HIGH TORQUE-TO-INERTIA SERVO
SYSTEM FOR STABILIZING
SENSOR SYSTEMS

CANDIDATE SYSTEMS INCLUDE MISSILE GUIDANCE,
SURVEILLANCE, AND TRACKING

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THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.
This is a technical report on the design and development of a high torque-to-inertia servo system for stabilizing a sensor system. The design philosophy leads to a low cost/high performance system. The stabilizing element developed is universal and has application for 1) missile guidance, 2) surveillance, and 3) tracking sensor systems. The servo design is based on math models and is used to develop performance specifications and perform evaluations.
OBJECTIVE

Establish the design and develop a servo system for space stabilizing and command positioning a sensor system. Provide growth potential in the design for alternate sensors or sensor weight to be added to the gimbal structure without servo system performance degradation.

RESULTS

1. A high torque-to-inertia servo system for space stabilizing a gimbaled sensor was developed. The high torque-to-inertia concept led to a low-cost design configuration with multisensor growth potential which would allow additional weight to be affixed to the gimbal structure.

2. Math model designs were formulated for the sensor system/stabilization platform in time- and frequency- domains.

3. A system control interface was developed to test and monitor the servo system in both the stabilization and slave command modes.

4. Performance levels were established for the stabilization and the slave command modes of operation.

RECOMMENDATION

Review some of the gimbal mechanical designs for minimizing friction.
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1. INTRODUCTION

This report covers the design and development of a universal servo system for space stabilizing a sensor system for tracking applications.

1.1 BACKGROUND ON SPACE STABILIZED SENSORS FOR MISSILE GUIDANCE

Space stabilized sensors for missile guidance are used for the following reasons:

1. They provide an inertial reference from which the line of sight rate can be measured.

2. Body motion is decoupled from the guidance signals.

3. The target is tracked (sensor system pointed at the target through a mechanical gimbal system) where the sensor system is maintained boresighted on the target.

Alternate ways of achieving the above characteristics have been derived using "fixed body" (sensor system centerline fixed relative to the missile body centerline) sensor systems. An example of the fixed body sensor is the electronic beam steering sensor by means of phased array antennas. The fixed body systems are rather atypical of tactical missiles and will not be specifically addressed. These beam steering systems are in many ways analogous to the gimbaled system as far as the resulting overall missile guidance is concerned.

This report presents in detail the design and development of a space stabilizing sensor system platform. The aspects of how and why gimbaled stabilization platforms achieve the above criterion (inertial reference, body motion decoupling, and target tracking) will be expanded upon.

The design philosophy is based on a high torque-to-inertia ratio which is detailed in the "Stabilized Platform Design" section.

The primary emphasis of this report relates to a space stabilization platform used for stabilizing a missile guidance sensor. The platform, however, is not restricted to this application. It is equally well suited to other applications such as stabilizing a sensor for surveillance and data gathering systems. In essence the development presented in this report relates to a state of the art/high technology space stabilizing sensor platform. Many systems, whether they be missile guidance, surveillance, or other type of tracking systems require that the tracking sensor be space-stabilized. This report covers the various phases of the development of a high torque-to-inertia space stabilizing sensor platform with universal respect to a number of applications. Reference 1 presents examples of systems that could utilize stabilized platforms for surveillance sensor or missile guidance systems.

2. MATH MODEL DEVELOPMENT

A math model for the stabilized platform was developed and from this model design specifications and predicted system performance were established.

2.1 SENSOR MEASUREMENT GEOMETRY

The fundamental function of a space-stabilized sensor is to measure and provide estimates of these measured guidance signals to the missile control section. The fundamental measurement made by the sensor is the angular position of the target relative to the sensor centerline or boresight. The angular information required for missile guidance control is the angular line of sight rate. This assumes that missile guidance is via proportional navigation. Missile guidance control can be structural around a number of different guidance laws, reference 2. The guidance law assumed in this document was proportional navigation.

Figure 1 illustrates the geometry for the sensor/missile/target angular relationships.

A block diagram can be generated based on the geometric relationships described in figure 1.

\[ e = \left(a - v_m - \theta_h \right) T_1 (s) \]  

where \( T_1 (s) \) is the sensor transfer function.

This equation can be translated to a diagram using two summing junctions as follows:

![Diagram](image)

Figure 2. Block diagram of equation 1.

Using rates rather than position quantities, figure 2 is changed as follows:

![Diagram](image)

Figure 3. Block diagram of measured boresight error signal. \( \dot{\omega}, \dot{\theta}_m \) and \( \dot{\theta}_h \) are rate quantities.

To be an inertial reference a device must maintain a fixed attitude in space or be space stabilized. Space stabilization is accomplished by means of a rate sensor mounted on the element to be fixed or stabilized in inertial space. Missile body motion or base motion, body rate \( \dot{\theta}_m \) is sensed by the rate sensor which generates an output voltage proportional to the missile body rate. The rate sensor output voltage is used to generate a head rate opposing the missile body rate. The net rate sensor input then becomes the difference between head rate and missile body rate. The purpose of this servo loop is to maintain zero head rate relative to inertial space. The degree to which stabilization can be achieved is determined by the loop gain. For stability, a compensation/shaping network is included in the loop which maintains a required phase/gain margin.
Incorporating the elements of the stabilization loop into a block diagram relative to the geometry of figure 1 is illustrated in figure 4.

![Block Diagram](image)

Figure 4. Stabilization loop.

Figures 3 and 4 are combined relative to the geometry of figure 1 which results in stabilized tracking sensor system which is shown in figure 5.

![Block Diagram](image)

Figure 5. Sensor tracker system.

2-3
The following definitions hold for the transfer functions and variables in figure 5:

\( T_1(s) \) is Sensor Transfer function

\( *T_2(s) \) is Torque Source – Servo Drive/Load Inertia transfer function

\( T_3(s) \) is the electronic servo gain and compensation networks Transfer functions

\( T_4(s) \) is the Transfer function of the rate Sensor which is physically mounted to the gimbal structure.

\( e = \hat{\phi} \) is the measured Line of Sight (LOS) rate or the estimated LOS rate.

\( \hat{\delta}_m \) is the measured value of the missile body angular rate in a particular plane.

\( \hat{\delta}_h \) is the measured value of the Sensor gimbal angular rate in a particular plane.

Figure 5 is the starting point for the design, development and analysis of a space-stabilized sensor system.

2.2 MATH MODEL

Figure 5 was formulated based on sensor/missile/target angular relationships. Figure 5 can be reconfigured as shown in figure 6.

*Usually consists of torque motor, torque motor servo amplifier, and load, which includes gears, gimbals and sensor load inertia.

Figure 6. Generalized block diagram of sensor/stabilized platform.
$T_4(s)$ has the units of volts/rad/sec.
$T_1(s)$ has the units of volts/radian.
$T_3(s)$ has the units of volts/volt.
$T_2(s)$ has the units of rad./sec./volt.

The stabilization and the track loops are the fundamental servo loops that are designed together since $t$, $y$ are the heart of the stabilized platform system. The slave loop is independent of the track loop and can be designed separately. Therefore, two models must be developed:

1. Stabilization/Track, and
2. Slave loop models.

The stabilization loop provides the body motion decoupling properties that were mentioned earlier as a desired requirement of the space stabilized platform. The stabilization loop, also referred to as the stab loop, is drawn separately as illustrated in figure 7.

![Figure 7. Generalized block diagram of stabilization loop.](image-url)
The transfer function of the gimbal movement ($\hat{\theta}_h$) as a function of steering commands ($\hat{\theta}$) is as follows:

$$\frac{\dot{\theta}_h}{\hat{\theta}} = \frac{T_3(s) T_2(s)}{1 + T_3(s) T_4(s) T_2(s)}$$  \(3\)

where the assumption is made that $\dot{\theta}_m$ is zero.

The transfer function of the gimbal movement ($\hat{\theta}_h$) as a function of body motion inputs is as follows.

$$\frac{\dot{\theta}_h}{\theta_m} = \frac{-T_4(s) T_3(s) T_2(s)}{1 + T_4(s) T_3(s) T_2(s)}$$  \(4\)

It is easily seen that for the magnitude of $T_2(s) T_3(s) T_4(s)$ large or

$$|T_2(s) T_3(s) T_4(s)| > 1$$  \(5\)

that

$$\theta_h \approx -\theta_m$$

This illustrates the body motion decoupling principle of a space stabilized platform. Another way of looking at the body motion decoupling properties of the space-stabilized platform is to examine figure 5. Body motion decoupling implies that the platform subtracts out the motion caused by missile body movement, $\theta_m$, from the desired estimated line of sight guidance signal, $\hat{\theta}$. Figure 5 is reconfigured as shown in figure 8.

By applying the condition of equation (5) to the transfer function block where body motion feeds into the platform loop, the block diagram illustrated in figure 8 can be simplified as shown in figure 9.

Note that the body motion term $\dot{\theta}_m$ cancels itself out of the steering command, $\hat{\theta}$. In section 4.0 a detailed analysis and data are presented on the level of body motion that is contained (or corrupts) the desired missile steering command ($\hat{\theta}$). Of course it is immediately obvious that the desired situation is as modeled in figure 8. There is no corruption of the steering command; however, this is not achievable in reality. Some level of body motion will corrupt the missile guidance steering command. It is this level of corruption that is expanded upon in section 4.0.

2.2.1 Torque Source Model – $T_2(s)$

The torque source and type of configuration are the basic determinations that must be made to quantify the gimbal drive element or the blocks that make up the $T_2(s)$ transfer function of figure 5. Actually the torque source is only one element of the $T_2(s)$ transfer function. $T_2(s)$ is the overall transfer function of the servo amplifier, torque source and gimbal/load. The torque source was selected as an electrical dc, armature controlled torque motor. Other alternatives could have been hydraulics, pneumatics, or electrical motor/clutch drive systems. For the particular guidance sensor environment, torque requirements
Figure 8. Block diagram of stabilized platform.

Figure 9. Simplified block diagram of stabilized platform.
and packaging constraints the dc armature controlled torque motor approach was selected. Once this selection was made the configurations as to the control of the torque source needed to be determined.

Basically there are two choices of drive control, voltage, or current. The parameters for relating the control (voltage or current drive) are: (1) system response, (2) velocity error constant, and (3) isolation to extraneous signals and torque sources. A model for each of the control drives is illustrated in figures 10 and 11 (10 is the Current Drive and 11 is the voltage drive servo control). Derivation of each of these models are presented in Appendix C.

In figure 11, the voltage control configuration, it is noted that the servo amplifier is outside the feedback path of the servo motor as compared to the configuration of figure 10, the current control servo drive system. The servo motor and the load for the system of figure 11 is relatively independent of the servo amplifier. The servo amplifier gain is strictly an electronic gain. Therefore, in a response comparison between the voltage drive and the current drive systems, the voltage drive system would consist of the motor/load elements while the current drive system would include the servo amplifier characteristics. Figure 12 shows a simplified block diagram of the servo drive and the electronic gain/compensation elements of a servo system.

For a comparison the transfer function $V_R/V_C$ must be evaluated for both types of drive systems.

The transfer functions for the two different servo drive controls are:

\[
T_4(s) = \frac{\dot{\theta}_h}{V_c} = \frac{A_0 K_T N}{L_a J_t r_a S^3 + (L_a r_a D + r_a J_t R_a + L_a J_t) S^2 + (D L_a + r_a D R_a + J_t R_a + A_0 R_{sb} J_t + K_T K_E N^2 r_a) S + R_a D + A_0 R_{sb} D + N^2 K_E K_T} \tag{6}
\]

for current drive, and

\[
T_4(s) = \frac{\dot{\theta}_h}{V_c} = \frac{A_{CL} K_T N}{L_a J_t S^2 + (J_t R_a + D L_a) S + R_a D + N^2 K_T K_E} \tag{7}
\]

for voltage drive.

The above equations (6 and 7) can be analyzed using inverse La Place transform theory. That is, the corresponding time responses for each type of system can be evaluated for identical inputs. A time domain analysis using an integration routine on the computer can also be used to evaluate the time domain response of the servo drive systems. The La Place transform technique does not take into consideration the nonlinearities of the system while the time integration procedure does. Both of these analysis techniques as well as the computer programs are contained in Appendix E. The current drive system is a third-order system (due to amplifier dynamics) and the voltage drive is a second-order system.
Figure 10. Block diagram of current drive servo control.
Figure 11. Block diagram of voltage drive servo control.

\[ A_{CL} = \frac{\beta - 1}{\beta} \]

WHERE \( \beta \) IS THE VOLTAGE FEEDBACK OR RATIO OF THE INPUT \( R_1 \) TO INPUT & FEEDBACK RESISTOR:

\[ T_a' = \frac{T_a}{\beta A_0 \left( \frac{R_i}{R_i + R_c} \right)} \]

Figure 12. Servo drive system.
The eigen values for each of these systems are evaluated. The location of the roots of these two different types of systems will specify the kind of performance that can be achieved. Specifically, the characteristic roots govern the behavior of the system; therefore, as part of the evaluation criterion to establish which servo control will be used a close look will be taken at where in general the roots lie for these two systems. The transfer functions in expanded form are presented in equations (6) and (7). A typical set of values for a candidate torque motor is as follows:

\[
\begin{align*}
Ra &= \text{armature resistance} = 3.0 \Omega \\
La &= \text{armature inductance} = .0014 \text{ Henries} \\
K_T &= \text{motor torque constant} = 24.8 \text{ in-oz/amp} \\
K_E &= \text{back EMF constant} = .177 \text{ V/Rad/sec}
\end{align*}
\]

The driving amplifier (servo amplifier) open loop parameters are:

\[
\begin{align*}
A_o &= \text{open loop gain} = 100,000 \text{ volt/volt} \\
\tau_o &= \text{time constant} = .02 \text{ sec}
\end{align*}
\]

The parameters that are not specifically known or defined, but are in a relative ballpark are the load inertia and viscous damping. Typical values of these parameters are in the following range:

\[
.5 \leq J_L \leq 5.0
\]

\[
.1 \leq D \leq 3.0
\]

The gear ratio is the parameter that is not fixed. This parameter can be selected to optimize the performance of the servo drive system. The range of values for the gear ratio (for practical considerations) range from direct to gear ratios of around 50.

Appendix E (figures E-6 and E-7) presents a set of data on gear ratio parameter variations to establish the optimum gear ratio.

Returning to equations (6) and (7) with the above values for the parameters it is seen that the low frequency eigen value for the current drive system is very close to zero and the other two roots, which are complex, are far-out roots and do not influence the response in the region of interest. For the outer gimbal the three eigen values are:

\[
S = -1.59
\]

and

\[
S = -1.096 \times 10^3 \pm 5.975 \times 10^4
\]

The dominant root is the real root at \(-1.59\).

The voltage drive configuration has two real roots. The dominant root sets at

\[
S = -34.1986
\]
with the other root at

\[ S = -2108.8168 \]

The dominant root for the current drive system is 215 times closer to the origin. This real root near the origin greatly increases the low frequency gain and therefore the velocity error constant of the current drive system as compared to the voltage drive system.

The time response data for each system with a step input is illustrated in figures E-2 and E-3. For the obvious reasons illustrated in these figures the current drive system was chosen as the type of servo drive for the stabilized sensor platform.

2.2.2 Stabilization/Track Loop Models

Basically the derivation of the math model for the stabilization/track loops comes from the generalized block diagram presented in figure 6. The three major elements that make up the stabilization loop are the rate sensor \( T_4(s) \), the torque source/load \( T_2(s) \), and the electronic gain/compensation \( T_3(s) \). The rate sensor is an off-the-shelf item. The specifications of the given rate sensor depends in large upon the specific requirements it must meet for required performance and operating environment. Appendix A presents in detail the definition of the rate sensor, its characteristics/specifications, and rationale for selection.

General specifications for the rate sensor chosen (Honeywell two-axis rate sensor, GG2500) are presented in table 1, and a cutaway view and an outline drawing showing size and configuration are shown in figure 13.

2.2.2.1 Rate Sensor Model \( - T_4(s) \). The transfer function for the rate sensor/readout electronics is

\[ T_4(s) = \frac{-K_{\text{MHD}} K_2}{(r_4 s + 1)^3} \]  

(8)

where \( K_{\text{MHD}} \) is the rate sensor gain in \( \text{VRMS/rad/sec.} \) and \( K_2 \) is the demodulator gain in \( \text{VDC/VRMS} \). The denominator of equation (8) defines a third-order low pass noise filter.

2.2.2.2 Stabilization/Track Loop Compensation Model \( - T_3(s) \). A lag-lead type of compensation was chosen to stabilize the servo loop and implement the high acceleration gain and set the bandwidth. The lag portion of the compensation allows a higher low frequency gain to be achieved and thereby a higher acceleration gain. The type of lead compensation chosen provides a minimum phase at a design frequency which sets the closed loop bandwidth. The gain is adjusted so that the zero dB crossover is at the minimum phase location. There is a fair amount of latitude available with this kind of design philosophy. If more loop gain is required for stabilization, isolation properties and acceleration properties, the lag portion of the compensation network can be adjusted. An integrator is incorporated
<table>
<thead>
<tr>
<th>Parameter</th>
<th>GG2500LC02: 15 ± 5% mv rms/deg/sec</th>
<th>GG2500LC03: 15 ± 1% mv rms/deg/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Factor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero Rate F. ror (includes run-up repeats)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linearity</td>
<td>0.1% of max. rate (max dev from best str line)</td>
<td>0.15 deg/sec max.</td>
</tr>
<tr>
<td>Cross Coupling (axis change vs. input rate)</td>
<td>0.5% of full scale (max dev from best str line)</td>
<td>(1)</td>
</tr>
<tr>
<td>Hysteresis</td>
<td>0.01 deg/sec max.</td>
<td></td>
</tr>
<tr>
<td>Threshold</td>
<td>0.01 deg/sec max.</td>
<td></td>
</tr>
<tr>
<td>Acceleration Sensitivity</td>
<td>0.05 deg/sec/g max.</td>
<td></td>
</tr>
<tr>
<td>Output Noise at Null</td>
<td>100 mv rms max. (using 1000-Hz bandwidth meter)</td>
<td></td>
</tr>
<tr>
<td>Rate Input Range</td>
<td>± 480 deg/sec</td>
<td></td>
</tr>
<tr>
<td>Frequency Response</td>
<td>100 Hz min. without electronics</td>
<td></td>
</tr>
<tr>
<td>Ref Gen Output</td>
<td>1V min. rms each axis</td>
<td></td>
</tr>
<tr>
<td>Ref Gen Phase Angle</td>
<td>90 ± 0.5 degrees</td>
<td></td>
</tr>
<tr>
<td>Performance Stability With Environments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zero Rate Error Stability Over All Environments</td>
<td>0.15 deg/sec</td>
<td></td>
</tr>
<tr>
<td>Acceleration Sensitivity Stability Over All Environments</td>
<td>0.03 deg/sec/g</td>
<td></td>
</tr>
<tr>
<td>Scale Factor Change - vs - Temperature</td>
<td>± 2%</td>
<td></td>
</tr>
<tr>
<td>Input Axis Change - vs - Temperature</td>
<td>± 0.5 deg</td>
<td></td>
</tr>
<tr>
<td>Excitation Requirements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motor</td>
<td>26 ± 2 volt rms 400 Hz 20°, 4 watts max.</td>
<td></td>
</tr>
<tr>
<td>Preamp</td>
<td>± 15 ± 3 Vdc, 4 ma max. with 500 mv max p-p ripple</td>
<td></td>
</tr>
<tr>
<td>Environments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overrange Capability</td>
<td>20,000 deg/sec</td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>-65°F to +160°F</td>
<td></td>
</tr>
<tr>
<td>Vibration</td>
<td>MIL-STD-810, Method 514, Proc II</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 hr/axis – time schedule V of Table 514-II, Curve H</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10g peak sine)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1/2 hr/axis – time schedule II of Table 514-II, Curve Q</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10g peak sine)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1/2 hr/axis – time schedule II of Table 514-II, Curves AH</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(11.9g rms random) and AK (20.7g rms random)</td>
<td></td>
</tr>
<tr>
<td>Shock</td>
<td>2 drops/axis each direction, 12 drops total each level: 40g, 18 ms; 400g, 1.5 ms; 100g, 6 ms; 500g, 0.75 ms; MIL-STD-810, method 516, proc IV</td>
<td></td>
</tr>
<tr>
<td>Acceleration</td>
<td>100g, each direction – each axis</td>
<td></td>
</tr>
<tr>
<td>Useless Life</td>
<td>Life tested to 1000 hours</td>
<td></td>
</tr>
<tr>
<td>Temperature Shock</td>
<td>MIL-STD-810, method 503, proc I + 71°C to -54°C to +71°C, four (4) hours each temp – 5 minutes between chambers</td>
<td></td>
</tr>
</tbody>
</table>

(1) When operated with amplifier-demodulator readout electronics. Deviation is expressed as a percent of opposite axis full scale.

