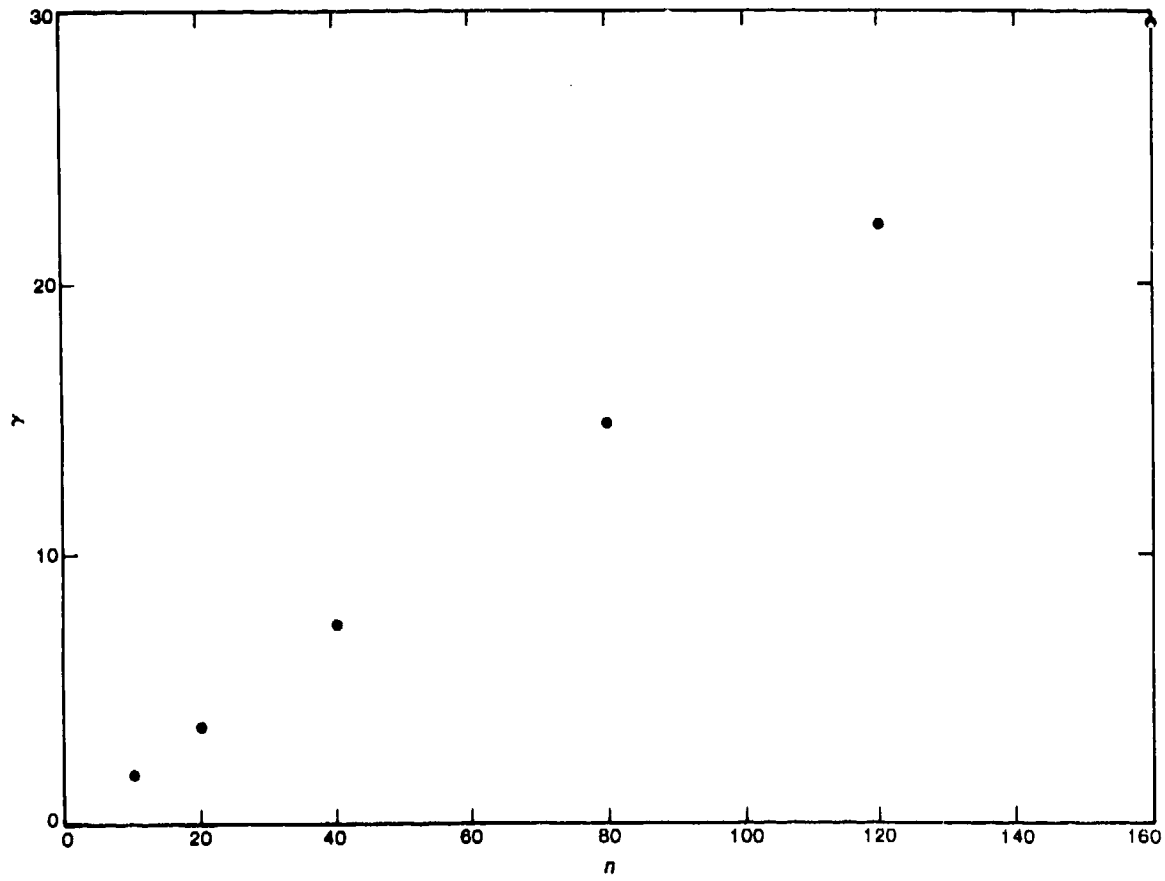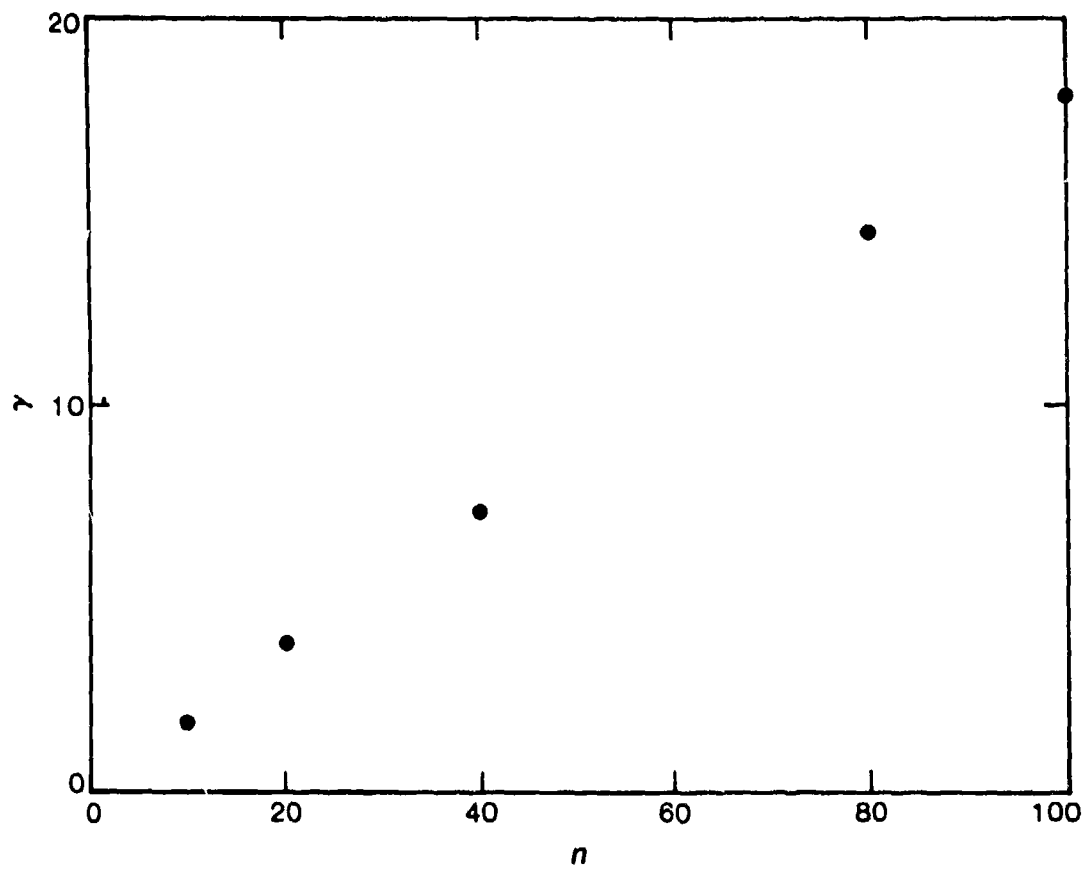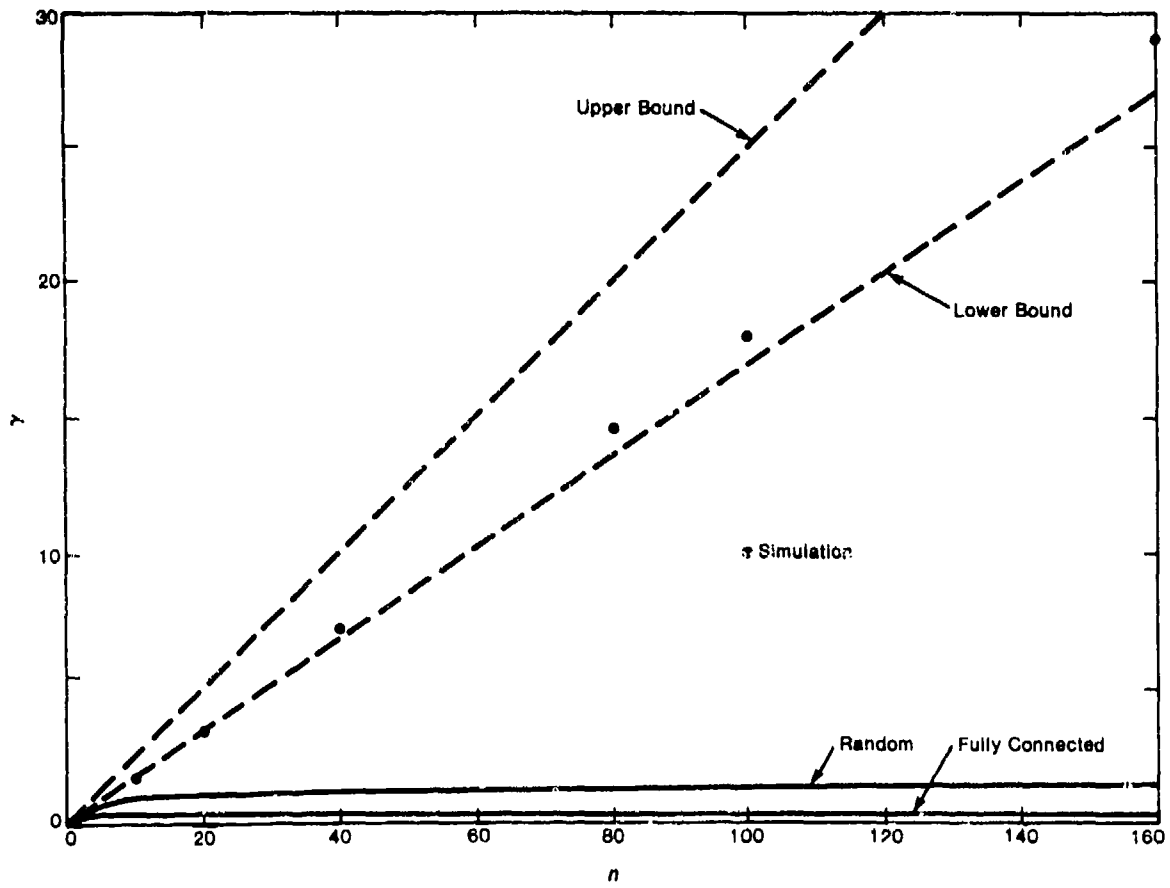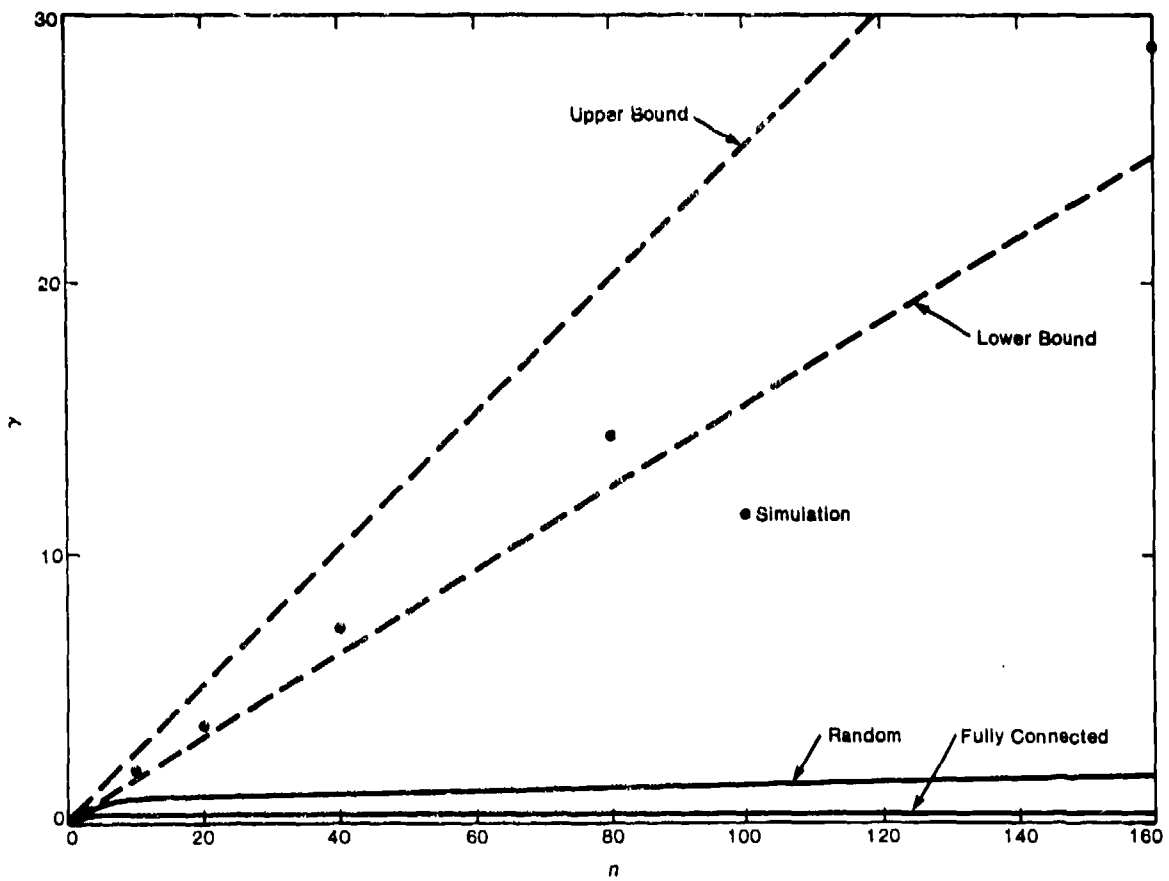As the networks that we consider are homogeneous, the throughput for all nodes is identically distributed; we can therefore drop the subscripts corresponding to the particular node under investigation. As noted before, the retransmission probability is only dependent on the number of nodes hit by a transmission. If we make the further assumption that both nodes of a partnership hit the same number of nodes* we obtain the following expression for the throughput:

$$\gamma_i = I \sum_{k=2}^{n} h_k \, \frac{1}{k} \left[1 - \frac{1}{k+2} \right] \tag{4.12}$$

Assumption: We assume that the interference heard by any node is independent of the number of nodes that he hits. With this assumption we can proceed with the computation of I.

$$I = \sum_{k=2}^{n} H_k (1-q)^{k-2} \tag{4.13}$$

where q is the expected transmission probability of a node that you hear, given by:

$$q = \sum_{k=2}^{n} \theta_k \, \frac{1}{k} \tag{4.14}$$

where θ_k are the 'adjusted hit probabilities', i.e., the probability that a node you hear hits k nodes when he transmits. We cannot use the hit probabilities as defined earlier, since a node is much more likely to hear a node that hits many other nodes than one that hits only a few. Thus using the 'sampled' distribution [KLEI 75c], we get:

$$\theta_{k+2} = \begin{cases} 0 & k = 0 \\ ck h_{k+2} & k \geqslant 1 \end{cases} \tag{4.15}$$

where c is a normalization constant such that $\sum \theta_k = 1$.

---

* As both nodes are transmitting at the same range, the *expected* number hit by a transmission will be the same, at least. As we saw for scheme v), using the transmission radius to estimate the transmission probability gave similar performance to scheme ii) (based on hitting degree). The policy of scheme v) is exactly what we model here.

We have thus reduced the problem of finding the throughput to that of determining the sets of probabilities h_i and H_i.

4.4.1 The Hitting Distribution

We know the distribution of the distance to the neighbor on your left (or right) and we must determine how many points are expected to fall in this distance on the other side of the connection, this being the number of points that will hear you. The distribution of the neighbor distance, x, is:

$$f(x)dx = \lambda e^{-\lambda x} dx \qquad (4.16)$$

The points that you hit are precisely those that fall in a distance x on your right (left). The number of points falling in this distance on the other side is Poisson distributed, thus:

$$\Pr\{i \text{ in a distance } x\} = e^{-\lambda x} \frac{(\lambda x)^i}{i!} \qquad (4.17)$$

So we have:

$$h_{i+2} = \int_0^\infty \frac{(\lambda x)^i}{i!} e^{-\lambda x} \lambda e^{-\lambda x} dx = \int_0^\infty \lambda^{i+1} x^i e^{-2\lambda x} dx$$

$$= (\tfrac{1}{2})^{i+1} \qquad (4.18)$$

We can derive this distribution in an alternate manner without having to rely on the exponential or Poisson distributions as follows.
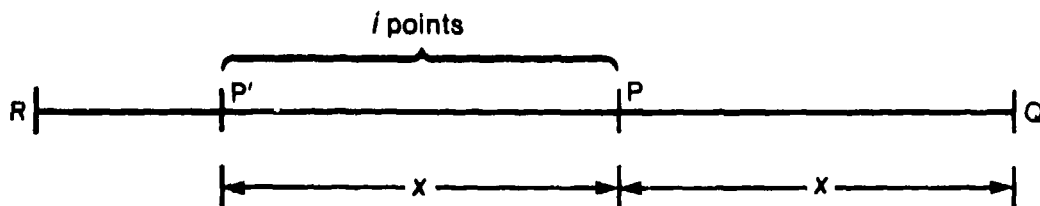
Figure 4.15 Derivation of 1-D ADJ Hitting Distribution

In Figure 4.15, suppose that node P (whose partner is Q) hits i excess nodes. Let R be the first point on the left that cannot hear P, for this to happen, P must be to the right of the midpoint of $\overline{QR}$; the probability of this event is $\frac{1}{2}$. If x is the distance from P to Q then consider a point P′ at a distance x to the left of P. Now all the i excess points must fall to the left of the midpoint of $\overline{QP'}$ (i.e. to the left

of P). The probability of this event is $(\frac{1}{2})^i$. Thus the probability that P hits i points is:

$$h_{i+2} = (\frac{1}{2})^{i+1} \tag{4.19}$$

From this we can determine θ_i:

$$\theta_{i+2} = \frac{ci}{2^{i+1}} \tag{4.20}$$

Summing these to obtain c we get:

$$\sum_{i=1}^{n-2} \frac{i}{2^{i+1}} = 1 - \frac{n}{2^{n-1}} \tag{4.21}$$

$$=> \quad c = \frac{2^{n-1}}{2^{n-1}-n} \tag{4.22}$$

$$\approx 1 \qquad \text{(for large } n\text{)} \tag{4.23}$$

From this we may determine the expected transmission probability of interfering nodes, q:

$$q = c \sum_{i=0}^{n-2} \frac{i}{2^{i+1}} \cdot \frac{1}{i+2}$$

$$= c \left[\sum_{i=0}^{n-2} \frac{i+2}{2^{i+1}(i+2)} + \sum_{i=0}^{n-2} \frac{2}{2^{i+1}(i+2)} \right]$$

$$= c \left[\sum_{i=0}^{n-2} (\frac{1}{2})^{i+1} - 4 \sum_{i=2}^{n} \frac{1}{i} (\frac{1}{2})^{i} \right]$$

$$= c \left[1 - (\frac{1}{2})^{n-1} + 4 \left(\log(\frac{1}{2}) + \frac{1}{2} + \sum_{i=n+1}^{\infty} \frac{1}{i} (\frac{1}{2})^{i} \right) \right]$$

$$= 3 - 4 \log(2) \qquad \text{(for large } n\text{)} \tag{4.24}$$

In order to proceed we must find the heading distribution.

