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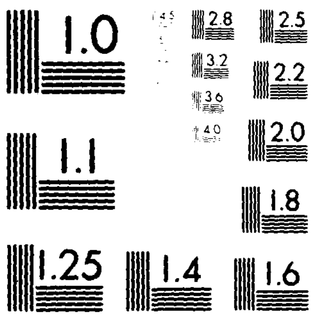

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ACCELERATED LIFE TESTS AND INFORMATION

by
RICHARD E. BARLOW

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ACCELERATED LIFE TESTS AND INFORMATION

by

Richard E. Barlow
Department of Industrial Engineering
and Operations Research
University of California, Berkeley

JUNE 1980 ORC 80-14

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ABSTRACT

This is a partly philosophical discussion of the problems of conducting accelerated life test experiments and analyzing data from such experiments. Our objective is to predict life at a specified stress level. The concept of information is presented and its quantification via entropy discussed.

ACCELERATED LIFE TESTS AND INFORMATION

by

Richard E. Barlow

1. INTRODUCTION

This is a partly philosophical discussion of the problems of conducting accelerated life test experiments and analyzing data from such experiments. An accelerated life test is one which is conducted at a stress level usually higher than that which we expect to occur in the use environment. Stress may be measured in pounds per square inch (psi) as it was for the pressure vessels to be analyzed. In a biological context it could be the number of rads of gamma radiation delivered to animals in radiation experiments designed to extrapolate lifetime test results at high stress or dose to lifetimes at low radiation dose environments. In setting up such experiments, the first problem is to select stress levels. Given test results, the main problem is to relate lifetimes at various stress levels and to predict life at a usually much lower stress level. At all stages we must assess information about quantities of interest.

Our objective is to predict life, say X, at a specified stress level. It is easier to think about parameters in a two or three parameter life distribution model, than it is to think about all possible life distribution models. Partly for this reason we have focused attention on a two parameter Weibull life distribution model. Given this model, information provided by accelerated life test experiments will be relative to parameters of the model. If we were certain in the beginning about the values of the parameters, no experiment would be informative relative to these parameters. There are many arguments which lead to the conclusion that

1. *Probability is the only admissible measure of uncertainty.* This is true whether the events about which we are uncertain can be observed, such as the life of an animal or events which cannot be observed, such as the event that a life distribution parameter exceeds 100. Admissibility can be defined relative to a number of scoring rules. See D. V. Lindley (1980) for a recent generalization of De Finetti's (1974) argument.

De Finetti (1974) also argues convincingly that

2. *Probability does not exist - except in the mind.*

When we analyze a scientific experiment we are in effect making a judgement. To avoid logical inconsistencies we must specify initial probabilities for uncertain quantities -- in this case the parameters of our life distribution model. These initial probabilities must then be updated using the calculus of probability to reach our final conclusions which must necessarily be stated in probabilistic terminology.

Information about uncertain quantities is then anything which *changes* our probability distribution for these uncertain quantities. Various ways of quantifying the change in probability distributions have been suggested and we will discuss one of these--namely change in expected entropy--when we discuss the problem of planning accelerated life tests.

2. ACCELERATED LIFE TEST DATA ANALYSIS

In Barlow, Toland and Freeman (1979) accelerated life test data on Kevlar strands and pressure vessels, partially constructed of these strands, was analyzed. For a given stress level s, they assume that

$$P[X > x \mid s,\alpha,\lambda] = e^{-(\lambda x)^{\alpha}} \tag{2.1}$$

for x, α, $\lambda > 0$ where X is the lifetime given stress s. The parameters α and λ also depend on stress s. This is a Weibull distribution with shape parameter α and scale parameter λ. Most certainly, neither strands nor pressure vessels have precisely Weibull life distributions. However, this is a parsimonious model (only two parameters) and these parameters summarize important properties of the life distribution model. Also it is an extreme value distribution and is sometimes appropriate for series type systems. Although neither the strands nor the pressure vessels are series systems, this might perhaps serve as a crude approximation.

