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SURFACE WAVES AND ANOMALOUS SOUND ABSORPTION

BY RONALD L. KLIGMAN

RESEARCH AND TECHNOLOGY DEPARTMENT

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FORWARD

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I. INTRODUCTION

The reflection and transmission of acoustic plane waves from a fluid-fluid interface is discussed routinely in a number of textbooks on sound.[1-3] The usual model assumes the interface to be at rest, in a tension-free state. Researchers[4] have recently raised questions concerning the effects of interfacial surface tension on the reflection and transmission of sound, and on anomalous sound absorption that could be tied to surface phenomena. One possible mechanism may be surface waves, supported by the interfacial surface tension. In the steady state, the surface waves are in equilibrium with the incident, reflected and transmitted acoustic waves. An obvious requirement for such a situation to exist is that the conventional boundary conditions applied to the normal acoustic particle velocity and acoustic pressure be modified to accommodate the more general situation of an interface that is free to move and support surface tension. A model is introduced that permits these generalizations. The interfacial or capillary layer itself contains additional degree of freedoms that will absorb sound. Capillary viscosity and thermal conduction effects serve as degrees of freedom that transfer energy from the acoustic wave to the surrounding interfacial region. The model established by Landau and Lifshitz[3] is modified to accommodate more generalized conditions. The fractional energy loss, i.e. the ratio of absorbed energy to incident energy is a function of the media's thermal conductivity, specific heats, surface or interfacial viscosity, and reflection coefficient.

II. PRESSURE AND VELOCITY BOUNDARY CONDITIONS

Consider a layered, two fluid system with the first fluid extending from $x = -\infty$ to $x = o$, the second filling the remaining semi-infinite space from $x = o$ to $x = +\infty$. The interface at $x = o$ is considered to be the time-averaged position of the actual interface. In a real physical situation with an acoustic plane wave incident obliquely on the interface, the surface can support waves, if it is in a state of tension. For the moment, let us consider the pressure conditions on either side of the interfacial layer in the absence of any external sound field, and later introduce the sound so as to modify the boundary conditions.

The curvature of the surface layer gives rise to a pressure difference between the two adjacent media. For the media to be in thermodynamic equilibrium with each other, certain conditions must be met. Landau and Lifshitz[3] show that the pressure difference is balanced by the product of the surface tension coefficient α, and the principal radii of curvature of the surface, R_1 and R_2, namely

$$p_1 - p_2 = \alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{2.1}$$

where p_1 and p_2 represent the pressures in the respective media 1 and 2. Eq. (2.1) is sometimes referred to as Laplace's formula. If we define the surface to be $\zeta = \zeta(x,y,t)$, whose position averaged over the spatial coordinate y is given by x = o, we can introduce the velocity potential of the surface as $\phi_s(x,y,z;t)$. If the surface wave propagates in the direction of the y axis and the pressure in medium two is taken as the zero reference level, then we may re-write Eq. (2.1) as

$$\rho_1 \frac{\partial\phi_s(x,y;t)}{\partial t} = \alpha\left(\frac{\partial^2\zeta}{\partial y^2} + \frac{\partial^2\zeta}{\partial x^2}\right) \tag{2.2}$$

Thus, there is a balance between the excess pressure across the interface and the surface tension residing within the surface layer. Figure 1 illustrates the case of a surface wave at an interface propagating along the y axis. Since the velocity potential additionally satisfies Laplace's equation one can obtain the dispersion relation the surface wave must satisfy, as a corollary to Eq. (2.2). The relationship between the frequency ω and the wave number k of the wave defines the relation. For this case it is rather simple, and is given by

$$\omega^2 = \alpha k^3/\rho_1 \tag{2.3}$$

where ρ_1 is the density of medium 1.

Now let us consider the case of a plane acoustic wave propagating through medium 1 and striking the interface at some angle θ_i. Such a case is shown in Figure 2. Let the incident, reflected and transmitted acoustic pressure fields be given by

$$p_\beta = A_\beta \exp i\left(\vec{k}_\beta \cdot \vec{r} - \omega t\right) \tag{2.4}$$

where β assumes the subscript value for the incident, reflected and transmitted waves, (i,r,t) and A_β represent the pressure amplitudes for these waves respectively. By demanding that the total pressure be continuous across the boundary we arrive at the pressure boundary condition at x = 0 and t = 0,

$$p_i + p_r + p_s = p_t - \alpha\left(\frac{\partial^2 \zeta}{\partial y^2} + \frac{\partial^2 \zeta}{\partial x^2}\right) \tag{2.5}$$

and

$$p_s = - \frac{\partial \phi_s}{\partial t} \tag{2.6}$$

The normal particle velocity boundary condition at x = 0 and t = 0 is

$$v_i^{(x)} + v_r^{(x)} = v_t^{(x)} = v_s^{(x)} \tag{2.7}$$

and

$$\vec{v}_s = \vec{\nabla}\phi_s \tag{2.8}$$

Eq. (2.7) states the equality between the surface wave velocity and the total acoustic wave to the left and to the right of the boundary layer.

As a consequence of these boundary conditions we shall obtain a generalized dispersion relation for those surface waves that "resonate" with the incident, transmitted and reflected acoustic waves. The reflection coefficient shall be obtained from eqs (2.5) and (2.7) as well.

III. THE REFLECTION COEFFICIENT

Since the pressure at the interface is given in terms of the velocity potential by the expression

$$\lim_{x \to 0-} p_s = - \rho_1 \frac{\partial \phi_s}{\partial t} \qquad (3.1)$$

and ϕ_s satisfies Laplace's equation

$$\nabla^2 \phi_s = 0 \qquad (3.2)$$

we obtain the result

$$p_s(y,t) = \rho_1 \omega_s A \cos k_s y \sin \omega_s t \qquad (3.3)$$

and A is the velocity potential amplitude. Eq. (3.3) is a representation of a standing wave rather than a traveling wave. This choice was made based on considerations put forth by Longuet-Higgins[5] who studied the effects of microseisms on the ocean floor induced by groups of surface waves travelling in opposite directions. The pressure fluctuation was found to have twice the frequency of the standing surface water wave. Thus the coupling mechanism between the two types of waves is non-linear. There are other examples of such non-linear couplings.[6-8] We shall make use of these results when we prescribe the resonance conditions between surface standing waves and acoustic waves.

If we substitute eq. (3.3) into the pressure boundary condition, eq. (2.5), and make use of eq. (2.7) the relation between the x component velocity potential gradient and the time rate of change of the surface displacement amplitude, i.e. $\frac{\partial \phi_s}{\partial x} = \frac{\partial \zeta}{\partial t}$, we obtain the expression

$$A_r = \left(\frac{\rho_1 c_1 \cos\theta_i - \rho_2 c_2 \cos\theta_t}{\rho_1 c_1 \cos\theta_i + \rho_2 c_2 \cos\theta_t}\right) A_i + A\rho_2 c_2 \cos\theta_t \sin\omega_s t \cos k_s y \quad x$$

$$x \quad \left(\frac{\alpha k_s^3}{\omega_s} - \rho_1 \omega_s\right) \exp\left[-i\left(k_i \sin\theta_i y - \omega t\right)\right] \Big/ \left(\rho_1 c_1 \cos\theta_i + \rho_2 \cos\theta_t\right) \qquad (3.4)$$

We impose the resonance conditions between the acoustic frequency ω and the standing surface wave frequency ω_s,

$$\omega = 2|\omega_s| \qquad (3.5)$$

and then between the projection of the acoustic wave vector along the interface and the surface wave number of the standing surface wave,

$$\left.\begin{aligned} k_i \sin\theta_i + k_s - k_s &= 0 \\ k_r \sin\theta_r + k_s - k_s &= 0 \\ k_t \sin\theta_t + k_s - k_s &= 0 \end{aligned}\right\} \qquad (3.6)$$

Figure 3 illustrates the various wave vector resonance conditions. Thus, we restrict our model to small angles of incidence, for arbitrary value of incident wave number. Averaging eq. (3.4) spatially over a wavelength $\lambda_s = 2\pi k_s^{-1}$, and then time-wise over a period $\tau_s = 2\pi\omega_s^{-1}$ we arrive at an expression for the reflection coeffecient R, the time and space averaged ratio of the energy flux densities in the reflected and incident acoustic waves.

$$R = \left(\frac{\rho_1 c_1 \cos\theta_i - \rho_2 c_2 \cos\theta_t}{\rho_1 c_1 \cos\theta_i + \rho_2 c_2 \cos\theta_t}\right)^2 + 4\left(\frac{\rho_2 c_2 \cos\theta_t}{\rho_1 c_1 \cos\theta_i + \rho_2 c_2 \cos\theta_t}\right)^2 \times$$

$$\times \left(\rho_1 \omega_s - \alpha k_s^3/\omega_s\right)^2 \Big/ \left[k_s^2 \sec^2\theta_i \sec^2\theta_t \left(\rho_1 c_1 \cos\theta_i + \rho_2 c_2 \cos\theta_t\right)^2 + \left(\rho_1 \omega_s - \alpha k_s^3/\omega_s\right)^2\right] \qquad (3.7)$$

The first term on the right hand side of eq. (3.7) is the usual expression for the reflection coefficient when surface tension from the interface is absent. The second term is a correction that is directly related to the surface tension at the boundary. For certain eigenfrequencies, the normal mode frequencies of the membrane, this correction is identically zero. In general, the second term is non-zero. For small angle incidence we find the correction term R_s approaches the following limiting expressions,

$$R_s \longrightarrow 0 \qquad (3.8)$$

where $\theta_i, \theta_t \longrightarrow 0$, $\rho_1 c_1 >> \rho_2 c_2$,

$$R_s \longrightarrow \frac{4\left(\rho_1 \omega_s - \alpha k_s^3/\omega_s\right)^2}{\left(k_s \rho_2 c_2\right)^2 + \left(\rho_1 \omega_s - \alpha k_s^3/\omega_s\right)^2} \qquad (3.9)$$

where $\theta_i, \theta_t \longrightarrow 0$, $\rho_2 c_2 >> \rho_1 c_1$,

$$R_s \longrightarrow \frac{\left(\rho_1 \omega_s - \alpha k_s^3/\omega_s\right)^2}{4\left(k_s \rho_2 c_2\right)^2 + \left(\rho_1 \omega_s - \alpha k_s^3/\omega_s\right)^2} \qquad (3.10)$$

where $\theta_i, \theta_t \longrightarrow 0$, $\rho_2 c_2 \longrightarrow \rho_1 c_1$

