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ION BEAM PROPAGATION IN A FILAMENTED CHANNEL.(U)

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Ion Beam Propagation in a Filamented Channel

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The production of intense focused ion beams has led to the consideration of using z-discharge plasma channels to transport them several meters to inertial fusion targets. Analysis has shown that current bunching will occur in the channel during transport. In this report ion beam propagation in a filamented channel is investigated in order to determine the effects on radial beam containment in channels and on the radial beam density profile.

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CONTENTS

ION BEAM PROPAGATION IN A FILAMENTED CHANNEL

I. INTRODUCTION

The production of intense focused ion beams[1] has led to the consideration of using z-discharge plasma channels to transport them several meters to inertial fusion targets[2]. Analysis has shown that although electrostatic beam-plasma streaming modes are stable[3], electromagnetic microinstabilities will occur during transport in such channels[4]. The fastest growing mode produces current bunching of the electrons in the channel. In this report ion beam propagation in a filamented channel is investigated in order to determine the effects on radial beam containment in channels and on the radial beam density profile.

In Sec. II, radial current bunching in the channel is considered and in Sec. III, azimuthal current bunching is considered. Finally the results of this work are summarized in Sec. IV.

II. RADIAL CURRENT BUNCHING IN THE CHANNEL

If $\underset{\sim}{k} = k\,\hat{e}_r$, then radial current bunching occurs in the channel and the net current density can be modeled by

$$\underset{\sim}{j} = \left[\bar{j}(r) + j_1\, H(r-r_b)\, J_o(kr)\right] \hat{e}_z \quad , \tag{1}$$

where

$$H(r-r_b) = \begin{cases} 1\ , & 0 \leq r \leq r_b \\ 0\ , & r > r_b \end{cases} ,$$

r_b is the beam radius and J_o is the zero order Bessel function which satisfies the wave equation in cylindrical coordinates. For

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a channel of radius r_c

$$I_o = 2\pi \int_o^{r_c} j(r)\, rdr = 2\pi \int_o^{r_c} \bar{j}(r)\, rdr$$

is the channel current which confines the ion beam. Here $k \equiv a_{1\ell}/r_b$ was used, where $a_{1\ell}$ is the ℓ^{th} zero of first order Bessel function J_1. The second term on the right hand side of Eq. (1) is then the residual current density arising from the bunched electron current superimposed on the unbunched ion beam current. Since $j(r)$ is established in an initially cold plasma and is driven by a capacitor bank on a time scale much longer than the beam pulse duration, when the beam is injected into the now highly conducting plasma the total net current resists change. Thus, in the absence of bunching $\underset{\sim}{j} = \underset{\sim}{j}_e + \underset{\sim}{j}_b = \bar{j}(r)\, \hat{e}_z$, where $\bar{j} << j_b$.

The magnetic field which determines the ion motion in a filamented channel is then found from

$$\frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) = \frac{4\pi}{c} \left[\bar{j}(r) + j_1 J_o(kr) \right] \quad , r \leq r_b. \tag{2}$$

Assuming $\bar{j}(r) = j_o (r/r_c)^{s-1}$ inside the channel ($s \geq 1$),

$$B_\theta = \frac{4\pi j_o r^s}{(s+1)\, c\, r_c^{s-1}} + \frac{4\pi j_1}{kc} J_1(kr) \; , \quad r \leq r_b, \tag{3}$$

where $r_c > r_b$. The parameter s is a magnetic field shaping factor which is equal to one at the front of the beam and increases as one moves toward the tail of the beam. The beam heated plasma in the channel at the tail of the beam expands radially carrying the channel current with it. This decreases the magnetic field (increases s) near the center of the beam and produces a sharply rising magnetic field at the edge of the beam.[5]

The total energy H, the axial cononical momentum P_z, and the canonical angular momentum P_θ are all constants of the motion for the ions. From $A_z = -\int B_\theta dr$ and Eq. (3), P_z is found to be

$$\frac{P_z}{m_i} = \dot{z} - \frac{2\omega_{cb} r_c}{(s+1)} \left(\frac{r}{r_c}\right)^{s+1} - \frac{2\omega_{cb}(s+1)j_1}{k^2 r_c j_o} \left[1 - J_o(kr)\right], \quad (4)$$

where $\omega_{cb} = 2\pi e r_c j_o / m_i c^2 (s+1)$. When $j_1/j_o > 0$, it is clear that the beam ion orbits are radially confined since $1 - J_o(kr) \geq 0$ and r^{s+1} is monotonically increasing. When $j_1/j_o < 0$, the ion orbits are also radially confined if $\left| j_1/j_o \right|$ is sufficiently small since $\left[1 - J_o(kr)\right] \leq \left[1 - J_o(a_{11})\right] \simeq 1.4$ and r^{s+1} is a monotonically increasing function of radius.