Life test data consisted of k, the number of observed failures in $[0,t]$ when testing stopped and

$$0 \le x_{(1)} \le x_{(2)} \le \cdots \le x_{(k)} \le t$$

the observed failure times. The likelihood function was computed in each case to provide information about the Weibull distribution model parameters. In particular,

$$L(\alpha,\lambda \mid D) \propto \alpha^{k}\lambda^{k\alpha}\left[\prod_{i=1}^{k} x_{(i)}\right]^{\alpha-1} \exp\left\{-\lambda^{\alpha}\left[\sum_{i=1}^{k} x_{(i)}^{\alpha} + (n-k)t^{\alpha}\right]\right\} .$$

Assuming a prior density $\pi_0(\alpha,\lambda)$ on α and λ, the posterior density is

$$\pi(\alpha,\lambda \mid D) \propto L(\alpha,\lambda \mid D)\pi_0(\alpha,\lambda) .$$

A flat or diffuse prior for α and λ (i.e., $\pi_0(\alpha,\lambda) = c$) was used to plot the contours in Figure 1. In particular, using three dimensional graphic computer programs

$$\pi(\alpha,\lambda \mid D) = \frac{L(\alpha,\lambda \mid D)}{\int_0^\infty\int_0^\infty L(\alpha,\lambda \mid D)d\alpha d\lambda}$$

was plotted. The outer contour was determined so that the volume over the contour and under $\pi(\alpha,\lambda \mid D)$ is 0.90. The contour plots are two dimensional "pictures" of the density $\pi(\alpha,\lambda \mid D)$.

The mean rupture stress of the strands was computed based on an initial stress rupture experiment. One hundred strands were put on life test at a stress equivalent to 90% of their mean rupture stress. Likewise, 100 were tested at 80%, 49 at 70%, 47 at 60% and 48 at 50% of mean rupture stress.

Although a flat (improper) prior was used for all contours, any proper prior which would be approximately constant within the region of the 95% contour and not "astronomically" large outside would have provided approximately the same posterior contour plot. The 90%, 80%, 70% and 60% data were sufficient to provide relatively "tight" contours. Of the 50% strands, only 3 had failed by the time of analysis. This partially accounts for the noninformative nature of the 50% contours. A proper prior based on prior test experience should have been used to analyze the 50% stress data.