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4.4.2 The Hearing Distribution

Figure 4.16 Derivation of 1-D ADJ Hearing Distribution

In Figure 4.16, suppose that between P and Q' there are $k-1$ points. In order for P to hear Q' (the partner of Q) all k points (the $k-1$ intervening points and Q' itself) must fall to the left of the midpoint of $\overline{PQ}$. The probability of this is easily determined to be:

$$\Pr\{P \text{ hears } Q\} = (\tfrac{1}{2})^k \tag{4.25}$$

We will say that Q' is at distance k from P if there are $k-1$ intervening points. We find that the points who interfere with P from the right are at distances 1,3,5,7, etc. We can use an identical argument for points on the left of P and we find that these points are at distances 2,4,6 etc. Let us therefore call the event of being hit by a point at distance k, E_k. Then:

$$\Pr\{E_k\} = (\tfrac{1}{2})^k \tag{4.26}$$

If P hears j points this means that exactly j of the set of events $\{E_k\}$ have occurred. We can therefore write the expressions for H_j. Let us first look at the probability that P does not hear any interference (this is H_2).

$$H_2 = \Pr\{P \text{ hears only his partner}\}$$

$$= \Pr\{\text{none of } E_k \text{ occur}\}$$

$$= \prod_{k=1}^{n-1} \left[1 - (\tfrac{1}{2})^k\right] \tag{4.27}$$

Unfortunately it appears that this product does not have a closed form. It is in fact related to the inverse of the partition function. We use the following identity of Euler to evaluate this expression, which converges extremely rapidly and also gives us a bound on the error (as it is an alternating monotonically decreasing series).

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$$\prod_{k=1}^{\infty} (1 - x^k) = \sum_{i=-\infty}^{\infty} (-1)^i \, x^{\frac{i(3i+1)}{2}} \qquad (4.28)$$

We find:

$$H_2 \approx 0.289 \qquad \text{(for large networks)} \qquad (4.29)$$

To find the probability that P hears one additional point we must find the probability that exactly one of the E_k occurs, and generalizing, if P hears j additional points then exactly j of the set $\{E_k\}$ must occur.

$$H_3 = \sum_{k=1}^{n-1} \frac{H_2}{1-(\frac{1}{2})^k} \qquad (4.30)$$

So, in general:

$$H_{j+2} = \sum_{k_1=1}^{n-1} \sum_{k_2=k_1+1}^{n-1} \cdots \sum_{k_j=k_{j-1}+1}^{n-1} \frac{H_2}{[1-(\frac{1}{2})^{k_1}][1-(\frac{1}{2})^{k_2}] \cdots [1-(\frac{1}{2})^{k_j}]} \qquad (4.31)$$

These have been evaluated by computer and the values are shown in Table 4.1 (for a 100 node network).

number	H_j		h_j	
j	analytical	simulation	analytical	simulation
0	0	0	0	0
1	0	0	0	0
2	0.289	0.302	0.500	0.504
3	0.440	0.446	0.250	0.250
4	0.209	0.212	0.125	0.117
5	0.036	0.037	0.063	0.068
6	0.003	0.002	0.031	0.030
7	0	0	0.016	0.014
8	0	0	0.008	0.009
9	0	0	0.004	0.004

Table 4.1 Hearing and Hitting for 1-D ADJ

From these we can evaluate I, the expected interference.

$$I - \sum_{j=2}^{n} H_j (1-q)^{j-2}$$

$$\approx 0.78924 \tag{4.32}$$

4.4.3 Alternate Evaluation of Interference

We can proceed from Equation 3.19 of Chapter 3 to determine this interference factor directly. Recalling,

$$I = e^{-\sum_{k=3}^{n} (k-2) h_k f_k} \tag{4.33}$$

We can substitute $h_k = (\frac{1}{2})^{k-1}$ and $f_k = 1/k$ to obtain:

$$I = \exp - \left\{ \sum_{k=3}^{n} (k-2) \, (\tfrac{1}{2})^{k-1} \, \frac{1}{k} \right\}$$

$$= \exp - \left\{ \sum_{k=3}^{n} (\tfrac{1}{2})^{k-1} - 2 \sum_{k=3}^{n} (\tfrac{1}{2})^{k-1} \, \frac{1}{k} \right\}$$

$$= \exp - \left\{ \frac{1}{2} + 4 \left(\log(1-\tfrac{1}{2}) + \frac{1}{2} + \frac{1}{8} \right) \right\} \tag{4.34}$$

This can be evaluated to give:

$$I = e^{-3} \, e^{-4\log(\frac{1}{2})} = 2^4 \, e^{-3} = 0.797 \tag{4.35}$$

This is a similar value to that found above. Note that we cannot use the approach of Chapter 3 to derive the hearing distribution itself in a simple way since there is dependency between the location of the node and whether it is even possible to hear it.

This gives the throughput for node J, γ_j:

$$\gamma_j = I \sum_{i=0}^{n-2} h_i \, \frac{1}{i+2} \left(1 - \frac{1}{i+2} \right)$$

$$= 2I \left[\sum_{i=2}^{n} \frac{1}{i} \, (\tfrac{1}{2})^i - \sum_{i=2}^{n} \frac{1}{i^2} \, (\tfrac{1}{2})^i \right]$$

$$\approx 2l \left[\log(2) + \frac{[\log(2)]^2}{2} - \frac{\pi^2}{12} \right] \quad \text{(for large } n\text{)}$$

$$= .1756 \tag{4.36}$$

Thus the total network throughput, γ, is given by:

$$\gamma \approx .176n \qquad \text{(for large } n\text{)} \tag{4.37}$$

Figure 4.17 shows the throughput predicted by this model and simulation results from the 'hitting degree' transmission scheme. We see very good agreement between analytical and simulation results.

4.5 Conclusions

In this chapter we have looked at the capacity of Packet Radio Networks for local traffic. In particular we were trying to determine what traffic pattern would allow us to achieve the highest throughput. We were to able to find some simple bounds on the performance of the the 'best' traffic matrix. We found that the total throughput γ for the optimal traffic matrix is bounded in one dimension by $\frac{2}{3} \frac{n}{4} \leqslant \gamma \leqslant \frac{n}{4}$ and in two-dimensions by $.62 \frac{n}{4} \leqslant \gamma \leqslant \frac{n}{4}$. We found that local traffic seemed to give low interference and so studied some specific configurations in more detail. For these local configurations we were able to achieve a capacity which is a linear function of the number of nodes in the network, exceeding the lower bound. We also studied various different transmission policies, finding that any of the policies which reduce the transmission probability of the links causing high interference allow us to achieve similar performance.

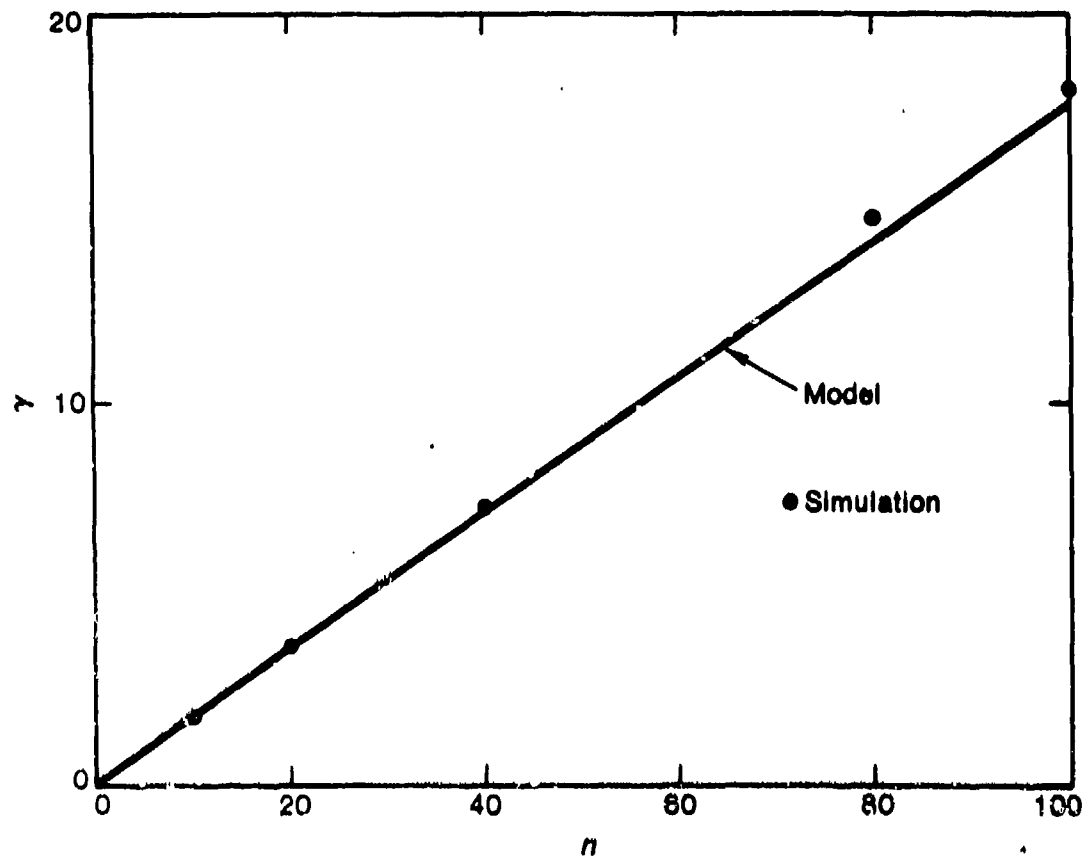Furthermore, we analyzed the one-dimensional case, showing $\gamma \approx .7 \frac{n}{4}$.

Figure 4.17 1-D ADJ

CHAPTER 5
REGULAR TOPOLOGIES

In this and following chapters we will be looking at multi-hop networks, where messages are forwarded from node to node following a path defined by the routing matrix.

5.1 Introduction

In this chapter we consider regular networks (i.e., those in which the nodes are regularly placed on a square grid, for example). These networks are much easier to analyze than the random networks discussed in Chapter 6 since the topology is fixed and the progress that can be made toward the destination in any hop is not dependent on any probabilistic argument.

We start by looking at one-dimensional networks. Loop networks, as discussed in section 5.2, are really one-dimensional line networks wrapped around a circle with the ends joined together. In section 5.3 we look at networks generated on the line. We follow this with a discussion of two-dimensional regular networks, such as the square lattice.

The traffic matrix that we use is uniform, i.e., each node splits its traffic between all possible destinations equally. With this traffic matrix and the uniformity of the topology, we assume that the traffic load on all links of the network is homogeneous. This seems to be a valid assumption for any reasonable routing algorithm in loop networks, since there are no edge effects to consider. In two-dimensional networks we neglect edge effects as they are of minor importance relative to the rest of the network (the perimeter of the network will contain $O(\sqrt{n})$ nodes).

5.2 One Dimensional Networks

One-dimensional networks consist of nodes uniformly spaced on a line. If we limit the length of the line we face a problem with edge effects, however. There are two classes of networks that we could consider which avoid this problem.

One approach is to consider placing the points on the circumference of a circle, generating a loop network. This gives us a one-dimensional network with no edge effects, which also has the nice property that distances are finite. We study this in section 5.3.

The other is to look at the infinite line, an approach similar to that found in [AKAV 79] The problem with this approach is that for a uniform traffic matrix the distances to travel are infinite. We must therefore assume some average distance that messages travel. We look at this in section 5.4.

5.3 Loop Networks

The networks considered in this section consists of n points uniformly (regularly) distributed around the circumference of a circle. Figure 5.1 shows a typical loop network, consisting of 8 nodes and average degree of 5 (each node can communicate with its two neighbors on either side). Each node in the network is identical in terms of traffic handled, degree and so on. We can therefore compute the number of successful transmissions per slot for the whole network (s_{net}), to be n times the probability of success for any particular node s_i.
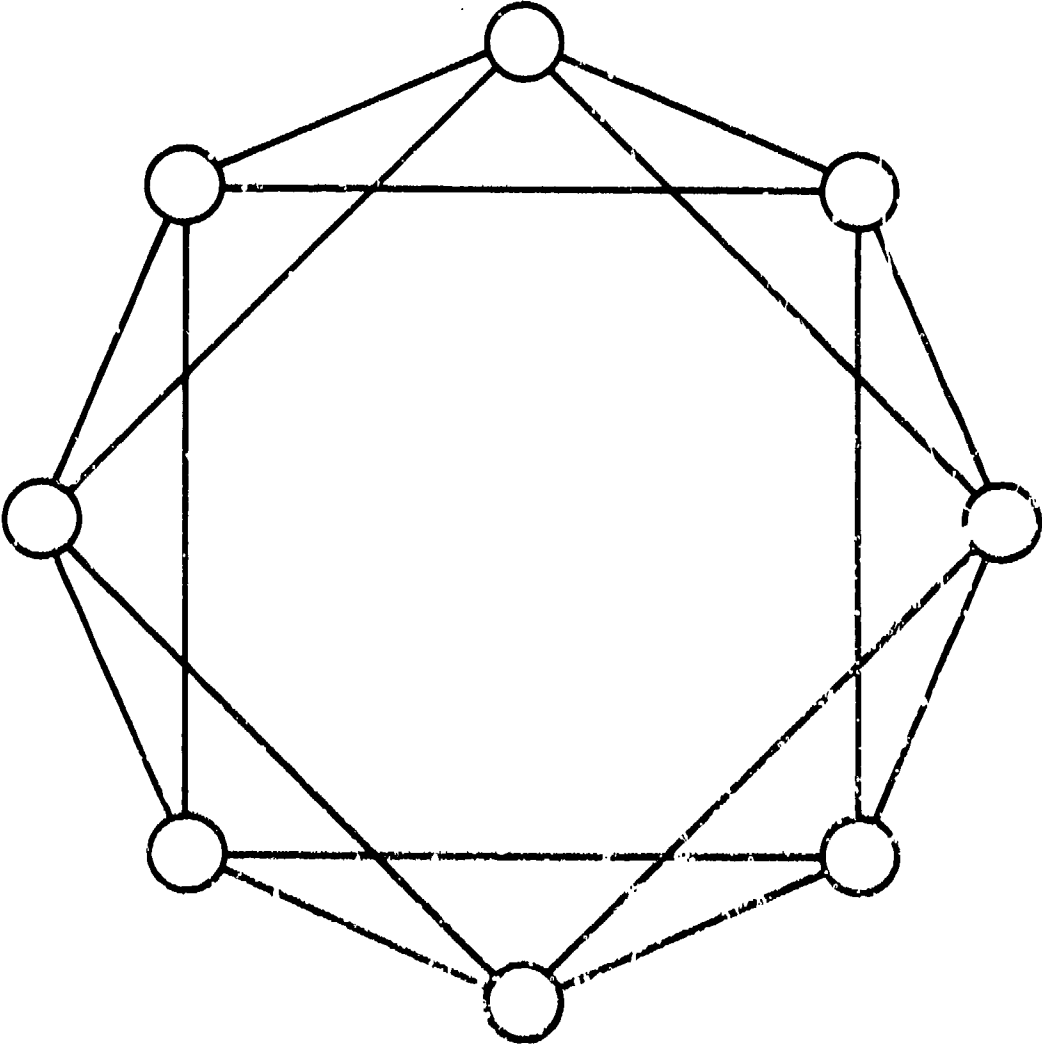
Figure 5.1 Regular Loop Network

5.3.1 Network Success Rate

We start, therefore, by determining the number of successful transmissions per slot for an arbitrary node (i) in the network. Let N denote the degree of any node in the network, counting the node itself (this corresponds to our earlier concept of average degree). We will use the term "one-hop throughput" to mean the rate at which any individual node can successfully transmit packets to the next node along the path to the destination. If we let s_i denote the probability of a successful transmission in any slot by node i (one-hop throughput) then:

$$s_i = \Pr\{\text{node } i \text{ successfully transmits}\} \tag{5.1}$$

$$= \Pr\{\text{node } i \text{ transmits and no other interfering node does}\}$$

$$= p \, (1-p)^{N-1} \tag{5.2}$$

where p is the transmission probability. Since all nodes carry the same traffic and have the same degree, we give every node the same transmission probability. In order to find the optimum value for this transmission probability we differentiate:

$$\frac{ds_i}{dp} = (1-p)^{N-1} - (N-1)p(1-p)^{N-2} \tag{5.3}$$

$$= (1-Np) \, (1-p)^{N-2}$$

For optimality we equate this to zero and find that:

$$p^* = \frac{1}{N} \tag{5.4}$$

This is exactly as found in [ABRA 70], as expected. We drop the asterisk and use p to represent the optimum value for the rest of this analysis. Rewriting the expression for success probability:

$$s_i = \frac{1}{N}\left(1 - \frac{1}{N}\right)^{N-1} \tag{5.5}$$

From this we can obtain the number of successful transmission per slot for the whole network s_{net}:

$$s_{net} = \frac{n}{N}\left(1 - \frac{1}{N}\right)^{N-1} \tag{5.6}$$

This represents the number of successful packets received per slot for the whole network. It does not correspond to the throughput since a multi-hop path will require many transmissions and we are counting each hop in this expression as a contribution to the throughput. When the network is fully connected $N = n$, we see that this reduces to the usual $1/e$ (path lengths all being one in this case).

