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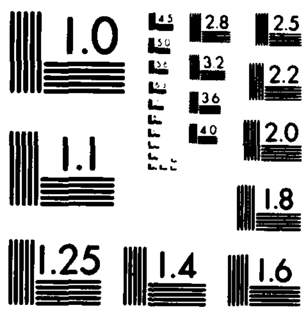

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A Functional For Finite Element Analysis of Low Intensity Magnetic Fields Including Permanent Magnetization Effects

John W. Frye
Office of Engineering Mechanics

17 June 1980

NUSC
Naval Underwater Systems Center
Newport, Rhode Island • New London, Connecticut

Preface

This report was prepared under NUSC Project No. A53223, "Finite Element Modeling of Electric and Electromagnetic Fields" (U), Principal Investigator, R. G. Kasper (Code 401), and Associate Principal Investigator, J. W. Frye (code 401), Navy Subproject and Task No. B-0005, 11221; Naval Industrial Funding from David W. Taylor Naval Ship Research and Development Center, Project Director, W. Andahazy (DTNSRDC, Code 2704); Program Manager, W. L. Welsh (NAVMAT PM-2-21).

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A functional is presented that is useful in the finite element analysis of low intensity, static magnetic field solutions. The variational principle is written in terms of a scalar potential function and, when minimized, leads to field equations and boundary conditions that characterize a magnetic field--accounting for the presence of current, permanent magnetization, and incident magnetic fields. This variational principle finds applications in the finite element analysis of magnetic field problems.

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A Functional for Finite Element Analysis of Low Intensity Magnetic Fields Including Permanent Magnetization Effects

Introduction

The analysis of static magnetic fields using scalar potential functions in association with the Biot-Savart law has been reported in the open literature by Zienkiewicz et al.[1] and Armstrong et al.[2] Wikswo[3] has indicated how widely distributed current distributions can be economically treated using this approach. Although the work of Zienkiewicz et al.[1] uses a general variational, there is no discussion on how a problem with permanent magnetization effects should be approached.

Solution Approach

If the magnetic fields are of low intensity or do not change rapidly over the magnetized body, the change in magnetic flux as a function of magnetic field intensity will be linear; i.e.,

$$\vec{B} = \vec{B}_R + \mu_\Delta \vec{H}, \tag{1}$$

where μ_Δ is the incremental permeability and $\vec{B}_R$ is the remanent magnetization measured by drawing a tangent from the operating point on the B-H curve to the H = 0 axis.

Following the usual scalar potential solution to magnetic field problems, we have

$$\vec{H} = \vec{H}_C + \vec{H}_I + \vec{\nabla}\phi, \tag{2}$$

where $\vec{H}_C$ is the Biot-Savart law magnetic field strength due to currents, $\vec{H}_I$ is the applied farfield magnetic field strength, and ϕ is an unknown continuous scalar potential function. For the purposes of this discussion, $\vec{H}_C$ and $\vec{H}_I$ are assumed to have either low amplitudes or to be changing slowly, so that μ_Δ can be taken as a constant for the problem.

Using the assumed solution of equation (2), we immediately satisfy one of Maxwell's equations involving the curl of the magnetic field strength $\vec{H}$ and the current density $\vec{J}$:

$$\vec{\nabla} \times \vec{H} = \vec{J}. \tag{3}$$

Furthermore, the continuity requirements on the scalar potential function provide for the satisfaction of the boundary condition between dissimilar materials involving the magnetic field strength components tangent to the boundary:

$$\vec{n} \times \vec{H}|_1 = \vec{n} \times \vec{H}|_2 , \tag{4}$$

where $\vec{n}$ is a vector normal to the boundary and the symbols $|_1$ and $|_2$ indicate that the operation is to be carried out in materials 1 and 2, respectively.

The relation requiring that the divergence of the flux density must vanish furnished the field relation for the determination of the scalar potential function:

$$\vec{\nabla} \bullet \vec{B} = 0. \tag{5}$$

Substitution of equations (1) and (2) into equation (5) gives

$$\vec{\nabla} \bullet \mu_\Delta \vec{\nabla} \phi = -\vec{\nabla} \bullet \mu_\Delta \vec{H}_C - \vec{\nabla} \bullet \mu_\Delta \vec{H}_I - \vec{\nabla} \bullet \vec{B}_R . \tag{6}$$

The boundary condition requiring that the normal component of the flux density be constant across the boundary must also be satisfied:

$$\vec{n} \bullet \vec{B}|_1 = \vec{n} \bullet \vec{B}|_2 . \tag{7}$$

A further boundary condition on ϕ is that it decay to zero as the distance from the permanently magnetized body becomes large:

$$\phi = 0 \text{ as } |\vec{r} - \vec{r}_B| \rightarrow \infty , \tag{8}$$

where $|\vec{r} - \vec{r}_B|$ is the distance between the point r at which ϕ is evaluated and the nearest point of the finitely sized permanently magnetized region.

