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MULTICHARACTERISTIC QUALITY CONTROL: A SURVEY. (U)

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The form is a rectangular grid with several rows and columns. A checkmark is visible in the top right corner. The letter 'A' is written in the bottom left corner. The text within the grid is mostly illegible due to fading and low resolution.

MULTICHARACTERISTIC QUALITY CONTROL:

A SURVEY

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ABSTRACT

Multicharacteristic quality control is concerned with inspecting the quality of an allotment of items and with monitoring production processes when the quality of an item or the state of a process is determined by a number of observable characteristics. This report reviews procedures currently available for multicharacteristic quality control when the characteristics are either variables or attributes. Attention is given to both location and dispersion parameters in the case of variables. Some limitations of these procedures are noted.

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## INTRODUCTION

The concepts of quality control using statistical procedures were originated in the works of Shewhart (28, 29) and Dodge and Romig (7) at the Bell Telephone Laboratories in the 1930's along with E. S. Pearson (25) in England. Quality control became an attempt to answer two types of questions.

- 1) Can we conclude from the observation of a sample of the product that the process is operating "normally"?
- 2) Can we conclude from the observation of a sample from a "lot" of procured product that the entire "lot" is acceptable?

As the quality characteristic could be either in a discrete or "attribute" form or a "variable" measurement, the problem can be subdivided into four cases as shown in Table 1 for ease of reference.

TABLE 1: Basic Types of Problems in Quality Control

Type of Activity	Type of Characteristic	
	Attributes	Variables
Process Monitoring	Case 1	Case 2
Acceptance Sampling	Case 3	Case 4

To a large extent the procedures developed for quality control are based on the observations or measurements of a single quality characteristic. However, it is not unusual that more than one characteristic may be important in the manufacture or purchase of a product. Hotelling (11)

gives the example of a bomb-sight where range and deflection were the two critical characteristics. Jackson (12) gives the example of a film developing solution where the concentrations of Hydroquinone and Elon were the two critical characteristics. Ghare and Torgersen (9) give the example of a pilot ejection device where the two critical characteristics were the force of explosion and the time delay of ejection. The importance of multiple characteristics has become more prominent in the 1970's as an increasing number of products are required to meet multiple specifications, some based on the use of the product and others based on the effects produced on the environmental, economic or social systems. For example, a buyer of corn may be concerned with the protein content as well as the dryness. A buyer of coal may be concerned with the sulfur content as well as the calorific value. An ore refiner may be concerned not only with the yield but also with the amount of hazardous chemical waste.

On the surface it would appear expedient to develop test procedures separately for all the characteristics. But this would lead to two problems. These problems were pointed out by Hotelling (11), Jackson (12), and Ghare and Torgersen (9).

The first problem stems from the fact that the characteristics may not be statistically independent. If the characteristics are correlated, these correlations may have to be estimated from a sample too small to give any reasonable confidence in the estimates. Any decision based on such estimates is likely, also, to be unreliable.

Second is the possibility that the characteristics may individually appear to be out of control when they jointly are actually in control.



This possibility increases when the characteristics are statistically independent or minimally dependent.

To illustrate these problems consider a product with two characteristics, each measured as a continuous variable. It is assumed that a Shewhart  $\bar{X}$  chart is used for each. These variables are assumed to obey a joint bivariate normal distribution with correlation  $\rho$ , and means and variances of the marginal distributions for both to be 0 and  $\sigma^2$ , respectively. If a  $\pm 2\sigma$  control limit is used to monitor the two characteristics individually the probabilities that both would appear to be in control are given in Table 2. The third column in Table 2 gives the average sample number (ASN) before rejection of the hypothesis that  $H_0: \mu_1 = 0, \mu_2 = 0$  is true.