IV. THE DISPERSION RELATION

In order to obtain eq. (3.7), one made use of the boundary conditions, eqs. (2.5) and (2.7) to find a relation between the velocity potential amplitude and the incident acoustic pressure amplitude. This constitutes a dispersion relation between the wave number and frequency of the acoustic waves and the surface waves. One obtains

$$\left\langle \langle|A/A_i|\rangle^2 \right\rangle_{\tau_s,\lambda_s} = 16k_s^{-2} \left[\left(\frac{\rho_1 c_1}{\cos\theta_t} + \frac{\rho_2 c_2}{\cos\theta_i}\right)^2 + \left(\rho_1\omega_s - \frac{\alpha k_s^3}{\omega_s}\right)^2\right]^{-1} \qquad (4.1)$$

The bracket represents a spatial and temporal average of the amplitudes over the reciprocal wave number k_s^{-1} and the period, $\tau_s = 2\pi\omega_s{}^{-1}$. Eq. (4.1) can be put into a more tractable form if we do a bit of rearrangement. We can re-express this as

$$\left(\rho_1 s\right)^2 + k_s^2\left(f - 2\alpha\rho_1 k_s\right)s + \alpha^2 k_s^6 - \frac{16\eta k_s^2}{a^2} = 0 \qquad (4.2)$$

$$f = \left(\frac{\rho_1 c_1}{\cos\theta_t} + \frac{\rho_2 c_2}{\cos\theta_i}\right)^2 \quad ,$$

$$s = \omega_s^2 \quad ,$$

$$\eta = |A_i|^2$$

and

$$a = |A|k_s/\omega_s$$

where a is the surface wave amplitude.

Eq. (4.2) is a simple quadratic in s with roots

$$s = \omega_s^2 = \omega_o^2 - \frac{fk_s^2}{2\rho_1^2} \pm \frac{k_s^2}{2\rho_1^2}\sqrt{f^2 - 4f\alpha\rho_1 k_s + \frac{64\rho_1^2\eta}{k_s^2 a^2}} \tag{4.3}$$

and $$\omega_o^2 = \alpha k_s^3 \Big/ 2\rho_1^2 \tag{4.4}$$

is the dispersion relation for an isolated capillary or surface wave that propagates along the interface. For large wave numbers the dispersion relation reduces to the usual surface wave dispersion relation, and the waves are "surface like". For small wave numbers the dispersion relation goes like the square root of the wave number.

For large values of k_s we have

$$\omega_s \propto k_s^{3/2} \tag{4.5}$$

and for small values of k_s

$$\omega_s \propto k_s^{1/2} \tag{4.6}$$

For intermediate values of k_s we specify that

$$f^2 >> 4f\alpha\rho_1 k_s + \frac{64\rho_1^2\eta}{k_s^2 a^2} \tag{4.7}$$

and

$$\omega_o^2 << \frac{fk_s^2}{2\rho_1^2} \tag{4.8}$$

Then eq. (4.3) becomes approximately

$$\omega_s^2 \approx \omega_o^2 - \frac{fk_s^2}{\rho_1{}^2} \approx - \frac{fk_s^2}{\rho_1{}^2} \tag{4.9}$$

$$|\omega_s^2|^{1/2} \approx k_{eff}\, c_2 \tag{4.10}$$

and $$k_{eff} = k_s \left(\frac{2\sin\theta_1}{\sin 2\theta_t} + \frac{\rho_2}{\rho_1 \cos\theta_1} \right) \tag{4.11}$$

In the intermediate wave number regime, the frequency of the wave becomes predominantly imaginary and the wave either grows or decays exponentially in time, a situation that may lead to instabilities for the surface wave. Incidentally eq. (4.10) resembles the dispersion relation for acoustic waves, except that there is a phase difference of approximately $\pi/2$ between frequency and wave number. Figure 4(a) is a qualitative sketch of the relationship between frequency and wave number for waves that can propagate along the surface. That portion of the k_s axis defined by the condition $k_s' \leq k_s \leq k_s''$ is an unstable zone. Specifically, in this wave number regime, frequency is a complex quantity. Rather than two surface waves of the type $\exp i(\omega_s t - k_s y)$ and $\exp i(\omega_s t + k_s y)$ one has waves that either grow or decay exponentially in time and are of the form $\exp i(\pm k_s y) \exp(-\omega_s'' t) \exp(i\omega_s' t)$ or $\exp i(\pm k_s y) \exp(\omega_s' t) \exp(i\omega_s'' t)$, where $\omega_s = \omega_s' \pm i\omega_s''$ with both ω_s' and ω_s'' real quantities. Figure 4(b) illustrates the dispersion relation for such waves.

V. NUMERICAL RESULTS FOR SPECIAL CASES

Corrections to the reflection coefficient are most significant when there is a very close match between the characteristic acoustic impedances of the two adjoining media as seen by inspecting eq. (3.7). The results of two special cases are presented here, the first for a two-fluid model consisting of castor oil and water, the second representing benzene and water. Densities, sound velocities and surface tensions for these media are shown in Table 1, and were evaluated at standard temperature and pressure, the room temperature taken to be T = 20°C, except for the sound velocity in castor oil which was measured at T = 18.6°C. Surface tension data at a castor oil-water interface was not available in the literature. however such data at a castor oil-air interface was found.[12] When one compares interfacial tensions between various organic liquids and water with those found between the same liquids and air, one observes a percentage change which varies from a minimum of 17.49% for the case of benzene to a maximum of 177% for hexane.[14] This gives an approximate upper and lower bound on the error introduced by using the surface tension value appropriate to a castor oil-air system in place of the actual interfacial tension between castor oil and water.

The incident pressure amplitude A_i will range from 10^{-1} to $10^{-2}\ \frac{\text{dynes}}{\text{cm}^2}$ where we assume a threshold of sound pressure in water to be 10^{-5} μbars or $10^{-6}\ \frac{\text{dynes}}{\text{cm}^2}$. The surface wave amplitude a will be given values from 10^{-8} to 10^{-5} cm, for $10^{-8} \leq k_s a \leq 10^{-3}$. For these specified ranges of values, the surface wave frequency is not that sensitive to the wave number k_s, but does depend strongly on the incident acoustic pressure, A_i. For example, if A_i is 10^{-1} dyne/cm^2 and k_s assumes values from 1 to 100 cm^{-1}, ω_s is approximately 268 H_z, for the entire range. If we increase A_i by a factor of 10 to 1 $\frac{\text{dyne}}{\text{cm}^2}$, we correspondingly increase the frequency by about a factor of 10. Examining eq. (4.3) shows that the last term under the radical dominates all other terms for the chosen parameter range and in this regime there is a linear relationship between incident pressure amplitude and surface wave frequency.

Table 2 lists the average surface wave frequency $\langle\omega_s\rangle_{\theta_i}$ for given values of wave number, surface wave amplitude, dimensionless parameter $k_s a$ and incident acoustic pressure amplitude. The average is calculated over a range of incident angle θ_i that extends from 0 to 10°, and the frequencies satisfy the dispersion relation, eq. (4.3). Blank entries indicate that the roots of the dispersion relation are imaginary for that choice of parameters.

Figures 5(a) through 5(d) show the behaviour of the reflection coefficient as a function of wave number, for various incident pressures, and surface wave amplitudes. The special cases for a castor oil/water and benzene/water two-fluid systems are plotted together, in all cases. The dependence on angle for $0 \leq \theta_i \leq 40°$ is relatively small. Therefore, only the results for normal incidence only are shown in the figures. Figure 5(a) shows the reflection coefficient rising sharply from values less than .02 at wave numbers less than 20 cm^{-1} to a near saturation value of .95 for castor oil with water, and.75 for benzene with water. Saturation takes place near 30 cm^{-1} for both cases. Figure 5(b) shows the reflection coefficient to have the same qualitative behavior as in Figure 5(a) for a different range of wave numbers. We see in Figure 5(c) a minimum in the reflection coefficient for both the castor oil and benzene cases, at 170 and 190 cm^{-1} respectively. Figure 5(d) shows behaviour similar to 5(c) except that the minimum occurs clearly at 870 cm^{-1} in the case of the benzene and water system while the minimum for the castor oil with water fluid layer occurs off scale at a wave number less than 800 cm^{-1}.

Figures 6(a) through 6(d) show that portion of the total reflection coefficient that arises from surface tension, namely the second term on the right hand side of eq. (3.7), R_s. These curves are almost indistinguishable from those shown in Figures 5(a) through 5(d). The total reflection coefficient differs from that portion associated with surface tension by an average of 5% for the case of benzene and water to an average of less than 1% for the castor oil with water case. This shows the overwhelming influence surface tension has on a surface's ability to reflect acoustic energy, when the contiguous media have characteristic acoustic impedances that are very similar in value.

The presence of the minimum in both the total and the "surface tension" reflection coefficient is not surprising if one simply inspects the 2nd term on the right hand side of eq. (3.7). The numerator in that term is exactly the dispersion relation for isolated surface waves when the numerator vanishes. Except for a small residual value, there is no reflection from the surface when the frequency of the incoming acoustic wave matches twice the frequency of a pure standing surface wave for a given wave number. This represents a "resonance" transmission phenomena and the surface becomes transparent to incoming acoustic radiation.