5.3.2 Path Length

In order to compute the network throughput, γ, we must divide s_{net} by the expected path length in hops $\bar{l}$. In order to find the average path length, we split the network into groups such that all the members of one group are equi-distant (in hops) from a given (typical) node. Thus the first group will be those with whom a node can directly communicate, the second group will be those that are two hops away and so on. Each group will have the same number of members, $N-1$, except possibly for the last group which will have the remainder if $(n-1)/(N-1)$ is not an integer. Let g represent the number of complete (not counting this remainder) groups, then:

$$g = \left\lfloor \frac{n-1}{N-1} \right\rfloor \tag{5.7}$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to x. There are $(n-1) - (N-1)g$ nodes in the last partial group, with path length $g+1$. This group is taken care of by the second term in the following expression for $\bar{l}$.

$$\bar{l} = \frac{1}{n-1} \left[(N-1) \sum_{i=1}^{g} i \;+\; (g+1)\,[(n-1) - g(N-1)\,] \right]$$

$$= \frac{1}{n-1} \left[(g+1)(n-1) - \frac{(N-1)g(g+1)}{2} \right]$$

$$= (g+1) - \frac{(N-1)g(g+1)}{2(n-1)} \tag{5.8}$$

Recalling the example of figure 5.1, we find the number of complete groups is:

$$g = \left\lfloor \frac{7}{4} \right\rfloor = 1 \tag{5.9}$$

and the average path length is:

$$\bar{l} = \frac{10}{7} \tag{5.10}$$

If there are no nodes in the special extra group, which is to say that there is no remainder in the division of $n-1$ by $N-1$, this reduces to the following 'clean' expression for $\bar{l}$.

$$\bar{l} = \frac{n+N-2}{2N-2} \tag{5.11}$$

This will also be a valid expression for $\bar{l}$ in the limit when the total number of nodes is large compared to the number in the last group. More precisely when,

$$\frac{(N-1)g}{2} \gg (n-1) - g(N-1) \tag{5.12}$$

5.3.3 Throughput

For the case of Equation 5.12, we can compute the network throughput, γ, From this we can compute the network throughput, γ, as the network success rate divided by the average path length.

$$\gamma = \frac{S_{net}}{l} \tag{5.13}$$

$$= \frac{n}{N}\left(1 - \frac{1}{N}\right)^{N-1}\frac{2N-2}{n+N-2} \tag{5.14}$$

$$= \frac{2n}{n+N-2}\left(\frac{N-1}{N}\right)^{N} \tag{5.15}$$

Let us evaluate this expression for two interesting cases, i) when $N=n$ (i.e a fully connected net), and ii) when $N=3$ (i.e. each node is only connected to his immediate neighbors). We notice that the average path length for the fully connected case is 1, and $\frac{n+1}{4}$ when each node is only connected to its neighbors. We denote the throughput for a network with $N=j$ by γ^j.

5.3.3.1 Fully Connected Network

For the fully connected network we have:

$$\gamma^n = \frac{n}{n}\left(1 - \frac{1}{n}\right)^{n-1}\frac{2n-2}{2n-2}$$

$$= \left(1 - \frac{1}{n}\right)^{n-1} \tag{5.16}$$

Taking the limit for large n, we find:

$$\lim_{n \to \infty}\left(1 - \frac{1}{n}\right)^{n-1} = \frac{1}{e} \tag{5.17}$$

We see that for large n we achieve the throughput which corresponds to the usual infinite ALOHA

population.

5.3.3.2 Neighbor Communication

Considering the other extreme where each node is only connected to its two neighbors, we find (neglecting the effect of the last group):

$$\gamma^3 = \frac{2n}{n+1}\left(\frac{2}{3}\right)^3$$

$$= \frac{16n}{27(n+1)} \tag{5.18}$$

For a large net the throughput for the neighbor case is therefore $\frac{16}{27}$ which is clearly greater than $\frac{1}{e}$. It may be that the maximum for γ is achieved for an intermediate value. We, therefore, proceed to investigate the behavior of Equation 5.15 for intermediate values of N.

5.3.4 Optimal Average Degree

Recall the throughput expression for γ^N, (neglecting the last group):

$$\gamma^N = \frac{2n}{n+N-2}\left(\frac{N-1}{N}\right)^N \tag{5.19}$$

Differentiating Equation 5.19 with respect to N, we have:

$$\frac{d\gamma}{dN} = \frac{2n}{(n+N-2)^2}\left(\frac{N-1}{N}\right)^N\left[-1 + (n+N-2)\left(\frac{1}{N-1} + \log\left(\frac{N-1}{N}\right)\right)\right] \tag{5.20}$$

To find the optimal N, we must solve the equation:

$$(n+N-2)\left(\frac{1}{N-1} + \log\left(\frac{N-1}{N}\right)\right) - 1 = 0 \tag{5.21}$$

Rewriting, we have:

$$N - 1 = n+N-2 + (N-1)(n+N-2)\log\left(1 - \frac{1}{N}\right) \tag{5.22}$$

Since $N > 1$ we can expand the log to obtain:

$$0 = n-1 - (N-1) \, (n+N-2) \left[\frac{1}{N} + \frac{1}{2} \frac{1}{N^2} + o\left(\frac{1}{N^3} \right) \right] \tag{5.23}$$

After some algebra we find:

$$\frac{n}{2N^2} + o\left(\frac{1}{N} \right) + o\left(\frac{n}{N^3} \right) = 1 \tag{5.24}$$

In order for this equation to balance for large n, we have:

$$N = \sqrt{\frac{n}{2}} \tag{5.25}$$

For large n, the second factor of the equation for γ will vary from $\frac{8}{27}$ (for $N=3$) to $\frac{1}{e}$ (for large N). The first term is equal to 2, provided that N grows at a slower rate than n. We found above that at the optimum N must be of the order of $\sqrt{n/2}$. For such a value of N, the throughput will be given by $2/e$. In fact, the exact value of N is not critical as long as it is greater than 3 and grows slower than n. We see, therefore, that the maximum throughput is $\frac{2}{e}$ and will be achieved for any moderate value of N. In Figure 5.2 we plot the throughput as a function of the average degree, as given by Equation 5.19, and see that the predicted behavior is achieved.

5.4 Line Networks

Another one-dimensional network of interest is the line network. The successful transmission rate for the network is the same.

$$S_{net} = \frac{n}{N} \left(1 - \frac{1}{N} \right)^{N-1} \tag{5.26}$$

Computation of path length is not so straightforward however. If we fix the average number of nodes that messages pass through at say k, then the path length l is:

$$l = \left\lceil \frac{k}{N/2} \right\rceil \tag{5.27}$$

The network throughput is, therefore:

$$\gamma = \frac{n}{2k} \left(1 - \frac{1}{N} \right)^{N-1} \tag{5.28}$$

which is (almost) independent of the average degree, provided that our approximation for the ceiling function is valid, i.e., $N \ll k$. If $N/2$ is close to k in value, then the network throughput is equal to S_{net}.
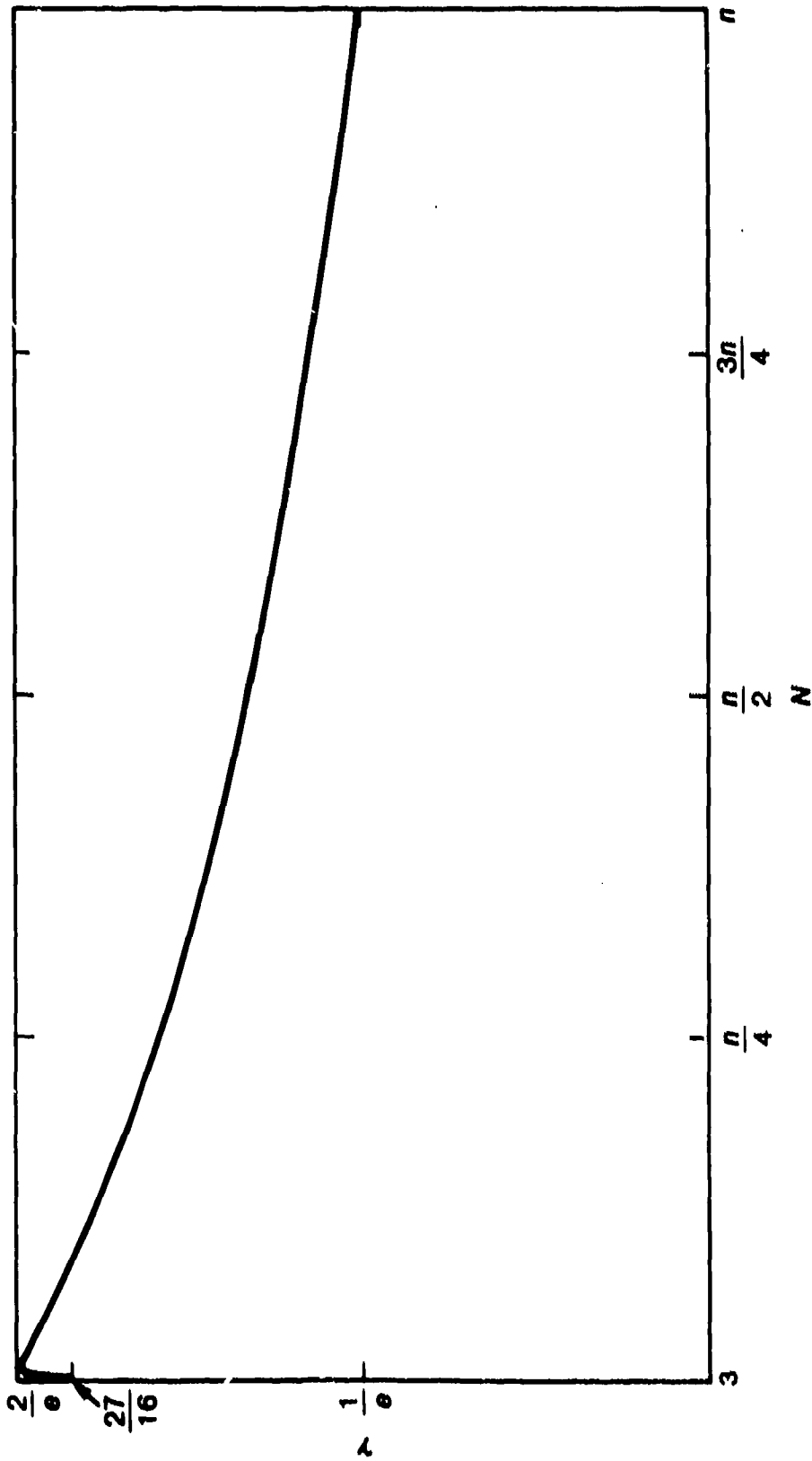
Figure 5.2 Throughput vs Average Degree

If we consider a traffic matrix which causes the average number of hops to be a function of n, a uniform traffic matrix for example, then the throughput is given by:

$$\gamma = c \left[1 - \frac{1}{N} \right]^{N-1} \tag{5.29}$$

where c is some constant, depending on the specific traffic requirement.

The important result here is that the throughput is independent of the average degree, provided that the average degree is smaller than the number of hops that messages take.

5.5 Two Dimensions

The one-hop throughput for a two-dimensional regular network with fixed degree N will be the same as for the one-dimensional case.

$$s_{net} = \frac{n}{N} \left[1 - \frac{1}{N} \right]^{N-1} \tag{5.30}$$

5.5.1 Path Length

The path length for a uniform traffic matrix will now be proportional to the square root of the number of nodes, rather than proportional to the number of nodes as we found for one-dimensional networks.

$$\bar{l} \; \alpha \; \frac{\sqrt{n}}{\sqrt{N}} \tag{5.31}$$

The throughput will be:

$$\gamma = d \sqrt{n/N} \left[1 - \frac{1}{N} \right]^{N-1} \tag{5.32}$$

where d is a proportionality constant depending on the particular topology of the network. The implication of this is that we should let N become as small as possible, since both $\sqrt{n/N}$ and $(1-1/N)^{N-1}$ increase as N decreases. The minimum value that N can take is 4 for a hexagonal tesselation (a 3-connected net). For small degrees ($N=4$ or 5), we must evaluate the proportionality constant (d). In [AKAV 79], Akavia makes this comparison and finds that the optimum network is the hexagonal tesselation mentioned above.

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5.5.2 Manhattan Nets

Let us consider the Manhattan (square grid) network in more detail. Figure 5.3 shows a sample Manhattan network for $N=5$ and $n=49$. The distance metric for this network is the sum of the differences in x and y coordinates (i.e., we can only move parallel to the x or y axes).

Thus the average distance between two arbitrary points in a square (of unit area) is:

$$d = 2\int_0^1 \left[\int_0^x (x-y)\ dy + \int_x^1 (y-x)\ dy \right] dx$$

$$= \frac{2}{3}$$

(5.33)

If we superimpose a grid of n points on this square the average path length will be:

$$\bar{l} = \frac{2}{3}\sqrt{n}$$

(5.34)

and the network throughput will be given by:

$$\gamma = \frac{n}{5}\left(1 - \frac{1}{5}\right)^4 \frac{1}{2/3\sqrt{n}}$$

$$= 0.123\sqrt{n}$$

(5.35)

In Figure 5.4 we plot this analytical expression and compare it to simulation results. We notice that there is a significant discrepancy between the model and simulation results. The reason for this is that the routing algorithm is incapable of producing truly balanced flow on all links (corresponding to the homogeneity assumption that we make in the analytical derivation). The central nodes will therefore be carrying higher traffic and thus experiencing higher interference. We do note that the throughput grows in a fashion similar to that predicted by the model, however. We also show a single plot corresponding to using a higher average degree on the same grid network. Using an average degree of $N=9$, we find that the performance is significantly degraded.

5.6 Concluding Remarks

We find that for one-dimensional regular networks the throughput is (basically) independent of the degree. We can achieve a throughput of c/e with the constant c depending of the form of the traffic matrix. For loop networks $c=2$ and we can thus achieve a capacity of $2/e$.

For two-dimensional networks we achieve a throughput proportional to the square root of the number of nodes in the network. The best degree to use is the minimum possible, i.e., 3.
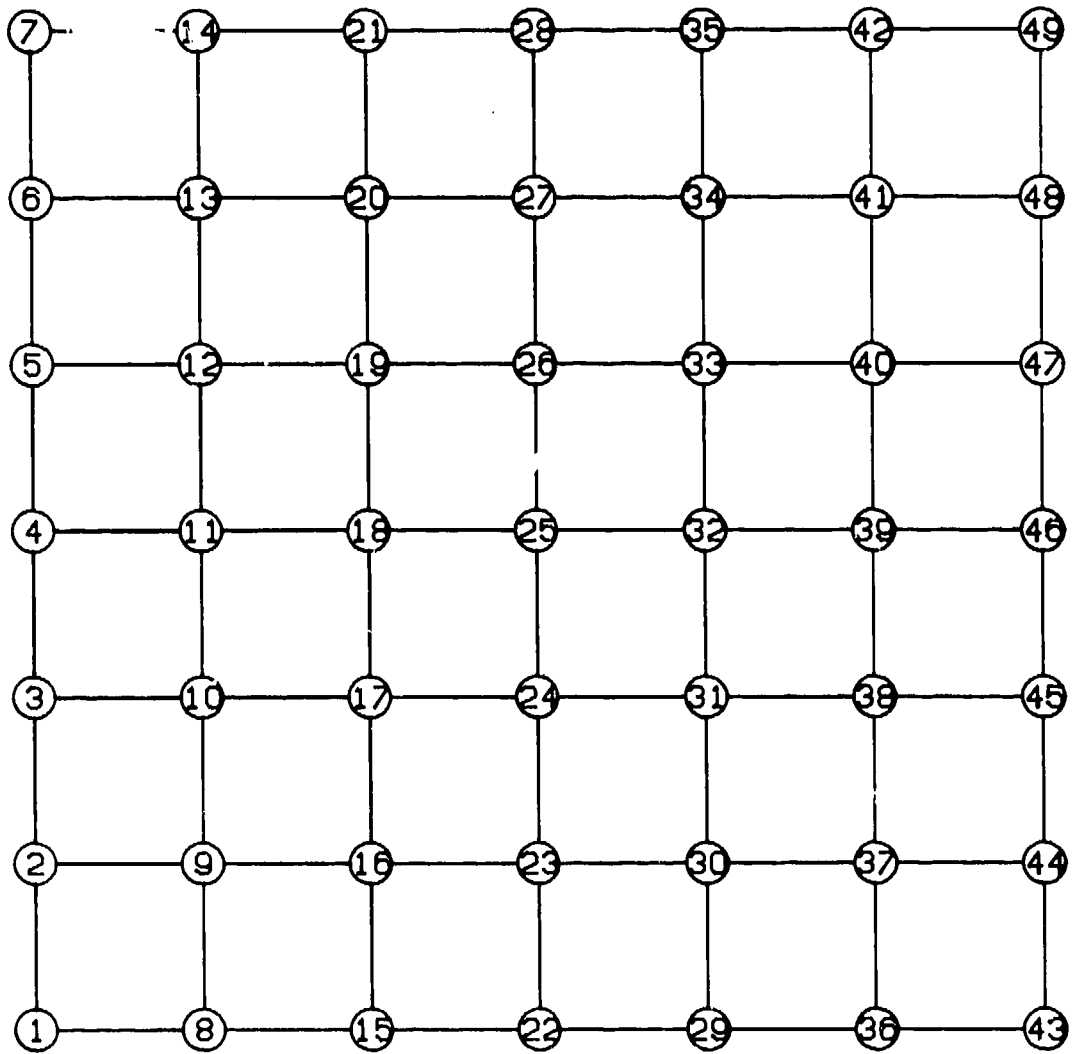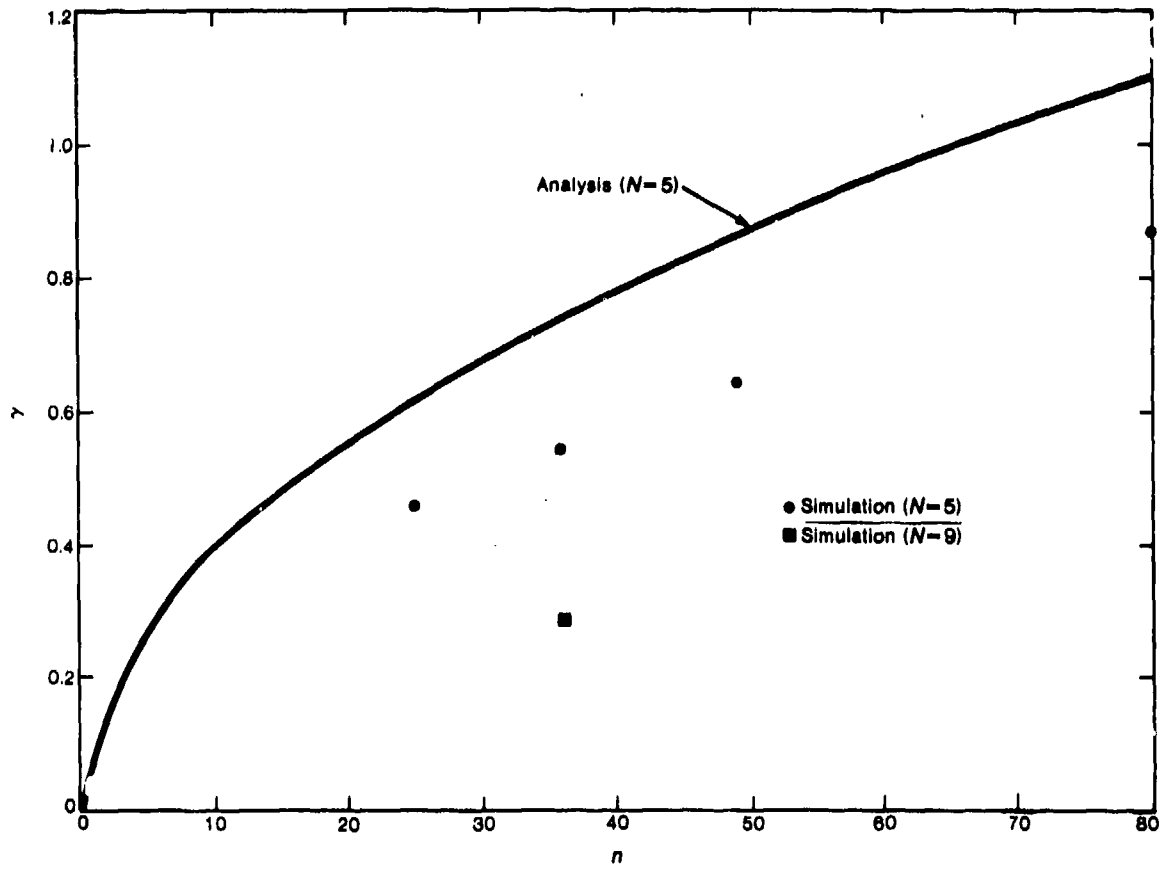
Figure 5.3 A Square Grid Network

Figure 5.4 Throughput for 2-D Regular Networks

CHAPTER 6
RANDOM PLANE NETWORKS

6.1 Introduction

In this chapter we study the capacity of packet radio networks in which the nodes are *randomly located*. This random location of nodes can be thought of as representing either an arbitrary network or a snapshot of a mobile one.

