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INVESTIGATION OF DISTURBANCE ACCOMMODATING CONTROLLER APPLICATI--ETC(U)
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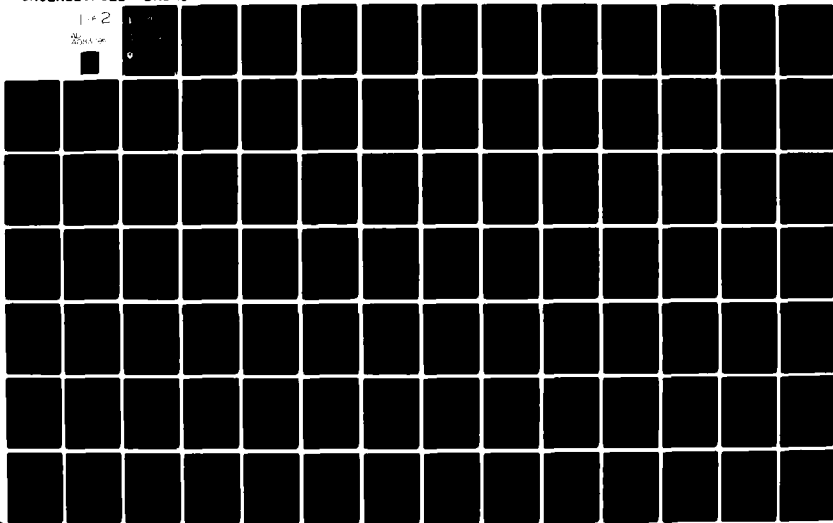
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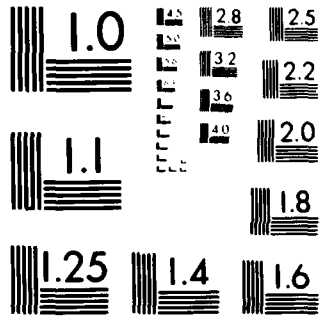
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TECHNICAL REPORT T-79-83

INVESTIGATION OF DISTURBANCE
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APPLICATION TO A MISSILE
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**U.S. ARMY
MISSILE
RESEARCH
AND
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COMMAND**

Wayne L. McCowan
Guidance and Control Directorate
Technology Laboratory

31 May 1979



Redstone Arsenal, Alabama 35809

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places in the plant, into a functioning unit which would still perform its overall purpose. However, the true test of how a DAC would function in a system application would be to implement one in a 6-DOF simulation and fly it with a severe program of varying disturbance vectors.

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1. INTRODUCTION

This report presents the results of an investigation into the feasibility of using Disturbance Accommodating Controller (DAC) design techniques, as developed by Dr. C. D. Johnson of the University of Alabama in Huntsville, to cancel out disturbance inputs to a missile autopilot channel.

The DAC method of design uses a combination of waveform-mode disturbance modeling and state-variable control techniques. As a tool for controller design, the DAC approach permits three primary modes of disturbance accommodation: (1) cancellation (absorption) of disturbance effects, (2) minimization of disturbance effects, or (3) constructive utilization of the disturbances as an aid in accomplishing the primary control task.

The purpose of this report is to determine if these techniques, specifically the cancellation and minimization modes, can be successfully applied to a missile system in such a manner as to cancel out the effects of disturbance inputs which would otherwise degrade system accuracy.

It is not the intent of this report to thoroughly cover all the background theory involved in the development of DAC design procedures. This theory can best be obtained by reading the original papers; see, for instance, *References 1-4*. For applications of DAC to several simple systems see *Reference 5*.

2. SOME BACKGROUND

The plant considered in this report is one which can be described by state equations of the form

$$\begin{aligned}\dot{\underline{x}} &= \underline{A}\underline{x} + \underline{B}\underline{u} + \underline{F}\underline{w} \\ \dot{\underline{y}} &= \underline{C}\underline{x} + \underline{E}\underline{u} + \underline{G}\underline{w}\end{aligned}\tag{1}$$

where

\underline{x} is the plant state vector,

\underline{u} is the plant control input vector,

\underline{w} is the vector of external disturbance acting on the plant,

\underline{y} is the system output vector, and

\underline{A} , \underline{B} , \underline{F} , \underline{C} , \underline{E} , \underline{G} are appropriate size, known matrices which are not necessarily constant.

Now, the external disturbances, $w(t)$, for which DAC theory is intended are characterized by the presence of "waveform structure," i.e., the functions $w(t)$ can be described by known differential equations which the $w(t)$ satisfy "almost everywhere." For the cases considered in this report, the disturbances will be assumed to be described by the following general set of linear disturbance state equations:

$$\begin{aligned} \underline{w} &= \underline{H}\underline{z} + \underline{L}\underline{x} \\ \dot{\underline{z}} + \underline{D}\underline{z} + \underline{M}\underline{x} + \underline{\sigma} & \end{aligned} \quad (2)$$

where

\underline{z} is the disturbance "state" vector,

$\underline{\sigma}$ is a sequence of randomly arriving vector impulses, and

\underline{D} , \underline{H} , \underline{L} , \underline{M} are known, time-invariant matrices.

In most practical applications, neither the complete set of plant state variables nor the various components $w_i(t)$ of the disturbance are available for direct on-line measurement. Therefore, the DAC is restricted to operate only on information in the available on-line measurements of the system outputs and commands and any disturbance components which may actually be available for direct measurement. In the case at hand, it is assumed that none of the disturbance components are measurable on-line and that the information available from the plant consists of the input command, command to the actuators and the measured pitch plane acceleration and rate components of the missile motion.

Since the idealized DAC control law is a function of the real-time system state, \underline{x} , and disturbance state, \underline{z} , the required on-line data for practical DAC implementation must be generated via use of state reconstructors (observers) operating on real-time system outputs \underline{y} and control inputs \underline{u} . Since the external disturbances $w(t)$ are assumed to have waveform structure and to be modeled by known linear state models, a state reconstructor can be designed to generate estimates $\hat{\underline{z}}$ of the instantaneous disturbance state \underline{z} . In addition, that same state reconstructor can be designed to produce estimates $\hat{\underline{x}}$ of the instantaneous system state \underline{x} .

Procedures have been developed to generate both "full-dimensional" observers of dimension $(n+\rho)$, where n is the order of \underline{x} and ρ the order of \underline{z} , and "reduced-dimensional" observers of dimension $(n+\rho-m)$, where n , ρ are as above and m is the rank of \underline{C} . The work performed in this study is concerned with "full-dimensional" observers.

For the form of the state equations given by Equation (1), the full-dimensional observer is expressed as

$$\begin{pmatrix} \dot{\hat{x}} \\ \dot{\hat{z}} \end{pmatrix} = \begin{bmatrix} \underline{A} + \underline{F}\underline{L} + \underline{K}_1(\underline{C} + \underline{G}\underline{L}) & | & \underline{F} + \underline{K}_1\underline{G} \underline{H} \\ \underline{M} + \underline{K}_2(\underline{C} + \underline{G}\underline{L}) & | & \underline{D} + \underline{K}_2\underline{G}\underline{H} \end{bmatrix} \begin{pmatrix} \hat{x} \\ \hat{z} \end{pmatrix} - \begin{bmatrix} \underline{K}_1 \\ \underline{K}_2 \end{bmatrix} \underline{y}(t) \\ + \begin{bmatrix} \underline{B} + \underline{K}_1\underline{E} \\ \underline{K}_2\underline{E} \end{bmatrix} \underline{u}(t) \quad (3)$$

where \underline{K}_1 , \underline{K}_2 , are gain matrices to be designed,
 \underline{A} , \underline{F} , \underline{L} , \underline{C} , \underline{G} , \underline{H} , \underline{D} , \underline{M} are as previously described.

Such a composite-type state reconstructor can be utilized to implement DAC control laws in the form

$$\underline{u} = f(\hat{x}, \hat{z}, t).$$

Of course, for acceptable performance the real-time estimation errors

$$\underline{\epsilon}_x = \underline{x} - \hat{\underline{x}}$$

$$\underline{\epsilon}_z = \underline{z} - \hat{\underline{z}}$$

must settle to zero rapidly in comparison to system settling times where $\underline{\epsilon}_x$ and $\underline{\epsilon}_z$ are given by

$$\begin{pmatrix} \dot{\underline{\epsilon}}_x \\ \dot{\underline{\epsilon}}_z \end{pmatrix} = \begin{bmatrix} \underline{A} + \underline{F}\underline{L} + \underline{K}_1(\underline{C} + \underline{G}\underline{L}) & | & \underline{F} + \underline{K}_1\underline{G} \underline{H} \\ \underline{M} + \underline{K}_2(\underline{C} + \underline{G}\underline{L}) & | & \underline{D} + \underline{K}_2\underline{G}\underline{H} \end{bmatrix} \begin{pmatrix} \underline{\epsilon}_x \\ \underline{\epsilon}_z \end{pmatrix} + \begin{pmatrix} \underline{0} \\ \underline{\bar{d}}(t) \end{pmatrix} \quad (4)$$

3. PLANT

The plant utilized for the studies detailed in this report is the pitch plane acceleration autopilot channel shown in block diagram form in *Figure 1*. An ideal accelerometer is assumed in the acceleration feedback loop and an ideal rate gyro is assumed in the rate feedback loop. Also, no actuator dynamics are considered.

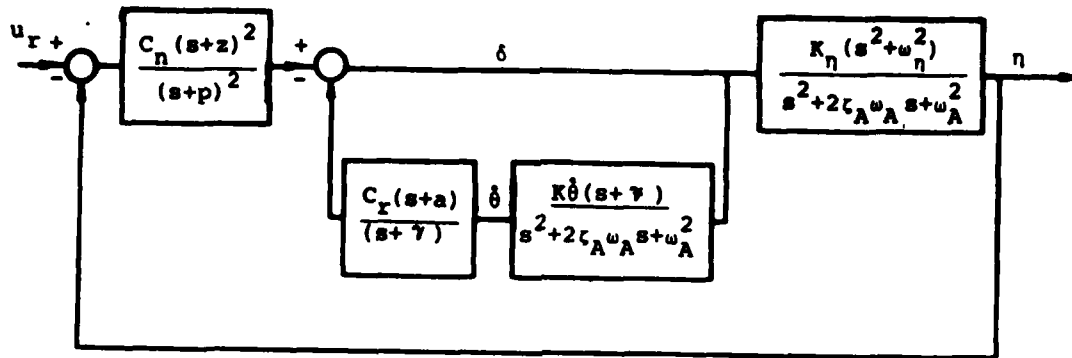


Figure 1. Pitch acceleration autopilot channel.

The transfer functions indicated in *Figure 1* are:

- a. Pitch rate per fin deflection in pitch:

$$\frac{\dot{\theta}(s)}{\delta(s)} = \frac{K_{\dot{\theta}}(s + \gamma)}{s^2 + 2\zeta_A \omega_A s + \omega_A^2}$$

- b. Lateral acceleration per fin deflection in pitch:

$$\frac{\eta(s)}{\delta(s)} = \frac{K_{\eta}(s^2 + \omega_{\eta}^2)}{s^2 + 2\zeta_A \omega_A s + \omega_A^2}.$$

with the terms in the transfer functions being determined from the aerodynamic characteristics of the missile at given points along a trajectory.

- c. Autopilot compensation terms:

$$\frac{C_R(s+a)}{s + \gamma} \quad \text{and} \quad \frac{C_{\eta}(s+z)^2}{(s+p)^2} \quad \text{are compensation terms}$$

which were designed into the autopilot to improve performance. Most of the terms are varied over a trajectory according to dynamic pressure.

Since the transfer function parameters and most of the autopilot compensation terms do vary along the missile trajectory, several representative points along a nominal trajectory were chosen as design points for use in this report. These points were chosen to cover as nearly as possible the entire range of values of the parameters involved. *Table 1* lists the time points and parameter values.

For the initial investigation, it was decided to look at several different configurations involving the plant, or some part of it, and a disturbance source. First, the entire loop was used with a disturbance assumed to be acting at the input. Next, the rate loop alone was considered with an assumed disturbance summed in with the $\dot{\theta}$ due to fin deflection. Third, the entire loop was again used, this time with a disturbance summed into the output. As a final case, the entire loop was used with disturbances at the input and the output. Each of these cases will be detailed later in this report.

4. DISTURBANCE MODEL

The disturbances modeled here in all cases were taken to be composed of constants plus ramps, i.e.,

$$w(t) = C_0 + C_1 t \quad (5)$$

where C_0 and C_1 are, in general, unknown a priori and can change value in a completely unknown random-like manner. This disturbance model was chosen because it is easy to work with but still illustrates the point.

To put (5) into the form (2), proceed as follows. First, take the Laplace Transform of $w(t)$,

$$w(s) = \frac{C_0}{s} + \frac{C_1}{s^2} = \frac{C_0 s + C_1}{s^2}$$

The characteristic polynomial associated with this is

$$\lambda^2 = 0. \quad (6)$$

TABLE 1. AIRFRAME/COMPENSATION PARAMETERS

FLIGHT TIME (SEC) / PARAMETER	9.85 (JUST AFTER BURNOUT)	18.0	50.5	66.7 (APOGEE)	103.3	111.4	135.8
ζ_A	0.0256	0.02	0.01	0.009	0.014	0.017	0.038
ω_A	14.54	8.7	2.216	1.77	3.87	5.21	9.48
$K_{\dot{\theta}}$	-107.	-50.	-6.56	-5.17	-14.68	-25.4	-118.6
γ	0.536	0.255	0.034	0.026	0.08	0.138	0.56
K_{η}	-317.8	-148.5	-19.5	-15.35	-43.6	-75.3	-352.2
ω_{η}^2	-540	-209.9	-18.16	-12.9	-48.	-85.6	-330.4
C_n	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$	$1.0471(10^{-4})$
z	10.	10.	10.	10.	10.	10.	10.
p	1.	1.	1.	1.	1.	1.	1.
CR	-0.1396	-0.1396	-0.4363	-0.4363	-0.4363	-0.4363	-0.4363

Therefore, choose

$$\begin{aligned}\underline{w} &= \underline{Hz} \\ \dot{\underline{z}} &= \underline{Dz} + \underline{g}\end{aligned}$$

(Note: no state dependence terms are included in this case) such that

$$\underline{z} = \underline{Dz}$$

has a characteristic polynomial $\lambda^2 = 0$ and \underline{Hz} has the general form $w = C_0 + C_1 t$.

So, let

$$\underline{w} = \underline{Hz} = (1 \ 0) \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

and

$$\dot{\underline{z}} = \underline{Dz} + \underline{g} = \begin{bmatrix} -\beta_2 & 1 \\ -\beta_1 & 0 \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \underline{g}.$$

Then,

$$\det |\underline{D} - \lambda \underline{I}| = \begin{vmatrix} -\beta_2 - \lambda & 1 \\ -\beta_1 & -\lambda \end{vmatrix} = \lambda^2 + \beta_2 \lambda + \beta_1 = 0. \quad (7)$$

Comparing (7) with (6), one must have $\beta_2 = \beta_1 = 0$.

Thus,

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \underline{g}$$

$$\underline{w} = (1 \ 0) \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \quad (8)$$

From this one has, therefore,

$$\underline{H} = (1 \ 0)$$

$$\underline{D} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} .$$

These two matrices are used throughout this report in the disturbance state model.

In order to give a feel for just what "disturbance" as an entity is insofar as the applications here are concerned, it could be any or all of: wind or wind gust, thrust misalignment, tipoff rates, biases, instrument drifts, target motion and more. An influencing agent which has waveform structure and which imposes an undesirable effect on the system may be considered a disturbance.

5. ACCELERATION LOOP WITH DISTURBANCE AT INPUT

A. DAC MODEL DEVELOPMENT

A block diagram representation of the autopilot/disturbance combination used in the development for this section is shown in *Figure 2*.

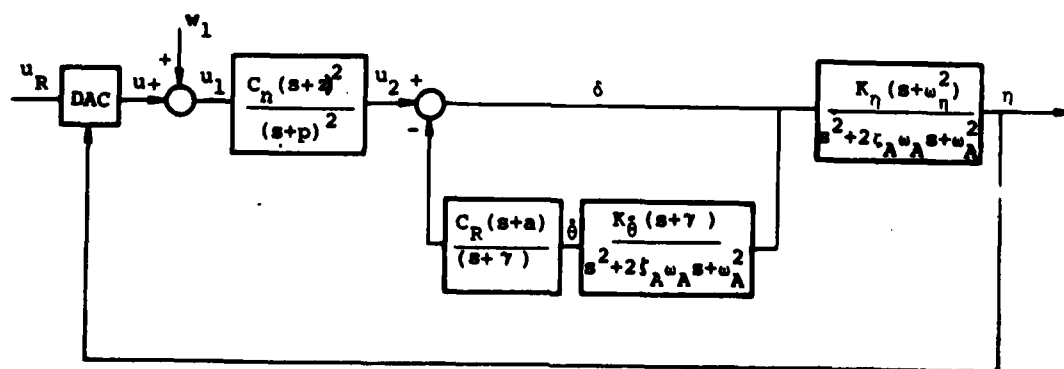


Figure 2. Pitch acceleration channel with disturbance at input.

The closed loop transfer function for the rate loop is

$$\frac{\delta(s)}{u_2(s)} = \frac{s^2 + 2\zeta_A \omega_A s + \omega_A^2}{s^2 + (2\zeta_A \omega_A + K_{\theta}^i C_R) s + (\omega_A^2 + K_{\theta}^i C_R a)} \quad (9)$$

Let

$$a_0 = 2\zeta_A \omega_A$$

$$a_1 = 2\zeta_A \omega_A + K_{\theta}^i C_R$$

$$a_2 = \omega_A^2 + K_{\theta}^i C_R a$$

Then

$$\frac{\delta(s)}{u_2(s)} = \frac{s^2 + a_0 s + \omega_A^2}{s^2 + a_1 s + a_2} \quad (10)$$

With this, the product of the transfer function blocks between u_1 and η is

$$\frac{K_{\eta} C_{\eta} (s+z)^2 (s^2 + \omega_{\eta}^2)}{(s+p)^2 (s^2 + a_1 s + a_2)} = \frac{K_{\eta} C_{\eta} [s^4 + 2zs^3 + (z^2 + \omega_{\eta}^2)s^2 + 2z\omega_{\eta}^2 s + \omega_{\eta}^2 s^2]}{s^4 + (2p + a_1)s^3 + (p^2 + 2pa_1 + a_2)s^2 + (a_1 p^2 + 2pa_2)s + a_2 p^2} \quad (11)$$

Let

$$b_0 = 2z$$

$$b_1 = z^2 + \omega_{\eta}^2$$

$$b_2 = 2z\omega_n^2$$

$$b_3 = \omega_n^2 z^2$$

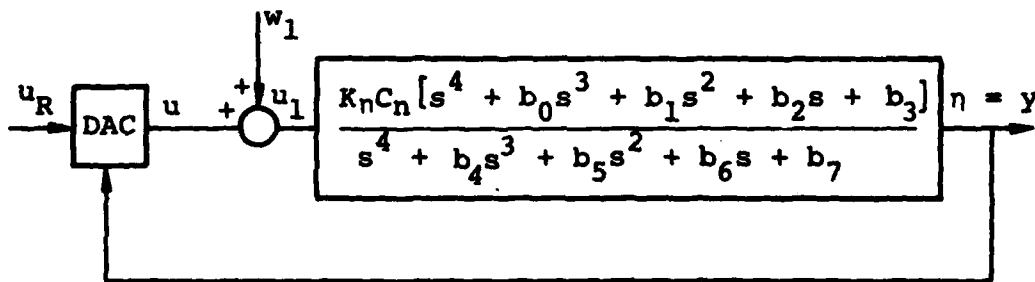
$$b_4 = 2p + a_1$$

$$b_5 = p^2 + 2pa_1 + a_2$$

$$b_6 = a_1 p^2 + 2pa_2$$

$$b_7 = a_2 p^2$$

The block diagram has thus been reduced to



In order to represent the plant in the form (1), proceed as follows.

$$\frac{y(s)}{u_1(s)} = \frac{K_n C_n [s^4 + b_0 s^3 + b_1 s^2 + b_2 s + b_3]}{s^4 + b_4 s^3 + b_5 s^2 + b_6 s + b_7}$$

Cross-multiplying gives

$$[s^4 + b_4 s^3 + b_5 s^2 + b_6 s + b_7] y(s) = K_n C_n [s^4 + b_0 s^3 + b_1 s^2 + b_2 s + b_3] u_1(s)$$

Solving for $y(s)$.

$$\begin{aligned}
 y(s) = & K_{\eta} C_n u_1 + \frac{1}{s} \left\langle K_{\eta} C_n b_0 u_1(s) b_4 y(s) + \frac{1}{s} \right\{ K_{\eta} C_n b_1 u_1(s) \\
 & - b_5 y(s) + \frac{1}{s} [K_{\eta} C_n b_2 u_1(s) - b_6 y(s)] \\
 & + \frac{1}{s} (K_{\eta} C_n b_3 u_1(s) - b_7 y(s)) \left. \right\rangle \quad (12)
 \end{aligned}$$

where $\frac{1}{s}$ denotes an integration.

This can be represented diagrammatically as

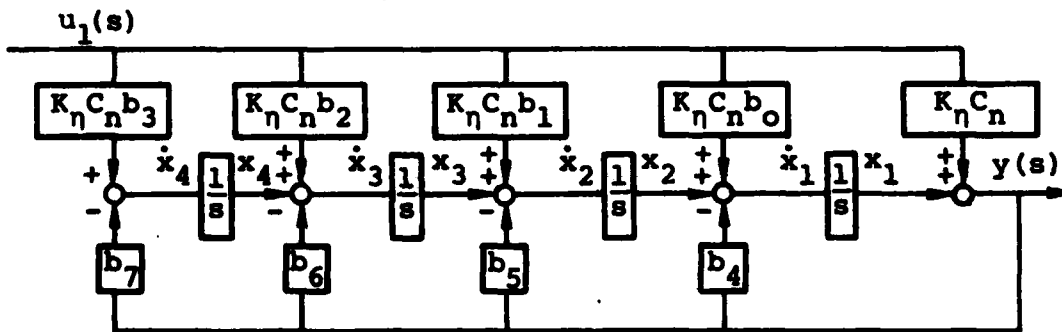


Figure 3. Plant state representation.

and from this, the equations for the states can be written directly as

$$\dot{x}_1 = x_2 + K_{\eta} C_n b_0 u_1 - b_4 y$$

$$\dot{x}_2 = x_3 + K_{\eta} C_n b_1 u_1 - b_5 y$$

$$\dot{x}_3 = x_4 + K_{\eta} C_n b_2 u_1 - b_6 y$$

$$\dot{x}_4 = K_{\eta} C_n b_3 u_1 - b_7 y$$

$$y = x_1 + K_{\eta} C_n u_1$$

However, for purposes of DAC design these equations need to be expressed as functions of \underline{x} , \underline{u} and \underline{w} . So, since $u_1 = u + w_1$,

$$\begin{aligned}
 y &= x_1 + K_{\eta} C_n (u+w_1) \\
 \dot{x}_1 &= -b_4 x_1 + x_2 + K_{\eta} C_n (u+w_1) (b_0 - b_4) \\
 \dot{x}_2 &= -b_5 x_1 + x_3 + K_{\eta} C_n (u+w_1) (b_1 - b_5) \\
 \dot{x}_3 &= -b_6 x_1 + x_4 + K_{\eta} C_n (u+w_1) (b_2 - b_6) \\
 \dot{x}_4 &= -b_7 x_1 + K_{\eta} C_n (u+w_1) (b_3 - b_7) \quad .
 \end{aligned} \tag{13}$$

or, in matrix form,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -b_4 & 1 & 0 & 0 \\ -b_5 & 0 & 1 & 0 \\ -b_6 & 0 & 0 & 1 \\ -b_7 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + K_{\eta} C_n \begin{bmatrix} b_0 - b_4 \\ b_1 - b_5 \\ b_2 - b_6 \\ b_3 - b_7 \end{bmatrix} \underline{u} \\
 + K_{\eta} C_n \begin{bmatrix} b_0 - b_4 \\ b_1 - b_5 \\ b_2 - b_6 \\ b_3 - b_7 \end{bmatrix} \underline{w}_1 \quad .$$

(14)

$$\underline{y} = [1 \ 0 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [K_n C_n] \underline{u} + [K_n C_n] \underline{w}_1 \quad (15)$$

Thus, Equations (14) and (15) define the remainder of the matrices needed for the DAC design.

Before proceeding, it is necessary to first check for the existence of a control, \underline{u} , which can totally counteract all the disturbance effects.

This control will exist if and only if $\underline{F} \equiv \underline{B} \underline{\Gamma}$ for some $\underline{\Gamma}$. Here,

$$K_n C_n \begin{bmatrix} b_0 - b_4 \\ b_1 - b_5 \\ b_2 - b_6 \\ b_3 - b_7 \end{bmatrix} \equiv K_n C_n \begin{bmatrix} b_0 - b_4 \\ b_1 - b_5 \\ b_2 - b_6 \\ b_3 - b_7 \end{bmatrix} \underline{\Gamma} \quad \text{for } \underline{\Gamma} = 1.$$

Such a control does, therefore, exist and will be

$$\underline{u}_c = -\underline{\Gamma} \underline{w}_1 = -\underline{w}_1 = -\hat{\underline{z}}_1.$$

Now, a full-dimensional composite state reconstructor in the form of Equation 3 must be designed to provide $\hat{\underline{z}}_1$. Starting with the error dynamics (Equation 4) in order to obtain \underline{K}_1 and \underline{K}_2 we have (since $\underline{L} = \underline{M} = 0$)

$$\begin{pmatrix} \dot{\underline{\epsilon}}_x \\ \dot{\underline{\epsilon}}_z \end{pmatrix} = \left[\begin{array}{c|c} \underline{A} + \underline{K}_1 \underline{C} & (\underline{F} + \underline{K}_1 \underline{G}) \underline{H} \\ \hline \underline{K}_2 \underline{C} & \underline{D} + \underline{K}_2 \underline{GH} \end{array} \right] \begin{pmatrix} \underline{\epsilon}_x \\ \underline{\epsilon}_z \end{pmatrix} + \begin{pmatrix} \underline{Q} \\ \underline{\sigma} \end{pmatrix}.$$

Substituting in the the appropriate matrix values:

$$\underline{\varepsilon} = \left[\begin{array}{c|c} \begin{bmatrix} -b_4 & 1 & 0 & 0 \\ -b_5 & 0 & 1 & 0 \\ -b_6 & 0 & 0 & 1 \\ -b_7 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} k_{11} & 0 & 0 & 0 \\ k_{21} & 0 & 0 & 0 \\ k_{31} & 0 & 0 & 0 \\ k_{41} & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} f_{11} + K_n C_n k_{11} \\ f_{21} + K_n C_n k_{21} \\ f_{31} + K_n C_n k_{31} \\ f_{41} + K_n C_n k_{41} \end{bmatrix} \\ \hline \begin{bmatrix} k_{12} & 0 & 0 & 0 \\ k_{22} & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} K_n C_n k_{12} & 1 \\ K_n C_n k_{22} & 0 \end{bmatrix} \end{array} \right] \underline{\varepsilon} + \begin{bmatrix} 0 \\ a \end{bmatrix}$$

where

$k_{11}, k_{21}, k_{31}, k_{41}$, are the elements of \underline{K}_1 ,

k_{12}, k_{22} , are the elements of \underline{K}_2 and

$f_{11}, f_{21}, f_{31}, f_{41}$ are the elements of \underline{F} .

Performing the matrix addition and multiplication indicated,

$$\underline{\varepsilon} = \left[\begin{array}{c|c} (k_{11}-b_4) & 1 & 0 & 0 & (f_{11} + K_n C_n k_{11}) & 0 \\ (k_{21}-b_5) & 0 & 1 & 0 & (f_{21} + K_n C_n k_{21}) & 0 \\ (k_{31}-b_6) & 0 & 0 & 1 & (f_{31} + K_n C_n k_{31}) & 0 \\ (k_{41}-b_7) & 0 & 0 & 0 & (f_{41} + K_n C_n k_{41}) & 0 \\ k_{12} & 0 & 0 & 0 & K_n C_n k_{12} & 1 \\ k_{22} & 0 & 0 & 0 & K_n C_n k_{22} & 0 \end{array} \right] \underline{\varepsilon} + \begin{bmatrix} 0 \\ a \end{bmatrix} \quad (16)$$

To simplify notation in the following development, let Equation (16) be represented as

$$\dot{\underline{\epsilon}} - \tilde{\underline{A}} \underline{\epsilon} + \begin{bmatrix} \underline{0} \\ \underline{0} \end{bmatrix} \quad \text{and let } \tilde{\underline{A}} \text{ be represented as}$$

$$\tilde{\underline{A}} = \begin{bmatrix} e_0 & 1 & 0 & 0 & e_6 & 0 \\ e_1 & 0 & 1 & 0 & e_7 & 0 \\ e_2 & 0 & 0 & 1 & e_8 & 0 \\ e_3 & 0 & 0 & 0 & e_9 & 0 \\ e_4 & 0 & 0 & 0 & e_{10} & 1 \\ e_5 & 0 & 0 & 0 & e_{11} & 0 \end{bmatrix}$$

Now, to solve for the gain matrices \underline{K}_1 and \underline{K}_2 , one must first find the eigenvalues of $\tilde{\underline{A}}$.

$$\det |\tilde{\underline{A}} - \lambda \underline{I}| = \underline{0}.$$

$$\det |\tilde{\underline{A}} - \lambda \underline{I}| = \begin{vmatrix} e_0 - \lambda & 1 & 0 & 0 & e_6 & 0 \\ e_1 & -\lambda & 1 & 0 & e_7 & 0 \\ e_2 & 0 & -\lambda & 1 & e_8 & 0 \\ e_3 & 0 & 0 & -\lambda & e_9 & 0 \\ e_4 & 0 & 0 & 0 & e_{10} - \lambda & 1 \\ e_5 & 0 & 0 & 0 & e_{11} & -\lambda \end{vmatrix} = \underline{0}$$

Expanding this determinant about the first column results in the expression

$$\begin{aligned} \det |\tilde{\underline{A}} - \lambda \underline{I}| &= \lambda^6 - (e_0 + e_{10}) \lambda^5 + (e_0 e_{10} - e_{11} - e_1 - e_4 e_6) \lambda^4 \\ &+ (e_0 e_{11} + e_1 e_{10} - e_2 - e_4 e_7 - e_5 e_6) \lambda^3 \\ &+ (e_1 e_{11} + e_2 e_{10} - e_3 - e_4 e_8 - e_5 e_7) \lambda^2 \\ &+ (e_2 e_{11} + e_3 e_{10} - e_4 e_9 - e_5 e_8) \lambda \\ &+ (e_3 e_{11} - e_5 e_9) \end{aligned} \quad (17)$$

If the desired roots of Equation (17) are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$, then the desired characteristic equation is

$$(\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3)(\lambda - \lambda_4)(\lambda - \lambda_5)(\lambda - \lambda_6) = 0 \quad (18)$$

Expanding Equation (18) and equating coefficients of like powers of λ between Equations (17) and (18) we see that

$$(a) \quad e_0 + e_{10} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 = A_0$$

$$(b) \quad e_0 e_{10} - e_{11} - e_1 - e_4 e_6 = \sum_{i=1}^5 \sum_{j=i+1}^6 \lambda_i \lambda_j = A_1$$

$$(c) \quad e_0 e_{11} + e_1 e_{10} - e_2 - e_4 e_7 - e_5 e_6 =$$

$$- \left[\sum_{i=1}^4 \sum_{j=i+1}^5 \sum_{k=j+1}^6 \lambda_i \lambda_j \lambda_k \right] = -A_2$$

$$(d) \quad e_1 e_{11} + e_2 e_{10} - e_3 - e_4 e_8 - e_5 e_7 =$$

$$\sum_{i=1}^3 \sum_{j=i+1}^4 \sum_{k=j+1}^5 \sum_{l=k+1}^6 \lambda_i \lambda_j \lambda_k \lambda_l = A_3$$

$$(e) \quad e_2 e_{11} + e_3 e_{10} - e_4 e_9 - e_5 e_8 =$$

$$- [\lambda_1 \lambda_2 \lambda_3 \lambda_4 (\lambda_5 + \lambda_6) + \lambda_1 \lambda_2 \lambda_5 \lambda_6 (\lambda_3 + \lambda_4)$$

$$+ \lambda_3 \lambda_4 \lambda_5 \lambda_6 (\lambda_1 + \lambda_2)] = -A_4$$

$$(f) \quad e_3 e_{11} - e_5 e_9 = \lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 \lambda_6 = A_5$$

Substituting the relations for e_0 through e_4 from Equation (16) into (a) through (f) and solving for the elements of \underline{K}_1 and \underline{K}_2 , we obtain

$$\begin{aligned}
 k_{11} &= -K_n C_n k_{12} + b_4 + A_0 \\
 k_{21} &= -K_n C_n (b_0 k_{12} + k_{22}) + b_5 - A_1 \\
 k_{31} &= -K_n C_n (b_0 k_{22} + b_1 k_{12}) + b_6 + A_2 \\
 k_{41} &= -K_n C_n (b_2 k_{12} + b_1 k_{22}) + b_7 - A_3 \\
 k_{12} &= (-b_2 K_n C_n k_{22} + A_4) / K_n C_n b_3 \\
 k_{22} &= -A_5 / K_n C_n b_3
 \end{aligned} \tag{19}$$

It is desirable that $e(t) \rightarrow 0$ rapidly, thus the characteristic roots $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6$ can be picked to best accomplish this depending on the problem at hand. Having picked the λ 's, the gain values (Equation 19) can then be calculated. The full-dimensional observer can now be implemented, giving

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \\ \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} (k_{11}-b_4) & 1 & 0 & 0 & K_n C_n (k_{11}+b_0-b_4) & 0 \\ (k_{21}-b_5) & 0 & 1 & 0 & K_n C_n (k_{21}+b_1-b_5) & 0 \\ (k_{31}-b_6) & 0 & 0 & 1 & K_n C_n (k_{31}+b_2-b_6) & 0 \\ (k_{41}-b_7) & 0 & 0 & 0 & K_n C_n (k_{41}+b_3-b_7) & 0 \\ k_{12} & 0 & 0 & 0 & K_n C_n k_{12} & 1 \\ k_{22} & 0 & 0 & 0 & K_n C_n k_{22} & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ z_1 \\ z_2 \end{bmatrix}$$

$$\begin{bmatrix} k_{11} \\ k_{21} \\ k_{31} \\ k_{41} \\ k_{12} \\ k_{22} \end{bmatrix} \underline{y} + \begin{bmatrix} K_n C_n (k_{11}+b_0-b_4) \\ K_n C_n (k_{21}+b_1-b_5) \\ K_n C_n (k_{31}+b_2-b_6) \\ K_n C_n (k_{41}+b_3-b_7) \\ K_n C_n k_{12} \\ K_n C_n k_{22} \end{bmatrix} \underline{u} \tag{20}$$

Figure 4 is a diagram of the composite plant-DAC system. On the diagram, $h_1 - h_5$ are the components of the last matrix on the right-hand side of (Equation 20). The remainder of the symbols have been previously defined.

So now we have a disturbance cancelling control term, $u_c = -\hat{z}_1$, and we have a composite state reconstructor which gives \hat{z}_1 . The questions now are:

- Can the λ 's be picked so that ϵ_x and ϵ_r settle to zero "rapidly"?
- If so, does u_c really cancel out the effects of w_1 ? If both of these can be answered in the affirmative, then

—How well does the DAC work if the plant parameters are varied from the design point?

—How do the DAC characteristics vary over the trajectory?

Simulation results should provide answers to these questions.

B. SIMULATION AND RESULTS

The composite system shown in *Figure 4* was simulated on a digital computer. The simulation was written so that the plant parameters could be arbitrarily varied around the point for which the DAC was designed. A listing of this simulation is given in Appendix A.

As a first cut at seeing how effective the DAC would be, several runs were made for the $t = 9.85$ sec and $t = 18$ sec points. *Figures 5* and *6* give the results. As can be seen, the DAC effectively cancels out an input disturbance ($w_1 = 1.0$) equal to the input command.

To check the sensitivity of the DAC to plant parameter variations, a series of runs were made with parameters varied around the $t = 18$ sec values. *Table 2* is a summary of the results obtained and *Figures 7* through *40* give the system output, y , and reconstructor state, \hat{z}_1 (disturbance estimate), for each case. The table shows how the peak value of y varied and how the peak value and settling time of \hat{z}_1 varied due to both individual and collective parameter changes. In all cases, the input command is 1 and the settling time is defined to be the time at which the response stays within $\pm 5\%$ of steady-state.

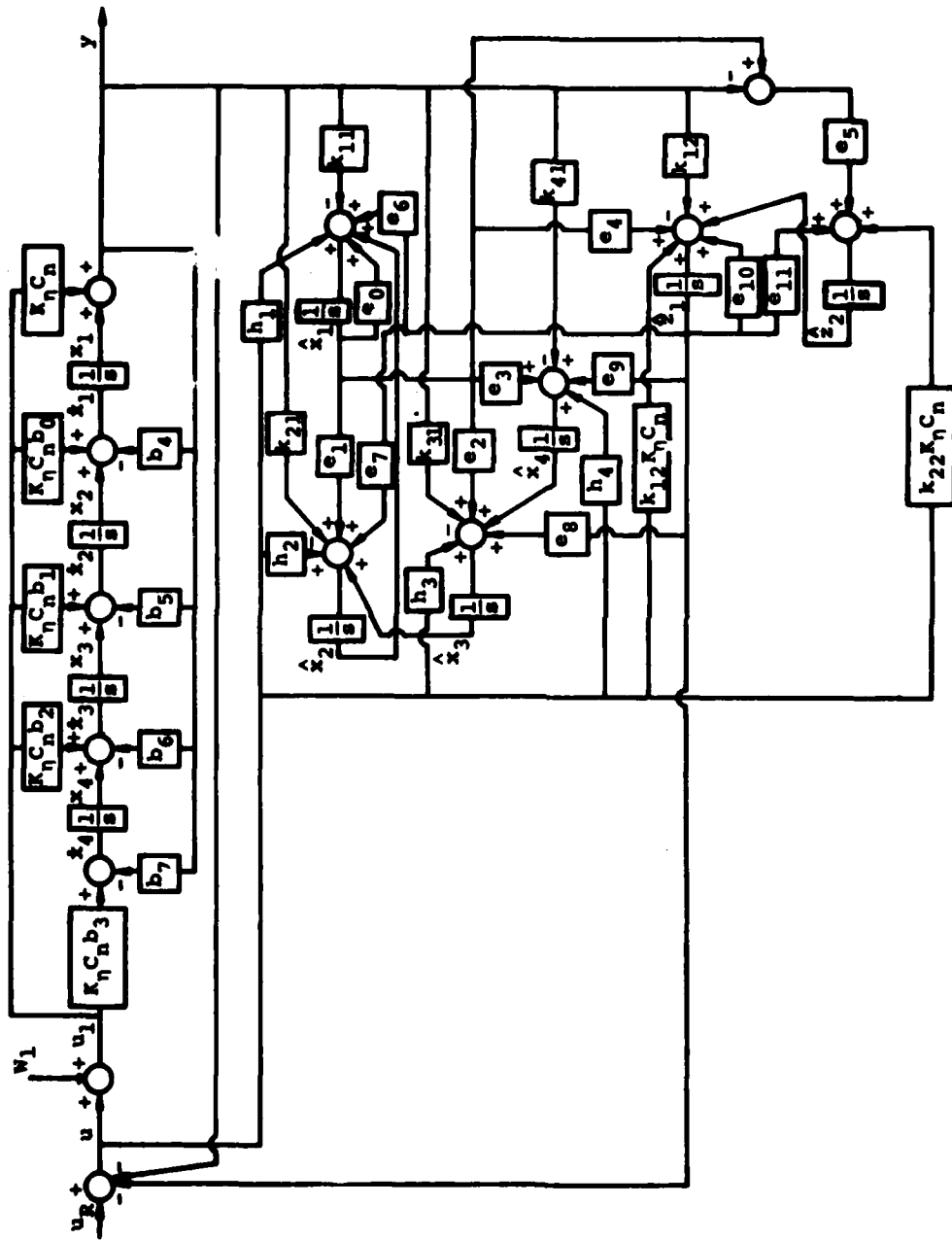


Figure 4. Plant-DAC composite diagram for acceleration loop with disturbance at input.