TABLE 2: Effect of the Correlation on the Probability of a  $\pm 2\sigma$  Rectangular Acceptance Region in the Bivariate Case

$\rho$	Pr (Accept)	ASN
0	0.9101	11.24
0.1	0.9113	11.27
0.2	0.9121	11.36
0.3	0.9132	11.52
0.4	0.9149	11.75
0.5	0.9172	12.07
0.6	0.9210	12.50
0.7	0.9237	13.11
0.8	0.9296	14.00
0.9	0.9357	15.56
0.95	0.9411	16.96

The procedures for quality control for any product would involve the monitoring of a random process: either the process of manufacture or the process that gave rise to the defect in purchased product. Usually it involves monitoring both the location and dispersion parameters of the random process. The monitoring takes the form of testing a hypothesis that the system is behaving as intended:

$H_0$ : The process is in control or the lot is acceptable.

$H_1$ : The process is out of control or the lot is not acceptable.

There are two assumptions about the random process which underlie all quality control techniques developed thus far.

Assumption 1: The sampling scheme gives rise to a hypergeometric distribution.

Assumption 2: The probability distribution can be approximated adequately by the Normal distribution for variable characteristics (referring to the Central Limit Theorem) and by the Binomial and Poisson distributions for attributes.

A further assumption that successive units of product are statistically independent, is also resorted to most of the time.

It is desirable that any monitoring procedure should detect the "out of control" state of a process as quickly as possible, yet it should permit the operation of the process to continue without false alarms when the

process is "in control". Hence the "run length" of successive samples before a signal becomes a very good standard for evaluating the effectiveness of any process monitoring procedure. Under the assumption that successive samples are independent, the run-length distribution is a Geometric distribution. The ASN column in Table 2 shows the expectations of the run lengths.

When there are more than one significant quality characteristics, the monitoring procedure is further complicated in that it is not sufficient merely to know that an out-of-control state exists. It becomes imperative to diagnose which of the characteristics are assignable causes. This problem of diagnosis has not been explored extensively thus far. Of the techniques for multicharacteristic quality control described in the literature, only qualitative statements can be made about the diagnostic capabilities.

In this paper we undertake to summarize the different techniques and procedures developed for Multicharacteristic Quality Control. These are grouped into four categories: Multicharacteristic Procedures, Sequential Procedures, Control of Dispersion Parameters, and Economic Acceptance Plans. This survey of techniques is followed by the description of some problems for further investigations.

#### MULTICHARACTERISTIC PROCEDURES

The development of multicharacteristic procedures for quality control depends on the joint distribution of several characteristic variables. These are multivariate continuous distributions for variable characteristics,

multivariate discrete distributions for attributes, and hybrid distributions when the types of characteristics are mixed. In Cases 2 and 4 (Table 1) it is possible to use a Multivariate Normal distribution for the sample averages by appealing to the Central Limit Theorem. Although the samples usually are not large enough to justify the use of this theory, the use is considered reasonable as the observed distributions themselves often are approximately Normal.

For Case 2, Hotelling proposed a "generalized Student ratio T" given by

$$T^2 = n(\bar{x} - \mu)' S^{-1}(\bar{x} - \mu)$$

where

$n$  = sample size

$\bar{x}$  = sample mean vector

$\mu$  = desired mean vector

and  $S$  = sample dispersion matrix.

The advantage of this statistic is that the distribution of  $T^2$  can be determined exactly, independently of any estimated "nuisance" parameters. When used for the purpose of process monitoring in Case 2, the in-control condition can be represented by

$$T_s^2 \leq T_{1-\alpha}^2$$

where  $T_s^2$  is the  $T^2$  value for the observed sample,  $\alpha$  is the assumed probability of type 1 error, and  $T_{1-\alpha}^2$  is the 100(1- $\alpha$ ) percentile of its

distribution. In the space of characteristics, the surface on which  $T^2$  is constant is an ellipsoid. This leads to two possible versions for the control charts. Figure 1a shows a chart of  $T^2$  with its control limit. In this chart the computed  $T^2$  behaves like a single characteristic in a Shewhart type chart. When there are only 2 characteristics of importance, it is possible to plot the characteristic measurements directly with  $T^2 = T_{1-\alpha}^2$ , a constant ellipse, acting as a control limit. This chart is shown in Figure 1b. One disadvantage is that the ellipse would change with each new estimate of  $\Sigma$ .

