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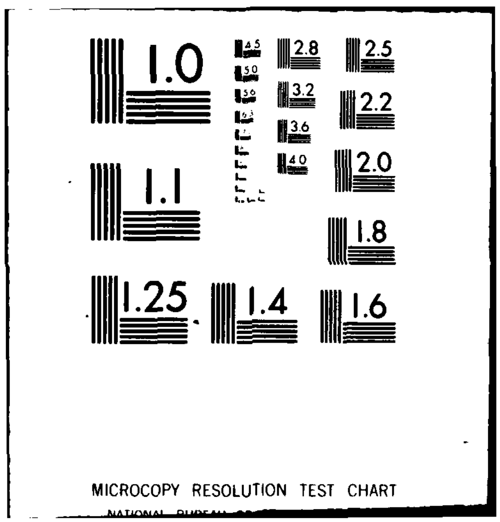
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INCOMPLETE BLOCK DESIGNS FOR COMPARING TREATMENTS WITH A CONTROL (V): OPTIMAL DESIGNS FOR p = 6, k = 3

by

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ABSTRACT

The present paper continues the study of balanced treatment incomplete block (BTIB) designs initiated in [1]-[4]. This class of designs was proposed for the problem of comparing simultaneously $p \geq 2$ test treatments with a control treatment when the observations are taken in blocks of common size $k < p+1$. The conjectured minimal complete class of generator designs, a catalog of admissible designs, and tables of optimal designs are given for $p = 6$, $k = 3$. The efficiency of the optimal BTIB design relative to that of replications of a BIB design is computed for situations in which both provide the same probability guarantee for the multiple comparisons with a control problem.

Key words and phrases: Multiple comparisons with a control, balanced treatment incomplete block (BTIB) designs, BIB designs, admissible designs, S-inadmissible designs, C-inadmissible designs, minimal complete class of generator designs, optimal designs.

1. INTRODUCTION

The present paper continues the study of balanced treatment incomplete block (BTIB) designs discussed in [1]-[4]. This class of designs was proposed for the problem of comparing simultaneously $p \geq 2$ test treatments with a control treatment when the observations are taken in blocks of common size $k < p+1$.

In [1] a general theory of BTIB designs was developed; in [2] optimal designs were given for the cases $p = 2$, $k = 2(1)6$ and $p = 3$, $k = 3$. In [3] optimal designs were provided for the cases $p = 4$, $k = 3$ and $p = 5$, $k = 3$ while in [4] optimal designs were given for the case $p = 4$, $k = 4$. In the present paper we give optimal designs for the case $p = 6$, $k = 3$; these optimal designs are subject to the same qualification as those given in [3] and [4]--namely that they are optimal relative to the generator designs known to us.

The reader is referred to [3] for the definitions of inadmissibility, S-inadmissibility and C-inadmissibility used in this paper. The reader is also referred specifically to Sections 2 and 3 of [2] and Sections 1 and 2 of [3] for an exact statement of the multiple comparison problem under consideration, expressions for the BLUE's of the treatment effect differences $\alpha_0 - \alpha_i$ $(1 \leq i \leq p)$, their variances $(\tau^2\sigma^2)$ and correlations (ρ), and an expression for the confidence coefficient (P) associated with joint one-sided confidence interval estimates of the $\alpha_0 - \alpha_i$ $(1 \leq i \leq p)$.

