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USING MULTICOMMODITY NETWORK MODELS FOR THE AIR FORCE LOGISTICS--ETC(U)
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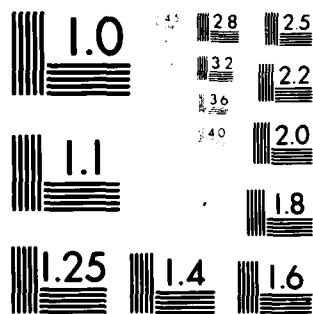
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6 USING MULTICOMMODITY NETWORK MODELS FOR THE
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A. I. / Ali
R. V. / Helgason
J. L. / Kennington

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Department of Operations Research
Southern Methodist University
Dallas, Texas 75275

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ABSTRACT

This paper reports on a successful application of mathematical programming for the Air Force Logistics Command. It presents a pair of multicommodity network flow models which represent the air freight network utilized by the Air Force to support sixty bases in the continental U. S. A. State-of-the-art software was developed to solve these models and this software is currently being used in an interactive model to aid Air Force personnel in making annual design changes in the route structure.

ACKNOWLEDGEMENT

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I. INTRODUCTION

The Air Force Logistics Command (AFLC) is responsible for the distribution system which connects the sixty Air Force Bases in the continental U. S. A. The Logistics Air Lift System (LOGAIR) is the principal air freight network utilized by AFLC to support its major installations. Civilian carriers are contracted each fiscal year to provide a fixed schedule of service to each air base in the system. These carriers make no routing or scheduling determinations; rather a system of routes and schedules is offered for bid each fiscal year. The route structure, schedule, and assignment of aircraft to routes historically have been determined via manual procedures. The total bid for the routing system for fiscal year 1980 was approximately \$50,000,000. The civilian carriers provide the aircraft, and crews; the Air Force supplies pallets and ground support equipment.

In this exposition we present a pair of optimization models which have been designed to assist Air Force personnel at Wright-Patterson AFB in the development of the LOGAIR route structure. One model is a multi-commodity network flow problem with approximately 15,000 variables and 3500 constraints. The companion model is a fixed charge multicommodity network flow problem with approximately 25 binary variables, 9000 continuous variables, and 3300 constraints. Routes generated by the first model together with routes generated manually by Air Force personnel provide the input to the integer program. This approach to routing and scheduling problems is novel in that a major component of the approximation occurs in the development of the model rather than in the development of the algorithm. Specialized software has been written for these models and computational experience is reported.

Finally, we discuss the implementation of the models and software at Wright-Patterson Air Force Base.

Although this exposition is directed toward the development of models and techniques for a particular application, we believe, that the modelling strategy and solution techniques presented are applicable to several other routing and scheduling problems.

II. SURVEY OF LITERATURE

Due to numerous applications, *routing and scheduling* problems have been extensively studied in the operations research literature. Unfortunately, simplifying assumptions are usually made to specialize a problem for a given situation (e.g. [11, 12, 20]). In school bus routing problems one is concerned with routing in a single period and with only a single destination. Problems in this class are almost always approached with a heuristic method based on a modification of the nearest unvisited city procedure developed for the traveling salesman problem (e.g. [4, 23, 27]).

Silman, Barzily, and Passy [25] present heuristic procedures for developing schedules for city buses. They propose a two phase approach for devising bus routes and schedules. Phase 1 obtains a set of potential routes while the second phase gives the frequency of travel. Their general approach is adaptable to the LOGAIR problem but their specific heuristic is specialized for only routing buses. Billheimer and Gray [8] address the general fixed-charge multicommodity network flow problem in the context of Mass Transportation Network Design. However, they assume that all arcs have infinite capacity which greatly simplifies the solution procedure.

Ferguson and Dantzig [10] present a model for assigning aircraft to routes. However, they assume the routes given and ignore all fixed charges. Bellmore, Bennington, and Lubore [5] present a model for assigning tankers to shipping routes to maximize a utility function. They view the tankers as the commodities and assume a possible loading after the tankers have been assigned to routes. Again the routes are assumed given and there are no fixed charges incurred for using a shipping lane. A similar study on the movement of train cars over a rail system was conducted by White

and Wrathall [28]. Unlike the LOGAIR problem, the network topology and schedules are input for their system. Geoffrion and Graves [13] solved a large warehouse location distribution problem for Hunt-Wesson Foods, Inc. However, their model is not applicable for the LOGAIR problem.