VI. ABSORPTION BY REFLECTION

A plane acoustic wave that is obliquely incident upon an interface separating two distinct media may lose energy through two mechanisms. The first is a viscous dissipation fluid particles in the wave experience when their tangential velocity must be brought to zero to match the velocity of the interface along the boundary. The second is a thermodynamic dissipation, which requires that the local temperature in the wave be the same as the temperature at the interface. Strictly speaking this temperature will not remain constant unless the specific heat in the interface is quite large compared to that found in either medium. Although we have no evidence this is the case, we shall assume it to be so and postulate a model that predicts sound absorption by reflection from a barrier, the barrier being the surface tension layer. The model is similar to one suggested by Landau and Lifshitz[15] for the case of sound absorption by reflection from a wall. Our model permits transmission as well as reflection of acoustic energy, which is a generalization of the Landau and Lifshitz result. Although some energy is taken from the incoming acoustic wave and transformed into surface wave energy, no attempt has been made to incorporate this feature into the model in this paper. The fraction of energy absorbed at the interface resulting from the reflection and transmission of acoustic plane waves at this boundary is given by

$$F = \sqrt{\omega_s/2}\left[1 + 2\sqrt{R} + R\right]\left[\sqrt{\nu_s}\sin^2\theta_1 + \sqrt{\chi_s}\left(\frac{c_p}{c_v} - 1\right)\right] \Big/ c\cos\theta_1 \quad (6.1$$

where c_p and c_v are the specific heats at constant pressure and volume in the interfacial region, ν_s is the kinematic viscosity in the boundary layer. (The kinematic viscosity is the ratio of the dynamic viscosity to the density of the medium), and χ_s is the thermometric conductivity in the interfacial region. (The thermometric conductivity is the ratio of the thermal conductivity κ to the product of medium density with the specific heat at constant pressure.) We shall use the values in water for the thermodynamic parameters, although there is some question as to the proper choice in the interfacial region. The specific heat at constant pressure c_p for water is found in the thermodynamic textbook by Callen[16] and has the value $4.1796 \frac{\text{Joule}}{\text{gm-°K}}$ at 20°C. Using the thermodynamic relation[17]

$$c_v = c_p - Tv\alpha_T^2/\kappa_T$$

where T is the temperature in °K, v is the specific volume, α_T is the coefficient of thermal expansion, and κ_T is the isothermal compressibility, we arrive at a value for the specific heat at constant volume at a temperature of 20°C, which is $4.1022 \frac{\text{Joule}}{\text{gm-°K}}$. Values for the isothermal compressibility and thermal conductivity of water were found in the chemical rubber handbook[18] and the specific volume of water in the AIP handbook[19]. The coefficient of thermal expansion for water was obtained from reference 15, p. 346. Table 3 summarizes the values taken for the parameters just discussed.

The kinematic viscosity ν_s in the interfacial layer is determined through an integral relationship[20],

$$2\int_0^\infty dx\ \eta_s(x) = e \qquad (5.2)$$

where e is defined as the "surface viscosity". The integral extends over the effective thickness of the boundary layer and in practice, a cut-off to the upper limit of x exists. With the imposition of a cut-off and the assumption that the kinematic viscosity does not strongly depend on its location within the layer we find

$$\eta_s \approx e/2d \qquad (6.3)$$

and d is the thickness of the interfacial regime. For distilled water, Klemm[20] with the aid of Alty's results was able to obtain a value of

$$e = 1.1 \times 10^{-4}\ \frac{\text{dyne-sec}}{\text{cm}}\ .$$

We shall take the thickness of the interfacial layer to be on the order of an angstrom based on arguments put forth by Eisenminger[22], although there is no overwhelming evidence that the thickness could not be ten's of angstroms. In fact, Klemm[19] assumed a thickness of 50Å. Using 1 Å for the thickness we arrive at a value of 5.50×10^3 dyne-sec for the dynamic viscosity, η_s and consequently $5.51 \times 10^3\ \frac{\text{cm}^2}{\text{sec}}$ for the kinematic viscosity, ν_s.

VII. NUMERICAL RESULTS FOR FRACTIONAL ABSORPTION

Using eq. (6.1) together with the thermodynamic parameters given in Table 3, the kinetic viscosity sited in Section VI and the reflection coefficient as calculated in Section V, we arrive at some graphical representations of the fractional energy loss ratio (i.e., the ratio of the acoustic energy absorbed at the interface to the incident energy), plotted against wave number in reciprocal centimeters. The incoming acoustic plane wave is assumed propagating in a water medium and strikes an interface with either benzene, or castor oil, at some incident angle θ_1 between 0 and 40°. Figure 7 graphically depicts the angular dependence of the loss ratio as well as the dependence on wave number. Not shown in the figures are the special cases of normal incidence, for which the viscosity related loss is identically zero and the thermodynamic contribution is much smaller than the smallest scale on the graph, and 5° incidence for which the loss is fairly constant having the value .004 in the wave number regime between 600 and 800 cm^{-1}. At 10° and 15° some structure appears between 700 and 800 cm^{-1} in the form of shallow minima for both the benzene and the castor oil cases. As the incident angle continues to increase up to 40°, the minima become sharper and deeper. Moreover, the respective minima for the cases of benzene and castor oil are spaced closer together. For all these cases, the incident pressure amplitude, A_1 is 100 $\frac{\text{dynes}}{\text{cm}^2}$ and the surface wave amplitude, a is 10^{-8} cm. The minima correspond to the minima in the total reflection coefficient, R, previously discussed in Section V, as can be seen by examination of eq. (6.1). In the most extreme case, the fractional energy loss ratio can decrease by nearly a factor of four, from a value of .24 to .065 in the neighborhood of 657 cm^{-1}, for the case of castor oil in contact with water, at a 40° angle of incidence. At these "resonance" values of wave number, there is minimal loss of sound energy due to reflection from the interface. The surface waves "resonate" with the incoming acoustic radiation and serve to couple this radiation to the adjacent medium, either castor oil or benzene, increasing the amount of transmitted energy.

VIII. CONCLUSIONS

The coupling of acoustic waves to surface waves through the interfacial surface tension gives rise to a modified expression for the reflection coefficient. Such a modification is most dominant when the specific acoustic impedances of the adjoining layers are very close in value. Sound absorption occurs when thermal and viscous conditions in the boundary layer take energy out of the incoming acoustic wave and deposit it into surface waves and surface dissipation. Resonance conditions exist which minimize this effect when the frequency of the incoming acoustic wave equals twice the frequency of a pure, standing surface wave (i.e., one that satisfies the surface wave dispersion relation) for some given wave number.

Future experiments are needed to ascertain the relationship between the driving acoustic waves at oblique incidence and the response surface waves. For example, for some given incident acoustic pressure amplitude (an input) one measures a surface wave amplitude (an output); the relationship between input and output being a "transfer function" which can be included in the model for energy absorption by reflection. Such a relationship in all likelihood would be a non-linear one. Surface tension measurements at a castor oil - water interface are needed since such data do not appear to be available in the literature.

An improved theoretical treatment of this problem must include specifically the mechanism for the interactions between the acoustic waves and surface waves ab initio rather than the present model which only produces coupling through the modified boundary conditions. Future efforts will be directed toward the application of these techniques to interfacial problems of the fluid-loaded plate and the acoustically-coated fluid-loaded plate.

ACKNOWLEDGEMENT

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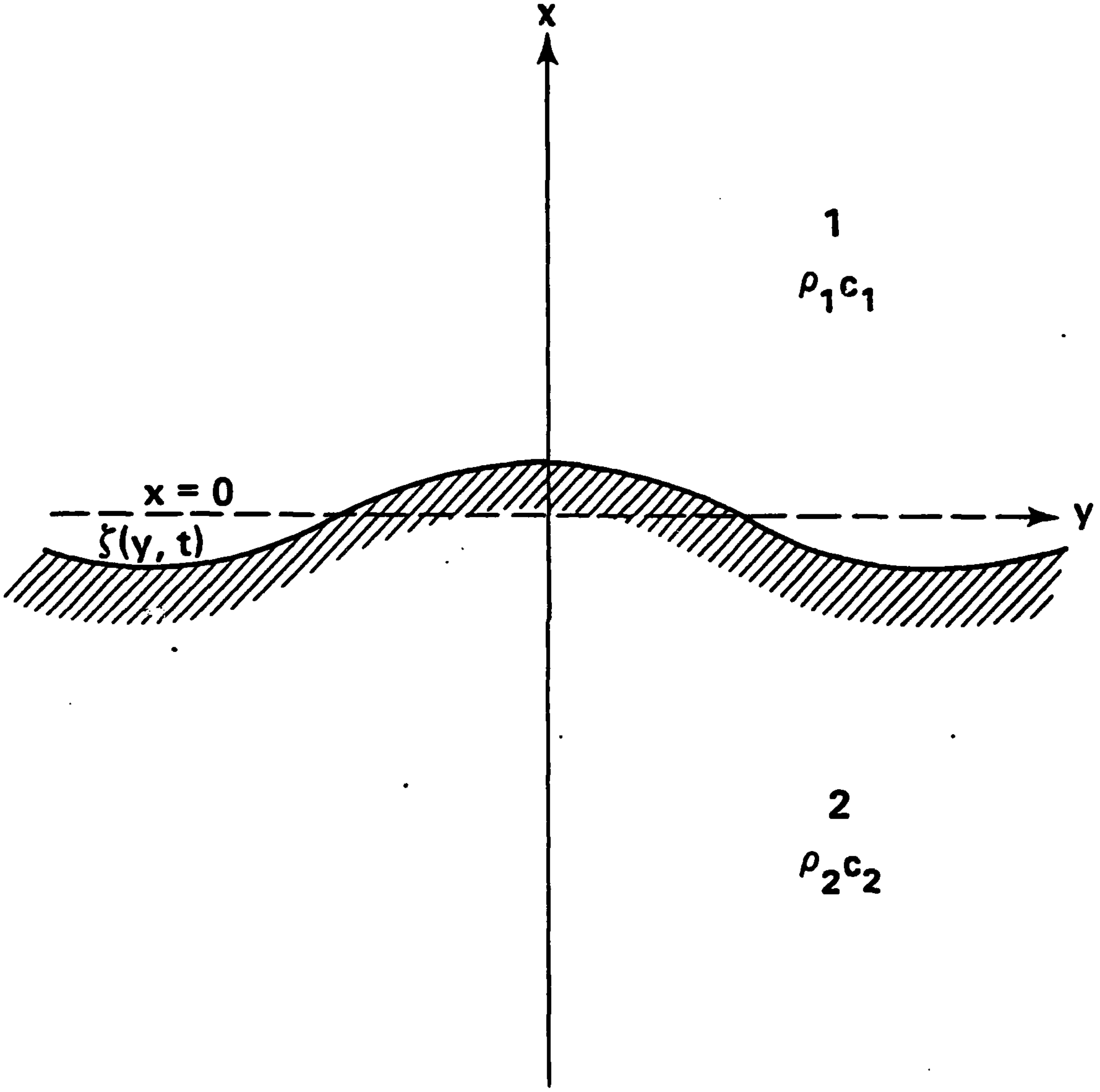