In the special case where the dispersion matrix  $\Sigma$  of the characteristics is known or sufficient data are available to assume that it is known, a Chi-squared chart can be used together with the statistic

$$n(\bar{x} - \mu)' \Sigma^{-1} (\bar{x} - \mu)$$

where  $n$ ,  $\bar{x}$ , and  $\mu$  are the same as before and  $\Sigma$  is the known dispersion matrix of  $x$ . The control charts of types shown in Figures 1a and 1b would remain unaltered, except that the shape and orientation of the control ellipse in Figure 1b would not change.

Control charts based on  $T^2$  and  $\chi^2$  distributions have been presented by Hotelling (11), Jackson (12) and Ghare and Torgersen (9). In the bomb-sight study by Hotelling, he partitions the sum of squares of the distances of bombs from the target,  $T_0^2$ , into the sum of squares of the distances from the mean point of impact,  $T_D^2$ , and the product of the number of bombs by the square of the distance from the mean point of impact to the target,  $T_M^2$ .

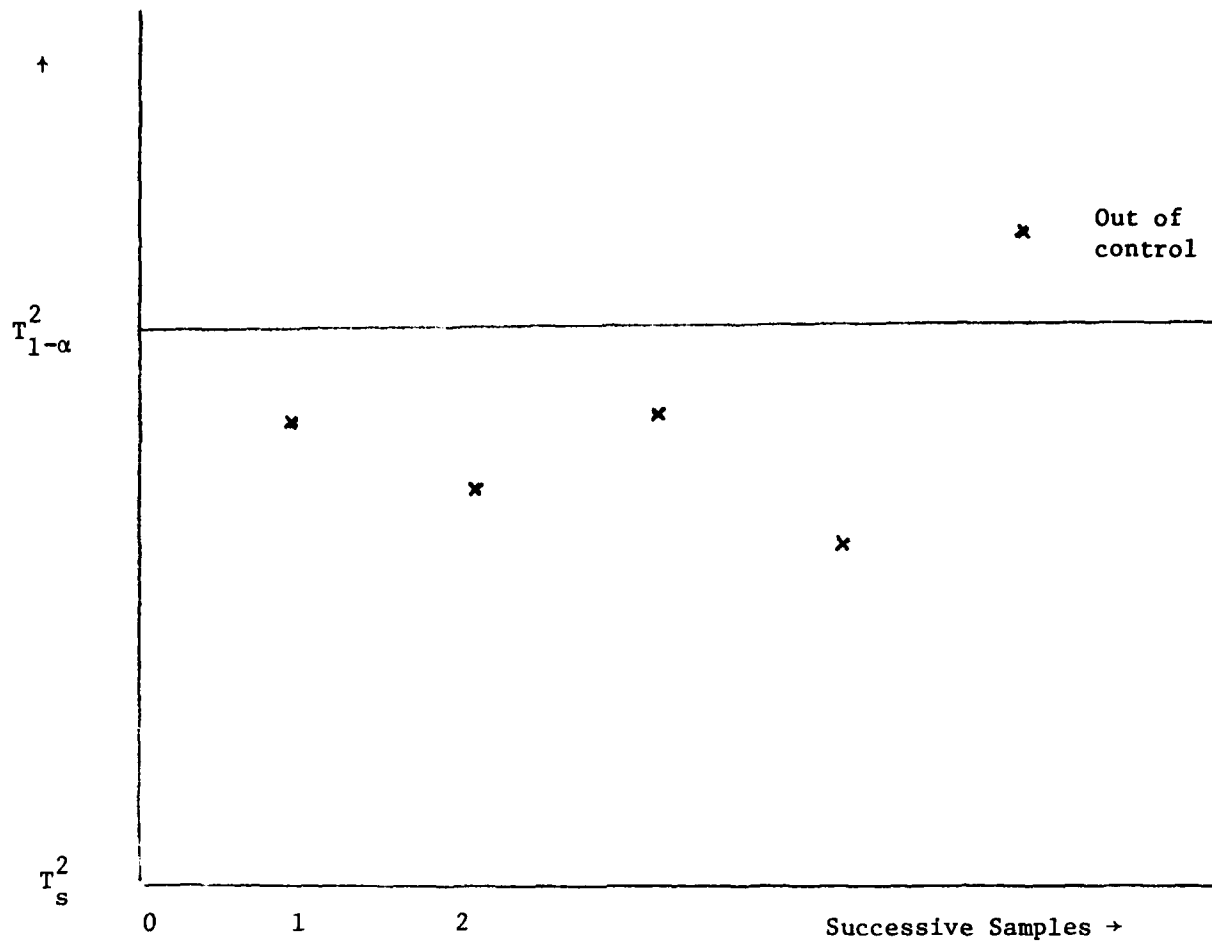


FIGURE 1a. A T<sup>2</sup> Control Chart

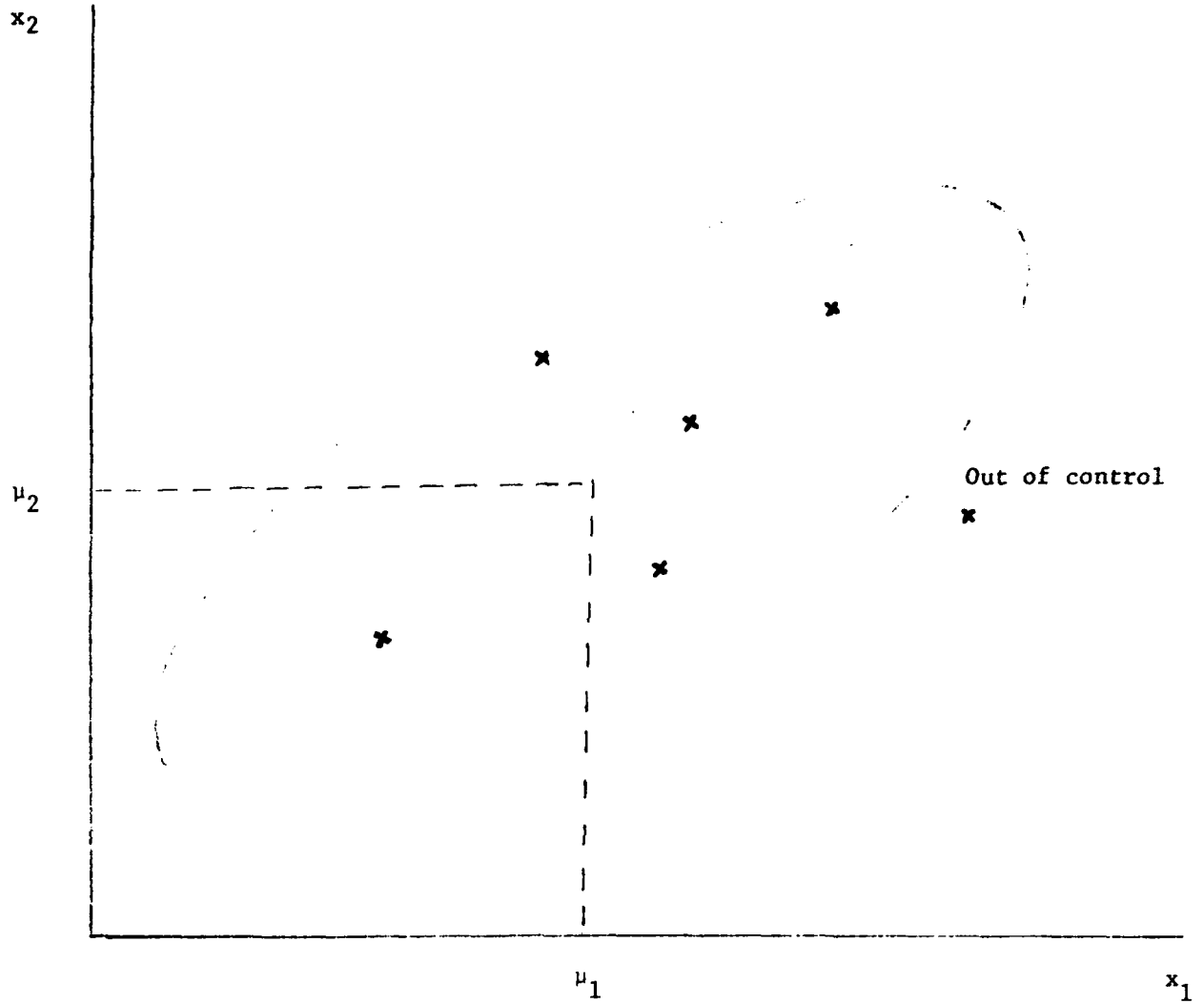


FIGURE 1b. A Bicharacteristic Chart for Use  
With a Fixed Value of S.