As before, we find that one of the major factors affecting the capacity is the transmission radius (average degree) that the nodes use. The randomness of the node location causes new problems, however. Although using a very large transmission radius gives a high degree of connectivity, there will be much interference and a corresponding loss of channel throughput. In the extreme case where we have a completely connected network we know that the (ALOHA) capacity for the entire network is only $1/e$. We can limit this interference and increase the capacity by reducing the transmission radius, but doing this implies a corresponding increase in the number of hops a message must take in order to arrive at its destination. This is similar to what we found in the last chapter for regular networks, but with small transmission radii we find additional connectivity problems not present in regular networks. This increased number of hops creates more internal traffic which tends to reduce the effective capacity of the network.

We analyze this tradeoff and find that there is a transmission radius which optimizes the capacity and that this radius allows us to achieve a throughput proportional to the square root of the number of nodes in the network. Kleinrock [KLEI 75b] also found that a critical radius exists when trying to minimize delay in an arbitrary point to point network and Akavia [AKAV 79] finds similar results in trying to minimize the cost of the network for a certain delay requirement. Both of these authors assume a continuum of sources (repeaters) throughout the network, the consequence being that a transmission will always progress toward the destination by a distance equal to the transmission radius. For small transmission radii (or sparse networks) this assumption is invalid and we must take the topology into consideration. We are unable to progress to the edge of our transmission radius for two reasons: firstly, the probability of finding a point close to the edge of our transmission radius decreases as the expected number of points within range is reduced; and secondly, the probability of finding someone in the direction in which we wish to travel is also diminished.

Rather than restrict ourselves to certain specific topologies (regular networks, for example), we will consider networks, consisting of a set of nodes *randomly* located in the plane. We consider these to be nodes in a distributed (i.e. not centralized) communication network. Such a network can be thought of as either representing a snapshot of a mobile network or as a representative sample of the set of all networks.

We presume the existence of a routing algorithm which allows packets to be forwarded from source to destination through the network. (This is much harder than for regular networks, especially as the networks best represented by random models are mobile.) Each packet radio unit is assumed to use a predetermined fixed radius for transmission (which determines the network structure). The performance of the netw--k will then be studied as the transmission radius is varied. Clearly if the transmission radius is too small some of the nodes will become isolated. In this chapter we restrict ourselves to consider only connected networks. By requiring that the transmission radius be large, we can make the probability of the network not being connected small.

As we increase the transmission radius we find that the degree of connectivity increases, each node being able to communicate with more nodes in one hop. In addition to varying the transmission radius we have an additional degree of freedom, namely the transmission probability. It will be necessary to reduce the transmission probability as the connectivity increases so that the environment around any node is not overloaded with traffic. In the following analysis we will optimize this transmission probability to give the best throughput.

6.2 General Model

The nodes of the network are considered to be uniformly distributed in (two-dimensional) space with density λ (that is, there will be an average of λ points per unit area). The access mode that will be used is slotted ALOHA with each node having a transmission probability p in a slot. The slots correspond to the transmission time of the longest packet used in the system. Each node will transmit with the same radius r, which will determine the connectivity (topology) of the network. Any nodes falling within the circle of radius r about a node will be able to hear that node and also be able to transmit to it. We only consider the heavy traffic case, in which every node is always busy and will transmit whenever permitted (the restraint being the transmission probability). We show a sample random network in Figure 6.1, having 20 nodes and an average degree of 8.

The traffic matrix we will study is uniform, each node wishing to communicate with all others on an equal basis. We will therefore consider each node to be equivalent, having the same transmission radius, transmission probability and traffic load. (We are assuming here that the edge effects and imbalance of traffic due to routing are of minor importance.)

We will find the capacity of the network, which is the maximum achievable throughput measured in terms of source destination messages. We start by studying the number of transmissions per unit time that can be handled by the network.

6.3 Per-Hop Traffic

Consider the number of successful transmissions per slot. This is a measure of the throughput if nodes are only talking to their neighbors. If, however, some traffic requires more than one hop, we will be counting each transmission along the path as a contribution to the throughput.

Consider an arbitrary node in the network. We define h_i to be the probability of hitting i other nodes by a transmission and H_i to be the probability of being in range of i other nodes. As the nodes are randomly distributed, the number of nodes that will be in a circle of radius r is Poisson distributed, i.e.,

$$h_i = \frac{(\lambda A)^i e^{-\lambda A}}{i!} \qquad (i=0,1,2,\dots) \tag{6.1}$$

where A is the area (volume) covered by the transmission.

$$A = \begin{cases} \pi r^2 & \text{for two dimensions} \\ 2r & \text{for one dimension} \end{cases} \tag{6.2}$$
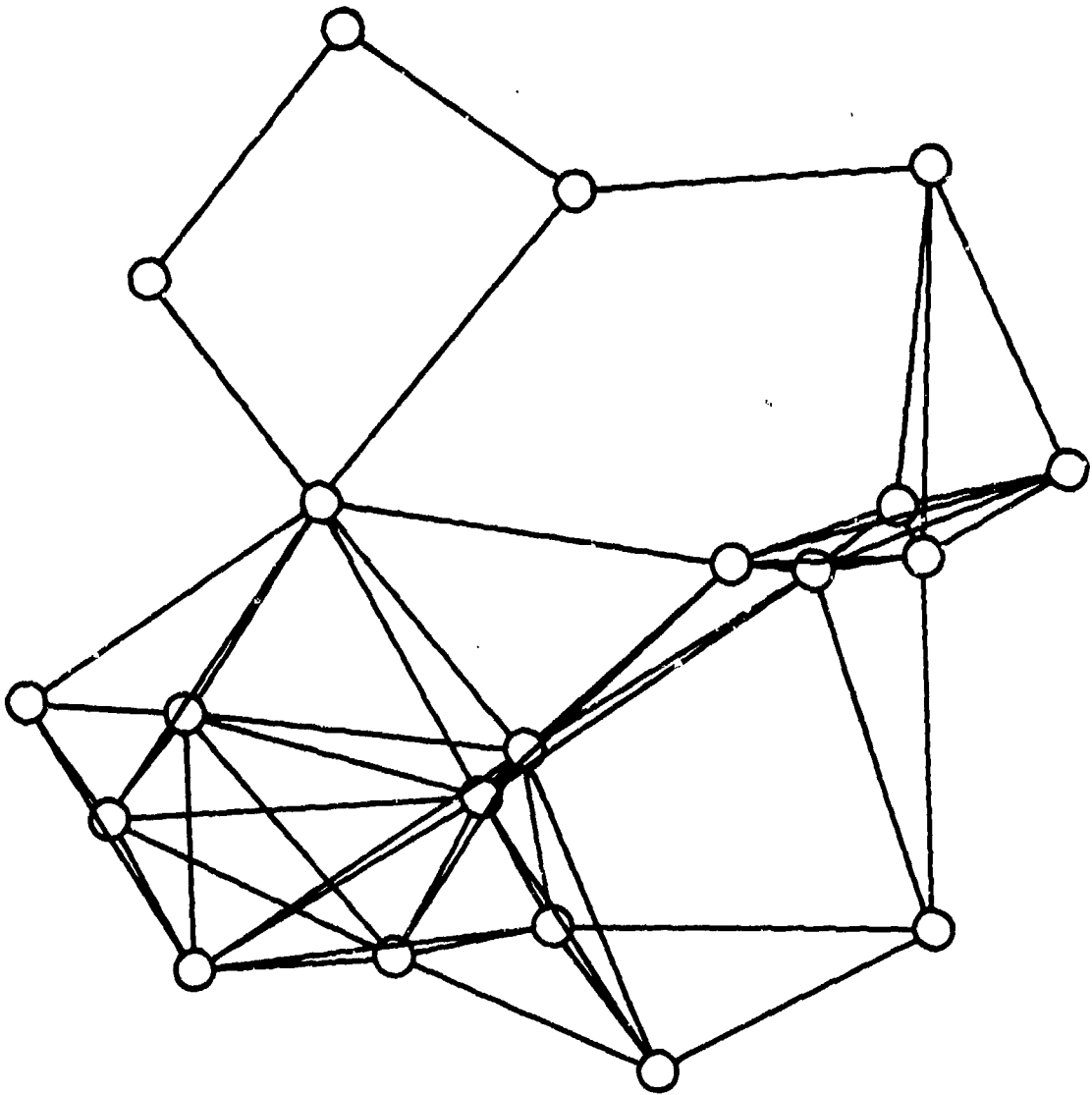
Figure 6.1 Random 2-D Network with 20 Nodes

We will find that the term λA continually crops up in our equations. This corresponds to the expected number of nodes in a transmission radius about any point. For convenience, therefore, let us define N to be this average degree.

$$N = \lambda A \qquad (6.3)$$

$$= \lambda \pi r^2 \qquad \text{(for two dimension)}$$

We can therefore rewrite Equation 6.1 in these terms.

$$h_i = \frac{N^i e^{-N}}{i!} \qquad (i=0,1,2,...) \qquad (6.4)$$

In the case where all nodes are using the same transmission radius, it is clear that you will hear precisely those nodes that hear you and, thus, H will have the same distribution as h, that is,

$$H_i = \frac{N^i e^{-N}}{i!} \qquad (i=0,1,2,...) \qquad (6.5)$$

We are interested in counting the number of successful transmissions in any slot. Let us, therefore, define q to be the probability of a node successfully receiving a packet in a slot, and q_i to be the same conditioned on the fact that this node hears i people. This is the probability that exactly one of the units that you hear transmits to you and you are silent. In slotted ALOHA these events are independent as there is no control and all nodes are considered constantly busy for the heavy traffic case. For simplicity let us assume that every node in the network uses the same transmission probability p. Let us define A_i to be the condition that a node is in hearing range of i other nodes. nodes.

We then have:

$$q_i = Pr\{a \text{ neighbor transmits to you and you do not transmit} \mid A_i\} \qquad (6.6)$$

$$= Pr\{exactly \text{ one of the } i \text{ units transmits} \mid A_i\}$$

$$*Pr\{addressed \text{ to you} \mid A_i\} *Pr\{you \text{ do not transmit} \mid A_i\}$$

$$= \binom{i}{1} p \ (1-p)^{i-1} \frac{1}{i} \ (1-p) \qquad (6.7)$$

$$= p(1-p)^i \qquad (6.8)$$

If we now uncondition on the number heard we can obtain the probability, s, of successfully receiving a packet in any particular slot.

$$s = \sum_{i=1}^{\infty} H_i q_i \qquad (6.9)$$

$$= \sum_{i=1}^{\infty} \frac{N^i e^{-N} p (1-p)^i}{i!} \qquad (6.10)$$

Summing and rewriting we obtain,

$$s = p e^{-Np} - p e^{-N} \qquad (6.11)$$

The e^{-N} in the second term corresponds to the probability of there being nobody in range. As we are only considering connected networks we will need an average degree large enough to ensure against this. Erdos and Renyi [ERDO 59] have considered the issue of connectivity for large random graphs (i.e. graphs not defined by a geometrical relationship) and found that if the average degree is $\log(n)+c$ then the probability of the graph being connected is $e^{-e^{-c}}$. The graphs that we are interested in, however, are Euclidean graphs where the existence of edges is not an independent process. The analysis of connectivity is much more complex and no simple results like those for random graphs are known. Dewitt [DEWI 77] finds a lower bound on the probability of connectedness. If the average degree is $4\log(n) + 4\log\log(n) + 4c$ then $\Pr\{\text{connected}\} \geq e^{-e^{-c}}$. He also suggests that $\log(n)+O(\log\log(n))$ should be sufficient for connectivity. These results are asymptotically true for large graphs and may or may not be exact for smaller graphs. In Table 1 we give the average degree necessary (based on these formulae) for the probability of connectedness to be 0.95. (We have found in our simulations that using an average degree of 5 we have always been able to generate a connected network in one or two tries for networks with less than 100 nodes.)

#nodes	Av. Deg. (Erdos)	Av. Deg. (DeWitt)
10	5.2	24
20	6.0	28
40	6.6	32
80	7.3	35
150	8.0	38

Table 6.1 Number of Edges Required for Connectivity

We see, therefore, that we will need a degree of at least four to have a connected network. From stability arguments (so that we do not overload the local channel) [LAM 74] we know that p must decrease as N increases and in fact, should be proportional to $1/N$. The second term in Equation 6.11 then becomes negligible compared to the first.

Rewriting, we obtain the approximation:

$$s = p e^{-Np} \qquad (6.12)$$

Optimizing for p we find:

$$\frac{ds}{dp} = e^{-Np} - Npe^{-Np} \qquad (6.13)$$

$$= e^{-Np}(1 - Np) = 0 \qquad (6.14)$$

Thus:

$$p_{opt} = \frac{1}{N} \qquad (6.15)$$

Substituting this value back into Equation 6.11 we see that for a connected net ($N>4$) our assumption to neglect the second term appears to be justified.

Which gives the local throughput s, (i.e. throughput per node):

$$s = \frac{1}{Ne} \qquad (6.16)$$

The fact that the optimum value of p is found to be $1/N$ is no surprise as it corresponds to setting the average traffic load G to be equal to one packet per slot in any local environment [ABRA 70, LAM 74].

6.4 Network Utilization

From s we can determine the expected number of successful transmissions per slot for the whole network, s_{net}, by multiplying by the total number of nodes n:

$$s_{net} = \frac{n}{Ne} \qquad (6.17)$$

If we set N to be equal to n, which is equivalent to allowing all nodes to hear each other (i.e. very large transmission radius), the throughput reduces to $1/e$ which is Abramson's result for such nets [ABRA 70] (the path lengths are 1 in this case).

6.5 Network Throughput

The quantity obtained above is a measure of the number of successful transmissions per slot and must be divided by the average path length to obtain the network throughput.

Clearly this average path length is dependent on the traffic matrix. In fact, if we consider a traffic matrix which only specifies nearest neighbor communication, we have that the number of successful transmissions is indeed equivalent to the network throughput. This is not the interesting case, however. We will consider the more general case in which we assume that each node wishes to communicate with every other node in the network on an equal basis. That is to say, for each message generated at a node, we will randomly select the destination from the set of other nodes in the network. This is a uniform traffic matrix.

We need therefore to find the traffic-weighted path length, which, for the uniform traffic matrix, is the same as the usual concept of path length in a graph. The determination of average path length in a random graph is hard, and so we proceed by calculating the expected progress per hop. If the points were infinitely dense (compared to the transmission radius) we would expect to always be able to reach the edge of our transmission range in any direction in which we wished to travel. As the radius decreases however we will find that the point which will allow us to make the most progress towards our destination will be further and further away from the circumference. Eventually, in fact, we will not be able to make any progress at all in the direction we wish (the graph is likely to be disconnected by this time).

Dividing the expected distance between a random pair of points in the graph by the expected progress in one hop, we find the expected number of hops to reach an arbitrary destination. This is equivalent to the average path length.

6.6 Expected Progress

Let us consider the expected progress in one hop, z. In Figure 6.2, P is the source having a message destined to Q (in fact P can also be one of the intermediate points along the path defined by the routing matrix). Any point on the arc centered at Q is equivalent in terms of progress, the distance z is then measured from P to this arc.
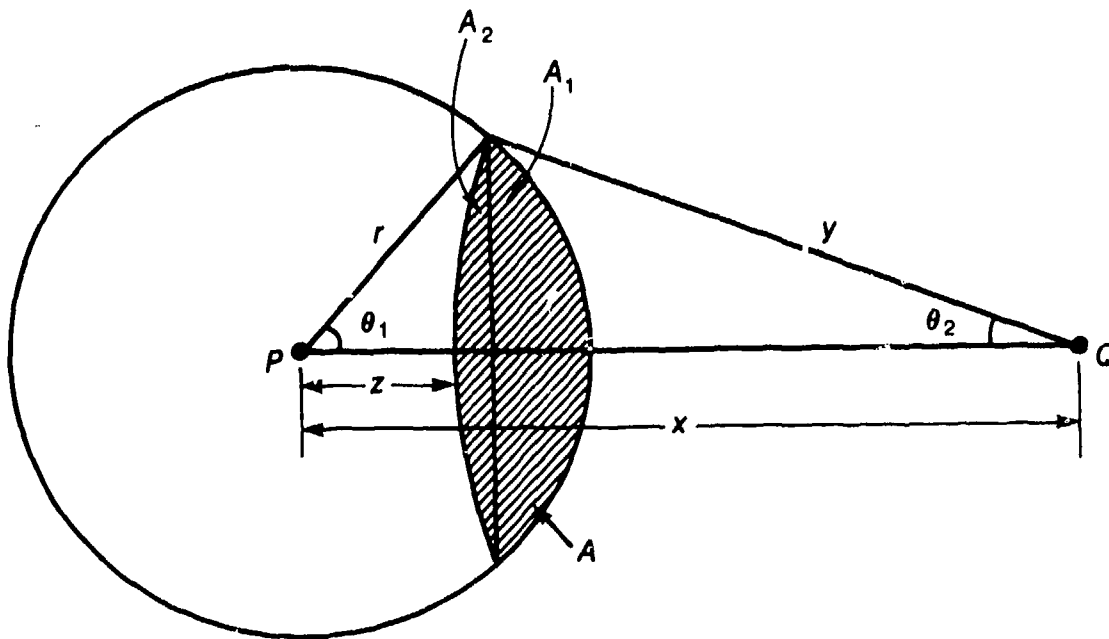

Figure 6.2 Progress in One Hop

Let us define:

$$F(z) = Pr\{Z \leqslant z\} \tag{6.18}$$

$$= e^{-\lambda A} \tag{6.19}$$

where A is the shaded area.
A is composed of two spherical caps A_1 and A_2:

$$A_1 = r^2\left[\theta_1 - \frac{\sin(2\theta_1)}{2}\right] \qquad A_2 = y^2\left[\theta_2 - \frac{\sin(2\theta_2)}{2}\right] \tag{6.20}$$

where the angles are given by:

$$\theta_1 = \cos^{-1}\left[\frac{r^2+x^2-y^2}{2rx}\right] \qquad \theta_2 = \cos^{-1}\left[\frac{y^2+x^2-r^2}{2xy}\right] \tag{6.21}$$

If P is sufficiently distant from Q we may neglect A_2 and for convenience, in the following we only consider A_1. We in fact did study the effect of including the correction term of A_2 and found that it made no significant difference to the average path length computation given below.