FIGURE 1 A SURFACE WAVE PROPAGATING ALONG THE INTERFACE OF MEDIA 1 AND 2

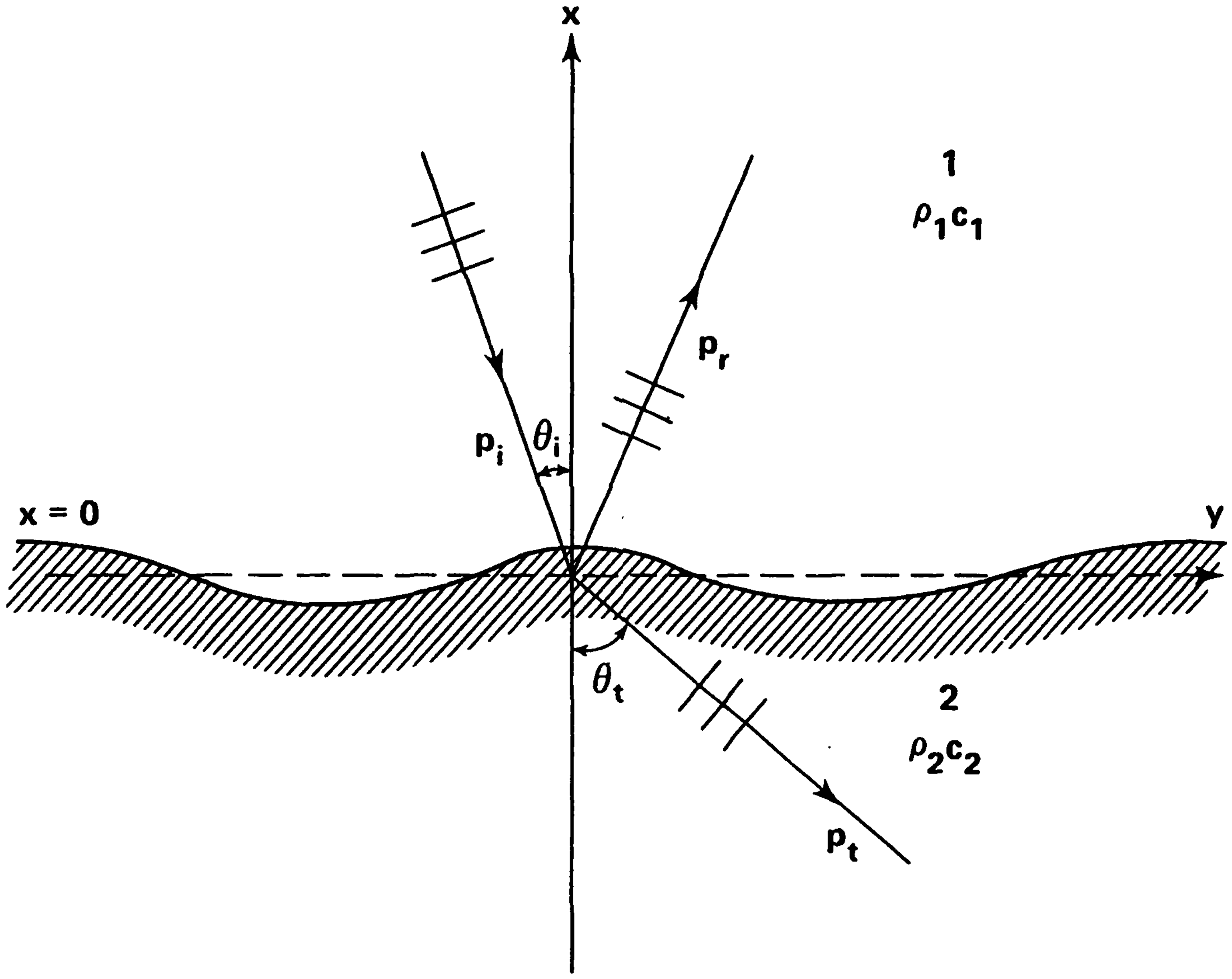

FIGURE 2 A PLANE ACOUSTIC WAVE STRIKING AN INTERFACE WITH A PROPAGATING SURFACE WAVE

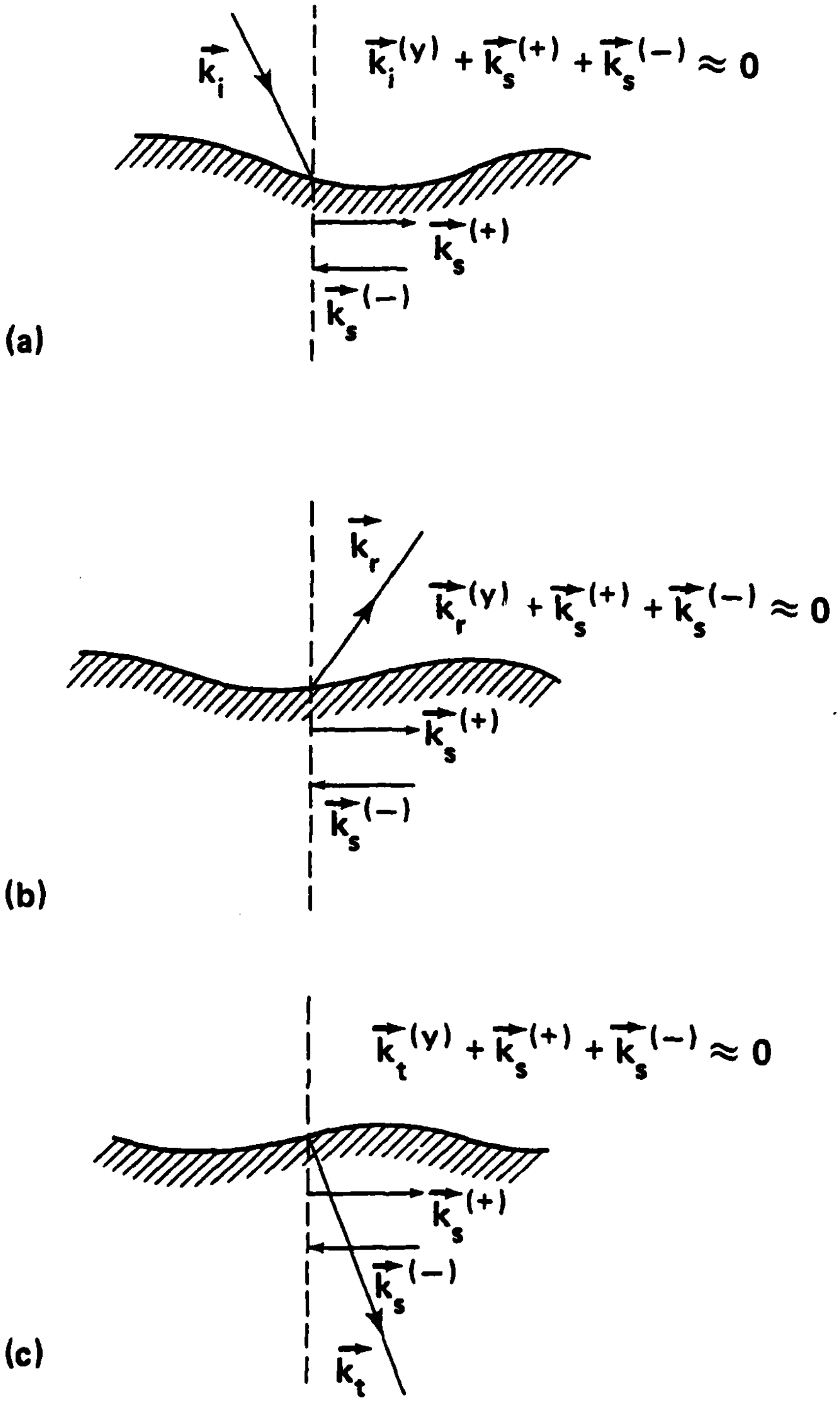

FIGURE 3 THE WAVE VECTOR RESONANCE CONDITIONS FOR THE (a) INCIDENT, (b) REFLECTED AND (c) TRANSMITTED WAVE

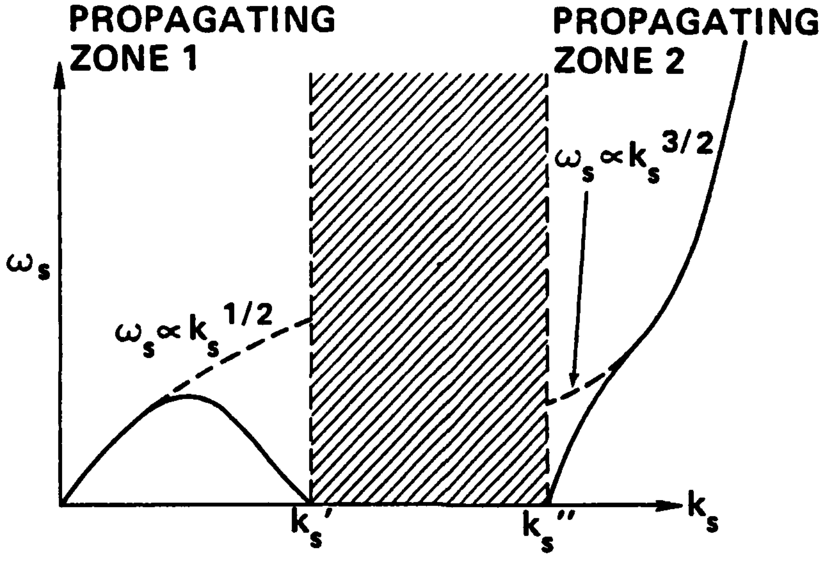

FIGURE 4(a) QUALITATIVE SKETCH OF DISPERSION RELATION FOR PROPAGATING SURFACE AND ACOUSTIC WAVES

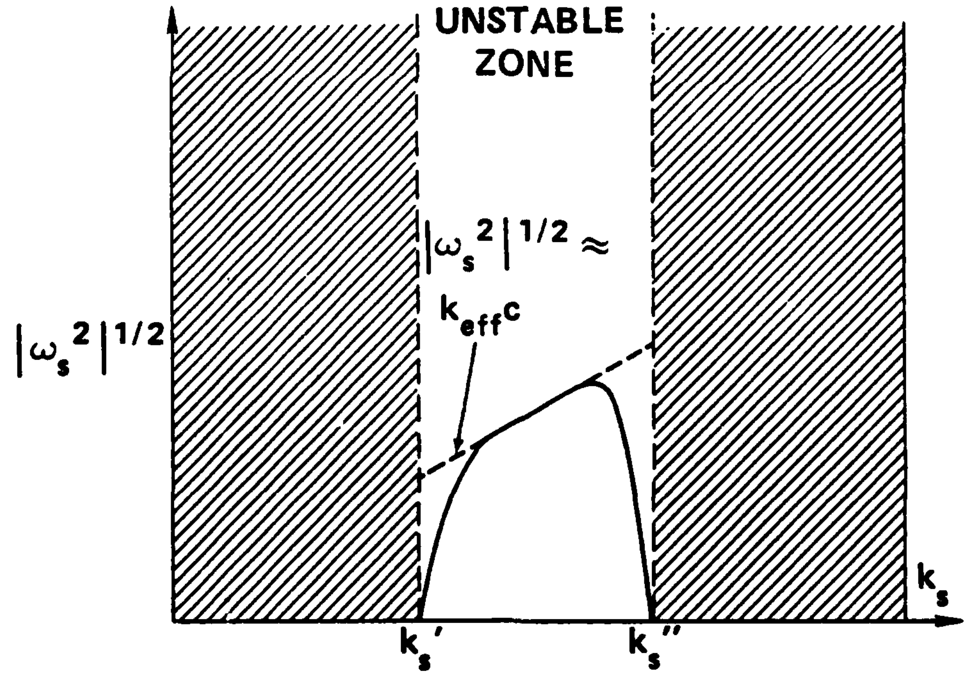

FIGURE 4(b) QUALITATIVE SKETCH OF DISPERSION RELATION FOR GROWING AND DECAYING SURFACE AND ACOUSTIC WAVES

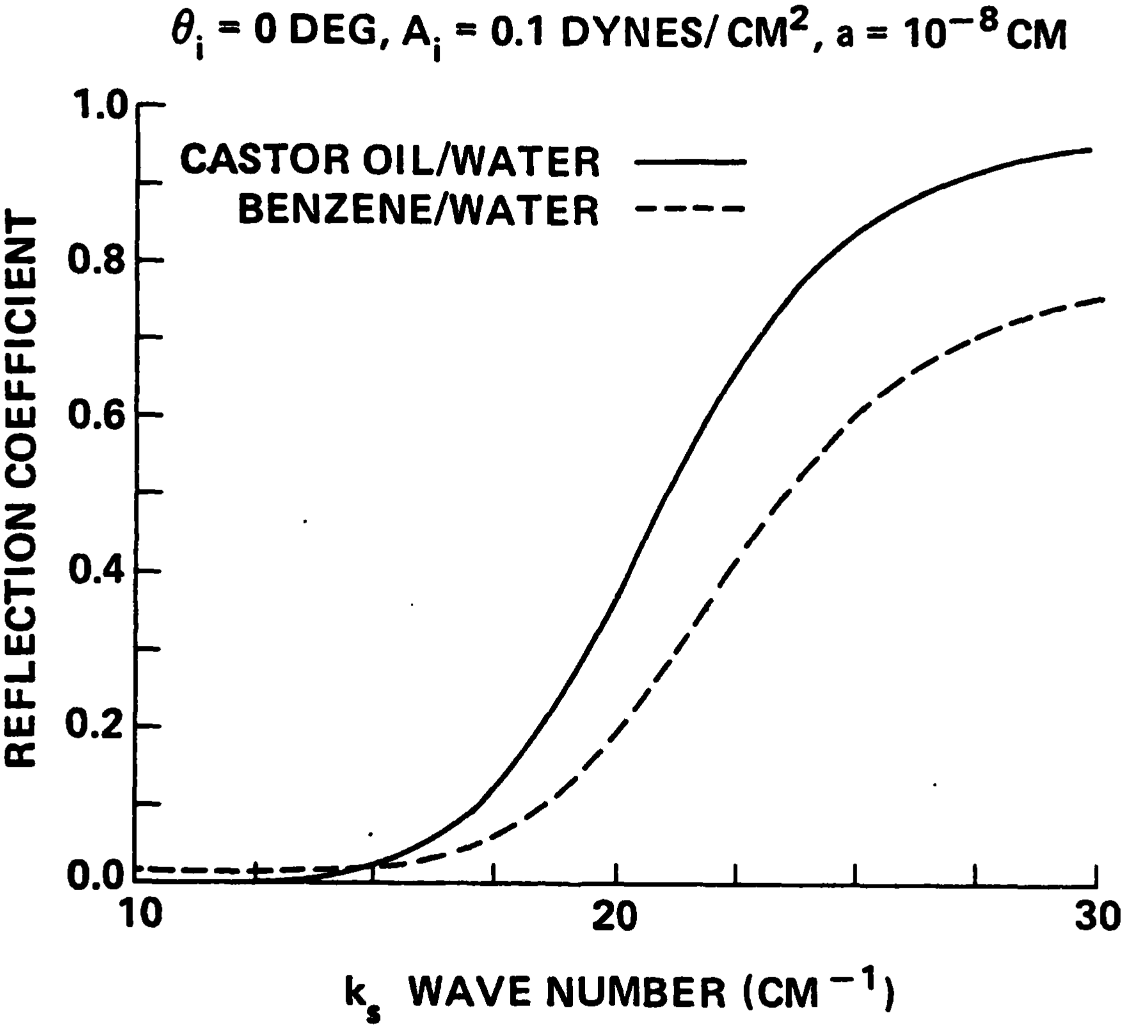

FIGURE 5(a) REFLECTION COEFFICIENT AND ITS DEPENDENCE ON WAVE NUMBER, k_s

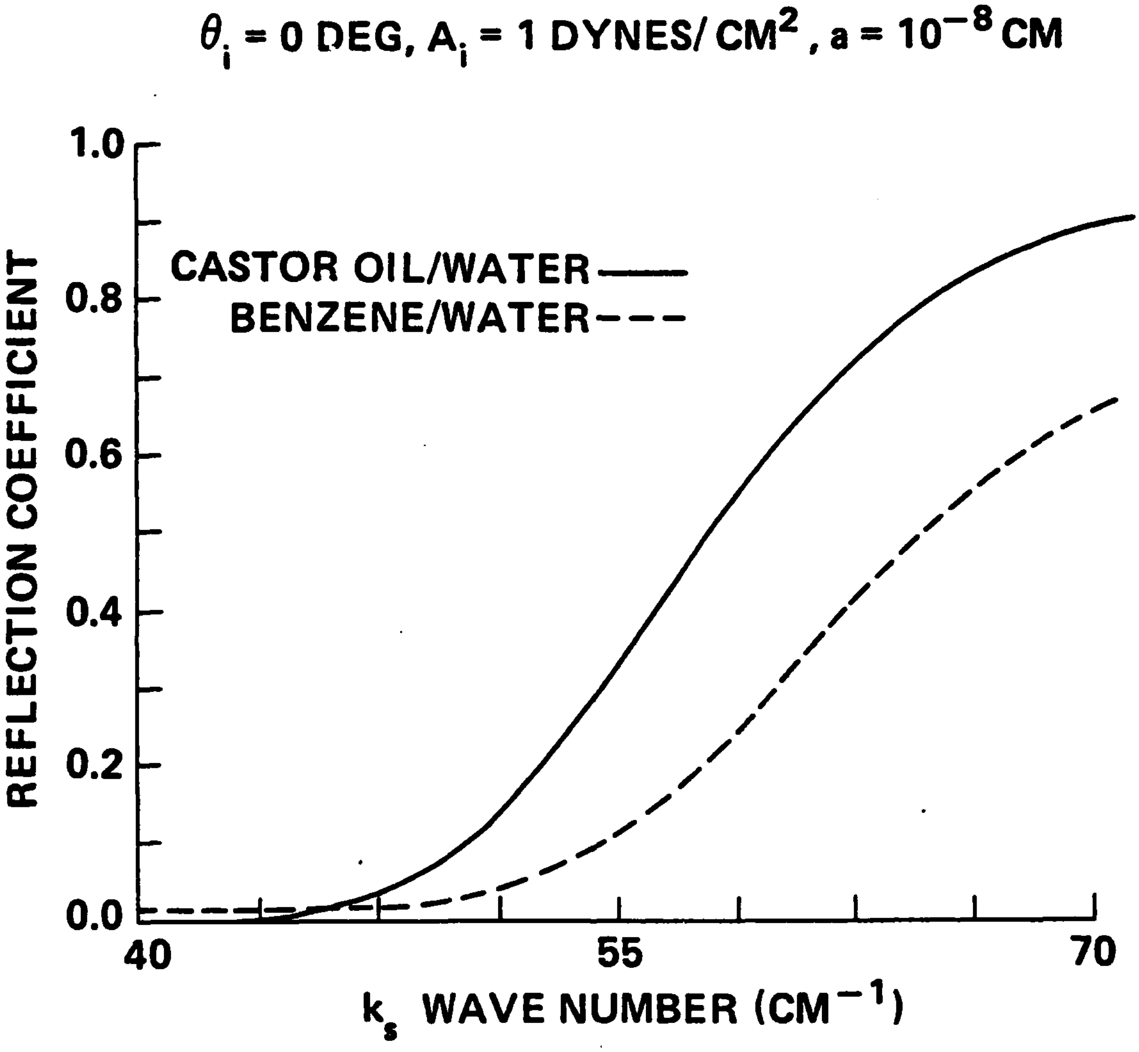

FIGURE 5(b) REFLECTION COEFFICIENT AND ITS DEPENDENCE ON WAVE NUMBER, k_s

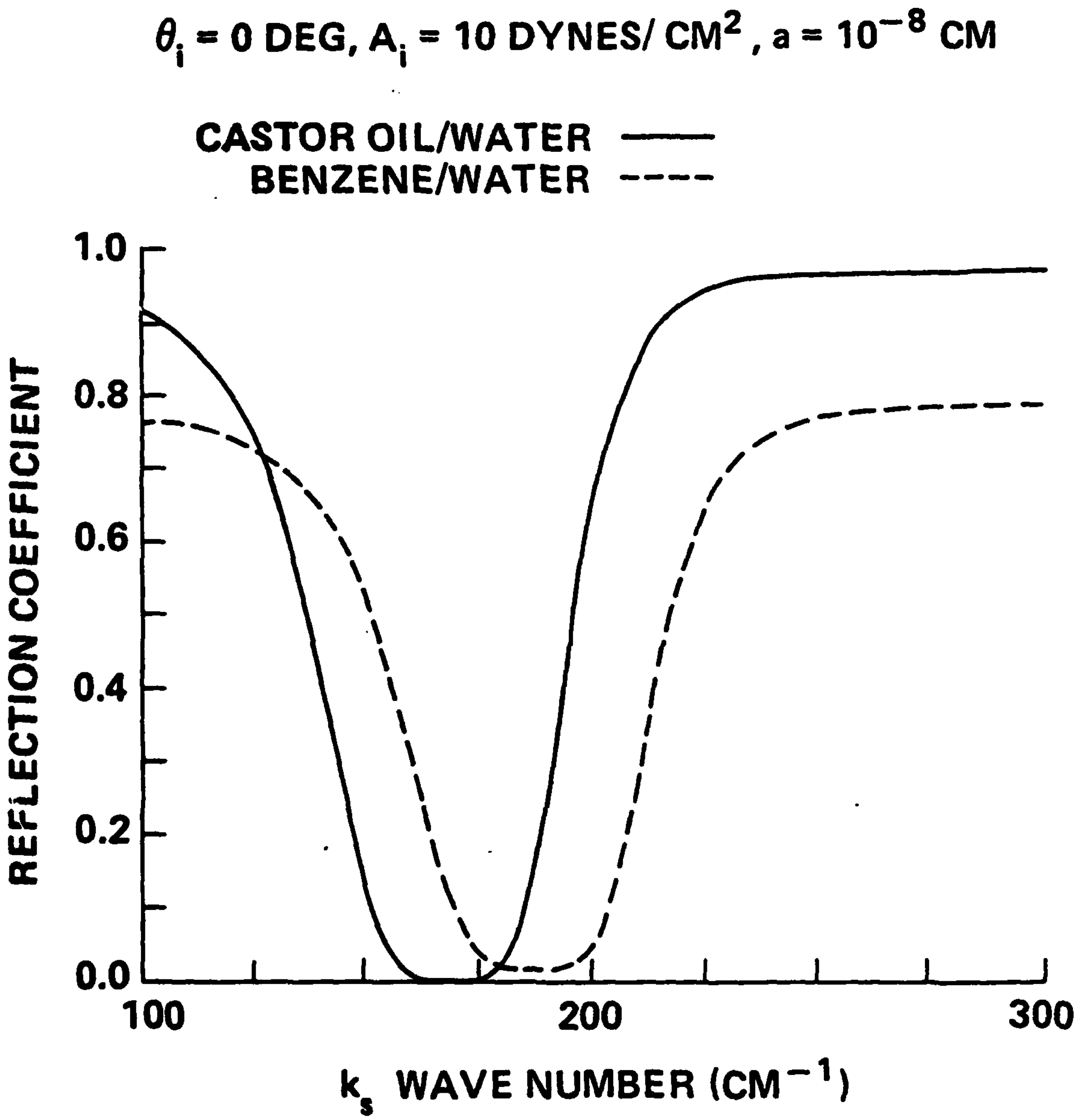