2. RESULTS FOR $p = 6$, $k = 3$

2.1 Conjectured minimal complete class of generator designs

In Table 2.1 we list for $p = 6$, $k = 3$ the generator designs in our

Table 2.1

Conjectured Minimal Complete Class of Generator Designs for p = 6, k = 3

Label	Design	b_i	$\lambda_0^{(i)}$	$\lambda_1^{(i)}$
D_1	$\left\{\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{matrix}\right\}$	6	2	0
D_2	$\left\{\begin{matrix} 0 & 0 & 0 & 1 & 1 & 2 & 3 \\ 1 & 2 & 4 & 2 & 5 & 3 & 4 \\ 3 & 6 & 5 & 4 & 6 & 5 & 6 \end{matrix}\right\}$	7	1	1
D_3	$\left\{\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 1 & 2 & 2 & 3 & 3 & 4 & 4 & 3 & 5 \\ 2 & 5 & 6 & 3 & 4 & 5 & 6 & 5 & 6 & 4 & 6 \end{matrix}\right\}$	11	3	1
D_4	$\left\{\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 & 3 & 3 & 3 & 4 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 & 3 & 4 & 5 & 6 & 4 & 5 & 6 & 5 & 6 & 6 \end{matrix}\right\}$	15	5	1
D_5	$\left\{\begin{matrix} 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 4 \\ 2 & 2 & 3 & 3 & 4 & 3 & 3 & 4 & 5 & 5 \\ 5 & 6 & 4 & 6 & 5 & 4 & 5 & 6 & 6 & 6 \end{matrix}\right\}$	10	0	2

conjectured minimal complete class. We prove in the Appendix that D_1 in that class is C-inadmissible for $b \geq 22$.

2.2 Catalog of admissible designs

A catalog of admissible designs has been prepared based on the conjectured minimal complete class of generator designs given in Table 2.1. This catalog is given in Table 2.2 for $b = 6,7,11,14,15,17,18,21,22,24(1)63$.

2.3 Tables of optimal designs

Optimal designs for $p = 6$, $k = 3$ are given in Table 2.3 for $d/\sigma = 0.1(0.1)1.0$ for $b = 6$-62. Optimal designs that achieve a specified confidence coefficient $1-\alpha$ are given as a function of $d/\sigma = 0.2(0.2)2.0$ for $1-\alpha = 0.75, 0.80, 0.85, 0.90, 0.95$ and 0.99 in Table 2.4. These designs were found by a complete computer search among all admissible designs.

3. EFFICIENCY OF THE OPTIMAL BTIB DESIGN RELATIVE TO THAT OF REPLICATIONS OF THE BIB DESIGN D_2 FOR THE MCC PROBLEM

It is of some interest to compare the efficiency of the optimal BTIB design relative to that of replications of the BIB design D_2 for the MCC (multiple comparisons with a control) problem when each is used to achieve the same confidence coefficient $1-\alpha$ for a given d/σ. We have computed the required number of blocks for the BIB design and given the results in Table 3.1 along with the corresponding results for the optimal BTIB design (the latter being abstracted from Table 2.4); also given in the table is the ratio of the number of blocks required by the BTIB design to that of the number of blocks required by the BIB design. This ratio, which we term the relative efficiency, must be less than or equal to unity since a BIB design is also a BTIB design. Thus the

Table 2.2

Catalog of admissible designs[1/] for $p = 6$, $k = 3$

No. of blocks (b)	D_1 $b_1 = 6$ $\lambda_0^{(1)} = 2$ $\lambda_1^{(1)} = 0$	D_2 $b_2 = 7$ $\lambda_0^{(2)} = 1$ $\lambda_1^{(2)} = 1$	D_3 $b_3 = 11$ $\lambda_0^{(3)} = 3$ $\lambda_1^{(3)} = 1$	D_4 $b_4 = 15$ $\lambda_0^{(4)} = 5$ $\lambda_1^{(4)} = 1$	D_5 $b_5 = 10$ $\lambda_0^{(5)} = 0$ $\lambda_1^{(5)} = 2$	λ_0	λ_1	τ^2	ρ
6	1	0	0	0	0	2	0	1.5000	0.000
7	0	1	0	0	0	1	1	0.8571	0.500
11	0	0	1	0	0	3	1	0.4444	0.250
14	0	2	0	0	0	2	2	0.4286	0.500
15	0	0	0	1	0	5	1	0.3273	0.167
17	0	1	0	0	1	1	3	0.6316	0.750
18	0	1	1	0	0	4	2	0.2813	0.333
21	0	0	1	0	1	3	3	0.2857	0.500
	1	0	0	1	0	7	1	0.2637	0.125
22	0	1	0	1	0	6	2	0.2222	0.250
24	0	2	0	0	1	2	4	0.3462	0.667
25	0	2	1	0	0	5	3	0.2087	0.375
26	0	0	1	1	0	8	2	0.1875	0.200
27	0	1	0	0	2	1	5	0.5806	0.833
28	0	1	1	0	1	4	4	0.2143	0.500
29	0	1	2	0	0	7	3	0.1714	0.300
30	0	0	0	2	0	10	2	0.1636	0.167

[1/] For each number of blocks, the number under D_i $(1 \leq i \leq 5)$ in the body of the table is the frequency f_i with which D_i appears in the design $D = \bigcup_{i=1}^{5} f_i D_i$.