Table 1. MHD rate sensor specifications (GG2500LC02 and GG2500LC03).
into the compensation network to yield a type one system. References 3, 4, and 5 present compensation techniques for these types of servo system. The compensation transfer function is

\[ T_2(s) = \frac{K_3 (\tau_2 S + 1) (\tau_2' S + 1) (\tau_5 S + 1)}{S (\tau_3 S + 1) (\tau_3' S + 1) (\tau_6 S + 1)} \]  

(9)

The lead time constants are \( \tau_2, \tau_2', \tau_3, \) and \( \tau_3' \) where

\[ \tau_2, \tau_2' > \tau_3, \tau_3' \]  

(10)

The lag time constants are \( \tau_5 \) and \( \tau_6 \) where \( \tau_5 < \tau_6 \).

2.2.2.3 Stabilization/Track Loop Math Model. The complete math model for the stabilization/track loops model is defined in the block diagram of figure 14. Included in this diagram is the saturation nonlinearities which are due to amplifier saturations of the various amplifiers in the servo amplifier drive system, the compensation network and the rate sensor feedback system. Appendix B presents data on the servo amplifier used in the system. This math model is the basis for the synthesis/design of the stabilized platform track mode. All of the analysis for performance evaluation was accomplished using this model.

A number of items need further exploration. The output of the sensor is the all important estimate of the line-of-sight rate, designated $\dot{\theta}$. This value is in units of volts, therefore, it is related to the angular values of radians through a scale factor. This scale factor is the product $K_2$ times $K_{MHD}$.

The estimated value of the line-of-sight is the quantity that is inputted to the flight control systems of a guided missile. Since this parameter is referenced to a scale factor; that is, a specific voltage equates to a given line-of-sight rate through a linear relationship, and any gains in the loop which are not constant would contribute to an error in the estimate of the line-of-sight rate. Gain variations are caused by a number of different phenomena such as temperature, acceleration bias, nonlinear seeker guidance measurements and radome error slopes. These items are mentioned at this time to point out that total missile performance is a function of the sensor/stabilization platform's ability to generate accurate line-of-sight measurements. The major contributor to the line-of-sight rate degradation relative to the platform design is the effect of acceleration bias on the rate sensor in the feedback loop. The sensor gains and radome error slope degradation to the line-of-sight rate is not a function of the platform design. The track loop model does include the sensor function but this is only for performance evaluations. There are numerous references on the sensor nonlinearity gain and radome error slope degradation of the line-of-sight rate, see references 6 and 7. Since the rate sensor is a major element of the stabilization platform, the effect of acceleration bias and how it affects the estimated line-of-sight rate ($\dot{\theta}$) will be examined. As an example refer to the data presented in table 1. We find that the acceleration sensitivity of the MHD rate sensors (GG2500LC02 and GG2500LC03) is 0.05 deg/sec/g max. For a 10 “G” acceleration the rate sensor will contribute a 0.5 deg/sec error in the estimated line-of-sight rate. In this example suppose that $K_2 = 14$ volts Dc/volts rms and that $K_{MHD} = 0.8595$ volts rms/rad./sec and assume that the platform is moving at a rate of two degrees/sec. The voltage being sent to the flight control system is then 0.4190255 volts. Now if the platform, while moving at a two deg/sec rate is subjected to a 10 “G” acceleration, the voltage being sent to the flight control system will be 0.3149986 volt which is an erroneous input to the autopilot since the estimated line-of-sight rate is actually two degrees/sec, but the autopilot is looking at a 1.5 degree/sec. signal which it takes as the line-of-sight rate.

2.2.3 Slave Loop Model

The slave loop, shown in figure 15, is a position type one servo system. The function of the slave loop is to position the sensor or slave the movement of the sensor to an

Figure 14. LADSS stabilized platform track/stabilization loops block diagram.
input command signal. For a position command following type of servo system, the final steady-state position error is a function of the position constant, $K_p$.

The steady-state error for a type 1 system for a position input is zero.

$$K_p = L(s)\big|_{s=0}$$  \hspace{1cm} (11)

where

$$L(s) = T_1(s)T_2(s)T_3(s)$$  \hspace{1cm} (12)

is the loop transfer function.

JJ D'Azzo, (ref. 8), presents a chapter on basic servo system characteristics relative to the steady-state performance of the system.

2.2.3.1 Slave Loop Compensation. The slave loop compensation block is essentially a lag-lead network. The electronic gain or loop gain adjustment is also contained in this block. This type of compensation allows the gain margin, phase margin, and bandwidth to be specified. Contained within the compensation block is a nonlinearity as a result of the limiting of the electronic amplifier used to synthesize the network. Figure 16 illustrates the model of the compensation network.

2.2.3.2 Slave Loop Math Model. The complete math model of the slave loop is presented in figure 17. This figure represents the elements comprising the position Type 1 servo system. The servo drive/torque source/load element is identical to that shown in figure 14, a block diagram of the track/stabilization loop.

Figure 16. Slave loop compensation network.
Figure 17. Slave loop of sensor servo platform.
3. STABILIZED PLATFORM DESIGN

3.1 DESIGN PHILOSOPHY

The paramount design criterion was a high torque-to-inertia ratio for the following reasons:

(1) Leads to a low cost design.
(2) Allows growth to multiple sensors on the stable member of the platform.
(3) Achieves high isolation from unwanted inputs such as base or body motion and extraneous torques.
(4) Has high acceleration and high velocity performance.

With the high torque-to-inertia ratio as the basic approach, the design philosophy addressed two major areas: 1) mechanical design, and 2) servomechanism design. The mechanical design addressed the gimbal type and drive arrangement. The servo design addressed the servomechanism performance and specifications for the track/stabilization and slave loops independently. Performance specifications for both the frequency and time domain were established. Once these specifications were established the problem was reduced to that of a synthesis problem or one of formulating a system that will meet the desired performance specifications. This, to say the least, is not an easy problem. There are many approaches to the synthesis problem each of which may lead to a unique solution. In control system engineering problems there are many solutions that could conceivably satisfy a set of performance specifications. It is the synthesis problem and the solution thereof that present the greatest challenges to the control system engineer. It is in this area that creativity and ingenuity applied with basic engineering are the ingredients for the desired solution.

3.2 MECHANICAL DESIGN

A two degree of freedom stabilized platform consists of a set of gimbals such that a member attached to the gimbal set can be positioned about a pivot point (virtual or fixed) in any of a number of positions within the gimbal travel limits. Figure 18 is a simplified representation of two degrees of freedom motion for a member attached at a pivot point on a fixed base. There are a number of ways that the two degrees of freedom of motion can be achieved through various gimbal schemes. Usually these gimbal arrangements are called inner and outer gimbals. Each gimbal has a separate drive source to move the gimbal. It turns out that for an inner/outer gimbal arrangement at least one of the gimbal drive torque sources must be physically displaced with the gimbal movement (for a geared torque drive system). The torque source that moves or is physically displaced when the gimbals move is the inner gimbal drive source. The outer gimbal drive source can be made stationary, i.e., it is not physically displaced with gimbal movement. The inner gimbal drive source can be attached to the inner gimbal and thus will be physically displaced when the inner gimbal moves, or it can be attached to the outer gimbal and will only be displaced when the outer gimbal moves. There are advantages to this last arrangement, inner gimbal drive physically attached to outer gimbal. Figure 19 illustrates this concept. The outer gimbal is a semicircle
Figure 18. Simplified pivot platform.

Figure 19. Inner/outer gimbal configuration showing inner gimbal torquer attached onto the outer gimbal.
(bail ring) which is driven, through a gear train, by a fixed stationary torque source. The bail ring choice for an outer gimbal has the following advantages:

1. It leads to a virtual pivot point which allows the sensor or load to be physically positioned at the pivot point. (In many sensor applications this is extremely important.)
2. It lends itself to larger/heavier sensor loads because volume within the semi-circle can be utilized for sensor packaging.
3. The sensor/load is physically attached at two points (inner gimbal bearing points) which allows the heavier/stiffer loads and maintains higher mechanical resonances.

The advantages of the inner gimbal drive source being physically attached to the outer gimbal is that the torque source can be made physically much larger yielding a larger inner gimbal torque value. Traditionally the inner gimbal torque source has been physically located on the inner gimbal. With this arrangement movement of the inner gimbal consisted of the load and the torquer. The weight of the torquer adds directly to the inner gimbal inertia and therefore is of prime concern since the driving design parameter is a high torque-to-inertia ratio. With this arrangement (inner gimbal torque source physically located on an inner gimbal) not only are smaller torque values achievable, but the total inner gimbal inertia is increased leading to a smaller torque-to-inertia ratio, rather than a larger torque-to-inertia ratio. In contrast, the design arrangement that places (physically mounts) the inner gimbal torque source on the outer gimbal leads to a high torque-to-inertia design. The inertia of the outer gimbal is increased by the addition of the inner gimbal torquer; however, the outer gimbal torquer is mounted to a reference that does not move with either inner or outer gimbal movement. It can therefore increase in size yielding a larger torque value to drive the outer gimbal with the added weight.

The inner/outer gimbal and torque drive arrangement shown in figure 19 has another paramount advantage – it lends itself to an optimum gearing arrangement. Figure 20 presents a typical curve showing speed of response to gear ratio. It is seen from this figure that a direct drive system is far from optimum. In fact it approaches the same performance as a very high gear ratio system. The semicircle bail ring/inner gimbal drive located on outer gimbal design is of a geometrical shape and configuration such that optimum gearing can be taken advantage of.

### 3.3 SERVO MECHANISM DESIGN

As mentioned earlier, two separate independent servo loops are used in the sensor platform: 1) Track/stabilization loop used for null tracking a target to generate an estimate of the line-of-sight rate and decouple base or body motion from the estimated line-of-sight rate and provide a space stabilized inertial reference, and 2) Slave loop used to position the sensor or slave the sensor to a commanded position input. Figures 15 and 17 are block diagrams of these two servo loops respectively. Since these two loops are independent, each will be treated as separate designs.
Figure 20. Typical response curve as a function of gear ratio and load inertia.

A. PERFORMANCE SPECIFICATIONS

Performance specifications (ref 9) may be divided into three sub groups:

1. Frequency-domain specifications.
2. Time-domain specifications.
3. Specifications on statistical bases.

The first two are the most popular and dominate the literature. These will be used to establish the design parameters and performance specifications of the servo loops.

B. FREQUENCY-DOMAIN SPECIFICATIONS

Frequency-domain specifications are those which relate to the relationships between the sinusoidal input and outputs of a servo mechanism. A list of the more common frequency-domain specifications found in the control systems literature are:

1. Bandwidth

2. Phase margin (and crossover frequency)
3. Gain margin
4. M peak (and peak frequency)
5. Deviation ratio
6. Error-constant-bandwidth ratio
7. Output impedance
8. Gain-bandwidth product

Not all of these specifications are mutually exclusive. A more concise list of the frequency-domain performance specifications are

1. Bandwidth BW
2. M peak M_p
3. Peak frequency \( \omega_p \)
4. Output impedance Z

Figure 21 presents a typical magnitude plot from which the definition of the frequency-domain performance specifications are illustrated.

In figure 21 BW_3 is the three dB bandwidth. This is the servo bandwidth normally referred to as just BW. The BW_6 bandwidth is referred to as the 6 dB bandwidth. |T(j\omega)| is the magnitude of the total closed loop system transfer function as a function of j\omega. (See ref 14 for bandwidth definitions.)

Each of the basic frequency-domain specifications will be discussed briefly.

---

Figure 21. Frequency-domain performance specification definitions.

C. BANDWIDTH BW

BW is probably the most significant performance specification as it gives indication of the speed of response. Horowitz [ref 3], pages 192 and 194, presents an empirical formula with the rise time to the bandwidth. However, noise rejection and price considerations require low BW. The choice of BW is thus a compromise affair that will differ from case to case.

M Peak, $M_p$, and Peak Frequency $W_p$

These quantities are basically stability specifications. The magnitude of the peak relates to the settling time (refer to time-domain specifications), i.e., the time required for the oscillations to die out. There is also a correlation between $M_p$ and the sharpness with which the magnitude falls off with the percent overshoot. Jaworski [ref 10] presents empirical transient formulae relationships between frequency-domain and time-domain performance specifications.

Output Impedance $Z$

A maximum specification on $Z$ will guarantee that the servo will perform properly over the expected load range. It is particularly important to realize that $Z$ will vary with frequency, and it is therefore necessary to specify its peak value. The corresponding time-domain specification, compliance, is obviously not sufficient to predict intolerable output oscillations that could result from periodic load variations if applied at a frequency corresponding to peak impedance.

It should be stressed that the impedance specifications make sense only in those cases where, in reality, we can expect load fluctuations.

Time-Domain Specifications

Probably the most common of all performance specifications are those that relate the transient output of a system to a test input, usually in the form of a step function. Conceivably, one could specify time-domain performance specifications in terms of many various types of test inputs, and it is therefore appropriate to give some of the reasons why we chose this particular one:

1. A step is easy to apply and is sufficiently drastic.
2. No physical system is capable of following a step completely.
3. A large amount of information is available in the literature relating to this type of test input.
4. From a knowledge of the step-function response, it is possible to compute the response for any arbitrary input.

The last fact is demonstrated in figure 22. We wish to compute the response at time $t$ for the general input function $i(t)$, assuming that we know, either from analysis or

Figure 22. A general signal \( i(\tau) \) can be considered as composed of elementary step functions.

From experiment, the response \( g(\tau) \) for a unit step input applied at \( t = 0 \). The input function \( i(\tau) \) can be considered composed of the infinitesimal step functions shown shaded in the figure.

By superposition, we obtain the output at time \( t \) from all these step functions of amplitude \( di \) plus the initial step of magnitude \( i(0) \):

\[
o(t) = i(0)g(t) + \int_0^t g(t - \tau) \, di + \int_0^t g(t - \tau) \frac{di}{d\tau} \, d\tau
\]

\[
= i(0)g(t) + \int_0^t g(t - \tau)i'(\tau) \, d\tau
\]

As in the case of frequency-domain performance specifications, we find that the literature abounds with suggested figures of merit. The five time-domain performance specifications which will be used are:

1. Delay time \( T_D \)
2. Rise time \( T_R \)
3. Percentage overshoot \( PO \)
4. Settling time \( T_S \)
5. Final (static) value of error \( FVE \).

These are illustrated in figure 23. Explanation of these time-domain performance specifications is expanded in the following paragraphs.
Delay Time $T_D$

This quantity is a measure of the "delay" of the servo and is defined as the time interval between the application of the input step and the moment when a substantial output is observed, usually defined as 50 percent of the step amplitude. The delay time is closely associated with the second item in the set of PS, the rise time.

Rise Time $T_R$

This quantity expresses the sharpness of the leading edge of the output. Several definitions exist, the one suggested here is based upon the rate of the pulse increase at the moment the output pulse "arrives," ie, at time $T_D$. Both delay and rise time are closely related to the bandwidth specification in the frequency domain.
Overshoot PO and Settling Time TS

These two quantities specify the degree of stability of the servo. They therefore are closely associated with M peak and peak frequency in the frequency domain. TS is defined as the time it takes for the output to settle down within a specified x percent of the final value.

Time-domain specifications are those which are generally used to specify system performance. In the design procedure, however, it is often easier to work in the frequency domain. The synthesis processes can then be one of relating frequency-domain specifications to time-domain specifications. Analytically this is quite easily done for a second order system. Melsa and Schultz [ref 11] present a thorough treatment of relating time-domain to frequency-domain specifications for second- and third-order systems. Figures 24(A) and (B) present frequency- and time-domain performance specifications with tolerances specified by the shaded areas. The synthesis procedure is to examine, in the frequency domain, the magnitude of the closed loop system response. By knowing the mapping from the frequency to time-domain, the time-domain specifications can be evaluated. If requirements are not met (in the time domain), system parameters can be changed and the frequency-domain specifications reexamined. As can be seen this is an iterative process. Since the mapping between frequency and time-domain specifications (for higher order systems) is based on empirical relations, the procedure is somewhat trial and error. Trial and error is probably the most widely used synthesis procedure. Elgerd [ref 9] classifies synthesis methods into three categories (trial-and-error, analytical, and optimal). For the classical control system problem the trial-and-error is the most useful and widely used.

3.3.1 Slave Loop Performance Specification

The designer has, for classical control system design, another means of specifying system performance. This is with the error constants. Basically those most commonly used are the acceleration-error constant, position-error constant and velocity-error constant. Referring to the time-domain specifications for a step input, the error defined in figure 24B as the final value of error relates directly to the error constant. The slave loop is a Type 1 system, therefore it has a zero final value error for a position input. As will be seen in the following section, the acceleration error constant will be of importance to the design of the track/stabilization loop. The desired performance specifications for the inner and outer gimbal for the slave loop design are presented in table 2.

3.3.2 Stabilization/Track Loop Performance Specifications

The stabilization/track loop is quite different from slave loop. Its function is to provide an estimate of the line-of-sight rate to the guidance computer by means of a rate tracking servo loop. The sensor is stabilized such that it is an inertial reference. This is required for low data rate systems. The sensor maintains its look angle in inertial space even during loss of tracking data. The stabilization loop is designed to have a very high acceleration constant and an infinite Kv.

Figure 24. Frequency- and time-domain performance specifications.
### Table 2. Slave loop design specifications.

<table>
<thead>
<tr>
<th>Design/Performance Parameters</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>.025 sec</td>
<td>.025 sec</td>
</tr>
<tr>
<td>Delay Time</td>
<td>.01 sec</td>
<td>.01 sec</td>
</tr>
<tr>
<td>Percent Overshoot</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Settling Time</td>
<td>.1 sec</td>
<td>.1 sec</td>
</tr>
<tr>
<td>Velocity Error Constant</td>
<td>40,000.00</td>
<td>40,000.00</td>
</tr>
<tr>
<td>BW</td>
<td>200 rad</td>
<td>200 rad</td>
</tr>
<tr>
<td></td>
<td>30 Hz</td>
<td>30 Hz</td>
</tr>
<tr>
<td>$M_p$</td>
<td>3 dB</td>
<td>3 dB</td>
</tr>
</tbody>
</table>

Infinite $K_v$ implies zero steady state rate tracking error. The more paramount design requirements placed on the stabilization/track loop is the isolation design requirements. These design requirements are isolation of the estimated rate tracking signal and pointing error from extraneous rate signals and disturbance torques. The higher the acceleration error constant, the better the isolation achieved.

The isolation properties from extraneous disturbance signals are functions of frequency. Therefore, the design requirements must specify the minimum isolation over a frequency region. This frequency region is usually tied to the airframe characteristics for the particular airframe in which the stabilized platform will be used. The short period poles of the airframe transfer function characterize the higher frequency natural resonances in the airframe/autopilot system. The design goal is to have the isolation of the disturbance signals reach their minimum values beyond the natural frequency responses of the airframe/autopilot.

The isolation properties for extraneous signals (for a type II system) all have the same general characteristics; see figure 25. For low frequencies isolation is large. It reaches a minimum and then increases with increasing frequencies.

The desired design specifications for the track/stabilization loop (inner and outer gimbals) are presented in tables 3 and 4.
Figure 25. Typical isolation curve.

<table>
<thead>
<tr>
<th>Design/Performance Parameters</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.005 sec</td>
<td>0.005 sec</td>
</tr>
<tr>
<td>Delay Time</td>
<td>0.002 sec</td>
<td>0.002 sec</td>
</tr>
<tr>
<td>Percent Overshoot</td>
<td>20%</td>
<td>20%</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.1 sec</td>
<td>0.1 sec</td>
</tr>
<tr>
<td>Acceleration Error Constant</td>
<td>200,000.00</td>
<td>250,000.00</td>
</tr>
<tr>
<td>BW</td>
<td>500 rad</td>
<td>500 rad</td>
</tr>
<tr>
<td>$M_p$</td>
<td>3 dB</td>
<td>3 dB</td>
</tr>
</tbody>
</table>

Table 3. Stabilization loop design specifications.
### Design/Performance Parameters

<table>
<thead>
<tr>
<th>Design/Performance Parameters</th>
<th>Inner Gimbal (Isolation – dB)</th>
<th>Outer Gimbal (Isolation – dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Tracking Rate</td>
<td>100 deg/sec</td>
<td>100 deg/sec</td>
</tr>
<tr>
<td>Minimum Isolation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{\delta}{\delta} S = 1; 10 \text{ rad}$</td>
<td>80; 60</td>
<td>80; 60</td>
</tr>
<tr>
<td>$\frac{\delta}{Td} S = 1; 10 \text{ rad}$</td>
<td>100; 60</td>
<td>100; 60</td>
</tr>
<tr>
<td>$\frac{\epsilon}{\delta} S = 1; 10 \text{ rad}$</td>
<td>100; 60</td>
<td>100; 60</td>
</tr>
<tr>
<td>$\frac{\epsilon}{Ta} S = 1; 10 \text{ rad}$</td>
<td>100; 80</td>
<td>100; 80</td>
</tr>
</tbody>
</table>

Table 4. Track loop design specifications.
4. STABILIZED PLATFORM DESIGN PERFORMANCE

The stabilized sensor platform was designed to meet the performance requirements as specified in the previous sections. This section briefly presents the design performed for each of the modes of the stabilized sensor platform.