Functional and Finite Element Equations

A functional that, when minimized, leads to the field and boundary conditions of equations (6) and (7) is given below.

$$X = \int_V 1/2\,(\vec{\nabla}\phi + \vec{H}_I + \vec{H}_C) \bullet (\mu_\Delta \vec{\nabla}\phi + m_D \vec{H}_I + \mu_\Delta \vec{H}_C + 2\,\vec{B}_R)\, dV . \tag{9}$$

This functional may be used as the basis of a finite element formulation. In such a formulation the scalar potential function is set equal to the sum of the products of interpolation functions with discrete unknowns:

$$\phi = \sum_{i=1}^{n} N_i \phi_i , \tag{10}$$

where $N_i = N_i$ (x, y, z) are the interpolation functions, ϕ_i are discrete unknown values of potential associated with various discrete points in the problem domain, and n is the number of discrete unknowns.

A discussion of the details of the finite element method is unnecessary here since such details are covered in any standard text on the subject.[4]

When equation (10) is substituted into the variational principle (9) and the functional is minimized with respect to the ϕ_i unknowns, a set of linear algebraic equations results that can be conveniently represented in matrix form:

$$[K]\{\phi\} + \{f_c\} + \{f_I\} + \{f_R\} = 0\,, \tag{11}$$

where

$$k_{ij} = \int_V \vec{\nabla} N_i \bullet \mu_\Delta \vec{\nabla} N_j \, dV \tag{12}$$

$$f_{c_i} = \int_V \vec{\nabla} N_i \bullet \mu_\Delta \vec{H}_c \, dV \tag{13}$$

$$f_{I_i} = \int_V \vec{\nabla} N_i \bullet \mu_\Delta \vec{H}_I \, dV \tag{14}$$

$$f_{R_i} = \int_V \vec{\nabla} N_i \bullet \vec{B}_R \, dV\,. \tag{15}$$

Equations (12) and (13) are identical to those given by Zienkiewicz et al.[1] Equation (14) shows the "loading" term relation that accounts for the incident magnetic field on the body. If we use this solution approach for a problem with an incident magnetic field, the finite element solution gives us the magnetic field disturbance caused by the body directly. Often the magnetic field disturbance is the topic of interest in such problems. Equation (15) shows the term accounting for permanent magnetization effects; the distribution of remanent flux density $\vec{B}_R$ must be known for this formulation.

Conclusions

Equations (11) through (15) allow determination of the magnetic field around a permeable body. The equations account for the effects of currents, incident fields, and permanent magnetization. Since the heat transfer elements in standard finite element programs have the same formulation as that given by equation (12), such programs may be used to solve a variety of magnetic field problems. A preprocessor must be developed, however, to calculate the loadings defined by equations (13) through (15), and a postprocessor must be used to perform the additional operation given by equation (2) in order to obtain the total field solution.

References

1. O. C. Zienkiewicz, John Lyness, and Dr. J. Owen, "Three-Dimensional Magnetic Field Determination Using a Scalar Potential - A Finite Element Solution," *IEE Transactions on Magnetics*, vol. 13, no. 5, September 1977, pp. 1649-1656.

2. A. G. Armstrong, et al., *The Solution of 3D Magnetostatic Problems Using Scalar Potentials*, Rutherford Laboratory Report RL-78-0888, Rutherford Laboratory, Chilton, Didcot, Oxon, 0X1100X, U.K., September 1978.

3. J. P. Wikswo, Jr., "The Calculation of the Magnetic Field from a Current Distribution: Application to Finite Element Techniques," *IEEE Transactions on Magnetics*, vol. 14, no. 5, September 1978, pp. 1076-1077.

4. O. C. Zienkiewicz, *The Finite Element Method in Engineering Science*, McGraw-Hill, London, 1971.

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