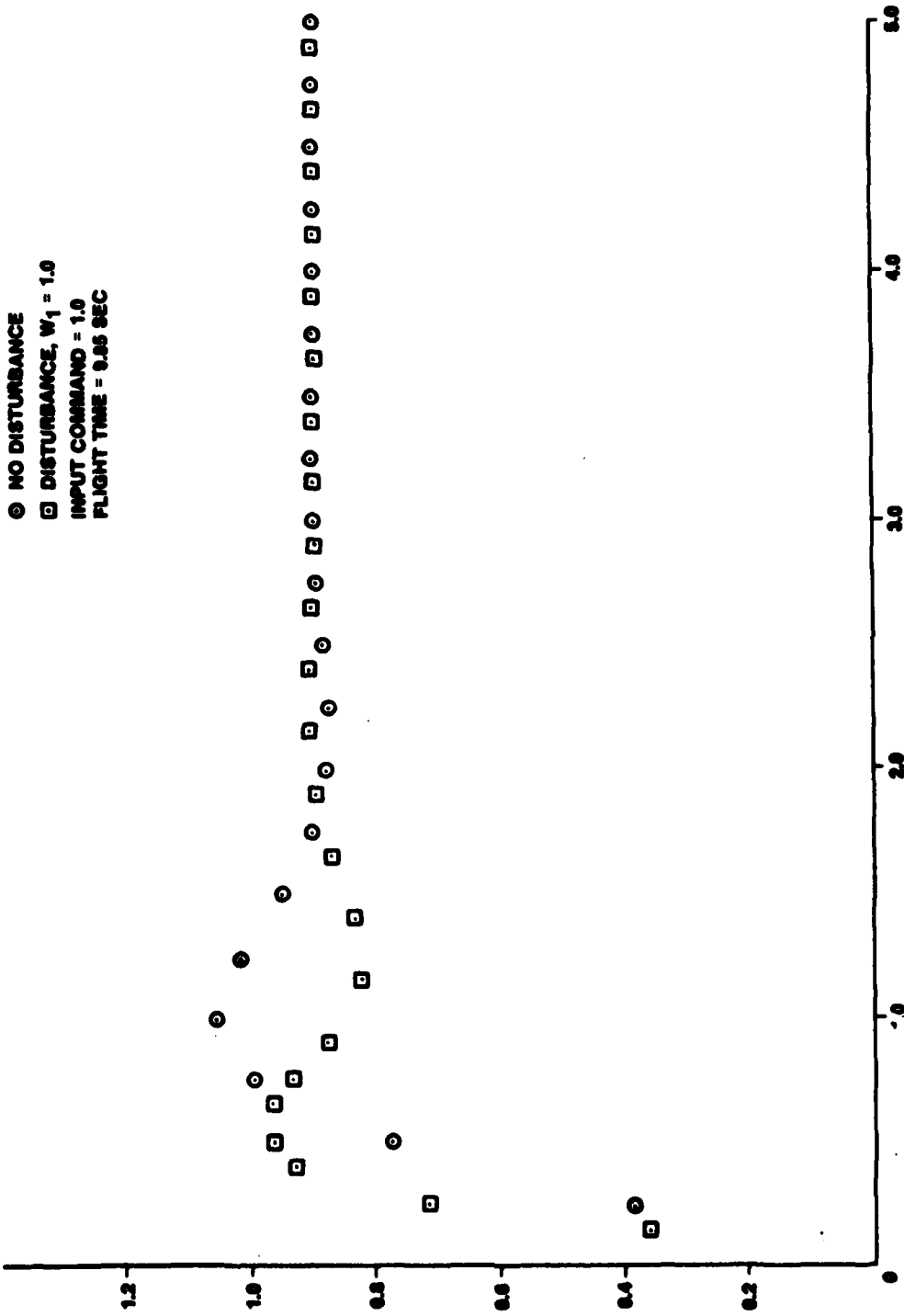


Figure 5. System outputs for $t = 9.85$ sec case, with and without disturbance.

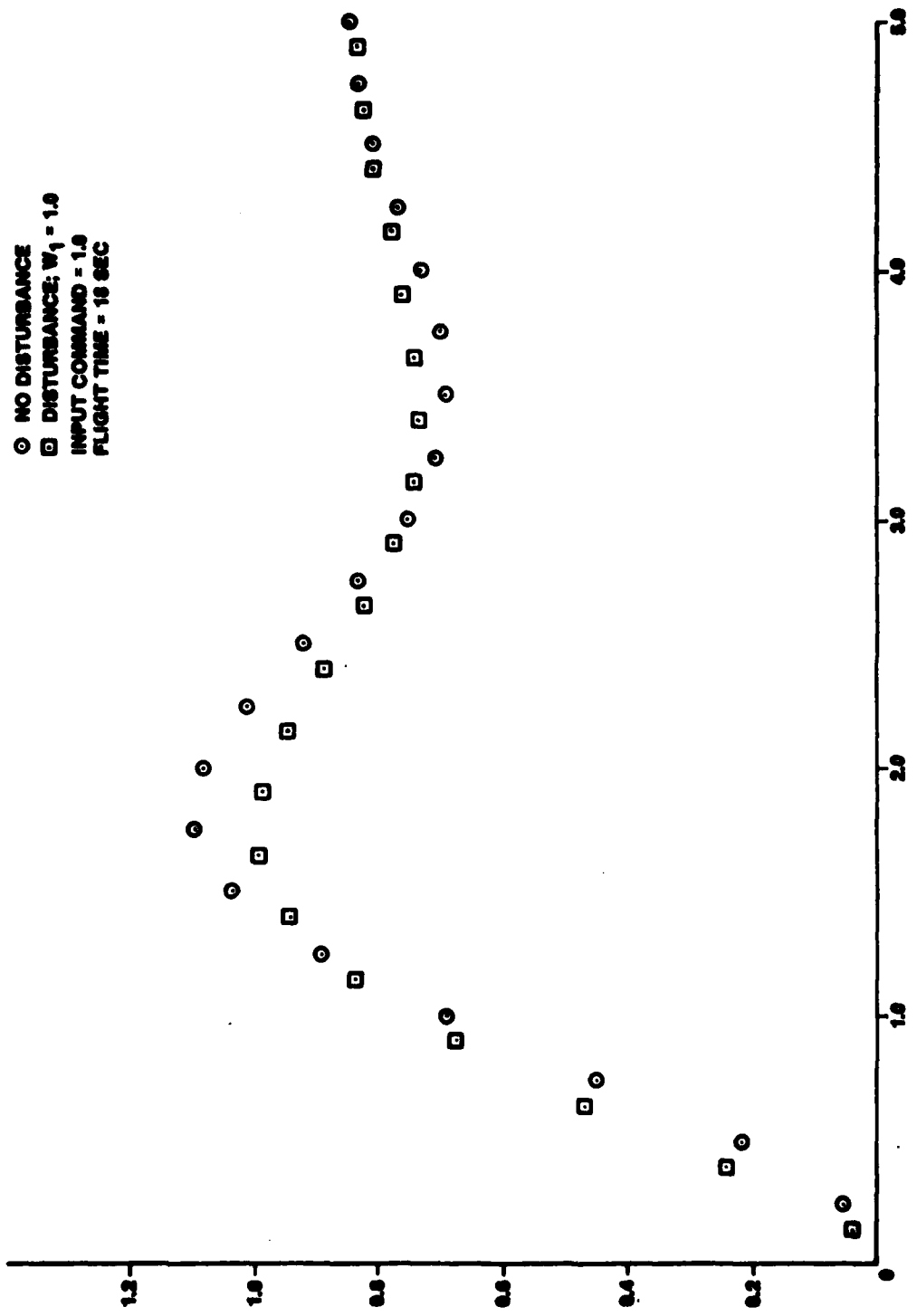


Figure 6. System outputs for $t = 18$ sec case, with and without disturbance.

TABLE 2. RESPONSE SENSITIVITY TO PLANT PARAMETER VARIATIONS

CONDITION	OUTPUT (y)		DISTURBANCE RECONSTRUCTOR (\hat{z}_1)		
	PEAK AMPL	TIME (SEC)	PEAK AMPL	TIME (SEC)	SETTLING TIME
NO DISTURBANCE (NOM)	1.097	1.82	—	—	—
WITH $W_1 = 1$:					
· NOMINAL	0.997	1.76	1.35	0.42	1.09
· + 10% K_f	0.999	1.725	1.29	0.50	1.32
· - 10% K_f	0.994	1.78	1.47	0.35	0.98
· + 10% γ	0.996	1.75	1.36	0.42	1.10
· - 10% γ	0.996	1.75	1.36	0.42	1.10
· + 10% ζ_A	0.99	1.80	1.36	0.45	1.15
· - 10% ζ_A	0.99	1.75	1.36	0.45	1.16
· + 10% ω_A	1.02	1.75	1.2	0.42	5.0
· - 10% ω_A	0.98	1.75	1.54	0.50	4.5
· + 10% $\omega_{\eta 2}$	0.987	1.78	1.49	0.40	1.18
· - 10% $\omega_{\eta 2}$	1.01	1.72	1.21	0.48	1.00
· + 10% C.R.	1.00	1.75	1.3	0.50	1.4
· - 10% C.R.	0.99	1.80	1.47	0.38	1.05
· + 10% K_{η}	0.98	1.80	1.50	0.40	1.20
· - 10% K_{η}	1.01	1.70	1.21	0.50	1.05
· + 10% ON ALL	0.99	1.75	1.32	0.45	1.12
· - 10% ON ALL	0.98	1.75	1.45	0.45	1.15
· + 20% ON ALL	0.99	1.8	1.27	0.47	1.1
· - 20% ON ALL	0.98	1.75	1.58	0.45	1.2

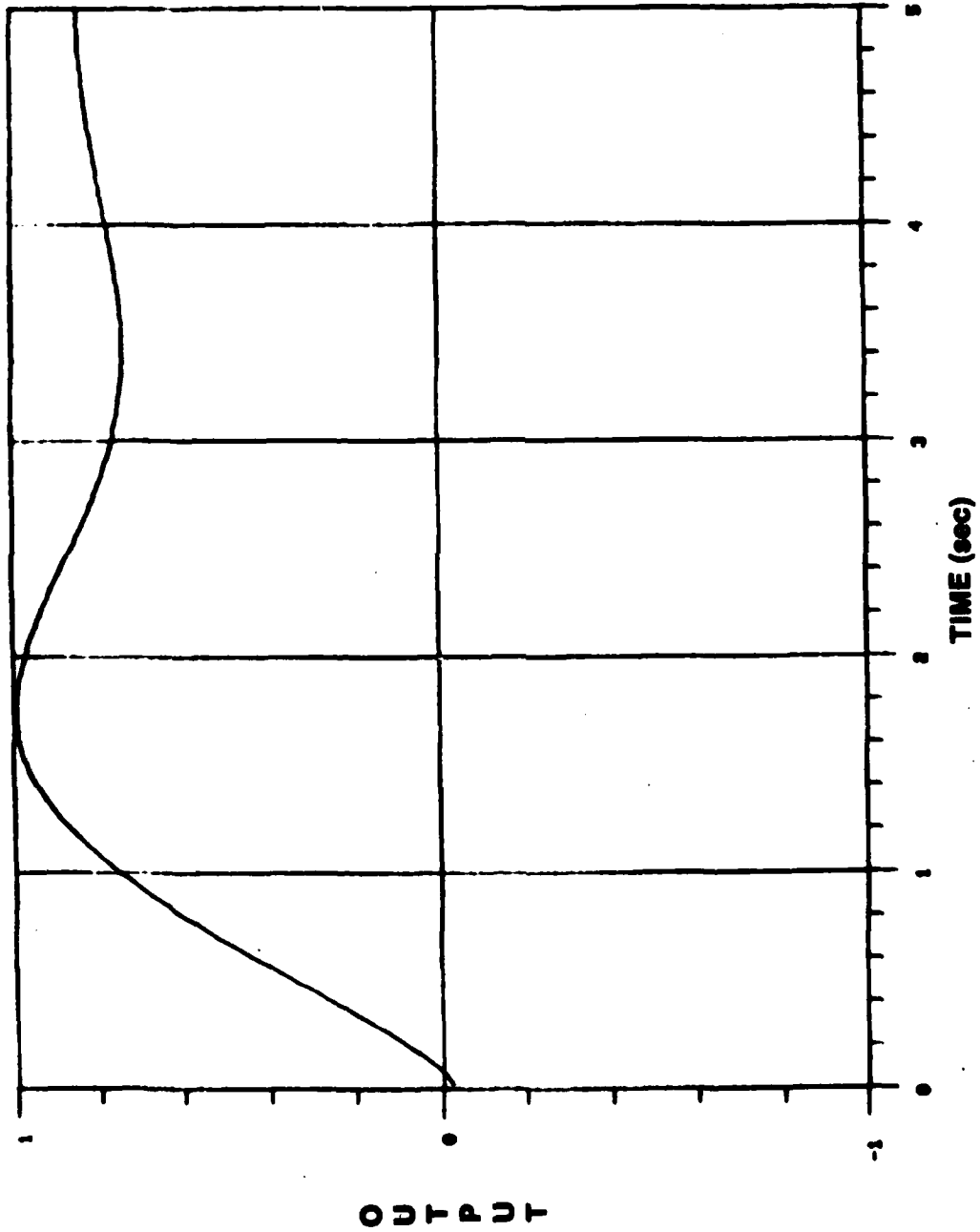


Figure 7. System output response (Y), $W_1 = 1$, nominal parameters.

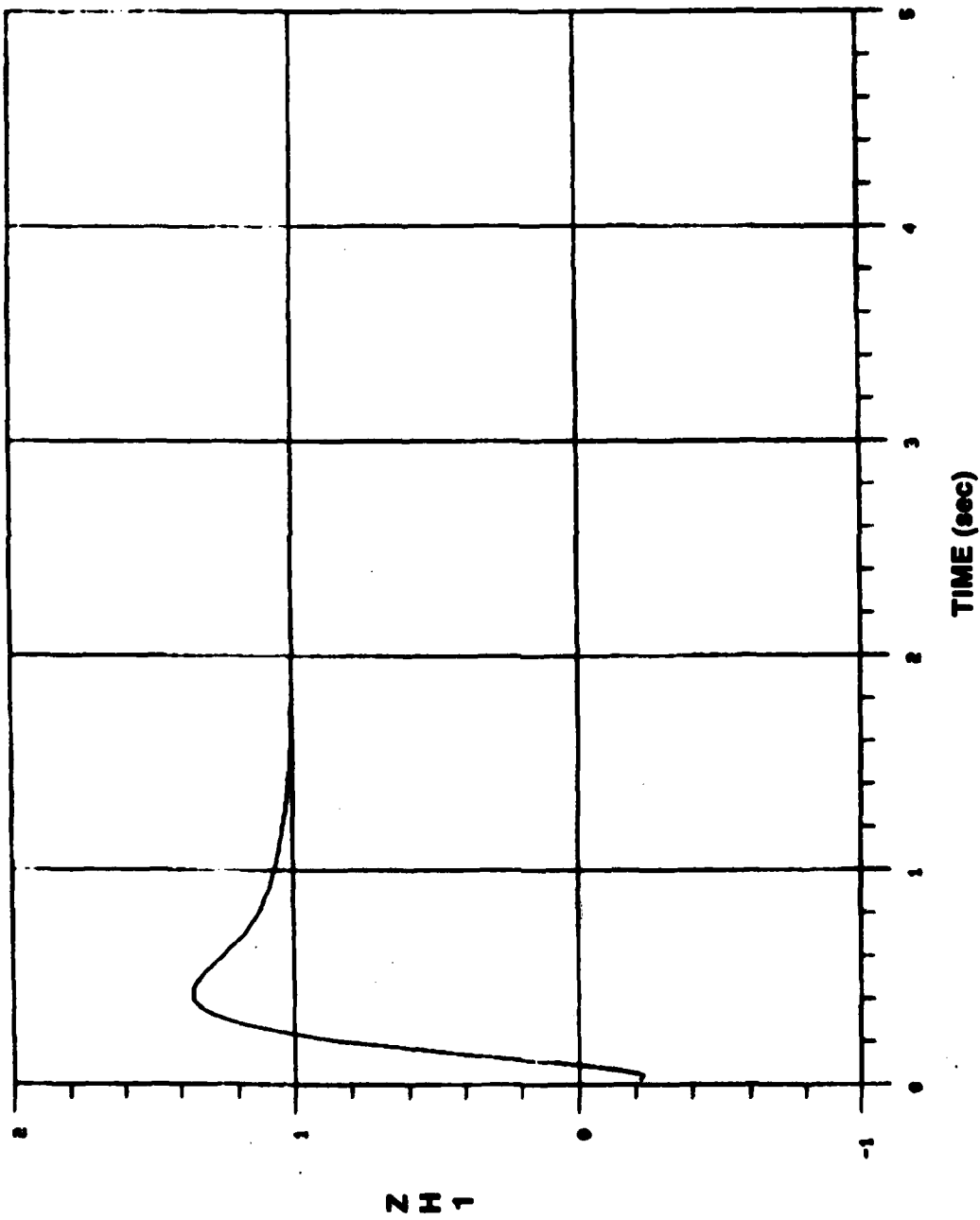


Figure 8. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, nominal parameters.

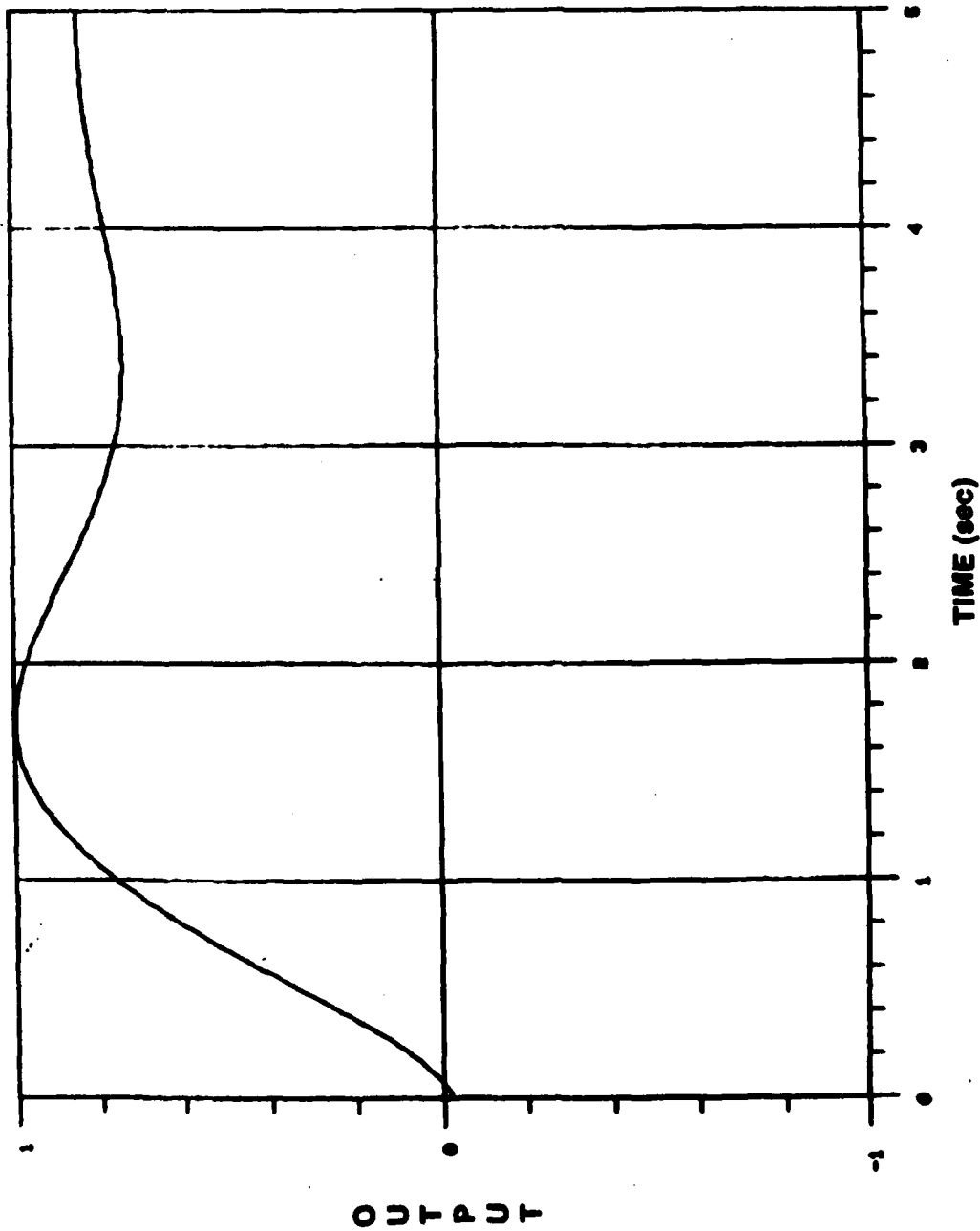


Figure 9. System output response (y), $W_1 = 1$, +10% variation on K_p .

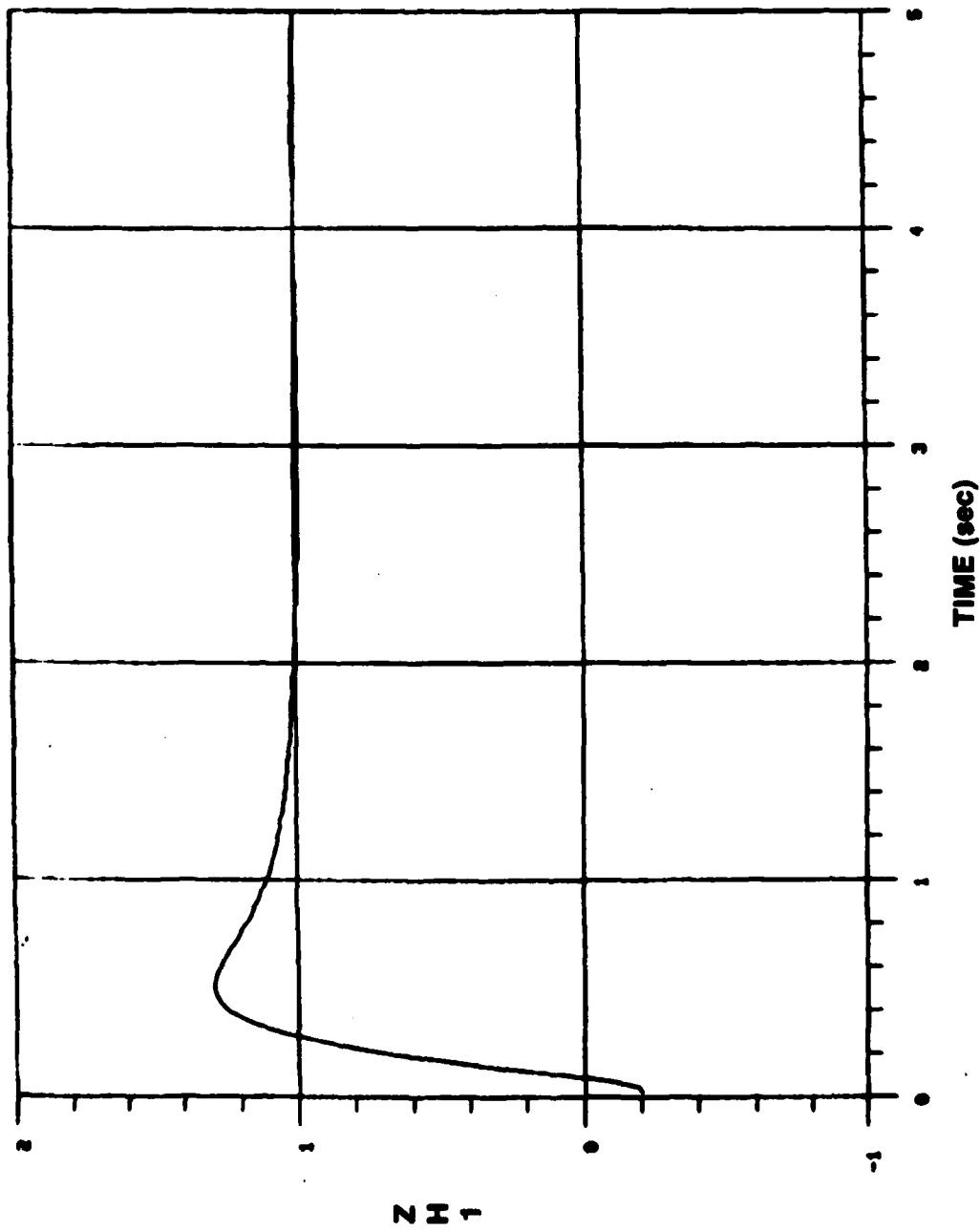


Figure 10. DAC disturbance estimate (Z_1), $W_1 = 1$, +10% variation on $K\phi$.

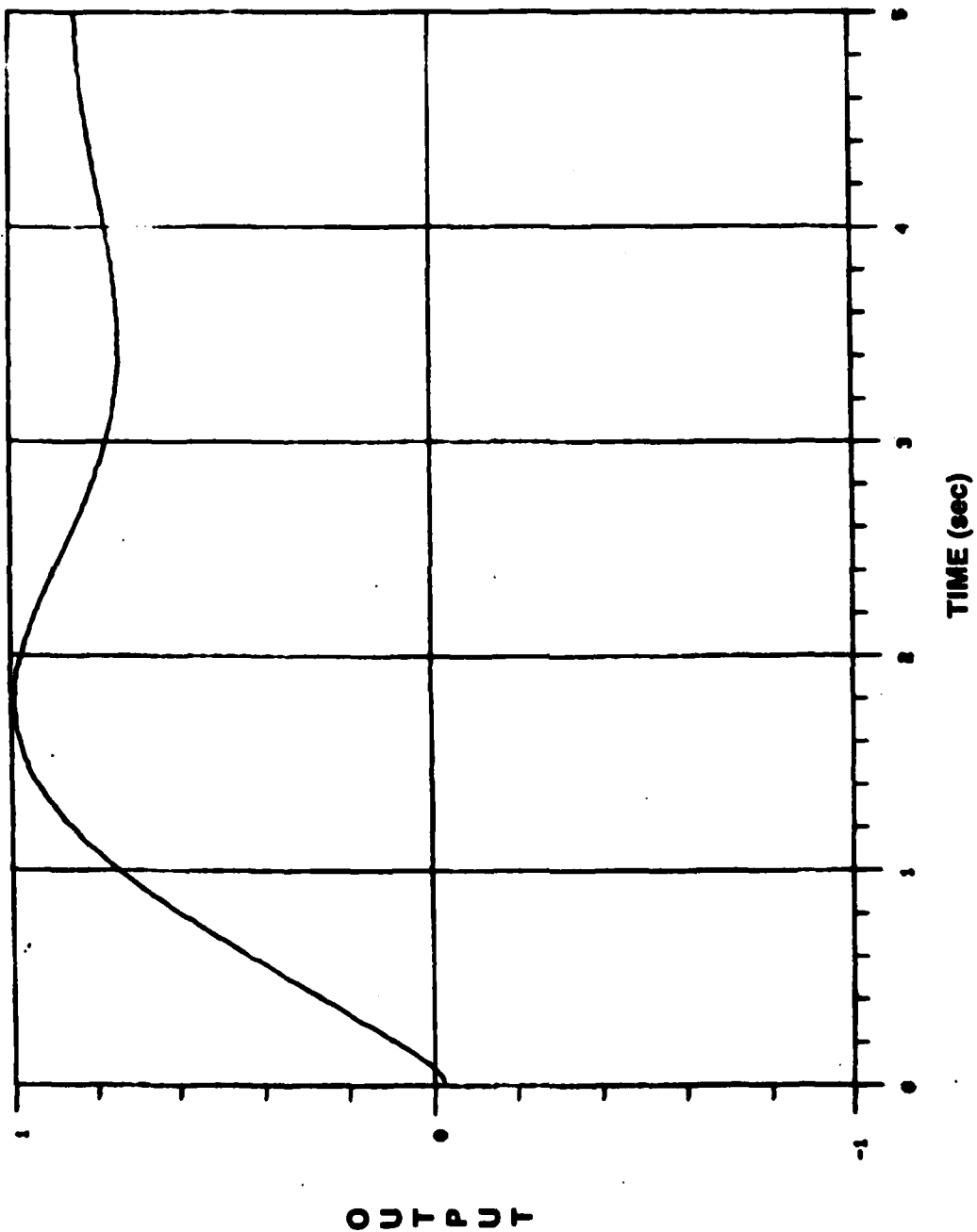


Figure 11. System output response (y), $W_1 = 1$, -10% variation on $K\hat{\phi}$.

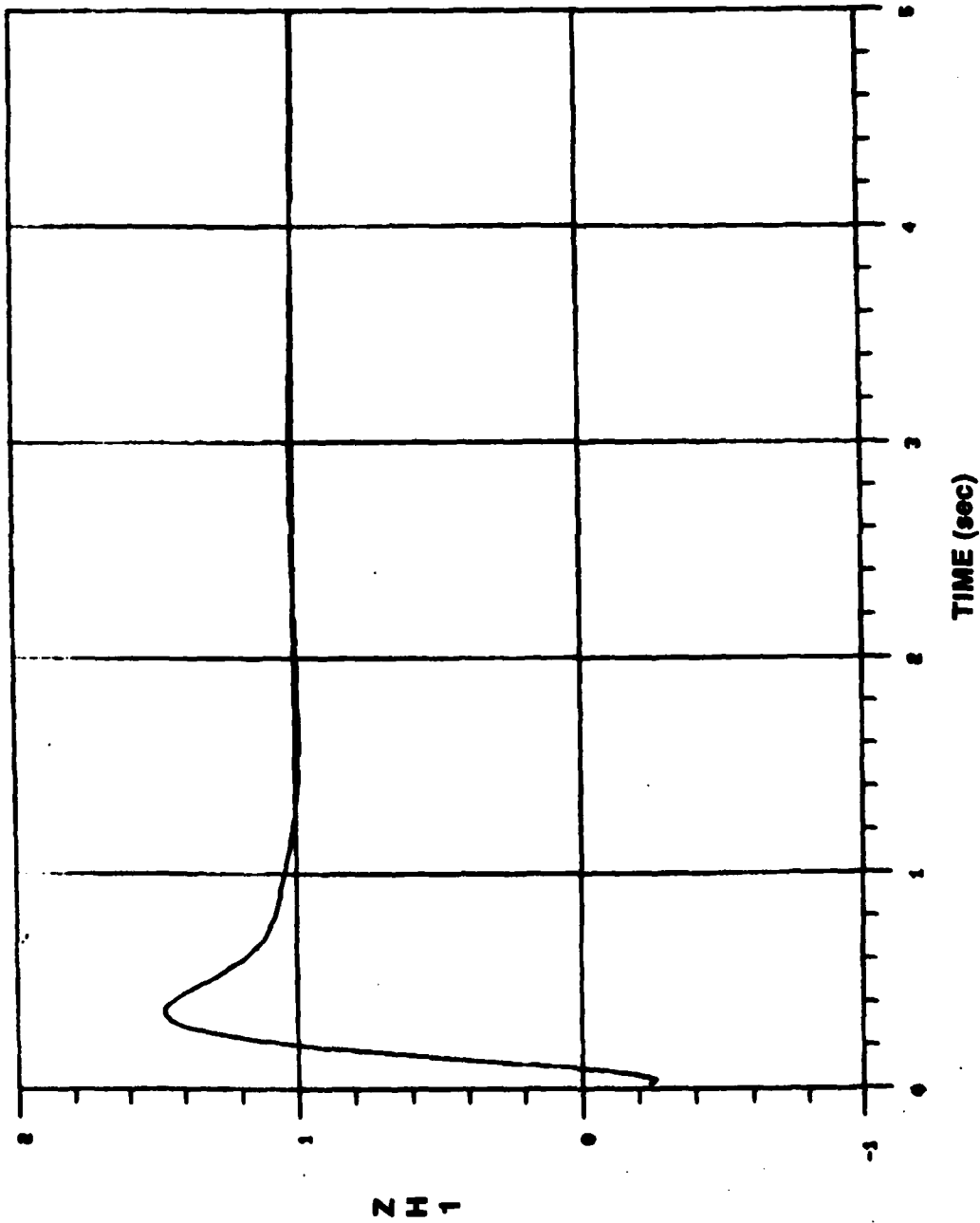


Figure 12. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, -10% variation on $K\phi$.

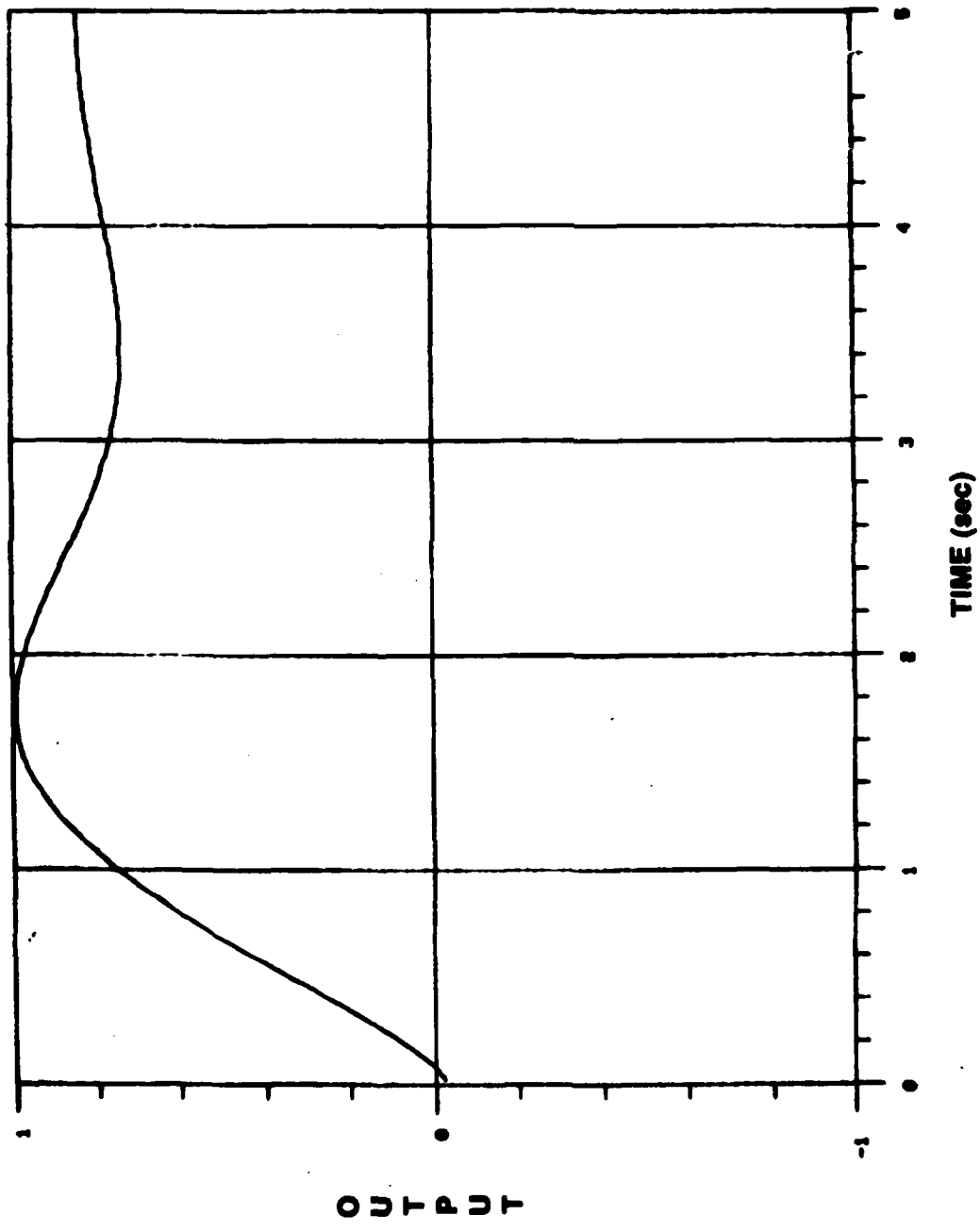


Figure 13. System output response (y), $W_1 = 1$, +10% variation on γ .

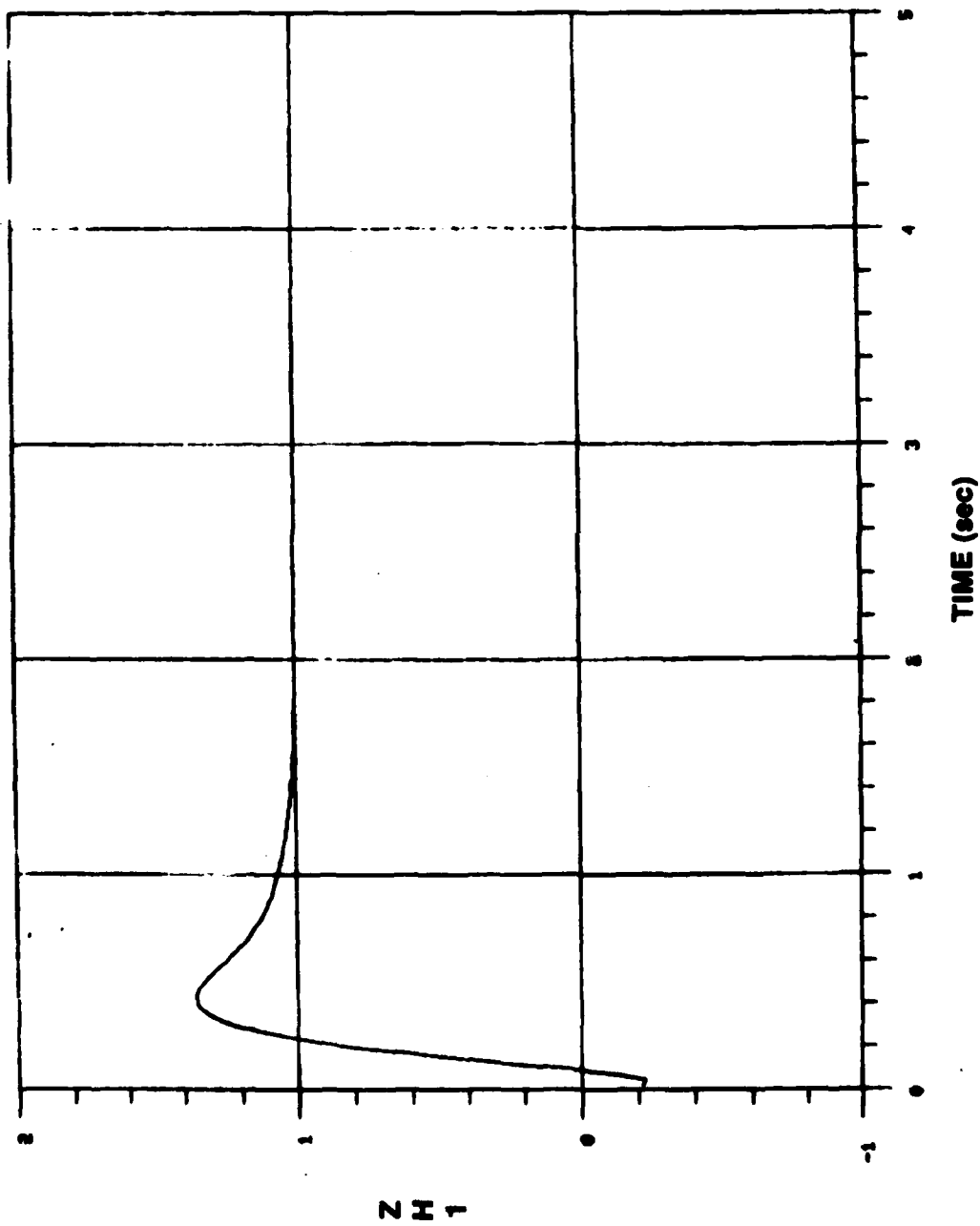


Figure 14. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, +10% variation on γ .

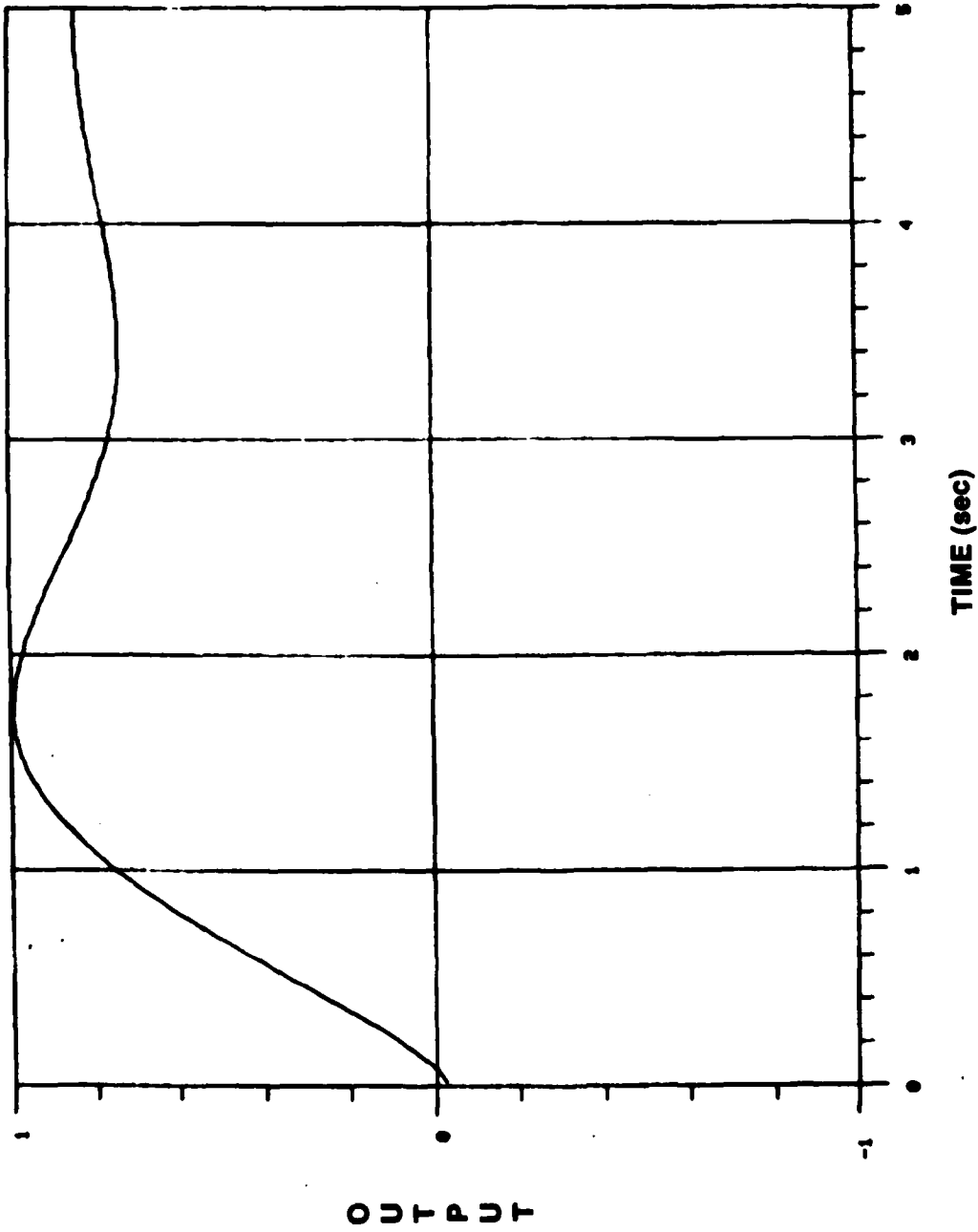


Figure 15. System output response (y), $W_1 = 1$, -10% variation on γ .