Table 2.2 (continued)

No. of blocks (b)	D_1 $b_1 = 6$ $\lambda_0^{(1)} = 2$ $\lambda_1^{(1)} = 0$	D_2 $b_2 = 7$ $\lambda_0^{(2)} = 1$ $\lambda_1^{(2)} = 1$	D_3 $b_3 = 11$ $\lambda_0^{(3)} = 3$ $\lambda_1^{(3)} = 1$	D_4 $b_4 = 15$ $\lambda_0^{(4)} = 5$ $\lambda_1^{(4)} = 1$	D_5 $b_5 = 10$ $\lambda_0^{(5)} = 0$ $\lambda_1^{(5)} = 2$	λ_0	λ_1	τ^2	ρ
31	0	3	0	0	1	3	5	0.2424	0.625
32	0	1	0	1	1	6	4	0.1667	0.400
33	0	1	1	1	0	9	3	0.1481	0.250
34	0	2	0	0	2	2	6	0.3158	0.750
35	0	2	1	0	1	5	5	0.1714	0.500
36	0	0	1	1	1	8	4	0.1406	0.333
37	0	1	0	0	3	1	7	0.5581	0.875
	0	1	0	2	0	11	3	0.1317	0.214
38	0	1	1	0	2	4	6	0.1875	0.600
39	0	1	2	0	1	7	5	0.1390	0.417
40	0	2	1	1	0	10	4	0.1235	0.286
41	0	3	0	0	2	3	7	0.2222	0.700
	0	0	1	2	0	13	3	0.1191	0.188
42	0	1	0	1	2	6	6	0.1429	0.500
43	0	1	1	1	1	9	5	0.1197	0.357
44	0	2	0	0	3	2	8	0.3000	0.800
	0	1	2	1	0	12	4	0.1111	0.250
45	0	2	1	0	2	5	7	0.1532	0.583
	0	0	0	3	0	15	3	0.1091	0.167
46	0	2	2	0	1	8	6	0.1193	0.429
47	0	1	0	0	4	1	9	0.5455	0.900
	0	1	0	2	1	11	5	0.1064	0.313
48	0	1	1	0	3	4	8	0.1731	0.667
	0	1	1	2	0	14	4	0.1015	0.222

Table 2.2 (continued)

No. of blocks (b)	D_1 $b_1 = 6$ $\lambda_0^{(1)} = 2$ $\lambda_1^{(1)} = 0$	D_2 $b_2 = 7$ $\lambda_0^{(2)} = 1$ $\lambda_1^{(2)} = 1$	D_3 $b_3 = 11$ $\lambda_0^{(3)} = 3$ $\lambda_1^{(3)} = 1$	D_5 $b_4 = 15$ $\lambda_0^{(4)} = 5$ $\lambda_1^{(4)} = 1$	D_5 $b_5 = 10$ $\lambda_0^{(5)} = 0$ $\lambda_1^{(5)} = 2$	λ_0	λ_1	τ^2	ρ
49	0	1	2	0	2	7	7	0.1224	0.500
50	0	2	1	1	1	10	6	0.1043	0.375
51	0	3	0	0	3	3	9	0.2105	0.750
	0	2	2	1	0	13	5	0.0966	0.278
52	0	1	0	1	3	6	8	0.1296	0.571
	0	1	0	3	0	16	4	0.0938	0.200
53	0	1	1	1	2	9	7	0.1046	0.438
54	0	2	0	0	4	2	10	0.2903	0.833
	0	1	2	1	1	12	6	0.0938	0.333
55	0	2	1	0	3	5	9	0.1424	0.643
	0	1	3	1	0	15	5	0.0889	0.250
56	0	2	2	0	2	8	8	0.1071	0.500
	0	0	1	3	0	18	4	0.0873	0.182
57	0	1	0	0	5	1	11	0.5373	0.917
	0	3	1	1	1	11	7	0.0926	0.389
58	0	1	1	0	4	4	10	0.1641	0.714
	0	1	1	2	1	14	6	0.0857	0.300
59	0	1	2	0	3	7	9	0.1124	0.563
	0	1	2	2	0	17	5	0.0826	0.227
60	0	2	1	1	2	10	8	0.0931	0.444
	0	0	0	4	0	20	4	0.0818	0.167
61	0	3	0	0	4	3	11	0.2029	0.786
	0	2	2	1	1	13	7	0.0839	0.350
62	0	1	0	1	4	6	10	0.1212	0.625
	0	1	0	3	1	16	6	0.0793	0.273
63	0	1	1	1	3	9	9	0.0952	0.500
	0	1	1	3	0	19	5	0.0773	0.208