FIGURE 5(c) REFLECTION COEFFICIENT AND ITS DEPENDENCE ON WAVE NUMBER, k_s

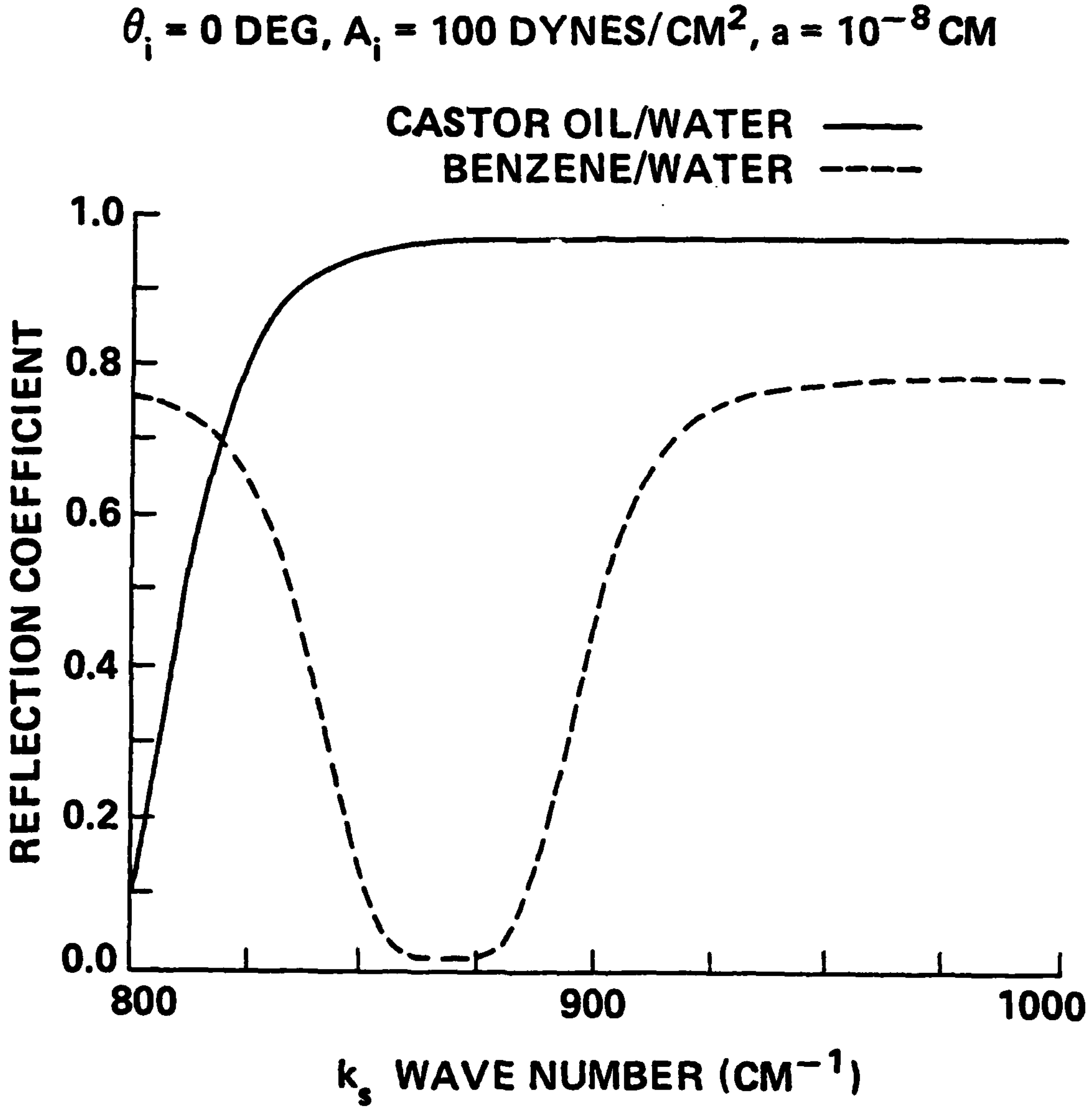

FIGURE 5(d) REFLECTION COEFFICIENT AND ITS DEPENDENCE ON WAVE NUMBER, k_s

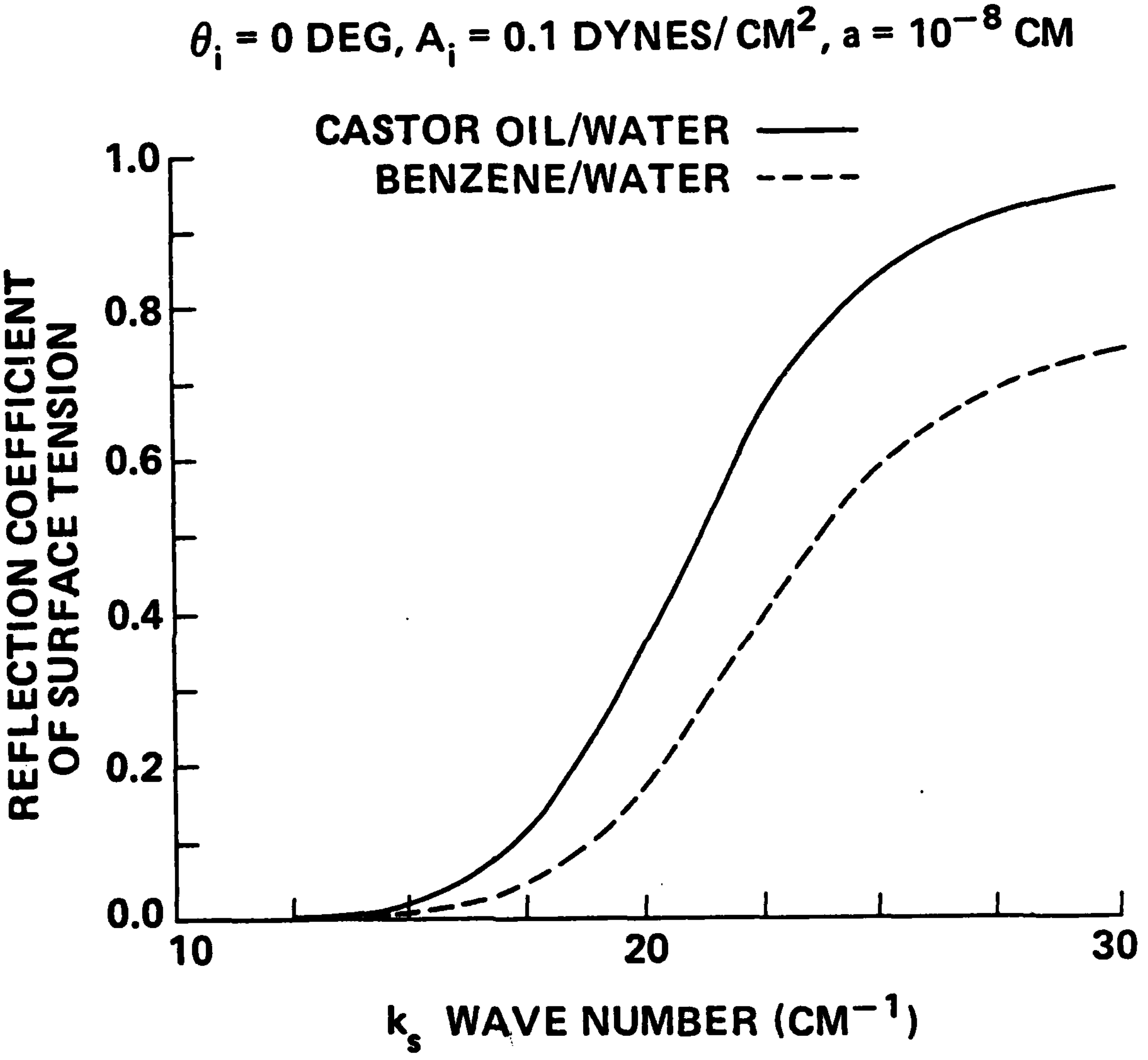

FIGURE 6(a) REFLECTION COEFFICIENT ARISING FROM SURFACE TENSION AND ITS DEPENDENCE ON WAVE NUMBER, k_s

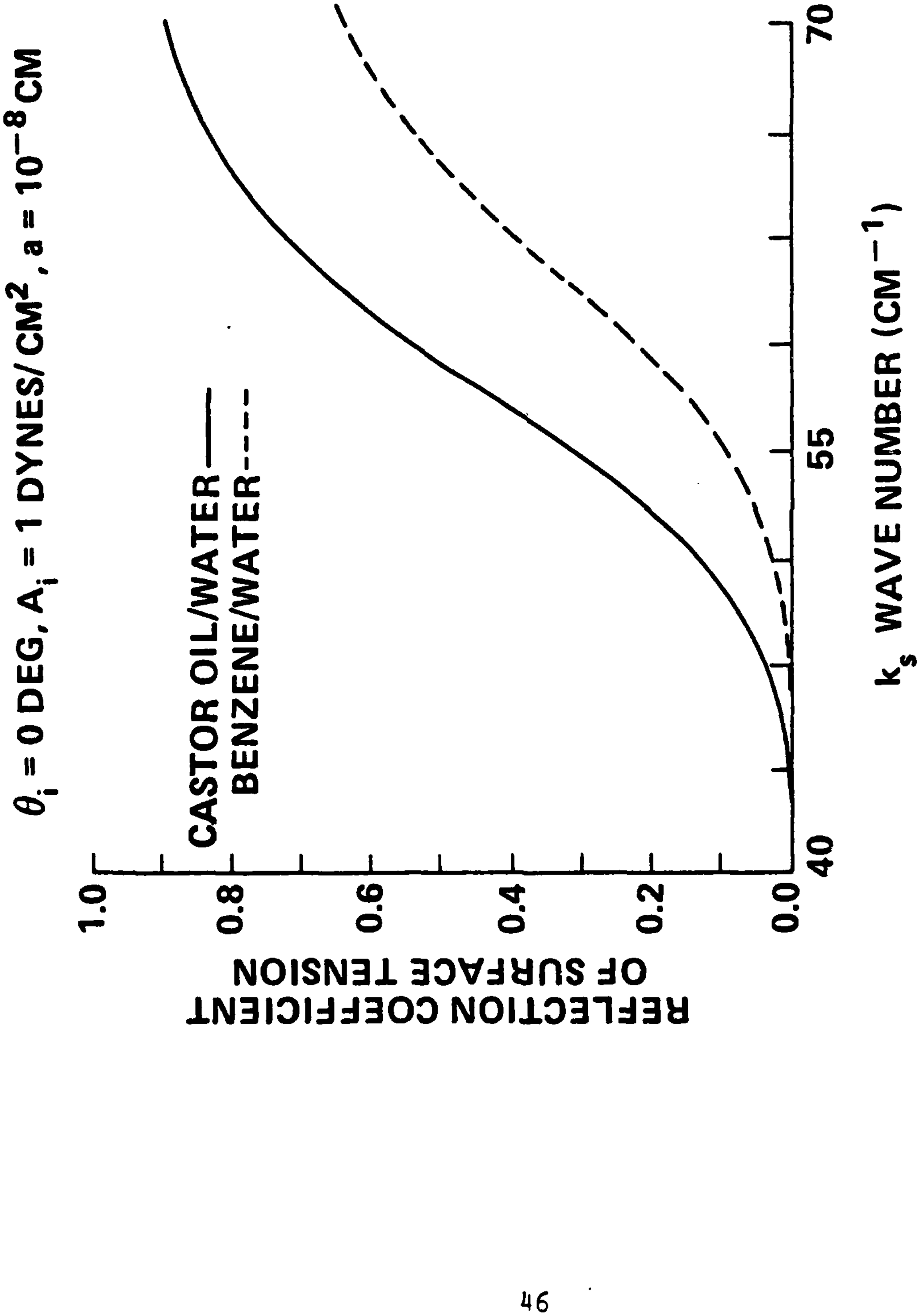

FIGURE 6(b) REFLECTION COEFFICIENT ARISING FROM SURFACE TENSION AND ITS DEPENDENCE ON WAVE NUMBER, k_s

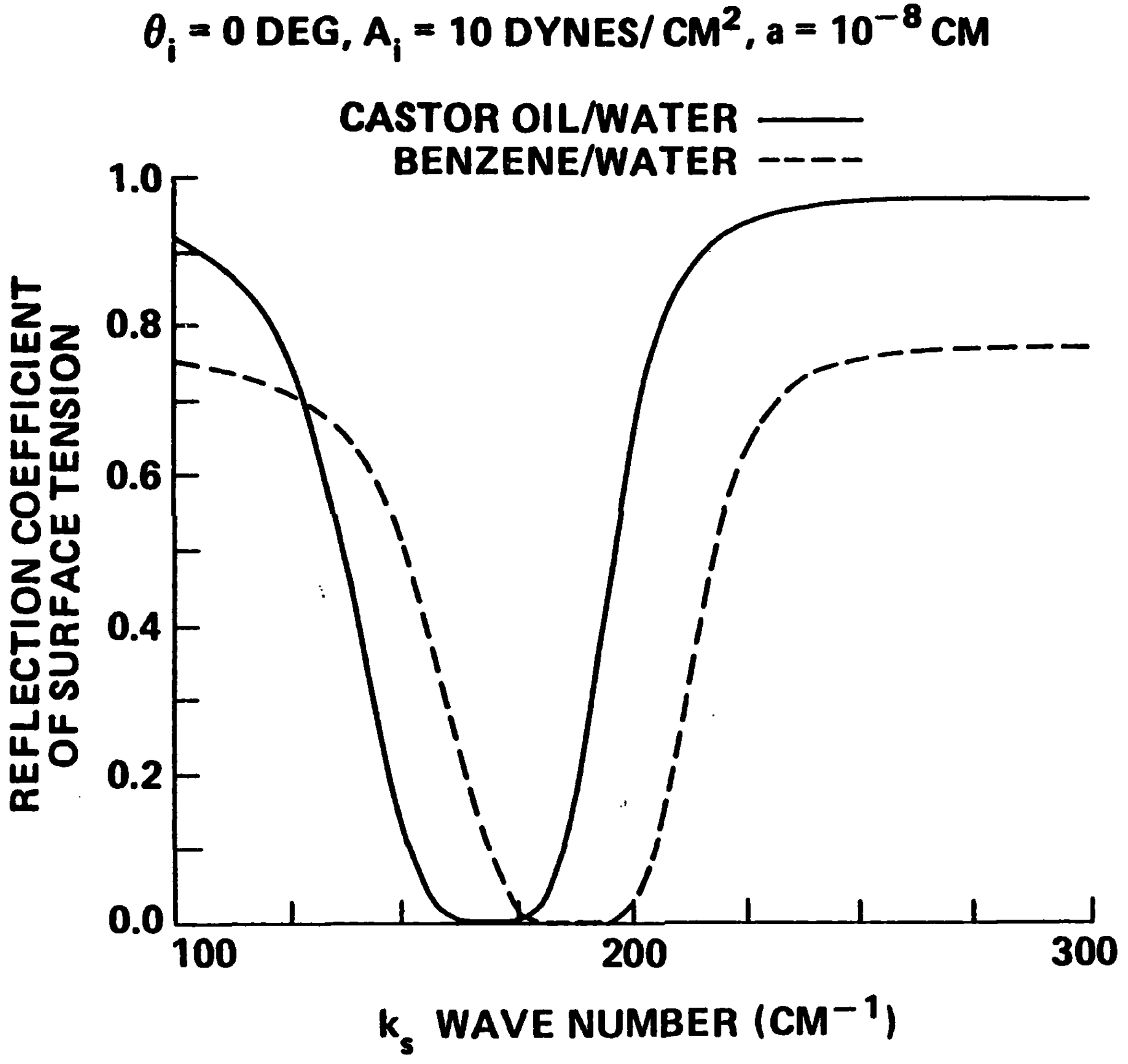

FIGURE 6(c) REFLECTION COEFFICIENT ARISING FROM SURFACE TENSION AND ITS DEPENDENCE ON WAVE NUMBER, k_s

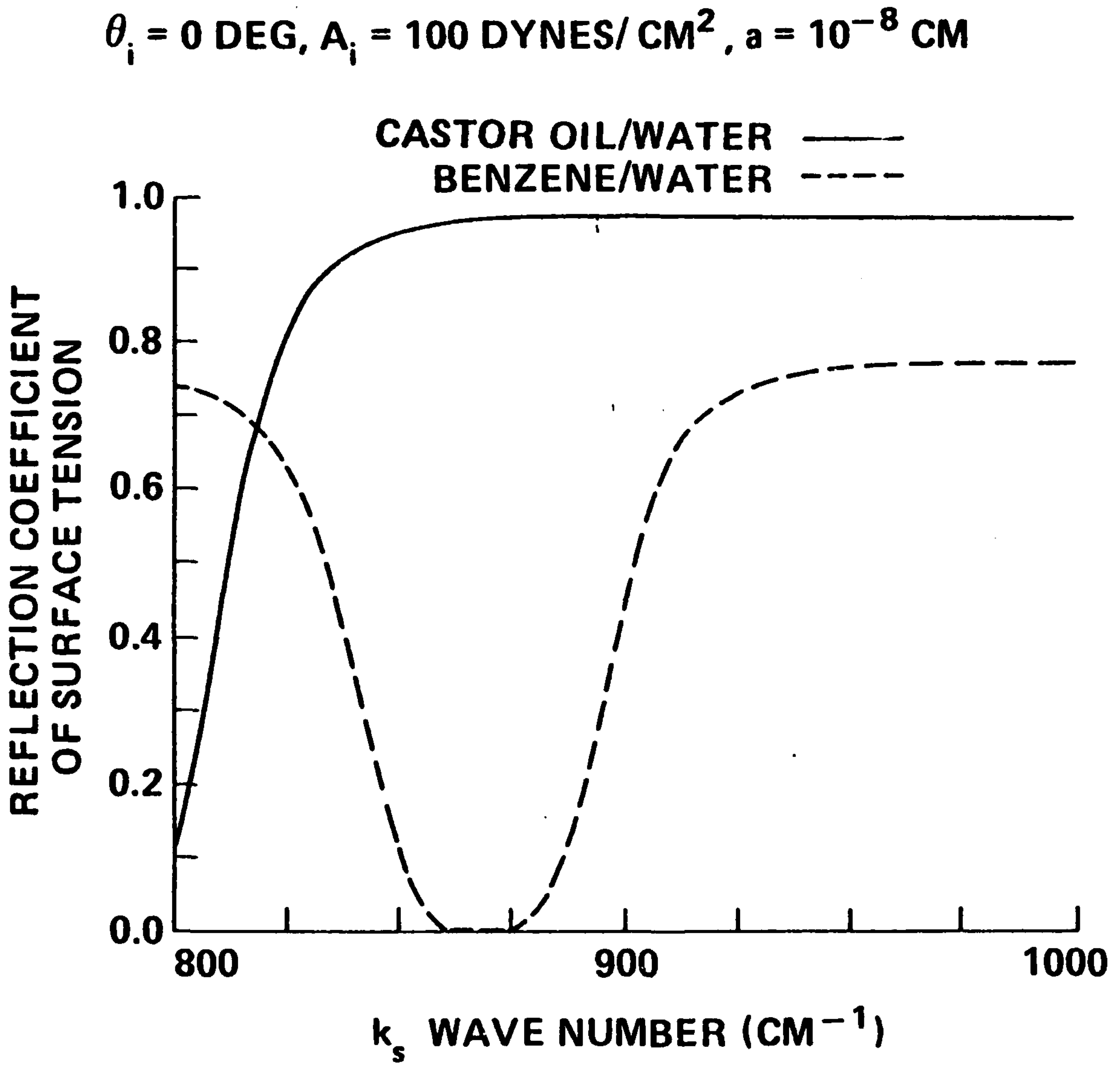

FIGURE 6(d) REFLECTION COEFFICIENT ARISING FROM SURFACE TENSION AND ITS DEPENDENCE ON WAVE NUMBER, k_s

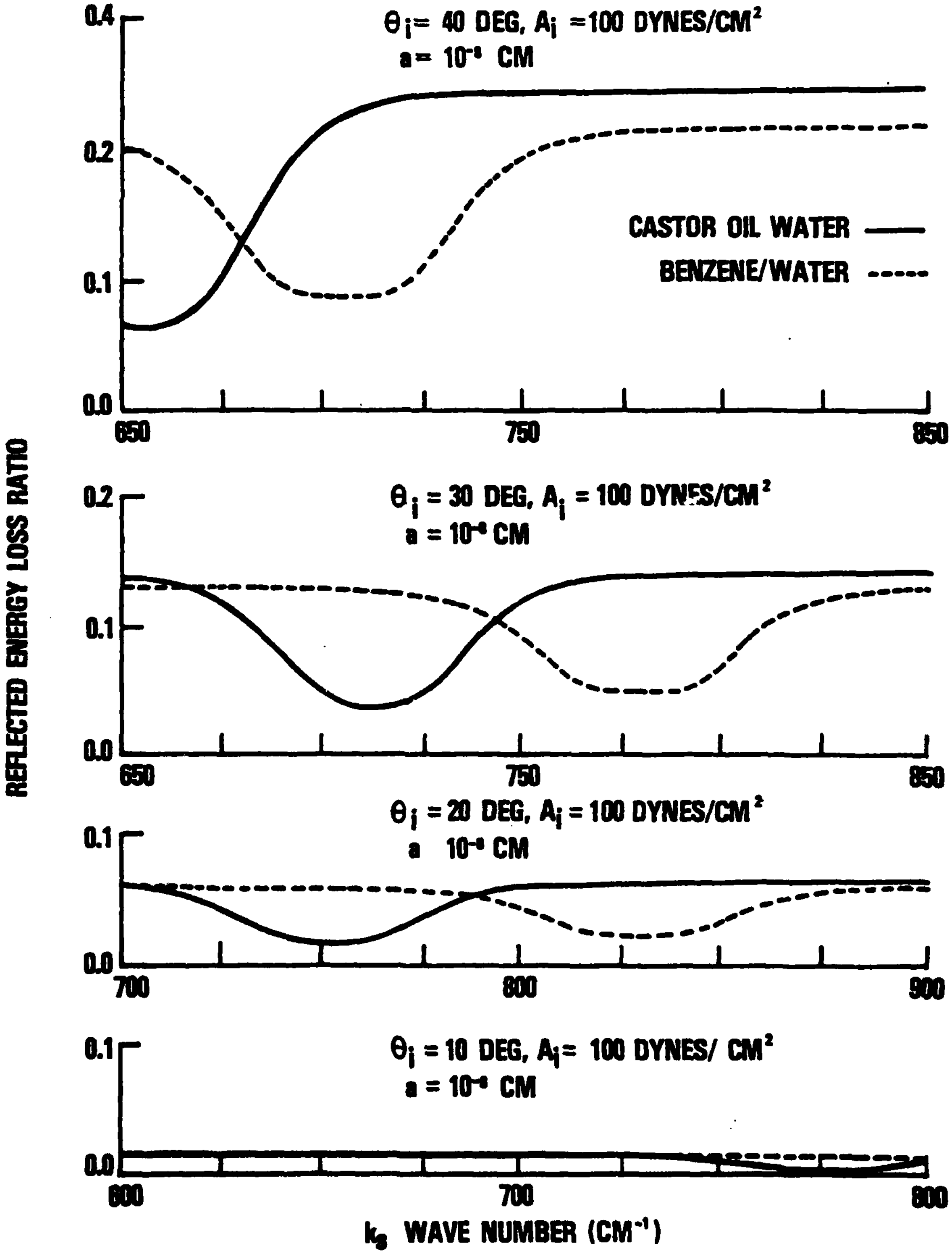

FIGURE 7 REFLECTED ENERGY LOSS RATIO AND ITS DEPENDENCE ON WAVE NUMBER k_s FOR VARIOUS ANGLES OF INCIDENCE, θ_i