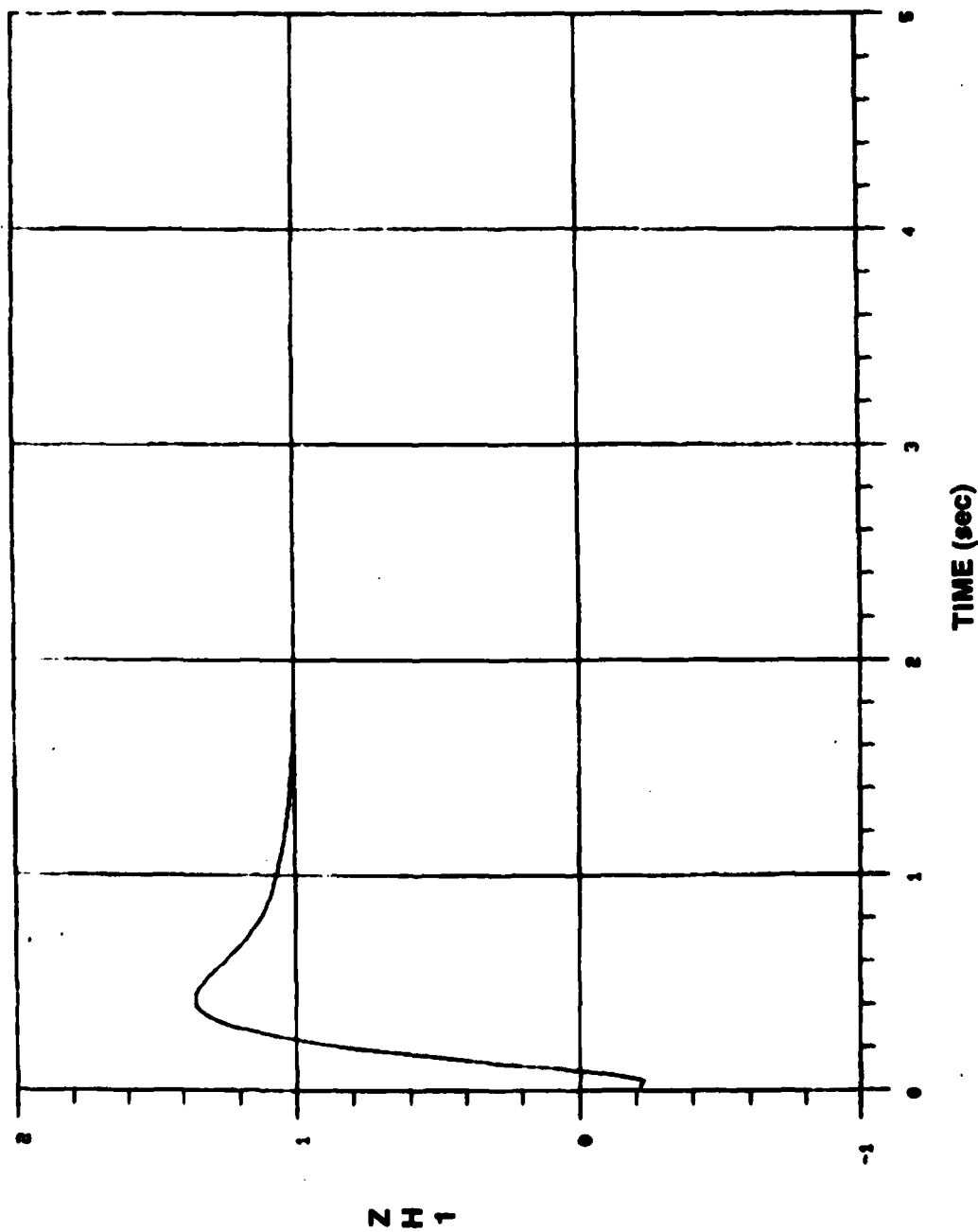


Figure 16. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, -10% variation on γ .

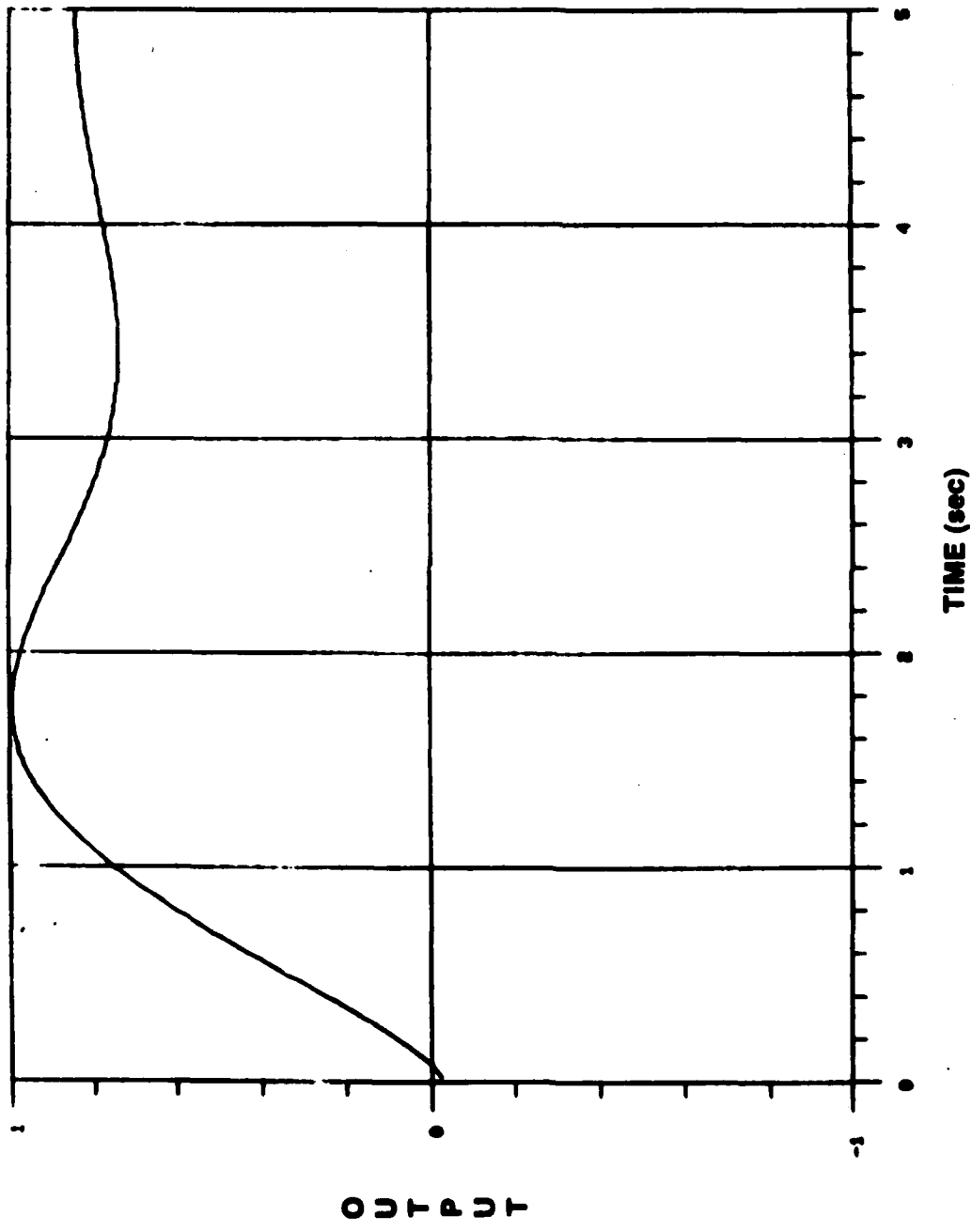


Figure 17. System output response (y), $W_1 = 1$, +10% variation on ζ_A .

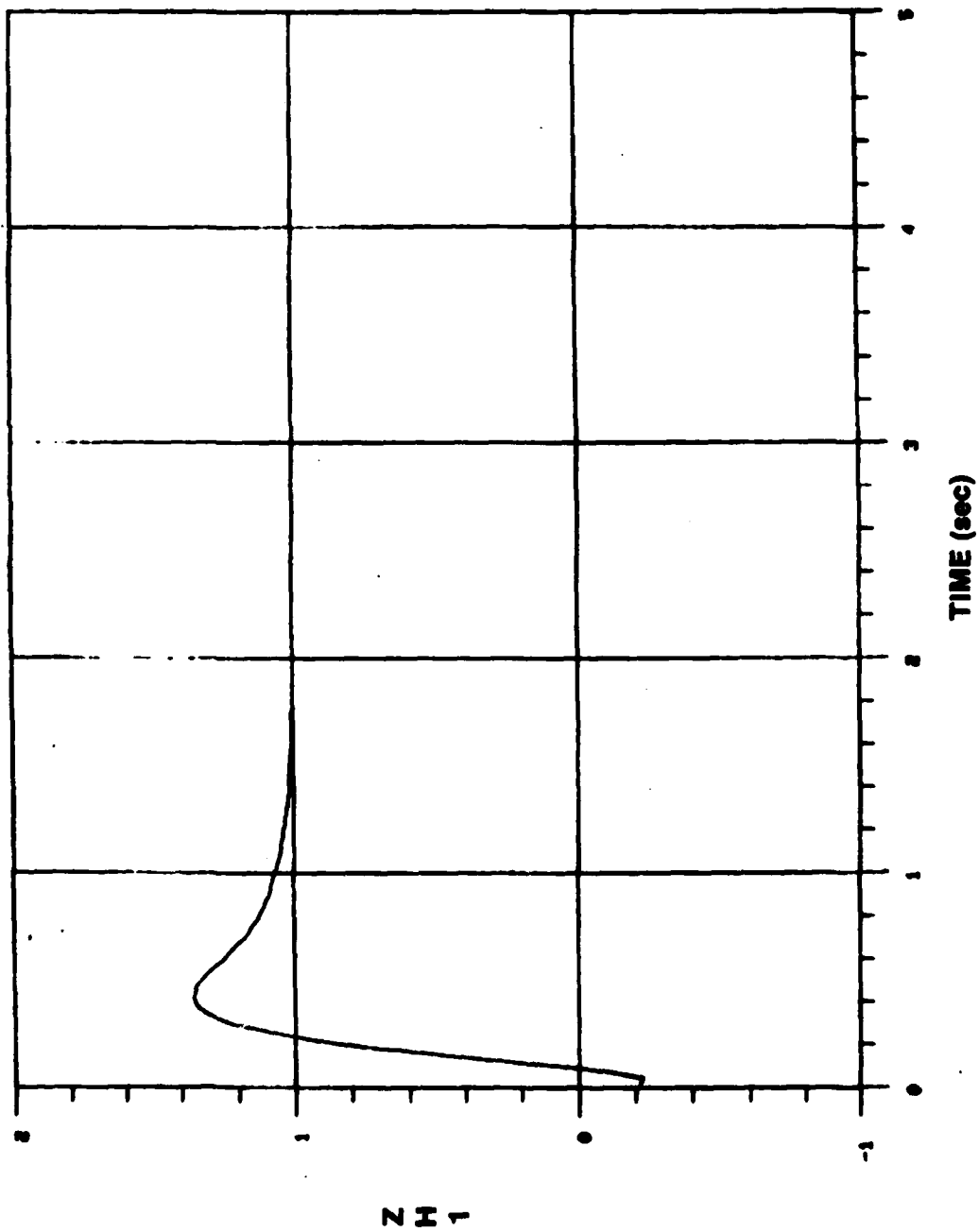


Figure 18. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, +10% variation on ζ_A .

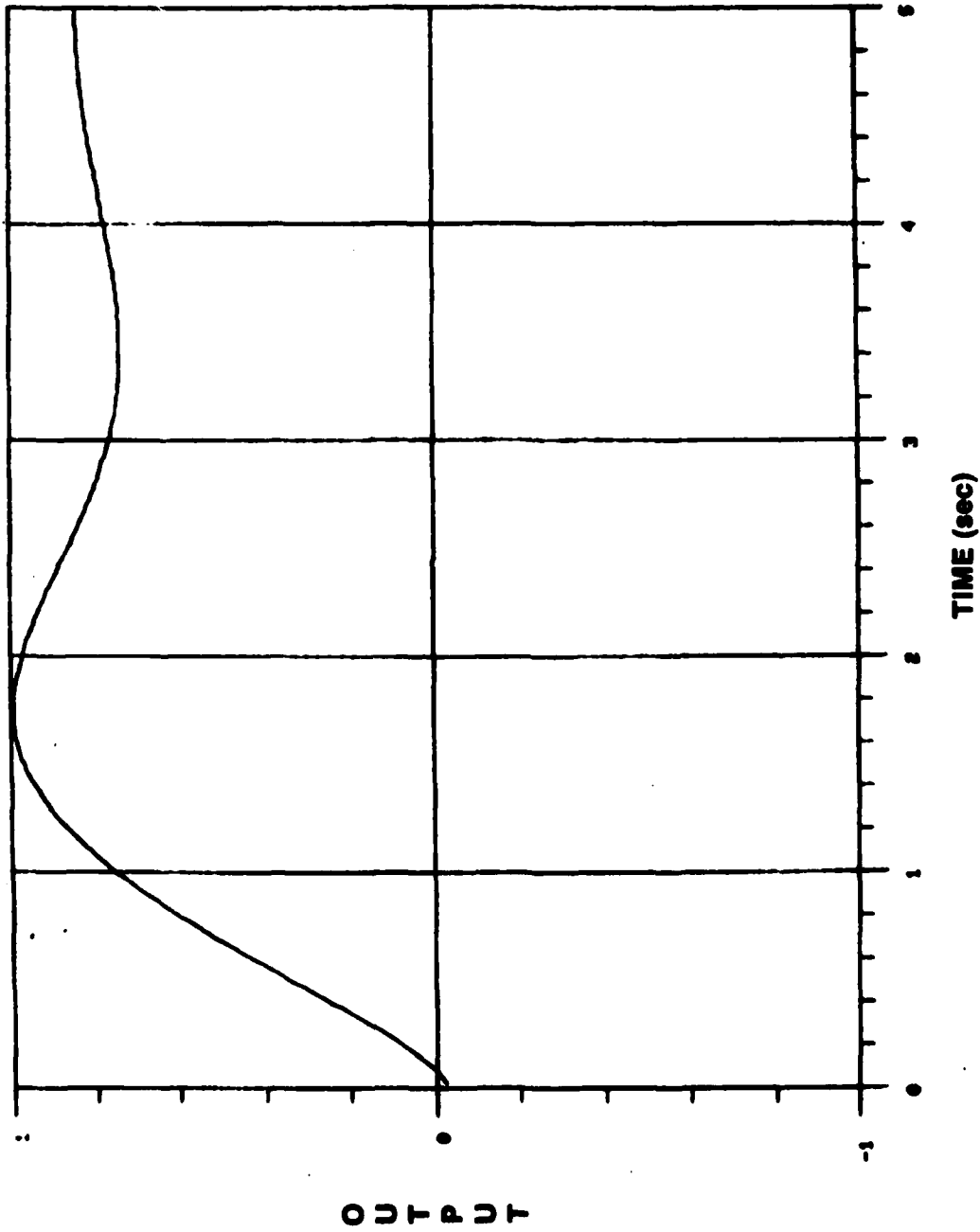


Figure 19. System output response (y), $W_1 = 1$, -10% variation on ζ_A .

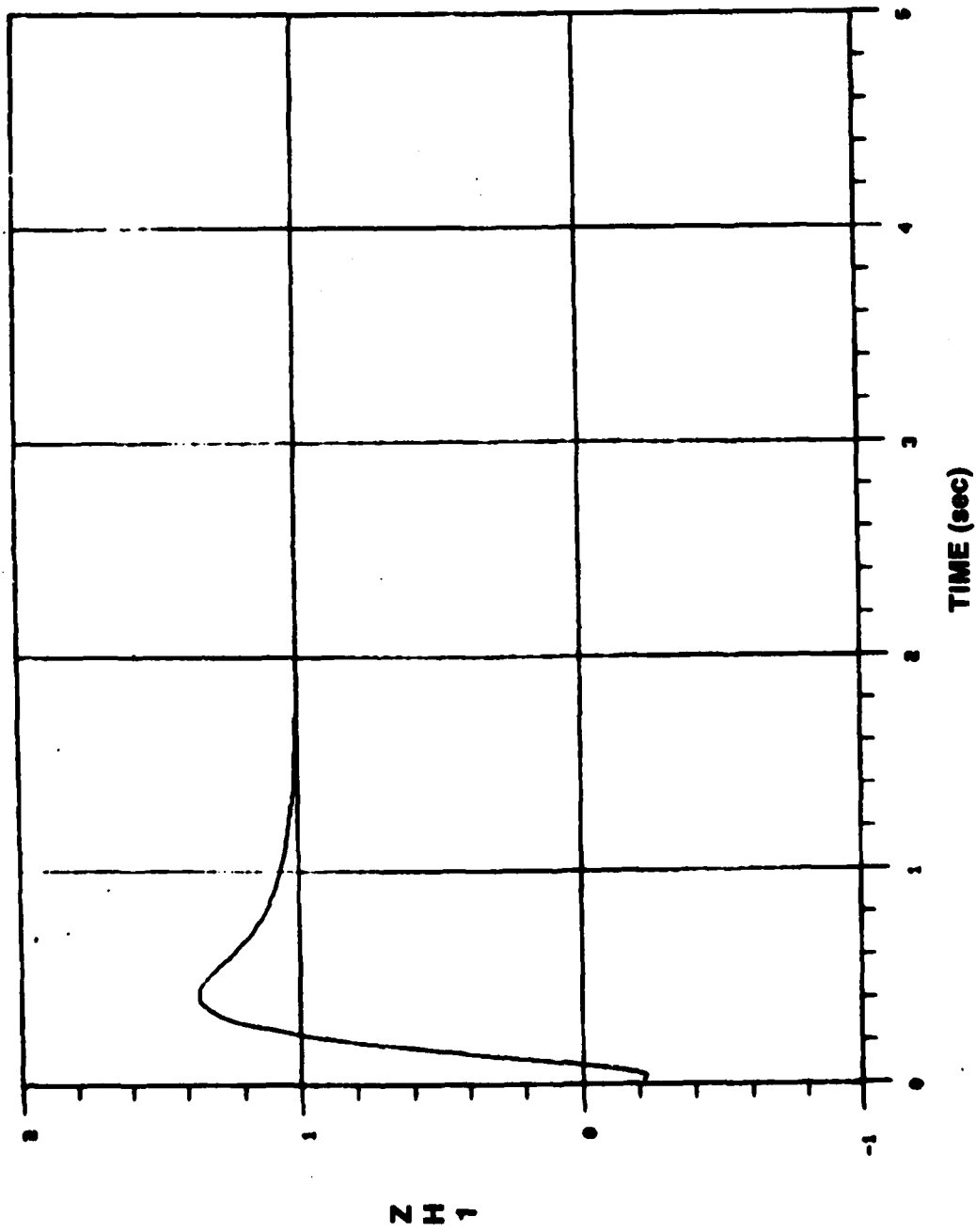


Figure 20. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, -10% variation on ζ_A .

Z H 1

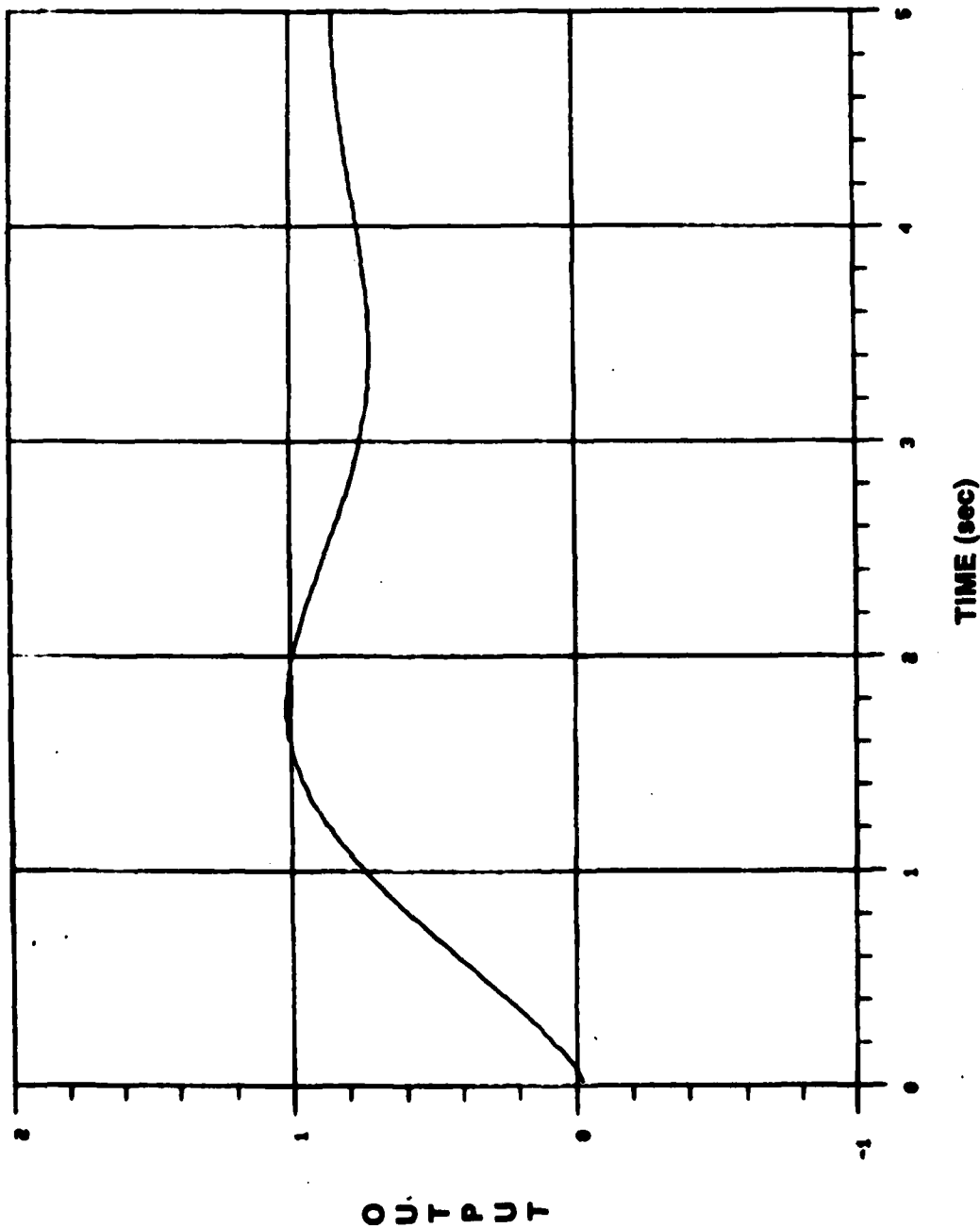


Figure 21. System output response (y), $W_1 = 1$, +10% variation on ω_A .

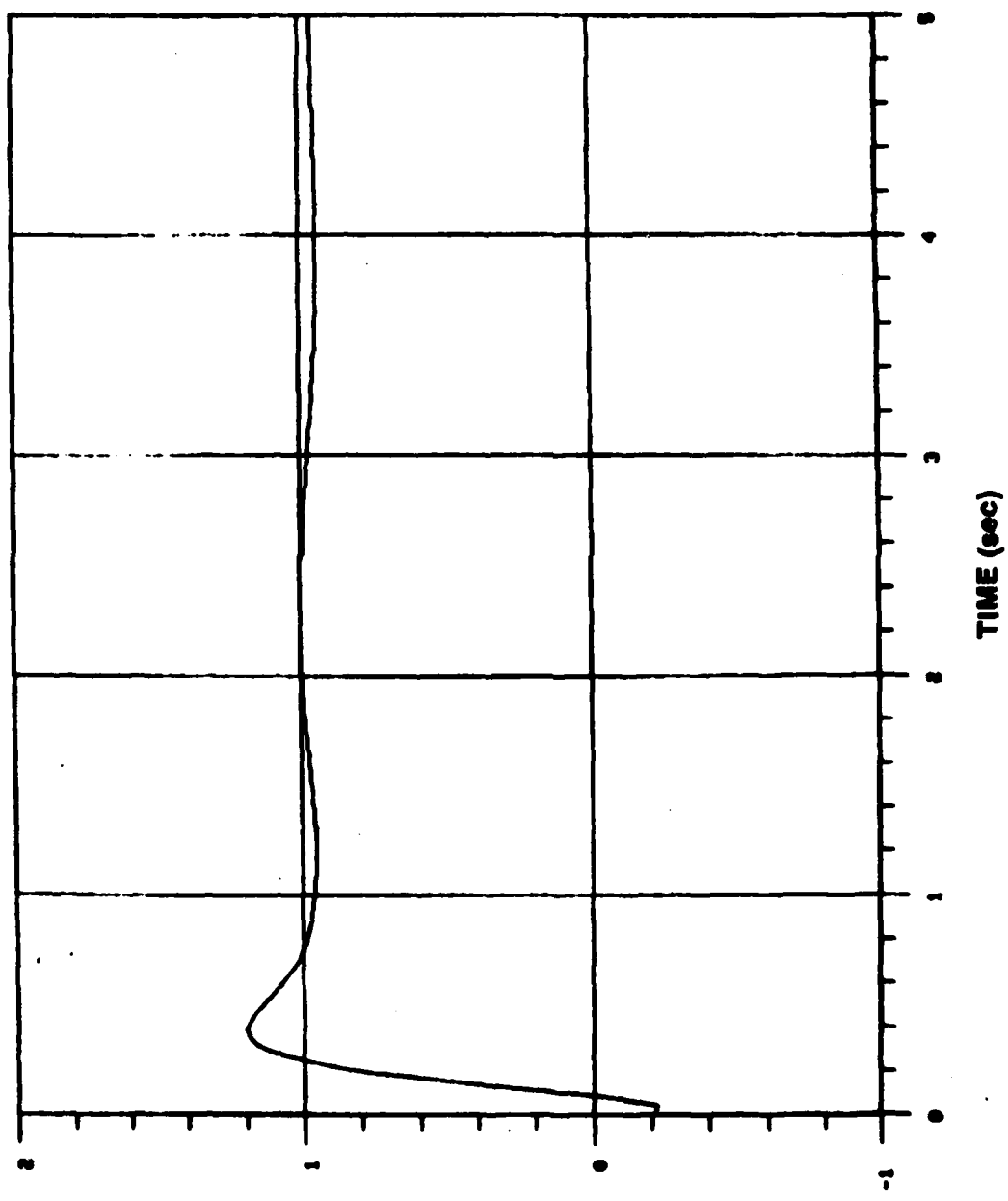


Figure 22. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, +10% variation on ω_A .

Z H 1

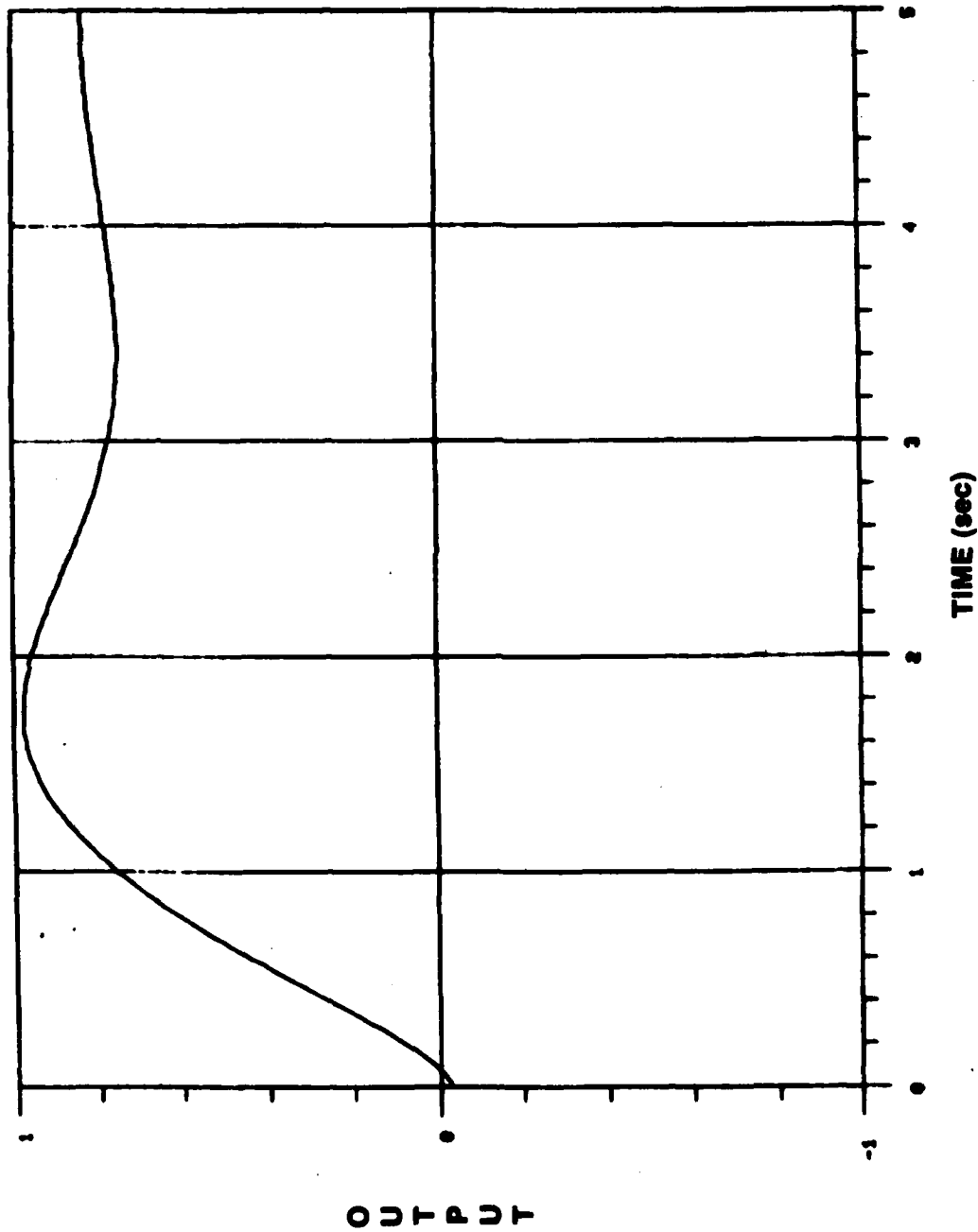


Figure 23. System output response (y), $W_1 = 1$, -10% variation on ω_A .

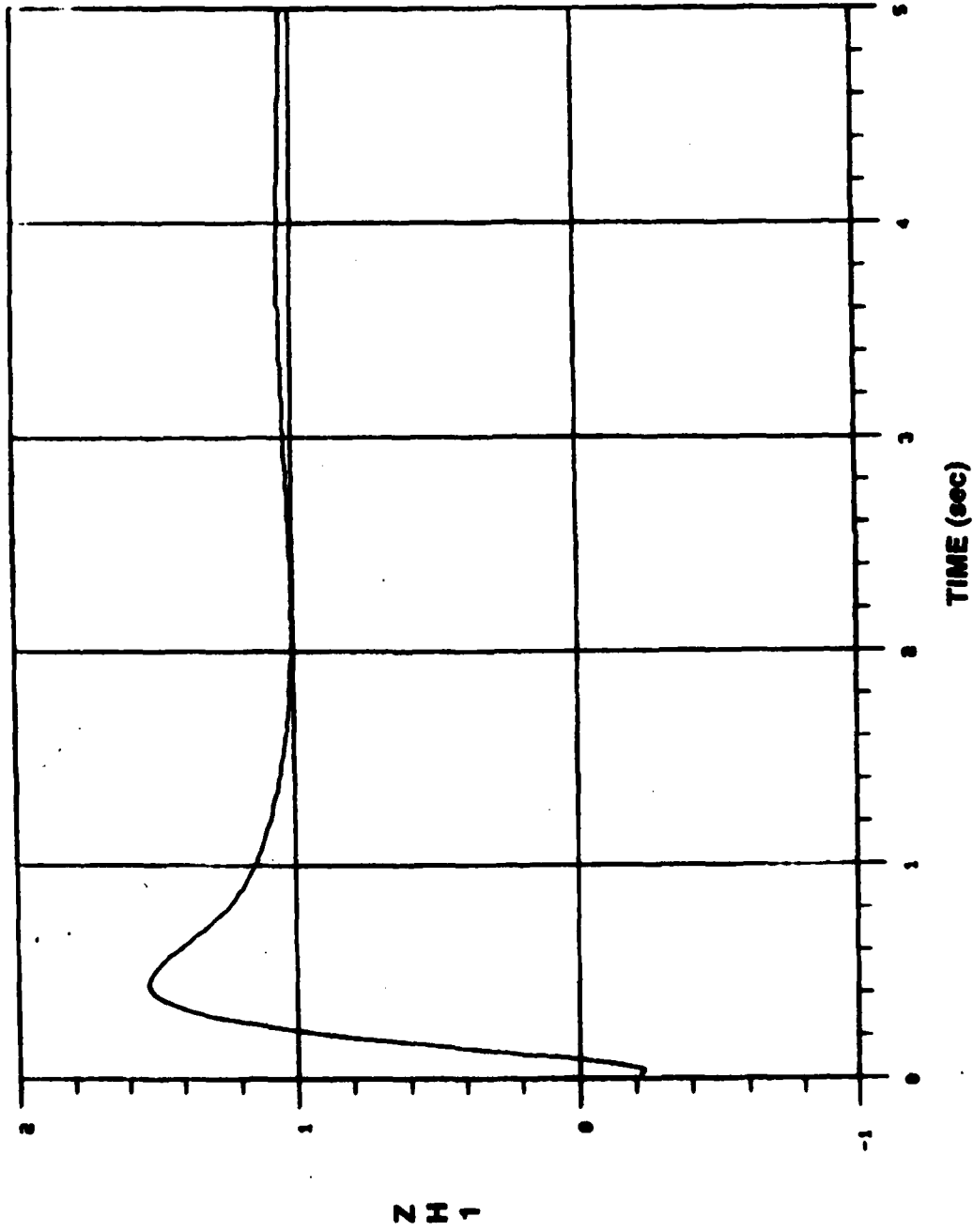


Figure 24. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, -10% variation on ω_A .

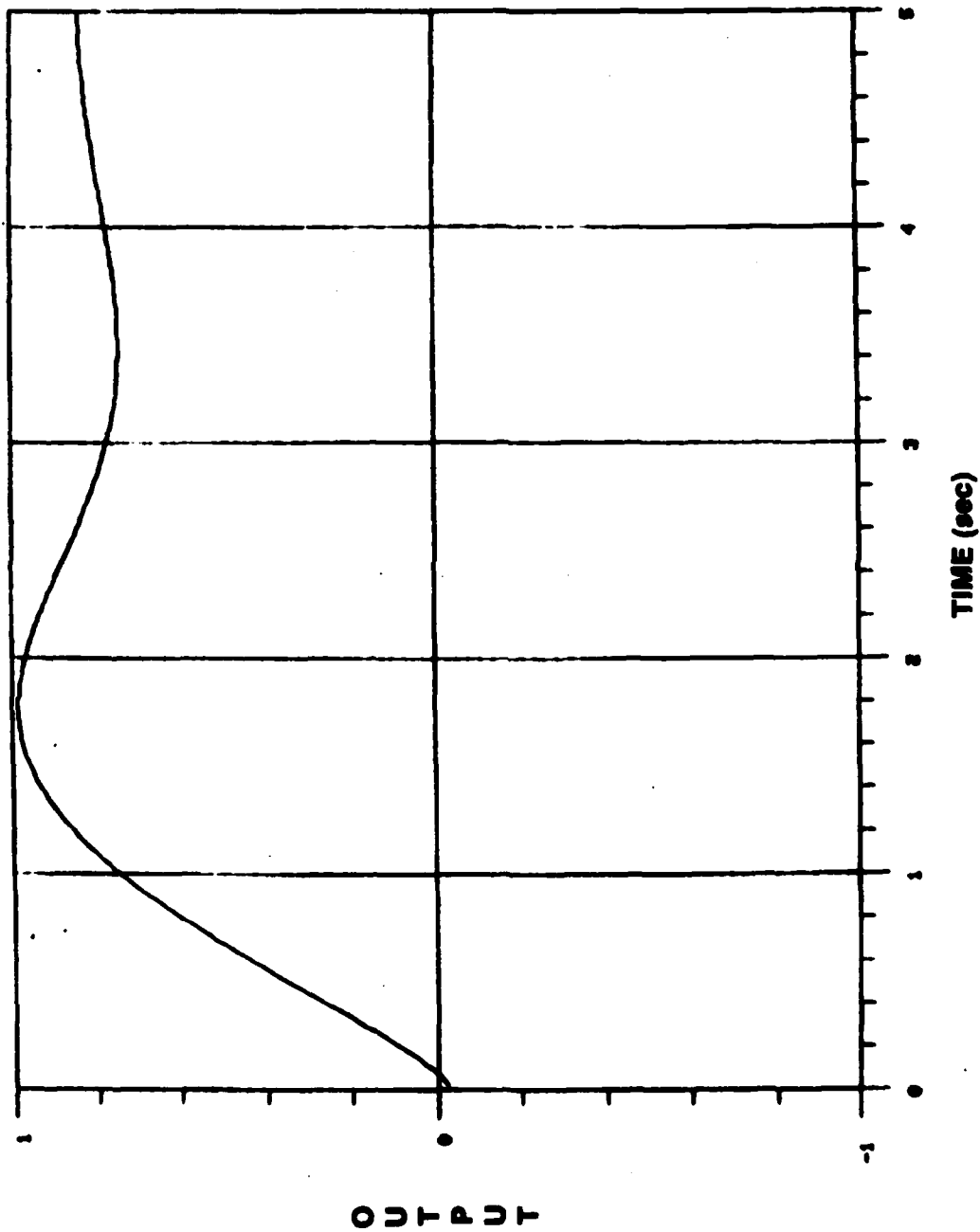


Figure 25. System output response (y), $W_1 = 1$, +10% variation on ω_1^2 .

OUTPUT

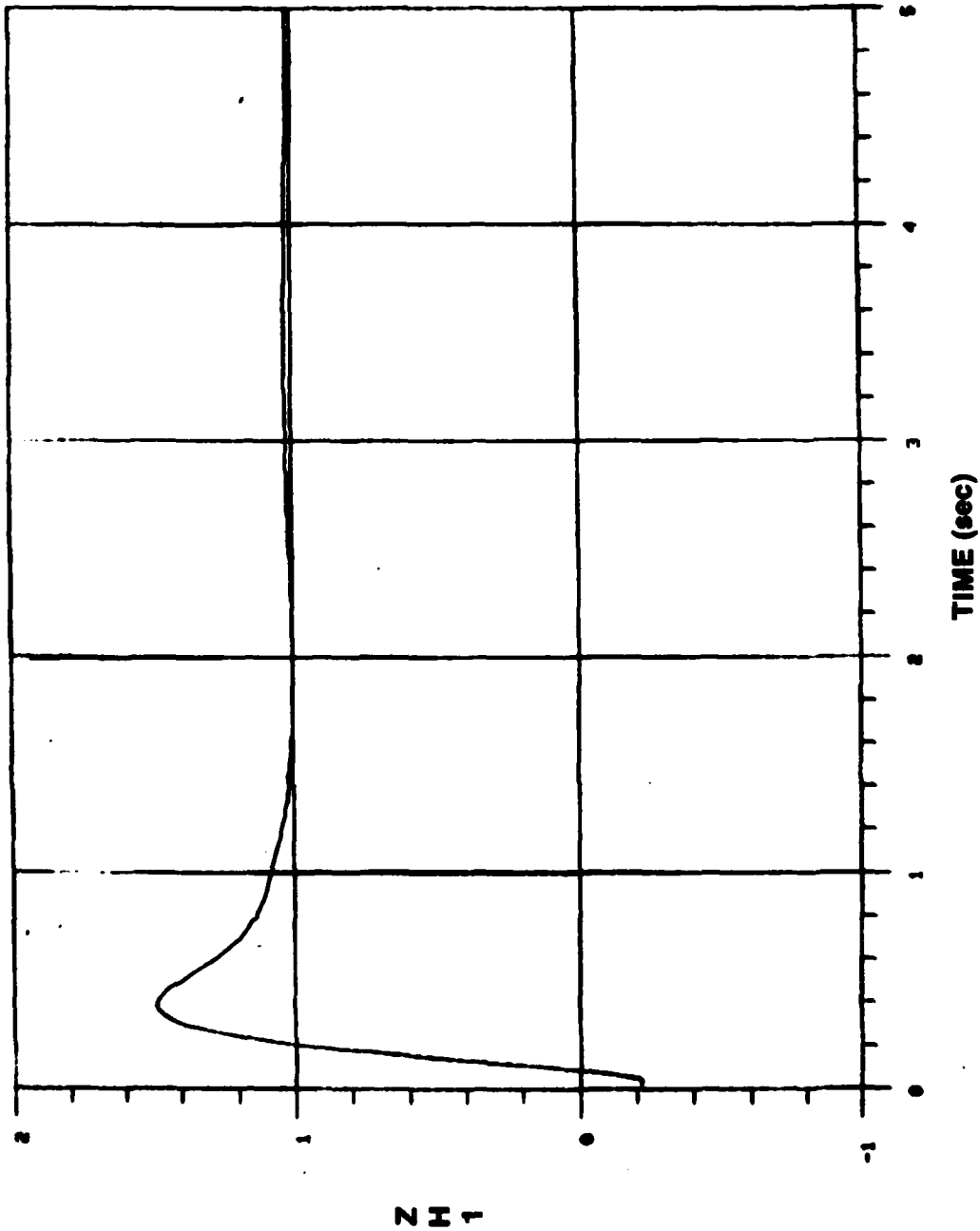


Figure 26. DAC disturbance estimate (Z_{H_1}), $W_1 = 1$, +10% variation on ω_1^2 .