4.1 SLAVE LOOP DESIGN PERFORMANCE

Figures 26 through 37 present the design and performance characteristics of the slave loop for both the inner and outer gimbals.

For the outer gimbal the open loop uncompensated frequency response is presented in figure 27. The loop frequency response after being compensated is shown in figure 29. This figure presents the gain and phase margins for the slave loop outer gimbal. The phase margin is 40 degrees and the gain margin is 20 dB. The closed loop frequency response (response from which the bandwidth can be evaluated) is presented in figure 30. The time response for the slave loop outer gimbal is shown in figure 31. The inner gimbal design performance data is presented in figures 32 through 37.

The summary section of this report presents the design configurations and the design performance for each of the servo loops. Much of the performance criterion presented in table form in the Summary section was summarized from the curves and figures presented in this section.

4.2 STABILIZATION LOOP DESIGN PERFORMANCE

Figures 38 through 47 relate to the stabilization loop design (inner and outer gimbals) and the design performance criterion. Specifically, figures 38 through 42 deal with the inner gimbal while figures 43 through 47 deal with the outer gimbal. Figure 38 is the frequency plot of the uncompensated loop. The magnitude of the frequency response reaches zero dB after the phase reaches a minus 180 degrees. In fact the phase has reached approximately a minus 240 degrees, thus, some form of compensation is required. A comparison of figures 33 and 38 (open loop frequency response curves for the slave loop and stabilization) shows that the compensation network of the stabilization loop must increase the phase considerably over that of the slave loop compensation network. In addition to a lead network to increase the phase at the 0 dB crossover a low frequency lag network is added to allow the gain to be increased and yield a higher acceleration constant. The magnitude and phase plots of the stabilization network frequency response curve is shown in figure 39. The phase reaches a peak at approximately 500 radians, therefore, the closed loop bandwidth can be approximately set at 500 radians. By examining figures 40 and 41, the open loop and closed loop frequency response curves, it is seen that the system is stable (conditionally stable) and that the bandwidth is approximately 750 radians. The phase margin is 59 degrees. The gain margin, for increasing gain, is 12 dB and is 20 dB for decreasing gain. The time response for the stabilization loop is shown in figure 42. All of the nonlinearities of the system are reflected in the time response curve these nonlinearities are not included in the frequency response curves because frequency response techniques deal only with linear systems.
4.3 TRACK LOOP DESIGN PERFORMANCE

Figures 48 through 59 present the track loop performance data. Briefly these figures present data of disturbance isolation, closed loop frequency and time response characteristics of the track loop. Figures 48 through 53 present the outer gimbal data and figures 54 through 59 present the inner gimbal data. The isolation curves are used to establish the level of body or base motion that can be tolerated for acceptable tracking or steering information and the level of mass imbalance that is tolerable. The level of tolerable mass imbalance can be established from the extraneous torque disturbance isolation curves, figures 49 and 51. The maximum tracking rates are derived from the frequency response curve of figure 52. It is seen that a flat response is maintained out to 2 radians; that is, the estimated line-of-sight rate follows the true line-of-sight rate out to 2 rad/sec with no appreciable degradation. Again both inner and outer gimbal data is presented. Figures 48 through 53 are for the outer gimbal and 54 through 59 for the inner gimbal. Comparative statements of the inner gimbal data can be made in the same light as was done for the outer gimbal data.
Figure 26. Servo amp/motor/load frequency response (inner gimbal).

Figure 27. Slave loop (open loop) uncompensated frequency response (inner gimbal).
Figure 28. Slave loop compensation network frequency response (inner gimbal).

Figure 29. Compensated slave loop (open loop) frequency response (inner gimbal).
Figure 30. Slave loop (closed loop) frequency response (inner gimbal).

Figure 31. Slave loop inner gimbal time response.
Figure 32. Servo amp/motor/load frequency response (outer gimbal).

Figure 33. Slave loop (open loop) uncompensated frequency response (outer gimbal).
Figure 34. Slave loop compensation network frequency response (outer gimbal).

Figure 35. Compensated slave loop (open loop) frequency response (outer gimbal).
Figure 36. Slave loop (closed loop) frequency response (outer gimbal).

Figure 37. Slave loop outer gimbal time response.
Figure 38. Stabilization loop frequency response (inner gimbal) uncompensated.

Figure 39. Stabilization loop compensation – inner gimbal.
Figure 40. Stabilization loop frequency response compensated (inner gimbal).

Figure 41. Stabilization closed loop frequency response (inner gimbal).
Figure 42. Stabilization loop time response (inner gimbal).

Figure 43. Stabilization loop frequency response (outer gimbal) uncompensated.
Figure 44. Stabilization loop compensation – outer gimbal.

Figure 45. Stabilization loop frequency response compensated (outer gimbal).
Figure 46. Stabilization closed loop frequency response (outer gimbal).

Figure 47. Stabilization loop time response (outer gimbal).
Figure 48. Frequency response of pointing error to body motion disturbance $e/\dot{\theta}$ (outer gimbal).

Figure 49. Frequency response of pointing error to torque disturbance $e/Td$ (outer gimbal).
Figure 50. Frequency response of estimated line-of-sight to body motion disturbance $\delta/\dot{\theta}$ (outer gimbal).

Figure 51. Frequency response of estimated line-of-sight to torque disturbance $\delta/T_d$ (outer gimbal).
Figure 52. Frequency response of track loop $\dot{\theta}/\dot{\theta}$ transfer function (outer gimbal).

Figure 53. Track loop time response (outer gimbal).
Figure 54. Frequency response of pointing error to body motion disturbance $e/\dot{\theta}$ (inner gimbal).

Figure 55. Frequency response of pointing error to torque disturbance $e/T_d$ (inner gimbal).
Figure 56. Frequency response of estimated line-of-sight to body motion disturbance $\hat{\theta}/\dot{\theta}$ (inner gimbal).

Figure 57. Frequency response of estimated line-of-sight to torque disturbance $\hat{\theta}/T_d$ (inner gimbal).
Figure 58. Frequency response of track loop $\hat{\theta}$/$\dot{\theta}$ transfer function (inner gimbal).

Figure 59. Track loop time response (inner gimbal).
5. SUMMARY

A stabilized sensor is required for modern day missile guidance applications. This is particularly essential for air-to-air guidance missiles. The stabilized sensor provides the following:

(1) An inertial reference from which line-of-sight rates can be measured.
(2) It decouples body motion and extraneous torque disturbances from the guidance signals.
(3) The target tracking sensor is maintained in an optimum boresight region of operation.

This report covers the design and development of such a space stabilized platform for a missile guidance or surveillance sensor. The design philosophy was based on a high torque-to-inertia ratio. This philosophy leads to a low cost system. The high torque-to-inertia system has excellent extraneous torque disturbance isolation. This allows higher tolerances on mass imbalances. It is the "Swiss watch" manufacturing process in the conventional space stabilized platforms which needed to be maintained to achieve the low mass imbalance requirements. Traditionally this Swiss watch type of manufacturing carries along with it high machining costs. Torque is essentially a cheap commodity. By maintaining higher torque level in the servo design, a design approach was established which led to overall lower system costs. In addition, the tracking and slew rate performance of the platform was greatly increased as a result of the high torque-to-inertia criterion.

The design performance of the platform is summarized in tables 5 through 7. Each table relates to a separate mode of operation or servo loop.

Figures 60 and 61 present the block diagrams of the finalized designs for the servo loops. Tables 8 and 9 present the values for the individual parameters shown in the block diagrams of figures 60 and 61. Figures 62 and 63 show the system hardware, ie, the antenna sensor mounted to the gimbal.

<table>
<thead>
<tr>
<th>Design/Performance Parameters</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.014 sec</td>
<td>0.01 sec</td>
</tr>
<tr>
<td>Delay Time</td>
<td>0.011 sec</td>
<td>0.008 sec</td>
</tr>
<tr>
<td>Percent Overshoot</td>
<td>12.64%</td>
<td>10.7%</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.065 sec</td>
<td>0.05 sec</td>
</tr>
<tr>
<td>Velocity Error Constant</td>
<td>40288.88 1/sec</td>
<td>46482.4 1/sec</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>250 rad</td>
<td>270 rad</td>
</tr>
<tr>
<td>Np</td>
<td>4 dB</td>
<td>3 dB</td>
</tr>
</tbody>
</table>

Table 5. Slave loop design performance.
<table>
<thead>
<tr>
<th>Design/Performance Parameters</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rise Time</td>
<td>0.002 sec</td>
<td>0.0025 sec</td>
</tr>
<tr>
<td>Delay Time</td>
<td>0.0015 sec</td>
<td>0.00175 sec</td>
</tr>
<tr>
<td>Percent Overshoot</td>
<td>28%</td>
<td>28%</td>
</tr>
<tr>
<td>Settling Time</td>
<td>0.06 sec</td>
<td>0.1 sec</td>
</tr>
<tr>
<td>Acceleration Error Constant</td>
<td>225568.21/sec²</td>
<td>300040.44/sec²</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>750 rad</td>
<td>550 rad</td>
</tr>
<tr>
<td>Mp</td>
<td>3.2 dB</td>
<td>4 dB</td>
</tr>
</tbody>
</table>

Table 6. Stabilized loop design performance.

<table>
<thead>
<tr>
<th>Design/Performance Parameters</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Track Rate</td>
<td>143.24</td>
<td>114.6</td>
</tr>
<tr>
<td>Minimum Isolation:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{d^2}{dt}S = 1; 10 \text{ rad/sec}$</td>
<td>92; 48</td>
<td>90; 48</td>
</tr>
<tr>
<td>$\frac{\ddot{3}}{\ddot{t}_d}S = 1; 10 \text{ rad/sec}$</td>
<td>75; 50</td>
<td>90; 66</td>
</tr>
<tr>
<td>$\frac{\ddot{e}}{\ddot{t}_d}S = 1; 10 \text{ rad/sec}$</td>
<td>104; 60</td>
<td>100; 60</td>
</tr>
<tr>
<td>$\frac{\ddot{e}}{\ddot{t}_d}S = 1; 10 \text{ rad/sec}$</td>
<td>90; 64</td>
<td>106; 82</td>
</tr>
</tbody>
</table>

Table 7. Track loop design performance.
Figure 60. Sensor servo platform, slave loop.
Figure 61. LADSS stabilized platform, stabilization/track loops.
### Sensor:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>K4</td>
<td>Vdc/Vdc</td>
<td>7.0</td>
<td>7.0</td>
</tr>
<tr>
<td>K5</td>
<td>Vdc/Rad/sec</td>
<td>7.0</td>
<td>7.0</td>
</tr>
<tr>
<td>( \tau_7 )</td>
<td>sec</td>
<td>0.025</td>
<td>0.025</td>
</tr>
</tbody>
</table>

### Compensation Network:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>K3</td>
<td>Vdc/Vdc</td>
<td>46.68</td>
<td>84.72</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>sec</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( \tau'_2 )</td>
<td>sec</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( \tau_5 )</td>
<td>sec</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( \tau_3 )</td>
<td>sec</td>
<td>0.0005</td>
<td>0.0005</td>
</tr>
<tr>
<td>( \tau'_3 )</td>
<td>sec</td>
<td>0.0005</td>
<td>0.0005</td>
</tr>
<tr>
<td>( \tau_6 )</td>
<td>sec</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Limiter: All limiters are ±20 volts.

### Rate Sensor Feedback:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>K2</td>
<td>Vdc/Vrms</td>
<td>14.0</td>
<td>14.0</td>
</tr>
<tr>
<td>( \tau_4 )</td>
<td>sec</td>
<td>0.0015</td>
<td>0.0015</td>
</tr>
<tr>
<td>K_{mhd}</td>
<td>Vrms/Rad/sec</td>
<td>0.8595</td>
<td>0.8595</td>
</tr>
</tbody>
</table>

Servo Drive System: Same as those of slave loop of table 9.

Table 8. Stabilization track/loop parameter values.
Compensation Network:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_3$</td>
<td>Vdc/Vdc</td>
<td>4.668</td>
<td>5.086</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>sec</td>
<td>.018</td>
<td>.0195</td>
</tr>
<tr>
<td>$\tau_2'$</td>
<td>sec</td>
<td>.018</td>
<td>.0195</td>
</tr>
<tr>
<td>$r_3$</td>
<td>sec</td>
<td>.0015</td>
<td>.0015</td>
</tr>
<tr>
<td>$r_3'$</td>
<td>sec</td>
<td>.0015</td>
<td>.0015</td>
</tr>
<tr>
<td>$\tau_6$</td>
<td>sec</td>
<td>.024</td>
<td>.020</td>
</tr>
</tbody>
</table>

Limiter: All limiters are ±20 volts.

Servo Drive/Torque Source/Load:
(These values are for slave and stabilization loops)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Vdc/Vdc</td>
<td>100,000.00</td>
<td>100,000.00</td>
</tr>
<tr>
<td>$r_a$</td>
<td>sec</td>
<td>.02</td>
<td>.02</td>
</tr>
<tr>
<td>$L_a$</td>
<td>Millihenries</td>
<td>.0027</td>
<td>.0014</td>
</tr>
<tr>
<td>$D$</td>
<td>oz-in-sec</td>
<td>.622</td>
<td>.706</td>
</tr>
<tr>
<td>$R_a$</td>
<td>ohms</td>
<td>9.317</td>
<td>3.0</td>
</tr>
<tr>
<td>$R_{b}$</td>
<td>ohms</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$J_m$</td>
<td>oz-in-sec$^2$</td>
<td>.0015</td>
<td>.016</td>
</tr>
<tr>
<td>$J_L$</td>
<td>oz-in-sec$^2$</td>
<td>2.47</td>
<td>3.30</td>
</tr>
<tr>
<td>$K_T$</td>
<td>oz-in/amp</td>
<td>19.75</td>
<td>24.6</td>
</tr>
<tr>
<td>$N$</td>
<td>Gear ratio</td>
<td>12.8</td>
<td>8.5</td>
</tr>
<tr>
<td>$K_E$</td>
<td>Vdc/rad/sec</td>
<td>.141</td>
<td>.177</td>
</tr>
</tbody>
</table>

Position Feedback:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Inner Gimbal</th>
<th>Outer Gimbal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_F$</td>
<td>Vdc/Vdc</td>
<td>1.5</td>
<td>2.295</td>
</tr>
<tr>
<td>$N$</td>
<td>Pot gear ratio</td>
<td>3.0</td>
<td>8.5</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Vdc/rad</td>
<td>4.776</td>
<td>1.592</td>
</tr>
</tbody>
</table>

Table 9. Slave loop parameter values.
Figure 62  Stabilization platform/sensor system showing outer and inner gumbal.
Figure 6.3  Stabilizing platform and sensor mounted to stable platform.
6. REFERENCES


APPENDIX A
TECHNICAL DESCRIPTION OF RATE SENSOR

This appendix is essentially a reprint of technical product data supplied to NOSC by Honeywell.* The technical description in this appendix covers the Honeywell GG 2500 rate sensor and the readout electronics (demodulator and filters) associated with the rate sensor.

*Honeywell, Avionics Division, Minneapolis, Minnesota, GG2500 MHD (Magnetohydrodynamic) Two-Axis Rate Sensor, February 1978
SECTION I
GG2500 MHD (MAGNETOHYDRODYNAMIC)
TWO-AXIS RATE SENSOR

The Honeywell GG2500 is a new concept subminiature, high performance, two-axis rate sensor specifically designed for large volume producibility. It has been qualified to environmental requirements of MIL-STD-810B for gyros installed in airplanes, helicopters, and air and ground launched missiles. It is ideally suited for tactical missile seekerhead stabilization, aircraft and missile autopilot application, and rate measuring for fire control systems.

Direct benefits to the user are:

- Subminiature size and weight; two-axis information from a unit only one-fourth the volume of two conventional rate gyros
- Excellent Linearity (< 0.1% FS)
- Negligible Hysteresis (<0.01 deg/sec)
- Low Temperature-Sensitivity
- Low G-Sensitivity
- Wide Dynamic Range ($10^6$)
- Frequency Response Independent of Temperature
- Over-Rate Capability 20,000 deg/sec
SECTION II
PERFORMANCE

The performance characteristics of the GG2500 are listed in Table 1. These characteristics, unless specified otherwise, apply for any of the environments shown in the table.

Table 1. MHD Rate Sensor Specifications (GG2500LC02 and GG2500LC03)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale Factor</td>
<td>GG2500LC02: 15 ± 5% mV rms/deg/sec</td>
</tr>
<tr>
<td></td>
<td>GG2500LC03: 15 ± 1% mV rms/deg/sec</td>
</tr>
<tr>
<td>Zero Rate Error (includes run-up repeats)</td>
<td>GG2500LC02: 0.5 deg/sec max.</td>
</tr>
<tr>
<td></td>
<td>GG2500LC03: 0.15 deg/sec max.</td>
</tr>
<tr>
<td>Linearity</td>
<td>0.1% of max. rate (max. dev. from best STR line)</td>
</tr>
<tr>
<td>Cross Coupling (axis change vs. input rate)</td>
<td>0.5% of full scale (max. dev. from best STR line)</td>
</tr>
<tr>
<td>Highness</td>
<td>0.01 deg/sec max.</td>
</tr>
<tr>
<td>Threshold</td>
<td>0.01 deg/sec max.</td>
</tr>
<tr>
<td>Acceleration Sensitivity</td>
<td>0.05 deg/sec/g max.</td>
</tr>
<tr>
<td>$g^2$ Sensitivity</td>
<td>$1 \times 10^{-3}$ deg/sec/g$^2$ max.</td>
</tr>
<tr>
<td>Output Noise at Null</td>
<td>100 mV rms max. (using 1000-Hz bandwidth meter)</td>
</tr>
<tr>
<td>Rate Input Range</td>
<td>To ±80 deg/sec</td>
</tr>
<tr>
<td>Frequency Response</td>
<td>100 Hz min. without electronics</td>
</tr>
<tr>
<td>Ref. Gen. Output</td>
<td>1 V rms min, each axis</td>
</tr>
<tr>
<td>Ref. Gen. Phase Angle</td>
<td>90 ± 0.5 degrees</td>
</tr>
<tr>
<td>Performance Stability with Environments</td>
<td>0.15 deg/sec</td>
</tr>
<tr>
<td>Zero Rate Error Stability over all Environments</td>
<td>0.03 deg/sec/g</td>
</tr>
<tr>
<td>Acceleration Sensitivity Stability over all Environments</td>
<td>±2%</td>
</tr>
<tr>
<td>Scale Factor Change - vs - Temperature</td>
<td>±0.5 deg</td>
</tr>
<tr>
<td>Input Axis Change - vs - Temperature</td>
<td></td>
</tr>
<tr>
<td>Excitation Requirements</td>
<td></td>
</tr>
<tr>
<td>Motor</td>
<td>26 ± 2 volt rms 400 Hz 2a, 4 watts max.</td>
</tr>
<tr>
<td>Preamp</td>
<td>±15 ± 3 Vdc, 4 mA max. with 500 mV max. p-p ripple</td>
</tr>
<tr>
<td>Weight</td>
<td>70 grams max.</td>
</tr>
</tbody>
</table>

(1) When operated with amplifier-demodulator readout electronics. Deviation is expressed as a percent of opposite axis full scale.
The GG2500 is designed to meet or exceed environmental requirements of MIL-STD-810B as it applies to gyros intended for installation in aircraft, helicopters, and air and ground launched missiles; and has been successfully qualified to the environmental levels listed in Table 2. Performance before, during, and after each exposure has been measured; these results have been used to establish performance capability. See Figures 1 and 2 for the installation drawing and sensor schematic.
Table 2. Environments

<table>
<thead>
<tr>
<th>Environments</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overrange Capability</td>
<td>20,000 deg/sec</td>
</tr>
<tr>
<td>Temperature</td>
<td></td>
</tr>
<tr>
<td>Operating</td>
<td>-65°F to +160°F</td>
</tr>
<tr>
<td>Non-Operating</td>
<td>-65°F to +220°F</td>
</tr>
<tr>
<td>Altitude</td>
<td>MIL-STD-810B, Method 500, Proc II to 60,000 Ft.</td>
</tr>
<tr>
<td>Temperature Shock</td>
<td>MIL-STD-810, Method 503, Proc 1 + 71°C to -54°C to +71°C, four (4) hours each temp -5 minutes between chambers.</td>
</tr>
<tr>
<td>Vibration</td>
<td>MIL-STD-810, Method 514, Proc II 2 hr/axis - Time Schedule V of Table 514-II, Curve H (10 G Peak Sine)</td>
</tr>
<tr>
<td></td>
<td>1/2 hr/axis - Time Schedule II of Table 514-II, Curve Q (10 G Peak Sine)</td>
</tr>
<tr>
<td></td>
<td>1/2 hr/axis - Time Schedule II of Table 514-II, Curves AH (11.9 G RMS Random) and AK (20.7 G RMS Random)</td>
</tr>
<tr>
<td>Shock</td>
<td>2 drops/axis each direction, 12 drops total each level: 40 G, 18 MS; 400 G, 1.5 MS; 100 G, 6 MS; 500 G, 0.75 MS; MIL-STD-810, Method 516, Proc IV</td>
</tr>
<tr>
<td>Acceleration</td>
<td>100 G, each direction - each axis</td>
</tr>
<tr>
<td>Magnetic Sensitivity</td>
<td>.05 deg/sec/ gauss max</td>
</tr>
<tr>
<td>EMI Susceptability</td>
<td>MIL-STD-461</td>
</tr>
<tr>
<td>Useful Life</td>
<td>Life tested to 1000 hours</td>
</tr>
</tbody>
</table>
Figure 1. Installation Drawing

Figure 2. Sensor Schematic
SECTION IV
TECHNICAL DESCRIPTION

OPERATING PRINCIPLE

The GG2500 Rate Sensor is a non-gyro sensor; it does not depend on the momentum of a spinning wheel for operation. An angular accelerometer is used in the basic sensor. By rotating the accelerometer at a constant speed about an axis perpendicular to its input axis, an input rate in a plane normal to the spin axis is changed to a time varying angular acceleration that is sensed by the accelerometer.