Table 2.3

Optimal Designs[1/] and Associated Confidence Coefficient (P) as a Function of

b and d/σ for p = 6, k = 3

No. of blocks (b)	d/σ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
6	1,0,0 0,0 0.0228	1,0,0 0,0 0.0325	1,0,0 0,0 0.0452	1,0,0 0,0 0.0614	1,0,0 0,0 0.0815	1,0,0 0,0 0.1060	1,0,0 0,0 0.1349	1,0,0 0,0 0.1685	1,0,0 0,0 0.2065	1,0,0 0,0 0.2485
7	0,1,0 0,0 0.1744	0,1,0 0,0 0.2100	0,1,0 0,0 0.2494	0,1,0 0,0 0.2924	0,1,0 0,0 0.3384	0,1,0 0,0 0.3869	0,1,0 0,0 0.4372	0,1,0 0,0 0.4883	0,1,0 0,0 0.5395	0,1,0 0,0 0.5900
11						0,0,1 0,0 0.4037	0,0,1 0,0 0.4837	0,0,1 0,0 0.5636	0,0,1 0,0 0.6402	0,0,1 0,0 0.7109
14	0,2,0 0,0 0.1887	0,2,0 0,0 0.2424	0,2,0 0,0 0.3033	0,2,0 0,0 0.3701	0,2,0 0,0 0.4408	0,2,0 0,0 0.5132	0,2,0 0,0 0.5850	0,2,0 0,0 0.6538	0,2,0 0,0 0.7177	0,2,0 0,0 0.7751
15									0,0,0 1,0 0.7276	0,0,0 1,0 0.7996
17	0,1,0 0,1 0.2918	0,1,0 0,1 0.3403	0,1,0 0,1 0.3916	0,1,0 0,1 0.4448	0,1,0 0,1 0.4990	0,1,0 0,1 0.5532	0,1,0 0,1 0.6063	0,1,0 0,1 0.6576		

[1/] The "matrix" in each cell is $\left\{\begin{matrix}\hat{f}_1,\hat{f}_2,\hat{f}_3\\ \hat{f}_4,\hat{f}_5\end{matrix}\right\}$ where $\hat{D} = \bigcup_{i=1}^{5} \hat{f}_i D_i$ with $b = \sum_{i=1}^{5} \hat{f}_i b_i$ is the optimal design for the given value of b and d/σ.

Table 2.3 (continued)

No. of blocks (b)	d/σ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
18						0,1,1 0,0 0.5560	0,1,1 0,0 0.6485	0,1,1 0,0 0.7322	0,1,1 0,0 0.8038	0,1,1 0,0 0.8620
21					0,0,1 0,1 0.5222	0,0,1 0,1 0.6094	0,0,1 0,1 0.6911	0,0,1 0,1 0.7640	0,0,1 0,1 0.8261	0,0,1 0,1 0.8764
22							0,1,0 1,0 0.7042	0,1,0 1,0 0.7916	0,1,0 1,0 0.8607	0,1,0 1,0 0.9116
24			0,2,0 0,1 0.4038	0,2,0 0,1 0.4785	0,2,0 0,1 0.5540	0,2,0 0,1 0.6274				
25						0,2,1 0,0 0.6571	0,2,1 0,0 0.7504	0,2,1 0,0 0.8276	0,2,1 0,0 0.8871	0,2,1 0,0 0.9299
26							0,0,1 1,0 0.7522	0,0,1 1,0 0.8376	0,0,1 1,0 0.9001	0,0,1 1,0 0.9422
27	0,1,0 0,2 0.3426	0,1,0 0,2 0.3947	0,1,0 0,2 0.4487	0,1,0 0,2 0.5037	0,1,0 0,2 0.5586					