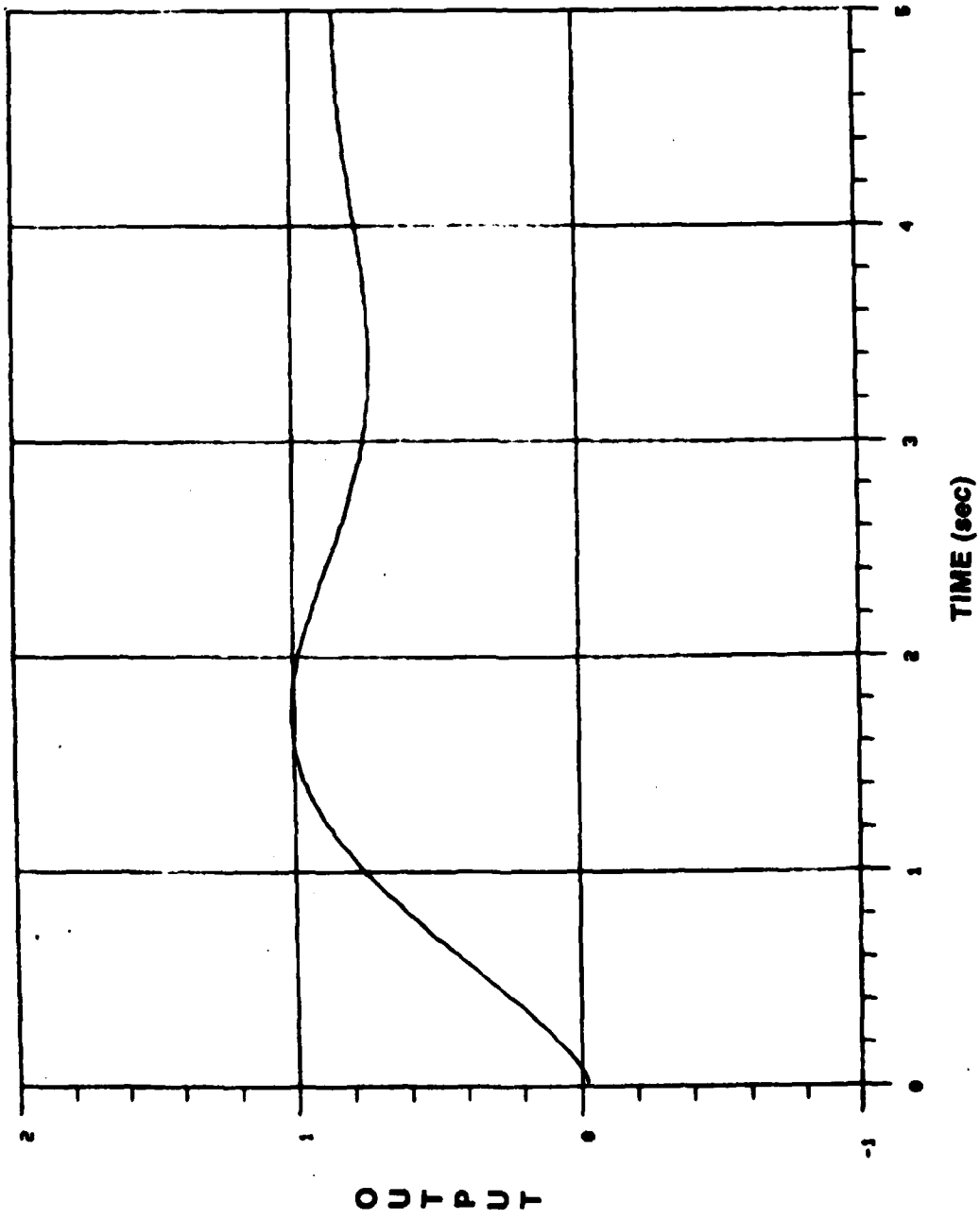


Figure 27. System output response (y), $W_1 = 1$, -10% variation on ω_n^2 .

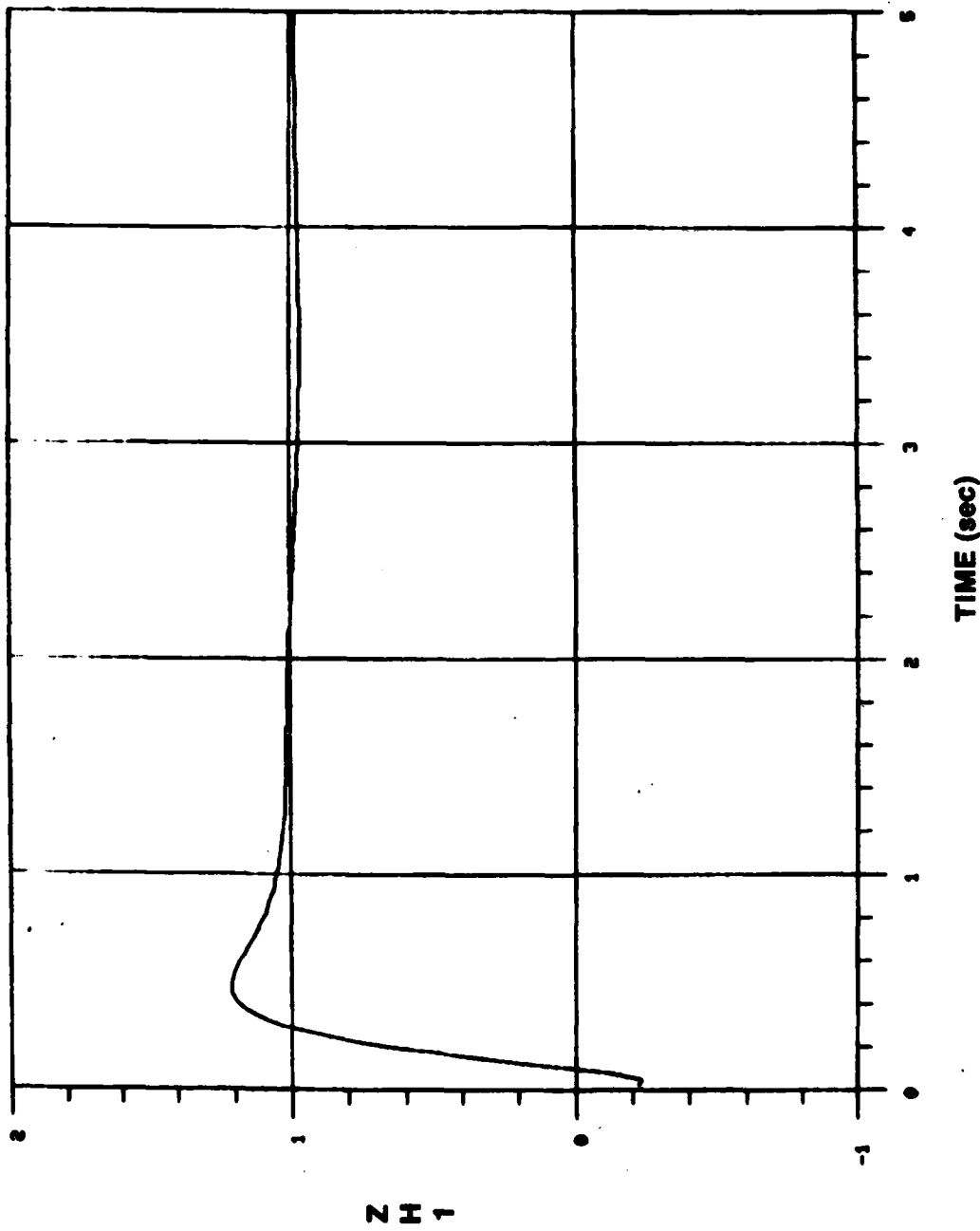


Figure 28. DAC disturbance estimate (\hat{z}_1). $W_1 = 1$, -10% variation on ω_1^2 .

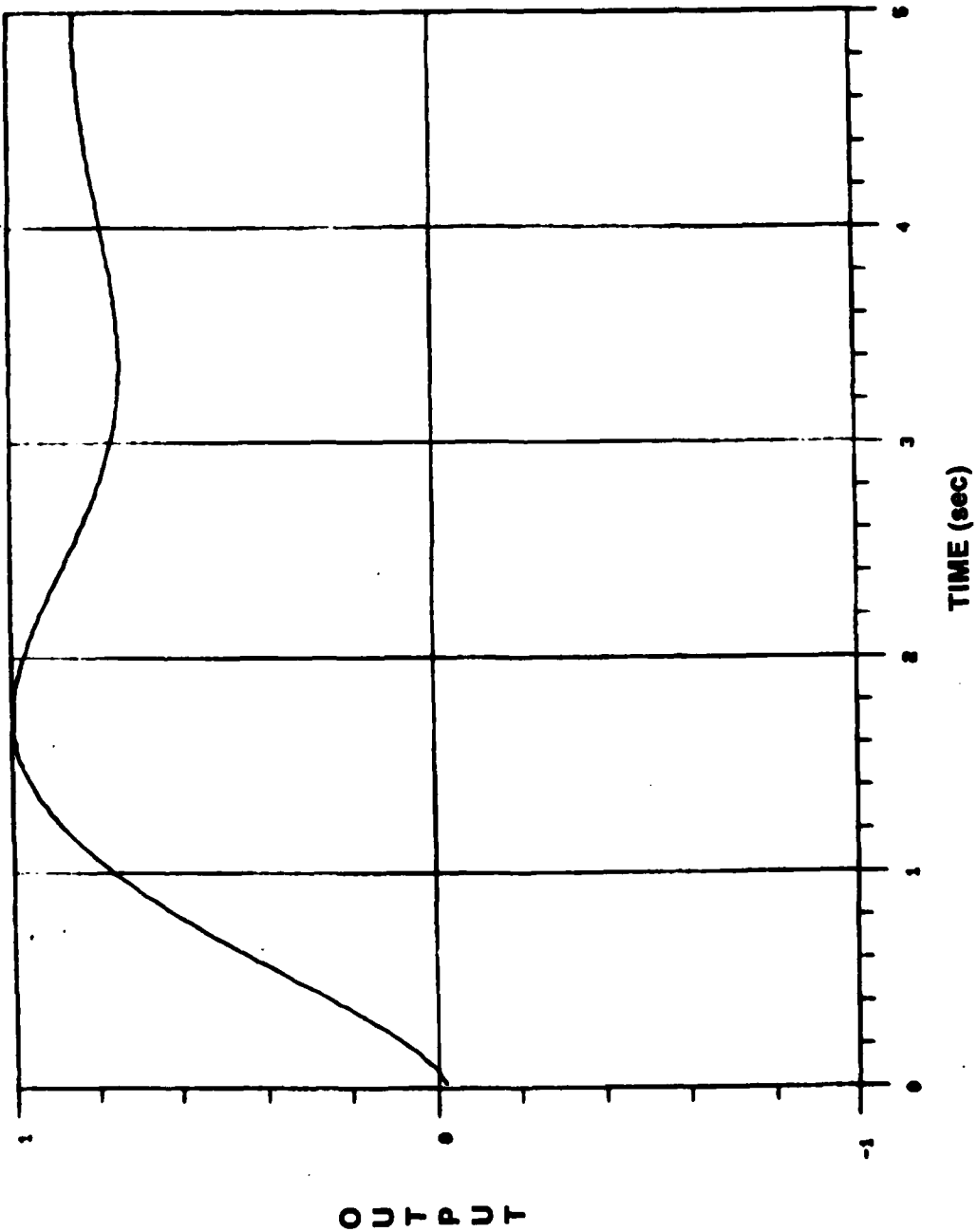


Figure 29. System output response (y), $W_1 = 1$, +10% variation on C_r .

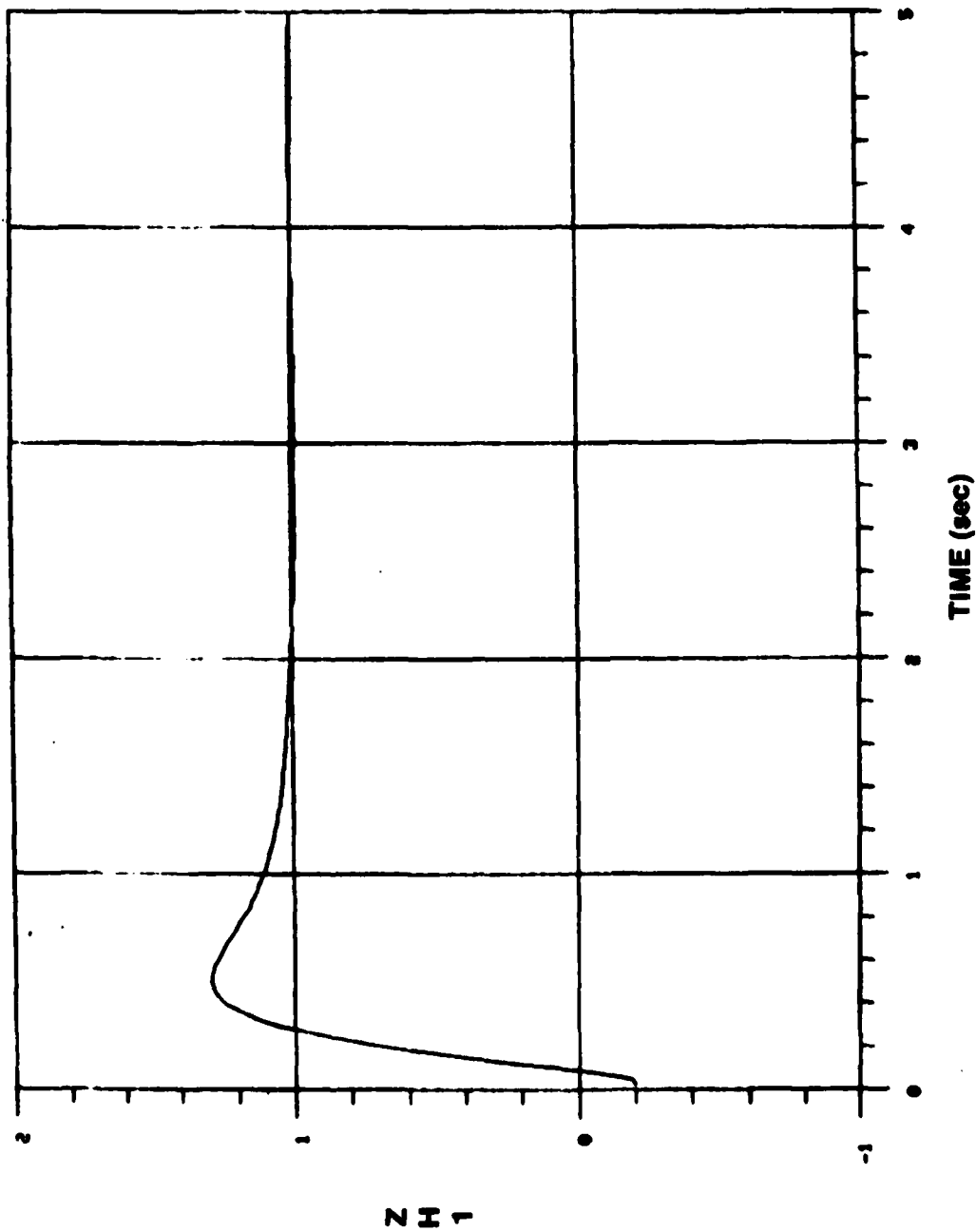


Figure 30. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, +10% variation on CR.

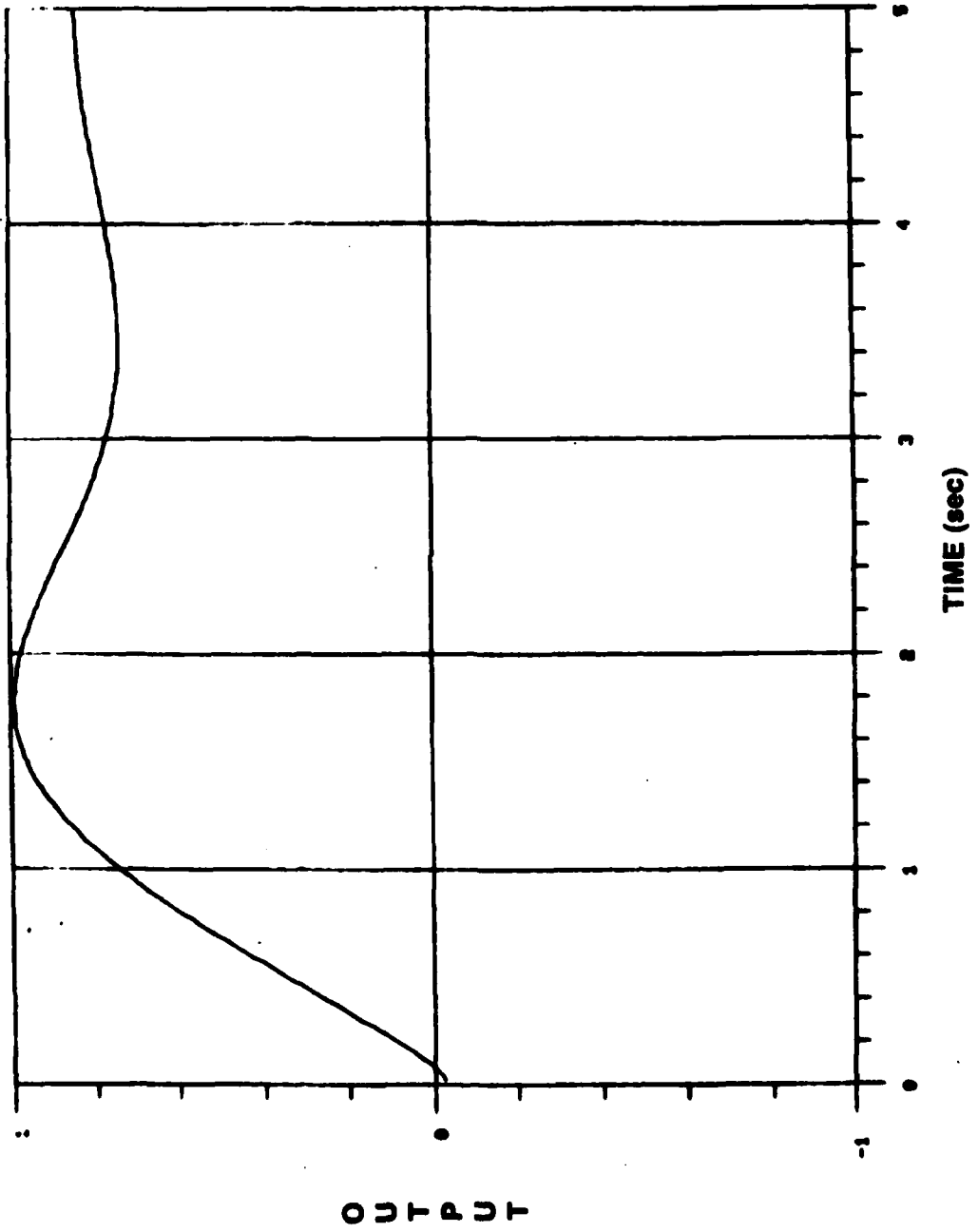


Figure 31. System output response (y), $W_1 = 1$, -10% variation on CR.

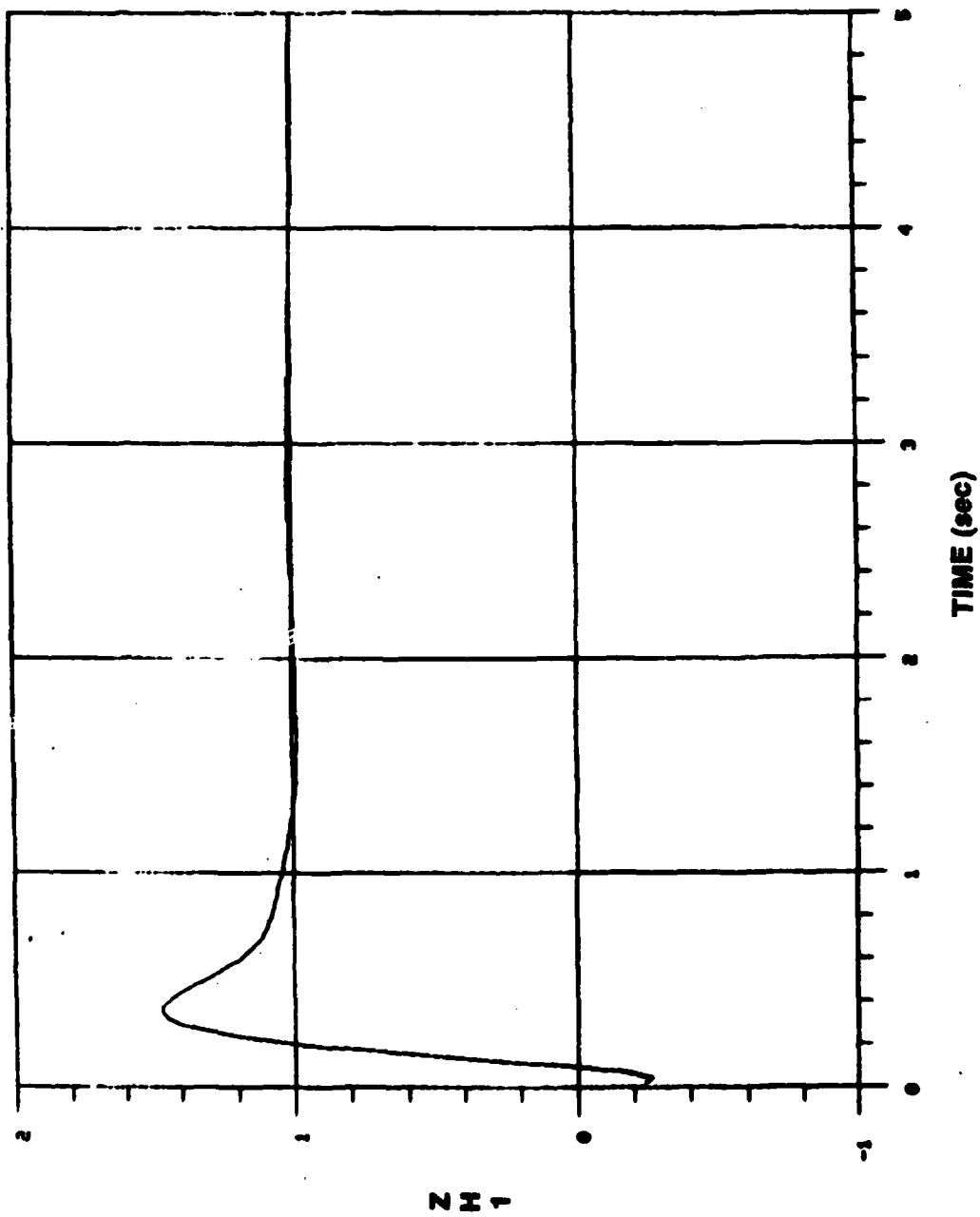


Figure 32. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, -10% variation on CR.

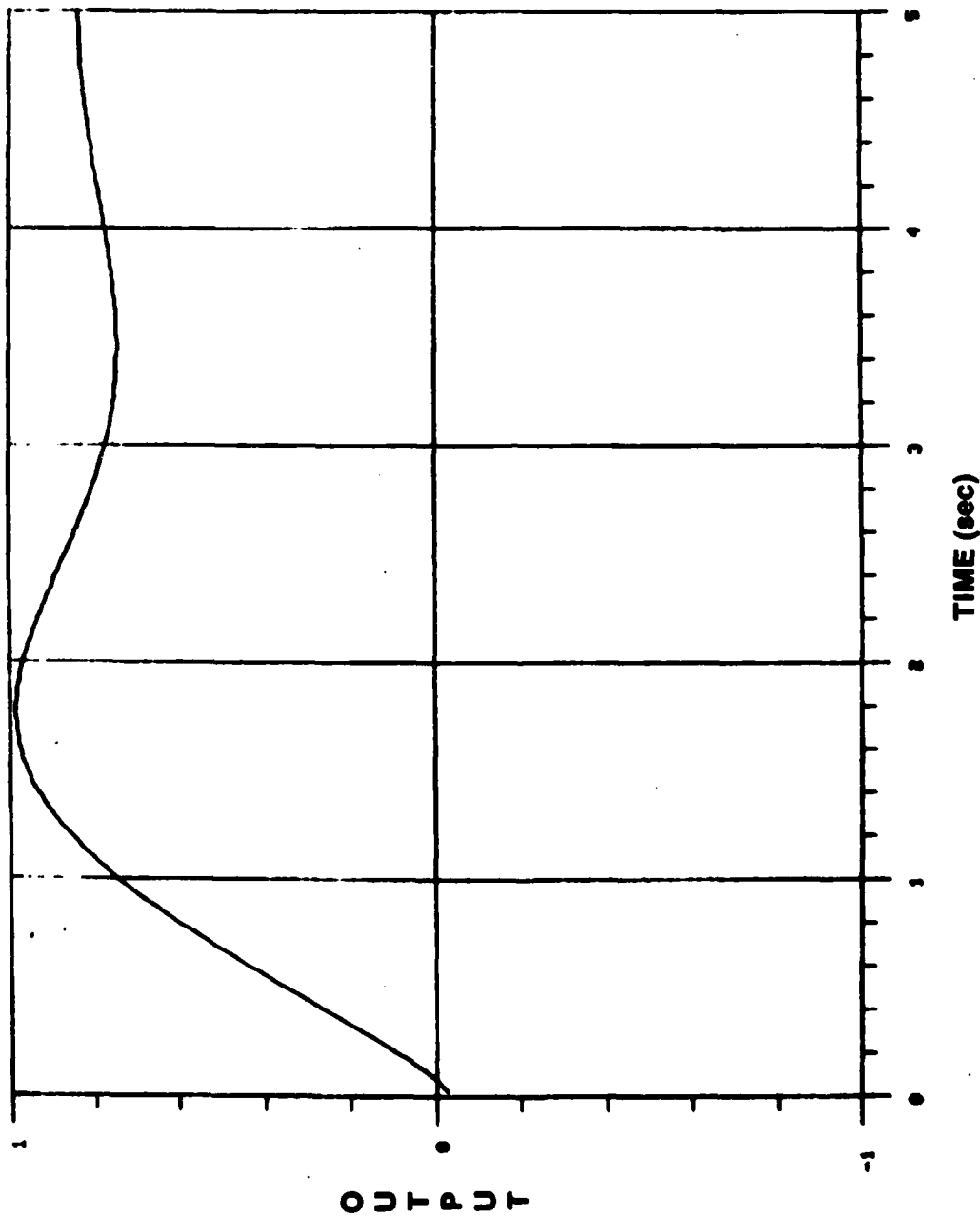


Figure 33. System output response (Y), $W_1 = 1$, +10% variation on K_T .

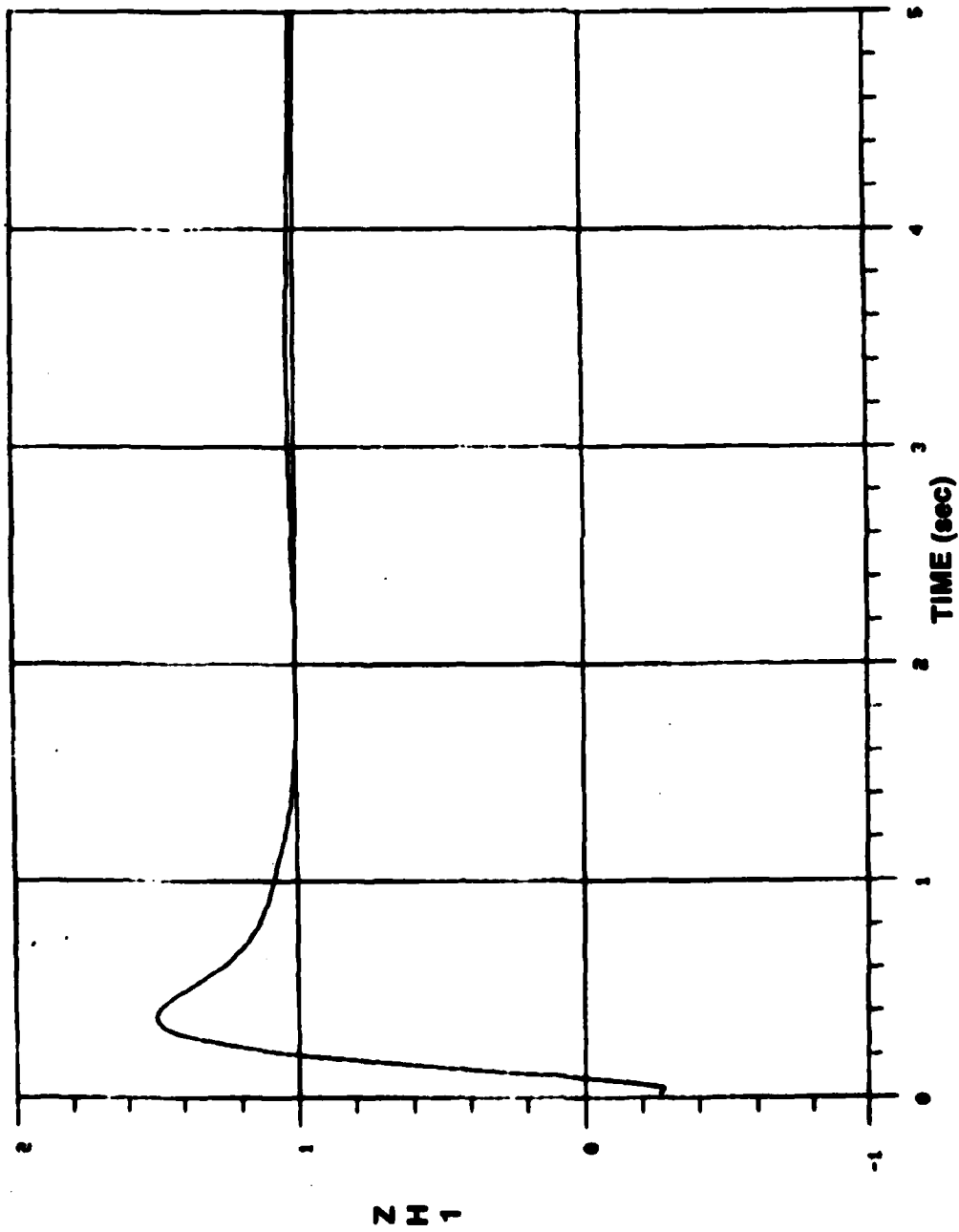


Figure 34. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, +10% variation on K_η .

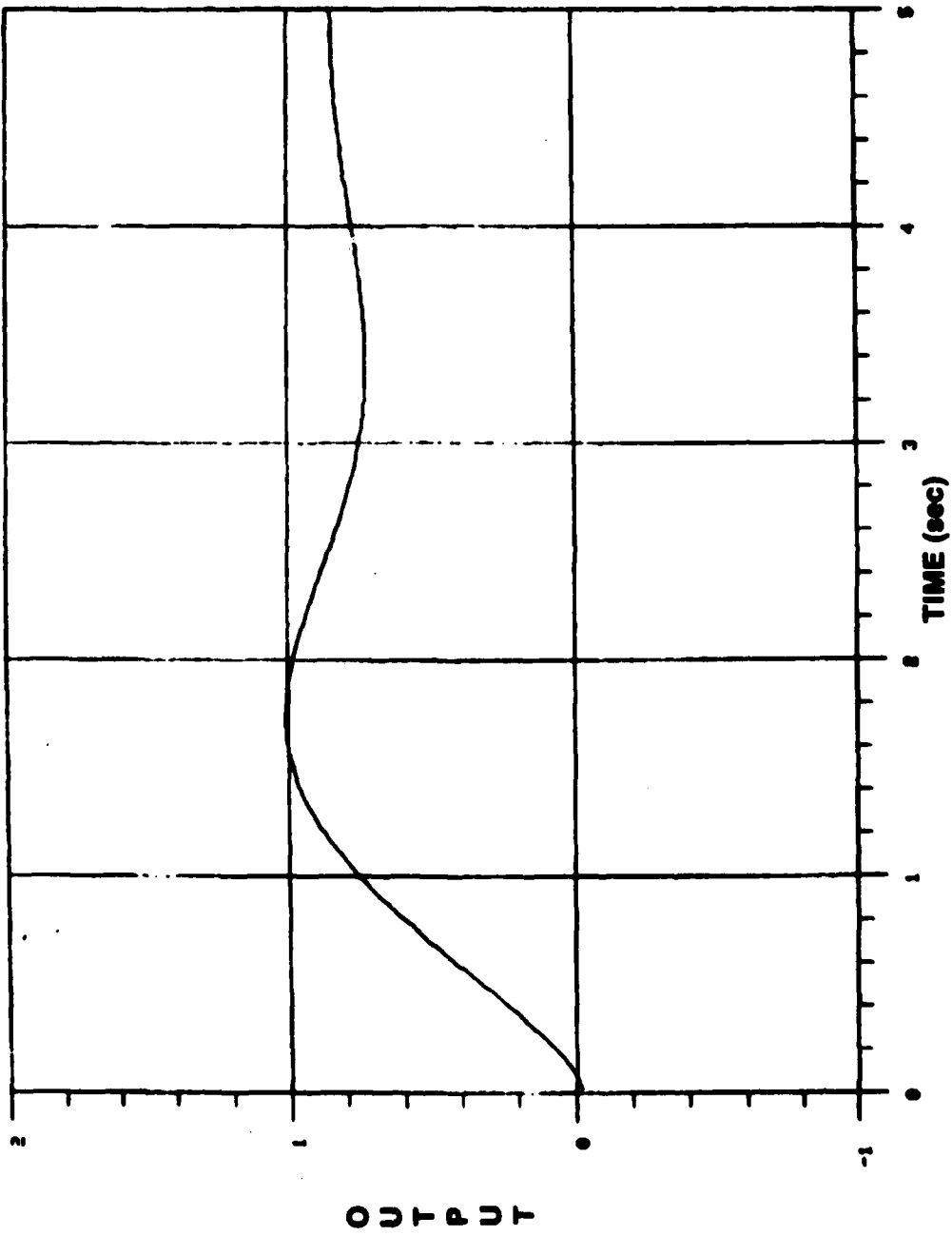


Figure 35. System output response (y), $W_1 = 1$, -10% variation on K_1 .

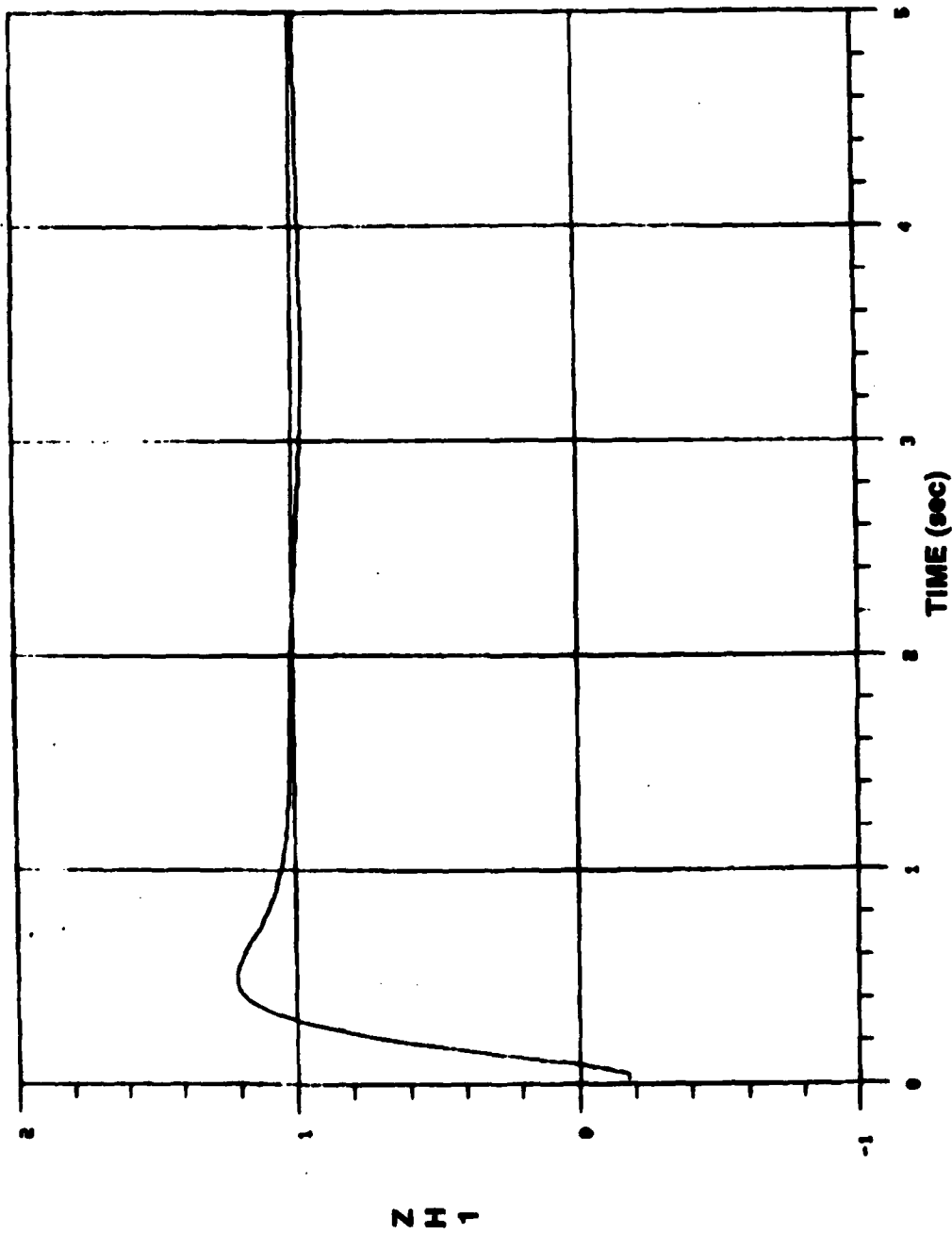


Figure 36. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, -10% variation on K_7 .

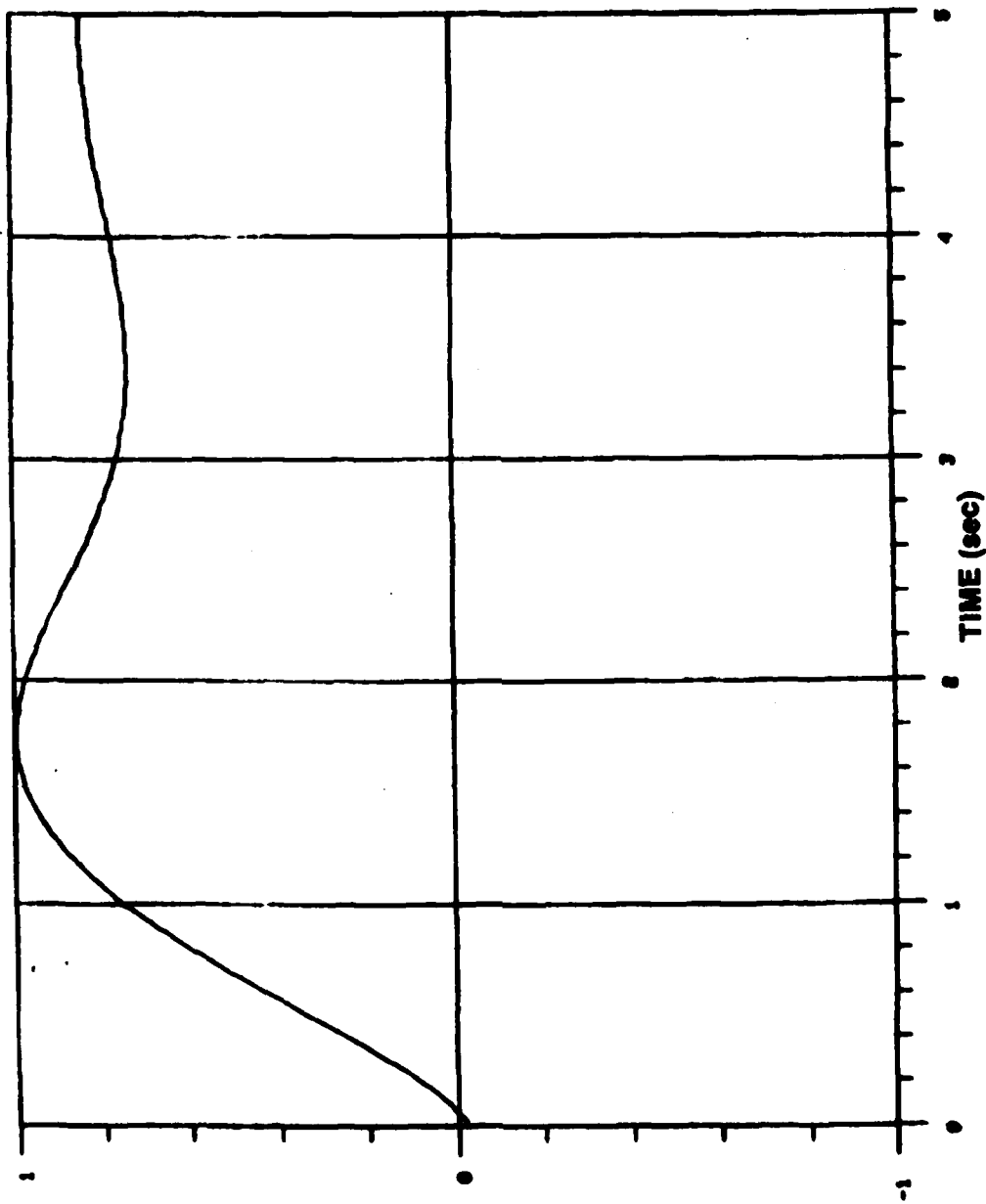


Figure 37. System output response (y), $W_1 = 1$, +20% variation on all parameters.