To obtain further insight into the operation, consider an angular accelerometer that is being rotated at a constant rate, $W_S$, about an axis perpendicular to the angular accelerometer input axis. If a rate exists perpendicular to this rotation axis, the instantaneous rate about the angular accelerometer input axis is:

$$\omega_o = \omega_x \sin \omega_s t$$

(see Figure 3)

Figure 3. MHD Theory of Operation
The angular acceleration about the input axis, therefore, is:

\[
\dot{\omega}_o = \frac{d\omega_o}{dt} = \omega_s \omega_x \cos \omega_s t
\]

By these means the input rate is changed to a time-varying angular acceleration. The rotating accelerometer acts as an integrator that provides an ac output voltage, which is directly proportional to rate, at a frequency equal to the rotation frequency.

PHYSICAL CONSTRUCTION

A cross section of the complete rate sensor is shown in Figure 4. The sensor consists of the angular accelerometer, a hysteresis-synchronous drive motor, a two-phase reference generator, a slip ring assembly to transfer the accelerometer output signal from the rotating element, and an integrally mounted accelerometer preamplifier. The entire sensor is fabricated from a high permeability nickel iron alloy that serves as an effective magnetic shield. Laser welding of all internal and external joints ensures structural integrity and hermaticity under severe environments.

![Figure 4. GG2500 Rate Sensor](image-url)
MOTOR DESIGN

The rotor drive is a two-phase hysteresis synchronous motor wound to operate with an excitation of 26 V at 400 Hz. Since the stator is attached directly to the rate sensor case, it does not enter into the dynamics of rotor balance and mass stability as in a conventional gyro. In addition, the heat generated in the motor windings is conducted directly to the sensor mounting surfaces without passing through the rotor bearings, thus minimizing motor temperature rise and thermal gradients across the sensor.

REFERENCE GENERATOR

The two-phase reference generator provides the demodulator reference signals to permit the composite rate signal to be resolved into two-axis information. The stator is positioned on the rotor spin axis and is wound in a standard two-phase configuration. A diametrically charged ring magnet attached to the rotor provides the lines of flux required to generate a voltage in each winding. The reference generator output, when loaded with a 10K or greater resistive load, is greater than 1.0 V rms at the rotor frequency of 200 Hz.

ANGULAR ACCELEROMETER

The angular accelerometer used in the device is depicted in Figure 5. An annular ring of mercury exists between the radially oriented permanent magnet and the magnetic case, which provides the magnetic path. The existence of a rate input results in a relative motion of the magnetic field with respect to the mercury. This motion through the phenomenon of magneto-hydrodynamics (MHD) causes a voltage gradient across the mercury at right angles to the magnetic field and the relative motion. Contacts on either side
of the mercury ring provide a single turn primary for the transformer. The voltage generated across the mercury causes a current to flow through the single turn primary, which, in turn induces a corresponding voltage in the secondary winding.

![Diagram of Angular Accelerometer with labels: Secondary Winding, Mercury, Transformer Core, Magnet]

Figure 5. Angular Accelerometer

The voltage induced in the mercury is:

$$E = Blv$$

where

- $B$ = flux density
- $l$ = length of moving conductor
- $v$ = velocity of conductor relative to the magnetic field

In terms of angular velocity:

$$e = Bl\omega r$$
where
\[ r = \text{mean radius of the mercury} \]
\[ \omega_r = \text{angular velocity of the mercury relative to the magnetic field (or sensor case)} \]

To determine the relationship between \( \omega_r \) and the input angular rate \( \omega_o \), the open-loop transfer function for the angular accelerometer is examined:

\[ \omega_r = \omega_o \left( \frac{I}{C} \right) \left[ \frac{S}{S+1} \right] \]

where
\[ \omega_r = \text{angular velocity of the mercury relative to the magnetic (or sensor case)} \]
\[ \omega_o = \text{angular input to case} \]
\[ I = \text{polar moment of inertia of mercury} \]
\[ C = \text{damping of mercury} \]
\[ S = \text{La Place operator} \]

In the practical case where \( \frac{1}{C} S \) is much greater than one, the quantity within the parenthesis is unity to within one part in \( 10^7 \). This means that the input rate and the rate between the magnetic field and the mercury are essentially identical and that the mercury is motionless about its input axis. Thus, the output of the MHD rate sensor is a true representation of the input rate.

Since both \( I \) and \( C \) are essentially constants over the operating temperature range, a method of temperature control to hold these parameters is not necessary.
ROTOR SUSPENSION

The rotor is mounted on two preloaded miniature precision ball bearings. Since the entire rotor and case structure are made from the same material, preload does not change as a function of ambient temperature changes. In a conventional gyro, mass balance instability can be caused by migration of lubricant from the rotor bearing, so lubrication is kept to an absolute minimum. Because the GG2500 does not operate by measuring precession torques, oil migration does not cause performance errors, so the lubricant can be applied copiously. The large amount of lubricant, coupled with inner race rotation and light loads, ensures long bearing life and stable device performance.

SLIP RING ASSEMBLY

The slip ring assembly is mounted on the rotor axis and, in conjunction with a case mounted brush block assembly, provides the means of coupling the accelerometer output signal to the preamplifier. Multiple brushes for each circuit result in extremely low contact resistance and noise free operation. Slip ring life tests have proven an operating life of greater than 2000 hours without degradation or any increase in contact resistance or slip ring noise.

PREAMPLIFIER ASSEMBLY

A thick-film hybrid circuit preamplifier is mounted integrally to the GG2500 Rate Sensor. This preamplifier functions as an interface between the low-level high-impedance sensor output and the external readout electronics. The input circuit of the preamplifier consists of a dual FET follower stage chosen for reasons of very low bias current (10 na) to minimize noise effects from slip ring resistance and dc offsets in the transformer core. The dual
FET stage drives an integrated circuit operational amplifier connected as a conventional non-inverting amplifier which furnishes the high-level low impedance output. The preamplifier assembly provides for scale factor and zero rate error (ZRE) calibration. A scale factor temperature compensation network has also been added to the preamplifier assembly.

A schematic of the GG2500 Rate Sensor is shown in Figure 2.
A dual-channel demodulator is required to resolve the output of the GG2500 Rate Sensor into two-axes of information. In addition, some filtering is required to shape the response characteristics of the device.

A block diagram of the Honeywell circuits is shown in Figure 6. The circuit consists of an input band-pass filter and demodulator driver amplifier, reference signal driver amplifiers, a two-channel demodulator, and third-order low-pass output filters.

Honeywell is in the final stages of developing a miniature readout electronics package (shown in Figure 7) using thick-film hybrid packaging techniques. Honeywell expects to fully qualify this package to the GG2500 Rate Sensor qualification levels and have it available for delivery in the last half of 1978.

Initially, the EG1030AD will be available in the following gains, and resultant full scale ranges when used with the GG2500 Rate Sensor:

<table>
<thead>
<tr>
<th>Gain Vdc/V</th>
<th>Full Scale Range Deg/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.8</td>
<td>57.3</td>
</tr>
<tr>
<td>3.33</td>
<td>100</td>
</tr>
<tr>
<td>1.11</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>370</td>
</tr>
</tbody>
</table>

Two versions of the output filter will be available; one with a 70-Hz bandwidth, the other with a 100-Hz bandwidth. The amplitude and phase
Figure 6. Block Diagram - GG2500LC Rate Sensor/EG1030AD Amplifier Demodulator Readout Electronics.
responses for these filters when used with a GG2500 Ratv. Sensor are shown in Figures 8 and 9.

The performance listed in Table 3 has been achieved using discrete components. It is expected that comparable performance will be realized with the thick-film circuitry. The quoted performance is for the EG1030AD when tested as a unit and does not include the contributions of the GG2500 MHD.
Figure 8. Amplitude and Phase Response - C2500/EG1390 (100 Hz Output Filter)
Figure 9. Amplitude and Phase Response - GG2500/EG1030 (70 Hz Output Filter)
Table 3. EG1030 Miniature Amplifier-Demodulator
Projected Performance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Projected Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply Voltage</td>
<td>±15 ±3 Vdc</td>
</tr>
<tr>
<td>Supply Current</td>
<td>15 mA at 60% F. S.</td>
</tr>
<tr>
<td>Output Range</td>
<td>±5 Vdc min.</td>
</tr>
<tr>
<td>Gain Set</td>
<td>±1% max.</td>
</tr>
<tr>
<td>Gain Stability (OTR)</td>
<td>±1% max. deviation</td>
</tr>
<tr>
<td>Offset</td>
<td>±1 mV max.</td>
</tr>
<tr>
<td>Offset Stability (OTR)</td>
<td>±0.10 deg/sec max. @ 5.8 gain</td>
</tr>
<tr>
<td></td>
<td>±0.10 deg/sec max. @ 3.33 gain</td>
</tr>
<tr>
<td></td>
<td>±0.15 deg/sec max. @ 1.11 gain</td>
</tr>
<tr>
<td>Linearity</td>
<td>±0.03% of F. S. (Max DEV from best STR line)</td>
</tr>
<tr>
<td>Cross coupling</td>
<td>±0.10% of F. S. (Max DEV from best STR line)</td>
</tr>
<tr>
<td>Phase Angle (OTR)</td>
<td>±0.15 deg. max. deviation</td>
</tr>
<tr>
<td>Output Noise</td>
<td></td>
</tr>
<tr>
<td>Input Shorted At 60% F. S.</td>
<td>2 mV rms</td>
</tr>
<tr>
<td>Operating Temperature</td>
<td>-65°F to +200°F</td>
</tr>
<tr>
<td>Frequency Response</td>
<td>-3 dB @ 70 Hz or</td>
</tr>
<tr>
<td></td>
<td>-3 dB @ 100 Hz</td>
</tr>
<tr>
<td>Dynamic Output Impedance</td>
<td>Less Than 1 ohm</td>
</tr>
<tr>
<td>Weight</td>
<td>16 grams</td>
</tr>
</tbody>
</table>

(1) Deviation is expressed as a percent of opposite channel full scale.

(2) This parameter represents the change in the phase relationship between the signal and the reference voltages at the input to the demodulators.
SECTION VI
SUPPORTIVE DATA

Typical data is presented to support and/or supplement the performance which was specified in Table 1.

FREQUENCY RESPONSE

The GG2500 is a wide-response device with an equivalent natural frequency of well beyond 100 Hz. The amplitude and phase response for the GG2500 is shown in Figure 10. It will be noted that as the input frequency approaches the spin frequency (200 Hz) there is considerable peaking before the output falls off to zero at 200 Hz. Therefore, Honeywell uses a third-order filter at the output of the amplifier-demodulator readout electronics to eliminate the peaking and yet at the same time maintain the maximum bandwidth. The amplitude and phase response for the combination of the GG2500 Rate Sensor and the EG1030 Amplifier-Demodulator Readout Electronics has previously been shown in Figure 8 for a 70-Hz filter and in Figure 9 for a 100-Hz filter.

SHORT-TERM ZRE STABILITY

A recording of the zero rate error (ZRE) from both channels over a 10-minute time period is shown in Figure 11. The maximum peak-to-peak excursion for either channel over the 10-minute time interval is 0.02 degree per second.
LINEARITY AND CROSS COUPLING

Honeywell defines linearity for the GG2500LC MHD Rate Sensor as the maximum deviation from a least-squares best-fit straight line expressed as a percentage of the full-scale input. In a like manner, cross coupling is defined as the maximum deviation from a least-squares best-fit straight line based on all output data points for input about the opposite axis and expressed as a percentage of the full-scale input of the opposite axis.

Since Honeywell specifies the full-scale range of the GG2500 MHD Rate Sensor at ±480 deg/sec, this gives rise to the question of what is the performance capability for lesser full-scale ranges.

Linearity and cross coupling data is presented here for one GG2500 Rate Sensor (S/N S-8) for input ranges of 0 to ±20 deg/sec, 0 to ±60 deg/sec, and 0 to ±480 deg/sec. In each case, the calculations were based on a least-squares best-fit straight line for that range. The data can be summarized as follows:

<table>
<thead>
<tr>
<th>Full-Scale Range</th>
<th>Linearity % Full-Scale (Spec = 0.1%)</th>
<th>Cross Coupling % Full-Scale (Spec = 0.5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>± 20 deg/sec</td>
<td>0.086</td>
<td>0.058</td>
</tr>
<tr>
<td>± 60 deg/sec</td>
<td>0.096</td>
<td>0.092</td>
</tr>
<tr>
<td>±480 deg/sec</td>
<td>0.070</td>
<td>0.223</td>
</tr>
</tbody>
</table>

ANGULAR ACCELERATION SENSITIVITY

Since the basic transducer of the GG2500 is an angular accelerometer, a question that frequently arises is that of the sensitivity of the GG2500 to angular accelerations. As a matter of fact the GG2500 is virtually insensitive to angular accelerations. This is because the I/C time constant of the mercury torus is so great that at the frequencies of interest the transducer is basically an angular rate sensor.
As proof, consider the frequency response curves of Figure 10. Since these curves are run using an oscillating table the MHD was subjected to angular accelerations during these tests. It will be noted that the amplitude response curves are flat out to frequencies approaching the rotor speed. If the MHD were sensitive to these angular accelerations then the response curves should exhibit a 6 db per octave rise. Such was not the case. Honeywell has also run angular acceleration tests about the spin axis. Again no measurable effect was observed.
PERFORMANCE HISTOGRAMS

Histograms of critical performance parameters from the most recent 68 production units are shown below. Histograms of threshold and hysteresis are not plotted since they are within 0.01 deg/sec sensitivity of the device.
QUALIFICATION TEST PERFORMANCE DATA

Acceleration Sensitivity Versus Temperature

Zero Rate Error Versus Temperature

Acceleration Sensitivity Versus Environments

Zero Rate Error Versus Environments

Linearity Deviation

Detailed data is available for discussion on any GG2500 parameter.

A-28
APPENDIX 3
TORQUE MOTOR AND SERVO AMPLIFIER SPECIFICATIONS

CLIFTON PRECISION
LITTON SYSTEMS, INC.

ACCEPTANCE TEST DATA SHEET

UNIT TYPE: DPH-3320-A-2T
FUNCTION: TORQUE MOTOR POTENTIOMETER ASSEMBLY

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>UNITS</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Voltage</td>
<td>Volts</td>
<td>29.5</td>
</tr>
<tr>
<td>Terminal Resistance</td>
<td>Ohms</td>
<td>3.06</td>
</tr>
<tr>
<td>Stall Current (I_s)</td>
<td>Amps</td>
<td>9.65</td>
</tr>
<tr>
<td>No Load Speed</td>
<td>Rad/Sec</td>
<td>159</td>
</tr>
<tr>
<td>Torque Sensitivity</td>
<td>Oz-In/Amp</td>
<td>24.9</td>
</tr>
<tr>
<td>Peak Torque @ I_s</td>
<td>Oz-In</td>
<td>240</td>
</tr>
<tr>
<td>Back E.M.F.</td>
<td>V/Rad/Sec</td>
<td>178</td>
</tr>
<tr>
<td>Inductance</td>
<td>Millihenries</td>
<td>1.4</td>
</tr>
<tr>
<td>Friction Torque</td>
<td>Oz-In</td>
<td>5</td>
</tr>
<tr>
<td>Armature Inertia</td>
<td>Oz-In-Sec^2</td>
<td>.016</td>
</tr>
<tr>
<td>Acceleration</td>
<td>Rad/Sec^2</td>
<td>15,000</td>
</tr>
<tr>
<td>Pot Resistance</td>
<td>Ohms</td>
<td>5014</td>
</tr>
<tr>
<td>Linearity</td>
<td>Percent</td>
<td>.22</td>
</tr>
</tbody>
</table>

UNIT TYPE: DPH-1990-B-2T
FUNCTION: TORQUE MOTOR POTENTIOMETER ASSEMBLY

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>UNITS</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Voltage</td>
<td>Volts</td>
<td>28</td>
</tr>
<tr>
<td>Terminal Resistance</td>
<td>Ohms</td>
<td>9.301</td>
</tr>
<tr>
<td>Stall Current (I_s)</td>
<td>Amps</td>
<td>3.11</td>
</tr>
<tr>
<td>No Load Speed</td>
<td>Rad/Sec</td>
<td>199.91</td>
</tr>
<tr>
<td>Torque Sensitivity</td>
<td>Oz-In/Amp</td>
<td>18.837</td>
</tr>
<tr>
<td>Peak Torque @ I_s</td>
<td>Oz-In</td>
<td>56.64</td>
</tr>
<tr>
<td>Back E.M.F.</td>
<td>V/Rad/Sec</td>
<td>.135</td>
</tr>
<tr>
<td>Inductance</td>
<td>Millihenries</td>
<td>3.21</td>
</tr>
<tr>
<td>Friction Torque</td>
<td>Oz-In</td>
<td>1.28</td>
</tr>
<tr>
<td>Armature Inertia</td>
<td>Oz-In-Sec^2</td>
<td>.0015</td>
</tr>
<tr>
<td>Acceleration</td>
<td>Rad/Sec^2</td>
<td>37,760</td>
</tr>
<tr>
<td>Pot Resistance</td>
<td>Ohms</td>
<td>5171</td>
</tr>
<tr>
<td>Linearity</td>
<td>Percent</td>
<td>&lt;.25</td>
</tr>
</tbody>
</table>
EM: 1800 Series
D.C. SERVO AMPLIFIERS
25W-1500W 28-48-60 volts

TTL logic level inhibit

Specialty Products Div.

B-2
D.C. SERVO AMPLIFIERS

- Single Power Supply (-15V bias supply not required for -B models).
- Aluminum Enclosure used for 25W Amplifier for Improved Heat Transfer.

**Description**

The EM1800 Series Linear DC Servo amplifiers are designed to drive DC torque motors, other DC servo motors and low inertia motors. Versatility of application is made possible by user connections to program for either voltage amplifier or current amplifier operation. Adjustable gain and current limiting can be user programmed with external resistors. A TTL logic-level inhibit gate can also be provided as an option for computer controlled shutdown applications. The requirement for a low-power negative bias supply has been eliminated without changing form, fit, or function of the plug-in modular amplifiers. Special circuitry reduces deadband thereby minimizing crossover distortion. Power output capabilities of the plug-in amplifiers range from 25 to 300 watts. Amplifier assemblies using the plug-in amplifiers as drivers have power output capabilities of 400 to 1500 watts.

Connectors on the plug-in amplifiers are in the form of twenty .040 inch diameter gold-plated pins that protrude from the encapsulated module. Easiest of application is made possible with the use of a mating socket (SO-1801) that fits all of the plug-in series amplifiers. The 25 watt amplifiers are encapsulated inside a black anodized aluminum shield. The 200 and 300 watt amplifiers have a surface ground aluminum plate on which is attached four NPN power transistors. This power bridge is driven by the basic 25 watt amplifier (see Fig. 1) and the entire circuit is encapsulated in a thermally-conductive epoxy.

Power dissipation capability of the amplifiers can be increased with the use of external heatsinks and/or forced air cooling. Most applications require the use of an external heatsink. The 200 and 300 watt amplifiers have a surface ground plate with four thru-holes to provide heatsink attachment. A heatsink (HS-1801) is available made from a black anodized aluminum extrusion, with four drilled and tapped holes, and having a thermal resistance of approximately 1.2°C/W to fit the 200 and 300 watt amplifiers.

Higher power outputs can also be obtained by using the 25 watt amplifier module as a driver for an external H-type power bridge consisting of NPN silicon power transistors (see Figure 2). Amplifier assemblies using this technique are available for power outputs of 400 to 1500 watts complete with plug-in driver amplifier, socket, power transistors mounted to integral heatsink, and forced air cooling where needed.