Under the assumption of large sample size,  $T_0^2$ ,  $T_M^2$  and  $T_D^2$  have asymptotic  $\chi^2$  distributions. Hotelling remarks that if the true mean point of impact is unknown or if we choose to ignore it, the best available measure of general degree of dispersion is  $T_D^2$  or a function of  $T_D^2$ ; if the true mean point of impact is assumed to be at the target, the best measure of general degree of dispersion would be  $T_0^2$  or a function of  $T_0^2$ ; for detection of bias in the mean point of impact, the use of  $T_M^2$  has advantages. These ideas have been discussed further by Jackson and Morris (17) and Jackson and Bradley (15).

The use of Hotelling's  $T^2$  statistic may lead to computational difficulty when the determinant of the sample dispersion matrix is near zero and may ignore significant variations. Jackson and his co-workers (13, 17) propose using the sum of squares of the largest  $k \leq p$  principal components together with a  $T^2$  chart ( $\chi^2$  chart when  $\Sigma$  is known) and the variability unexplained is monitored through the residual sum of squares by means of its asymptotic Chi-squared distribution (17, 18).

An acceptance sampling procedure using a  $T^2$  chart for the fraction defective with a given specification region is developed by Shakun (27). The sampling plan specifying the sample size and the control limit is designed to achieve desired operating characteristic.