OUTPUT

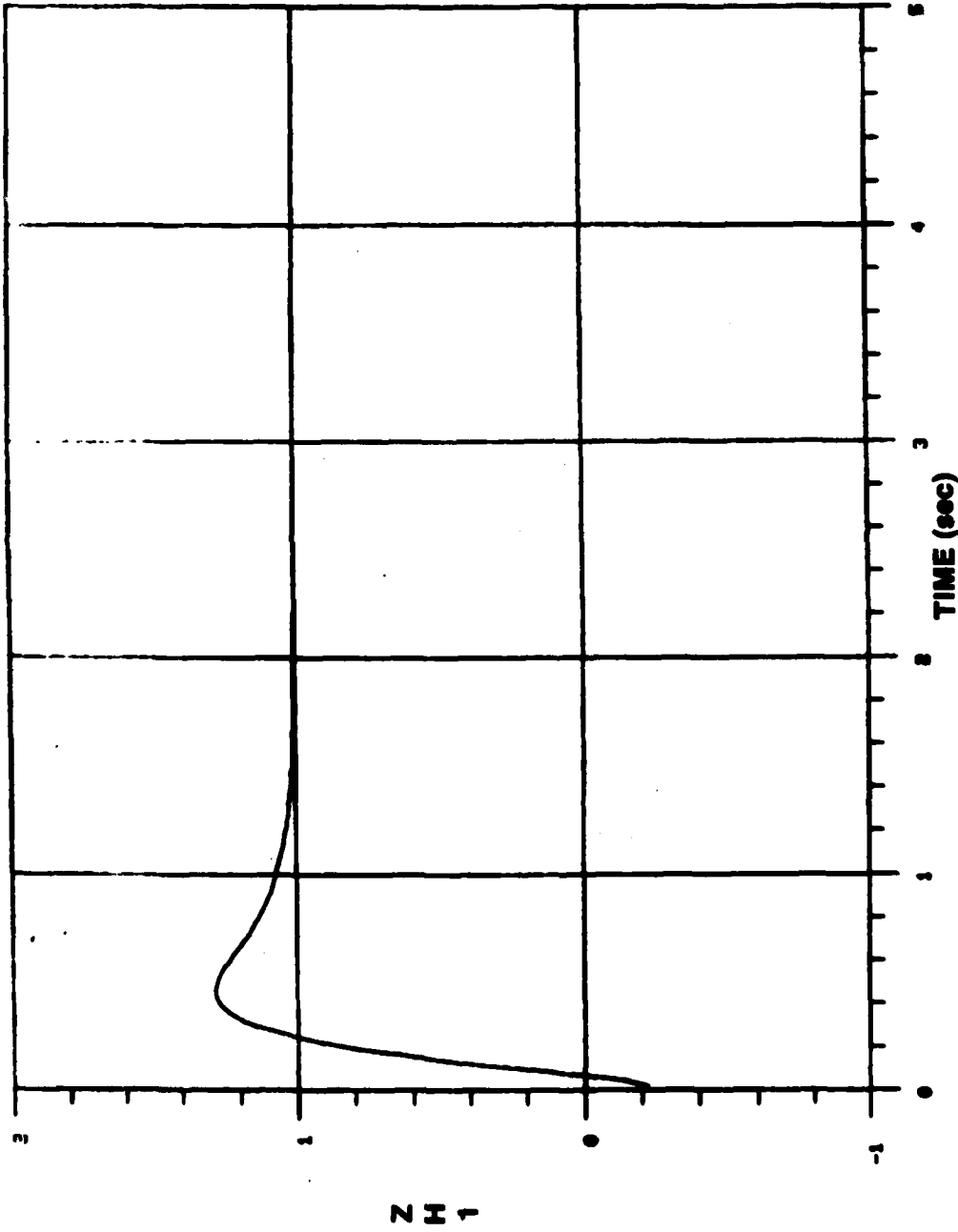


Figure 38. DAC disturbance estimate (\hat{z}_1), $W_1 = 1$, +20% variation on all parameters.

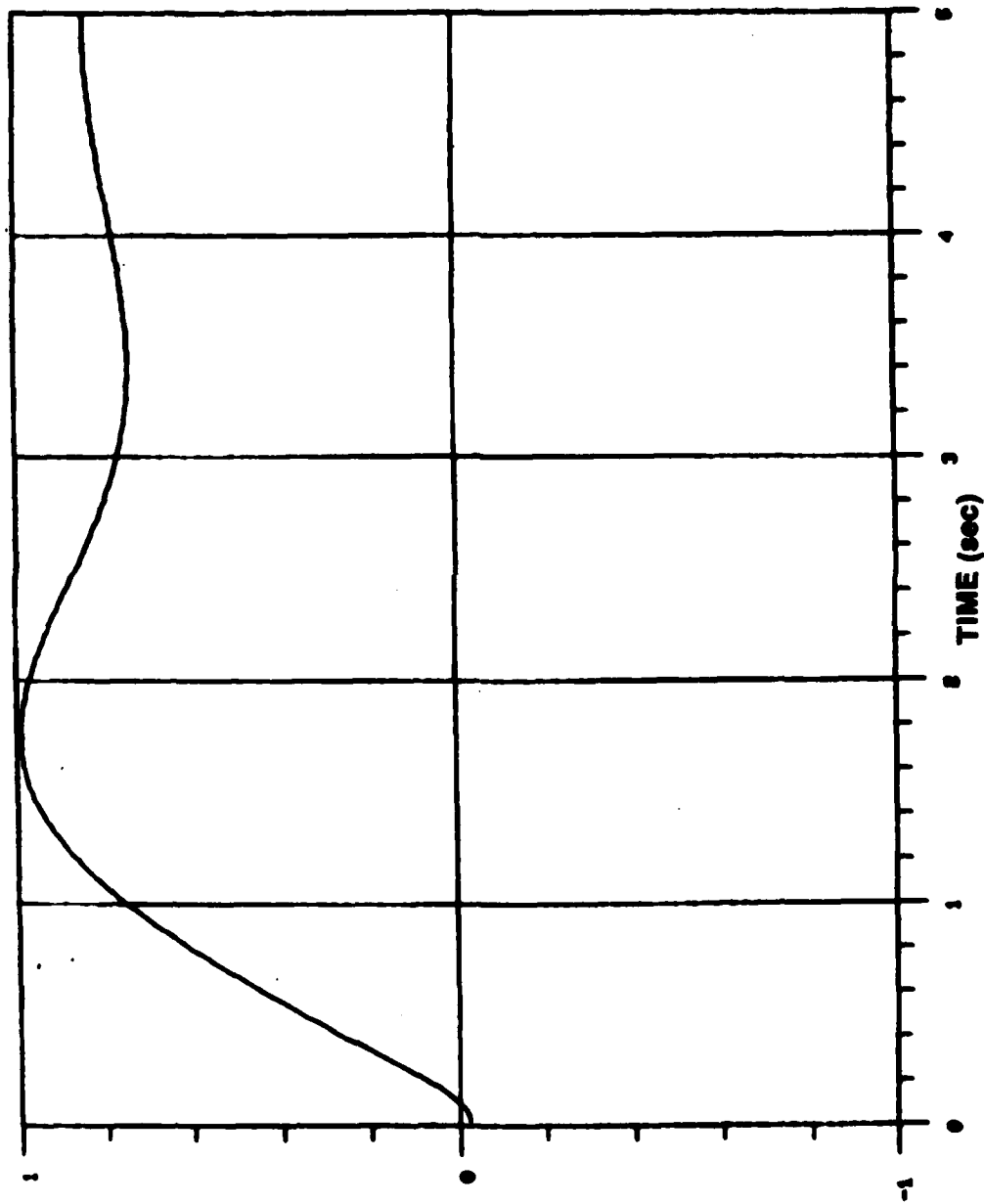


Figure 39. System output response (y), $W_1 = 1$, -20% variation on all parameters.

OUTPUT

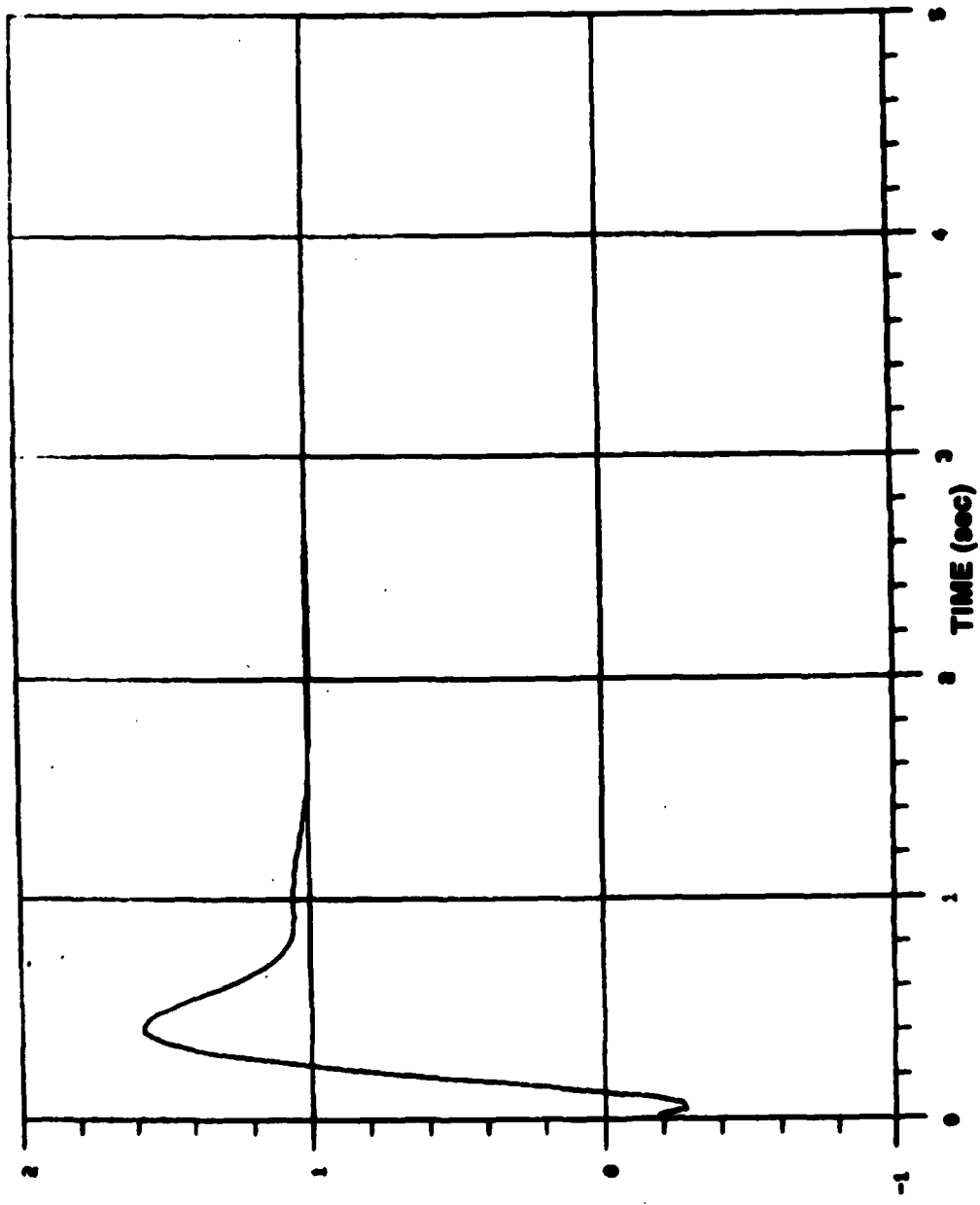


Figure 40. DAC disturbance estimate (Z_{H1}), $W_1 = 1$, -20% variation on all parameters.

Z H 1

From these results, the first two questions previously posed can indeed be answered in the affirmative. The answer to the third question would seem to be that the DAC works well with at least up to 20% variation of plant parameters and possibly for larger variations. For a given system, though, this should be thoroughly verified by checking at all critical times along a trajectory, i.e., burnout, apogee, etc.

In order to answer the fourth question, three of the time points shown in *Table 1* were used in the simulation. The roots of Equation (18), which were used to settle out the state

TABLE 3. ROOTS FOR DETERMINING DAC GAIN MATRICES

ROOT TIME POINT (SEC)	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
9.85	-5.	-6.	-10.	-10.	-12.	-15.
66.7	-0.5	-0.5	- 1.	- 1.	- 1.5	- 1.5
111.4	-3.	-4.	-7+j2	-7-j2	- 8.	-10.

reconstructor in each case and to calculate the components of the DAC gain matrices, are shown in *Table 3*. The components of K_1 and K_2 are shown in *Table 4*.

TABLE 4. DAC GAIN MATRIX COMPONENTS

GAIN VALUE TIME POINT (SEC)	9.85	66.7	111.4
k ₁₁	-45.77	-1.716	-30.38
k ₂₁	-1237.14	-5.85	-660.19
k ₃₁	-15951.2	-7.36	-4197.9
k ₄₁	-51488.6	-8.16	-13735.5
k ₁₂	-155.26	-1.935	-588.51
k ₂₂	-300.51	-0.271	-640.07

Three simulation runs were made at each time point:

- with nominal airframe parameters, no disturbance,
- with nominal airframe parameters and a disturbance and
- with a 20% variation on airframe parameters in the direction of increasing flight time, with a disturbance. The results are presented in *Figures 41 through 58*.

From these results and the DAC parameters shown in *Tables 3 and 4*, it is evident that a DAC designed at one point of a trajectory will not perform as well as needed over large portions of the trajectory. Gain switching, similar to an autopilot gain switch program, will be required for DAC implementation.

C. CONCLUSIONS

For this case, with the disturbance at the input, it was possible to find a control u_c which could be implemented and which, theoretically, would totally cancel the disturbance. In a practical application, it was found that the control did cancel the disturbance very well, that the DAC would continue to function well within a band about the design point and that, with gain switching, the DAC should perform its function as the plant parameters vary over an entire trajectory. As can be seen from *Table 4* the DAC gains do have a wide range.

For the apogee case, since the system is so sluggish, the DAC did not offer much in the way of disturbance cancellation, i.e., the estimation errors did not settle out rapidly enough. One reason is due to the nearness of the eigenvalues of the \tilde{A} matrix to zero. This allowed the large overshoot. However, these eigenvalues had to be maintained in this region because moving them to more negative positions caused an instability to develop. Overall, it might be advantageous to zero out the DAC gains near apogee.

6. RATE LOOP WITH DISTURBANCE ON OUTPUT

A. DAC MODEL DEVELOPMENT

The missile from which this autopilot channel was taken uses an attitude control during boost, so it was of interest to consider the rate loop alone, with a disturbance included, to see if a DAC would be useful in taking out effects due to external rate perturbations.

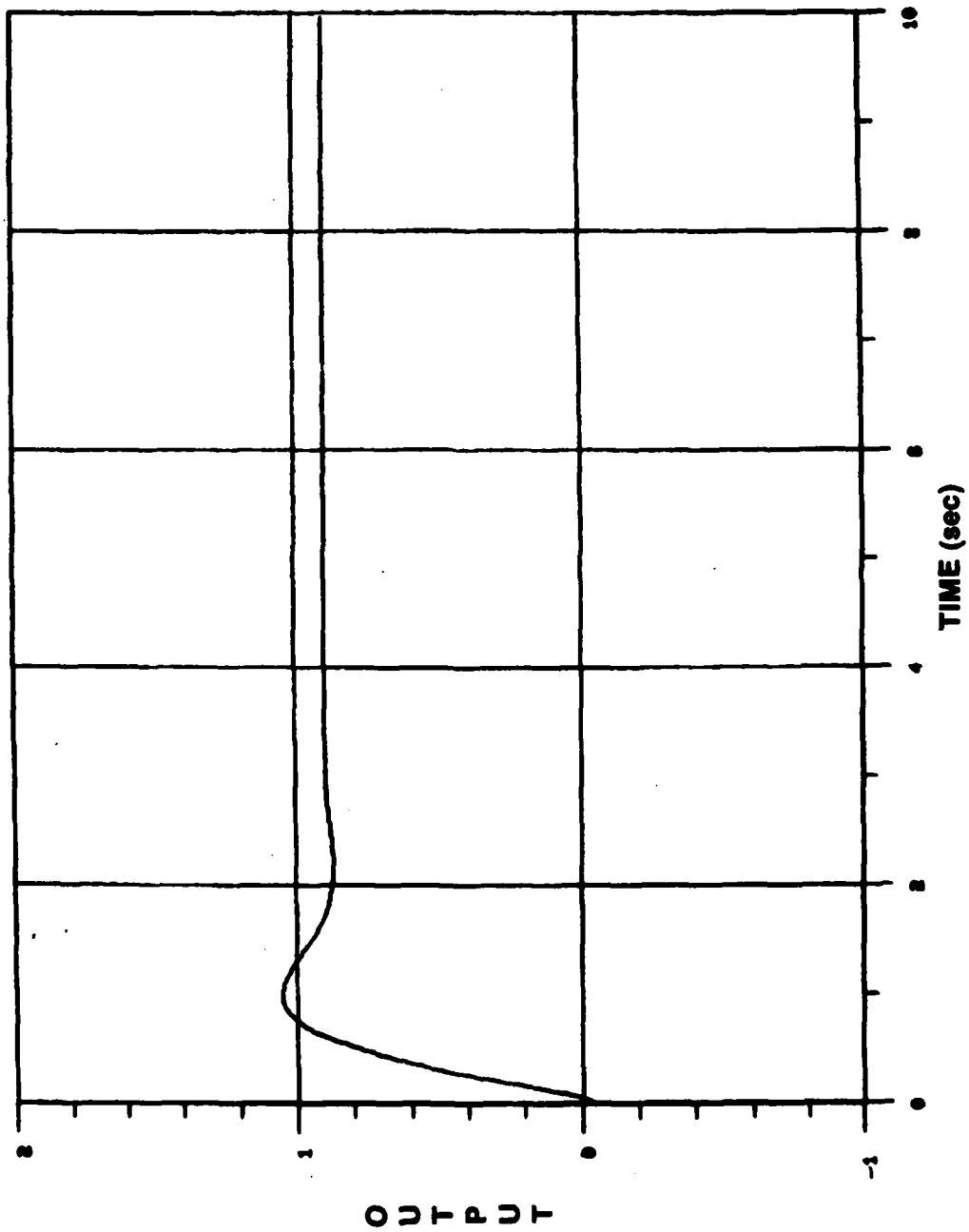


Figure 41. System output response, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_1 = 0$.

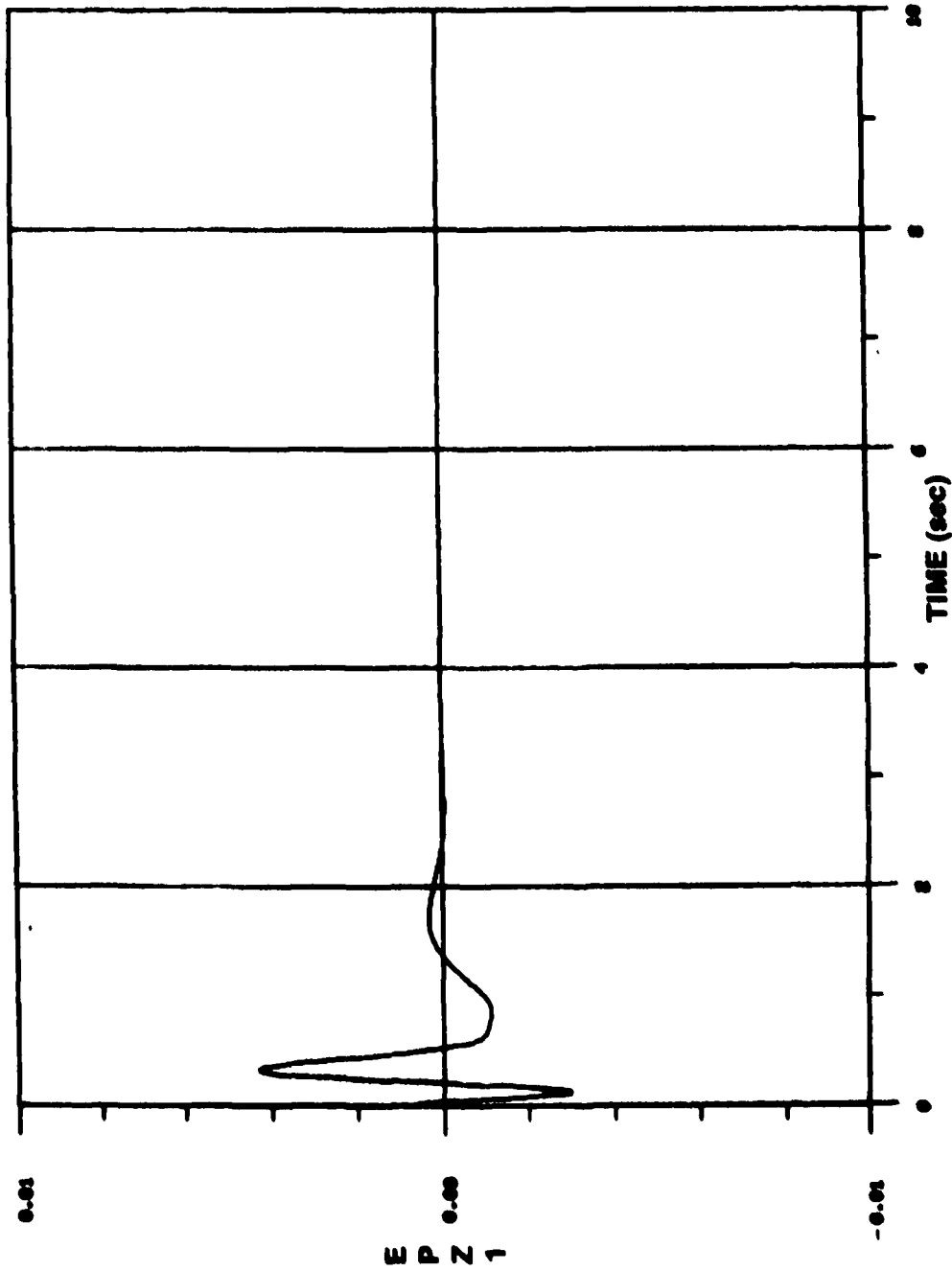


Figure 42. DAC disturbance estimation error, $t_f = 9.95$ sec, $P_{GO} = 1$, $W_1 = 0$.

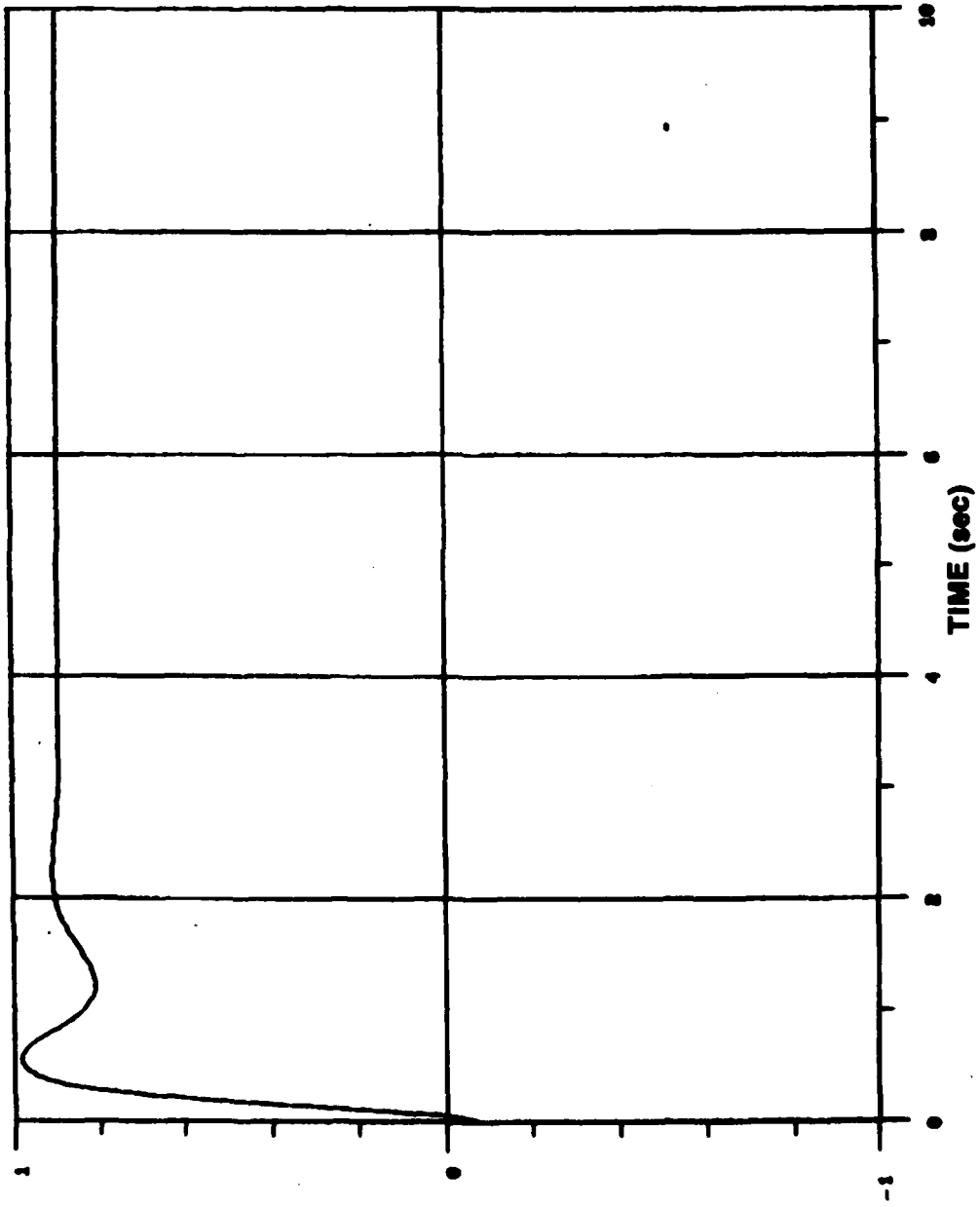


Figure 43. System output response, $t_f = 9.85$ sec, $PGO = 1$, $W_1 = 1.0$.

OUTPUT

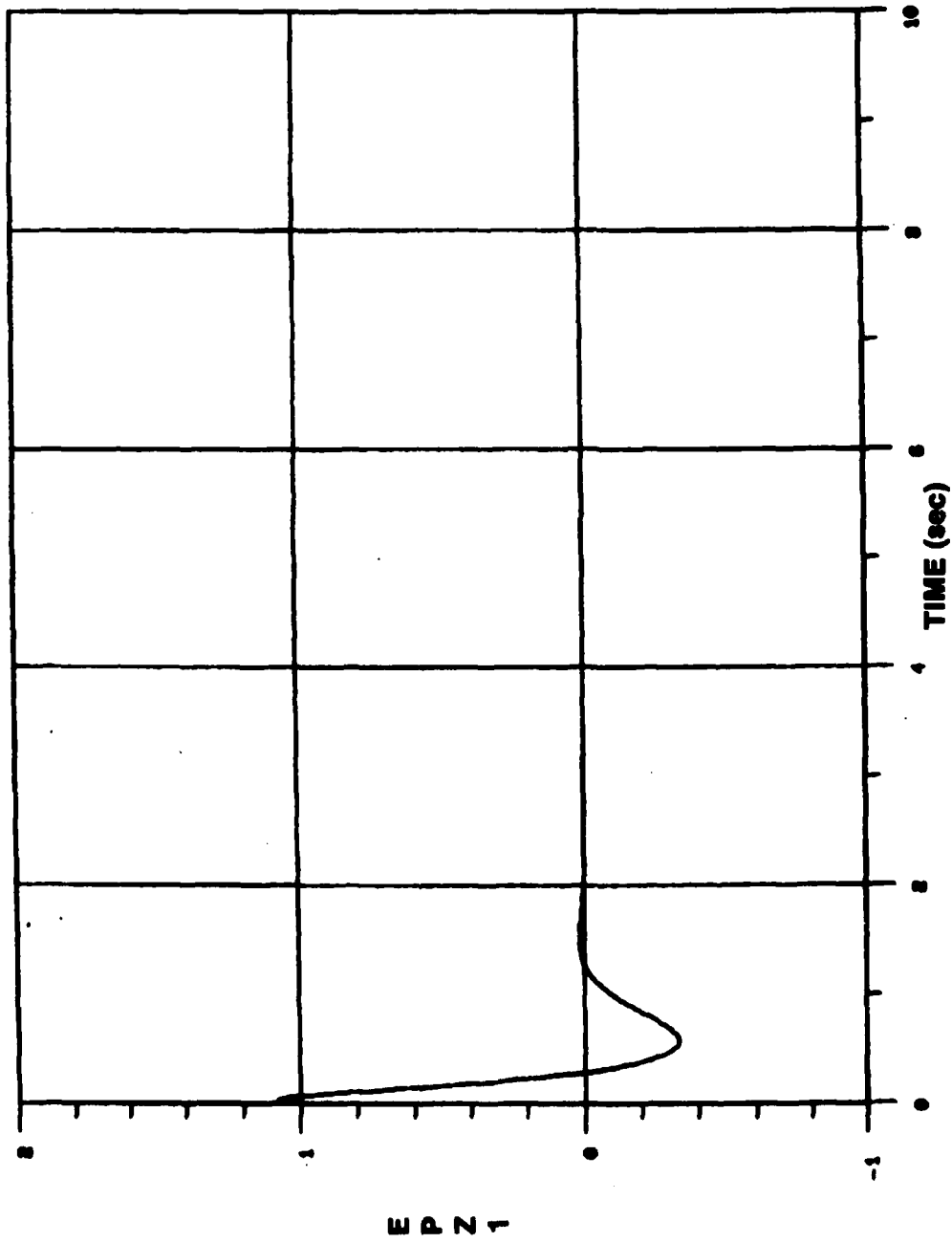


Figure 44. DAC disturbance estimation error, $t_f = 9.85$ sec, $PGO = 1$, $W_1 = 1.0$.

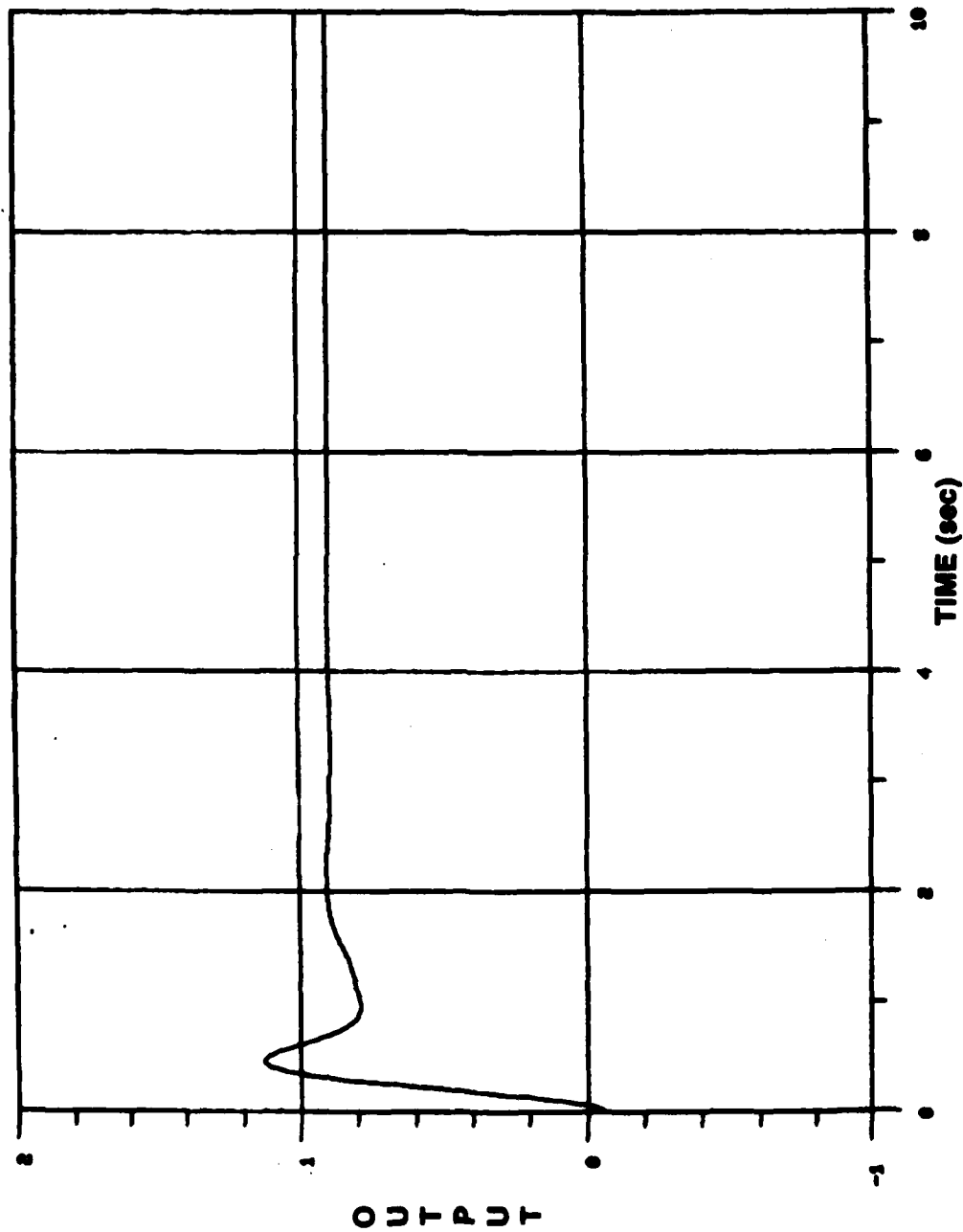


Figure 45. System output response, $t_r = 0.85$ sec, $W_1 = 1.0$, -20% variation on airframe parameters.

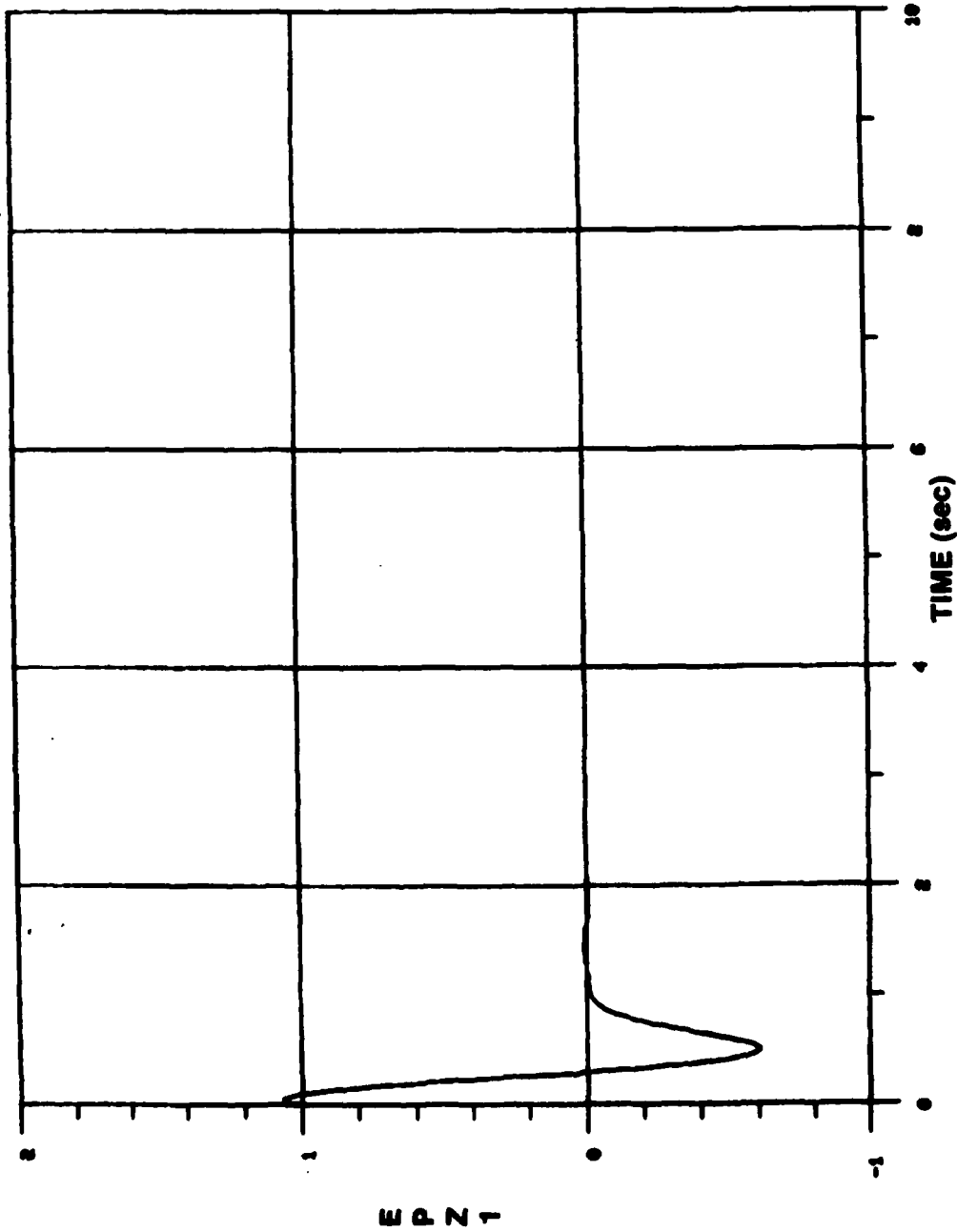


Figure 46. DAC disturbance estimation error, $t_f = 9.85$ sec, $W_1 = 1.0$,
 -20% variation on airframe parameters.

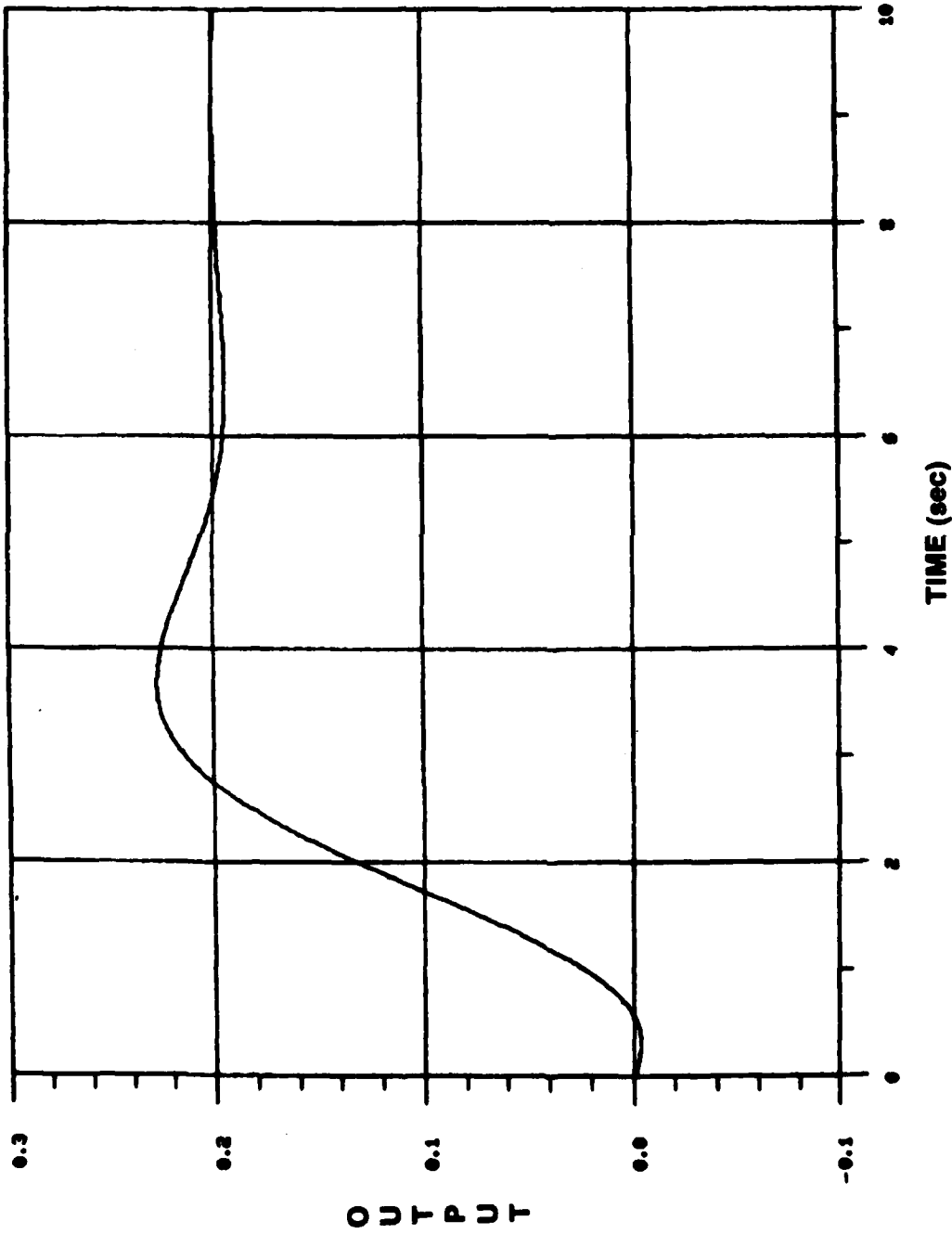


Figure 47. System output response, $t_f = 66.7$ sec, $PGO = 0.5$, $W_1 = 0$.