**Operation**

**General**

The input signal is applied to pins 4 and 7. Pin 7 is the offset adjust and is usually connected directly to pin 9 or through offset circuitry depending on whether the amplifier is connected for voltage or current operation. (See discussion under “Offset Adjustment”). Close-tolerance 10KΩ input resistors make up part of the total summing junction to a high gain differential amplifier. This input pre-amplifier generates differential output signals which drive an H-type power bridge. Special circuitry assures equal power dissipation in the bridge transistors under normal operating conditions where current output is below the set limit value. The bridge construction allows the use of a single-polarity high-voltage source and enables the output pins, 11 and 20, to switch polarity while the load floats above ground potential.

**Single Power Supply**

The -B designation identifies the amplifier model that does not require a -15V bias supply. The -A model should be selected for those applications where -15V bias supply is readily available. Both models have the same physical dimensions and are pin-for-pin interchangeable (Pin 1 has no connection on the -B models).
Supply Voltage Selection

Three amplifier families are available for operation with supply voltages of +28 Volt, +48 Volt, and +60 Volt. This expanded choice of supply voltages allows for better matching to minimize internal dissipation in the amplifier. MIL-STD-704 is applicable to the +28 Volt amplifiers.

Current Limit

An internal current-limit circuit senses and clamps the output current at 0.2 amperes. The value of the current-limit \( I_{cl} \) is adjustable to obtain higher currents by the use of an external current-limit resistor \( R_L \) which is connected between pins 17 and 18 with pin 17 jumpered to pin 6. The value of \( R_L \) is calculated by the formula:

\[
R_L = \frac{2}{I_{cl} - 0.4} \quad \text{Eq. 1}
\]

NOTE: The dissipation rating of \( R_L \) must be considered:

\[
P_{D_{R_L}} = (I_{cl})^2 R_L \quad \text{(Double for safety margin)} \quad \text{Eq. 2}
\]

Voltage Or Current Amplifier Operation

Voltage Amplifier

Programming for a voltage or a current amplifier becomes a simple matter of connecting the proper feedback to the input summing junctions. Whenever the 25 watt amplifier is used as a motor driver, it should be connected as shown in Figure 3 before the feedback connections are applied. Otherwise, to achieve the voltage amplifier mode, jumper pin 20 to pin 2 and pin 11 to pin 8. In the voltage amplifier mode, pin 14 and pin 15 should be connected to power ground (pin 13) to avoid unnecessary power dissipation in the current sense resistors (see Figure 4). The voltage gain is factory set for 20V/V. Standard operational amplifier techniques can be used to change the gain within reasonable limits.

Current Amplifier

For the current amplifier mode, jumper pin 3 to pin 14 and pin 5 to pin 15 (see Figure 5). The current gain is factory set for the plug-in amplifiers at 0.5A/V. Standard operational amplifier feedback techniques can be used to change the gain within reasonable limits.
APPENDIX C
TORQUE MOTOR MODEL DERIVATION FOR CURRENT AND VOLTAGE DRIVE

CURRENT DRIVE

The amplifier/motor control configuration for the current drive system is shown in figure C-1.

\[
V_{in} \quad \frac{A}{\tau_a s + 1} \quad \text{ARMATURE CONTROLLED MOTOR}
\]

\[
\text{PERMANENT MAGNET FIELD}
\]

\[
R_F \quad R_S \quad L_a \quad R_a \quad K_E \quad \delta_H
\]

\[
\text{DC MOTOR MODEL}
\]

\[
\text{MOTOR BACK EMF} = K_E \delta_H
\]

\[
\text{ARMATURE RESISTANCE} = R_a
\]

\[
\text{ARMATURE INDUCTANCE} = L_a
\]

\[
R_S = \text{CURRENT SENSING RESISTOR}
\]

\[
\tau_a = \text{AMPLIFIER TIME CONSTANT}
\]

\[
A = \text{OPEN LOOP AMPLIFIER GAIN}
\]

Figure C-1. Current drive amplifier/servo motor.

Figure C-2, the amplifier/motor configuration, shows the motor model incorporated into the diagram.

Let \( A' = \frac{A}{\tau_a s + 1} \)

then

\[
V_2 = \frac{E_S R_F}{R_1 + R_F} = E_S b
\]

where

\[
b = \frac{R_F}{R_1 + R_F}
\]
Figure C.2. Amplifier/motor diagram.

\[ E_0 = (V_1 - V_2) A' \]

\[ \therefore V_2 = - \left( \frac{E_o}{A'} - V_1 \right) \]  \quad (1)

\[ I_L = \frac{E_o - E_{BEMF}}{Z_L} \text{ assuming } E_s \text{ is small} \]

\[ E_s = I_L R_s \]

\[ \therefore V_2 = \frac{(f_o \cdot E_{BEMF}) b R_s}{Z_L} \]  \quad (2)

Combining (1) and (2) and solving for \( E_o \)

\[ \frac{E_o}{A'} + V_1 = \left( \frac{E_o - E_{BEMF}}{Z_L} \right) b R_s \]

\[ V_1 = \frac{E_o b R_s}{Z_L} - \frac{E_{BEMF} b R_s}{Z_L} + \frac{E_o}{A'} \]
\[
V_1 = E_o \left( \frac{A R_s b + Z_L}{A' Z_L} \right) - \frac{E_{BEMF} R_s b}{Z_L}
\]

Also
\[
E_o = I_L Z_L + E_{BEMF}
\] (4)

Now combine (3) and (4) and solve for \( I_L \)
\[
V_1 + \frac{E_{BEMF} R_s b}{Z_L} = \frac{I_L Z_L + E_{BEMF}}{A' R_s b + Z_L}
\]

\[
I_L Z_L = \frac{A' Z_L V_1 + E_{BEMF} A R_s b}{A' R_s b + Z_L} - E_{BEMF}
\]

\[
I_L Z_L = \frac{A' Z_L V_1 + E_{BEMF} A R_s b - E_{BEMF} A R_s b - E_{BEMF} Z_L}{A' R_s b + Z_L}
\]

\[
I_L Z_L = \frac{A' V_1 - E_{BEMF}}{A' R_s b + Z_L}
\] (5)

Now, replacing \( I_L \) by \( I_M \)
\[
Z_L \text{ by } S L_a + R_a
\]

and
\[
E_{BEMF} \text{ by } K_E N \dot{\theta}_H
\]

\[
I_M = \frac{A' V_{in} - K_E N \dot{\theta}_H}{A' R_s b + S L_a + R_a}
\] (6)

The load torque, \( T_L \), is given by \( J_T \, S^2 \dot{\theta}_H + D S \, \dot{\theta}_A \) (7)
where

\[ J_T = N^2 J_M + J_L \]

is the total inertia and

\[ N \]

is the gear ratio

\[ J_M \]

is the motor inertia

\[ J_L \]

is the load inertia

\[ D \]

is the viscous damping

The desired torque is given by:

\[ T_d = N K_T J_M \]

where \( K_T \) is the torque sensitivity constant

(8)

Now we can equate the load torque (7) with the desired torque (8) to obtain an expression for the amplifier/motor/load transfer function \( \frac{H}{V_{in}} \).

\[ N K_T J_M = J_T S^2 \theta_H + DS \theta_H \]

\[ N K_T J_M = \theta_H \left( J_T S^2 + D \right) \]

Now substitute expression (6) for \( J_M \)

\[ N K_T \left[ \frac{A V_{in} - K_E N \theta_H}{A R_s b + S L_a + R_a} \right] = \theta_H \left( J_T S^2 + D \right) \]

\[ \frac{A V_{in} - N^2 K_E K_T \theta_H}{A R_s b + S L_a + R_a} = \theta_H \left( J_T S^2 + D \right) \]

\[ \frac{A V_{in}}{A R_s b + S L_a + R_a} = \theta_H \left[ N^2 K_E K_T + J_T S A R_s b + S^2 J_T L_a + S J_T R_a + D A' R_s b \right] \]
Thus the amplifier/motor/load block diagram is as follows:

$$\dot{\theta}_H = \frac{N K_T A}{V_{in} (r_a S + 1) (L_a S + R_a) (J_T S + D) + A R_s b (J_T S + D) + N^2 K_E K_T (r_a S + 1)}$$

(11)

Figure C-3. Amplifier/motor/load block diagram for current drive servo control.

VOLTAGE DRIVE

The model for the DC armature controlled motor is shown in figure C-4.

$$V = i_a R_a + \frac{d}{dt} i_a L_a + K_E \dot{\theta}_M$$

(12)
Taking the LaPlace transform of each side of (12) we get

\[ V(s) = I_a(s) R_a + S L_a(s) L_a + SK_E \theta_m(s) \] (13)

The motor torque is

\[ T_M = N K_T I_a \] (14)

and the load torque is

\[ T_L = J_T S^2 \theta_H(s) + D S \theta_H(s) \] (15)

where

\[ J_t \] is the total inertia

\[ J_t = N^2 J_m + J_L \] (16)

and

\[ D \] is the total viscous damping

\[ D = N^2 D_m + D_L \] (17)

and

\[ \theta_H \] is the output shaft angle of the gear train (gimbal displacement)

Substituting (14) into (13)

\[ V(s) = N K_T N K_T I_a + SK_E N \theta_H \] (18)

and equating motor torque to load torque (18) becomes

Figure C-5. Motor/load gear ratio diagram.
from which the motor load transfer function is derived

\[
\frac{\theta_H(s)}{V(s)} = \frac{N K_T}{S \left[ J_t L_a S^2 + (J_t R_a + D L_a) S + D R_a + N^2 K_T K_E \right]}
\]  

or

\[
\frac{\delta_H}{V} = \frac{N K_T}{J_t L_a S^2 + (J_t R_a + D L_a) S + D R_a + N^2 K_T K_E}
\]

The block diagram for the voltage drive servo control is shown in figure C-6.

Figure C-6. Voltage drive servo control block diagram.
APPENDIX D
LOAD INERTIA DERIVATION

This appendix deals with the mathematical derivation of the load inertia for both inner and outer gimbals. Figure D-1 shows the components that comprise the total load inertia. All of the components, excluding the hat, are exact calculations of inertia. The moment of inertia of the hat was an approximation due to its complex geometric shape. Ref 14 was used in the derivation of the following load inertias.

1. OUTER GIMBAL

The moment of inertia for the moving parts of the outer gimbal will be found by calculating the moments for each of the nine parts in figure 1, and adding them. The outer gimbal pivots about a line through the center of mass of the plate or through the axis of the bails ring. All moments will be referred to this line and then summed. The method of calculation for the moment of each part \( I_z \) will be to calculate the moment \( (I_{CM}) \) about the line through the center of mass of the part and parallel to the axis of rotation, then substitute this into the formula

\[
I_z = I_{CM} + md^2, \tag{1}
\]


![Diagram of LADSS platform moment of inertia.](image)
where $m$ is the mass of the part and $d$ is the perpendicular distance from the center of mass of the part to the axis of rotation. Formulas for volumes, centroids and moments of inertia were found in CRC Standard Mathematical Tables 17th Edition. All parts are made of aluminum (density $= 2700 \text{ Kg/m}^3$) except the inner gimbal driven gear which is brass (density $= 8700 \text{ Kg/m}^3$). Note: all outer gimbal moments will be calculated for the zero position of the inner gimbal. The moment of inertia for the hat will not be calculated directly due to its complexity.

A. Plate

The center of mass of the plate lies on the axis of rotation, thus the following formula applies. $I_z = m/12 (a^2 + b^2)$. The dimensions of the plate are .1524 m $\times$ .1524 m $\times$ .0254 m.

$$m = \rho V = (2700 \text{ Kg/m}^3) (.1524 \text{ m})^2 (.0254 \text{ m}) = 1.592823 \text{ Kg}$$

$$I_z = \frac{1.592823 \text{ Kg}}{12} \left[ (.1524 \text{ m})^2 + (.0254 \text{ m})^2 \right]$$

$$I_z = 3.16851 \times 10^{-3} \text{ Kg-m}^2$$

B. Posts

There are four cylindrical posts that hold the hat in place on the plate. The position of the posts is symmetric about the axis of rotation, therefore the total moment of inertia for all posts is four times the moment for one post.

![Diagram of plate and posts]

Dimensions

- $l = .04572 \text{ m}$
- $r = .00635 \text{ m}$
- $a = .0635 \text{ m}$
- $b = .03556 \text{ m}$

The moment of inertia about the center of mass ($I_{CM}$) is given by

$$m \left( \frac{r^2 + g^2}{4} \right)$$

$$m = \rho V = \rho \pi r^2 \ell = (2700 \text{ Kg/m}^3) (\pi) (.00635 \text{ m})^2 (.04572 \text{ m})$$
In = .015637 Kg  Total mass for 4 posts = .062548 Kg

Then

\[ I_{CM} = (0.015637 \text{ Kg}) \left( \frac{(0.00635 \text{ m})^2 + (0.04572 \text{ m})^2}{4} \right) \]

\[ I_{CM} = 2.88158 \times 10^{-6} \text{ Kg-m}^2. \]

\[ I_z = (2.88158 \times 10^{-6} \text{ Kg-m}^2) + (0.015637 \text{ Kg}) \left( (0.0655 \text{ m})^2 + (0.03556 \text{ m})^2 \right) \]

\[ I_z(\text{for one post}) = 8.57071 \times 10^{-5} \text{ Kg-m}^2. \]

For all four posts

\[ I_z = 3.42828 \times 10^{-4} \text{ Kg-m}^2 \]

C. Pot Mount

![Figure D-2](image)

\[ a = 0.00508 \text{ m} \]
\[ b = 0.0254 \text{ m} \]
\[ x = 0.07366 \text{ m} \]
\[ y = 0.0254 \text{ m} \]
\[ D = 0.0381 \text{ m} \]

The moment of inertia about the center of mass is given by

\[ I_{CM} = \frac{m}{12} (a^2 + b^2) \]
\[ m = \rho V = \rho abD = (2700 \text{ Kg/m}^3) \times 0.0508 \text{ m} \times 0.0254 \text{ m} \times 0.0381 \text{ m} \]
\[ m = 0.0132735 \text{ Kg} \]
\[ \therefore I_{CM} = \left( \frac{0.0132735 \text{ Kg}}{12} \right) \left( (0.00508 \text{ m})^2 + (0.0254 \text{ m})^2 \right) \]
\[ I_{CM} = 7.42174 \times 10^{-7} \text{ Kg-m}^2 \]
Then $I_z$ is given by $I_z = I_{CM} + md^2$ where $d = \sqrt{x^2 + y^2}$

$$I_z = (7.42174 \times 10^{-7} \text{ Kg-m}^2) + (0.0132735 \text{ Kg}) [(0.07366 \text{ m})^2 + (0.0254 \text{ m})^2]$$

$$I_z = 8.1325 \times 10^{-5} \text{ Kg-m}^2$$

D. Support Block

![Support Block Diagram](image)

$$a = 0.01778 \text{ m},$$
$$b = 0.0254 \text{ m},$$
$$D = 0.0254 \text{ m},$$
$$d = 0.08382 \text{ m},$$
$$m = \rho V = \rho abD = (2700 \text{ Kg/m}^3) (0.01778 \text{ m}) (0.0254 \text{ m})^2$$

$$m = 3.09716 \times 10^{-2} \text{ Kg}$$

$$I_{CM} = \frac{m}{12} (a^2 + b^2) = \frac{3.09716 \times 10^{-2} \text{ Kg}}{12} [(0.01778 \text{ m})^2 + (0.0254 \text{ m})^2]$$

$$I_{CM} = 2.48105 \times 10^{-6} \text{ Kg-m}^2$$

$$I_z = I_{CM} + md^2 = (2.48105 \times 10^{-6} \text{ Kg-m}^2) + (3.09716 \times 10^{-2} \text{ Kg}) (0.08382 \text{ m})^2$$

$$I_z = 2.20081 \times 10^{-4} \text{ Kg-m}^2$$

E. Inner Gimbal Drive Gear

![Inner Gimbal Drive Gear Diagram](image)

This gear is half of a circular plate imbedded in the aluminum plate. The moment of inertia will be calculated for the full gear even though the slot was not allowed for in calculating the moment of the plate. The calculation will be as follows.
1. Calculate the mass, \( m = \rho V \), using \( \rho = 8700 \text{ Kg/m}^3 \) for brass.

2. Calculate the moment of inertia about the diameter, \( I_d \).

3. Find the location of the center of mass, \( \bar{y} \) (perpendicular distance from diameter)

4. Find the moment of inertia about the line parallel to the diameter, through the center of mass, \( I_{CM} \).

5. Calculate \( I_2 \) using Eq 2.

Dimensions

\[
\begin{align*}
  r &= .04953 \text{ m} \\
  t &= .00254 \text{ m (thickness)} \\
  x &= .03302 \text{ m} \\
  \rho &= \frac{1}{2} \pi r^2 t
\end{align*}
\]

\[
m = (8700 \text{ Kg/m}^3) \left( \frac{\pi}{2} \right) (.04953 \text{ m})^2 (.00254 \text{ m})
\]

\[
m = .085155 \text{ Kg}
\]

2. Moment of inertia about diameter
   element of mass is given by
   \[
   \rho t r dr d\theta
   \]
   Second moment of element about
diameter is given by
   \[
   (\rho t r dr d\theta) (r \sin \theta)^2
   \]
   Thus
   \[
   I_d = \int_0^{\pi} \int_0^{r} \rho t r^3 \sin^2 \theta \, dr \, d\theta
   \]
   \[
   I_d = \rho t \int_0^{\pi} \frac{r^4}{4} \sin^2 \theta \, d\theta
   \]
   \[
   I_d = \frac{\rho tr^4}{4} \left[ \frac{\pi}{2} - \frac{1}{4} \sin 2\pi - \left( 0 - \frac{1}{4} \sin \theta \right) \right] = \frac{\rho tr^4}{4} \left( \frac{\pi}{2} \right)
   \]
   \[
   I_d = \frac{\rho tr^4}{8} = 5.22259 \times 10^{-5} \text{ Kg-m}^2
   \]

3. Distance of center of mass from diameter \( (\bar{y}) \) element of mass = \( \rho t r dr d\theta \)
   First moment of element about diameter is given by \( (\rho t r dr d\theta) (r \sin \theta) \)
   
   D-5
Then
\[ M_x = \int_{\theta=0}^{\pi} \int_{r=0}^{r} \rho t r^2 \sin \theta \, dr \, d\theta \]
\[ M_x = \rho t \int_{\theta=0}^{\pi} \frac{r^3}{3} \sin \theta \, d\theta \]
\[ M_x = \frac{\rho t r^3}{3} \left[ -\cos \pi + \cos 0 \right] = \frac{2\rho t r^3}{3} \]

Now,
\[ \bar{y} = \frac{M_x}{m} = \frac{\frac{2\rho t r^3}{3}}{\rho \pi r^2 t} = \frac{4r}{3\pi} \]
\[ \bar{y} = .021021 \text{ m} \]

4. To find the moment of inertia about the center of mass, \( I_{CM} \) use Eq. 2 with \( I_z = I_d, d = \bar{y} \) and solve for \( I_{CM} \).
\[ I_d = I_{CM} + My^2 \Rightarrow I_{CM} = I_d - My^2 \]
\[ I_{CM} = (5.22259 \times 10^{-5} \text{ Kg-m}^2) - (.085155 \text{ Kg})(.021021 \text{ m})^2 \]
\[ I_{CM} = 1.45974 \times 10^{-5} \text{ Kg-m}^2 \]

5. Find \( I_z \) using Eq. 2 with \( d = \sqrt{x^2 + y^2} \)
\[ I_z = I_{CM} + m (x^2 + y^2) \]
\[ I_z = (1.45974 \times 10^{-5} \text{ Kg-m}^2) + (.085155 \text{ Kg}) [(0.03302 \text{ m})^2 + (.021021 \text{ m})^2] \]
\[ I_z = 1.45072 \times 10^{-4} \text{ Kg-m}^2 \]

F. Inner Gimbal Motor

\[ m = .2575 \text{ Kg (measured)} \]
\[ r = .0254 \text{ m} \]
\[ \ell = .0381 \text{ m} \]
\[ d = .05842 \text{ m} \]

Figure D-6
\[ I_{CM} = m \left( \frac{r^2}{4} + \frac{g^2}{12} \right) = (0.2575 \text{ Kg}) \left[ \frac{(0.0254 \text{ m})^2}{4} + \frac{(0.0381 \text{ m})^2}{12} \right] \]

\[ I_{CM} = 7.26813 \times 10^{-5} \text{ Kg-m}^2 \]

Then

\[ I_z = I_{CM} + md^2 = (7.26813 \times 10^{-5} \text{ Kg-m}^2) + (0.2575 \text{ Kg}) (0.05842 \text{ m})^2 \]

\[ I_z = 9.515 \times 10^{-4} \text{ Kg-m}^2 \]

G. Motor Mount

\[ a = 0.02032 \text{ m} \]
\[ b = 0.0381 \text{ m} \]
\[ c = 0.0381 \text{ m} \]
\[ d = 0.085 \text{ m} \]

\[ m = \rho V = \rho abc = (2700 \text{ Kg/m}^3)(0.02032 \text{ m})(0.0381 \text{ m})(0.0381 \text{ m}) \]

\[ m = 0.079641 \text{ Kg} \]

\[ I_{CM} = \frac{m}{12}(a^2 + b^2) = \frac{0.079641 \text{ Kg}}{12} \left[ (0.02032 \text{ m})^2 + (0.0381 \text{ m})^2 \right] \]

\[ I_{CM} = 1.23743 \times 10^{-5} \text{ Kg-m}^2 \]

\[ I_z = I_{CM} + md^2 = (1.23743 \times 10^{-5} \text{ Kg-m}^2) + (0.079641 \text{ Kg}) (0.085 \text{ m})^2 \]

\[ I_z = 5.87781 \times 10^{-4} \text{ Kg-m}^2 \]

H. Bail Ring

\[ r_1 = 0.09525 \text{ m} \]
\[ r_2 = 0.106363 \text{ m} \]
\[ h = 0.03683 \text{ m} \]
\[ m = \rho V = \rho \frac{200}{360} (\pi r_2^2 h - \pi r_1^2 h) = (\rho \pi h) \left( \frac{200}{360} \right)(r_2^2 - r_1^2) \]

\[ m = (2700 \text{ Kg/m}^3) (\pi) (.03683 \text{ m}) \left( \frac{200}{360} \right) [(0.106363 \text{ m})^2 + (0.09525 \text{ m})^2] \]

\[ m = .388859 \text{ Kg} \]

\[ I_2 = \frac{m}{2} (r_2^2 - r_1^2) = \frac{.388859 \text{ Kg}}{2} [(0.106363 \text{ m})^2 + (0.09525 \text{ m})^2] \]

\[ I_2 = 3.96358 \times 10^{-3} \text{ Kg-m}^2 \]

I. HAT

The moment of inertia of the hat assembly will be approximated by a segment of a spherical shell. The dimensions of the shell were chosen such that, if it were made of aluminum, the mass would coincide with the measured mass, 1.06 Kg. Also the dimensions were chosen so that the center of mass of the shell is located at the estimated center of mass of the hat assembly. A cross section of the shell model is shown in figure D-9, with dimensions.

\[ r_1 = .132801 \text{ m} \]
\[ r_2 = .137595 \text{ m} \]
\[ h_1 = .09398 \text{ m} \]
\[ h_2 = .098774 \text{ m} \]
\[ c_1 = .254 \text{ m} \]
\[ c_2 = .26401 \text{ m} \]
\[ \alpha = 2.558909 \text{ rad} \]
\[ s = .038821 \text{ m} \]
\[ C_M = \text{center of mass} \]
\[ C_R = \text{center of rotation} \]
The following formulae were used in calculating some of the dimensions.

\[ s = r_1 - h_1 \]

\[ \alpha = \sin^{-1} \frac{c_1}{2r_1} + \sin^{-1} \frac{c_2}{2r_2} \]

\[ c = \sqrt{4h(2r - h)} \]

To find the moment of inertia of the shell, \( I_s \), the following procedure will be used. First an expression for the mass will be found. Then the center of mass will be located using the expression \( z = M_p/m \) where \( M_p \) is the first moment about the plane A-A in figure D-9. Next the moment of inertia about the center of mass, \( I_{CM} \), will be calculated and finally the moment of inertia about the center of rotation, \( I_{CR} \), using Eq. 1.