The expected progress, $\bar{z}$, is given by:

$$\bar{z} = \int_0^r [1 - F(x)] \, dx - \int_{-r}^0 F(x) \, dx + re^{-\lambda\pi r^2} \tag{6.22}$$

The last term in this expression corresponds to the probability of nobody being in range, and the second integral corresponds to the case where no progress can be made (i.e. we must move away from our destination). It could be argued that this term should not be included (depending on the routing strategy used), but we include it for completeness in the geometrical argument. It will have a negligible contribution to the computation for the range of degrees that we shall consider (i.e. those that will guarantee connectivity). Making the substitution $t=\cos(\theta_1)$ we have:

$$\bar{z} = r\left[1 + e^{-\lambda\pi r^2} - \int_{-1}^1 e^{-\lambda r^2\left[\cos^{-1}(t)-t\sqrt{1-t^2}\right]} \, dt\right] \tag{6.23}$$

$$= \left(\frac{N}{\lambda\pi}\right)^{1/2}\left[1 + e^{-N} - \int_{-1}^1 e^{-\frac{N}{\pi}\left[\cos^{-1}(t) - t\sqrt{1-t^2}\right]} \, dt\right] \tag{6.24}$$

If we consider the progress factor (normalized with respect to the radius) $f=z/r$, we find that it is a

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function depending only on N rather than explicitly on the radius. Figure 6.3 shows the expected progress factor as a function of the expected degree N.
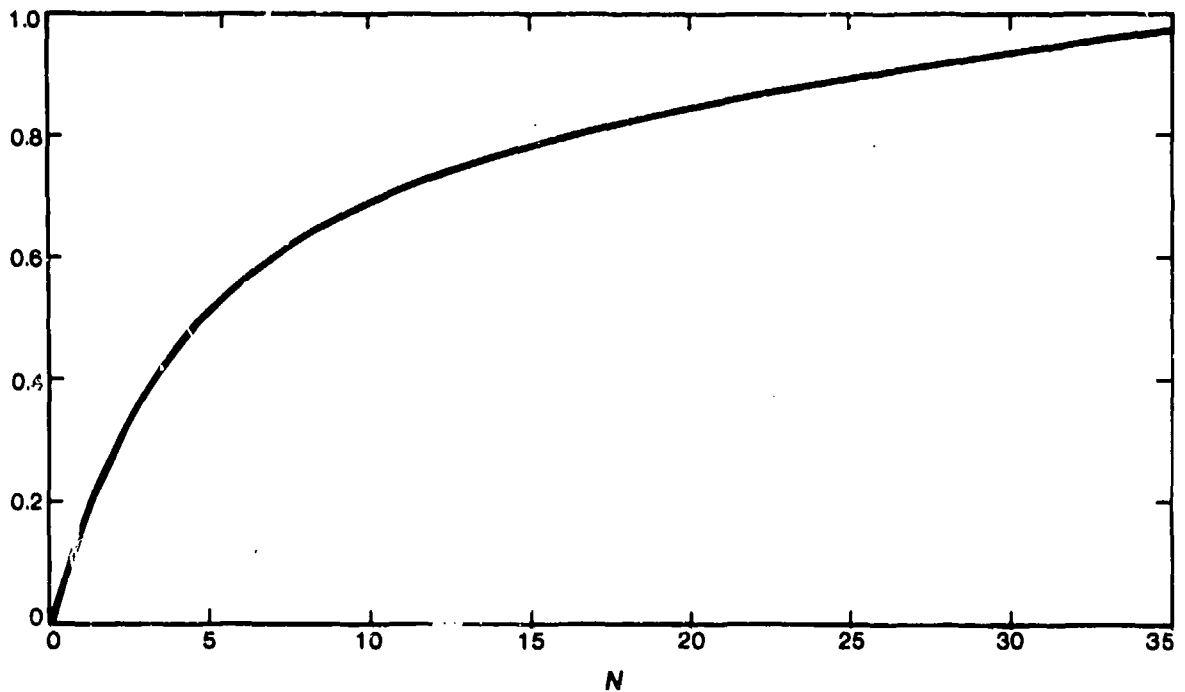

Figure 6.3 Expected Progress

Although we show the curve for small values of N, the curve probably does not represent the true progress that would be made in a real network, due to connectivity limitations and that the routing procedure may not allow motion away from the destination.

6.7 Expected Path Length

In order to determine the average path length we need to find the average distance between any two points in the network. This is equivalent to finding the distance between two points randomly located inside the area in which the network is enclosed. If we assume that the network is situated inside a disc of radius R, then the expected distance, d, between any two points randomly located within this disc is given by [KEND 63].

$$d = \frac{128}{45\pi} R \qquad (6.25)$$

We need to express R in terms of the density and total number of nodes.

$$\lambda \pi R^2 = n$$

$$\Rightarrow R = \left[\frac{n}{\lambda\pi}\right]^{1/2} \tag{6.26}$$

We can, thus, find the average number of hops $\bar{l}$ to be:

$$\bar{l} = \frac{d}{z} = \frac{128}{45\pi}\left[\frac{n}{N}\right]^{1/2} \cfrac{1}{1 + e^{-N} - \int\limits_{-1}^{1} e^{-\frac{N}{\pi}\left[\cos^{-1}(t) - t\sqrt{1-t^2}\right]}dt} \tag{6.27}$$

6.8 Network Throughput

We can now determine the true network throughput, γ, by dividing the number of successful transmissions (Equation 6.17) by the number of times a packet is repeated (the average path length given in Equation 6.27).

$$\gamma = \frac{45\pi}{128e}\left[\frac{n}{N}\right]^{1/2}\left[1 + e^{-N} - \int\limits_{-1}^{1} e^{-\frac{N}{\pi}\left[\cos^{-1}(t)-t\sqrt{1-t^2}\right]}dt\right] \tag{6.28}$$

This equation is the main result of this chapter, showing the network throughput as a function of the average degree. It expresses the tradeoff between small transmission radii (many hops) and large transmission radii (too much interference). If the average degree is a constant we see that the throughput is proportional to the square root of the number of nodes in the network. If the degree is an increasing function of the number of nodes however, the capacity will grow at a rate slower than $\sqrt{n}$. We show in Figure 6.4 the normalized network throughput $\frac{\gamma}{\sqrt{n}}$. The value of N which maximizes the throughput is 5.89, at which point the optimal network throughput γ^* is given by:

$$\gamma^* = .0976\sqrt{n} \tag{6.29}$$

which should be compared to the ALOHA (fully-connected) throughput of $1/e$ independent of the network size. We also notice that the throughput is extremely sensitive to reduction in degree from this optimum, whereas the capacity is relatively insensitive to the use of larger degrees.

Figure 6.5 shows the network throughput given by Equation 6.28 as a function of the number of nodes, for various average degrees. For comparison purposes we show the curve for a completely connected ALOHA network which is asymptotic to $1/e$ for large nets (slightly exceeding this for small nets [ABRA 70]). The curves for γ^* are only valid for average degrees less than the network size, as the performance reduces to that of the completely connected net for degrees close to the number of nodes. The reason that Equation 6.28 is not valid for average degrees comparable to the network size is that we must use a more sophisticated computation for path length to incorporate edge effects and the area A_2 mentioned in section 6.6.
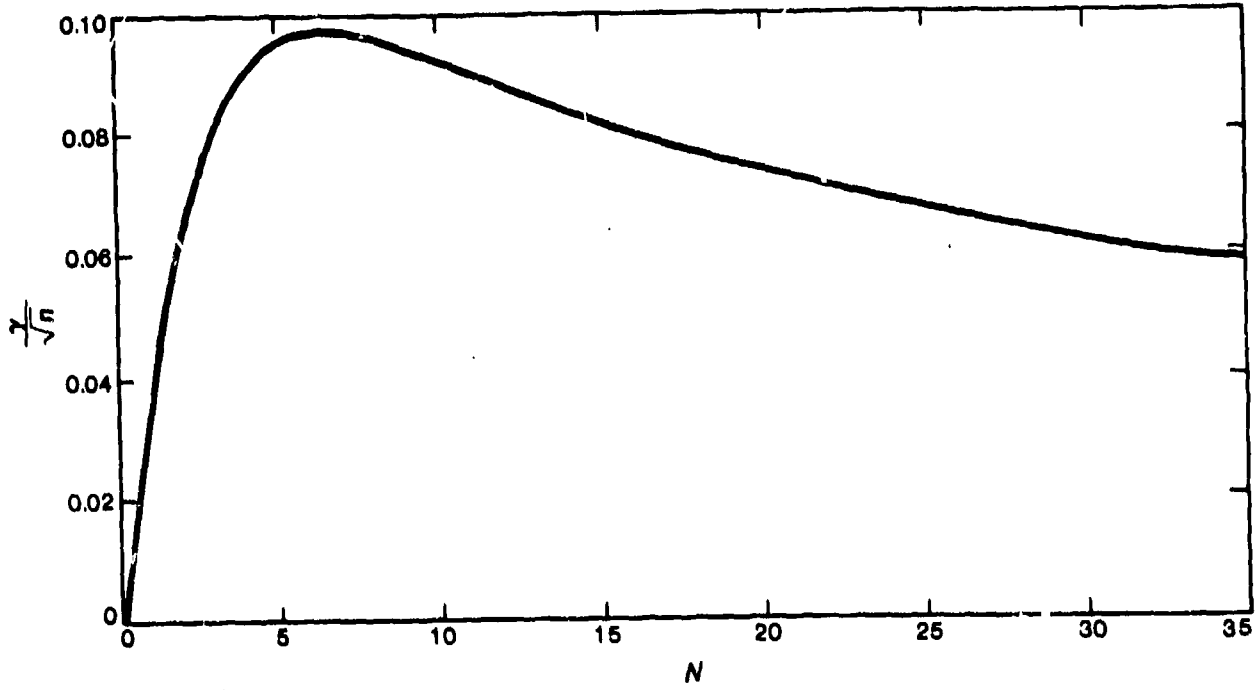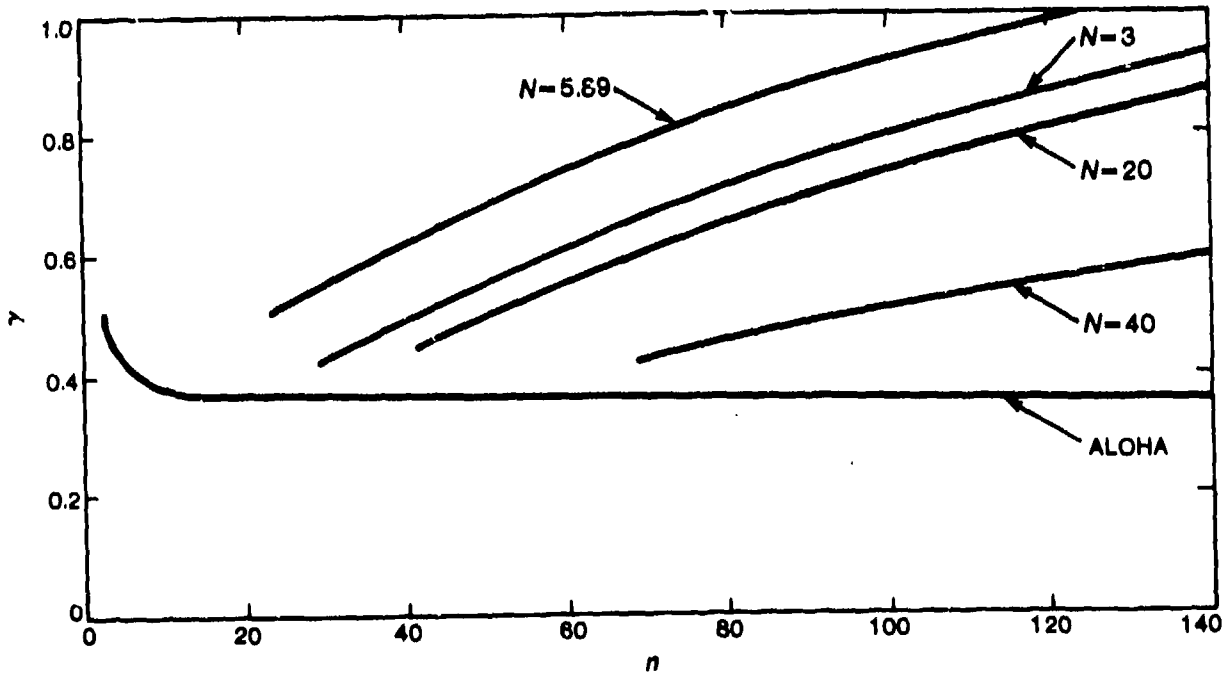
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Figure 6.4 Normalized Network Throughput

Figure 6.5 Network Throughput

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6.9 Conclusions

We have shown that for a constant average degree in a random network we can obtain a throughput proportional to the square root of the number of nodes on the network. We have also shown that the optimal average degree is approximately 6. Using a degree less than 6 causes drastic reduction in capacity of the network (the network also becomes disconnected), whereas exceeding 6 causes only gradual degradation (provided we do not have a degree which is an increasing function of the number of nodes). When an average degree of 6 is used the network throughput is $.0976\sqrt{n}$, as opposed to $1/e$ for a fully connected network.

CHAPTER 7
SIMULATION RUNS

In this chapter we will look at various simulation runs that we have made.

7.1 The Network Generator

In order to check the validity of the models of Chapter 6 we wrote a simulation program. This program has two major parts: i) Network Generation; and ii) Performance Computation. The network generation phase causes a random network to be generated with the specified number of nodes and average degree. The performance computation part then takes this network, imposes a uniform traffic matrix, determines the flow requirements and finally evaluates the performance. The initial strategy was to use transmission probabilities equal to the reciprocal of the average degree for each node. We found, however, that this produced very poor performance since the network load is not uniform. In section 7.4 we will describe a modified approach.

In order to determine the loads on each link we had to implement a routing algorithm. Initially we used the 'most progress' concept as presented in Chapter 6. A node forwards messages to that node that is closest (geographically) to the final destination of the message. We found, however, that this tends to worsen the non-uniformity of the loading and so we proceeded to try other routing algorithms. Section 7.3 describes the various routines that we used.

The programs were written in PL/1 and were run on the IBM 360/91 at UCLA. They were written in a modular fashion to facilitate changing parts of the simulation (such as the routing algorithm) without having to recompile the whole package.

7.1.1 Sample Networks

We show some sample networks that were generated by the program in Figures 7.1a, b, c, and 7.2a, b, c. Each series of figures shows the same node set for different values of the average degree.

Figure 7.1a has an average degree of 8 and we notice (contrary to our assumptions) that the topology is decidedly non-uniform. In particular we can identify two major features: i) there is one area of the net where the nodes are heavily clustered (nodes 1, 15, 33, 7, 18, 28, 17 etc) - we anticipate that the interference in this region will be high; and ii) the connections between {30, 11} and {19, 16, 25} must carry all of the traffic from the nodes in the dense area to the other half of the net (nodes 36, 39, 6, 24 etc) - we anticipate that these links will become heavily loaded causing a bottleneck.

In Figure 7.1b, the average degree has been increased to 12. We notice that the bottleneck has disappeared, since there are now additional paths between the two groups. We also notice that the size of the dense region has increased, reducing the throughput in this region.

Increasing the degree to 16 (Figure 7.1c) does not change the network structure in any significant way, except that the dense region has again increased in size. We expect that this will further reduce throughput. For this particular set of nodes we anticipate that the maximum performance will be achieved when the average degree is 12 (i.e., no bottleneck but no large dense region).