Now consider this result in more detail. Since $P_\theta = 0$[6], the minimum axial canonical momentum is given by

$$P_z^o = m_i \left[V_o \cos\alpha_o - \frac{2\omega_{cb} r_c}{(s+1)} \left(\frac{r_s}{r_c}\right)^{s+1}\right], \quad (5)$$

when $j_1 = 0$. Here α_o is the maximum ion injection angle, $V_o = (2H/m_i)^{1/2}$ and r_s is the beam focal spot size at the point of injection into the channel. The beam envelop radius is then

$$r_b^o = r_c \left[\frac{(s+1)}{2\omega_{cb} r_c} \left(V_o - \frac{P_z^o}{m_i}\right)\right]^{\frac{1}{s+1}} \quad (6)$$

where $r = r_b^o$ when $\dot{z} = V_o$ for an ion injected with $P_z = P_z^o$. When $s = 1$ the usual result is recovered[6]

$$r_b^o = r_c \left[\frac{V_o}{\omega_{cb} r_c} \left(1 - \frac{P_z^o}{m_i V_o}\right)\right]^{1/2} \quad (7)$$

When $j_1 \neq 0$, the minimum axial canonical momentum is given by

$$P_z^1 = P_z^o - \frac{2m_i j_1 \omega_{cb}(s+1)}{j_o k^2 r_c} \left[1 - J_o(kr_s)\right], \tag{8}$$

where this result only strictly applies for $\left|j_1/j_o\right| < kr_c(r_s/r_c)^s/(s+1)$. In this case the beam envelope radius is found from

$$\frac{2}{(s+1)}\left(\frac{r_b}{r_c}\right)^{s+1} + \frac{2j_1(s+1)}{j_o k^2 r_c^2}\left[1-J_o(kr_b)\right] = \frac{V_o}{\omega_{cb} r_c}\left(1-\frac{P_z^1}{m_i V_o}\right). \tag{9}$$

For $\left|j_1/j_o\right| \ll 1$ this reduces to

$$r_b \approx r_b^o\left(1 - \frac{j_1}{j_o}\,\frac{(s+1)}{(kr_b^o)^2}\left(\frac{r_c}{r_b^o}\right)^{s-1}\left[2 - J_o(kr_b^o) - J_o(kr_s)\right]\right), \tag{10}$$

so that for $j_1 < 0$ the beam radius expands, whereas for $j_1 > 0$ the beam radius decreases. For larger $\left|j_1/j_o\right|$ Eq. (9) must be solved numerically.

When $j_1 < 0$, beam ions can be magnetically trapped and prevented from reaching the axis if $\left|j_1/j_o\right|$ is sufficiently large. Thus the density profile could be depressed on axis. For a given ion injected into the channel at a radius r_o and at an injection angle α trapping occurs if

$$\left|\frac{j_1}{j_o}\right| > \frac{k^2 r_c^2}{\left[1-J_o(kr_o)\right]}\left[\frac{V_o(1-\cos\alpha)}{2\,\omega_{cb} r_c(s+1)} + \frac{(r_o/r_c)^{s+1}}{(s+1)^2}\right]. \tag{11}$$

When $s = 1$ and $\alpha \ll 1$, this reduces to

$$\left|\frac{j_1}{j_o}\right| > \frac{k^2 r_c^2/4}{\left[1-J_o(kr_o)\right]}\left[\frac{V_o\,\alpha^2}{2\,\omega_{cb} r_c} + \left(\frac{r_o}{r_c}\right)^2\right] \geq 1, \tag{12}$$

so that trapping begins when the bunched current density exceeds the magnitude of the channel current density which is required to confine the beam. Furthermore, Eq. (9) shows that the equilibrium beam radius is also significantly modified if $\left| j_1/j_o \right| > 1$.