A. The mass of the shell is given by \( \rho V \) where \( \rho = 2700 \text{ Kg/m}^3 \) (the density of aluminum) and \( V \) is the volume. Using spherical coordinates, an element of mass is given by

\[ \rho r^2 \sin \phi \, d\theta d\phi. \]

Thus the mass is given by

\[ m = \rho \int_{\phi=0}^{\phi=\alpha/2} \int_{\theta=0}^{2\pi} \int_{r=r_1}^{r=r_2} r^2 \sin \phi \, dr d\theta d\phi \]

\[ m = \frac{2\pi}{3} \left( r_2^3 - r_1^3 \right) (1 - \cos \alpha/2) \]

Using the dimensions in figure 9, \( m = 1.06 \text{ Kg.} \)
B. Due to symmetry the center of mass lies on the line B-B in figure D-9. Thus the distance \( z \) must be calculated. To find \( z \), the first moment about the polar plane, \( M_p \), must be calculated. Again, an element of mass is given by

\[
\rho r^2 \sin \phi \, drd\theta d\phi
\]

and its distance from the polar plane is given by

\[ r \cos \phi. \]

Thus

\[
M_p = \int_{\phi=0}^{\phi=\alpha/2} \int_{\theta=0}^{2\pi} \int_{r=r_1}^{r_2} \rho r^3 \sin \phi \cos \phi \, drd\theta d\phi
\]

\[
M_p = \frac{\pi \rho}{4} \left( \frac{r_2^4 - r_1^4}{r_2^3 - r_1^3} \right) \sin^2 \left( \frac{\alpha}{2} \right)
\]

Then

\[
\bar{z} = \frac{M_p}{m} = \frac{3}{8} \frac{(r_2^4 - r_1^4)}{(r_2^3 - r_1^3)} \left( 1 - \cos \frac{\alpha}{2} \right)
\]

Substitution yields

\[ \bar{z} = 0.087034 \text{ m} \]

C. The moment of inertia about the line through the center of mass, perpendicular to line B-B, is found by summing the second moments of the mass elements. That is, the mass element times the distance from the line (\( x \)) squared. The mass element is as before and the distance squared is given by

\[ x^2 = r^2 \sin^2 \phi + (r \cos \phi - \bar{z})^2 \]

Thus the moment of inertia about the CM line is given by

\[
I_{CM} = \int_{\phi=0}^{\phi=\alpha/2} \int_{\theta=0}^{2\pi} \int_{r=r_1}^{r_2} \rho r^2 \sin \phi \left[ r^2 \sin^2 \phi + (r \cos \phi - \bar{z})^2 \right] drd\theta d\phi
\]

\[
I_{CM} = 3m \left[ \frac{1}{5} \frac{r_2^5 - r_1^5}{r_2^3 - r_1^3} \frac{\bar{z}}{4} \left( \frac{r_2^4 - r_1^4}{r_2^3 - r_1^3} \right) \left( 1 + \cos \frac{\alpha}{2} \right) + \bar{z}^2 \right]
\]

\[
I_{CM} = 1.13559 \times 10^{-2} \text{ Kg-m}^2
\]
D. The total moment of inertia about the axis of rotation can now be found using Eq. 1 with \( d = \overline{Z} - s \).

\[
I_{CR} = I_{CM} + m(\overline{Z} - s)^2
\]

\[
I_{CR} = 1.381986 \times 10^{-2} \text{ Kg-m}^2.
\]

II. INNER GIMBAL

The elements that make up the inner gimbal inertia are the plate, posts, pot mount, inner gimbal drive gear and the hat (see figure D-1). Again, the moment of inertia of the hat will not be computed directly. The moments of the plate and posts need not be recomputed due to symmetry.

J. Pot Mount

\( a = .00508 \text{ m} \)
\( b = .0254 \text{ m} \)
\( c = .0381 \text{ m} \)
\( y = .0254 \text{ in} \)

The moment of inertia about the center of mass \((I_{CM})\) is given by:

\[
I_{CM} = m\left(\frac{b^2 + c^2}{12}\right)
\]

\( m = .0132735 \text{ Kg} \) (from section I.C. pg D-3)

\[
I_{CM} = (.0132735 \text{ Kg})\left(\frac{(.0254 \text{ m})^2 + (.0381 \text{ m}^2)}{12}\right) = 7.42174 \times 10^{-7} \text{ Kg-m}^2
\]

Then

\[
I_z = I_{CM} + my^2
\]

\[
I_z = 7.42174 \times 10^{-7} \text{ Kg-m}^2 + (.0132735 \text{ Kg}) (.0254 \text{ m})^2
\]

\[
I_z = 1.088323 \times 10^{-5} \text{ Kg-m}^2
\]

K. Gear Drive
Since the center of rotation passes through the center of the diameter of the semi-circle, the moment of inertia is given by:

\[ I_z = \frac{m r^2}{2} \]

\[ m = 0.085155 \text{ Kg} \quad \text{(from section I.E.1. pg D-5)} \]

\[ r = 0.04953 \text{ m} \]

\[ \therefore I_z = (0.085155 \text{ Kg}) \left( \frac{(0.04953 \text{ m})^2}{2} \right) \]

\[ I_z = 1.04452 \times 10^{-4} \text{ Kg-m}^2 \]

III. TOTAL MOMENT OF INERTIA

1. Summary for Outer Gimbal

<table>
<thead>
<tr>
<th>Part</th>
<th>Mass (Kg)</th>
<th>Inertia (Kg-m^2)</th>
<th>Inertia (oz-in-sec^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Plate</td>
<td>1.592823</td>
<td>3.16851 \times 10^{-3}</td>
<td>.448661</td>
</tr>
<tr>
<td>B. Posts</td>
<td>.062548</td>
<td>3.42828 \times 10^{-4}</td>
<td>.048544</td>
</tr>
<tr>
<td>C. Pot Mount</td>
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<td>8.1325 \times 10^{-5}</td>
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<td>D. Support Blk</td>
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<td>2.20081 \times 10^{-4}</td>
<td>.031163</td>
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<td>E. Gear</td>
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<td>G. Mount</td>
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<td>5.87781 \times 10^{-4}</td>
<td>.08323</td>
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<td>H. Bail Ring</td>
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<td>.561243</td>
</tr>
<tr>
<td>I. HAT</td>
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<td>1.381986 \times 10^{-2}</td>
<td>1.96037</td>
</tr>
<tr>
<td>Total</td>
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2. C. Summary for Inner Gimbal

<table>
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<th>Part</th>
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<th>Inertia (Kg-m^2)</th>
<th>Inertia (oz-in-sec^2)</th>
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</thead>
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<td>K. Gear</td>
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<tr>
<td>Total</td>
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APPENDIX E
TIME-DOMAIN ANALYSIS OF AMPLIFIER/MOTOR/LOAD

This appendix derives the equations used for time-domain analysis of the servomotor system. The analysis was structured such that both the current drive and voltage drive systems could be evaluated with only changes to the input data for both versions of the time-domain computer programs (LaPlace Transform and Time Integration versions).

Figure E-2 presents comparative data between the two types of gimbal drive systems (current vs voltage) for both the inner and outer gimbals. The gimbal rate as a function of time is plotted for the comparative data. It is quite obvious from the curves that the current drive system is superior in response to that of the voltage drive system.

Two types of time-domain analysis were conducted to obtain the time-domain response data. The inverse LaPlace transform method, which is explained in detail in this section, was used to generate the comparative data for gear ratio optimization. The inverse LaPlace transform technique was very useful in obtaining the large amounts of data needed for the gear ratio optimization. Figures E-4 and E-5 illustrate the effects on the gimbal rate as a function of drive shaft gear ratio. It is quite obvious that a direct drive system is quite sluggish. As the gear ratio increases so does the gimbal response time—up to a point. As the gear ratio increases beyond some optimum value, the response starts to slow down. Figures E-6 and E-7 summarize the gear ratio optimization for the inner and outer gimbal system. The point to be noted about the inverse LaPlace transform technique is that it requires very little computer time to generate a large amount of time-domain data; thereby, being an economic means of generating this kind of data. The inverse LaPlace transform technique does not take into account the system nonlinearities. Therefore, a time integration program was written to incorporate the nonlinearities (limiters) that are shown in the diagram of figure E-1. The data presented in figures E-2 and E-3 was generated with the time integration computer program. Ref. 13 was used for the LaPlace transform analysis formulae.

Following is a derivation of the LaPlace transform methodology for generating time-domain analysis for both the current and voltage servo drive systems.

I. BLOCK DIAGRAMS

CURRENT DRIVE VERSION

VOLTAGE DRIVE VERSION

Figure E-1. Current and voltage block diagrams.

II. TRANSFER FUNCTIONS

a. Current drive version (rate output)

\[
\frac{X_R}{P} = \frac{A K_T N}{(J_T S + D) (\tau_a S + 1) (L_a S + R_a) + A \cdot R_s b (J_T S + D) + N^2 K_E K_T (\tau_a S + 1)}
\]

or

\[
\frac{X_R}{P} = \frac{A K_T N/L_a J_T \tau_a}{S^3 + \left( \frac{L_a \tau_a D + J_T L_a + J_T \tau_a R_a}{L_a J_T \tau_a} \right) S^2 + \left( \frac{D L_a + \tau_a D R_a + J_T R_a + A R_b b J_T + N^2 K_E K_T \tau_a}{L_a J_T \tau_a} \right) S + \left( \frac{R_a D + A R_b b D + N^2 K_E K_T}{L_a J_T \tau_a} \right)}
\]

(1)
b. Voltage drive version (rate output)

\[
\frac{X_R}{P} = \frac{A' K_T N}{(J_T S + D) (r_a' S + 1) (L_a S + R_a) + N^2 K_E K_T (r_a' S + 1)}
\]

or

\[
\frac{X_R}{P} = \frac{A' K_T N / L_a J_T r_a'}{S^3 + \left( \frac{L_a r_a' D + J_T L_a + J_T r_a' R_a}{L_a J_T r_a'} \right) S^2 + \left( \frac{D L_a + r_a' D R_a + J_T R_a + N^2 K_E K_T r_a'}{L_a J_T r_a'} \right) S} + \left( \frac{R_a D + N^2 K_E K_T}{L_a J_T r_a'} \right)
\]

(2)

III. DESCRIPTION OF GAINS AND TIME CONSTANTS

\( A \) – Amplifier gain (v/v)

\( \tau_a \) – Amplifier time constant (sec.)

\( L_a \) – Motor inductance (henries)

\( R_a \) – Armature resistance (ohms)

\( R_s b \) – Current feedback sensing resistor (ohms)

\( K_T \) – Torque sensitivity (oz-in/amp)

\( N \) – Gear ratio (gimbal-to-motor)

\( J_T \) – Total inertia. This is given by \( J_T = N^2 J_M + J_L \) where \( J_M \) is the motor inertia and \( J_L \) is the load inertia

\( D \) – Friction (oz-in-sec) (viscous friction)

\( K_E \) – Back EMF (v/rad/sec)

\( A' \) – Amplifier gain for voltage driven motor.

Referring to the properties of an inverting op amp

\( A' = \frac{1 - \beta}{\beta} \) where \( \beta = \frac{R_l}{R_l + R_f} \)

and \( R_l \) is the input resistor, \( R_f \) is the feedback resistor. Choosing nominal values of \( R_l = 5K \) and \( R_f = 20K \) yields a \( \beta \) of .2 and \( A' = 4.0 \).

\( r_a' \) – Time constant for voltage driven motor. Again referring to properties of inverting op amps

\( r_a' = \frac{r_a}{1 + \beta A} \)
Note that, with the appropriate calculation of $A'$ and $r_d$ the voltage drive transfer function is the same as the current drive with $R_s b = 0.0$.

IV. METHOD OF OBTAINING TIME RESPONSE

A hand check of the poles of equations (1) and (2), using preliminary gains and time constants, showed that the roots could get quite large (negatively). This fact eliminates numerical integration as an efficient cost effective method of calculating the time response, due to the extremely small integration interval needed to show the effects of the large roots. Thus, the LaPlace transform method was chosen.

Both the current drive (1) and voltage drive (2) transfer functions contain cubic functions of $S$ in the denominator (for the rate output). To obtain the position output, (1) and (2) must be multiplied by $1/S$. Finally, to obtain the response to a unit step input, (1) and (2) must again be multiplied by $1/S$.

Therefore, we have two basic forms for the denominator: a cubic multiplied by $S$ for rate output with unit step input and a cubic multiplied by $S^2$ for position output with unit step input. In order to comply with standard forms of LaPlace transform pairs, we must divide these forms into two cases. Case 1 occurs when the cubic equation has three real roots and Case 2 when there are one real root and two conjugate complex roots.

V. LAPLACE TRANSFORM PAIRS

a. Three real root case

1. Rate output

\[
F(s) = \frac{1}{S (1 + T_1 S) (1 + T_2 S) (1 + T_3 S)}
\]

\[f(t) = 1 - \frac{T_1^2}{(T_1 - T_2)(T_1 - T_3)} e^{-t/T_1} - \frac{T_2^2}{(T_2 - T_1)(T_2 - T_3)} e^{-t/T_2} - \frac{T_3^2}{(T_3 - T_1)(T_3 - T_2)} e^{-t/T_3}
\]

2. Position output

\[
F(s) = \frac{1}{S^2 (1 + T_1 S) (1 + T_2 S) (1 + T_3 S)}
\]

\[f(t) = 1 - (T_1 + T_2 + T_3) - \frac{1}{(T_1 - T_2)(T_2 - T_3)(T_3 - T_1)} \left[ T_1^3 (T_2 - T_3) e^{-t/T_1} + T_2^3 (T_3 - T_1) e^{-t/T_2} + T_3^3 (T_1 - T_2) e^{-t/T_3} \right]
\]
b. One real, two complex root case

1. Rate output

\[
F(s) = \frac{1}{S (1 + TS) \left( 1 + \frac{2\xi}{\omega} S + \frac{1}{\omega^2} S^2 \right)}
\]
\[
f(t) = 1 - \frac{T^2 \omega^2}{1 - 2T\xi \omega + T^2 \omega^2} e^{-t/T} + \frac{\sin \left( \omega \sqrt{1 - \xi^2} t - \psi_R \right)}{\sqrt{1 - \xi^2} (1 - 2\xi T\omega + T^2 \omega^2)^{1/2}} e^{-\xi \omega t}
\]

where

\[
\psi_R = \tan^{-1} \left( \frac{\sqrt{1 - \xi^2}}{-\xi} \right) + \tan^{-1} \left( \frac{T \omega \sqrt{1 - \xi^2}}{1 - T\xi \omega} \right)
\]

2. Position output

\[
F(s) = \frac{1}{S^2 (1 + TS) \left( 1 + \frac{2\xi}{\omega} S + \frac{1}{\omega^2} S^2 \right)}
\]
\[
f(t) = t - T - \frac{2\xi}{\omega} + \frac{T^3 \omega^2}{1 - 2\xi T\omega + T^2 \omega^2} e^{-t/T} \frac{\sin \left( \omega \sqrt{1 - \xi^2} t - \psi_p \right)}{\omega \sqrt{(1 - \xi^2) (1 - 2\xi T\omega + T^2 \omega^2)^{1/2}}} e^{-\xi \omega t}
\]

where

\[
\psi_p = 2 \tan^{-1} \left( \frac{\sqrt{1 - \xi^2}}{-\xi} \right) + \tan^{-1} \left( \frac{T \omega \sqrt{1 - \xi^2}}{1 - T\xi \omega} \right)
\]

c. Note that, in all cases, \(f(t) = \mathcal{L}^{-1} (F(s))\). Also, the cubic equations in (1) and (2) must be put into the form of the appropriate \(F(s)\) in order to evaluate the inverse Laplace transform. Gain terms (numerator) in (1) and (2) multiply \(f(t)\) since the Laplace transform is linear.

VI. FINDING THE ROOTS OF THE CUBIC

Any calculation, involving real numbers, done on a digital computer results in an approximation to the desired result due to truncation. Due to the large numbers encountered in the transfer functions under consideration, this approximation was not a good one. This was caused by addition and subtraction of very large numbers that had their first few significant digits in common.
Since the closed form solution of the cubic did not yield satisfactory results, a check was made using Newton's method. It was found that this method converged rapidly and yielded a better approximation to the actual roots.

A subroutine was written to find the roots of a cubic polynomial using Newton's method. The algorithm used can be found in "Elementary Numerical Analysis: an Algorithmic Approach" by Conte and de Boor, McGraw-Hill, 1972, pg 69. Essentially the algorithm uses Newton's method to find one real root of the cubic, where successive values of the polynomial and its derivative are calculated using nested multiplication. This method yields the coefficients of the quotient quadratic which can be solved to find the other two roots. Whether these roots are real or complex is indicated by a flag.

VII. TRANSFORMING THE TRANSFER FUNCTIONS INTO THE FORM USED IN THE LAPLACE TRANSFORM PAIRS

a. The transfer functions for both the current and voltage drive versions are of the form

\[ F(s) = \frac{G'}{S^3 + pS^2 + qS + r} \]

where \( G' \) is the gain term \( A K_T \frac{N}{L_a} \tau_a J_T \) or \( A' K_T \frac{N}{L_a} \tau'_a J_T \). These must be put into the form used in the Laplace transform pairs. Two cases are necessary depending on whether there are three real roots, or one real, two complex.

b. Three real root case

The transfer function

\[ F(s) = \frac{G'}{S^3 + pS^2 + qS + r} \]

where

\[ G' = \frac{A K_T N}{L_a \tau_a J_T} \quad \text{or} \quad \frac{A' K_T N}{L_a \tau'_a J_T} \]

must be changed to the form

\[ F(s) = \frac{G}{(1 + T_1 S)(1 + T_2 S)(1 + T_3 S)} \]

Let \( R_1, R_2 \) and \( R_3 \) be the real roots of the denominator. Then we can write

\[ F(s) = \frac{G'}{(S - R_1)(S - R_2)(S - R_3)} \]

\[ F(s) = \frac{G'}{-R_1 R_2 R_3 (1 + S/R_1)(1 + S/R_2)(1 + S/R_3)} \]

E-6
Let \( T_1 = -\frac{1}{R_1} \), \( T_2 = -\frac{1}{R_2} \) and \( T_3 = -\frac{1}{R_3} \).