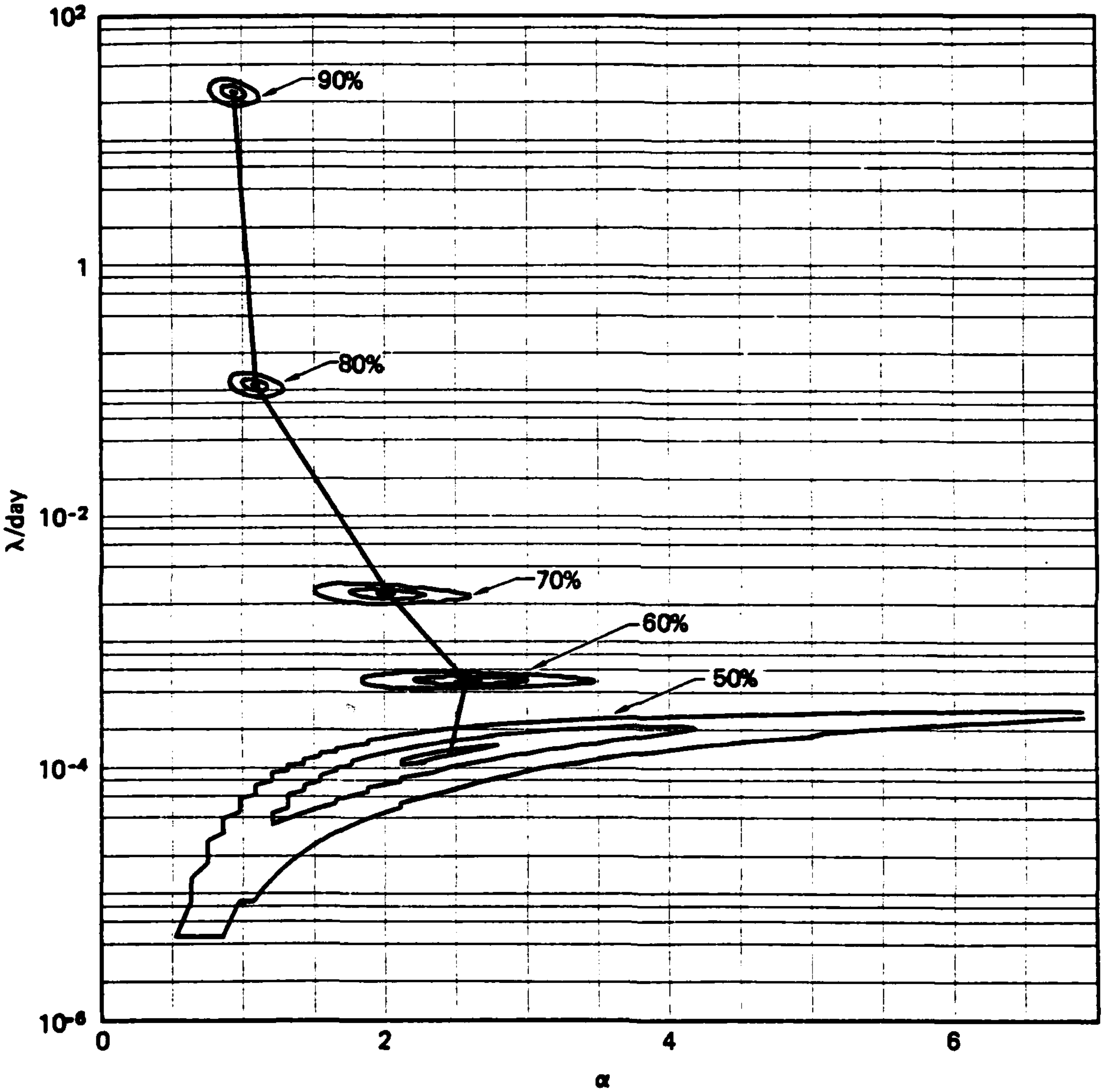

Fig. 1. Weibull posterior density confidence contours for α and λ for Kevlar strand life test data compiled as of August 1978.

From the contour plots, it was concluded that α , the shape parameter increased with decreasing stress. Since the mean life is $\Gamma\left(1 + \frac{1}{\alpha}\right)/\lambda$ for this model and in general lifetime increases with decreasing stress, it is natural that λ should decrease with decreasing stress.

Most statistical papers in accelerated life testing; i.e., Singpurwalla (1975) assume the shape parameter does *not* depend on the stress level. This study showed the danger of assuming a convenient functional relationship between life and stress, without first carefully analyzing the data at each stress level.

Kevlar pressure vessels were similarly life tested at 86%, 80%, 74% and 68% of their mean rupture strength. Figure 2 shows corresponding contour plots based in each case on a flat prior. The maximum likelihood estimates are joined by straight lines. Fewer vessels were tested and the test interval was also shorter.

Since our objective was to predict life at low stress levels, the maximum likelihood estimates of mean life were plotted as a function of decreasing stress, for both strands and spherical vessels. By a change of variable, the posterior density for mean life can be numerically computed. Credible intervals (95%) on mean life can be computed from this density. These are the vertical bars in Figure 3. An empirical projection for mean life at low stress can be computed from Figure 3.

Figure 4 superimposes contours for 68% stress data on pressure vessels. The inner contours are updated contours based on additional test data. The original contours correspond to our prior density which was used to generate the posterior contours (light curves). This figure was computed by Agrawal (1980).