Table 2.3 (continued)

No. of blocks (b)	d/σ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
28					0,1,1 0,1 0.5900	0,1,1 0,1 0.6855	0,1,1 0,1 0.7696	0,1,1 0,1 0.8391		
29						0,1,2 0,0 0.6993	0,1,2 0,0 0.7966	0,1,2 0,0 0.8713	0,1,2 0,0 0.9238	0,1,2 0.0 0.9578
30								0,0,0 2,0 0.8734	0,0,0 2,0 0.9279	0,0,0 2,0 0.9617
31				0,3,0 0,1 0.5184	0,3,0 0,1 0.6087					
32					0,1,0 1,1 0.6235	0,1,0 1,1 0.7310	0,1,0 1,1 0.8197	0,1,0 1,1 0.8868	0,1,0 1,1 0.9335	0,1,0 1,1 0.9635
33						0,1,1 1,0 0.7366	0,1,1 1,0 0.8335	0,1,1 1,0 0.9027	0,1,1 1,0 0.9474	0,1,1 1,0 0.9736
34			0,2,0 0,2 0.4579	0,2,0 0,2 0.5347						

Table 2.3 (continued)

No. of blocks (b)	d/σ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
35				0,2,1 0,1 0.5367	0,2,1 0,1 0.6475	0,2,1 0,1 0.7465				
36					0,0,1 1,1 0.6547	0,0,1 1,1 0.7686	0,0,1 1,1 0.8567	0,0,1 1,1 0.9182	0,0,1 1,1 0.9568	0,0,1 1,1 0.9790
37	0,1,0 0,3 0.3723	0,1,0 0,3 0.4260	0,1,0 0,3 0.4810			0,1,0 2,0 0.7696	0,1,0 2,0 0.8634	0,1,0 2,0 0.9259	0,1,0 2,0 0.9631	0,1,0 2,0 0.9831
38				0,1,1 0,2 0.5578	0,1,1 0,2 0.6586					
39					0,1,2 0,1 0.6811	0,1,2 0,1 0.7872	0,1,2 0,1 0.8685			
40					0,2,1 1,0 0.6839	0,2,1 1,0 0.8004	0,2,1 1,0 0.8851	0,2,1 1,0 0.9397	0,2,1 1,0 0.9711	0,2,1 1,0 0.9873
41				0,3,0 0,2 0.5688			0,0,1 2,0 0.8877	0,0,1 2,0 0.9432	0,0,1 2,0 0.9738	0,0,1 2,0 0.9890

Table 2.3 (continued)

No. of blocks (b)	d/σ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
42				0,1,0 1,2 0.5799	0,1,0 1,2 0.6966					
43					0,1,1 1,1 0.7110	0,1,1 1,1 0.8198	0,1,1 1,1 0.8977	0,1,1 1,1 0.9471	0,1,1 1,1 0.9751	0,1,1 1,1 0.9893
44			0,2,0 0,3 0.4926			0,1,2 1,0 0.8275	0,1,2 1,0 0.9073	0,1,2 1,0 0.9551	0,1,2 1,0 0.9803	0,1,2 1,0 0.9921
45				0,2,1 0,2 0.5950			0,0,0 3,0 0.9076	0,0,0 3,0 0.9562	0,0,0 3,0 0.9813	0,0,0 3,0 0.9927
46				0,2,2 0,1 0.6013	0,2,2 0,1 0.7287	0,2,2 0,1 0.8309				
47	0,1,0 0,4 0.3924	0,1,0 0,4 0.4469	0,1,0 0,4 0.5025		0,1,0 2,1 0.7378	0,1,0 2,1 0.8466	0,1,0 2,1 0.9194	0,1,0 2,1 0.9619	0,1,0 2,1 0.9837	0,1,0 2,1 0.9937
48				0,1,1 0,3 0.6026		0,1,1 2,0 0.8508	0,1,1 2,0 0.9248	0,1,1 2,0 0.9661	0,1,1 2,0 0.9863	0,1,1 2,0 0.9950

Table 2.3 (continued)