Then

\[
F(s) = \frac{G T_1 T_2 T_3}{(1 + T_1 S)(1 + T_2 S)(1 + T_3 S)}
\]

and we have

\[
F(s) = \frac{G}{(1 + T_1 S)(1 + T_2 S)(1 + T_3 S)}
\]

where

\[
G = \frac{A K_T N \cdot T_1 T_2 T_3}{L_a \tau_a J_T} \quad \text{or} \quad \frac{A' K_T N T_1 T_2 T_3}{L_a \tau_a' J_T}
\]

\[
T_1 = -\frac{1}{R_1}
\]

\[
T_2 = -\frac{1}{R_2}
\]

\[
T_3 = -\frac{1}{R_3}
\]

Now equation (4) can be used to ascertain the rate output time response, and equation (6) for the position output time response.

c. One real, two complex root case

The transfer function

\[
F(s) = \frac{G'}{S^3 + p S^2 + q S + r}
\]

where

\[
G' = \frac{A K_T N}{L_a \tau_a J_T} \quad \text{or} \quad \frac{A' K_T N}{L_a \tau_a' J_T}
\]

must be changed to the form

\[
F(s) = \frac{G}{(1 + TS) \left(1 + \frac{2 \omega_c}{\omega} S + \frac{1}{\omega^2} S^2\right)}
\]

Let \( R \) and \( \alpha \pm \beta i \) be the roots of the denominator. Then we can write

\[
F(s) = \frac{G'}{(S - R) \left[S - (\alpha + \beta i)\right] \left[S - (\alpha - \beta i)\right]}
\]

E-7
Let $T = -\frac{1}{R}$, then

$$F(s) = \frac{G'T}{(1 + TS) [S^2 - 2\alpha S + (\alpha^2 + \beta^2)]}$$

$$F(s) = \frac{G'T}{(1 + TS) (\alpha^2 + \beta^2) \left(1 - \frac{2\alpha}{\alpha^2 + \beta^2} S + \frac{1}{\alpha^2 + \beta^2} S^2\right)}$$

Let $\omega^2 = \alpha^2 + \beta^2$, $\xi = -\frac{\alpha}{\omega}$ and $G = G'T/\omega^2$.

Then

$$F(s) = \frac{G}{(1 + TS) \left(1 + \frac{2\xi}{\omega} S + \frac{1}{\omega^2} S^2\right)}$$

where

$$T = -\frac{1}{R}$$
$$\omega^2 = \alpha^2 + \beta^2$$
$$\xi = -\frac{\alpha}{\omega}$$

$$G = \frac{A K_T N T}{L_a J_T \tau_a \omega^2} \text{ or } \frac{A' K_T N T}{L_a J_T \tau'_a \omega^2}$$

VIII. ALGORITHM FOR COMPUTING TIME RESPONSE OF AMPLIFIER/MOTOR/LOAD

a. 1. Read all gains and time constants, stop time and time increment.
   2. Compute $p$, $q$ and $r$, the coefficients of a cubic polynomial that make up the denominator of the polynomial.
   3. Call subroutine to compute roots of the cubic
   4. If (one real, two complex roots: $r, \alpha \pm \beta i$) then
      5. Compute $T$, $\omega$, $\xi$, $G$, $\psi_R$, $\psi_P$
      6. While time $\leq$ stop time do
         7. increment time (1)
         8. compute rate and position using eq (8) & (10)
         9. output
   10. end do
   11. else (three real root case)

E-8
12. compute $T_1, T_2, T_3$ & $G$
13. while time $<$ stop $t$ do
14. increment time ($t$)
15. compute rate and position using eq (4) & (6)
16. output
17. end do
18. end if
19. stop

b. Notes:

1. Notice that the algorithm does not refer to current or voltage drive. The choice of types of drive is made by appropriately setting the input variables. Set $A$ to $A'$, $\tau_a$ to $\tau_a'$ and $R_s$ b to 0.0. See Section III.

2. The algorithm was implemented in FORTRAN. In the implementation, the names of the variables correspond to the variables used in this write-up (e.g., $ZETA = \xi$, $OMEGA2 = \omega^2$, $T = t$, $TT = T$ etc.)

3. The subroutine cubic referred to in step 3 returns three values, $R_1 \beta_2$ & $R_3$, and a flag. If the flag = 1 then $R_1$ is the real root $R$ in Section VII.c. and $R_2 = \alpha, R_3 = \beta$. If the flag = 0 then $R_1, R_2$ & $R_3$ are the three real roots referred to in Section VII.b.

4. In the implementation, intermediate variables are used to store portions of the inverse LaPlace transform formulas (4), (6), (8), (10). E.g., $TRM1 = 1 - 2T\xi \omega + T^2 \omega^2$. This is done to reduce the size of the assignment statements for rate and position and to reduce the number of computations done in the loop.

TIME INTEGRATION TIME-DOMAIN ANALYSIS

The time integration methodology was required to take into account the nonlinearities of the system. The drawback of this analysis procedure is the cost in running the computer model. Basically the time integration methodology takes an $n^{th}$ order differential equation and breaks it down into $n$ first order differential equations or in other words the system is defined by a system of state variables. The system of first order differential equations defining the servo drive system is summarized in Figure E-8. These equations are integrated by an integration routine every sampling period.

Figure E-8 is a block diagram of the servo drive system which is a math model used to formulate the system of "first order" differential equations used in the time-domain analysis (time integration) computer program.

Computer Program listings and sample of output data for both the LaPlace and time integration analysis procedures are included in the following section LADSS*AMLTR and LADSS*AML 201.

---

1. State variables may be defined as the minimum set of variables that provide full knowledge of the system’s behavior.
Figure F-2. Time response — comparison between voltage and current drives for inner gimbal.

Figure E-3. Amplifier/motor load time response comparison between voltage and current drives for outer gimbal.
Figure E-4. Amplifier/motor/load time response as function of gear ratio (inner gimbal current drive).

Figure F-5. Amplifier/motor/load time response as function of gear ratio (outer gimbal current drive).
Figure E-6. Gear ratio vs acceleration – inner gimbal – current drive.

Figure E-7. Gear ratio vs acceleration – outer gimbal/current drive.
FIRST ORDER DIFFERENTIAL EQUATIONS USED TO GENERATE THE TIME INTEGRATION TIME RESPONSE IN COMPUTER PROGRAM LADSS*AML301.

\[
E = P - R_d \cdot V \\
\dot{\theta} = (E \cdot A_0 - R) / \tau a \\
U = R - N \cdot K_E \cdot X_R \\
\dot{V} = (U - V \cdot R_a) / L_a \\
W = V \cdot K_T \cdot N \\
\dot{X}_R = (W \cdot X_R \cdot F_T) / J_T \\
\ddot{X} = X_R
\]

Figure E-8. Block diagram/equations of servo drive system/load for time integration computer program
LADSS* AMLTR

1. C** NAME: LADSS STABILIZED PLATFORM AMPLIFIER/MOTOR/LOAD TIME RESPONSE
2. C** USAGE: THE FOLLOWING CONTROL CARDS WILL EXECUTE THIS PROGRAM
3. C** EXE: LADSSAMLR,PROG
4. C** PURPOSE: THIS PROGRAM MODELS IN THE TIME DOMAIN
5. C** THE AMPLIFIER/MOTOR/LOAD PORTION OF THE LADSS
6. C** STABILIZED PLATFORM. THE TIME RESPONSE IS OBTAINED
7. C** BY CALCULATING THE INVERSE LAPLACE TRANSFORM OF THE
8. C** A/M/L TRANSFER FUNCTION.
9. C** LIMITATIONS: NONE
10. C** WARNINGS: NONE
11. C** SUBPROGRAMS REQUIRED:
12. C** CUBIC - COMPUTES THE ROOTS OF A CUBIC POLYNOMIAL
13. C** ARGUMENTS: NONE
14. C** NOTES:
15. C** 1. THIS PROGRAM RUNS ON THE UNIVAC 1110 IN ASCII FORTRAN
16. C** CPU TIME IS APPROXIMATELY (NUM / 100) SECONDS
17. C** 2. EACH INPUT VARIABLE MUST BE ON A SEPARATE LINE WITH 2 SPACES
18. C** AVAILABLE AT THE BEGINNING OF THE LINE FOR THE VARIABLE NAME
19. C** 3. THE INPUT TO THE A/M/L IS A UNIT STEP FUNCTION,
20. C** THIS IS DUE TO THE CHOICE OF LAPLACE TRANSFORM PAIRS
21. C** THAT ARE USED.
22. C** 4. TWO TYPES OF SYSTEMS CAN BE MODELED BY THIS PROGRAM:
23. C** A) A VOLTAGE DRIVEN MOTOR
24. C** B) A CURRENT DRIVEN MOTOR
25. C** THE CURRENT DRIVEN VERSION IS OBTAINED BY
26. C** SETTING INPUT VARIABLE RSB TO THE SENSING RESISTOR VALUE.
27. C** TO GET THE VOLTAGE DRIVEN RESPONSE SET RSB TO 0.0
28. C** AND ADJUST INPUT PARAMETERS A & B AS INDICATED IN WRITE UP
29. C** 5. THE AMPLIFIER IS A CURRENT OR VOLTAGE DRIVEN AMPLIFIER.
30. C** THE DC TORQUE MOTOR IS A PERMANENT MAGNET, ARMATURE
31. C** CONTROLLED DEVICE.
32. C** THE LOAD IS THE ANTENNA PLUS MOTOR INERTIA REFERENCED
33. C** TO THE GIMBAL SHAFT.
34. C** PROGRAMMER/ORGANIZATION: Daryl E. Smith CSC DEPT 551
35. C** ALGORITHM:
36. C** 1. READ AND ECHO INPUTS
37. C** 2. COMPUTE COEFFICIENTS OF CUBIC POLYNOMIAL
38. C** 3. CALL CUBIC TO FIND ROOTS
39. C** 4. COMPUTE VARIABLES USED IN INVERSE LAPLACE TRANSFORM
40. C** 5. EVALUATION
41. C** 6. DO FOR "HOP" TIME INCREMENTS
42. C** 7. EVALUATE INVERSE LAPLACE TRANSFORMS
43. C** 8. OUTPUT
44. C** END DO
118. READ (4,100) I /TUA1/1,HA,HP,HT,HI,HL,IL,KE
119. 100 FORMAT (7X,F10.0)
120. READ (5,200) NUM,TINC,ITER
121. 200 FORMAT (15,F10.6,15)
122. JT = NINT(JT) + JL
123. WRITE (16,300) U,TAU1,14,2,AS,KT,HI,HL,IL,KE
124. 300 FORMAT (3A1,F10.3,/,5A1,F10.5,/,1A1,F10.5,/,)
125. 6 * RA = F10.5,*/ JT,*/ F10.5,*/
126. 6 * RA = F10.5,*/ JT,*/ F10.5,*/
127. 6 = ( L3 = TAU1 = JT = 0 + F10.5,*/ KE,*/ F10.5 )
128. P = ( L3 = TAU1 = JT = 0 + F10.5,*/ KE,*/ F10.5 )
129. Q = ( L3 = TAU1 = JT = 0 + F10.5,*/ KE,*/ F10.5 )
130. 6 = ( L3 = TAU1 = JT = 0 + F10.5,*/ KE,*/ F10.5 )
131. WRITE (5,400) P,G,E
132. CALL CUREC ( F,DA,ITER,KEI,HL,RR,RR2,RR3 )
133. IF ( ITER.EQ.0 ) THEN
134. NUM = 0
135. END IF
136. IF ( KFLG.EQ.1 ) THEN
137. C ONE REAL, TWO COMPLEX ROOT CASE
138. C
139. C
140. ALPHA = RR2
141. BETA = RR3
142. RR = RR1
143. RR = RR2
144. WRITE (4,500) RR,ALPHA,BETA
145. 500 FORMAT ( 4,5D12 )
146. FORMAT ( ' ONE REAL ROOT, TWO COMPLEX ROOTS' )
147. 100 FORMAT ( 20,ALPHA,RH) = ( D20.12,3X )
148. TT = 1.000 / -RR
149. TTSQ = TT * TT
150. OMEGA = ALPHA+BETA+ BETA
151. OMEGA = DSO3 ( OMEGA )
152. ZETA = ( ALPHA / OMEGA )
153. ZETA**2 = D7A2 ( ZETA )
154. OMEGA**2 = D8A2 ( OMEGA )
155. TRM1 = 1.000 = 2.000 + ZETA + OMEGA
156. TRM1 = DSO3 ( TRM1 )
157. TRM2 = TTSQ + OMEGA
158. TRM2 = DSO3 ( TRM2 )
159. TRM3 = ( TT**OMEGA+RH**2 ) / ( 1.000 = TT**OMEGA )
160. TRM3 = DATAN ( TRM3 )
161. TRM4 = CLAN ( TRM3 )
162. TRM4 = DSO3 ( TRM4 )
163. GAIN = ( ALPHA + TT ) / ( OMEGA + TRM1 )
164. WRITE (4,600) TT,OMEGA,TRM4,TRM3,GAIN,ZETA
165. 600 FORMAT ( 5X,T20.12,2X)
166. 6 = O GAIN = ' D20.12,10X ' ZETA = ' D20.12 '}
167. 6 = O GAIN = ' D20.12,10X ' ZETA = ' D20.12 '}
168. 6 = O GAIN = ' D20.12,10X ' ZETA = ' D20.12 '}
169. 6 = O GAIN = ' D20.12,10X ' ZETA = ' D20.12 '}
170. WRITE (4,800) PHM,PHM
171. 800 FORMAT ( 1X,PHM = ' D20.12,10X ' PHM = ' D20.12 ')
172. RR,RR2,RR3,RR4
173. RR = TINC = FLOAT ( 1 )
174. TRM1 = DSO3 ( ZETA + OMEGA )
SUBROUTINE CUBIC ( P, Q, R, A, N, I, KFLG, X, R1, R2, R3 )

PURPOSE: THIS ROUTINE CALCULATES THE ROOTS OF THE CUBIC POLYNOMIAL

x^3 + P + x^2 + Q + R = 0, BY NEWTON'S METHOD.

LIMITATIONS: THE NEWTON ITERATION SCHEME WILL STOP AFTER ITER TIMES.

WARNINGS: IF THE ITERATION SCHEME DOES NOT CONVERGE, A MESSAGE

IS PRINTED, THEN CONTROL IS RETURNED TO THE CALLING PROGRAM.

THE RESULTS, AT THIS TIME WILL BE ERRONEOUS.

SUBPROGRAMS REQUIRED: NONE

ARGUMENTS:

P = COEFFICIENT OF SQUARED TERM OF CUBIC
Q = COEFFICIENT OF X TERM OF CUBIC
R = CONSTANT TERM OF CUBIC
I = MAXIMUM NUMBER OF TIMES TO ITERATE.
KFLG = IF ITERATION SCHEME DOES NOT CONVERGE.
KFLG = 0 = THREE REAL ROOTS
KFLG = 1 = ONE REAL, TWO COMPLEX
KFLG = 2 = TWO CASES
KFLG = 3 = SECOND REAL ROOT
KFLG = 4 = REAL PART OF COMPLEX ROOT
KFLG = 5 = THIRD REAL ROOT
KFLG = 6 = IMAG PART OF COMPLEX ROOTS

NOTES: ALL VARIABLES IN CUBIC ARE REAL, EXCEPT I, ITER, KFLG.

THIS INCLUDES P, Q, R, R1, R2, R3; IN THE CALL PARAMETERS

PROGRAMMER/ORGANIZATION: DARYL C. SMITH CSC DEPT 551

ALGORITHM: NEWTON'S METHOD IS USED TO FIND THE REAL ROOT.

EVALUATION OF THE POLYNOMIAL AND ITS DERIVATIVE IS BY THE
FACTORED METHOD. IF: I, (X+P) * X + Q = X + R.

THE RESOLVED QUADRATIC (X+X + B2*X + B1 = 0) IS THEN SOLVED
USING THE QUADRATIC FORMULA.

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78. 1  =  0
80.  x = -2.00-1
81.  m = 1.000
82.  C3 = 83
83.  t = 1 + 1 = 1
84.  B2 = X+3)
85.  C2 = 87 + 1*C3
86.  B1 = 0 + X+62
87.  C1 = B1 + X+62
88.  B0 = m + X+61
89.  Dx = A0 / C1
90.  X = X + DX
91.  WRITE ( 6,20 ) X,DX,R0,B1,R2,C1,C2
92.  Z0 FORMAT ( ' X,DX,R0; 4,5 ( D20.12,3X ) )
93.  G = R1,B2,C12-C21 / S ( D20.12,3X )
94.  C CHECK FOR END OF ITERATION
95.  C CHECK FOR CONVERGENCE
96.  C IF ( I + LE. ITER ) THEN
97.  C CONVERGENCE: FIND ROOTS
98.  C K FLG = 0
99.  C CHECK FOR POSITIVE DISCRIMINANT
100. C IF ( DESC ,GE. 0.000 ) THEN
101. C THREE REAL ROOTS
102. C RR2 = 1.00 + DSQRT ( DESC ) / 2.000
103. C RR3 = 1.00 - DSQRT ( DESC ) / 2.000
104. ELSE
105. C ONE REAL & TWO COMPLEX
106. C RR2 = .000 / 2.000
107. C RR3 = DSQRT ( -DESC ) / 2.000
108. ELSE
109. END IF
110. ELSE
111. C NO CONVERGENCE
112. C WRITE ( 4,30 )
113. C FORMAT ( ' D10-****** CURRIC ITERATION .GT. ITER ' )
114. C ITER = 0
115. ELSE
116. RETURN
117. END
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- **P, w, R2:**
  - -2.398503326506007, 2.1430678400810, 5.1776409600011
- **U1, U2, C1, C2:**
  - -2.415533592090002, 3.6649436335004, 7.2140963317005
  - -2.415533592090002, 5.6788644765032, 11.869530709002
- **U1, U2, C1, C2:**
  - -2.1186224754010, 1.0071128460007, 2.0946582946410, 0.100209469328007

### Real Roots

- **RX, RX2, RX3:**
  - -2.1533590200000, 2.1186228067004, -0.9969998436006
- **SIGMA, SIGMA2, SIGMA3:**
  - 0.409301518730001, 0.51401852396001, -0.70953190653003

### Gain

- **GAIN:**
  - 0.602693236484000

### Type

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SUBROUTINE EQUIT

COMMON /STEP/, DT, HIT, NSTS, H024

DIMENSION C(4), FSAV(30,4)

DATA C=-9.0, 0.37, 0.2, -9.0, 0.65, 0.4

DATA FSAV(120*4)

IF HIT GE 31 GO TO 20

C USE EULERS METHOD TO START

DO 10 I=1,NSTS

FSAV(I,1) = FSAV(I,2)

FSAV(I,2) = FSAV(I,3)

FSAV(I,3) = FSAV(I,4)

FSAV(I,4) = Y(I)

Y(I) = Y(I) + DT*FSAV(I,4)

10 CONTINUE

GO TO 90

C ANALYSIS - RASHIFORTH

DIV = 30 Go 30 I=1,NSTS

SUM = 0.0

FSAV(I,1) = FSAV(I,2)

FSAV(I,2) = FSAV(I,3)

FSAV(I,3) = FSAV(I,4)

SUM = SUM + C(I)*FSAV(I,1)

SUM = SUM + C(I)*FSAV(I,2)

SUM = SUM + C(I)*FSAV(I,3)

SUM = SUM + C(I)*FSAV(I,4)

10 CONTINUE

Y(I) = Y(I) + H024*SUM

30 CONTINUE

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APPENDIX F
FREQUENCY RESPONSE ANALYSIS

Figure F-1 presents the fundamental block diagram on which the frequency domain analysis will be based.

![Block Diagram](image)

Figure F-1. Stabilized platform block diagram.

where

\[
T_1(S) = \frac{K_4K_5}{\tau_7S + 1}
\]

\[
T_2(S) = \frac{K_3(\tau_2S + 1)^2(\tau_5S + 1)}{S(\tau_3S + 1)^2(\tau_6S + 1)}
\]

\[
T_3(S) = \frac{1}{L_aS + R_a}
\]

\[
T_4(S) = \frac{1}{J_TS + D}
\]

\[
T_5(S) = \frac{K_{MHD}K_2}{(\tau_4S + 1)^3}
\]

Also let

\[
T_m(S) = \frac{AKTN}{(\tau_aS + 1)(L_aS + R_a)(J_TS + D) + AR_b(J_TS + D) + (\tau_aS + 1)N^2K_EK_T}
\]
which is the full torque motor transfer function. Six transfer functions will be derived for the frequency analysis:

\[
1) \frac{\dot{\delta}}{\delta}, \quad 2) \frac{\dot{\delta}}{T_d}, \quad 3) \frac{\dot{\delta}}{\theta_M}, \quad 4) \frac{\dot{\delta}}{\theta_H}, \quad 5) \frac{e_T}{\theta_M}, \quad \text{and} \quad 6) \frac{e_T}{T_d}.
\]

All of these transfer functions are derived as per the conical form of figure F-2.