Denny and Brant [9], in an early paper, were the first to model the LOGAIR problem. Their model was a large linear program with GUB constraints. Agin and Cullen [1] present a model for the general vehicle routing and scheduling problem, and Richardson [24] presents a routing model for commercial airline schedule planning. Unfortunately, these models when applied to the LOGAIR problem produces a mixed integer program for which there is little hope of finding an exact solution.

III. DECOMPOSITION OF THE LOGAIR PROBLEM

The LOGAIR Distribution Problem when viewed as a single optimization problem is far beyond the state-of-the-art of mathematical programming. Consequently, we decomposed the problem into a pair of optimization models. These models will be referred to as the Route Generator Model, and the Route Selector Model. The Route Generator Model produces a set of routes which are combined with routes designed by Air Force personnel to define the set of potential routes. The Route Selector Model chooses the actual routes to be flown from the set of potential routes. Due to national security considerations and agreements with the Navy, certain routes are forced into the solution.

We now present the notation used to define a route in the context of the LOGAIR System. A network $G = [N, A]$ consists of a node set $N = \{1, \dots, N\}$ and an arc set $A = \{e_1, e_2, \dots, e_M\} \subset \{(i, j) : i, j \in N, i \neq j\}$. A finite sequence $P = \{s_1, e_{j_1}, s_2, e_{j_2}, \dots, e_{j_n}, s_{n+1}\}$ having at least one arc is defined to be a directed path in a network G if elements $s_i \in N$ are distinct and $e_{j_1} = (s_1, s_{1+1})$. A finite sequence $T = \{s_1, e_{j_1}, s_2, e_{j_2}, \dots, e_{j_n}, s_{n+1}\}$ having at least two arcs is called a circuit in G if the subsequence $\{s_1, e_{j_1}, s_2, e_{j_2}, \dots, s_{n+1}\}$ is a directed path in G , $e_{j_n} = (s_n, s_{n+1})$ and $s_1 = s_{n+1}$. For the LOGAIR problem, a route is equivalent to a circuit. Therefore, the output of the Route Generator Model is a set of circuits.

Let A denote the node-arc incidence matrix for a network and let C denote the set of arcs in some circuit in the network. Let $\underline{y}$ be any vector such that $A\underline{y} = \underline{0}$. Such a vector has been referred to as a flow by Berge and Ghouila-Houri [7]. Let

$$z_j = \begin{cases} 1, & \text{if } a_j \in C \\ 0, & \text{otherwise.} \end{cases}$$

Then the vector $\underline{z}$ is a flow and will be referred to as a vector-circuit corresponding to C .

For the LOGAIR problem, we use the Route Generator Model to obtain a vector $\underline{y}$ satisfying $A\underline{y} = \underline{0}$ and $\underline{y} \geq \underline{0}$. We then apply a simple labeling algorithm to decompose $\underline{y}$ into a set of vector-circuits and nonnegative multipliers such that $\underline{y} = \sum_{i=1}^p \alpha_i \underline{z}^i$. It may be easily shown that such a decomposition exists [7].

Most mathematical programming models which have been used to generate a circuit or a set of circuits have resorted to the use of binary variables. Typical examples include models for the travelling salesman problem and the m -travelling salesman problem. The unique feature of the Route Generator Model is that it is a specially structured linear program. More precisely, it assumes the form of a multicommodity network flow problem.