TABLE 1. Acoustic Parameters

	WATER(9)	CASTOR OIL	BENZENE
MASS DENSITY (gm/cm^3)	.99821	.9603(10)	.87901(13)
SOUND SPEED (cm/sec)	1.4823×10^5	† 1.500×10^5 (11)	1.3210×10^5 (11)
SURFACE TENSION (dyne/cm)		* 39.0(12)	35.000(13)
CHARACTERISTIC ACOUSTIC IMPEDANCE $(gm/cm^2\text{-sec})$	1.4796×10^5	1.44×10^5	1.1610×10^5

† This data was taken at a temperature T = 18.6°C

* The surface tension value for castor oil is for a castor oil - air interface. No data for a castor oil - water interface was found in the literature.

TABLE 2. Surface wave frequencies satisfying dispersion relation

k_s (cm^{-1})	a(cm)	$k_s a$	$A_i \left(\frac{\text{dyne}}{\text{cm}^2}\right)$	$\langle\omega_s\rangle\ \theta_i$ (Hz) castor oil/water	$\langle\omega_s\rangle\ \theta_i$ (Hz) benzene/water
1	10^{-8}	10^{-8}	10^{-1}	2.68 x 10^2	1.50 x 10^2
10	"	10^{-7}	"	"	"
10^2	"	10^{-6}	"	2.69 x 10^2	"
1	"	10^{-8}	1	2.68 x 10^3	1.49 x 10^3
10	"	10^{-7}	"	"	"