No. of blocks (b)	d/σ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
49				0,1,2 0,2 0.6187	0,1,2 0,2 0.7388					
50				0,2,1 1,1 0.6221	0,2,1 1,1 0.7565	0,2,1 1,1 0.8587	0,2,1 1,1 0.9262			
51			0,3,0 0,3 0.5097		0,2,2 1,1 0.7621	0,2,2 1,0 0.8689	0,2,2 1,0 0.9359	0,2,2 1,0 0.9721	0,2,2 1,0 0.9891	0,2,2 1,0 0.9962
52				0,1,0 1,3 0.6297		0,1,0 3,0 0.8708	0,1,0 3,0 0.9388	0,1,0 3,0 0.9743	0,1,0 3,0 0.9904	0,1,0 3,0 0.9968
53				0,1,1 1,2 0.6408	0,1,1 1,2 0.7686					
54			0,2,0 0,4 0.5170	0,1,2 1,1 0.6421	0,1,2 1,1 0.7810	0,1,2 1,1 0.8810	0,1,2 1,1 0.9427	0,1,2 1,1 0.9755	0,1,2 1,1 0.9907	
55			0,2,1 0,3 0.5187		0,1,3 1,0 0.7841	0,1,3 1,0 0.8877	0,1,3 1,0 0.9487	0,1,3 1,0 0.9793	0,1,3 1,0 0.9926	0,1,3 1,0 0.9976

Table 2.3 (continued)

No. of blocks	d/σ									
(b)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
56				0,2,2 0,2 0.6538		0,0,1 3,0 0.8880	0,0,1 3,0 0.9500	0,0,1 3,0 0.9804	0,0,1 3,0 0.9932	0,0,1 3,0 0.9979
57	0,1,0 0,5 0.4072	0,1,0 0,5 0.4622		0,3,1 1,1 0.6614	0,3,1 1,1 0.7940	0,3,1 1,1 0.8886				
58			0,1,1 0,4 0.5288		0,1,1 2,1 0.8027	0,1,1 2,1 0.8994	0,1,1 2,1 0.9551	0,1,1 2,1 0.9824	0,1,1 2,1 0.9939	0,1,1 2,1 0.9981
59				0,1,2 0,3 0.6618	0,1,2 2,0 0.8041	0,1,2 2,0 0.9035	0,1,2 2,0 0.9587	0,1,2 2,0 0.9845	0,1,2 2,0 0.9949	0,1,2 2,0 0.9985
60				0,2,1 1,2 0.6757			0,0,0 4,0 0.9591	0,0,0 4,0 0.9849	0,0,0 4,0 0.9951	0,0,0 4,0 0.9986
61			0,3,0 0,4 0.5345	0,2,2 1,1 0.6809	0,2,2 1,1 0.8160	0,2,2 1,1 0.9071				
62			0,1,0 1,4 0.5406		0,1,0 3,1 0.8220	0,1,0 3,1 0.9145	0,1,0 3,1 0.9644	0,1,0 3,1 0.9871	0,1,0 3,1 0.9959	0,1,0 3,1 0.9989

Table 2.4

Optimal Design[1/] to Achieve a Specified Confidence Coefficient as a Function of d/σ for $p = 6$, $k = 3$