Thus when radial current bunching occurs in the channel, good beam propagation is still expected unless $\left| j_1/j_o \right|$ exceeds unity. If $\left| j_1/j_o \right| > 1$, the beam will be expelled from the center of the channel when $j_1 < 0$ or the beam will pinch when $j_1 > 0$. If r_b becomes larger than r_c the analysis breaks down and a more sophisticated analysis is required, however, it has been established here that it is desirable to maintain $\left| j_1/j_o \right| < 1$. For $j_1 \approx j_n \exp(\gamma \tau_b)$, good beam propagation requires $j_n < j_o \exp(\gamma \tau_b)$. Here γ is the growth rate of the instability, τ_b is the beam pulse duration and j_n is the initial level of noise in the electron current. Since the predominate source of j_n is the nonuniformities in the radial profile of the ion beam current density (remember $j_e \simeq j_b + \bar{j}$), jitter in the ion beam current density must be kept below

$$\frac{j_{bn}}{j_b} < \frac{j_o}{j_b} \exp(-\gamma \tau_b). \qquad (13)$$

Typically $\gamma \tau_b \sim 1.0 - 2.0$ and $j_o/j_b \sim 0.1$ for proposed fusion systems.[4] Thus the jitter in the beam current density must be kept below 1 - 4%. Since the filamentation instability is nonconvective[4], these results only apply at the tail of the beam. At the front of the beam the limit on j_{bn}/j_b stated in Eq. (13) is less severe and can be found by replacing $\gamma \tau_b$ by $\gamma \tau_b x/x_b$ where x is the distance from the front of the beam for a beam of length x_b.

III. AZIMUTHAL CURRENT BUNCHING IN THE CHANNEL

If $\underset{\sim}{k} = k\hat{e}_\theta$ then azimuthal current bunching occurs in the channel and the net current density can be modeled by

$$\underset{\sim}{j} = (\bar{j}(r) + j_1 H(r-r_b)\sin m\theta)\hat{e}_z \ , \qquad (14)$$

where m = 3,4,5,.... will be considered. Special treatment is required for m = 1,2 which is not considered here since $m \approx 3\text{-}7$ in typical cases.[4] Again

$$I_o = \int j(r,\theta)\ rdr = 2\pi \int \bar{j}(r) r\ dr$$

is the channel current which confines the ion beam and $j_1 \sin m\theta$ is the residual current density arising from the bunched electron current superimposed on the unbunched ion beam current. The magnetic field which determines the ion motion in a filamented channel is then

$$B_\theta = \frac{4\pi j_o r^s}{(s+1) c r_c^{s-1}} - \frac{8\pi j_1 r}{c(m^2-4)} \sin m\theta \ , \ r \le r_b , \qquad (15)$$

$$B_r = \frac{4\pi j_1 r}{c(m^2-4)} \cos m\theta \ , \qquad r \le r_b \qquad (16)$$

where again $\bar{j}(r) = j_o (r/r_c)^{s-1}$ was assumed.

In this case H and P_z are constants of the motion but P_θ is not. From Eq. (15) P_z is found to be

$$\frac{P_z}{m_i} = \dot{z} - \frac{2\omega_{cb} r_c}{(s+1)^2} \left(\frac{r}{r_c}\right)^{s+1} + \frac{2\omega_{cb} r^2 j_1 \sin m\theta}{r_c (m^2-4) j_o} \ , \ r \le r_b . \qquad (17)$$

Solving Eq. (17) for $\dot{z}$ shows that as long as

$$\left|\frac{j_1}{j_o}\right| \leqslant \frac{m^2-4}{(s+1)^2} \ , \qquad (18)$$

where $r_c \geqslant r_b$, all ion orbits are confined within the channel. For larger $|j_1/j_o|$ ions can escape from the channel and the beam density is gradually depleted.

Even for small $|j_1/j_o|$ the beam density on axis decreases in time since $|P_\theta|$ increases in time for most ions. In fact P_θ is constant only for those few ions which are injected at $\theta_o = n\pi/m$ ($n = 1,2,3,\ldots,2m$). For $|j_1/j_o| << 1$

$$(R^2\theta')' = - R(R'' - R\theta'^2)\left(\frac{2mj_1\cos m\theta}{j_o(m^2-4)}\right) , \tag{19}$$

where $R = Kr$, $\tau = \Omega t$, $K^2 = m_i\omega_{cb}/P_z r_c$, $\Omega^2 = P_z\omega_{cb}/m_i r_c$ and a prime signifies differentiation with respect to τ. Thus, to zero order in the small parameter $|j_1/j_o|$

$$(R^2\theta')_o = K^2 P_\theta/\Omega = \text{const.} \tag{20}$$

If P_θ is initially zero and $R_o \approx A\cos(\tau + \Phi)$ (implying small injection angles,[6] i.e. $\alpha << 1$) the time averaged increase in $|P_\theta|$ can be expressed as

$$\frac{<|P_\theta|>}{r_o V_o \alpha} = t/\mathcal{T} , \tag{21}$$

where $r_o V_o \alpha \approx r(z=o)\dot{r}(z=o)$ and

$$\mathcal{T} = \frac{r_o\alpha(m^2-4)|j_o/j_1|}{mV_o \cos m\theta_o}\left(\alpha^2 + \frac{r_o^2\omega_{cb}}{V_o r_c}\right)^{-1} . \tag{22}$$