![Figure F-2. Conical form block diagram.](image)

where the transfer function of input to output is

\[
\frac{R}{C} = \frac{G(S)}{1 + G(S) H(S)}.
\]

(1)

For the first transfer function, \(\dot{\delta}/\delta\), the block diagram of figure F-1 is rearranged as illustrated in figure F-3.

![Figure F-3. Block diagram for \(\dot{\delta}/\delta\) transfer function.](image)

where

\[
G_1(S) = \frac{-T_1(S)}{S}
\]

(2)
\[
H_1(S) = \frac{T_M(S) T_2(S)}{1 + T_M(S) T_2(S) T_8(S)} \tag{3}
\]

\[
\frac{\hat{\theta}}{\hat{T}} = \frac{1}{SF} \cdot \frac{G_1(S)}{1 + G_1(S) H_1(S)} \tag{4}
\]

When the scale factor is incorporated into Eq. 4 \( \hat{\theta} \) is in the units of rad/sec. The scale factor is

\[
SF = K_{MHD} \cdot \eta_2. \tag{5}
\]

For the second transfer function, \( \hat{\theta}/T_d \), the block diagram of figure F-1 is rearranged as illustrated in figure F-4.

\[
G_2(S) = \frac{-T_1(S) T_4(S)}{S}, \tag{6}
\]

\[
H_2(S) = \frac{K_{TN} T_3(S) [T_A(S) T_2(S) (T_1(S) - ST_5(S)) - SNK_E]}{T_1(S) [1 + T_A(S) R_{sb} T_3(S)]} \tag{7}
\]

and

\[
\frac{\hat{\theta}}{\hat{T}_d} = \frac{1}{SF} \cdot \frac{G_2(S)}{1 + G_2(S) H_2(S)}. \tag{8}
\]

For the third transfer function, \( \hat{\theta}/T_M \), the block diagram of figure F-1 is rearranged as illustrated in figure F-5.
Figure F-5. Block diagram for \( \hat{\theta}/\theta_M \) transfer function.

where

\[
G_3(S) = \frac{-T_1(S)}{S} ,
\]

\[
H_3(S) = T_M(S) T_2(S) \left[ 1 - \frac{S T_5(S)}{T_1(S)} \right],
\]

and

\[
\hat{\theta}/\theta_M = \frac{1}{S^2} \cdot \frac{G_3(S)}{1 + G_3(S) H_3(S)}
\]

For the fourth transfer function, \( \hat{\theta}/\theta_H \), the block diagram of figure F-1 is rearranged as illustrated in figure F-6.

Figure F-6. Block diagram for \( \hat{\theta}/\theta_H \) transfer function.

where

\[
G_4(S) = T_2(S) T_M(S),
\]

\[
H_4(S) = T_5(S),
\]

and

\[
\hat{\theta}/\theta_H = \frac{1}{S^2} \cdot \frac{G_4(S)}{1 + G_4(S) H_4(S)}
\]

For the fifth transfer function, \( \varepsilon_T/\theta_M \), the block diagram of figure F-1 is rearranged as illustrated in figure F-7.
For the sixth transfer function, $e_T / T_d$, the block diagram of figure F-1 is rearranged as illustrated in figure F-8.

![Block diagram of $e_T / T_d$ transfer function.](image)

where

$$G_6(S) = \frac{-T_4(S)}{S}, \quad (18)$$

and

$$H_6(S) = \frac{K_T N T_3(S) \left[ T_A(S) T_2(S) \left( T_1(S) - ST_5(S) \right) - SNK_E \right]}{1 - T_A(S) R_s T_3(S)}. \quad (19)$$

and

$$\frac{e_T}{T_d} = \frac{G_6(S)}{1 + G_6(S) H_6(S)} \quad (20)$$

Figure F-7. Block diagram of $e_T/\delta_M$ transfer function.

where

$$G_5(S) = \frac{-1}{S}, \quad (15)$$

$$H_5(S) = T_M(S) T_2(S) [T_1(S) - ST_5(S)], \quad (16)$$

and

$$\frac{e_T}{\delta_M} = \frac{G_5(S)}{1 + G_5(S) H_5(S)} \quad (17)$$

For the sixth transfer function, $e_T / T_d$, the block diagram of figure F-1 is rearranged as illustrated in figure F-8.
I Z, V, 0:11 – a C C 0, 4 ft 0 K 4, * n. 4 X N V, - . 2 I - C t n O - x - U L h - . 4 A 411. 041 0 32002 S - r t r 12 44 5 0 0 I - L 2 L 00. 0 U W 00 0 1. 0 W 1-2 1 F-7
OFTH,RODS L1
FTN BR:,DN1/14,01-14:111331
1. C This function computes H for S
2. C canonical form of the control
3. C Subsystems listed in comments. Choice of subsystems is
4. C indicated by variable I in common area GHTYPE.
5. C
6. C
7. C
8. C
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53. C
54. C
55. C

 Torque disturbance = to Sigma dot H.

 BODY MOTION {THETA DOT M1} TO Sigma dot H.

 Torque disturbance = to pointing error.

 BODY MOTION {THETA DOT M1} TO pointing error.
QFTU,num5 LET

FTN ARR: 80/16/30-14:14-1A

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2. IMPLICIT HEAL (J,K,L,N)
3. COMPLEX S,ONE
5. G, A,TAU4,LA,NA,RSQ,KE,N,GT,DE2,EHDO,K2,KG,TAU4
6. DATA ONE / (1.0,0.0) /
7. T1 = KS + K5 / 1.0 + S + ONE
8. RETURN
9. END
DFTU, ROOS L.T4
FTH 8R6 -04/16/80-14:18:10

COMPLEX FUNCTION TV (S)

COMPLEX 5, ONE
COMMON /CONST/ K5, K4, TAU7, K3, TAU2, TAU0, TAU3, TAU4, TAU5, TAU6,

A, TAU1, LA, RA, RSR, KT, N, JT, DK, KE, KMHD, K2, KE, TAUN

DATA ONE (1.0, 0.0)

RETURN

END
I, M, r, co, u, I, ii, vi, U, J, UVN16
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**BOU HOT = SIG DIH Y LADSS PLATFORM**

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**Notes:**
- All calculations are based on the given frequency range and loop gain values.
- The real and imaginary parts of G and H are calculated accordingly.
- The values are rounded to the nearest possible precision.
APPENDIX G
TIME-DOMAIN ANALYSIS

TIME-DOMAIN ANALYSIS DERIVATION

The time-domain analysis was carried out using time integration computer programs. The advantage of using this program is that nonlinearities can very easily be incorporated into the computer model. The approach is straightforward. First the transfer functions are broken down into first order. This is illustrated in figures G-1 and G-2, the slave and track loops respectively. Each of the first order blocks can then be equated to a first order differential equation. These first order differential equations are listed on figures G-1 and G-2. Once the system of first order differential equations has been defined, they are incorporated into a computer program that uses an integration routine for a step by step time integration of a system of first order differential equations.
Figure G-1. Time-domain analysis block diagram of slave loop.
Figure G-2. Time-domain analysis block diagram of track loop.
LADSS* TRKLPTR

WITHIN ROM MAIN
FINISH 6/10/70-16:32 (1w)
1. **NAME=LADSS.TRKLPTR PLATFORM TRACK LOOP TIME RESPONSE
2. **
3. ** USAGL: THE FOLLOWING CONTROL CARDS WILL EXECUTE THIS PROGRAM
4. **
5. ** DEFT LADSS.TRKLPTR.PHLG
6. **
7. ** PURPOSE: THIS PROGRAM MODELS THE TRACK LOOP OF THE LADSS SIMULATED PLATFORM BY NUMERICAL INTEGRATION
8. ** TO A GIVEN INPUT SIGNAL.
9. **
10. ** LIMITATIONS: THE RC GAIN (A) AND THE ABNORMAL INDUCTANCE (LA) IS OMITTED.
11. **
12. ** WARNINGS: THIS PROGRAM MAY USE CONSIDERABLE COMPUTER TIME. IF THE
13. ** INPUTS STPNT AND DT ARE CHosen INCORRECTLY.
14. **
15. ** PROCEDURES REQUIRED:
16. **
17. ** EUNIT - PERFORMS THE NUMERICAL INTEGRATION
18. **
19. ** ARGUMENTS: NONE
20. **
21. ** NOTES:
22. **
23. ** 1: THIS PROGRAM RAN ON THE UNIVAC 1110 IN ASCII FORTRAN
24. **
25. ** 2: EACH INPUT VARIABLE IS ON A SEPARATE LINE WITH A SPACE FOLLOWING THE VARIABLE NAME.
26. **
27. ** 3: OUTPUT IS GENERATED EVERY DP SECONDS OF SIMULATED TIME.
28. **
29. ** 4: THE INPUT SIGNAL SIGD is CHosen AT EITHER
30. **
31. ** 5: A SINUSOID WHOSE FREQUENCY AND AMPLITUDE ARE GIVEN BY
32. **
33. ** 6: THE CHOICE IS MADE BY SETTING LOGICAL VARIABLE SIGF PROPERLY
34. **
35. ** 7: SIGF = TRUE YIELDS A SINUSOID WAVE
36. **
37. ** 8: SIGF = FALSE YIELDS A STEP FUNCTION.
38. **
39. **
40. ** PROGRAMMER/ORGANIZATION: DARYL E. SMITH CSC DEPT 551
41. **
42. ** ALGORITHM:
43. **
44. ** 1: READ AND ECDF INPUTS
45. **
46. ** 2: INITIALIZE CONTROL VARIABLES
47. **
48. ** 3: DO WHILE I EC. STPNT
49. **
50. ** 4: CALL-EQUI
51. **
52. ** CHECK FOR OUTPUT
53. **
54. **
55. ** Record of Modifications:
56. **
57. ** START EDIT PAGE
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**Example:**

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- **A3** | LIMIT ON R
- **A4** | LIMIT ON S
- **A5** | LIMIT ON T
- **A6** | LIMIT ON U
- **A7** | LIMIT ON V
- **A8** | LIMIT ON W
- **A9** | LIMIT ON X
- **A10** | LIMIT ON Y
- **A11** | LIMIT ON Z
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- **B2** | LOGICAL SIGG
- **B3** | LOGICAL SIGH
- **B4** | LOGICAL SIGI
- **B5** | LOGICAL SIGD
- **B6** | LOGICAL SIGC
- **B7** | LOGICAL SIGB
- **B8** | LOGICAL SIGA
- **B9** | LOGICAL SIG
HEAL U
HEAL V
HEAL R
HEAL X2
HEAL X3
HEAL X4
HEAL X60
HEAL X48
HEAL X60
HEAL X96
eq
SCL CLOCK DIAGRAM
= NOT0K CURRENT OUTPUT AMPS
= NOT0K E05NE OUTPUT. DI-IN-SEC2
= SCL CLOCK DIAGRAM
= SCL CLOCK DIAGRAM
= DERIVATIVE OF X5
= SCL CLOCK DIAGRAM
= START-LOOP OUTPUT
= DERIVATIVE OF X6
= OUTPUT OF NUMERICAL INTEGRATION
=DERIVATIVES INPUT TO NUM INTG
= EQUIVALENCE ( Y ( 1 ) , EPS ) , ( YD ( 1 ) , EPSD )
= EQUIVALENCE ( Y ( 2 ) , SIGH ) , ( YD ( 2 ) , SIGHMD )
= EQUIVALENCE ( Y ( 3 ) , D ) , ( YD ( 3 ) , DD )
= EQUIVALENCE ( Y ( 4 ) , C ) , ( YD ( 4 ) , CD )
= EQUIVALENCE ( Y ( 5 ) , D ) , ( YD ( 5 ) , DD )
= EQUIVALENCE ( Y ( 6 ) , D ) , ( YD ( 6 ) , PD )
= EQUIVALENCE ( Y ( 7 ) , X4 ) , ( YD ( 7 ) , X4D )
= EQUIVALENCE ( Y ( 8 ) , X5 ) , ( YD ( 8 ) , X5D )
= EQUIVALENCE ( Y ( 9 ) , X6 ) , ( YD ( 9 ) , X6D )
= EQUIVALENCE ( Y ( 10 ) , X4 ) , ( YD ( 10 ) , X4D )
= START-STOP PAGE
= READ AND ECHO INPUTS.
= C
= C
A
J

!w

K

\[ \text{FORMAT (} \text{X} \text{,} \text{X}\text{,} \text{X}\text{)} \text{, FOR THE VARIABLE NAME} \]

\[ \text{RAKE.,} \text{RAK}, \text{KPRE,} \text{KPRE,} \text{KPRE,} \text{KPRE,} \text{KPRE,} \]

\[ \text{FORMAT (} \text{X} \text{,} \text{X}\text{,} \text{X}\text{,} \text{X}\text{,} \text{X}\text{,} \text{X}\text{)} \text{, FOR THE VARIABLE NAME} \]

\[ \text{RAKE.,} \text{RAK}, \text{KPRE,} \text{KPRE,} \text{KPRE,} \text{KPRE,} \text{KPRE,} \]

\[ \text{FORMAT (} \text{X} \text{,} \text{X}\text{,} \text{X}\text{,} \text{X}\text{,} \text{X}\text{,} \text{X}\text{)} \text{, FOR THE VARIABLE NAME} \]

\[ \text{RAKE.,} \text{RAK}, \text{KPRE,} \text{KPRE,} \text{KPRE,} \text{KPRE,} \text{KPRE,} \]
SUBROUTINE EQUIT ( D , Y , YD , Y0 , Y )

DESCRIPTION OF D:

D = INPUT

DESCRIPTION OF Y:

Y = INPUT

DESCRIPTION OF YD:

YD = INPUT

DESCRIPTION OF Y0:

Y0 = INPUT

DESCRIPTION OF Y:

Y = OUTPUT

USING CALL EQUIT ( D , Y , YD , Y , Y0 )

PURPOSE: THIS ROUTINE PERFORMS A NUMERICAL INTEGRATION ON A SYSTEM

OF FIRST-ORDER LINEAR DIFFERENTIAL EQUATIONS.

LIMITATIONS: THE NUMBER OF EQUATIONS IN THE SYSTEM IS LIMITED TO 30

THE DIMENSION OF Y, YD, Y0, AND D IS N.

WARNINGS: NONE

SUBPROGRAMS REQUIRED: NONE

ARGUMENTS:

INPUT: D = REAL INTEGRATION INTERVAL

M = INTEGER COUNTS NUMBER OF TIMES EQINT IS CALLED.

U = USED TO FLAG SWITCH FROM EULER TO ADAMS-BASHFORTH

HYS = NO. OF EQUATIONS IN SYSTEM

YD = NUMERICAL INTEGRATION

OUTPUT: Y = NUMERICAL INTEGRATION

NOTES: THE DIFFERENTIAL EQUATIONS MUST BE CALCULATED BEFORE THE

CALL TO EQUIT, AND THE RESULTS STORED IN ARRAY YD.

THE RESULTS OF THE INTEGRATION ARE CALCULATED BY EQINT.

AND STORED IN ARRAY Y, THERE IS A ONE-ONE CORRESPONDENCE

BETWEEN THE ELEMENTS OF Y AND Y;

OF INTEGRATING YD(1).

PROGRAMMER/ORGANIZATION: DARYL E. SMITH, CSC DEPT 551

ALGORITHM: THE METHOD OF INTEGRATION IS THE STANDARD

ADAMS-BASHFORTH. FOR THE FIRST THREE CALLS TO EQINT, EULER'S

METHOD IS USED AND THE RESULTS STORED IN ARRAY FSAV. AFTER THAT

ADAMS-BASHFORTH IS USED.

CODE MODIFICATIONS:

START EDIT PAGE
90  \texttt{REAL (14) /-0.37,-\ldots,2.5/}  \texttt{ADAMS-BASHFORTH COEFFICIENTS}
97  \texttt{REAL D1}  \texttt{INTEGRATION INTERVAL}
98  \texttt{REAL DT*0.1}  \texttt{DT / 24.0 MULTIPLIER}
99  \texttt{REAL FSAVE(+4,6) /120.0/}  \texttt{INTERMEDIATE STORAGE FOR A-U}
100 \texttt{REAL I}  \texttt{LOOP COUNTER}
101 \texttt{REAL NINT \ldots}  \texttt{COUNTS \# OF INTEGRATIONS}
102 \texttt{REAL NSYS}  \texttt{\# OF EQUATIONS IN SYSTEM}
103 \texttt{REAL SUM}  \texttt{ADAMS-BASHFORTH SUM}
104 \texttt{REAL Y(50)}  \texttt{OUTPUT OF NUMERICAL INTEGRATION}
105 \texttt{REAL Y(250)}  \texttt{DERIVATIVES INPUT TO NUM-INTG}

107 \texttt{IF} \texttt{C-LST = L.3.3 \ldots \ldots}  \texttt{THEN \ldots}
108 \texttt{DO 30 J = 1,240}  \texttt{USE EULER'S TO START}
109 \texttt{DO 30 K = 1,240}  \texttt{\ldots}
1010 \texttt{FSAV (1,2) = FSAVE (1,2)}  \texttt{\ldots}
1011 \texttt{FSAV (1,3) = FSAVE (1,3)}  \texttt{\ldots}
1012 \texttt{FSAV (1,4) = Y(L-1)}  \texttt{\ldots}
1013 \texttt{Y (1) = Y (1) + DT * FSAVE (1,4)}  \texttt{\ldots}
1014 \texttt{CONTINUE}
1015 \texttt{DO 30 J = 1,240}  \texttt{\ldots}
1016 \texttt{DO 30 K = 1,240}  \texttt{\ldots}
1017 \texttt{FSAV (1,1) = Y(L-1)}  \texttt{\ldots}
1018 \texttt{FSAV (1,2) = FSAVE (1,2)}  \texttt{\ldots}
1019 \texttt{FSAV (1,3) = FSAVE (1,3)}  \texttt{\ldots}
1020 \texttt{FSAV (1,4) = Y(L-1)}  \texttt{\ldots}
1021 \texttt{Y (1) = Y (1) + DT * FSAVE (1,4)}  \texttt{\ldots}
1022 \texttt{CONTINUE}
1023 \texttt{DO 30 J = 1,240}  \texttt{\ldots}
1024 \texttt{DO 30 K = 1,240}  \texttt{\ldots}
1025 \texttt{FSAV (1,1) = Y(L-1)}  \texttt{\ldots}
1026 \texttt{FSAV (1,2) = FSAVE (1,2)}  \texttt{\ldots}
1027 \texttt{FSAV (1,3) = FSAVE (1,3)}  \texttt{\ldots}
1028 \texttt{FSAV (1,4) = Y(L-1)}  \texttt{\ldots}
1029 \texttt{Y (1) = Y (1) + DT * FSAVE (1,4)}  \texttt{\ldots}
1030 \texttt{CONTINUE}
1031  
1032 \texttt{END IF}
1033 \texttt{RETURN}
1034 \texttt{END}
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**Inflation Interval**: 0.0003100

Run for 1.60 seconds, print every 0.010 seconds.

Input is a function of amplitude 1.0000.
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| f_1.10h | 7.7574 | 14.061 | -7500-002 | 1.0065 | 1.5438 | 19.042 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.10h | 7.9455 | 16.328 | -64107-002 | 1.1456 | 1.6647 | 18.224 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.20h | 7.9795 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.30h | 8.2550 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.40h | 8.4350 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.50h | 8.6150 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.60h | 8.7950 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.70h | 8.9750 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.80h | 9.1550 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.90h | 9.3350 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.00h | 9.5150 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.010h | 9.6950 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.020h | 9.8750 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.030h | 10.055 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.040h | 10.235 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.050h | 10.415 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.060h | 10.595 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.070h | 10.775 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

| f_1.080h | 10.955 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |

<p>| f_1.090h | 11.135 | 11.265 | -5127-002 | 1.1231 | 1.4705 | 17.439 | 0.0520 | 0.1642 | -0.1409 | -0.1305 | 0.0520 | 0.1642 |</p>
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- $\|\cdot\|_2$: 1.1091
- $\|\cdot\|_\infty$: 1.1091

Matrix condition numbers:
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- $\text{cond}(\cdot, 2) = 1.1091 / 0.0439 = 25.1399$
- $\text{cond}(\cdot, \infty) = 1.1091 / 0.0439 = 25.1399$
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**Integration Interval**: 0.00010

**Run for 1.00 seconds, print every 0.0100 seconds**

**Input is a linear function of amplitude**: -1.0000

**Input**: 

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**Output**:

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**Summary:**

- The table contains data for SIGD, IPS, SIGDH, and XR for different time intervals.
- The values range from 0.297 to 0.3573 for TIME.
- The values for SIGD, IPS, SIGDH, and XR vary significantly across these time intervals.

**Further Analysis:**

- The data might be related to a specific experiment or study involving these variables.
- More context or additional data would be necessary to fully understand the implications of these values.
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