10^2	"	10^{-6}	"	"	"
10^3	"	10^{-5}	"	2.62 x 10^3	1.47 x 10^3
1	"	10^{-8}	10	2.64 x 10^4	1.49 x 10^4
10	"	10^{-7}	"	2.68 x 10^4	"
10^2	"	10^{-6}	"	"	"
10^3	"	10^{-5}	"	"	"
1	"	10^{-8}	10^{-2}	1.74 x 10^5	1.34 x 10^5
10	"	10^{-7}	"	2.64 x 10^5	1.49 x 10^5
10^2	"	10^{-6}	"	2.68 x 10^5	"
10^3	"	10^{-5}	"	"	"
10^4	"	10^{-4}	"	2.63 x 10^5	"
1	10^{-7}	10^{-7}	10^{-1}	2.7 x 10^1	1.49 x 10^1
10	"	10^{-6}	"	"	"
1	"	10^{-7}	1	2.70 x 10^2	1.50 x 10^2
10	"	10^{-6}	"	2.68 x 10^2	"
10^2	"	10^{-5}	"	"	1.51 x 10^2
1	"	10^{-7}	10	2.68 x 10^3	1.49 x 10^3
10	"	10^{-6}	"	"	"
10^2	"	10^{-5}	"	"	"
10^3	"	10^{-4}	"	2.62 x 10^3	"
1	"	10^{-7}	10^2	2.64 x 10^4	1.50 x 10^4
10	"	10^{-6}	"	2.68 x 10^4	"
10^2	"	10^{-5}	"	"	"
10^3	"	10^{-4}	"	"	"

TABLE 2. (cont'd) Surface wave frequencies satisfying dispersion relation

$k_s (cm^{-1})$	a(cm)	$k_s a$	$A_i \left(\frac{dyne}{cm^2}\right)$	$\langle\omega_s\rangle_{\theta_i}$ (Hz) castor oil/water	$\langle\omega_s\rangle_{\theta_i}$ (Hz) benzene/water