Thus in a time $\mathcal{T}$, $|P_\theta|$ increases from zero to $r_o V_o \alpha$. For the average ion $<|\cos m\theta_o|> = 2/\pi$, $r_o \approx r_b/2$, $\alpha \approx \alpha_o/2$ and

$$\bar{\mathcal{T}} = \frac{\pi r_b\alpha_o(m^2-4)|j_o/j_1|}{2mV_o}\left(\alpha_o^2 + \frac{r_b^2\omega_{cb}}{V_o r_c}\right)^{-1} . \tag{23}$$

Typically $r_b \approx 0.4$ cm, $\alpha_o \sim 0.2$ rad, $m \approx 5$, $V_o \sim 3.1 \times 10^9$ cm/sec and $\omega_{cb} \approx 4 \times 10^8$ sec^{-1} so that

$$\bar{\tau} \approx 2.3 \times 10^{-9} \left| j_o/j_1 \right| \text{ sec.} \tag{24}$$

Thus the beam hollows out on a time scale of $\bar{\tau}$ even for small $\left| j_1/j_o \right|$, although the beam is still confined within the channel. In order to prevent this hollowing out from occurring during beam propagation one needs $\tau_t < \bar{\tau}$ or

$$j_n < \frac{\pi r_b \alpha_o (m^2-4) j_o}{2mL} \left(\alpha_o^2 + \frac{r_b^2 \omega_{cb}}{V_o r_c} \right)^{-1} \exp(-\gamma \tau_b) \quad , \tag{25}$$

where $\tau_b = L/V_o$ is the beam transit time in a channel of length L and again j_n is the initial noise level in the electron current. Thus as argued at the end of Sec. II this implies that the jitter in the ion beam current density must be kept below

$$\frac{j_{bn}}{j_b} < \frac{\pi r_b \alpha_o (m^2-4)}{2mL} \frac{j_o}{j_b} \left(\alpha_o^2 + \frac{r_b^2 \omega_{cb}}{V_o r_c} \right)^{-1} \exp(-\gamma \tau_b) \quad . \tag{26}$$

Typically this is found to be $j_{bn}/j_b \sim 1 - 5 \times 10^{-4}$ for proposed fusion systems. Since this low level of noise is probably not achievable, some hollowing out of the beam is likely if current bunching occurs, however the beam will still be confined in the channel for small $\left| j_1/j_o \right|$. This effect will be most pronounced at the tail of the beam where largest growth of the nonconvective filamentation instability occurs.[4]

For $\left| j_1/j_o \right| \gtrsim (m^2-4)/(s+1)^2$ the beam density is gradually depleted as increasingly more ions become unconfined. This depletion will occur in addition to the hollowing out of the beam and also occurs predominately at the tail of beam. In order to prevent this beam density depletion one needs

$$\frac{j_{bn}}{j_b} < \frac{(m^2-4)}{(s+1)^2} \frac{j_o}{j_b} \exp(-\gamma\tau_b) \tag{27}$$

For $m \sim 5$, $s \sim 4$, $j_o/j_b \sim 0.1$ and $\gamma\tau_b \sim 1 - 2$ at the tail of the beam, $j_{bn}/j_b \leqslant 1 - 4\%$ is required. At the front of the beam the condition is much less severe with $\gamma\tau_b$ replaced by $\gamma\tau_b x/x_b$ where x is the distance from the front of the beam for a beam of length x_b. Eq. (26) can be similarly modified for the front portion of the beam.

IV. CONCLUSIONS

From the analysis presented here it can be concluded that good beam transport in a filamented z-discharge channel is possible as long as the current bunching remains below certain levels. For radial current bunching it was found that $|j_1/j_o|$ should not exceed unity, which implies that $j_n < j_o \exp(-\gamma\tau_b)$. If $|j_1/j_o|$ exceeds unity, the beam will be expelled from the center of the channel when $j_1 < 0$ or the beam will pinch when $j_1 > 0$.

For aximuthal current bunching the beam hollows out on a time scale $\bar{\tau}$ defined in Eq. (23). This hollowing out occurs for all values of $|j_1/j_o|$ but will not reach a significant level if $\tau_t < \bar{\tau}$ (see Eq. (25). For $|j_1/j_o| \geqslant \frac{m^2-4}{(s+1)^2}$ the beam density is also gradually depleted as increasingly more ions become unconfined.

These results set limits on the level of j_{bn} which can be tolerated without seriously affecting beam transport due to current bunching effects.

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