Figure 7.2a shows a different set of nodes with an average degree of 8. We notice that the network topology is similar to that of Figure 7.1a. In particular we still have dense regions and a bottleneck (nodes {7,21} and {15,26,39}).
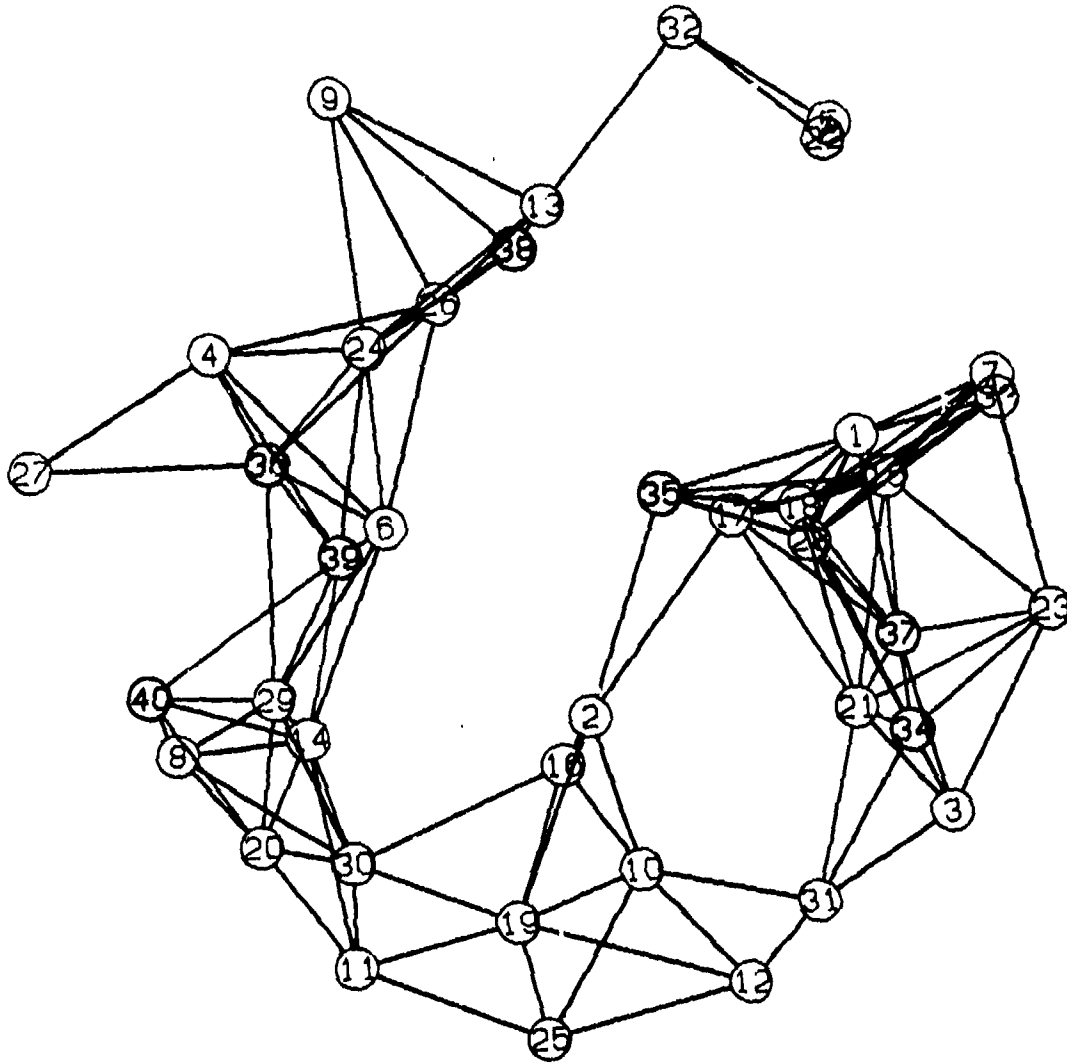
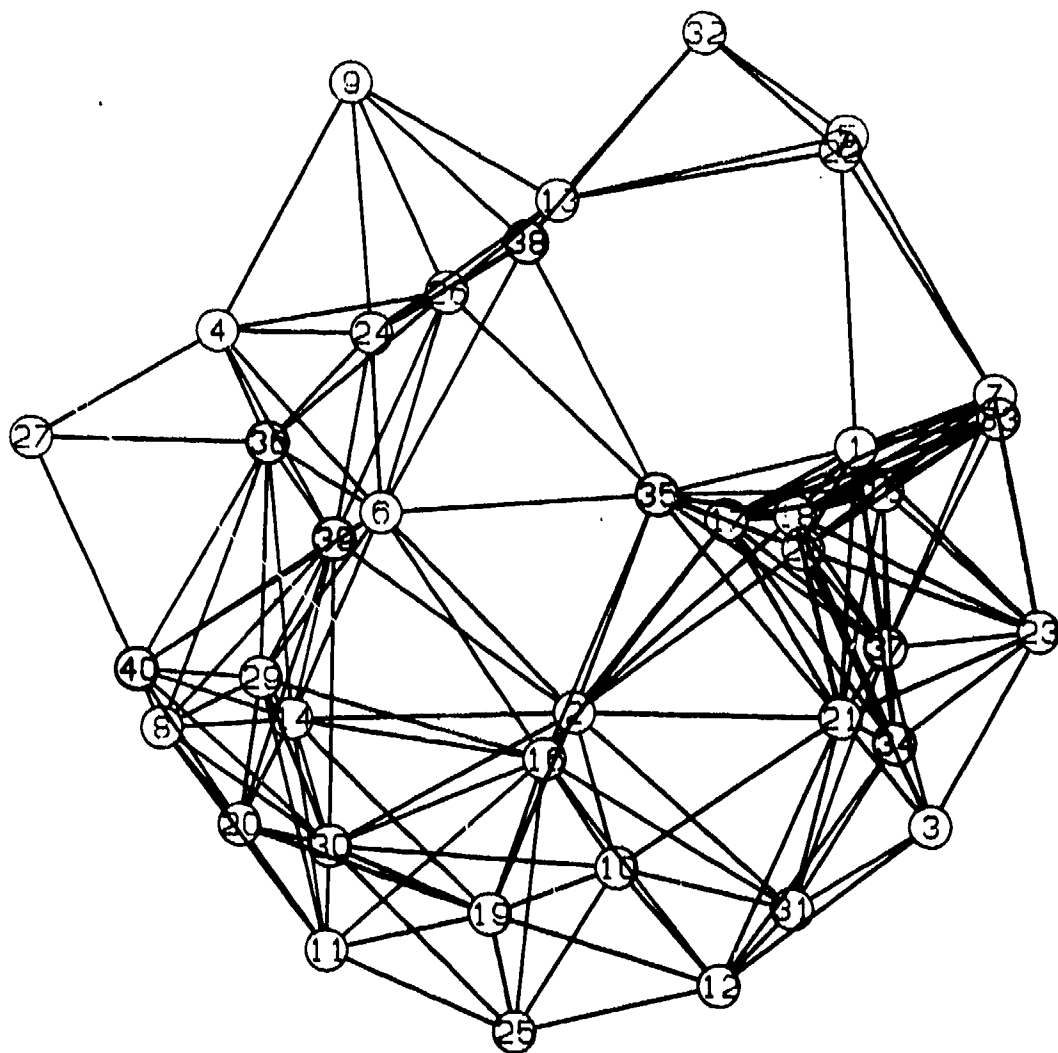Figure 7.1a 40 Node Net with Average Degree of 8
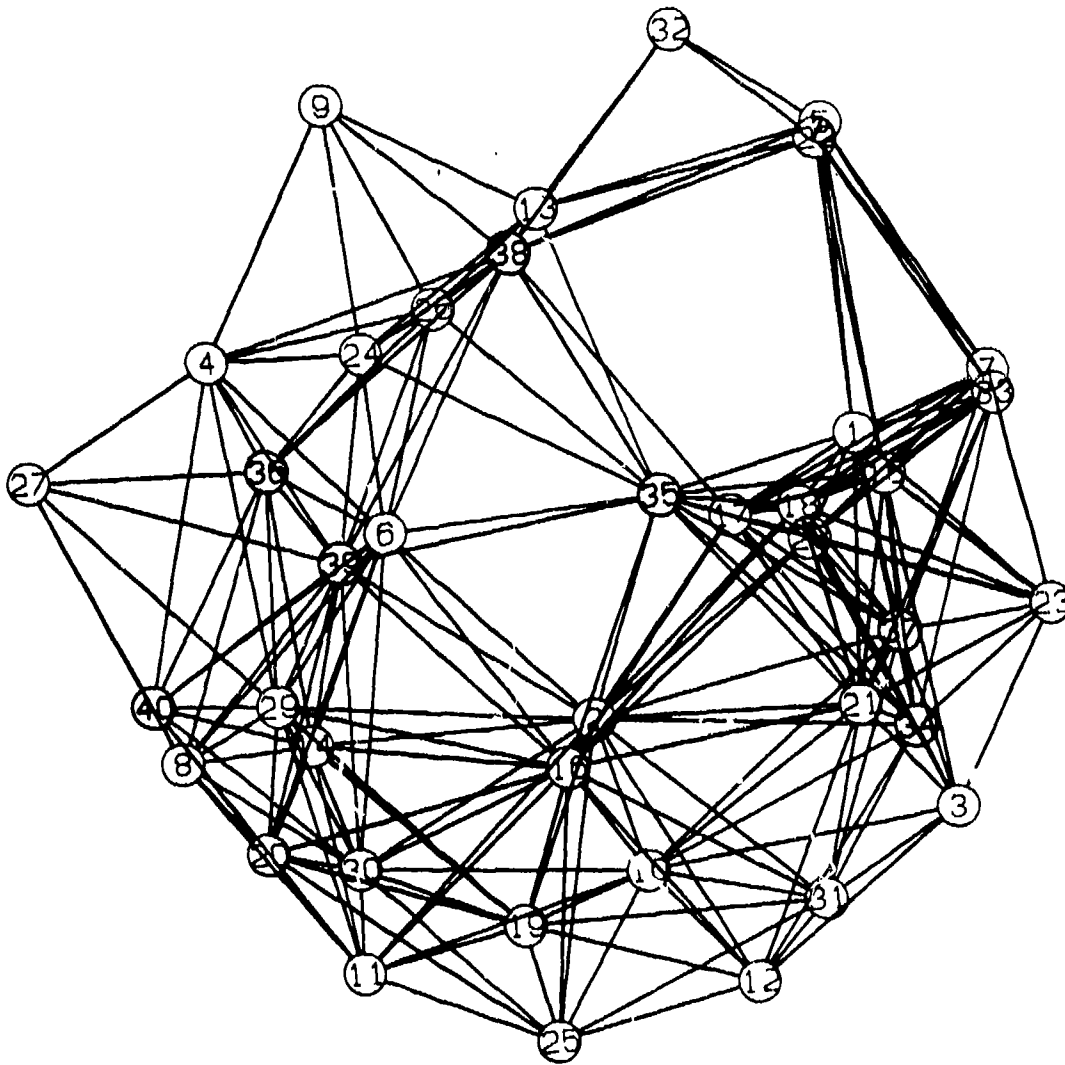
Figure 7.1b 40 Node Net with Average Degree of 12
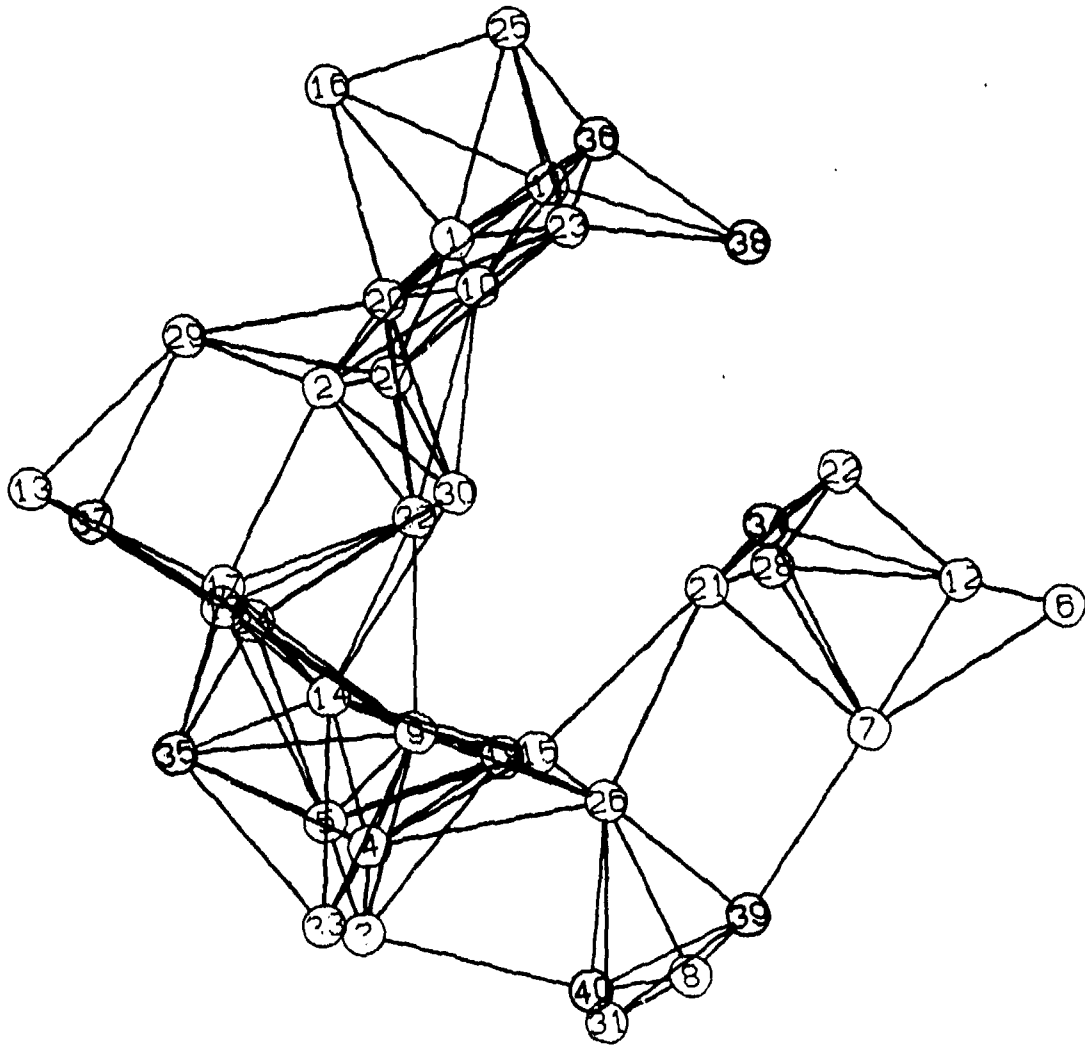
Figure 7.1c 40 Node Net with Average Degree of 16

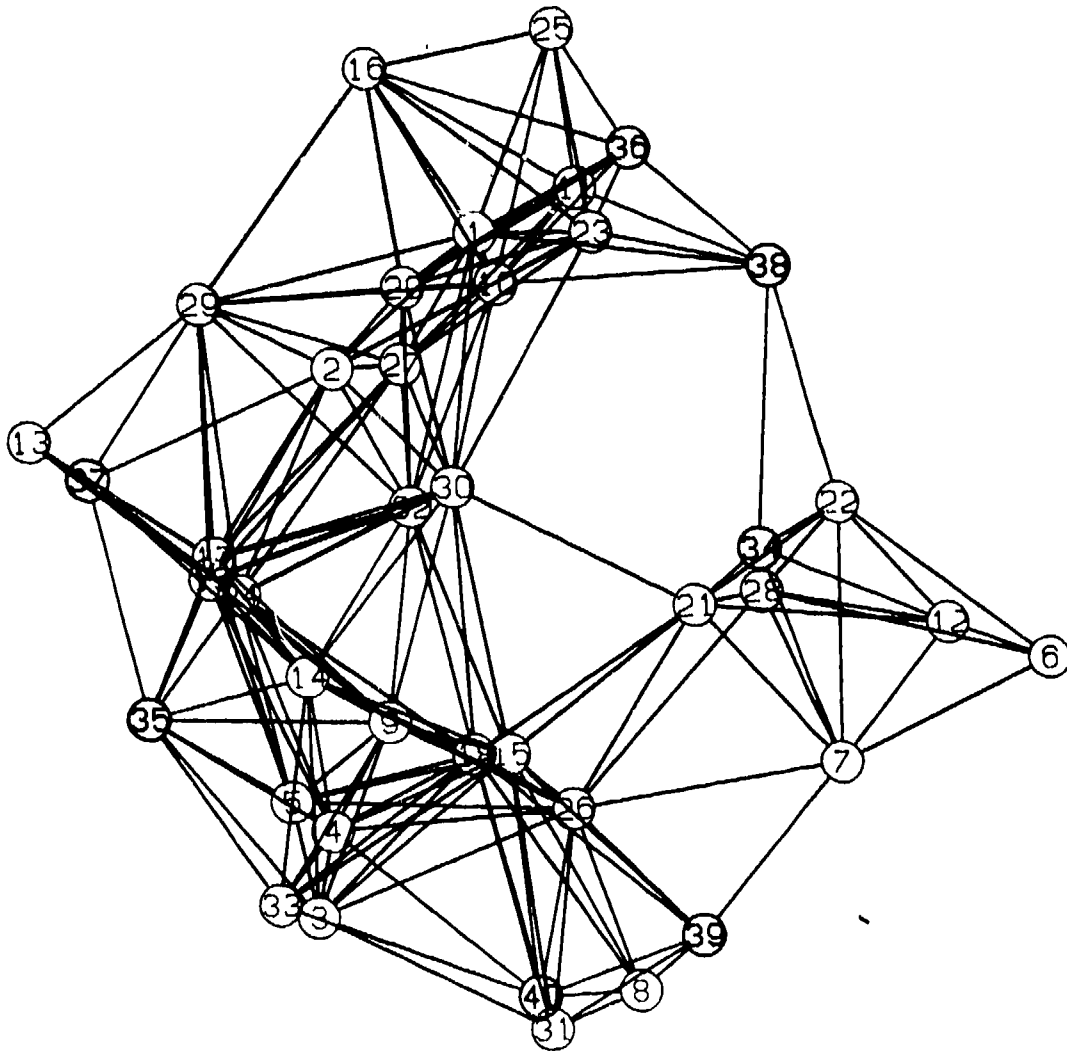Figure 7.2a Different 40 Node Net, Average Degree = 8
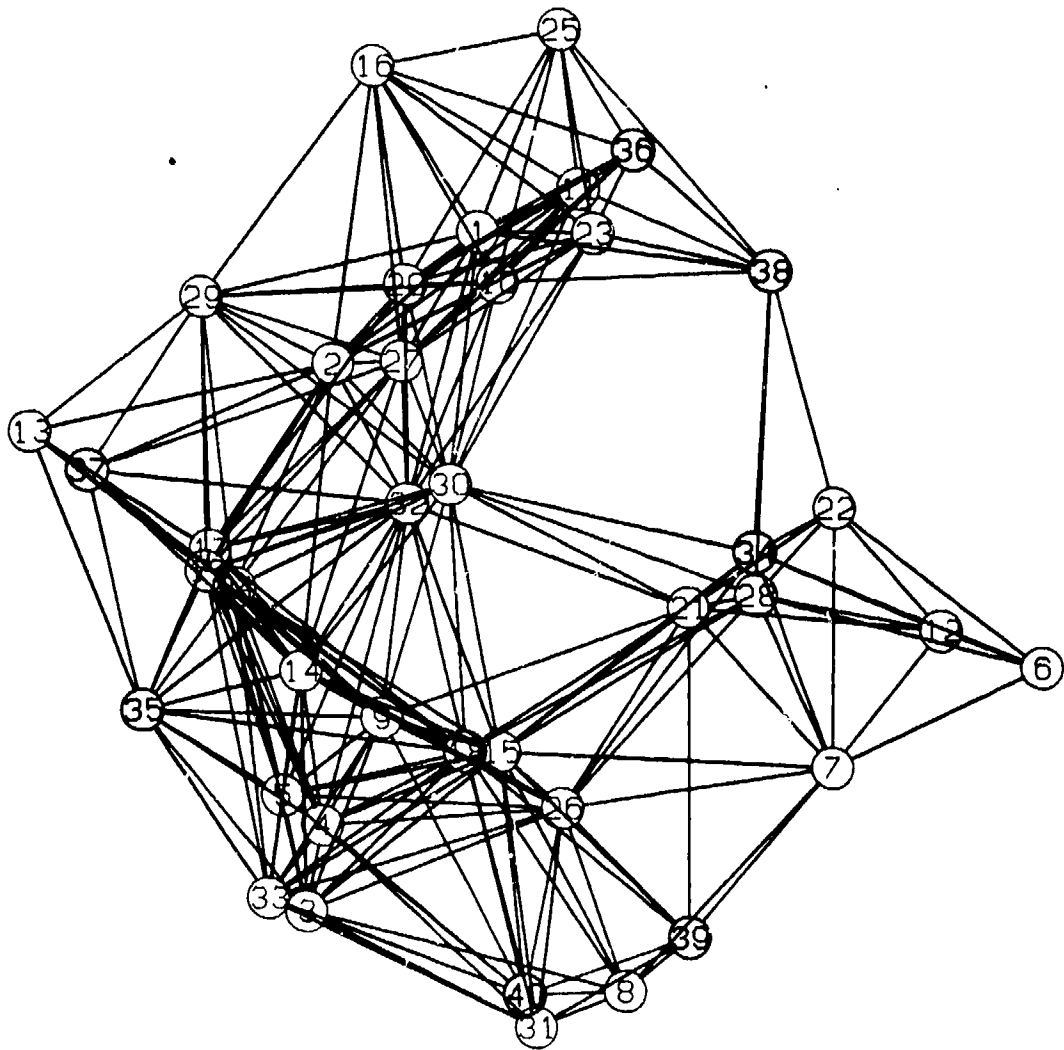
Figure 7.2b 40 Node Net, Average Degree = 12

Figure 7.2c 40 Node Net, Average Degree = 16

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Again when the average degree is increased to 12 the bottleneck disappears and the dense region becomes larger (Figure 7.2b). Figure 7.2c shows the same set of nodes with an average degree of 16. The major differences being that the dense area has increased in size.