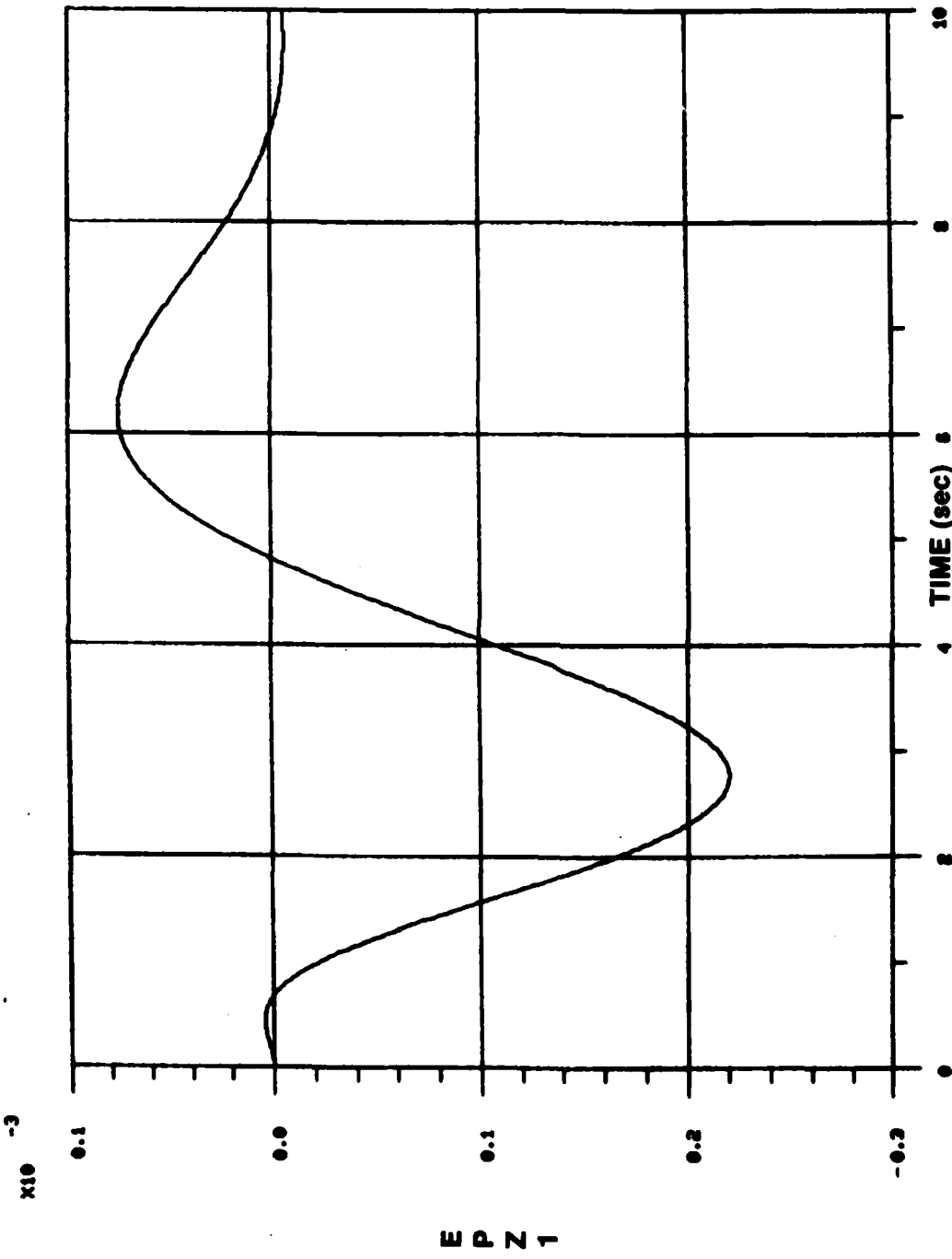


Figure 48. DAC disturbance estimation error, $t_f = 66.7$ sec, $PGO = 0.5$, $W_1 = 0.5$.

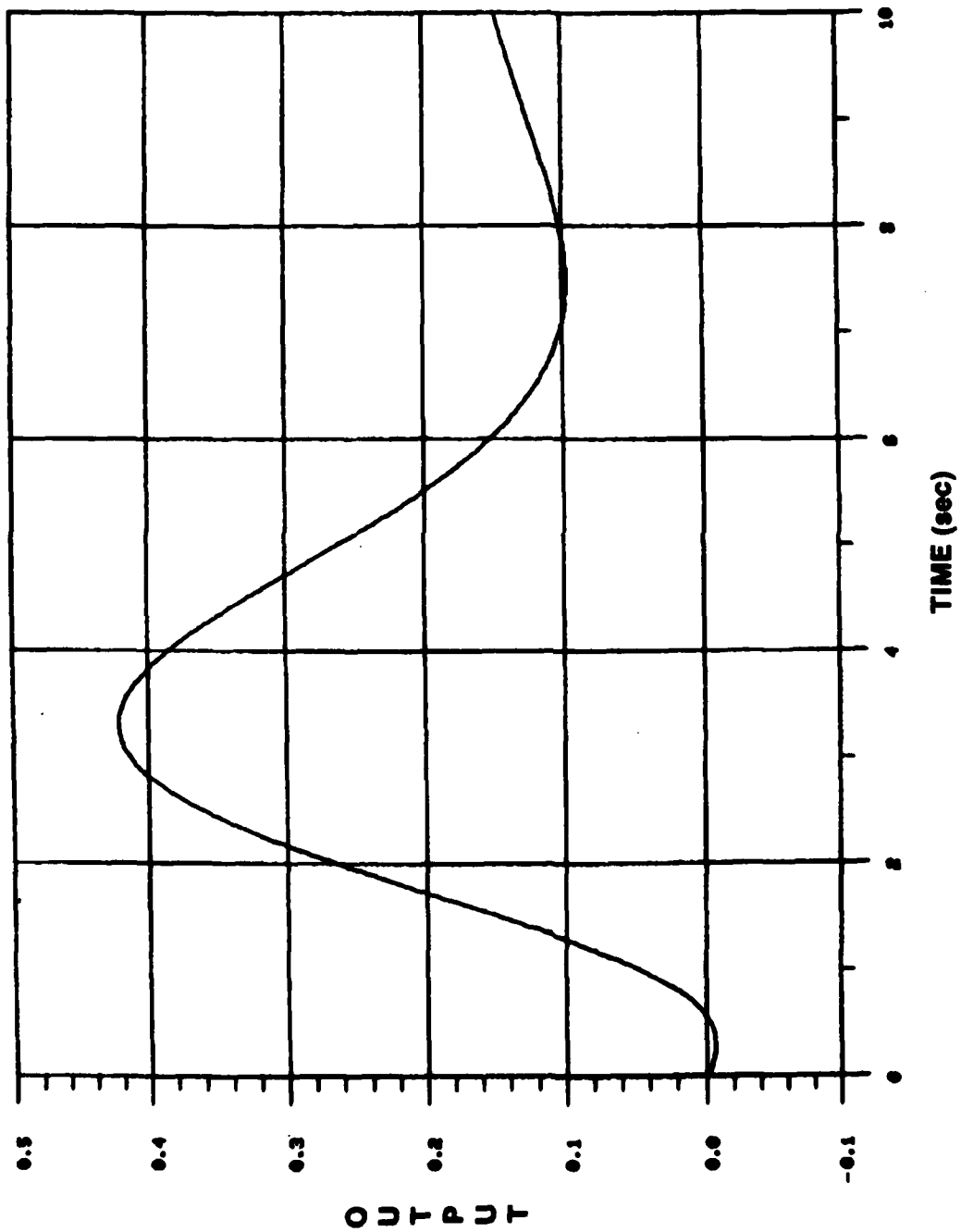


Figure 49. System output response, $t_f = 66.7$ sec, $P_{GO} = 0.5$, $W_1 = 0.5$.

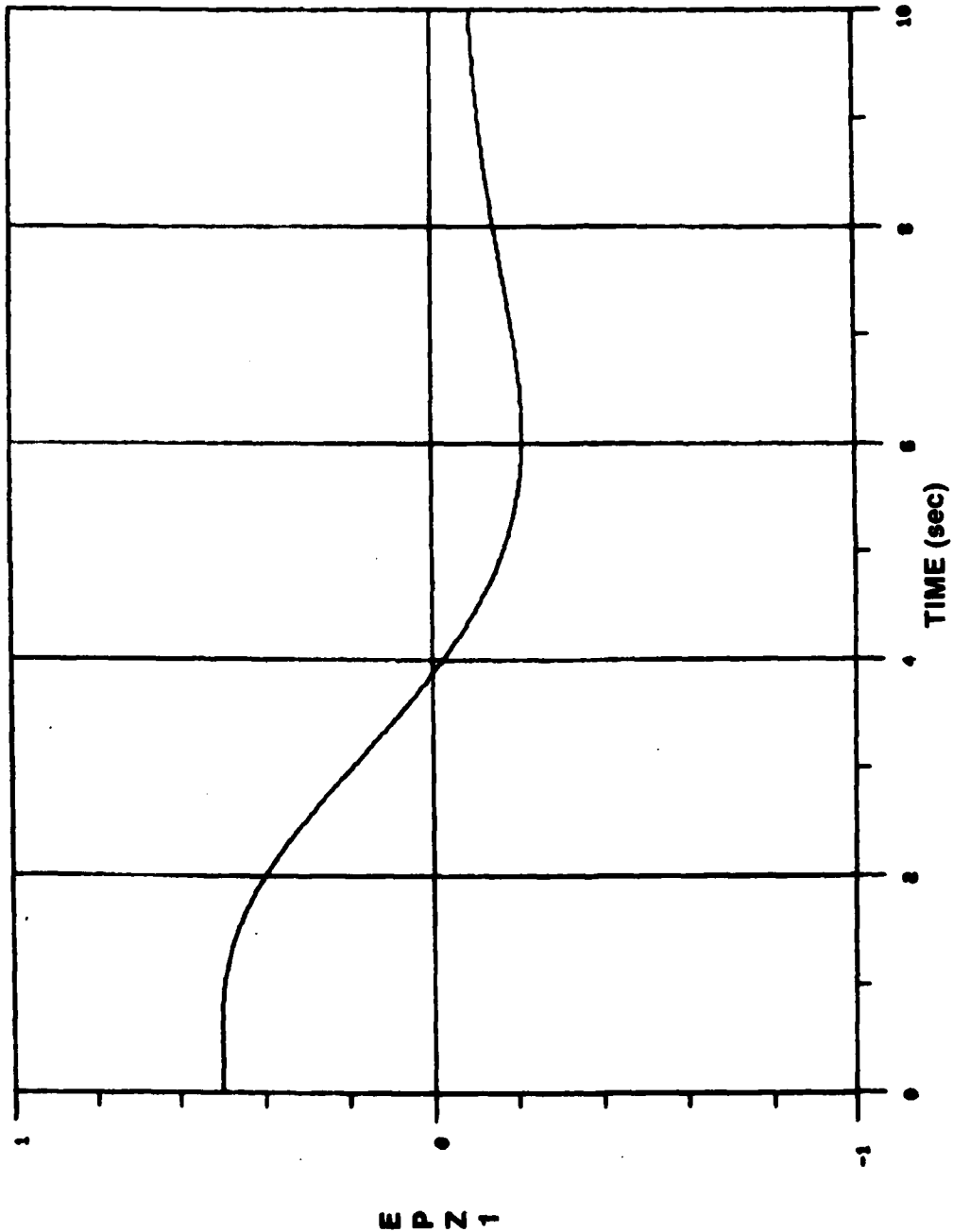


Figure 50. DAC disturbance estimation error, $t_1 = 66.7$ sec, $PGO = 0.5$,
 $W_1 = 0.5$.

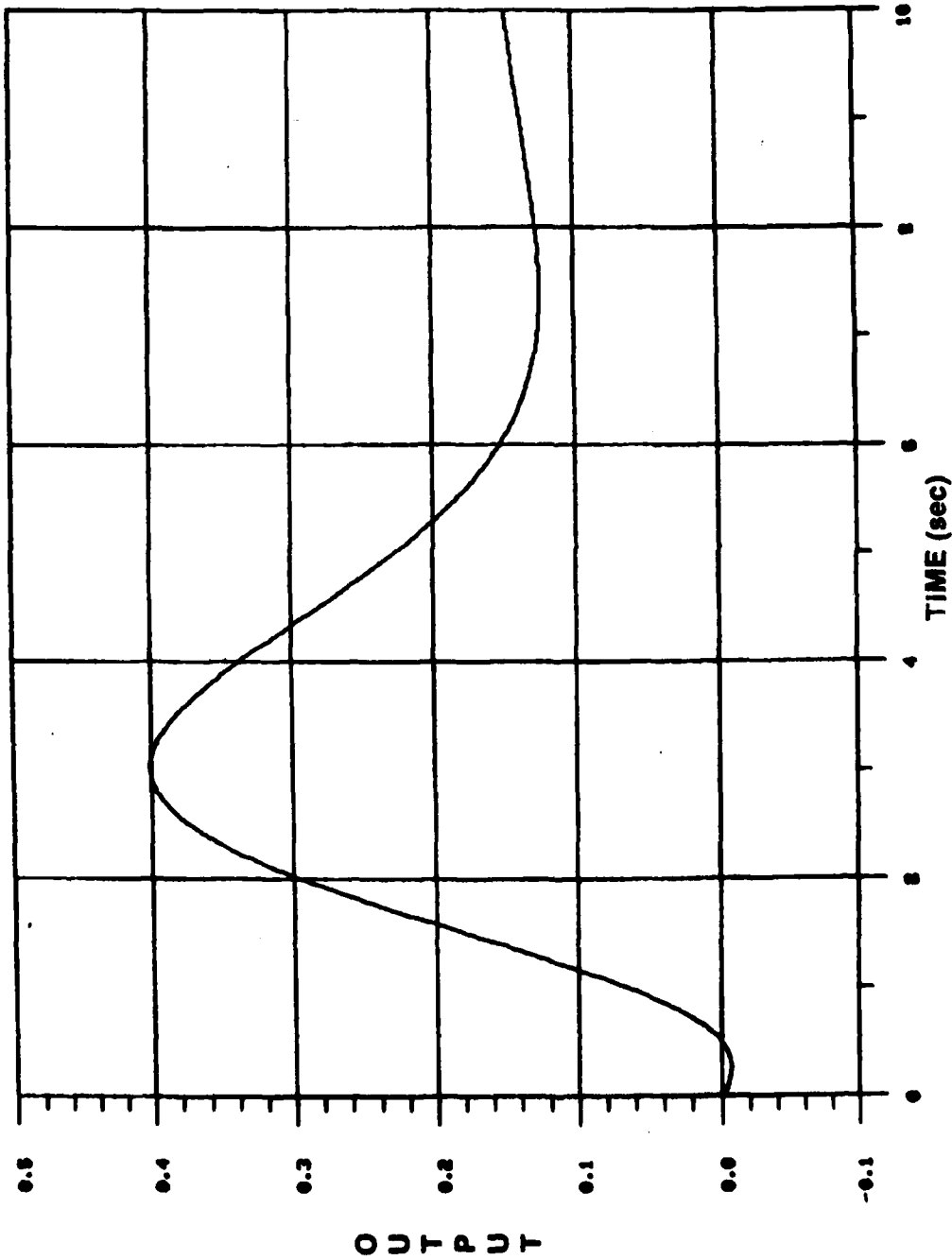


Figure 51. System output response, $t_f = 66.7$ sec, $PGO = 0.5$, $W_1 = 0.5$,
 + 20% variation on airframe parameters.

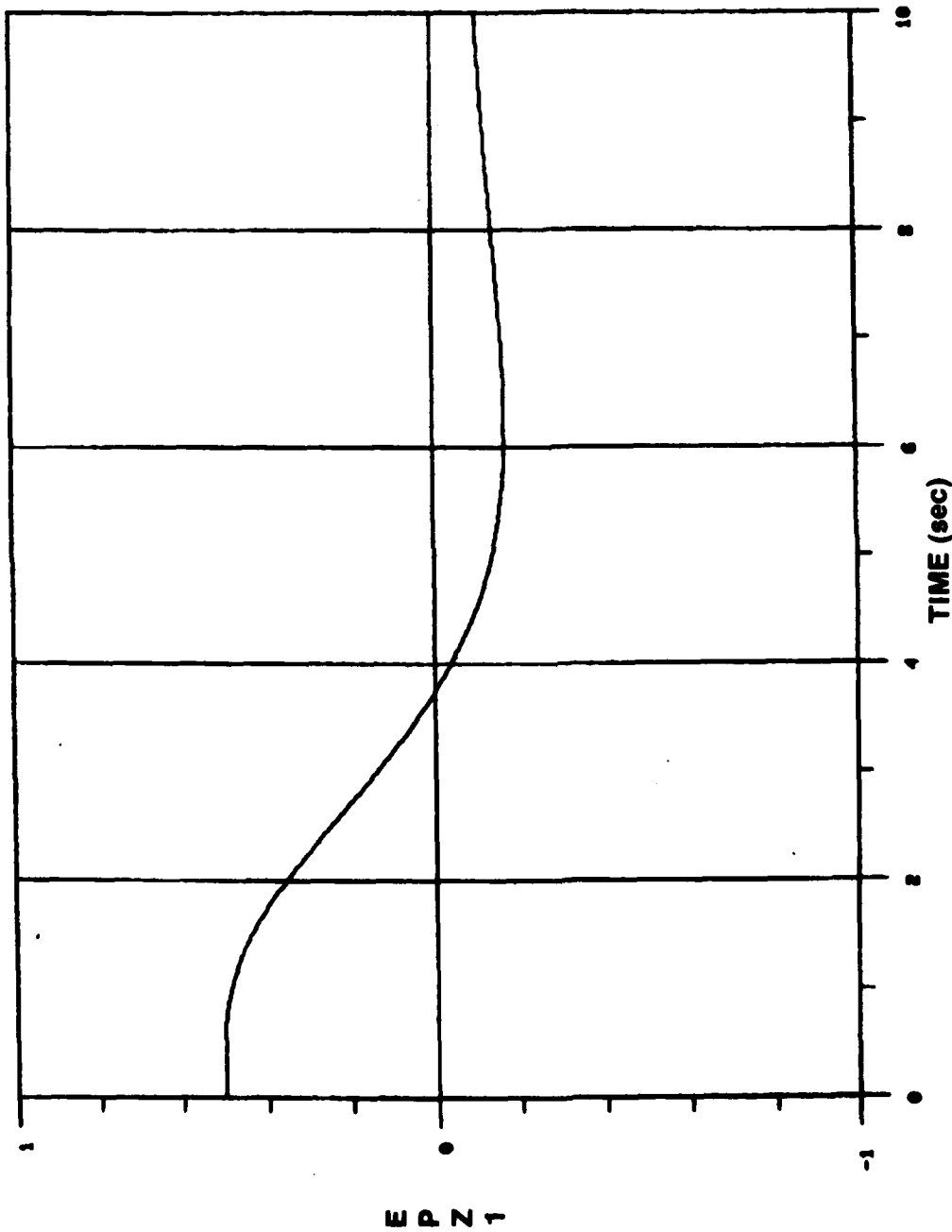


Figure 52. DAC disturbance estimation error, $t_f = 66.7$ sec, $PGO = 0.5$, $W_1 = 0.5$, +20% variation on airframe parameters.

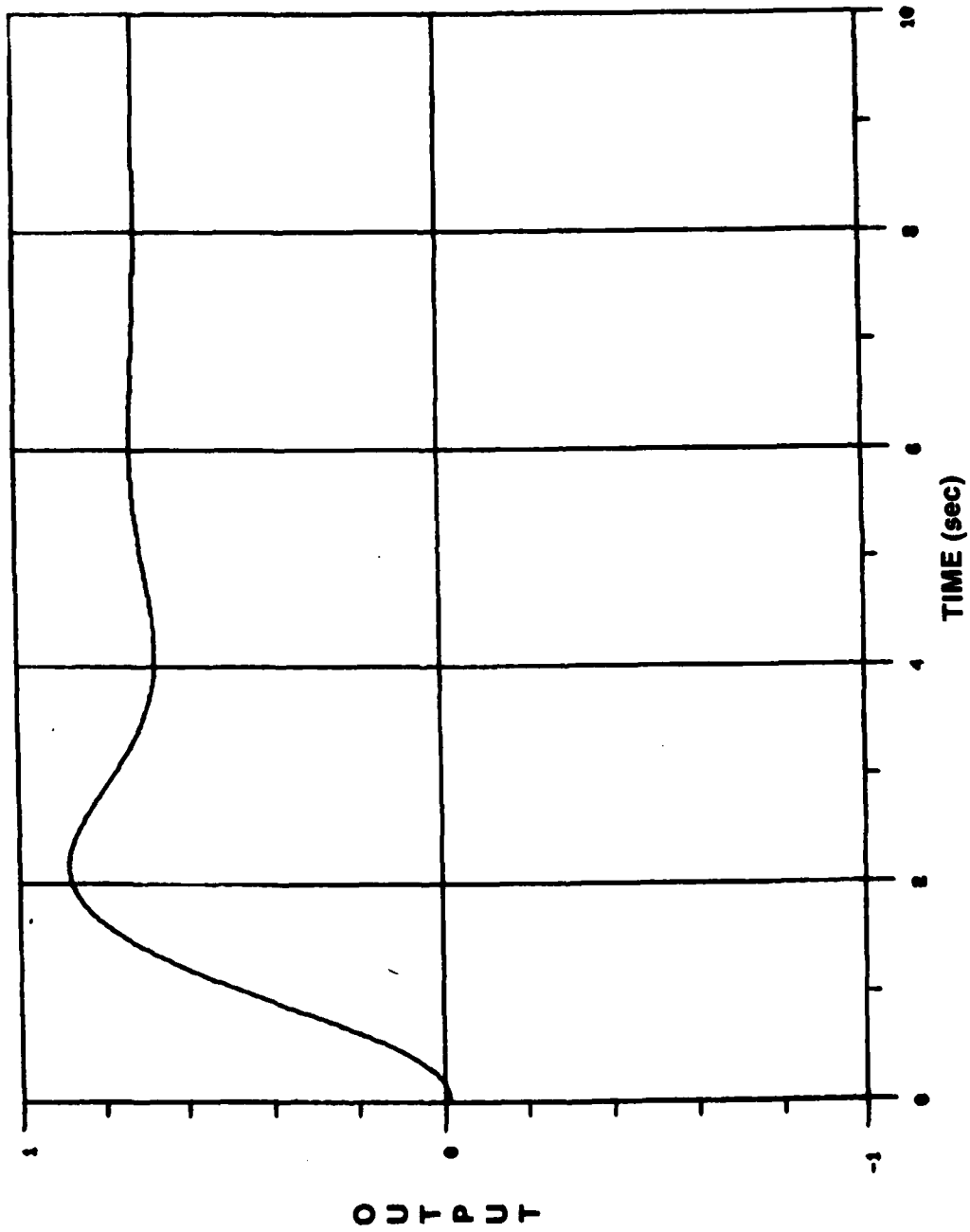


Figure 53. System output response, $t_f = 111.4$ sec, $PGO = 1.0$, $W_1 = 0.0$.

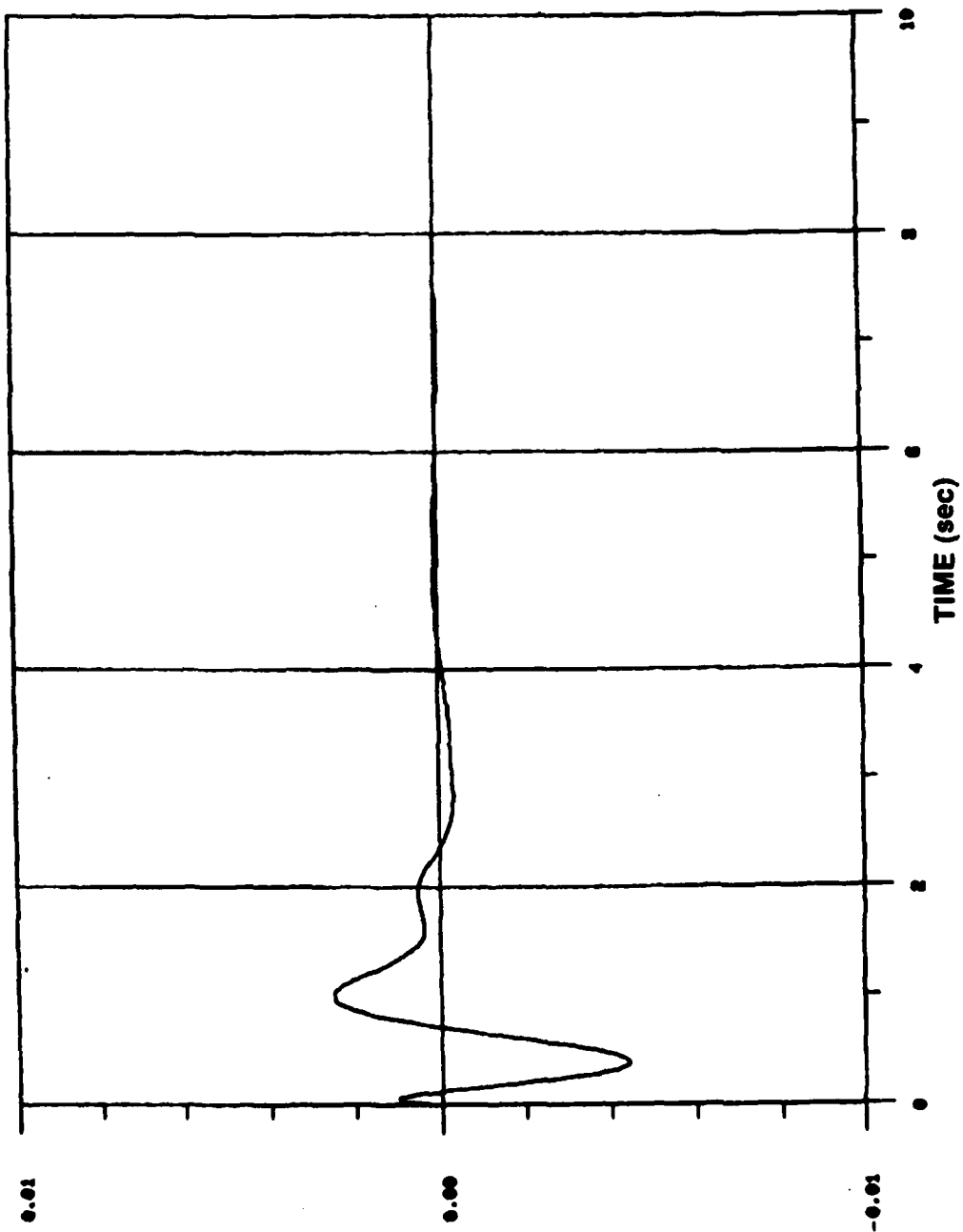


Figure 54. DAC disturbance estimation error, $t_f = 111.4$ sec, $PGO \approx 1.0$, $W_1 = 0.0$.

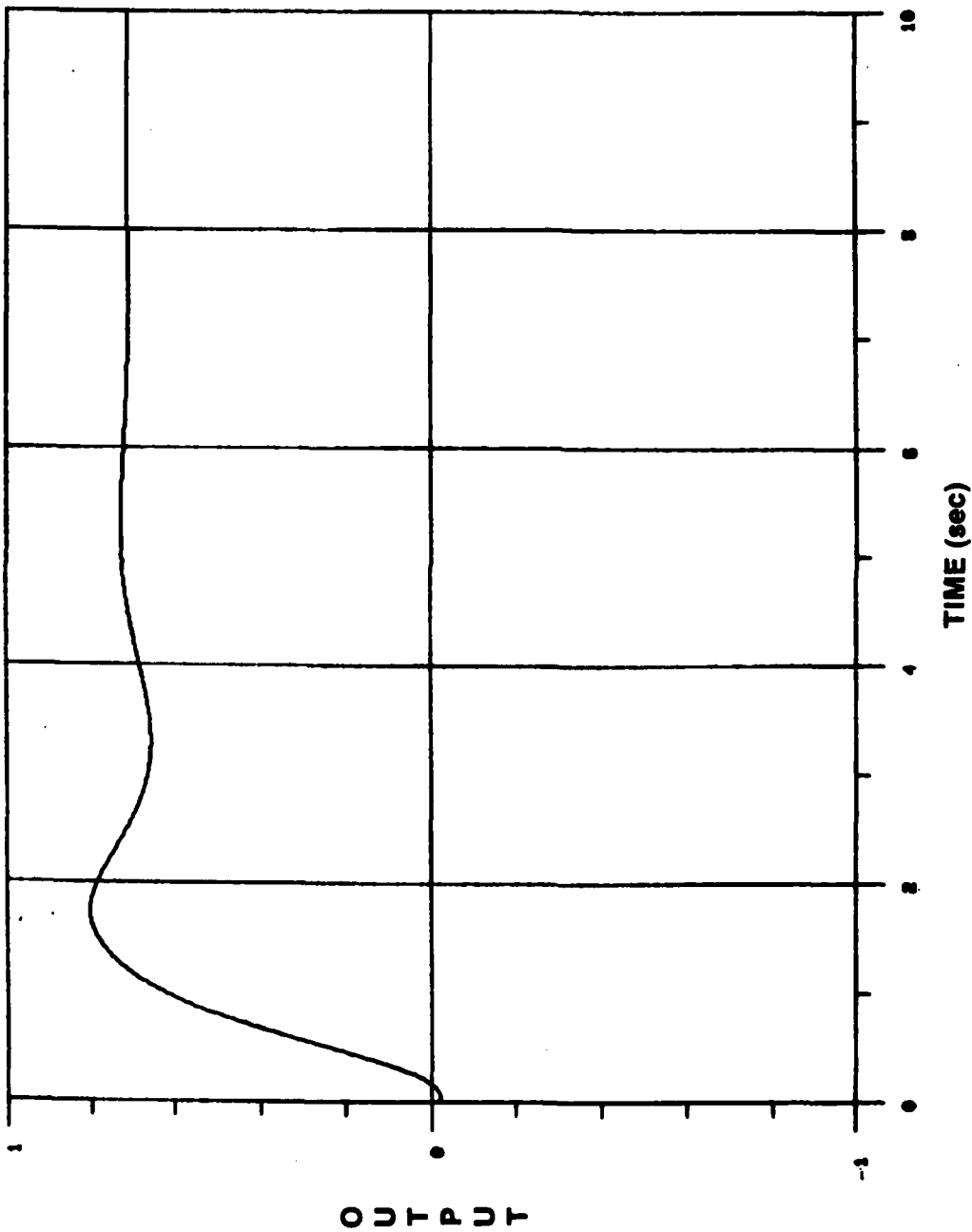


Figure 55. System output response, $t_f = 111.4$ sec, $PGO = 1.0$, $W_1 = 1.0$.

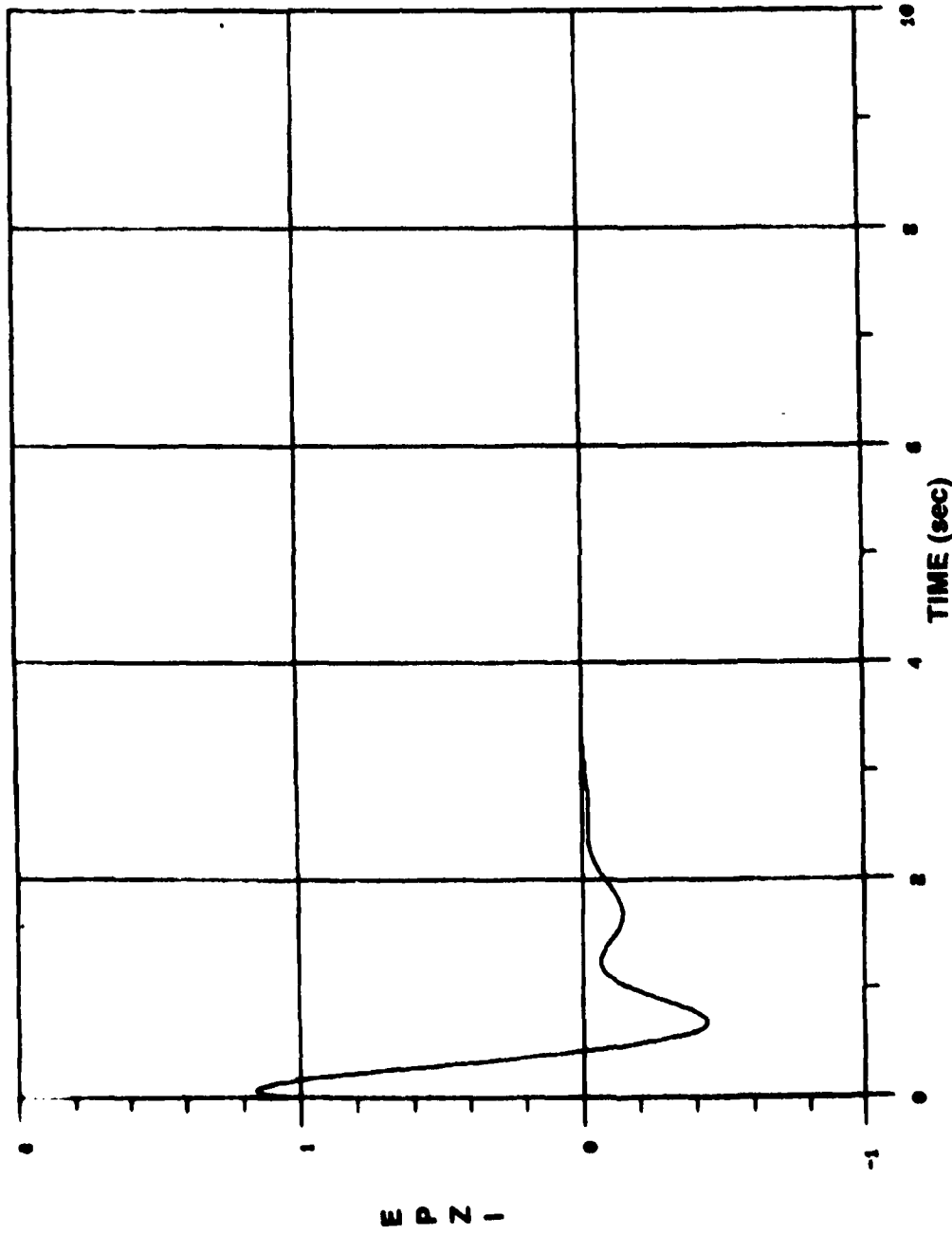


Figure 56. DAC disturbance estimation error, $t_f = 111.4$ sec, $PGC = 1.0$, $W_1 = 1.0$.

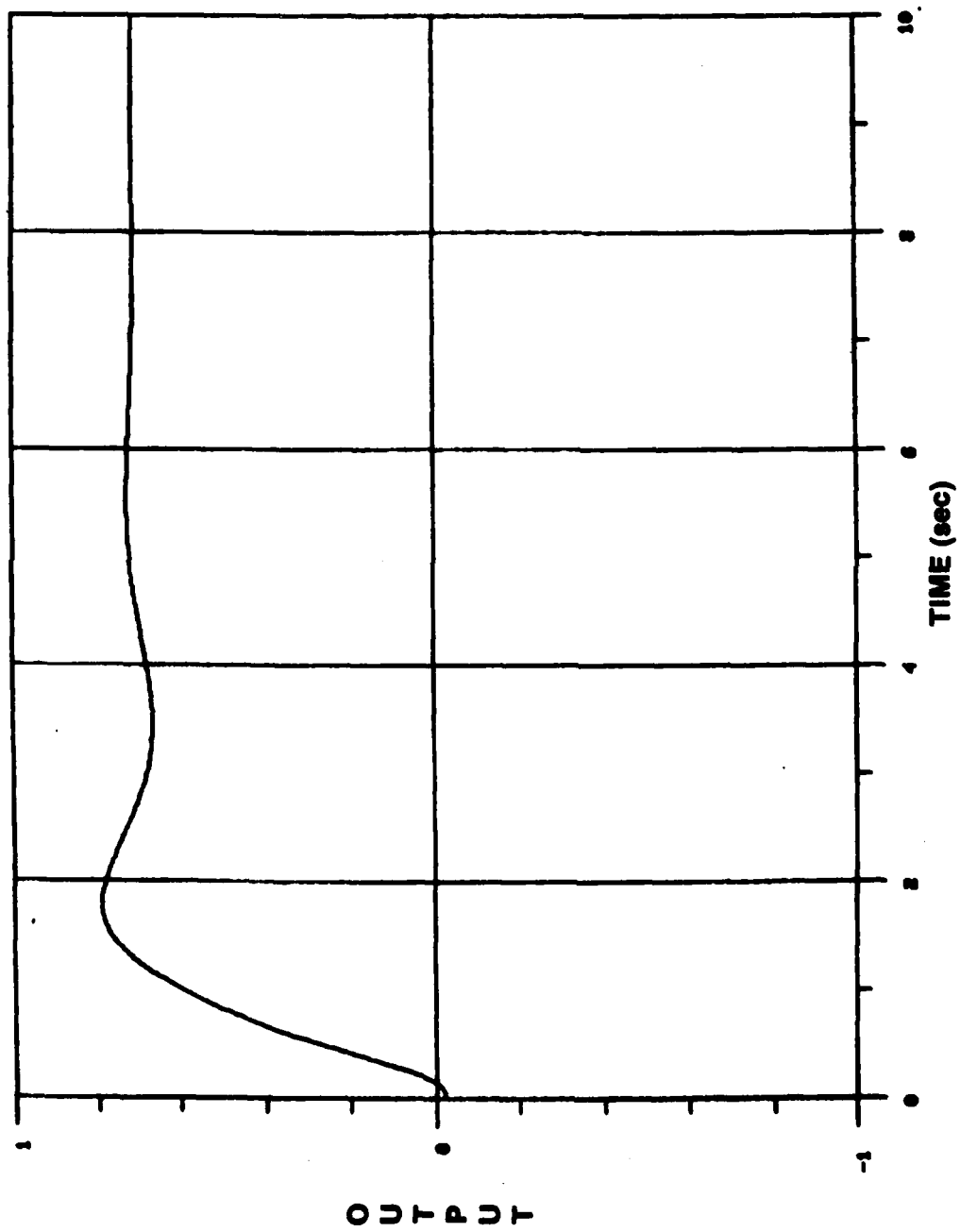


Figure 57. System output response, $t_f = 111.4$ sec, $PGO = 1.0$, $W_1 = 1.0$,
 +20% variation on airframe parameters.

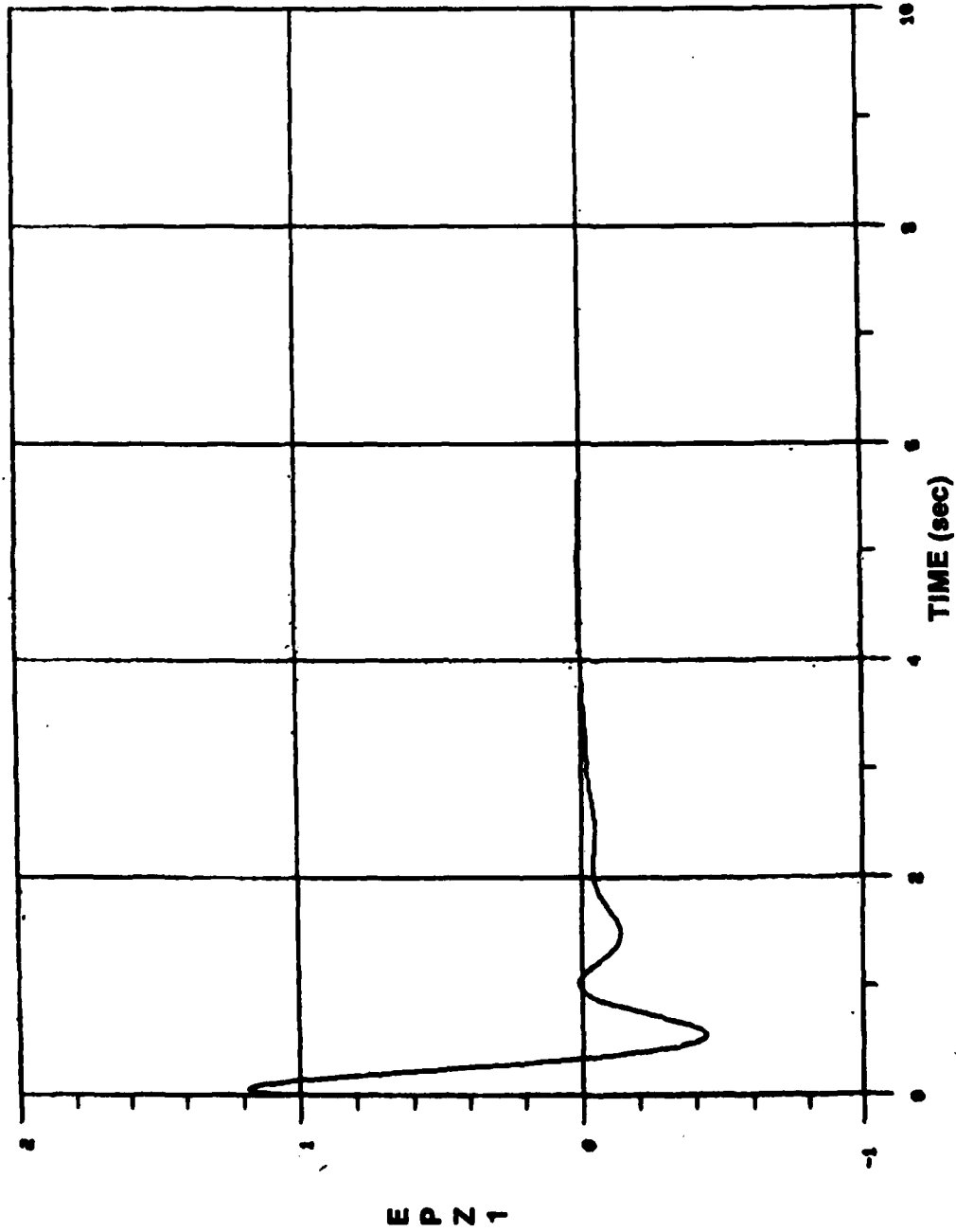


Figure 58. DAC disturbance estimation error, $t_f = 111.4$ sec, $PGO = 1.0$, $W_1 = 1.0$, +20% variation on airframe parameters.