1	10^{-6}	10^{-6}	10^{-1}	2.68	——
"	"	"	1	2.68×10^1	1.50×10^1
10	"	10^{-5}	"	"	"
1	"	10^{-6}	10	2.68×10^2	1.50×10^2
10	"	10^{-5}	"	"	"
10^2	"	10^{-4}	"	"	"
1	"	10^{-6}	10^2	2.68×10^3	1.50×10^3
10	"	10^{-5}	"	"	"
10^2	"	10^{-4}	"	"	"
10^3	"	10^{-3}	"	2.62×10^3	1.49×10^3

1	10^{-5}	10^{-5}	1	2.69	——
"	"	"	10	2.68×10^1	1.50×10^1
10	"	10^{-4}	"	2.66×10^1	"
1	"	10^{-5}	10^2	2.68×10^2	1.50×10^2
10	"	10^{-4}	"	"	"
10^2	"	10^{-3}	"	"	1.51×10^2

Table 3. Thermodynamic Parameters for Water at T = 20°C

C_p	C_v	κ	κ_T	α_T	v	χ_s
$4.1796 \frac{\text{Joule}}{\text{gm-°K}}$	$4.1022 \frac{\text{Joule}}{\text{gm-°K}}$	$1.43\text{x}10^{-3} \frac{\text{cal}}{\text{cm-sec-°K}}$	$49.1\text{x}10^{-6}\text{atmos}^{-1}$	$3.574\text{x}10^{-4}$	$1.0018 \frac{\text{cm}^3}{\text{g}}$	$1.44\text{x}10^{-3} \frac{\text{cm}^2}{\text{sec}}$

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