For the multivariate discrete case, there is a lack of distributions analyzed as extensively as the Multivariate Normal distributions. For this reason Patel (24) suggested the use of a Multivariate Normal approximation and the use of a  $\chi^2$  chart of the type shown in Figure 1a, to handle cases where the characteristics have Multivariate Binomial or Multivariate Poisson distributions. Multivariate distributions of these types



are discussed by Krishnamoorthy (19). Patel also develops transformation of observations under certain assumptions in the case when successive observations are time dependent.

#### SEQUENTIAL PROCEDURES

Multicharacteristic sequential inspection procedures were developed by Jackson and Bradley (14, 15, 16) as a multivariate extension of Wald's (31, 32) sequential probability ratio tests. As in the univariate case, for stated values of Type I and Type II errors the multivariate sequential tests could be shown to terminate with a probability of 1. The inspection procedures lead to a sequential  $\chi^2$  test when the dispersion matrix is known and to a sequential  $T^2$  test when the dispersion matrix has to be estimated from the observed data. The operating characteristics for these tests were studied by Appleby and Freund (2) using Monte Carlo techniques.

A general proof of termination with probability 1 for invariant sequential ratio tests based on Multivariate Normal distributions was given by Wijsman (33). A complete description of and the theoretical background on sequential probability ratio tests can be found in Sirjaev (30) when parameters are, at least partly, unknown.