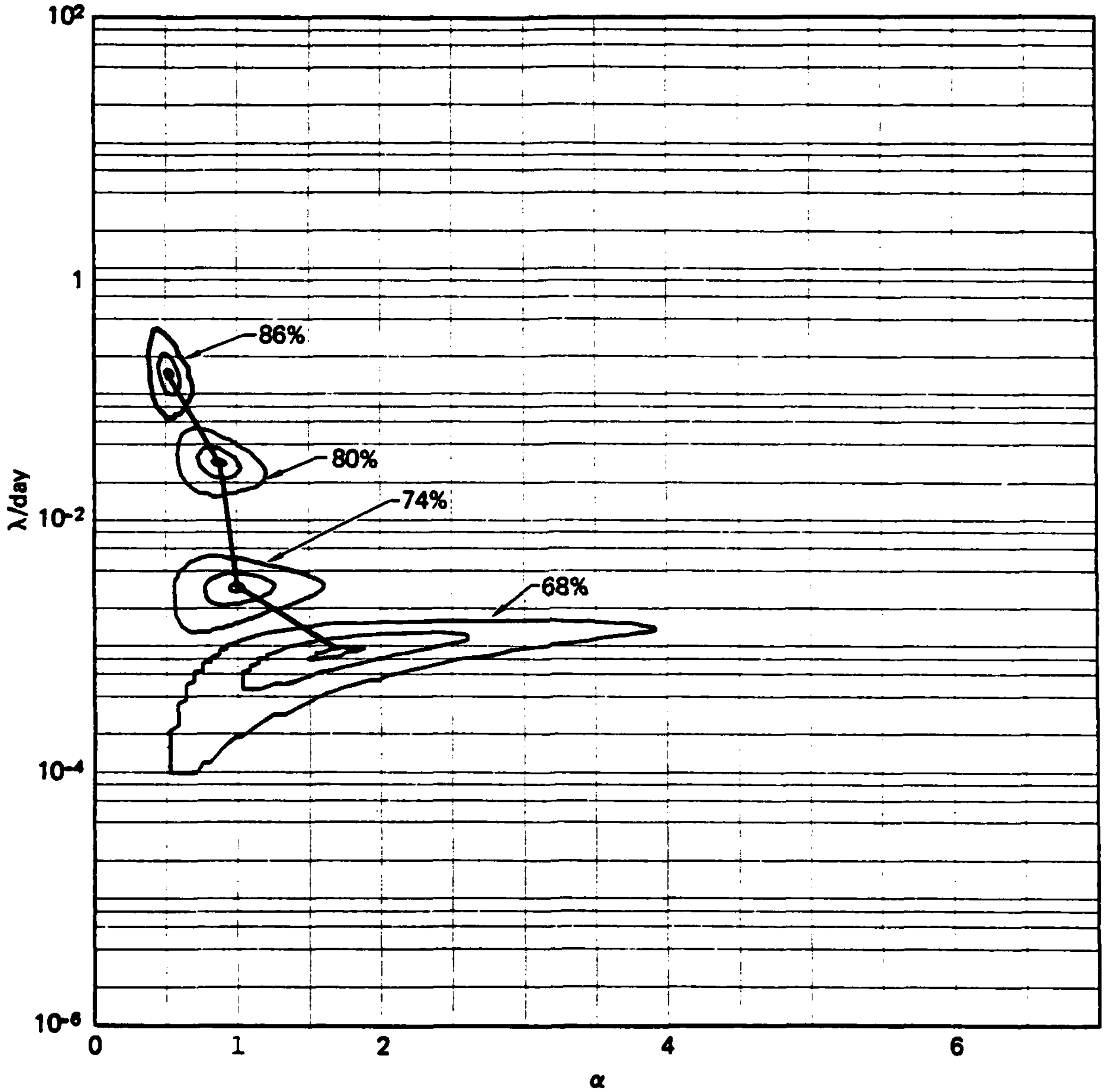

Fig. 2. Weibull posterior density confidence contours for α and λ for Kevlar pressure vessel life test data compiled as of December 1978.