Ir order to produce a more uniform topology we tried connecting each node to exactly so many other node .. In Figure 7.3 we show such a network for 20 nodes and an average degree of 6. It is important to realize that in this network the fact that nodes A hears node B does not imply that node B will hear node A. This has important ramifications on the acknowledgement scheme as we pointed out in Chapter 2. The resulting network is therefore *directed.* The topology is indeed more uriform than that produced by just using a fixed transmission radius, and we expect, therefore, that the performance will be higher.

7.2 Basic Results

Figure 7.4 shows simulation results for an 80 node network with various average degrees. We see that the maximum achievable throughput of the net is very low. In fact, for the range of average degrees shown we cannot even achieve a throughput of $1/e$. We do note, however, that the performance curve peaks at an average degree of about 9. We also note that it has the same shape as predicted by our analytical model. The actual performance is however very much lower than that predicted (an 80 node net should according to the model, have a throughput of about 0.8).

We believe that this degradation is due to three factors: i) the routing algorithm is not producing uniform traffic loads; ii) the retransmission probabilities do not take this non-uniformity into account; iii) the model does not account for the topological irregularities found in any *particular* random network.

In Figure 7.5 we show similar results for various different network sizes. We see that the optimum (low degree) average degree for all three nets is between 7 and 10, and that the capacity grows as a function of net size. The 25 node net exhibits unusual behavior due to edge effects being an important fraction of the net. The 40 and 80 node nets exhibit a significant reduction in throughput as the average degree is increased - until we approach full connectivity when the throughput will, of course, be $1/e$.

When the average degree is about half the network size we note that the performance is worst. This is important since we expect that many nets will operate in this range of connectivity. We see that for this range of average degrees we do not even achieve a performance of the fully connected net.

Figure 7.6 shows the performance of the exact average degree network (similar to Figure 7.3) with 80 nodes. We see that this produces throughputs which are much closer to the model's predictions. This is due to the more uniform networks that are created by this procedure.

7.3 Different Routing Algorithms

Part of the reduction in performance found above was due to the routing algorithm. The most progress algorithm will tend to select some routes frequently causing heavy traffic on those links. In order to reduce this effect we implemented an algorithm which randomly selects between the set of all shortest (hop) paths between nodes. The simulation results for this algorithm are shown in Figure 7.7. We see similar behavior to that produced by the most progress algorithm. The levels of throughput are increased however (from .15 to about .17 at an average degree of 9).
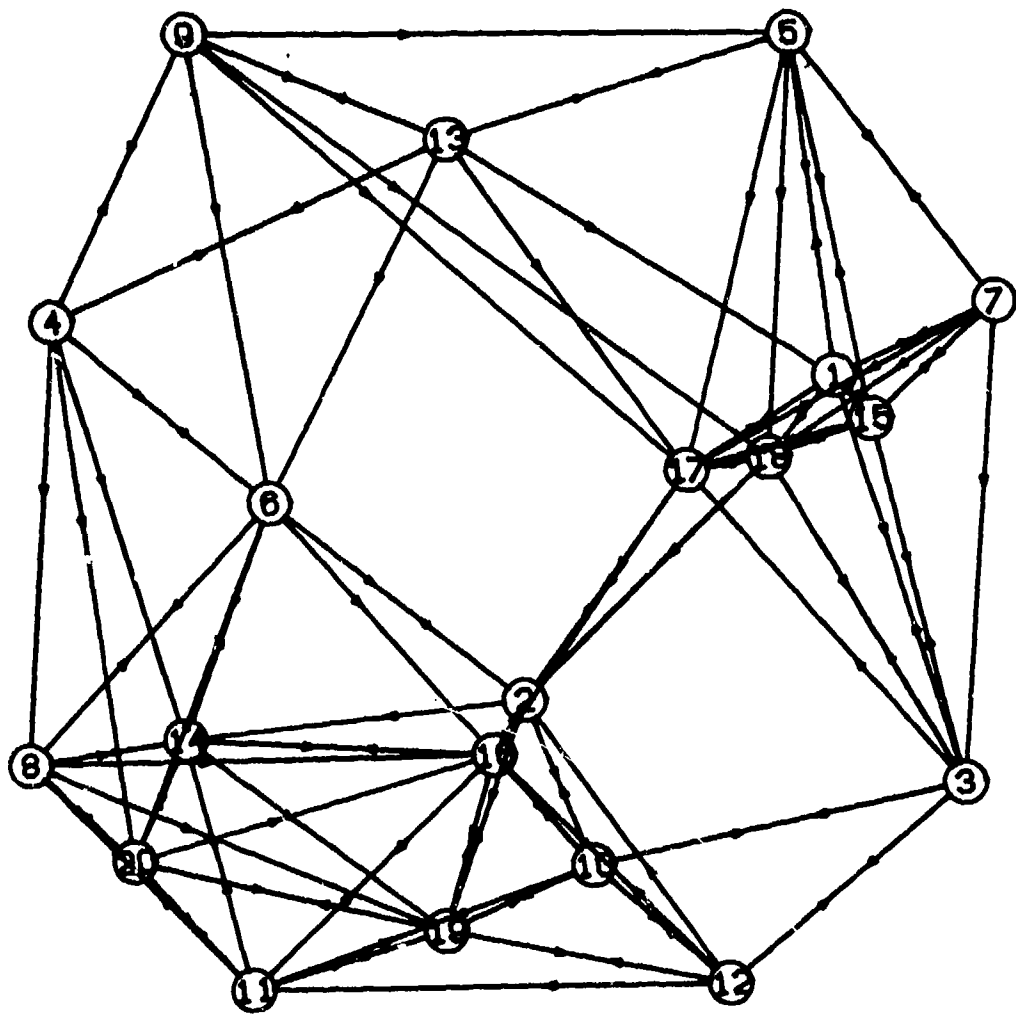
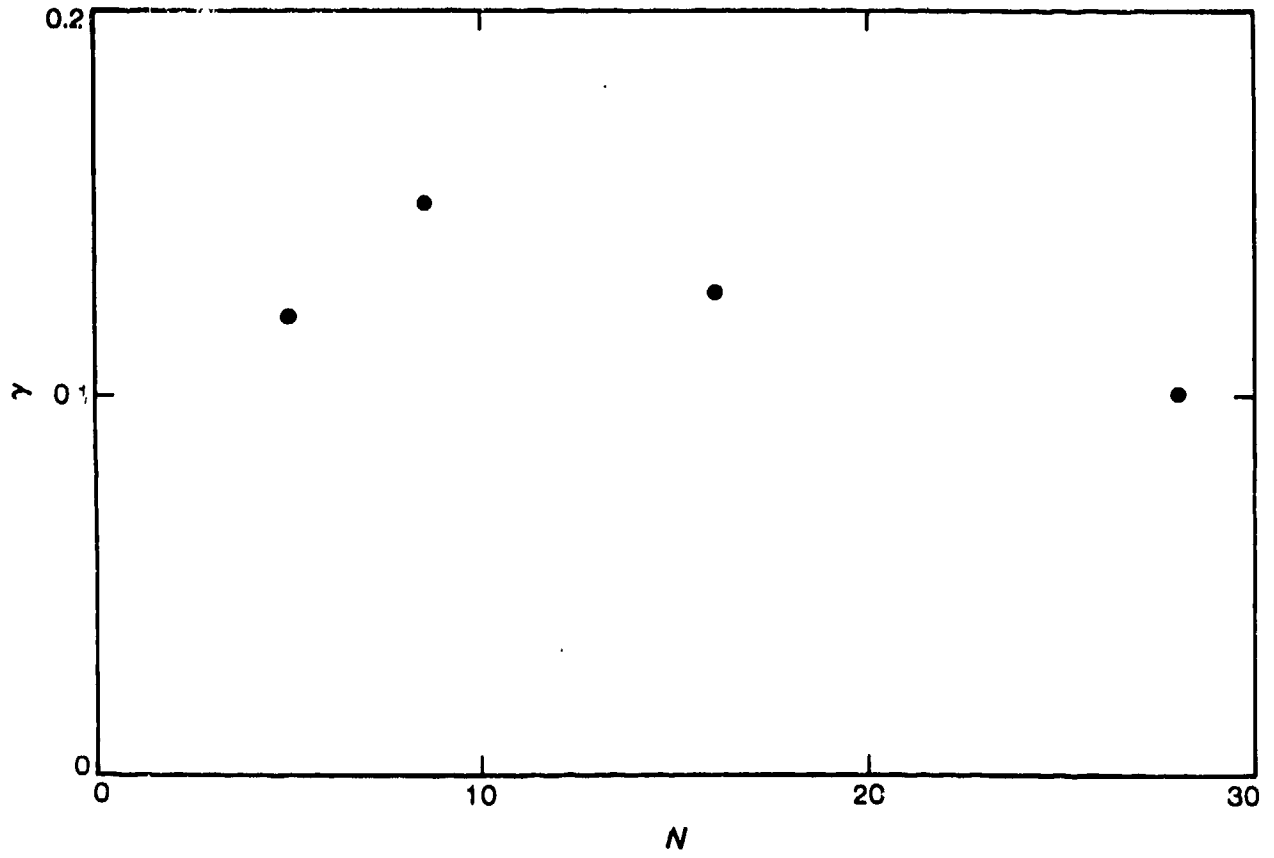Figure 7.3 20 Node Net with Exact Degree = 6

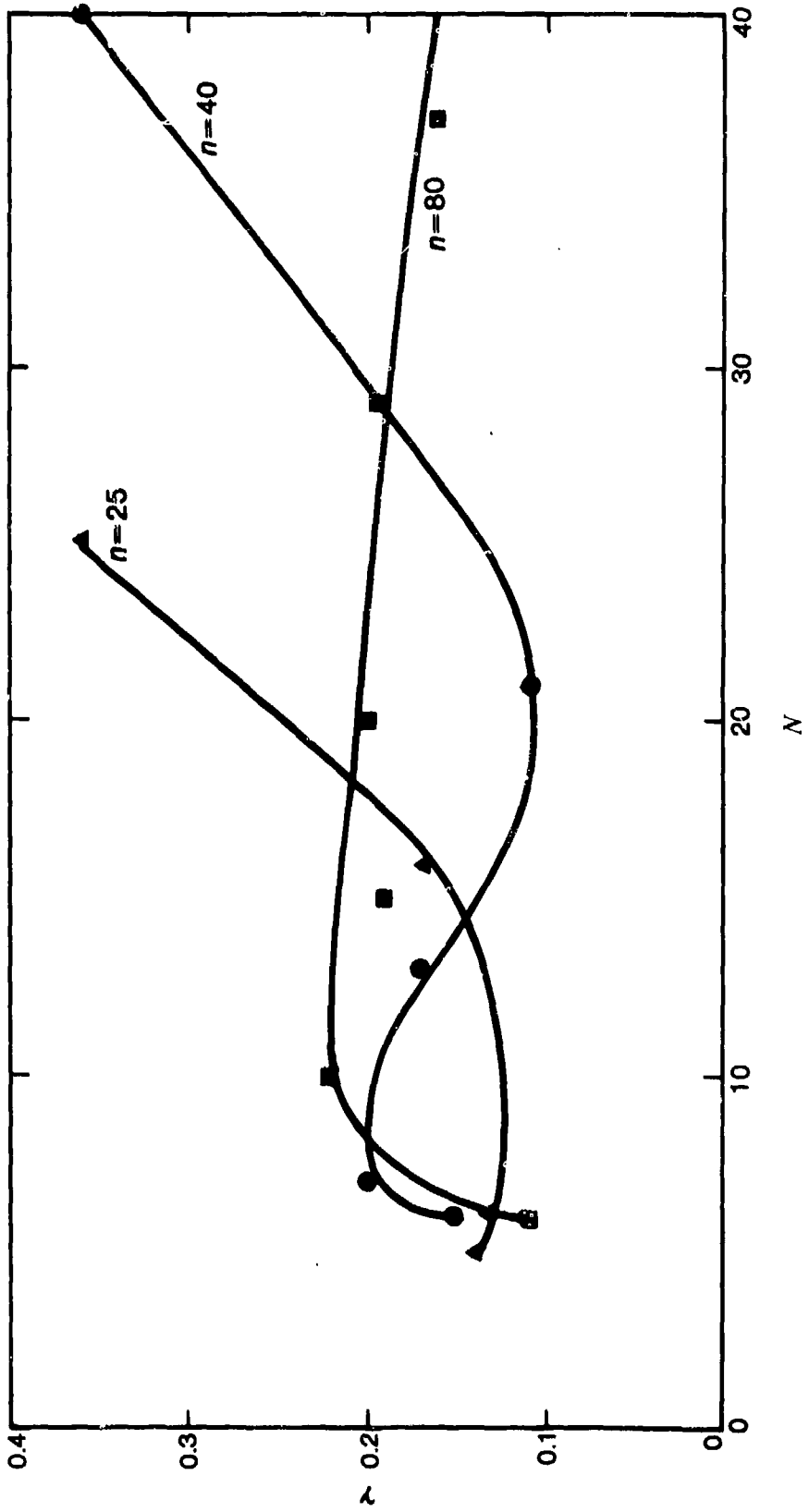Figure 7.4 80 Node Net Using Most Progress Routing

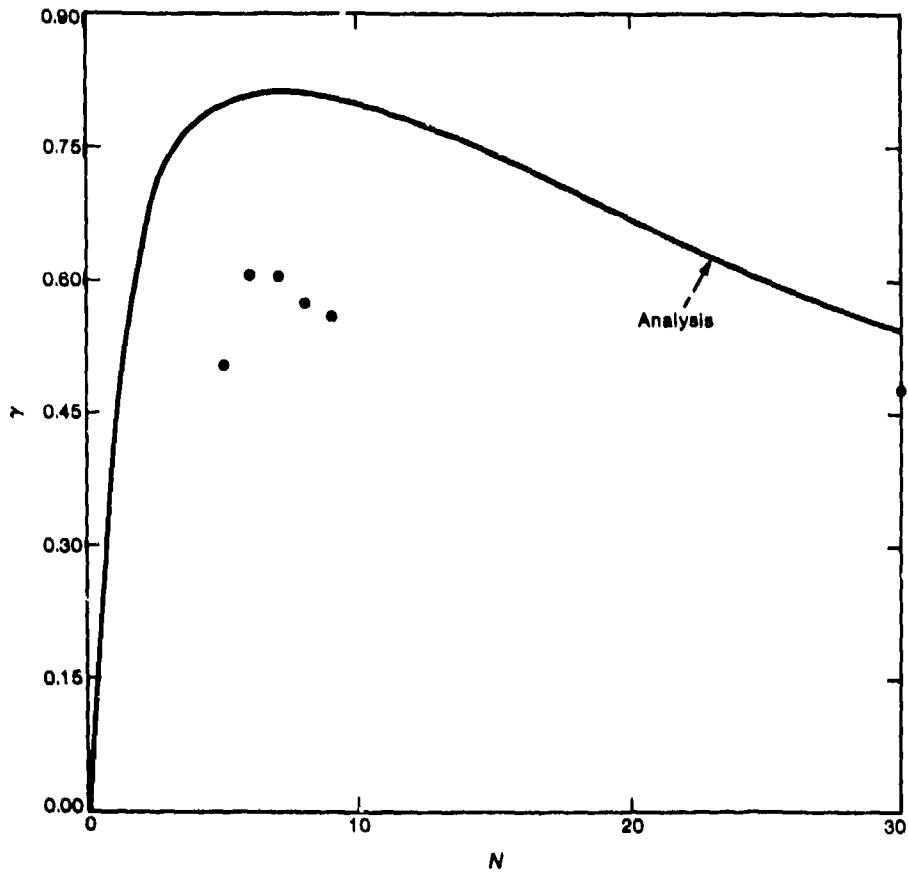Figure 7.5 Performance of 25, 40 and 80 Node Nets