For an air freight distribution system having N bases, there are a potential of $N(N-1)$ arcs (i.e. total arcs in a complete network). Suppose $M \leq N(N-1)$ arcs are selected for consideration and let A denote the corresponding N by M node-arc incidence matrix. Let c_j for $j = 1, \dots, M$, denote the flying distance associated with each of these arcs, and let $\underline{c}$ denote the vector of distances. For each pair of bases, (i, j) , let d_{ij} denote the total quantity of parts and equipment to be shipped from base i to base j in units of pounds per day. Since the capacity of the aircraft must be shared by all goods with various origin-destination pairs, these must be distinguished in the model. The typical method for modelling problems of this type is as a multicommodity problem with the commodities associated with either the nodes of origin or destination. For our models the commodities are associated with the nodes of origin. We let the node

length vector $\underline{r}^k$ denote the requirement vector for commodity k . If $r_1^k > 0$, then node 1 is called a supply point for commodity k with supply of r_1^k . If $r_1^k < 0$, then node 1 is called a demand point for commodity k with demand of $|r_1^k|$. For the LOGAIR problem the requirement vectors are defined as follows:

$$\underline{r}_1^k = \begin{cases} -d_{k1}, & k \neq 1 \\ \sum_{\substack{j=1 \\ j \neq 1}}^N d_{1j}, & k = 1 \end{cases} \quad \begin{matrix} k = 1, \dots, N, \\ i = 1, \dots, N. \end{matrix}$$

Letting x_j^k denote the flow of commodity k in arc e_j with corresponding vector $\underline{x}^k$, the Route Generator Model may be stated as follows:

$$\left. \begin{aligned} \min \quad & \underline{c}\underline{y} \\ \text{s.t.} \quad & \underline{A}\underline{x}^k = \underline{r}^k, \underline{x}^k \geq \underline{0}, \quad k = 1, \dots, N \\ & \underline{A}\underline{y} = \underline{0}, \underline{0} \leq \underline{y} \leq \underline{u} \\ & \sum_{k=1}^N \underline{x}^k \leq \underline{y} \end{aligned} \right\} \quad (1)$$

Given an optimal solution to (1), say $(\underline{x}^1, \underline{x}^2, \dots, \underline{x}^N, \underline{y}^*)$, we decompose $\underline{y}^*$ into a set of vector-circuits, $\underline{z}^1, \dots, \underline{z}^p$, and positive multipliers $\alpha_1, \dots, \alpha_p$, such that $\underline{y}^* = \sum_{i=1}^p \alpha_i \underline{z}^i$. The vector-circuits define potential routes for the LOGAIR system. The algorithm used to obtain the vector-circuits is presented in Section IV.

Given the notation used above, we now present the Route Selector Model. Suppose there are L routes in the set of potential routes. Let the set $R_\ell = \{e_{j_1}, e_{j_2}, \dots, e_{j_q}\}$ denote the arcs in route ℓ . Let the arc set be given by $\hat{A} = \bigcup_{\ell=1, \dots, L} R_\ell$. Then the network used in the Route

Selector Model is $[N, \hat{A}]$ where $N = \{1, \dots, N\}$. Let $\hat{A}$ denote the node-arc incidence matrix associated with $[N, \hat{A}]$.

Table 1 About Here

There are two major types of aircraft which are currently being used on these routes, the Lockheed L100 and L188. The L100's are the larger of the two and are used primarily on routes which service the six largest bases; Hill, Kelly, McCellan, Robins, Tinker, and Wright-Patterson. The capacities and cost characteristics which have been determined by AFLC are as shown in Table 1. From this data, costs for flying a route have been partitioned into the fixed and variable components, where the variable component includes only fuel cost. After assigning aircraft types to the L routes, we obtain the fixed charge (see Table 1 G.), the variable cost (see Table 1 H.), and the route capacity (see Table 1 E.). Letting f_ℓ and b_ℓ denote the fixed charge and aircraft capacity for route ℓ , respectively; the Route Selector Model is given by

$$\begin{aligned}
 \min \quad & \sum_{k=1}^N \underline{c}x^k + \sum_{\ell=1}^L f_\ell y_\ell \\
 \text{s.t.} \quad & \hat{A}\underline{x}^k = \underline{r}^k, \quad k = 1, \dots, N \\
 & \sum_{k=1}^N x_j^k \leq b_\ell, \quad \text{for all } e_j \in R_\ell \text{ and } \ell = 1, \dots, L \\
 & \sum_{e_j \in R_\ell} \sum_{k=1}^N x_j^k \leq M^* y_\ell, \quad \ell = 1, \dots, L \\
 & y_\ell \in \{0, 1\}, \quad \ell = 1, \dots, L \\
 & \underline{x}^k \geq 0, \quad k = 1, \dots, N,
 \end{aligned} \tag{2}$$

where M^* is a large positive number. The above model is a multicommodity fixed charge network flow problem. Solution of (2) provides a set of optimum routes from the set of L potential routes which if flown daily will guarantee that the daily demand is met subject to aircraft capacity constraints. The underlying assumptions associated with this model are as follows:

- (i) All cargo has the same priority.
- (ii) Loading and unloading costs have been ignored.
- (iii) Cargo volume restrictions have been ignored.
(However, these can be incorporated into the model at the expense of increasing the number of constraints).
- (iv) Circuitous routing is allowed to meet the demand constraints.

Since the model is to be used as a planning tool, the Air Force personnel involved feel that the above assumptions are reasonable for a planning tool of this type.

IV. VECTOR-CIRCUIT GENERATOR

In this section we consider the problem: Given a flow $\underline{y} \geq \underline{0}$ on a network $G = [N, A]$, determine a set of vector-circuits, say $\{\underline{z}^1, \underline{z}^2, \dots, \underline{z}^p\}$, and a set of nonnegative scalars $\{\alpha_1, \alpha_2, \dots, \alpha_p\}$ such that

$$\underline{y} = \sum_{i=1}^p \alpha_i \underline{z}^i.$$

This problem is related to the problem of finding the fundamental basis of cycles on an undirected graph and that of finding the basis of circuits on a directed graph. The former has been studied extensively in the literature (see [2, 6, 21, 26]). The latter has been studied by Murchland [22].

Since the dimension of the null space $\{\underline{y} : A\underline{y} = \underline{0}, \underline{y} \geq \underline{0}\}$ is $M - N + 1$ it follows directly that an upper bound on p is $M - N + 1$. The problem of obtaining a decomposition of $\underline{y}$ in which p is minimized may be expressed as a fixed charge problem. For our purposes, a much simpler algorithm which successively locates circuits with maximal flow on each suffices.

Our algorithm begins with the network G and deletes all arcs having flow equal 0. Next all nodes which have no incident arcs are also deleted. An iteration of the algorithm consists of selecting an arc $e_m = (i, j)$ such that

$$y_m = \max_i y_i$$

and then finding a directed path, P , of maximum flow from node j to node i . The path P together with the arc e_m forms a circuit. The smallest flow on any arc in the circuit is called the blocking flow and determines the capacity of the circuit. This circuit is retained as one of the routes, and the blocking flow is subtracted from each arc in the circuit. All arcs whose flows are reduced to zero are deleted and any nodes which become

isolated are deleted. This completes one iteration of the procedure. Since each iteration reduces the size of the graph by at least one arc, the algorithm is finite. An example is illustrated in Figure 1.

FIGURE 1
ABOUT HERE

V. COMPUTATIONAL EXPERIENCE

The primal partitioning code for solving multicommodity network flow problems reported in [3] has been specialized for the Route Generator Model (see [17] for a complete description of the primal partitioning algorithm). This system carries the inverse of the working basis in product form using the technique described in [15]. The reinversion routine is based on the work of Hellerman and Rarick [19] and uses the spike swapping procedure described in [16]. The route selector algorithm (Section 4), has also been coded. Both codes are written in standard FORTRAN and have been run on a CDC Cyber 73.

The Civil Aeronautics Board provided the distance matrix for the 60 Air Force Bases in the continental U. S. A. and the Air Force Logistics Command provided the point-to-point demands (d_{ij}) for the fiscal years 1979 and 1980. From this data two test problems for each year were generated. The two test problems differ only in the number of arcs used to define the network used in the Route Generator Model.

The termination criterion used for problems 2 and 4 was to check the objective function every 1000 iterations and terminate if the objective function value became less than $.5E+9$. This number was selected arbitrarily, though keeping in mind that the routes generated by AFLC personnel for 1980 yielded a cost of $.2225E+11$ pound-miles. The two smaller problems, problems 1 and 3, were solved to optimality. Table 2 summarizes relevant information obtained in solving these problems. Note that the vector-circuit generator takes only a few seconds while the multicommodity code requires more than 20 minutes to obtain an acceptable solution.