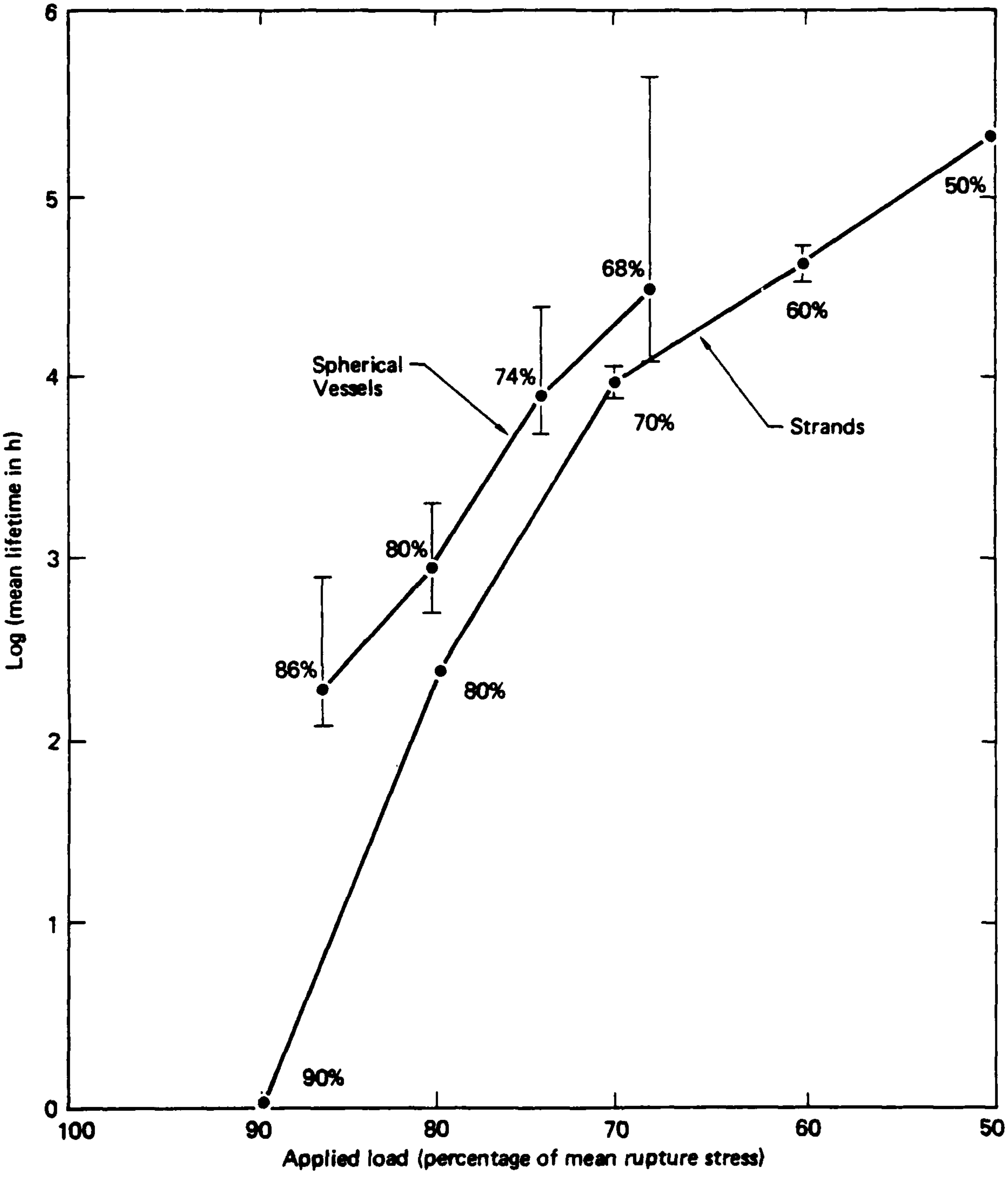

Fig. 3. Comparison of log mean life of strands and spherical pressure vessels. Vertical intervals are 95% credible intervals.