The rate loop shown in *Figure 2* was rearranged and simplified as shown in *Figure 59*. The compensation term was reduced to C_R since no actuator dynamics are considered. The disturbance is shown as a rate imposed on the airframe in addition to that due to a given fin deflection. Therefore, the total body rate,

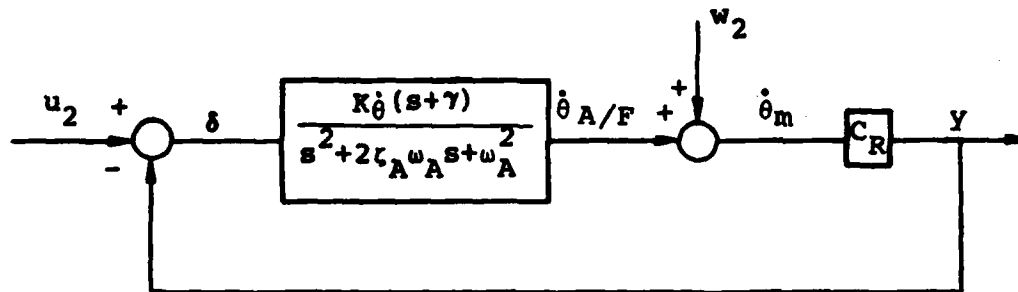


Figure 59. Rate loop block diagram.

assumed to be measured by an ideal rate gyro, would be

$$\dot{\theta}_m = \dot{\theta}_{A/F} + w_2$$

From *Figure 59* one has

$$\frac{\dot{\theta}_{A/F}(s)}{\delta(s)} = \frac{K_{\theta}(s+\gamma)}{s^2 + 2\zeta_A\omega_A s + \omega_A^2}$$

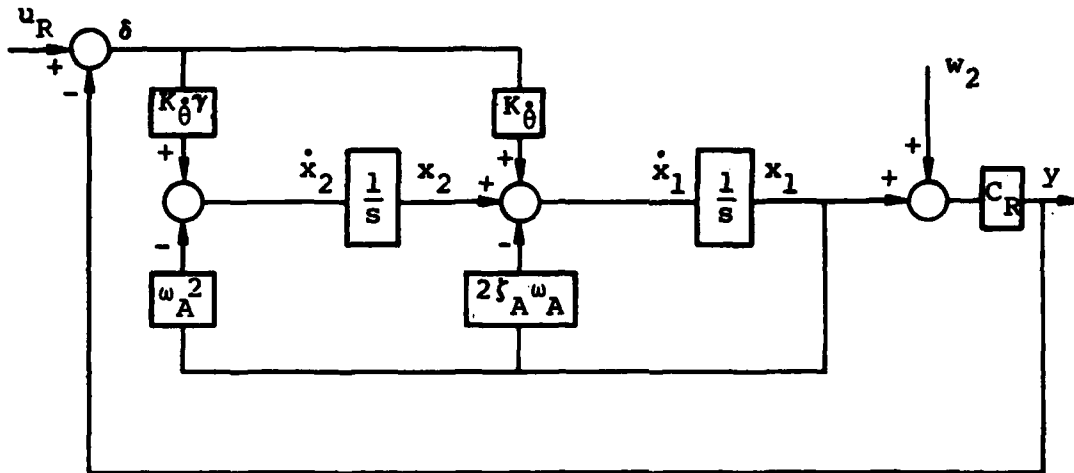
Therefore,

$$s^2\dot{\theta}_{A/F}(s) + 2\zeta_A\omega_A s\dot{\theta}_{A/F}(s) + \omega_A^2\dot{\theta}_{A/F}(s) = K_{\theta}s\delta(s) + K_{\theta}\gamma\delta(s)$$

and

$$\dot{\theta}_{A/F}(s) = \frac{1}{s} [K_{\theta}\delta(s) - 2\zeta_A\omega_A\dot{\theta}_{A/F}(s) + \frac{1}{s} (K_{\theta}\gamma\delta(s) - \omega_A^2\dot{\theta}_{A/F}(s))]$$

This appears, along with the rest of the loop, as



Writing the state space equations directly from this and substituting for δ gives:

$$\dot{x}_2 = K_\theta \gamma \delta - \omega_A^2 x_1 = -(K_\theta \gamma C_R - \omega_A^2) x_1 + K_\theta \gamma U_R - K_\theta \gamma C_R w_2$$

$$\begin{aligned} \dot{x}_1 = x_2 + K_\theta \delta - 2\zeta_A \omega_A x_1 = & -(K_\theta C_R + 2\zeta_A \omega_A) x_1 \\ & + x_2 + K_\theta U_R - K_\theta C_R w_2 \end{aligned}$$

$$y = (x_1 + w_2) C_R$$

$$\delta = u_R - y$$

(21)

Expressing these in the form (1),

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} -(K_\theta C_R + 2\zeta_A \omega_A) & 1 \\ -(K_\theta \gamma C_R + \omega_A^2) & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} K_\theta \\ K_\theta \gamma \end{bmatrix} u_R + \begin{bmatrix} -K_\theta C_R \\ -K_\theta \gamma C_R \end{bmatrix} w_2$$

$$\underline{y} = [C_R \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] u_R + [C_R] w_2$$

Now, in this case, $F \equiv B \Gamma$ for $\Gamma = [-C_R]$. Therefore, the theoretical total absorption controller would be $u_c = [C_R] w_2$. If this is implemented, however, it does not remove w_2 from the output, y . But, the desired action in this case is to remove any disturbance rates imposed on the missile so that it will maintain a given attitude during boost. On examining the plant, it can be seen that the disturbance rate w_2 imposed on the body can be related to an "equivalent" fin deflection δ_D , i.e., it can be considered as an additional body rate which would have resulted from an additional fin deflection command δ_D . In other words,

$$\dot{\theta}_m = \dot{\theta}_{A/F} + w_2 = (\delta + \delta_D) \frac{\dot{\theta}}{\delta}$$

So, if the proper gain can be found, and if a state observer can be designed which will reconstruct \hat{z}_1 , then $u_c = -C_{RG} \hat{z}_1$ can perhaps be used as a partial absorption (minimization) control on the effects of w_2 , where C_{RG} is the sought gain.

Again, Equation (4) is used to obtain the gain matrices K_1 , K_2 . In this case,

$$\dot{\underline{\epsilon}} = \left[\begin{array}{c|c} \left[\begin{array}{cc} -(K_{\dot{\theta}} C_R + 2\zeta_A \omega_A) & 1 \\ -(K_{\dot{\theta}} \gamma C_R + \omega_A^2) & 0 \end{array} \right] + \left[\begin{array}{c} k_{11} \\ k_{21} \end{array} \right] [C_R \ 0] & \left[\begin{array}{cc} k_{11} - K_{\dot{\theta}} & \\ k_{21} - K_{\dot{\theta}} \gamma & \end{array} \right] [C_R \ 0] \\ \left[\begin{array}{c} k_{12} \\ k_{22} \end{array} \right] [C_R \ 0] & \left[\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array} \right] + \left[\begin{array}{c} k_{12} \\ k_{22} \end{array} \right] [C_R \ 0] \end{array} \right] \underline{\epsilon} + \begin{pmatrix} 0 \\ \sigma \end{pmatrix} \quad (22)$$

so,

$$\begin{pmatrix} \dot{\epsilon}_x \\ \dot{\epsilon}_z \end{pmatrix} = \begin{bmatrix} k_{11} C_R - 2\zeta_A \omega_A - K_{\dot{\theta}} C_R & 1 & (k_{11} - K_{\dot{\theta}}) C_R & 0 \\ k_{21} C_R - \omega_A^2 - K_{\dot{\theta}} \gamma C_R & 0 & (k_{21} - K_{\dot{\theta}} \gamma) C_R & 0 \\ k_{12} C_R & 0 & k_{12} C_R & 1 \\ k_{22} C_R & 0 & k_{22} C_R & 0 \end{bmatrix} \begin{pmatrix} \epsilon_x \\ \epsilon_z \end{pmatrix} + \begin{pmatrix} 0 \\ \sigma \end{pmatrix} \quad (23)$$

Let Equation (23) be written

$$\underline{\dot{\epsilon}} = \underline{\tilde{B}}\underline{\epsilon} + \begin{pmatrix} 0 \\ \underline{\sigma} \end{pmatrix}$$

and $\underline{\tilde{B}}$ be written as

$$\underline{\tilde{B}} = \begin{bmatrix} B_1 & 1 & B_5 & 0 \\ B_2 & 0 & B_6 & 0 \\ B_3 & 0 & B_7 & 1 \\ B_4 & 0 & B_8 & 0 \end{bmatrix}$$

Solve for the eigenvalues of $\underline{\tilde{B}}$.

$$\det |\underline{\tilde{B}} - \lambda \underline{I}| = 0$$

$$\det |\underline{\tilde{B}} - \lambda \underline{I}| = \begin{vmatrix} B_1 - \lambda & 1 & B_5 & 0 \\ B_2 & -\lambda & B_6 & 0 \\ B_3 & 0 & B_7 - \lambda & 1 \\ B_4 & 0 & B_8 & -\lambda \end{vmatrix} = 0 .$$

This can be expanded to give

$$\begin{aligned} & \lambda^4 - (B_1 + B_3)\lambda^3 + (B_1B_3 - B_1B_4 - B_3B_5 - B_2) \lambda^2 \\ & + (B_1B_4 - B_4B_5 + B_2B_3 - B_3B_6) \lambda \\ & + (B_4B_2 - B_6B_4) = 0 . \end{aligned} \quad (24)$$

If the desired roots of Equation (24) are $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, then the desired characteristic equation is

$$(\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3)(\lambda - \lambda_4) = 0 . \quad (25)$$

Expanding Equation (25), equating coefficients of like powers of λ between Equations (24) and (25), substituting back in for B_1 through B_4 and solving for k_{11} , k_{21} , k_{12} , k_{22} gives

$$k_{11} = (2\zeta_A \omega_A - k_{12} C_R + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + K_\theta C_R) / C_R$$

$$k_{21} = [2\zeta_A \omega_A C_R (k_{12} - k_{22}) + k_{11} k_{12} C_R^2 - \omega_A^2 - K_\theta k_{22} C_R^2 + \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_1 \lambda_4 + \lambda_2 \lambda_3 + \lambda_2 \lambda_4 + \lambda_3 \lambda_4] / (-C_R)$$

$$k_{12} = (-2\zeta_A \omega_A C_R k_{22} + \lambda_1 \lambda_3 \lambda_4 + \lambda_2 \lambda_3 \lambda_4 + \lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \lambda_4) / (C_R \omega_A^2)$$

$$k_{22} = -\lambda_1 \lambda_2 \lambda_3 \lambda_4 / C_R \omega_A^2$$

Again, the λ 's are picked such that $\epsilon(t) \rightarrow 0$ rapidly. With these, the gains can be determined.

The full-dimensional observer, in the form of Equation (3), for this case is

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{z}}_1 \\ \dot{\hat{z}}_2 \end{bmatrix} = \begin{bmatrix} k_{11} C_R - 2\zeta_A \omega_A & 1 & k_{11} C_R & 0 \\ k_{21} C_R - \omega_A^2 & 0 & k_{21} C_R & 0 \\ k_{12} C_R & 0 & k_{12} C_R & 1 \\ k_{22} C_R & 0 & k_{22} C_R & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} - \begin{bmatrix} k_{11} \\ k_{21} \\ k_{12} \\ k_{22} \end{bmatrix} y + \begin{bmatrix} K_\theta \\ K_\theta \gamma \\ 0 \\ 0 \end{bmatrix} \delta \quad (26)$$

So, the question posed here is: Can the proposed gain C_{R0} be found so that, in conjunction with the state reconstructor, the effects of y_2 can be minimized?

For answers to these questions, a simulation is again used.

B. SIMULATION AND RESULTS

Figure 60 is a diagram of the composite plant-DAC system which was simulated on a digital computer. A listing of this simulation is shown in Appendix B.

The controller in this loop, with disturbance as shown, would probably be used only during the attitude controlled boost phase of this missile. However, for illustrative purposes, two of the time points shown in Table 1 (9.85 sec, 135.8 sec) will be used for this investigation.

Figures 61 through 72 present the results obtained for the time points above. For each point, three runs are presented: (1) nominal run with no disturbance; (2) run with w_2 equal to the plant steady state response (x_1) due to the input command, from (1), but with $C_{RG} = 0$; (3) same as (2) except C_{RG} is given an appropriate value. This value is determined from the ratio $(\delta/x_1)_u$ from the undisturbed case, as would be expected.

For the 9.85 sec point, $C_{RG} = -3.54$, and it can be seen from comparison of Figures 63 and 65 that the effects of w_2 are largely removed. For the $t = 135.8$ sec point, $C_{RG} = -1.45$, and similar conclusions are reached (compare Figures 69 and 71).

The gain matrix components for the DAC in the two cases above are given in Table 5 where the eigenvalues of matrix \underline{B} were taken to be $\lambda_1 = -3$, $\lambda_2 = -5$, $\lambda_3 = -4 + j1$, $\lambda_4 = -4 - j1$.

TABLE 5. DAC GAIN COMPONENTS

TIME POINT (SEC)	9.85	135.8
k_{11}	101.18	29.55
k_{21}	-726.94	72.5
k_{12}	8.11	5.47
k_{22}	7.62	4.59

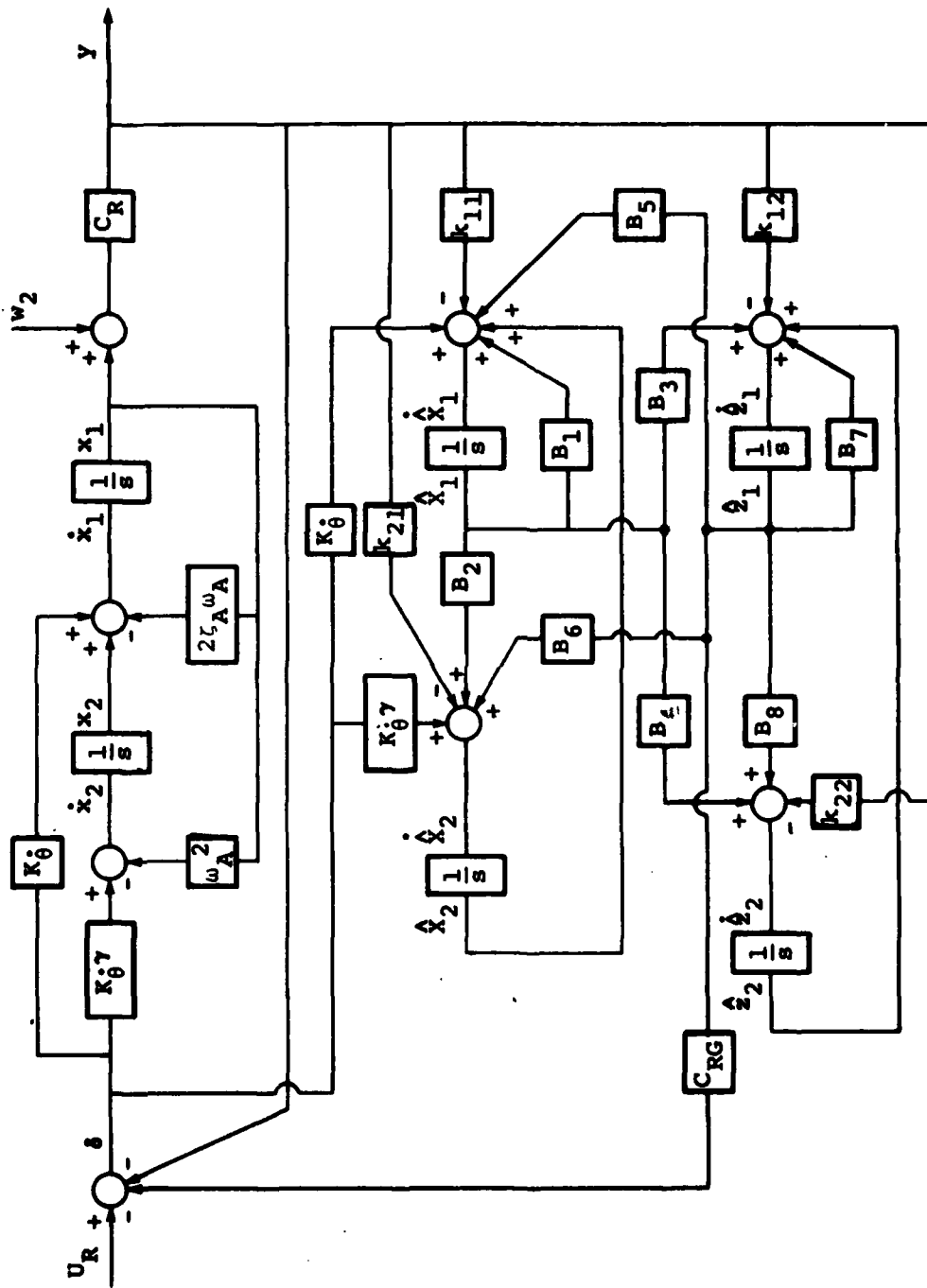


Figure 60. Plant-DAC composite diagram for rate loop with disturbance at output.

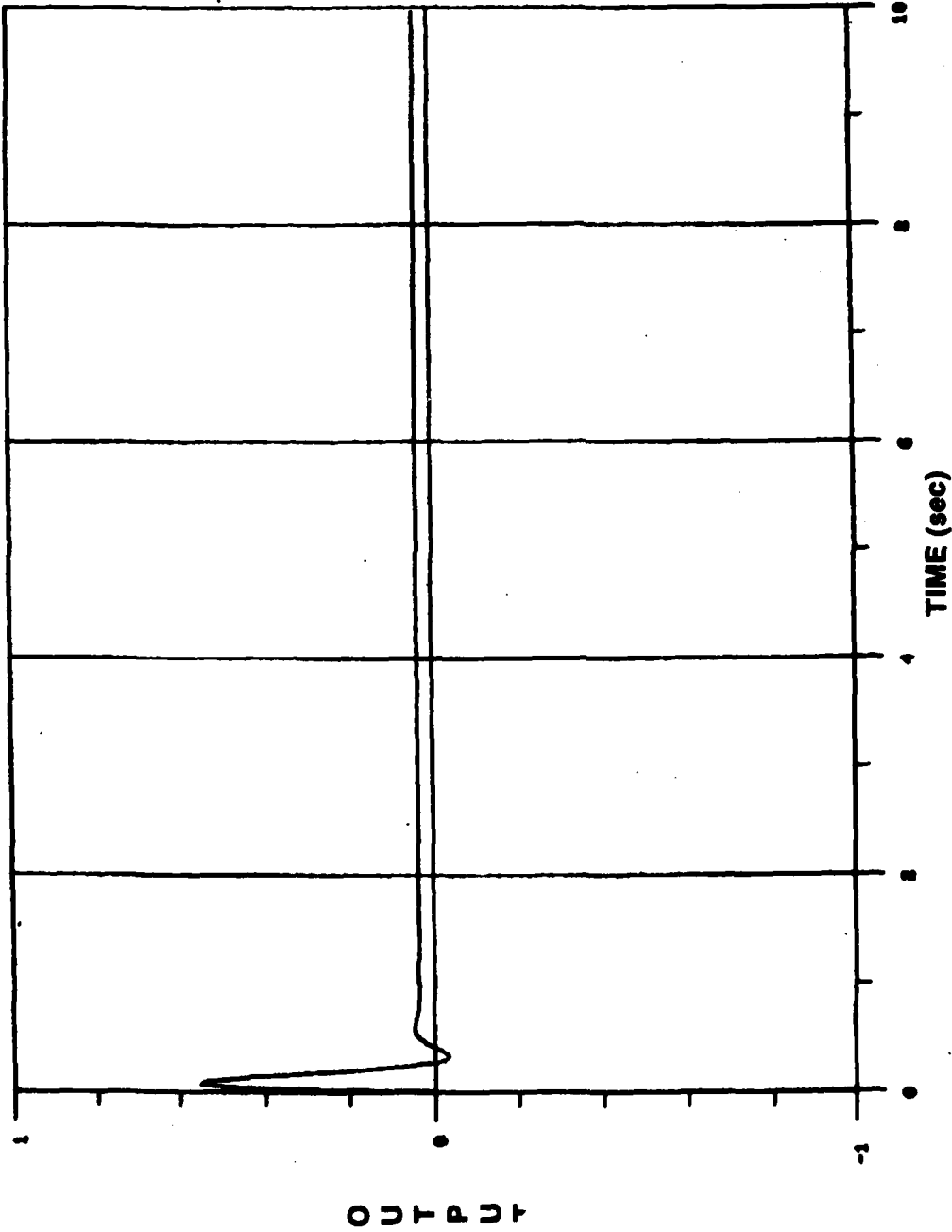


Figure 61. Rate loop response, $t_f = 9.85$ sec, $U_R = 1$, $W_2 = 0$.

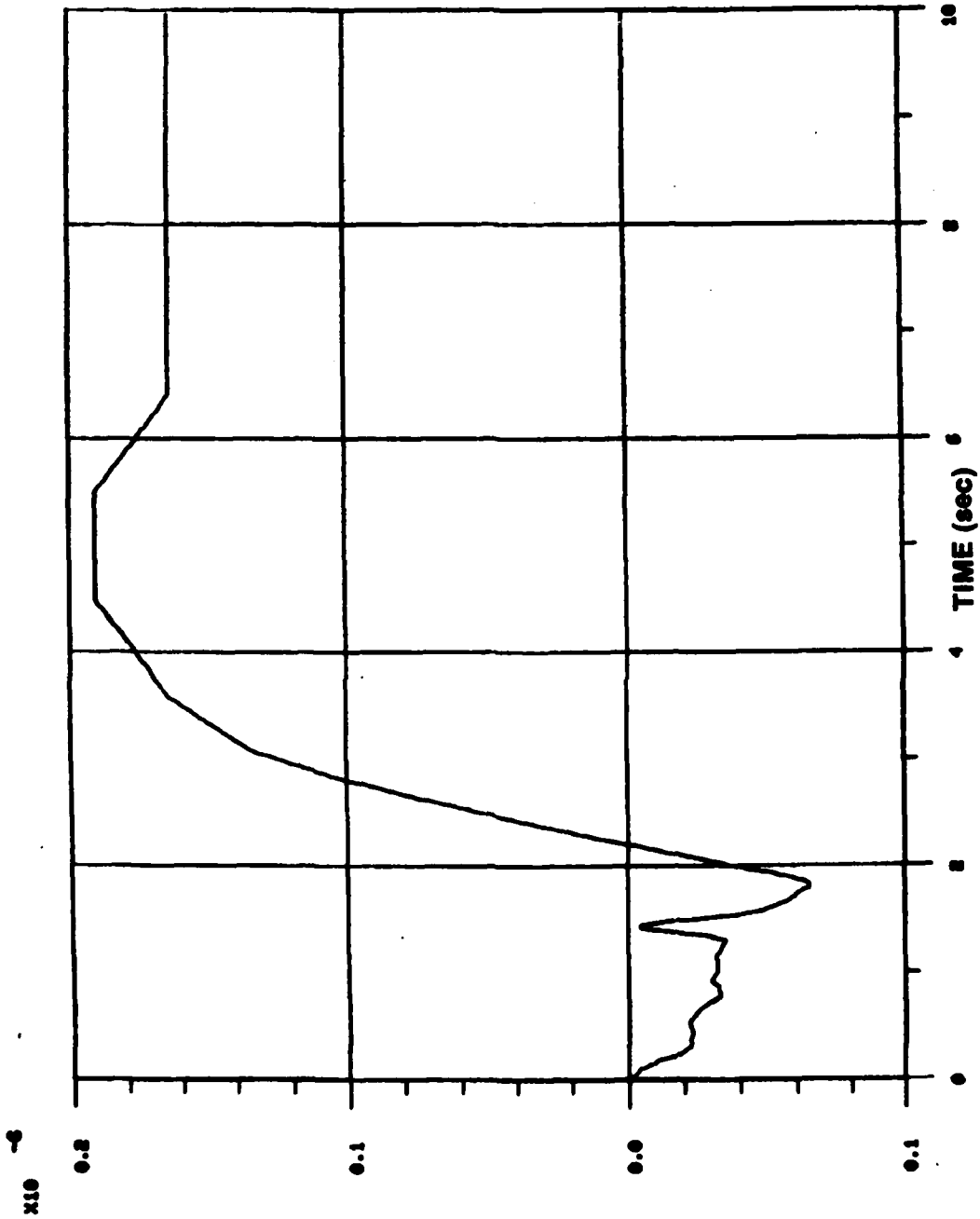


Figure 62. DAC disturbance estimation error, $t_1 = 9.85$ sec, $U_R = 1$, $W_2 = 0$.

E P Z 1

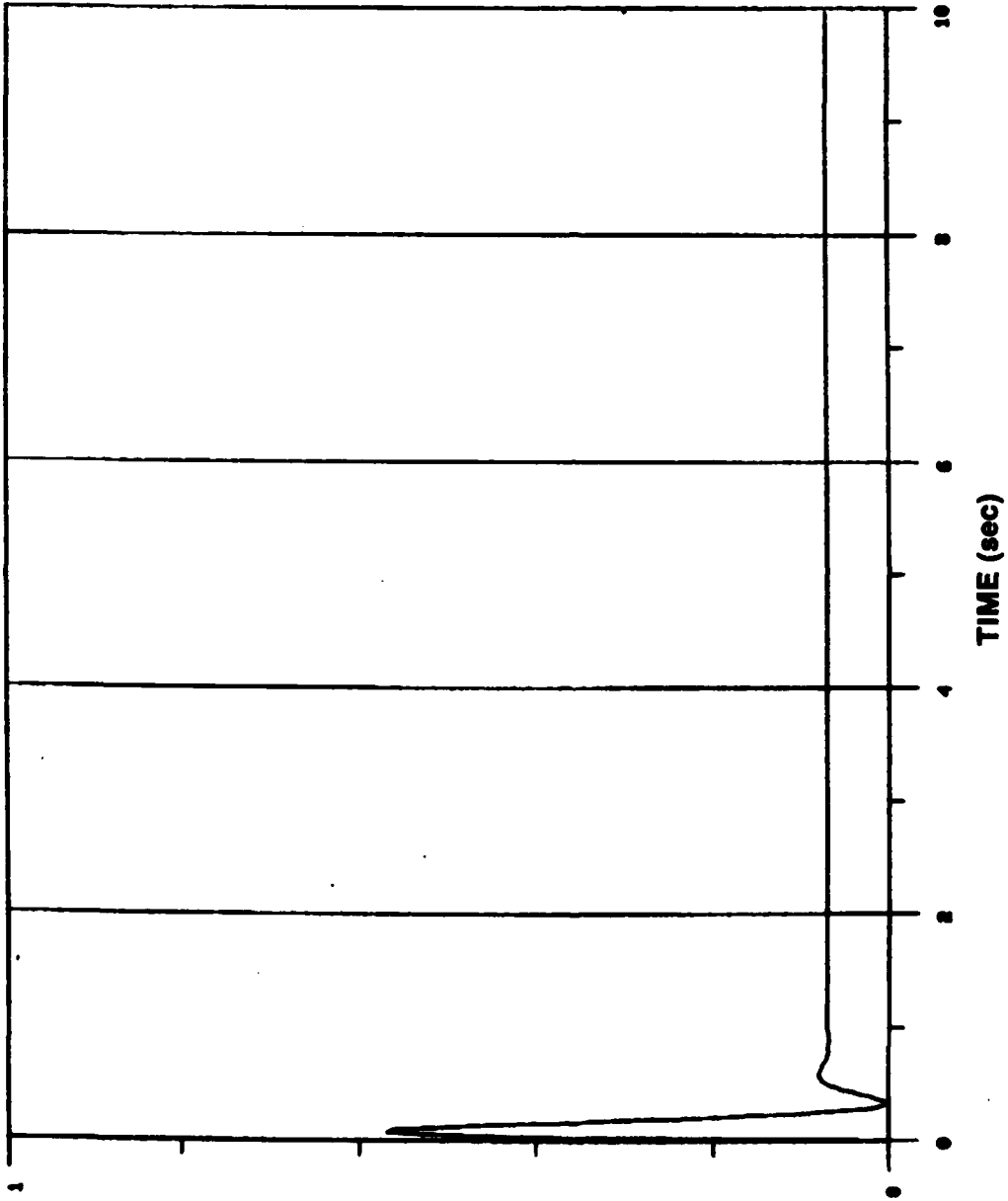


Figure 63. Rate loop response, $t_f = 9.85$ sec, $U_R = 1$, $W_2 = 0.26$, $CRG = 0$.

OUTPUT

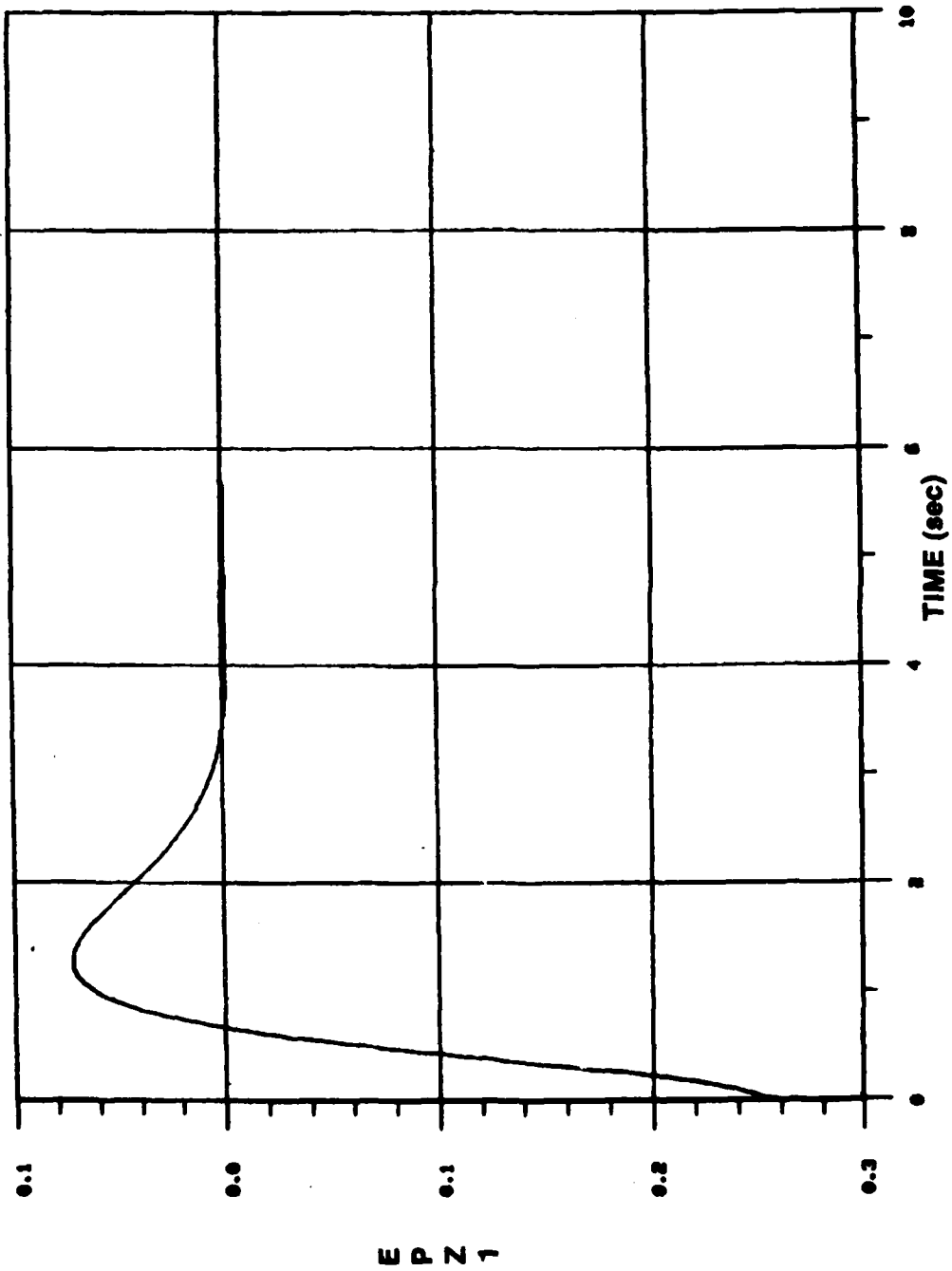
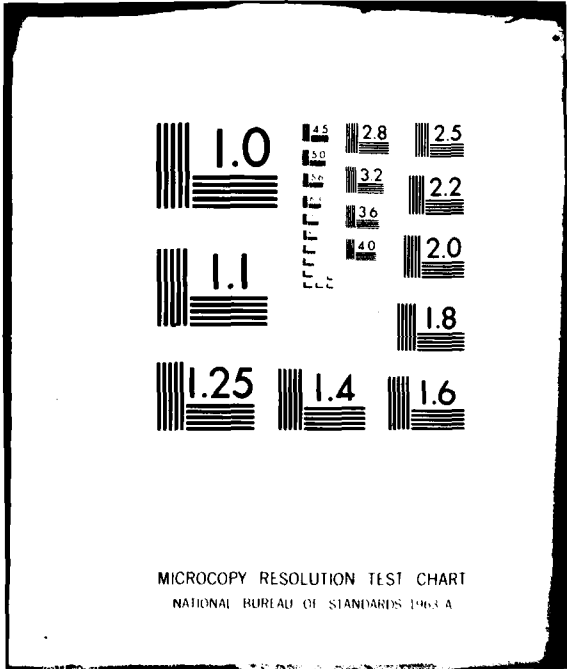


Figure 64. DAC disturbance estimation error, $t_f = 9.85$, $CRG = 0$, $U_R = 1$, $W_2 = -0.26$.



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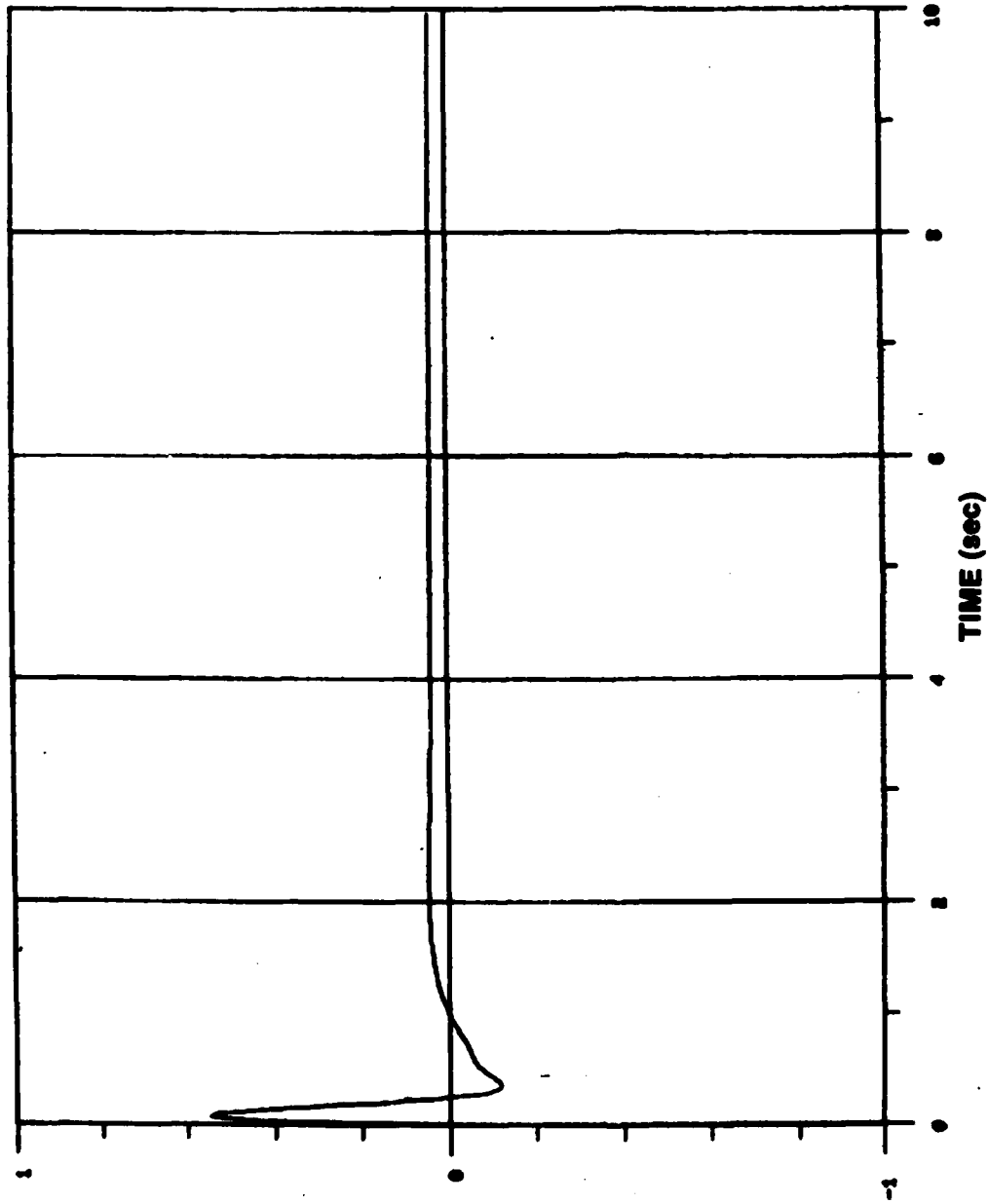


Figure 65. Rate loop response, $t_1 = 9.85$ sec, $U_R = 1$, $W_2 = -0.26$, $CRG = -3.54$.

OUTPUT

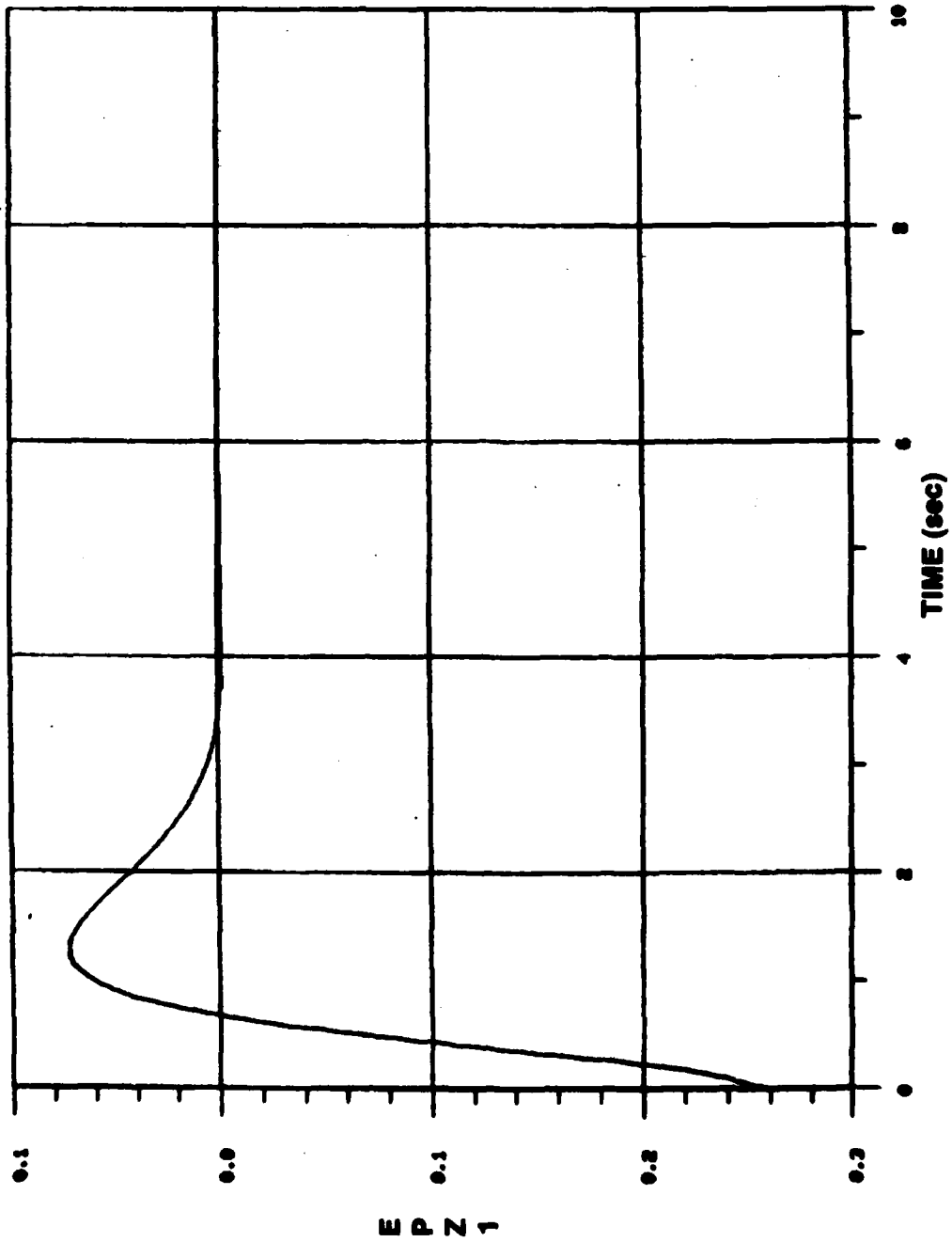


Figure 66. DAC disturbance estimation error, $t_f = 9.85$ sec, $U_R = 1$, $W_2 = -0.26$, $CRG = -3.54$.

