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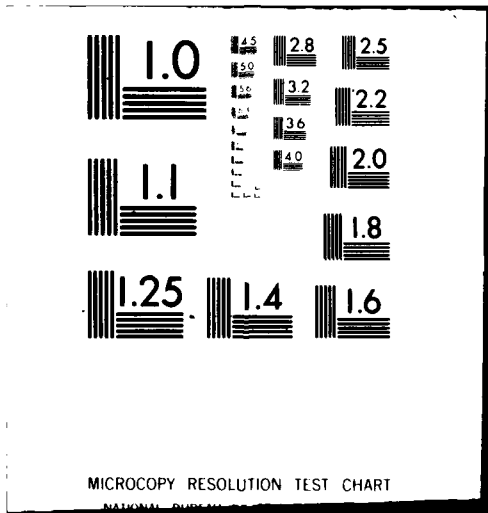
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| This project has investigated the costs imposed by the need for communication between user and machine and between components of the machine in solving data-processing problems. There have been three principal subtasks. The first, exploration of the minimal costs of storing and accessing information in simple data structures, is the oldest and results have been reported in several publications. The second is the design of minimax optimal universal codeword sets which can be used to represent any message set efficiently by assigning messages | | | |

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20. Abstract (continued from previous page)

in order of decreasing probability to codewords in order of increasing length. A manuscript giving results of this research is almost ready for publication. The third is the exploration of switching networks which can be used for communication between a number of processors engaged in a common computational task: that work is still in an early stage and is not yet ready for publication.

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BODY

When computers carry out interactive data processing tasks, communication is necessarily involved. Commands must be communicated from the user to the processor, results must be communicated from the processor to the user, and in the course of carrying out the computation there must be communication between the processor and the memory hierarchy. If multiple processors are involved in carrying out portions of a common task then they must communicate with one another. The basic objective of the research carried out in this project has been to explore the contribution of these communications requirements to the difficulty of carrying out data-processing tasks.

The storage and retrieval of information presents a class of problems for which such an investigation is especially appropriate, since the costs of a storage and retrieval problem are largely the costs of communication between processor and memory and the costs of storing information in memory. Information theory provides results and techniques which are appropriate for investigating such problems, and the principal published work resulting from this research has been an analysis of the trading relations between storage and access costs for best-possible representations of simple data structures. A review paper by Elias [1], reporting work performed under a previous ARO contract, gives results in special cases. The doctoral thesis of Donna Brown, issued in [2] as a report of the Laboratory for Computer Science, explores trading relations for linear data structures- lists, stacks and queues- in detail. Those and related results have been published by Brown in [3] and [4], with acknowledgement of ARO support under this contract, and another publication by Brown is in preparation. So is a publication by Elias, which generalizes the results in [1].

One obvious application of information theory to storage and retrieval problems concerns the choice of representation for data values. An entry in a data base which is selected from a fixed set can be assigned a codeword which is a sequence of symbols, and if the set of possible values is large and values occur with different frequencies then Huffman encoding can be used to reduce the average amount of storage space required by using the shortest codewords for the most frequent values. A difficulty with this approach is that in data processing the exact knowledge of frequencies of different values which is assumed in information theory and is available for English text may not be available for a given application. A second difficulty is that a different codebook must be consulted in looking up each value selected from a different set. Both of these problems are alleviated by using a universal codeword set, as introduced in [5]. A universal set of codewords is defined to have the property that if messages in a message set are assigned in order of decreasing probability to codewords in order of increasing length, the ratio of the

resulting average codeword length to the length of a best possible (Huffman) encoding of that source is bounded by a constant for all message sets whose entropy is neither zero nor infinite. In research reported in [5] and supported by a predecessor ARO contract I constructed infinite universal codeword sets and showed that the ratio of the average codeword length for such a set to the average length of a Huffman code was bounded by 3 for the worst possible probability distribution. However that bound was not shown to be the actual value of the ratio in the worst case (which turns out to be 2, not 3) nor was that set of codewords shown to minimize the maximum of that ratio over all message sets. More recent research by Rissanen [6] and by Davisson and Leon-Garcia [7] used different measures of performance than that ratio, and restricted their attention to finite rather than infinite codeword sets, but were able to find codeword sets which were minimax optimal by their measures on the sets of message probability distributions they considered. In recent work [8] now being prepared for publication I have found fast algorithms for the design of minimax optimal universal codeword sets by the ratio cost measure: the average codeword length for such a code is at most $253/160 \sim 1.58$ times the average codeword length for a Huffman code for the worst message distribution.

One other topic has also been explored, by a graduate student Andrew Boughton who has been supported in part under this contract. Boughton has investigated the communications problems involved in using a large number of processors in parallel to solve a single problem. The obvious way of making it possible to connect the output of one processor to the input to any other is a crossbar, which takes a number of elements growing like the square of n to connect n processors, which becomes expensive for large n . Other connection network techniques are informationally more efficient and require only $n \log n$ elements to connect n devices. However such networks may require long wires, and thus both significantly greater delay and significantly greater implementation cost when built as integrated circuits, in which wire is as expensive as devices. This work is also not yet ready for publication, but Boughton's doctoral thesis proposal will be complete shortly.

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