TABLE 2.
ABOUT HERE

We also designed and implemented a large-scale FORTRAN computer code to be used in obtaining the solution of the Route Selector Model. The code employs a branch-and-bound scheme with separation and candidate selection guided by heuristic rules. The free integer variable furthest from an integral value is chosen for separation. The candidate subproblems most recently created are chosen first with preference given to those whose separation variable was fixed at 1 when created. The branch-and-bound tree was kept on disk in groups of 16 nodes, making use of the CDC mass storage input/output subroutines. The system was designed to allow the user to terminate a run with the current branch-and-bound tree and later restart with that tree.

It is shown in [18] that the continuous relaxation of a candidate subproblem can be formulated as a minimum cost multicommodity network flow problem. Thus we make use of a specialization of the primal partitioning code of [3] for efficient solution of the relaxed candidate subproblems. The route selector system was tested on the 1980 data using 17 routes supplied by AFLC personnel and 8 routes supplied by the Route Generator Model. This yielded a fixed charge multicommodity model having 25 binary variables, 9349 continuous variables, and 3355 constraints.

Beginning with an initial feasible solution supplied by AFLC personnel, the system was used to generate a branch-and-bound tree having 1023 nodes. This required 46 restarts and took approximately 23 hours of computer time over a 3 week period. At the termination of the run, there were 15 nodes remaining in the candidate list. Only one new incumbent was developed during the computation but the estimated cost savings of this incumbent was approximately \$800,000. The new route structure involved the substitution of one of the 8 routes generated by the Route Generator Model for one of the original 17 supplied by the Air Force.

VI. IMPLEMENTATION

The two models and specialized software systems described above evolved over the period 1976 - 1980. All code development was done at Southern Methodist University by the authors for the Directorate of Transportation (LOT) located at Wright-Patterson Air Force Base in Dayton, Ohio. LOT personnel had many years of experience with the LOGAIR system, but they had little background in mathematical analysis and no background in either mathematical or computer programming. Even though the client was a naive user of optimization models, the problem was ideally suited for operations research analysis. The important characteristics which made this study feasible are as follows:

- (i) The problem was well-defined.
- (ii) It was a planning (as opposed to an operational) problem in which the plan was reevaluated annually.
- (iii) The problem involved a large cash outlay, \$50,000,000. Hence a 1% savings was very significant.
- (iv) Most of the data was already being collected and stored on magnetic tape. There was essentially no new data which had to be collected by the client.
- (v) The client had been attempting to solve the problem manually and had an appreciation for the complexity of the problem.

Rather than implement both models simultaneously, we chose to install the system in three phases. The first phase involves the Route Selector Model in which all binary variables are fixed by the user. The user

selects the routes and the system loads the routes to optimally satisfy the demand. An elaborate report generator was attached to this system to provide the client with detailed information about flow in the system. In particular, legs of routes running at 100% capacity and underutilized legs are highlighted. This system has been implemented at Wright-Patterson and was used to help develop the LOGAIR route structure for fiscal year 1981. The second phase involved using the Route Selector Model in a partially interactive branch-and-bound mode. This involves some decision making on the part of the client and has been delayed until we gain more experience with the phase 1 model. The final phase involves coordinating the Route Generator Model with the Route Selector Model.

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TABLE 1. AIRCRAFT CHARACTERISTICS (1980 DATA)