Confidence Coefficient $(1-\alpha)$	d/σ									
	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.99	b=1044 0,1,2 59,13	b=261 0,1,4 12,3	b=117 0,1,5 3,1	b=66 0,1,4 1,0	b=44 0,1,2 1,0	b=30 0,0,0 2,0	b=22 0,1,0 1,0	b=18 0,1,1 0,0	b=15 0,0,0 1,0	b=11 0,0,1 0,0
0.95	b=680 0,1,3 36,10	b=171 0,1,4 6,3	b=77 0,1,0 4,1	b=44 0,1,2 1,0	b=29 0,1,2 0,0	b=22 0,1,0 1,0	b=15 0,0,0 1,0	b=11 0,0,1 0,0	b=11 0,0,1 0,0	b=11 0,0,1 0,0
0.90	b=522 0,1,5 24,10	b=131 0,1,4 4,2	b=59 0,1,2 2,0	b=33 0,1,1 1,0	b=22 0,1,0 1,0	b=15 0,0,0 1,0	b=11 0,0,1 0,0	b=11 0,0,1 0,0	b=11 0,0,1 0,0	b=7 0,1,0 0,0
0.85	b=428 0,1,1 20,11	b=108 0,1,1 4,3	b=48 0,1,1 2,0	b=29 0,1,2 0,0	b=18 0,1,1 0,0	b=14 0,2,0 0,0	b=11 0,0,1 0,0	b=11 0,0,1 0,0	b=7 0,1,0 0,0	b=7 0,1,0 0,0
0.80	b=360 0,1,3 16,8	b=90 0,1,3 2,2	b=40 0,2,1 1,0	b=25 0,2,1 0,0	b=18 0,1,1 0,0	b=11 0,0,1 0,0	b=11 0,0,1 0,0	b=7 0,1,0 0,0	b=7 0,1,0 0,0	b=7 0,1,0 0,0
0.75	b=306 0,1,4 11,9	b=78 0,1,1 2,3	b=36 0,0,1 1,1	b=21 0,0,1 0,1	b=14 0,2,0 0,0	b=11 0,0,1 0,0	b=7 0,1,0 0,0	b=7 0,1,0 0,0	b=7 0,1,0 0,0	b=7 0,1,0 0,0

[1/] The "matrix" in each cell is $\left\{\begin{matrix}\hat{f}_1,\hat{f}_2,\hat{f}_3\\ \hat{f}_4,\hat{f}_5\end{matrix}\right\}$ where $\hat{D} = \bigcup_{i=1}^{5} \hat{f}_i D_i$ with $b = \sum_{i=1}^{5} \hat{f}_i b_i$ is the optimal design for the given value of $1-\alpha$ and d/σ.

Table 3.1

Efficiency of a BIB Design Obtained by Replicating D_2

Relative to the Optimal BTIB Design

as a Function of d/σ

for p = 6, k = 3

Confidence Coefficient (1-α)	d/σ				
	0.2	0.4	0.6	0.8	1.0
0.99	b = 1044 b = 1253 0.8332	b = 261 b = 315 0.8286	b = 117 b = 140 0.8357	b = 66 b = 84 0.7857	b = 44 b = 56 0.7857
0.95	b = 680 b = 791 0.8597	b = 171 b = 203 0.8424	b = 77 b = 91 0.8462	b = 44 b = 56 0.7857	b = 29 b = 35 0.8286
0.90	b = 522 b = 588 0.8878	b = 131 b = 147 0.8912	b = 59 b = 70 0.8429	b = 33 b = 42 0.7857	b = 22 b = 28 0.7857
0.85	b = 428 b = 469 0.9126	b = 108 b = 119 0.9076	b = 48 b = 56 0.8571	b = 29 b = 35 0.8286	b = 18 b = 21 0.8571
0.80	b = 360 b = 385 0.9351	b = 90 b = 98 0.9184	b = 40 b = 49 0.8163	b = 25 b = 28 0.8929	b = 18 b = 21 0.8571

1/ The top number in each cell is the smallest number of blocks required by the optimal BTIB design to achieve 1-α; this number is taken from Table 2.4.

2/ The middle number in each cell is the smallest number of blocks required by the BIB design obtained by replicating D_2 to achieve 1-α.

3/ The bottom number in each cell is the relative efficiency, i.e., the ratio of the top number in each cell to the middle number in that cell.

optimal BTIB design never requires more blocks for the MCC problem than does replications of the BIB design; the smaller the ratio, the more efficient is the optimal BTIB design.

We note from the table that as $1-\alpha$ approaches unity for fixed d/σ the relative efficiencies decrease (i.e., the BTIB design becomes relatively more efficient) while as d/σ approaches zero for fixed $1-\alpha$ the relative efficiencies increase (i.e., the BTIB design becomes relatively less efficient). This apparent inconsistency can be explained as follows: For $1-\alpha$ approaching unity when d/σ $(d/\sigma > 0)$ is fixed, it is known that (in the limit) the optimal design is the one that minimizes τ^2; for d/σ approaching zero when $1-\alpha$ $(1-\alpha < 1)$ is fixed, it is known that (in the limit) the optimal design is the one that maximizes ρ. These two limiting optimal BTIB designs are different, and the limiting relative efficiencies are bounded away from zero and one, respectively.