#### CONTROL OF DISPERSION

The multicharacteristic procedures described earlier are designed to monitor the means of processes under the assumption that patterns of variances and covariances are stationary. However, unlike the  $T^2$  for location parameters, there is no unanimous choice of monitoring procedure for the dispersion matrix. Hotelling (11) in analyzing the overall aims of

bomb sights partitioned the variation  $T_0^2$  into the components  $T_M^2$  and  $T_D^2$ . Of these  $T_D^2$  represented the variation about the mean and having a Chi-squared distribution asymptotically. So a Chi-squared chart could be used for monitoring  $T_D^2$ . This concept was adopted by Jackson and Bradley (14, 15) to develop the test statistic

$$X_D^2 = (n-1) \operatorname{tr} \underset{\sim}{S} \underset{\sim}{\Sigma}_0^{-1}$$

where

$\underset{\sim}{S}$  is the sample dispersion matrix

and

$\underset{\sim}{\Sigma}_0$  is the dispersion matrix, when the dispersion is in control.

This statistic has a Chi-squared distribution. Jackson and Bradley (14, 15) also derived a sequential test using the statistic  $X_D^2$  to test the hypothesis

$$H_0: \lambda_D^2 = \lambda_{D_0}^2$$

against the alternative

$$H_1: \lambda_D^2 > \lambda_{D_0}^2$$

where  $\lambda_D^2 = \operatorname{tr} \underset{\sim}{\Sigma} \underset{\sim}{\Sigma}_0^{-1}$ ,

$\underset{\sim}{\Sigma}$  is the population dispersion matrix and  $\underset{\sim}{\Sigma}_0$  its control value

and  $\lambda_{D_0}^2$  is specified.

This procedure is aimed at testing that  $\Sigma_{\sim} = \Sigma_{\sim 0}$ .

A different monitoring procedure is suggested by Montgomery and Wadsworth (23). The test statistic used is the logarithm of the determinant of the sample dispersion matrix, i.e.,  $\log |S_{\sim}|$ . This statistic has an asymptotic Normal distribution and a control chart can be constructed on that basis.

There are many statistical tests available in the literature for testing the variance and covariance structure and these can be applied readily to monitoring the dispersion parameters. However the difficulties with diagnostics, with computations and the unavailability of special aid tables limit their use in multicharacteristic quality control.

#### ECONOMIC ACCEPTANCE PLANS

Besides the statistical monitoring procedures, economic models for the selection of optimal control plans have been studied. A total expected cost model for the  $T^2$  control chart is presented by Montgomery and Klatt (21, 22), applicable to Case 4 (Table 1). The expected cost per unit of product associated with the monitoring procedure is the sum of the expected cost per unit of inspecting, the expected cost per unit of investigating and adjusting the process when it is indicated to be out of control, and the expected cost per unit of producing defective units. It is assumed that the process is in control when  $\mu_{\sim} = \mu_{\sim 0}$  and that there is only one out-of-control state for which  $\mu_{\sim} = \mu_{\sim 1}$ . The product is said to be nondefective if the quality characteristics lie within their specification limits. The time at which the process shifts from  $\mu_{\sim 0}$  to  $\mu_{\sim 1}$  is assumed to be governed by an exponential distribution. A Markov chain is developed to model the process

and the steady-state distribution is used in the expected costs. The optimal control plan which stipulates sampling numbers, sampling interval and control limit of the  $T^2$  chart to minimize the total expected cost per unit is chosen by a search technique. Heikes, Montgomery and Young (10) further develop the cost model for  $T^2$  control charts using Geometric, Poisson and Logarithmic Series distributions to characterize the time to failure. These plans apply to Case 3 (Table 1).

Several economic models of acceptance sampling were presented by different authors under the assumption that the characteristics are distributed independently according to some known distribution. Acceptance plans are designed separately for each quality characteristic and the behavior of each mean is modeled by a Markov process. The process is said to be in control (or the lot to be acceptable) if all quality characteristics are in control according to the separate control plans. The economic model usually defines the total cost as

$$TC = IC + RC + AC + CC$$

where

TC = Total expected cost

IC = Expected cost of Inspection

RC = Expected cost of Rejection

AC = Expected cost of Adjustments to the process

and CC = Expected cost of Continuation of the process (or Acceptance of the lot).

Under this type of model an economic control plan can be developed which minimizes the total cost.