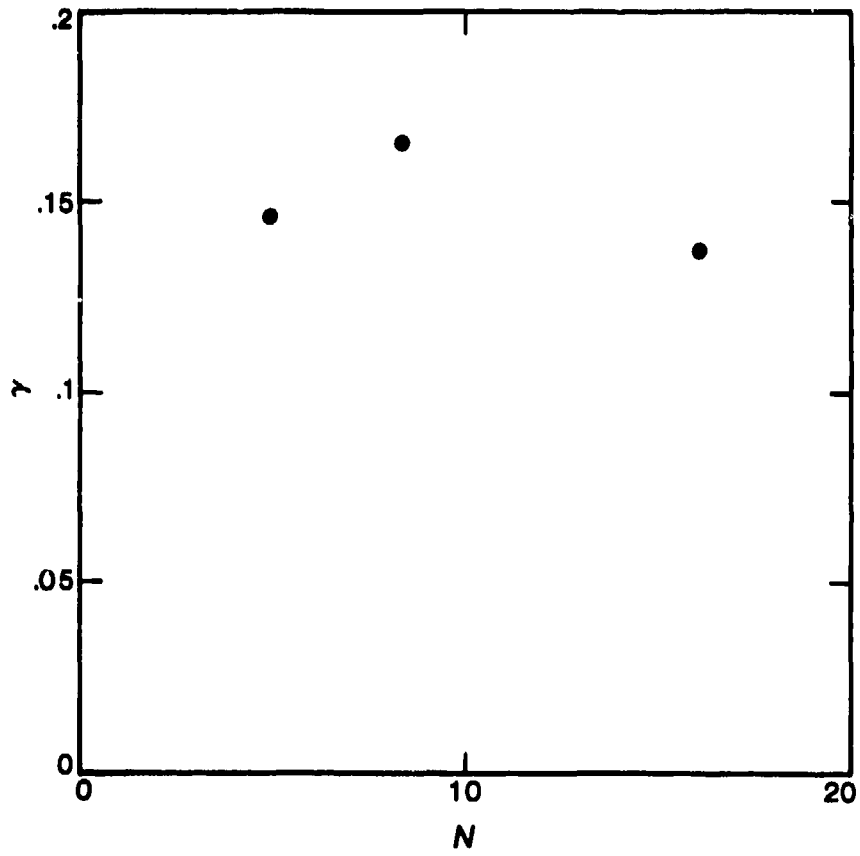Figure 7.6 Performance of the Exact Degrees Net

Figure 7.7 Random Path Selection

Even more improvement was obtained by using an algorithm that selects the (shortest path) route which has the least load. The most heavily loaded node on each of the shortest paths is determined and whichever path has the least load for the heavily loaded node is then selected as the route. We show the performance of this algorithm in Figure 7.8. We notice significant improvements over both of the other algorithms. Since this has the best performance of the algorithms that we studied we investigated it further and used it in all later experiments. It would be interesting to run the same experiments using an optimal routing algorithm - such an algorithm does not currently exist however. The reason that we cannot use the standard Flow Deviation Algorithm [GERL 73] is that adding traffic to a particular path affects all other nodes within range of nodes on this path. ·

7.4 Load Weighted Retransmission Probabilities

The other factor that tends to reduce throughput is incorrect selection of transmission probabilities. As noted in Chapter 2 we should use some optimization procedure in selecting these. Due to the complexity of this we have so far simply used the reciprocal of the average degree (which is correct for a uniform network). We use a transmission probability based on the fraction of the total load in the node's environment that is due to his traffic. That is,

$$p_i = \frac{f_i}{\sum_{j \in N_i} f_j} \tag{7.1}$$

where N_i is the set of nodes that hear i's transmission. We show the performance of this scheme in Figure 7.9, and notice additional improvement.

The traffic loads that are produced by any of the above algorithms result in some nodes being more or less busy than others. In computing the throughputs, though, we are assuming heavy traffic, i.e., all nodes are always busy. This will cause us to underestimate the throughput that can be achieved. In Figure 7.10 we plot the throughputs where the probability of a node being busy has been incorporated into the probability that he transmits in any slot. This causes an additional increase in the throughputs with the peak for this 80 node network occurring at an average degree of 9, at which point the throughput is about 0.48. The model predicts a maximum achievable throughput of about 0.8, however. The discrepancy is due to the fact that the actual topology generated by the random network is not as uniform as we assumed in the model.

7.5 Conclusions

We have presented various simulation studies that we made. We found that the performance found by simulation has the same characteristics as that predicted by the model (peaking for a small average degree, increasing with network size), but that the actual values that are achieved are significantly less than predicted. By considering flow balanced routing and more intelligent choice of transmission probabilities, and estimating the probability that a node is busy, we were able to produce results that had the same shape as the model but reduced in amplitude to about 60%.

We believe that using an optimal routing algorithm would reduce this gap, but that some of it is due to the particular random topology that is produced by the random network - a factor that is ignored by our model. We feel that the model can be used to predict performance trends but not the actual performance of any particular network.
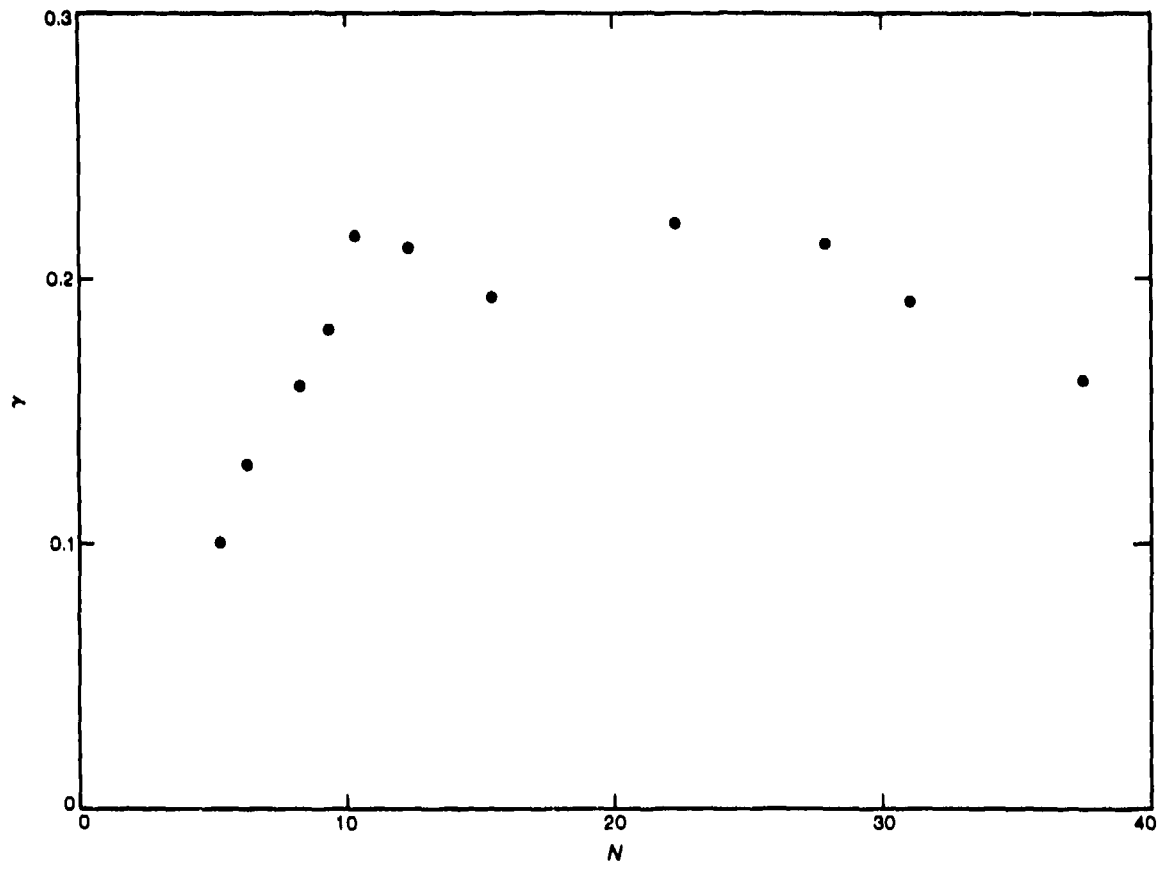
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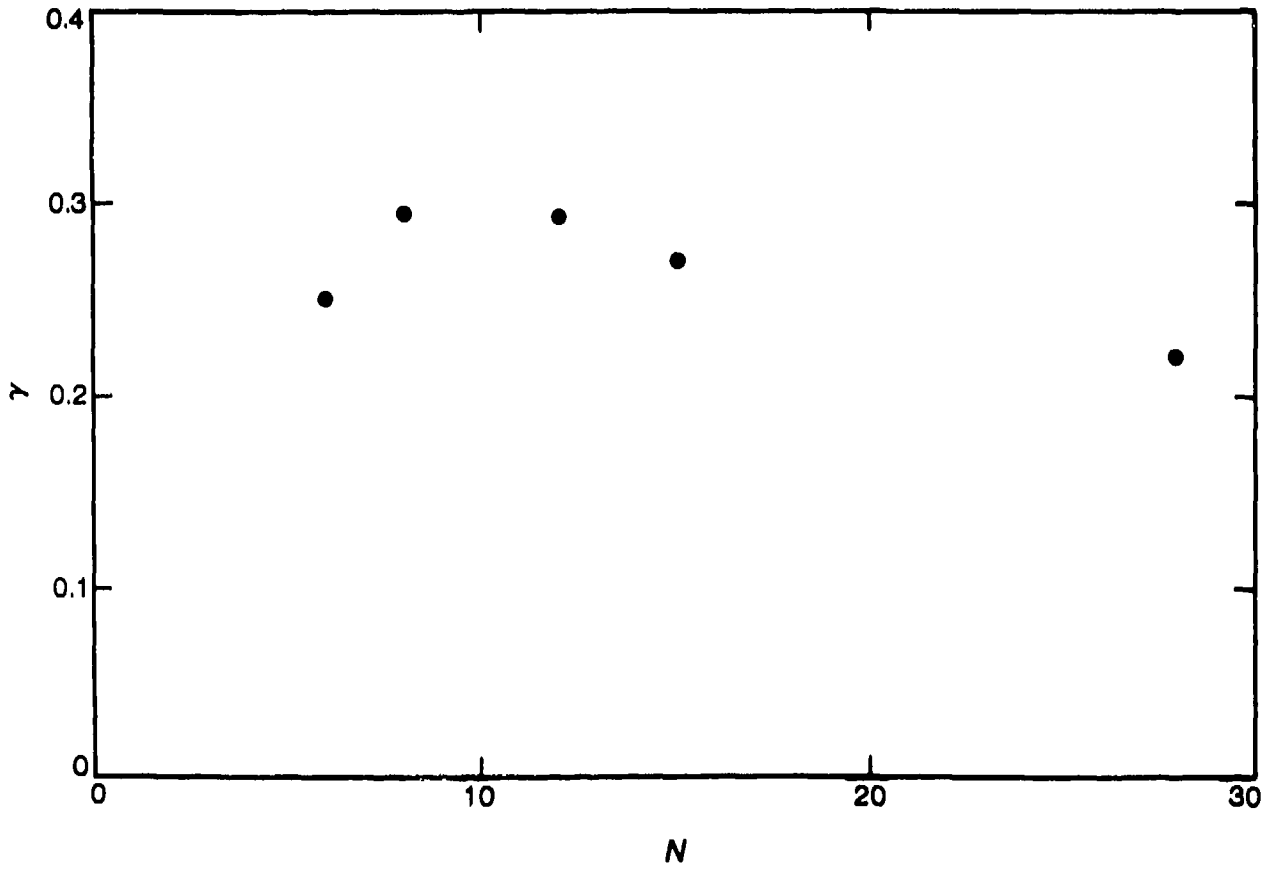Figure 7.8 Balanced Flow Routing

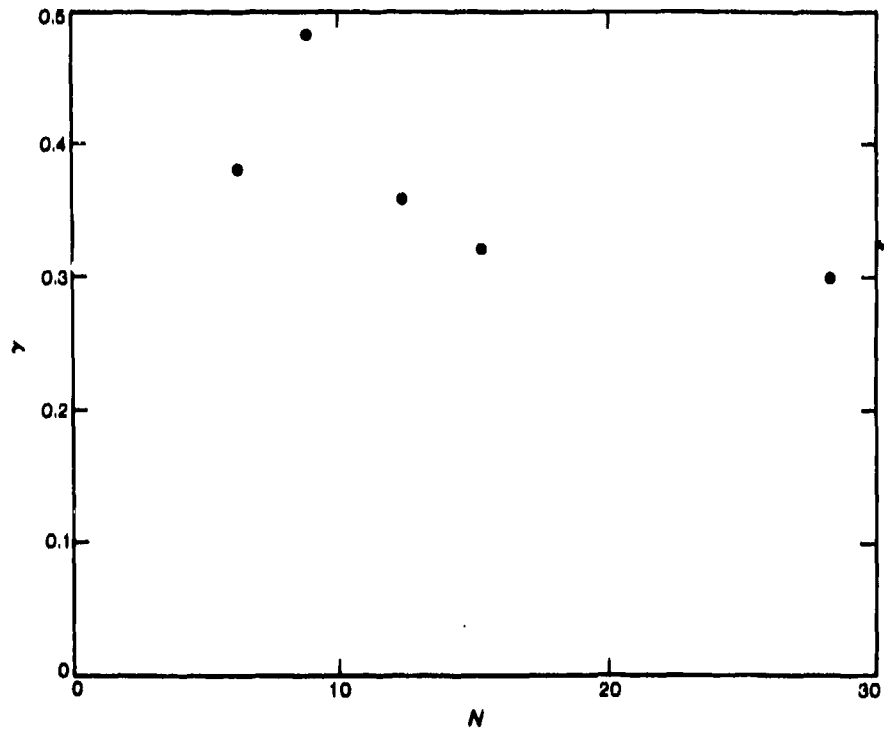Figure 7.9 Performance of 80 Node Net

Figure 7.10 Performance Incorporating Busyness Probabilities

CHAPTER 8
CONCLUSIONS

8.1 Summary of Results

In this dissertation we have mainly been concerned with determining the capacity of Packet Radio Networks operating in Slotted ALOHA mode. We have studied various different configurations and presented models for the capacity. In many cases we find that the capacity is a function of the average degree of the nodes - i.e., how many nodes are within the transmission range of a particular node. For many of the configurations studied we found the optimal transmission range that allows the highest throughput.

We start out by considering single-hop point to point networks and find that we can achieve a throughput which is logarithmically proportional to the number of nodes in the network, by restricting the range of the transmitters so that the receiver is just able to receive the message. This is a great improvement over the fixed $1/e$ capacity of the fully connected networks that have been extensively studied elsewhere.

We then attempt to find the optimum traffic matrix. We can find upper and lower bounds on its capacity which are linear with respect to the number of nodes and hence we know that the capacity of the best possible traffic matrix is also a linear function of the number of nodes. We also exhibit schemes which have linear behavior falling between the bounds. The constant of proportionality is found to be approximately $1/2e$. We give an analytical model for one of the schemes and although we do not find the $1/2e$ expression, show that the capacity is very close to this. In the course of this particular study we investigate various schemes for selecting the transmission probabilities and find that the best schemes always set $p=1/k$ where k is some measure of the traffic that is interfered with by the transmission.

For one-dimensional regular networks we find that the transmission range is unimportant. The capacity of these networks is $c/2e$ where the constant of proportionality depends on the traffic matrix. In two-dimensional regular networks, we find that we can achieve a throughput proportional to the square root of the number of nodes in the network, and that we should use the minimum degree that connects the network.

For random networks, one of the major problems turns out to be the connectivity issue. We find at first that we get a result similar to that found for regular networks (i.e., proportional to the square root of the number of nodes), but minimizing the average degree causes connectivity problems. In analyzing this we find that the optimum transmission radius to use is about 6. We also find that underestimating the average degree is disastrous, but overestimating causes only gradual loss of throughput.

When we simulate these random networks we find that the low connectivities have poor performance due to imbalance in the load on the links of the network. For this reason we find that the capacity reaches a peak at an average degree of about 10.

In all cases (except the local traffic models in chapter 4), we have considered what might be considered the 'worst' traffic matrix -i.e., the uniform or random one (in fact, the worst case is when every node wants to communicate with the most distant node - forcing full connectivity). We expect even greater benefits from restricted range if the traffic matrix has some locality, which would be expected in a real network.

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8.2 Suggestions for Future Research

Many research questions have arisen during the course of this research which are worthy of further study.

i) *Clustering and multi-level Organization*

In many real networks, we may find that the nodes are clustered into groups. Our models cannot handle these cases and indeed, it would seem that the idea of every node using fixed radii is completely wrong for these situations. Probably the best approach would be to separate the clusters and use a hierarchical strategy for communication between clusters. In is interesting to consider whether we can improve on performance for *random* networks by these techniques.

ii) *Delay*

We have not consider delay at all in this dissertation. Many of the optimization problems that we have studied can be considered from a delay viewpoint - it would be interesting to see a study of this. In [AKAV 79], Akavia studies similar networks and attempts to find minimum cost designs that satisfy certain delay constraints. His costs are a function of the bandwidth that is needed and so his results are related to ours. Some of our results (with appropriate assumptions), be shown to be equivalent to his.

iii) *Distributed Routing Algorithm*

In a real network it is desirable from reliability (and efficiency) viewpoints to have a distributed routing algorithm. This is difficult to implement for a point to point network and even more so for a mobile network. It seems that it is necessary to have some fixed nodes to give points of reference for the mobile elements of the network, or some way for the nodes to determine their geographical location.

iv) *Routing --Speed of Update vs. Topology Change*

If we have a mobile network it is necessary to send routing updates around the network. The frequency of these updates may have a significant impact on the network performance. A simple computation shows that a packet transmission time is of the order of 20ms (50K bits per second channel). The topology changes due to mobility will occur every time a device leaves the range of its nearest repeater. Assuming that repeater range is about 10 kilometers, then if the device were an aeroplane it would cross the range of a repeater in about 1 minute. If we were to restrict repeater range (as is suggested by this dissertation) to say one block (about 200 meters) the traversal time would be about 1 second (about 50 packet lengths), causing serious routing problems. A study of how to handle rapidly changing topologies would be interesting.

v) *Expected Topology of Random Networks*

An interesting side issue is to investigate the expected structure of random Euclidean networks.

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vi) *Different Traffic Matrices*

We have always considered uniform traffic matrices and we expect different results for other traffic matrices. One particular case of interest would be to determine the capacity of a band of repeaters across which all traffic is flowing.

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