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APPENDIX

Proof of C-inadmissibility of D_1 for $b \geq 22$

We note that D_1 is admissible for $b = 6$, and $D_1 \cup D_4$ is admissible for $b = 21$. We further note that $D_1 \cup D_2$ $(b = 13, \lambda_0 = 3, \lambda_1 = 1)$ is S-inadmissible wrt $D_3(11,3,1)$; $D_1 \cup D_3$ $(17,5,1)$ is S-inadmissible wrt D_4 $(15,5,1)$ and $D_1 \cup D_5$ $(16,2,2)$ is S-inadmissible wrt $2D_2$ $(14,2,2)$. Therefore it only remains to show that every design of the form $f_1D_1 \cup f_4D_4$ is inadmissible except $(f_1,f_2) = (1,0)$ or $(1,1)$ which are admissible.

Case 1 ($f_1 = 2m$ with $m \geq 1$): We shall show that $D = 2mD_1 \cup f_4D_4$ is inadmissible wrt $D' = mD_3 \cup f_4D_4$ for $m \geq 1$, $f_4 \geq 0$. Let $(b,\lambda_0,\lambda_1,\rho,\tau^2)$ $((b',\lambda_0',\lambda_1',\rho',\tau'^2))$ denote the parameters associated with D (D'). Then we have $b = 12m + 15f_4$, $\lambda_0 = 4m + 5f_4$, $\lambda_1 = f_4$ and $b' = 11m + 15f_4$, $\lambda_0' = 3m + 5f_4$, $\lambda_1' = m + f_4$. Thus $b > b'$. We have $\rho \leq \rho'$ iff $f_4/(4m + 6f_4) \leq (m + f_4)/(4m + 6f_4)$ which holds with strict inequality when $m \geq 1$. Finally, $\tau^2 \geq \tau'^2$ iff

$$\frac{4m + 6f_4}{(4m + 5f_4)(4m + 11f_4)} \geq \frac{4m + 6f_4}{(3m + 5f_4)(9m + 11f_4)}$$

which holds with strict inequality. This completes the proof for Case 1.

Case 2 ($f_1 = 2m+1$ with $m \geq 1$): We shall show that $D = (2m+1)D_1 \cup f_4D_4$ is inadmissible wrt $D' = mD_3 \cup D_2 \cup f_4D_4$. Let $(b,\lambda_0,\lambda_1,\rho,\tau^2)$ $((b',\lambda_0',\lambda_1',\rho',\tau'^2))$ denote the parameters associated with D (D'). Then we have $b = 12m + 15f_4 + 6$, $\lambda_0 = 4m + 5f_4 + 2$, $\lambda_1 = f_4$ and $b' = 11m + 15f_4 + 7$, $\lambda_0' = 3m + 5f_4 + 1$, $\lambda_1' = m + f_4 + 1$. Thus $b \geq b'$

when $m \geq 1$. We have $\rho \leq \rho'$ iff $f_4/(4m + 6f_4 + 2) \leq (m + f_4 + 1)/(4m + 6f_4 + 2)$ which holds with strict inequality when $m \geq 1$. Finally, $\tau^2 \geq \tau'^2$ iff

$$\frac{4m + 6f_4 + 2}{(4m + 5f_4 + 2)(4m + 11f_4 + 2)} \geq \frac{4m + 6f_4 + 2}{(3m + 5f_4 + 1)(9m + 11f_4 + 7)}$$

which holds with strict inequality. This completes the proof for Case 2, and hence of the desired result.

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The present paper continues the study of balanced treatment incomplete block (BTIB) designs initiated previously ~~in [1]-[4]~~. This class of designs was proposed for the problem of comparing simultaneously $p \geq 2$ test treatments with a control treatment when the observations are taken in blocks of common size $k < p+1$. The conjectured minimal complete class of generator designs, a catalog of admissible designs, and tables of optimal designs are given for $p = 6$, $k = 3$. The efficiency of the optimal BTIB design relative to that of replications of a BIB design is computed for situations in which both provide the same probability guarantee for the multiple comparisons with a control problem.

[1] Bechhofer, R.E. and Tamhane, A.C. (1979a). Incomplete block designs for comparing treatments with a control: General theory. Accepted for publication in Technometrics.

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