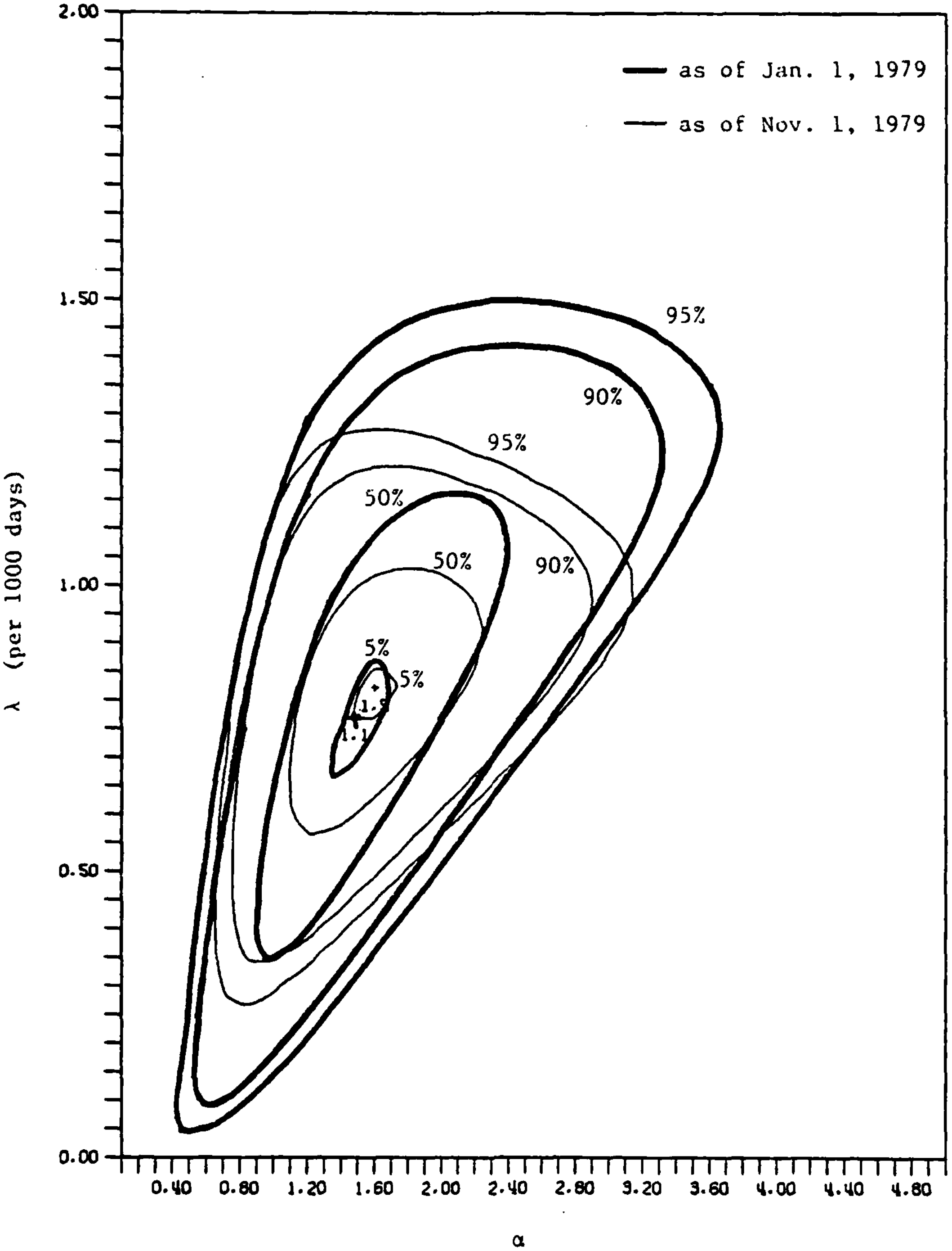

FIGURE 4

HIGHEST PROBABILITY DENSITY CONTOURS FOR α AND λ FOR KEVLAR/EPOXY PRESSURE VESSEL LIFE TEST DATA. THE PRESSURE VESSELS WERE TESTED AT 68% STRESS LEVEL.

Currently, work is in progress on formulating mathematical models relating life length and stress history. These models are based on the physics of failure of strands and pressure vessels as it is presently understood.

3. EXPECTED ENTROPY AS A MEASURE OF INFORMATION

A qualitative measure of the information gained by an additional year of testing can be deduced from the tighter contours in Figure 4. However, for the purpose of planning new experiments we need a quantitative measure of information which will allow us to compare alternative strategies. This can be obtained by specifying a utility function as a function of unknown parameters and decisions to be taken. If costs cannot be easily incorporated into a utility function, Lindley (1956) and Bernardo (1979) suggest using entropy.