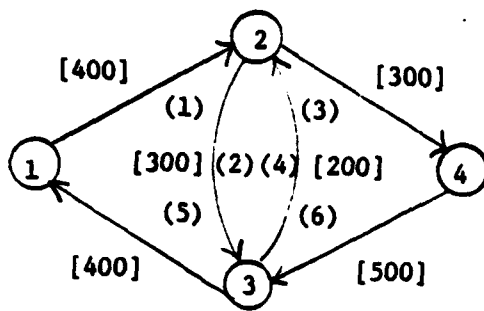
Aircraft Characteristics	Aircraft	
	L100	L188
A. Transportation Cost (\$/mile)	3.5260	2.4128
B. Fuel Consumption Cost (\$/mile)	0.9936	1.0304
C. Empty Weight (lbs)	74,746	56,013
D. Full Weight (lbs)	120,746	86,538
E. Usable Cargo Capacity (lbs)	43,160	28,640
F. Total Cargo Capacity (Cargo plus pallets) (lbs)	46,000	30,525
G. Variable Component of Cost (\$/mile) $\frac{F - B}{E - D}$	0.87703×10^{-5}	1.2691×10^{-5}
H. Fixed Component of Cost (\$/mile) $A - \frac{F - B}{D}$	3.1475	2.0493

Table 2. COMPUTATIONAL EXPERIENCE WITH ROUTE GENERATOR MODEL

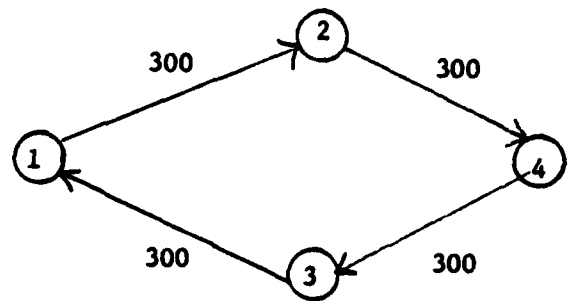
Row Description	Problem Number			
	1	2	3	4
PROBLEM CHARACTERISTICS				
Nodes	60	60	56	56
Arcs	249	313	237	305
Commodities	61	61	57	57
LP rows	3967	4031	3483	3551
LP columns	15009	18913	14277	18425
Linking Constraints	246	310	234	302
Data Source Year	1979	1979	1980	1980
SOLUTION STATISTICS				
Objective function value x E + 9	.36235	.41256	.15632	.44889
Termination Criterion *	optimal	conditional	optimal	conditional
Time in CP minutes	21	29	19	26
Iterations	11,605	13,000	11,349	12,000
Reinversions	77	87	76	76
Time for vector-circuit selector in seconds	6.3	8.1	6.2	7.8

* Conditional termination when obj value < 5E + 9.

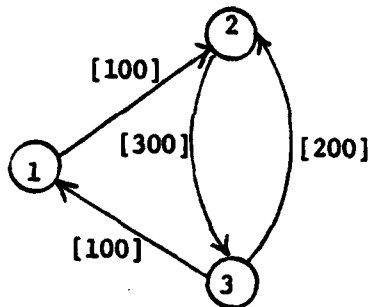
[flow]
(arc number)



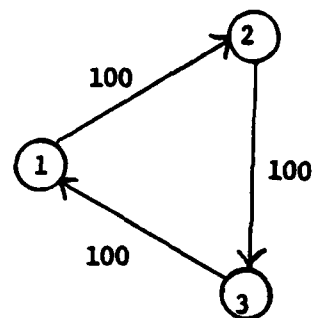
(a) Original Network
 $\underline{y} = [400 \ 300 \ 300 \ 200 \ 400 \ 300]$



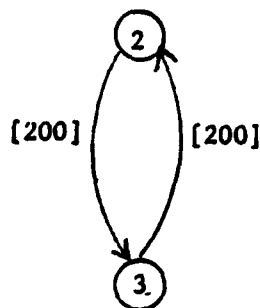
(b) Route 1
 $\alpha_1 \underline{z}^1 = 100 [1 \ 0 \ 1 \ 0 \ 1 \ 1]$



(c) Reduced Network



(d) Route 2
 $\alpha_2 \underline{z}^2 = 100 [1 \ 1 \ 0 \ 0 \ 1 \ 0]$



(e) Reduced Network and Route 3

Figure 1. Sample Flow Network

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This paper reports on a successful application of mathematical program- ming for the Air Force Logistics Command. It presents a pair of multicommodity network flow models which represent the air freight network utilized by the Air Force to support sixty bases in the continental U. S. A. State-of-the-art software was developed to solve these models and this software is currently being used in an interactive model to aid Air Force personnel in making annual design changes in the route structure.		

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