Optimal plans for Cases 2 and 4 of Table 1 for continuous quality characteristic variables were studied by Latimer, Schmidt and Bennett (20) and Schmidt and Bennett (26). Bennett and McCaslin (3), Case, Schmidt and Bennett (5) as well as Chapman (6) considered Cases 1 and 3 of Table 1 with attribute characteristics. A mixed design with both variable and attribute characteristics is presented by Ailor, Schmidt and Bennett (1).

Since the optimality of the control plans depends heavily on the cost coefficients and on the model assumptions, these authors have also studied the sensitivity of the economic models to variations in their components. In general it was found that the models are quite sensitive to changes in cost coefficients and are relatively robust to the assumptions about their distributions.

#### TOPICS FOR FURTHER STUDY

In single-characteristic quality control the question of diagnostics and corrective procedures are answered in a straightforward manner, there being little room for confusion. For example, if the observations of characteristic  $X$  are found to be above the upper control limit, the inference not only is that the characteristic  $X$  is out of control, but that the remedial action would entail altering the production process to reduce the magnitude of  $X$ . If in multicharacteristic quality control a bicharacteristic observation  $\underset{\sim}{X} = (X_1, X_2)$  is found to be outside the acceptance region, it could be that either  $X_1$  is out of control or  $X_2$  is out of control or both. Again the corrective action could entail the alteration of  $X_1$  or  $X_2$

or both. Furthermore it is entirely possible that the most economic remedy when  $X_1$  is out of control, would be to modify  $X_2$ . When more than two characteristics are involved the possible avenues for correction may be enormous.

One method suggested for diagnosis is to use acceptance regions in the shape of n-dimensional rectangular sets, i.e., the regions formed by the Cartesian products of the usual acceptance regions for individual characteristics. Such regions will yield a diagnosis when coupled with the understanding that any characteristic will be considered out of control if the observation on it lies outside its own acceptance region. One drawback to this usage is that the probability of acceptance is a function of the matrix of the correlation coefficients which usually is unknown. This difficulty has been illustrated in Table 2.

Another possible approach could be to eliminate diagnostics per se and attempt to devise the least expensive corrective measures. This procedure could be promising in situations where the probability distribution under the alternative hypothesis is known or can be estimated. So far very little has been done to explore this approach.

The question of robustness of quality control procedures has a practical concern to their users. One surely wants to know whether a procedure retains its operating characteristics when assumptions of the model fail. In practice it is common that some parameters or target values are estimated from a baseline period and these estimates are used as if the parameters are known. For example in the  $T^2$  control chart the sample mean values in a base period may be used as the target values and the sample dispersion matrix in a base period may be used subsequently assuming stationary variance and covariance

structure. Operating characteristics of the usual procedures will be different due to the dependency of test statistics on successive occasions and their run-length distributions will no longer be Geometric even in Shewhart type control charts. Disturbances in these run-length distributions deserve further study.

In another instance, unknown parameters may be estimated in a base period and then be used in lieu of nuisance parameters in the distributions of test statistics. For example, estimated correlation parameters may be used in establishing  $n$ -dimensional rectangular acceptance regions. This will lead to the study of stability and convergence properties of distributions parametrized in part using sample values.

Of particular interest in robustness studies are those cases for which Normal theory holds asymptotically, for either variables or attributes, by virtue of Central Limit Theory. For such cases bounds of the type of Berry (4) and Esseen (8) on rates of convergence to the Normal law are needed, particularly for multicharacteristic quality control problems. Such bounds would provide useful guidelines in practice regarding the use of normal-theory approximate procedures.

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18. SUPPLEMENTARY NOTES  The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Statistical control charts, lot inspection plans, multicharacteristic procedures, a review.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  Multicharacteristic quality control is concerned with inspecting the quality of an allotment of items and with monitoring production processes when the quality of an item or the state of a process is determined by a number of observable characteristics. This report reviews procedures → next page		

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currently available for multicharacteristic quality control when the characteristics are either variables or attributes. Attention is given to both location and dispersion parameters in the case of variables. Some limitations of these procedures are noted.  
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