Suppose a parameter θ takes values $\theta_1, \theta_2, \ldots, \theta_N$. In practice, N depends on the accuracy with which we wish to know θ and N must be specified in advance. Let $\pi(\theta_i)$, $i = 1,2, \ldots, N$ be the initial probabilities and $\pi(\theta_i \mid D, E)$ be the posterior probability of θ_i given data D, from an experiment E. The entropy corresponding to π is

$$-\sum_{i=1}^{N} [\ln \pi(\theta_i)]\pi(\theta_i) \; .$$

It is well known that the maximum is attained with $\pi(\theta_i) = \frac{1}{N}$ for $i = 1,2, \ldots, N$ so that

$$-\sum_{i=1}^{N} [\ln \pi(\theta_i)]\pi(\theta_i) \leq \ln N \; .$$

Perfect information about θ would correspond to the density $\pi_0(\theta_{i_0}) = 1$ and $\pi_0(\theta_i) = 0$, $i \neq i_0$. The corresponding entropy is

$$-\sum_{i=1}^{N} [\ln \pi_0(\theta_i)]\pi_0(\theta_i) = 0$$

with the convention that

$$0 \ln 0 = 0 \ .$$

The expected (negative) change in entropy due to performing an experiment $\mathcal{E}$ is

$$I(\mathcal{E}) = -E\left[\sum_{i=1}^{N} [-\ln \pi(\theta_i \mid D,\mathcal{E})]\pi(\theta_i \mid D,\mathcal{E})\right]$$
$$+ \sum_{i=1}^{N} [-\ln \pi(\theta_i)]\pi(\theta_i) \ .$$

Expectation is with respect to the sample space. Lindley (1956) proposed this measure of information of $\mathcal{E}$ and showed that

$$I(\mathcal{E}) \geq 0 \ .$$

Bernardo (1979) showed its connection with expected utility. If costs are to be considered, then other measures would be more appropriate.

If we were to perform the perfect experiment (e.g. infinite sample size) then the posterior distribution would be degenerate and in this case

$$I(\mathcal{E}) = -\sum_{i=1}^{N} [\ln \pi(\theta_i)]\pi(\theta_i) \leq \ln N \ .$$

We call this the expected value of perfect information, EVPI. This provides a standard against which we can judge the worth of our experiment.

Lindley shows that $I(\mathcal{E})$ as a function of sample size, n, is concave. Hence we can plot $I(\mathcal{E})$ as a function of n as in Figure 5.

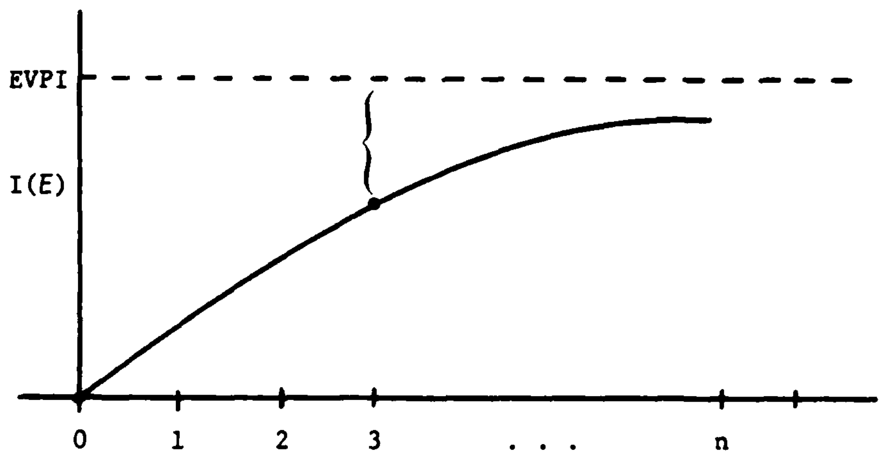

FIGURE 5

INFORMATION, I(E) , AS A FUNCTION OF SAMPLE SIZE, n

The worth of an experiment might be judged by $EVPI - I(E)$ or perhaps as a percentage of the EVPI.

Lindley (1956) defines information for probability densities with respect to Lebesgue measure. Unfortunately, the expected value of perfect information is infinite in this case so that we have no meaningful standard for comparison purposes. For this reason we have confined discussion to discrete measures.

SUMMARY

Accelerated life test data can be fully analyzed using posterior density contour plots. The change in contour plots as we gather more data is a measure of information gained. Expected entropy provides a kind of nonparametric expected utility which disregard costs. This can be used as a measure of expected information to be gained as a result of performing a planned experiment. However, in order that the expected value of perfect information be finite our prior and posterior distributions must be discrete. The expected value of perfect information is always less than ℓnN where N is the number of parameter values we wish to distinguish.

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