EPZ1

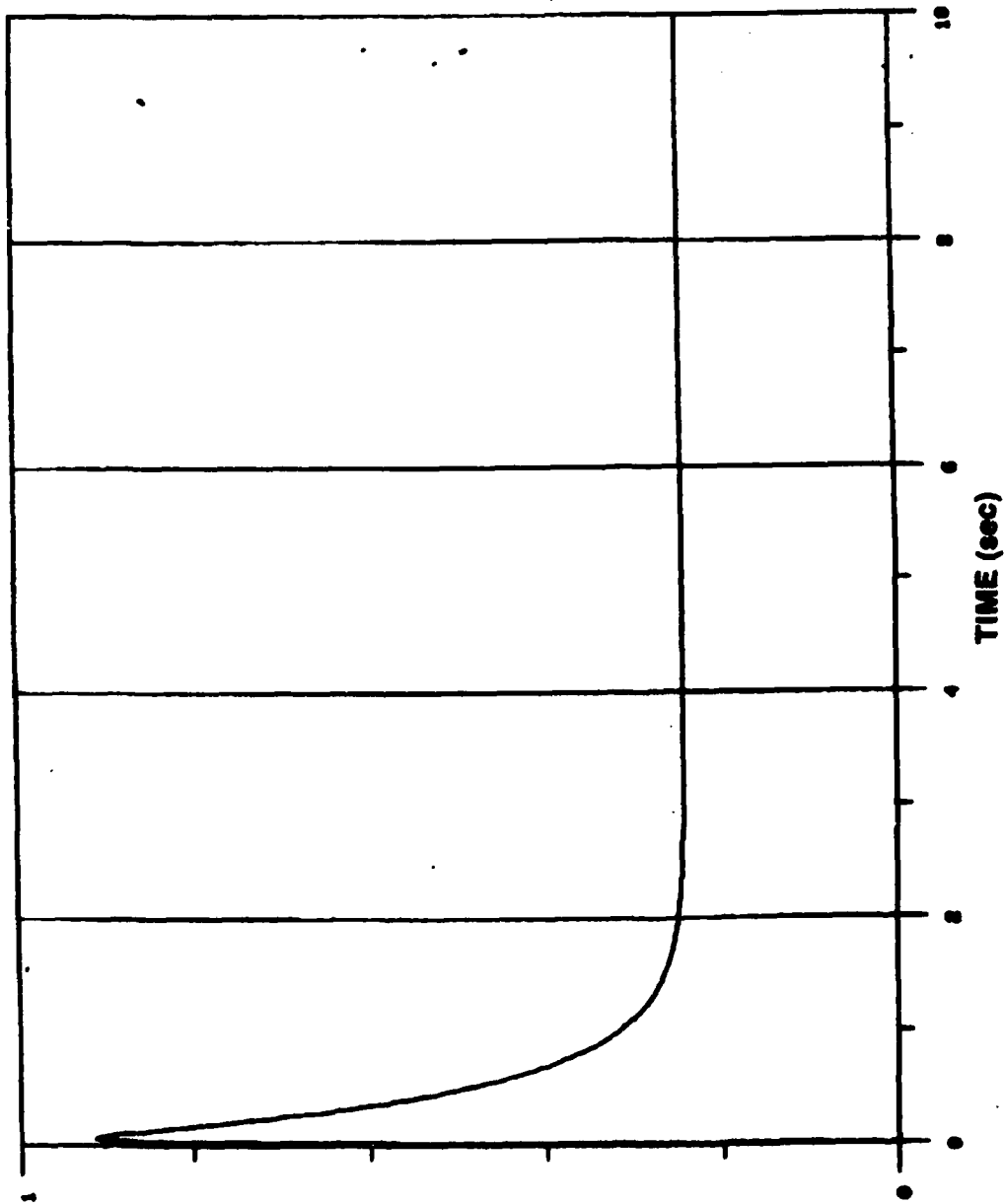
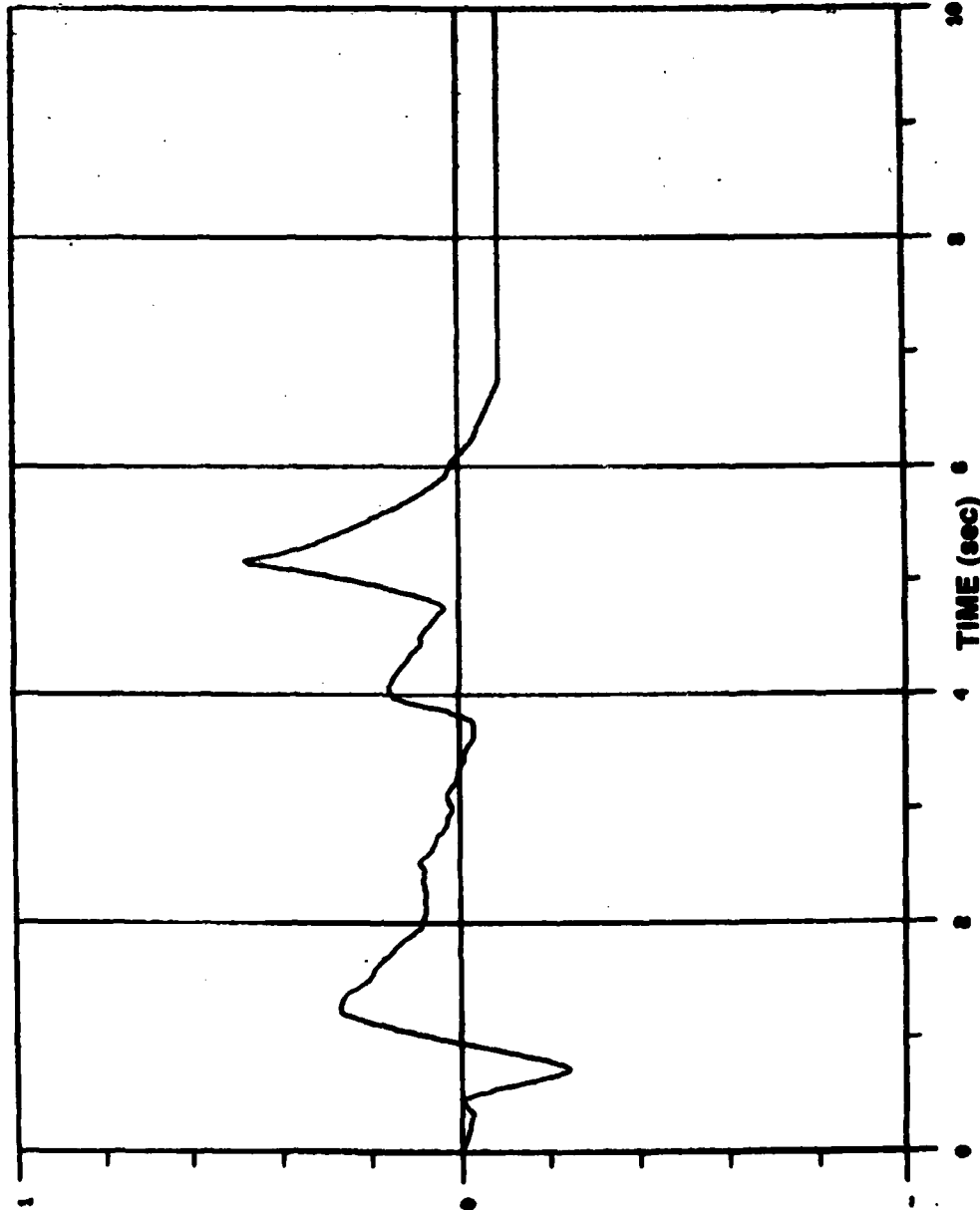


Figure 67. Rate loop response, $t_f = 135.8$ sec, $U_R = 1$, $W_2 = 0$.

OUTPUT

x10⁻⁶



EPZ 1

Figure 68. DAC disturbance estimation error, $\gamma = 135.8$ sec, $U_R = 1$, $W_2 = 0$.

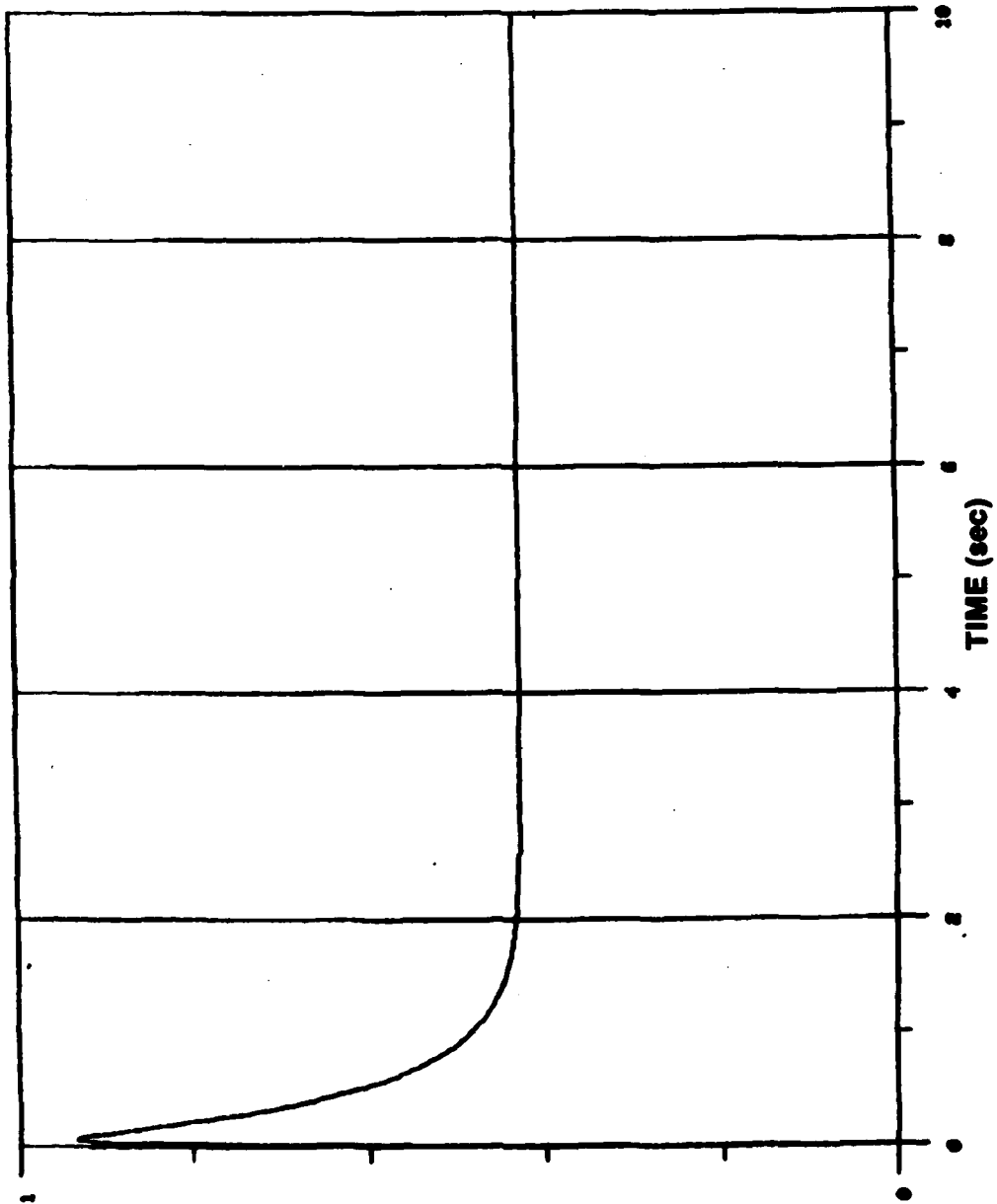


Figure 69. Rate loop response, $t_f = 135.8$ sec, $U_R = 1$, $W_2 = -0.5588$, $CRG = 0$.

OUTPUT

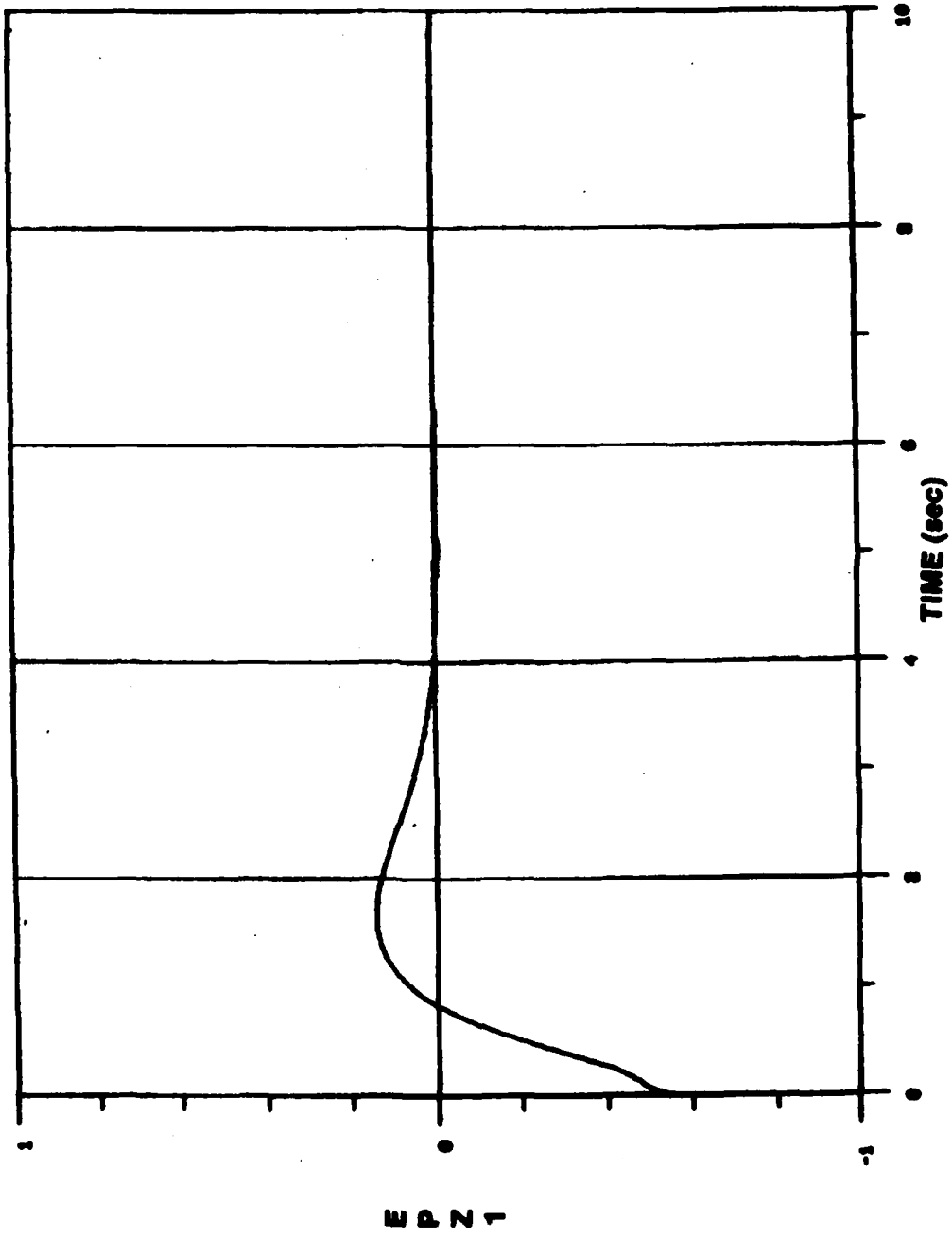


Figure 70. DAC disturbance estimation error, $t_f = 135.8$ sec, $U_R = 1$, $W_2 = -0.5588$, $CRG = 0$.

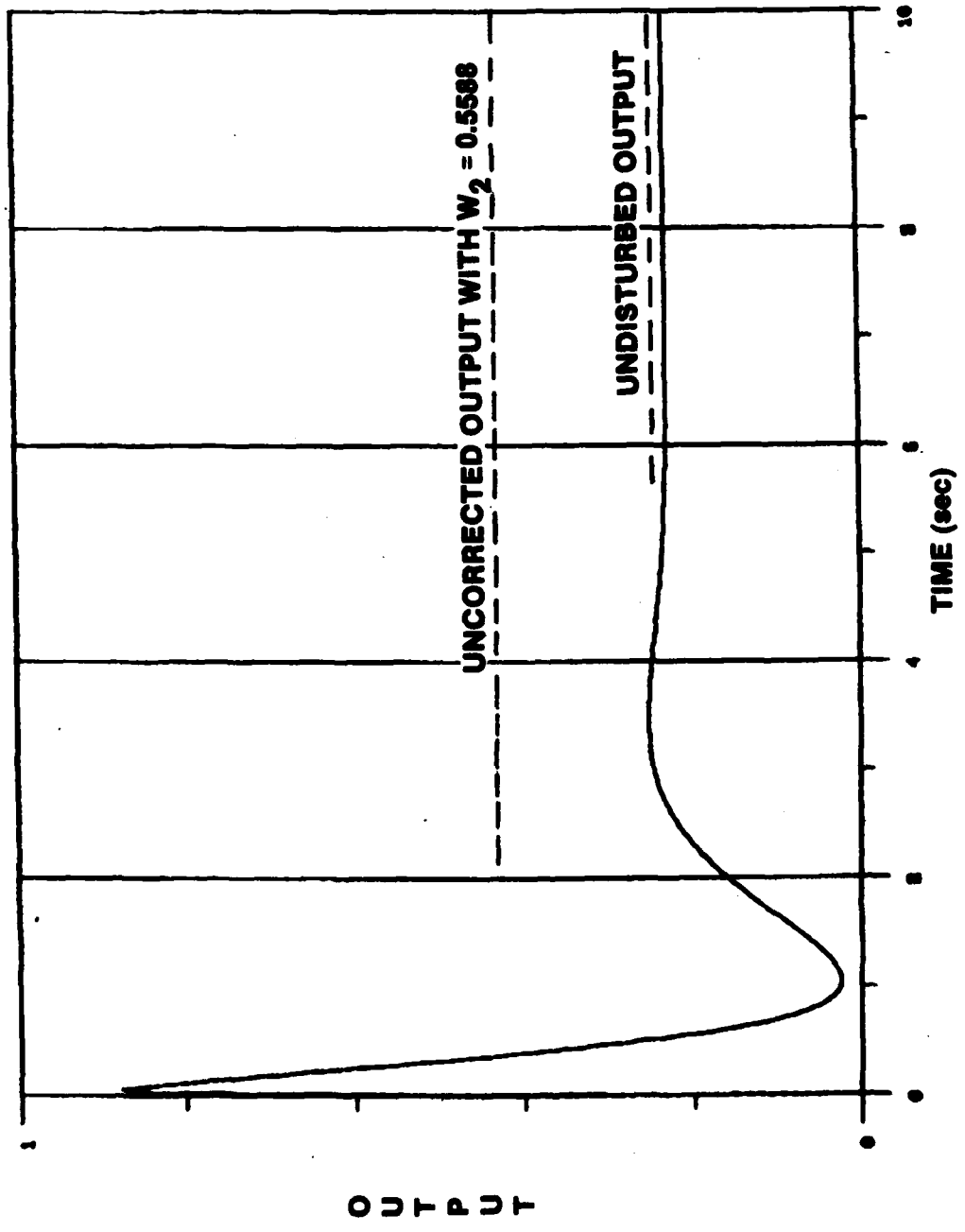


Figure 71. Rate loop response, $t_f = 135.8$ sec, $U_R = 1$, $W_2 = -0.5588$, $CRG = -1.45$.

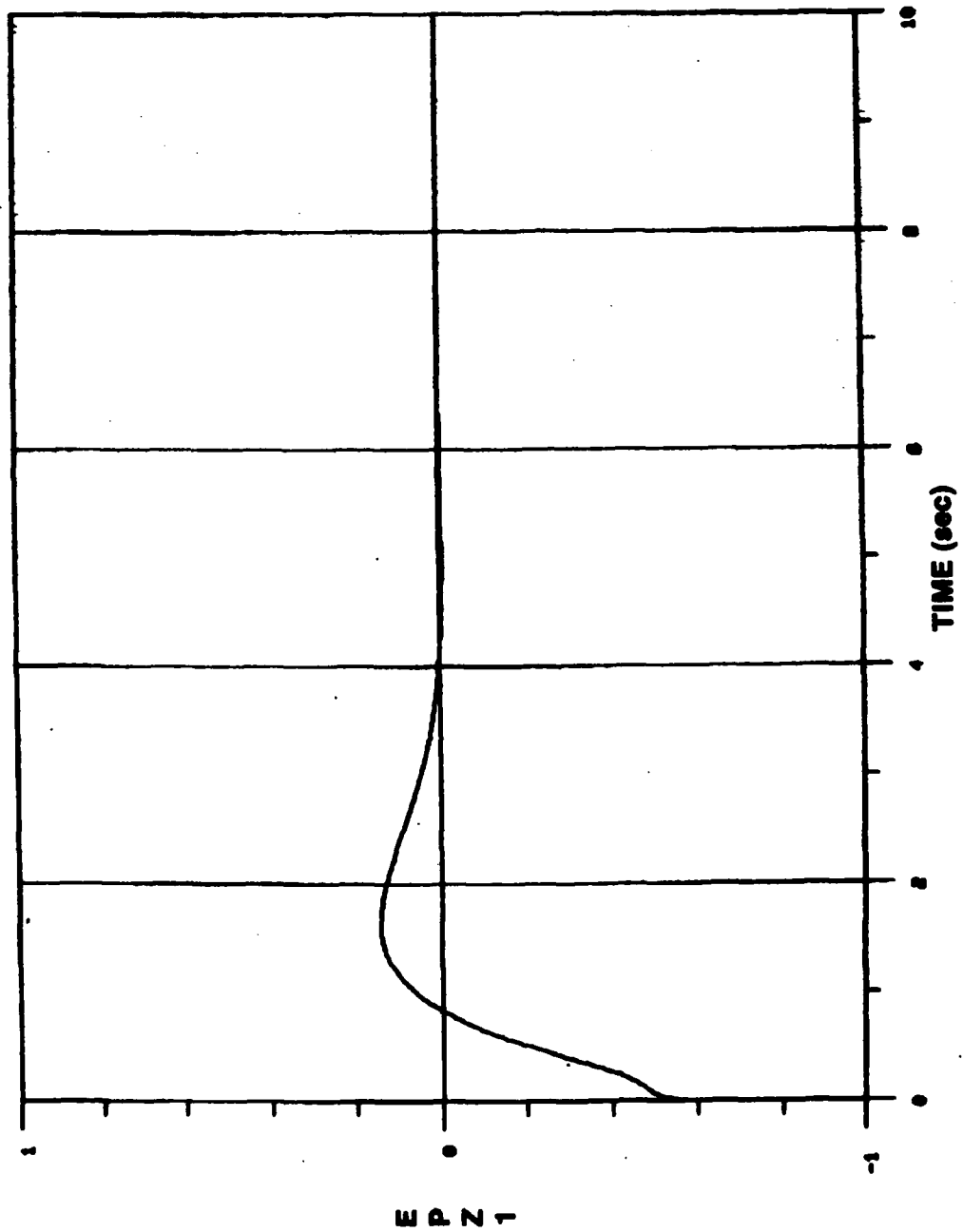


Figure 72. DAC disturbance estimation error, $t_f = 135.8 \text{ sec}$, $U_R = 1$, $W_2 = -0.5588$, $CRG = -1.45$.

C. CONCLUSIONS

From the results obtained here, even though there was no total absorption control u_c which would exactly cancel w_2 , by using a state reconstructor to estimate the value of the disturbance an implementation was possible whereby the effects of w_2 could be minimized.

7. ACCELERATION LOOP WITH DISTURBANCE ON OUTPUT

A. DAC MODEL DEVELOPMENT

As with the rate loop development in Section 6, this section will consider again a disturbance summed with the output, this time for the acceleration loop. This case is of much interest since the acceleration autopilot is used in the control loop from burnout onwards. A block diagram of this loop is shown in *Figure 73*.

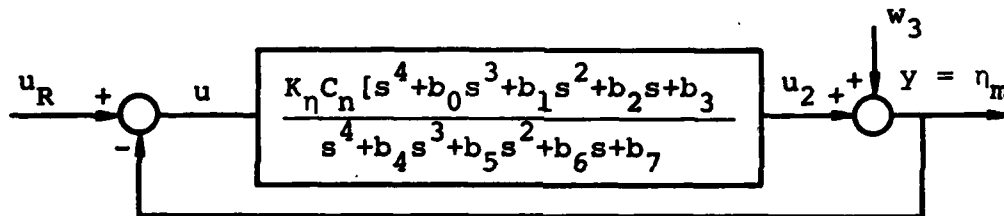


Figure 73. Acceleration loop with disturbance on output.

The transfer function from u to u_2 can be represented identically as shown in *Figure 3* with the same states and parameters. However, in this case, the matrix representation in the form of Equation (1) will be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -b_4 & 1 & 0 & 0 \\ -b_5 & 0 & 1 & 0 \\ -b_6 & 0 & 0 & 1 \\ -b_7 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + K_n C_n \begin{bmatrix} b_0 - b_4 \\ b_1 - b_5 \\ b_2 - b_6 \\ b_3 - b_7 \end{bmatrix} \underline{u} + [0] \underline{w}_3 \quad (27)$$

Solving for the eigenvalues of \tilde{C} gives

$$\begin{aligned} & \lambda^6 + (b_4 - k_{11} - k_{12}) \lambda^5 + (b_5 - b_4 k_{12} - k_{22} - k_{21}) \lambda^4 \\ & + (b_6 - b_4 k_{22} - b_5 k_{12} - k_{31}) \lambda^3 + (b_7 - b_5 k_{22} - b_6 k_{12} - k_{41}) \lambda^2 \\ & + (-b_6 k_{22} - b_7 k_{12}) \lambda - b_7 k_{22} = 0 \end{aligned} \quad (29)$$

In the same manner as for the first two cases, one can solve for the components of the gain matrices, \underline{K}_1 and \underline{K}_2 . Doing so gives the following.

- (a) $k_{11} = b_4 - k_{12} + A_0$
- (b) $k_{21} = b_5 - b_4 k_{12} - k_{22} - A_1$
- (c) $k_{31} = b_6 - b_4 k_{22} - b_5 k_{12} + A_2$
- (d) $k_{41} = b_7 - b_5 k_{22} - b_6 k_{12} - A_3$
- (e) $k_{12} = (b_6 k_{22} - A_4) / b_7$
- (f) $k_{22} = -A_5 / b_7$

where A_1 through A_5 are as defined for use in Equation (19). The full dimensional observer can now be expressed as

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \\ \dot{\hat{z}}_1 \\ \dot{\hat{z}}_2 \end{bmatrix} = \begin{bmatrix} (k_{11} - b_4) & 1 & 0 & 0 & k_{11} & 0 \\ (k_{21} - b_5) & 0 & 1 & 0 & k_{21} & 0 \\ (k_{31} - b_6) & 0 & 0 & 1 & k_{31} & 0 \\ (k_{41} - b_7) & 0 & 0 & 0 & k_{41} & 0 \\ k_{12} & 0 & 0 & 0 & k_{12} & 1 \\ k_{22} & 0 & 0 & 0 & k_{22} & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \\ \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} - \begin{bmatrix} k_{11} \\ k_{21} \\ k_{31} \\ k_{41} \\ k_{12} \\ k_{22} \end{bmatrix} \underline{y} + \underline{K}_\eta \underline{C}_n \begin{bmatrix} b_0 - b_4 + k_{11} \\ b_1 - b_5 + k_{21} \\ b_2 - b_6 + k_{31} \\ b_3 - b_7 + k_{41} \\ k_{12} \\ k_{22} \end{bmatrix} \underline{u} \quad (30)$$

The question for this case is of the same type as for the rate loop, i.e., can a gain, C_{RAL} , be found and used in conjunction with the state reconstructor output \hat{z}_1 , to minimize the effects of the disturbance?

B. SIMULATION AND RESULTS

The diagram for this composite system, with proposed control, is shown in *Figure 74*. The e's and h's are as defined for *Figure 3* with the following exceptions:

$$e_6 = k_{11}$$

$$e_7 = k_{21}$$

$$e_8 = k_{31}$$

$$e_9 = k_{41}$$

$$e_{10} = k_{12}$$

$$e_{11} = k_{22}$$

A listing of the digital simulation is given in Appendix C.

For this loop, the gain, C_{RAL} , is determined initially from the ratio $\frac{u}{y}$ (see *Figure 74*) from the undisturbed case and is then iterated, if necessary, to obtain a final value.

Several of the time points from *Table 1* were used to analyze this case. *Figures 75* through *84* give results for the 9.85 sec airframe parameters. By comparing *Figures 75, 77* and *79*, it can be seen that the effects of the disturbance are cancelled for a disturbance magnitude equal to the input command. *Figures 81* and *82* show results for w_3 equal to twice the input command magnitude and *Figures 83* and *84* are results for w_3 equal to five times the input command.

For $t_r = 66.7$ sec (apogee), an input command of 0.5 was used with $w_3 = 0.5$. *Figures 85* through *89* give the results for this time point. As can be seen from *Figures 85, 87* and *89*, the disturbance effects were again cancelled out although, since the system is sluggish, it takes longer to settle out than the previous case.

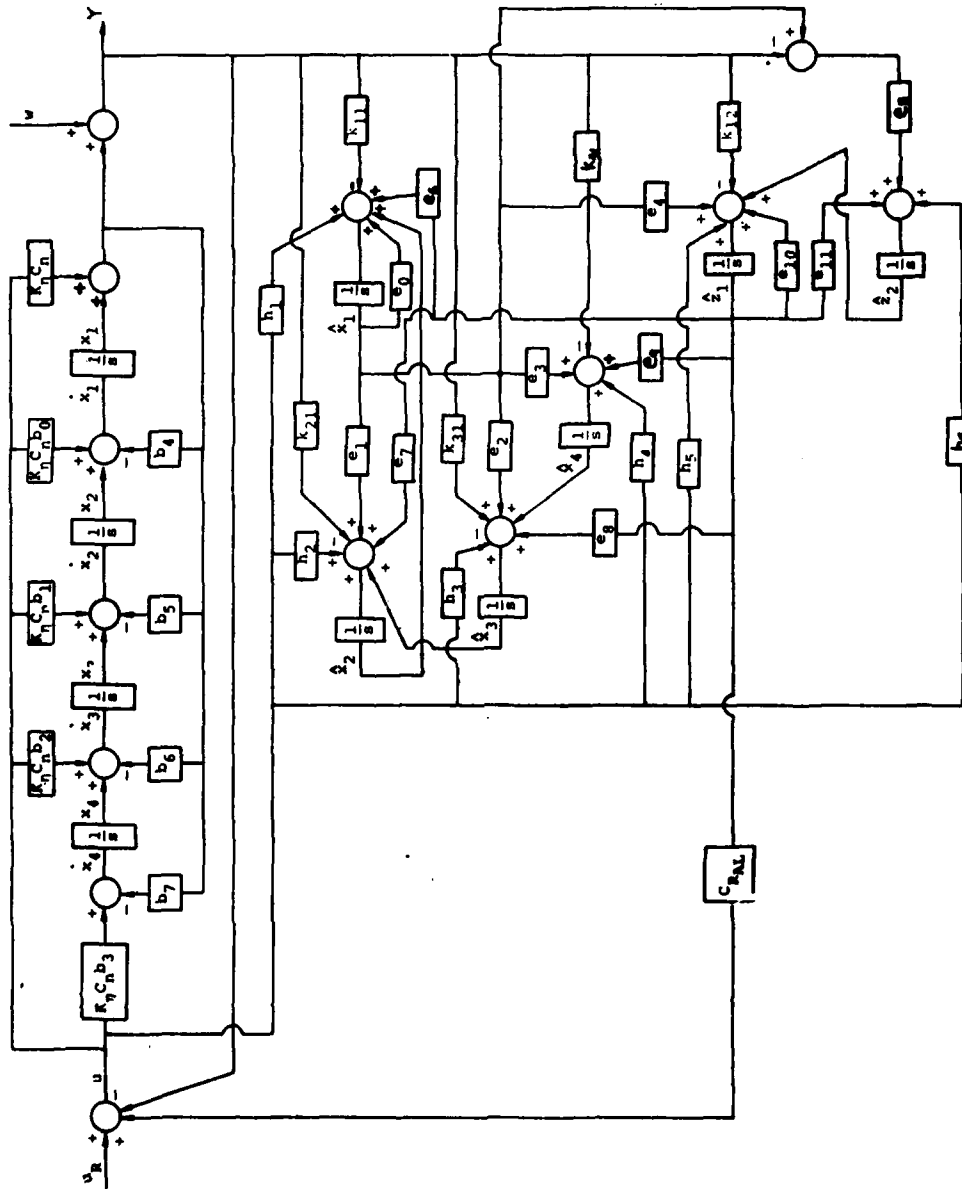


Figure 74. Plant-DAC composite for acceleration loop with disturbance at output.

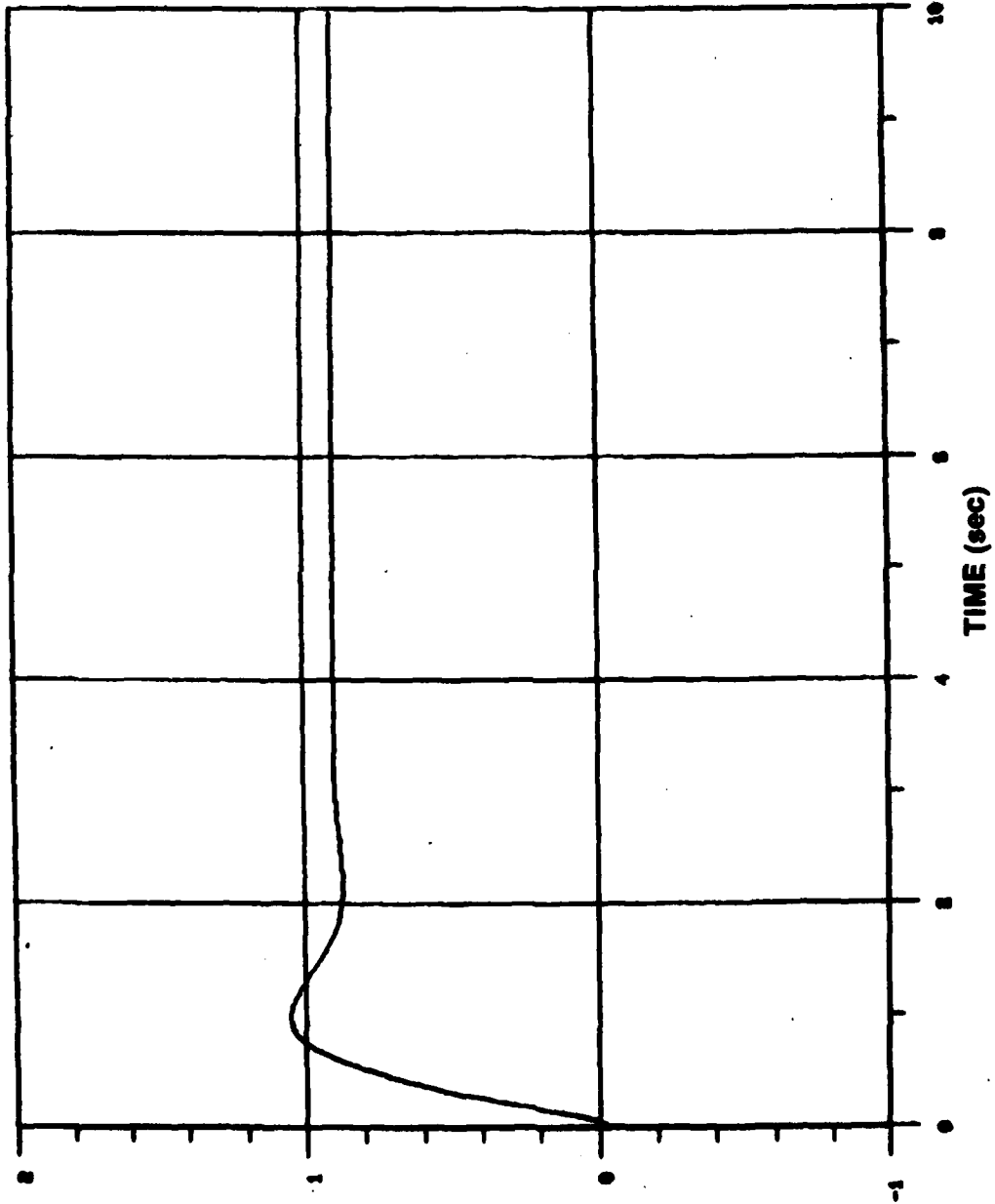


Figure 75. Acceleration loop response, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_3 = 0$.

ACCELERATION

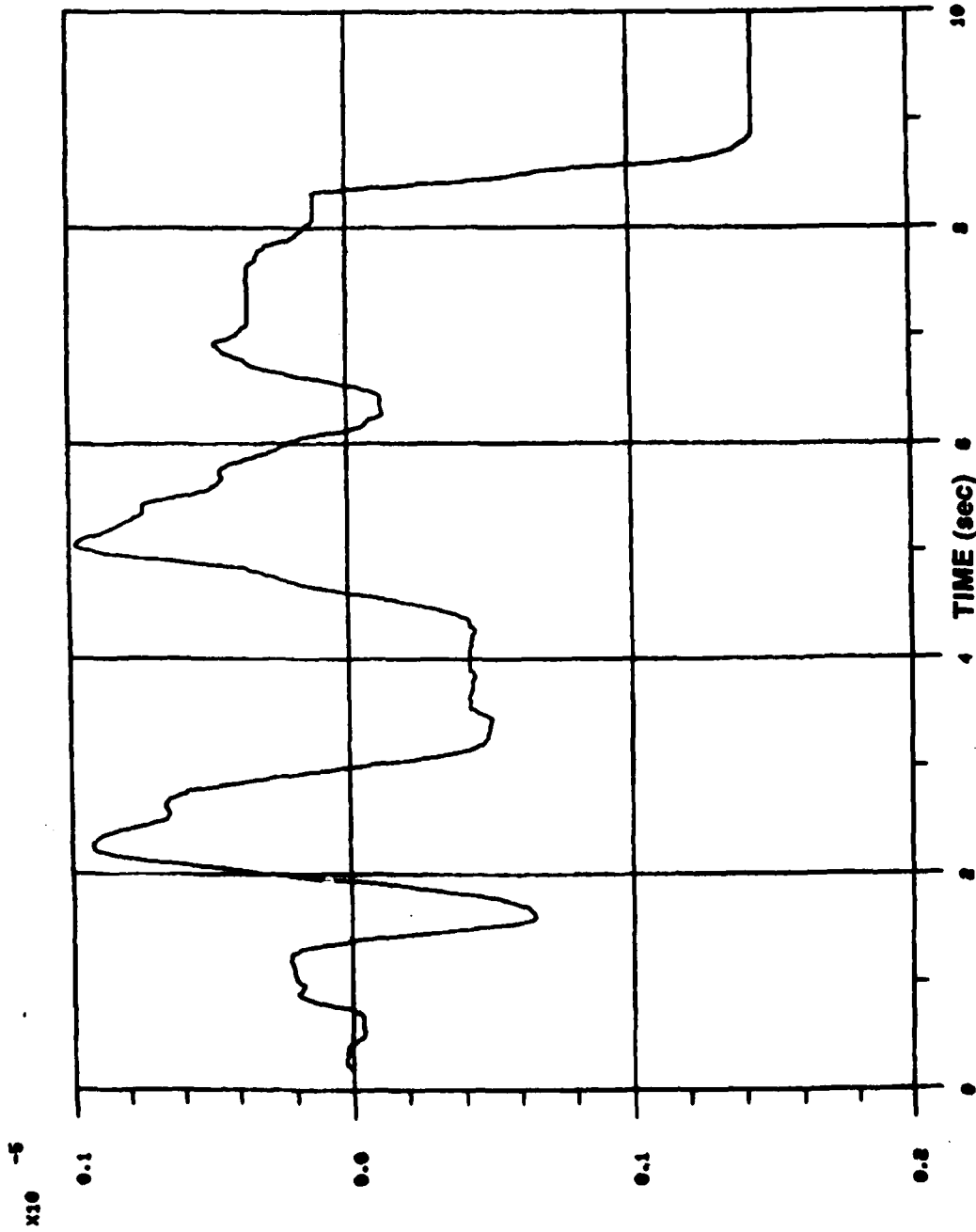


Figure 76. DAC disturbance estimation error, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_3 = 0$.

E P Z 1

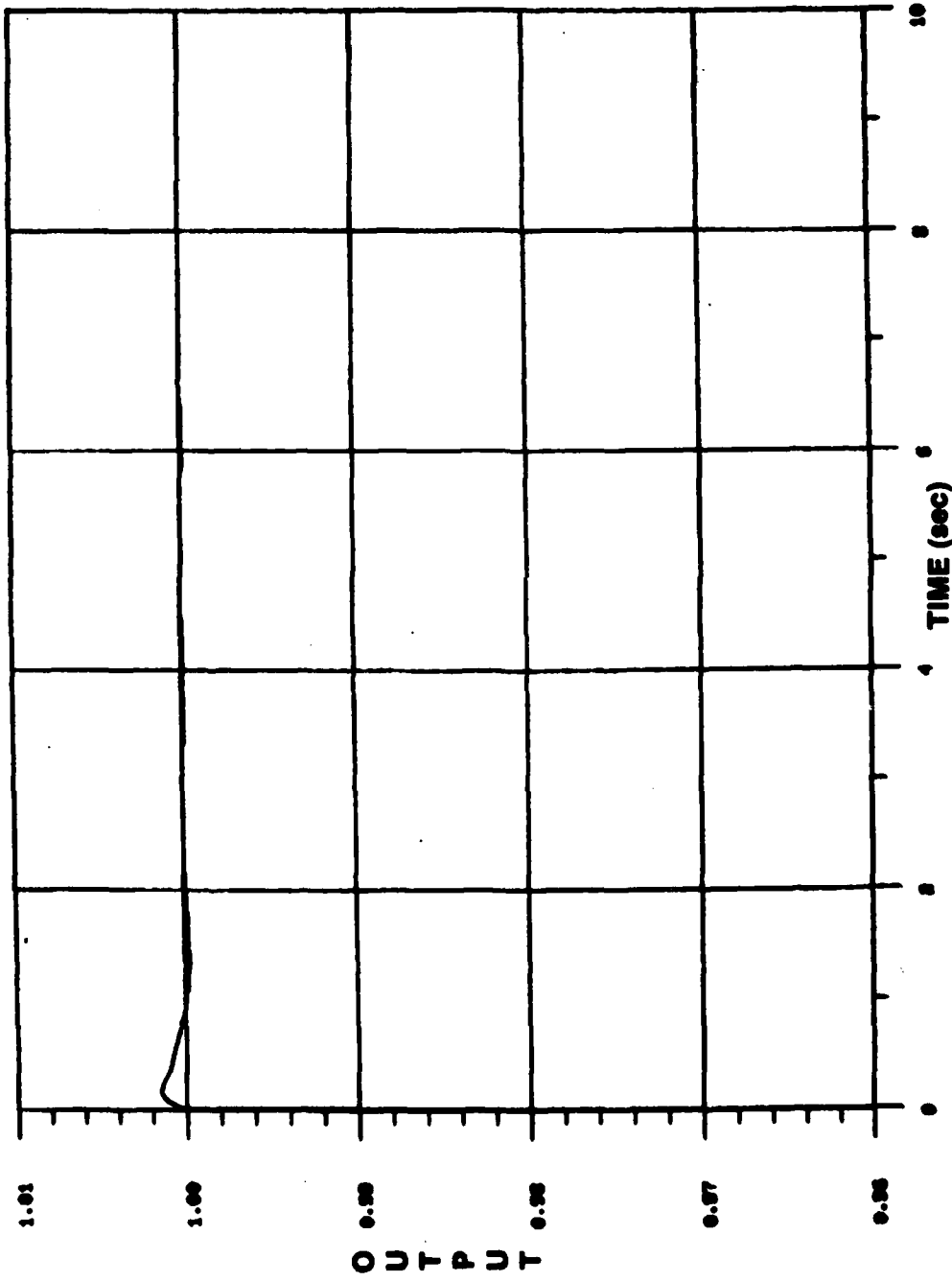


Figure 77. Acceleration loop response, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_3 = 1$, $CRAL = 0$.

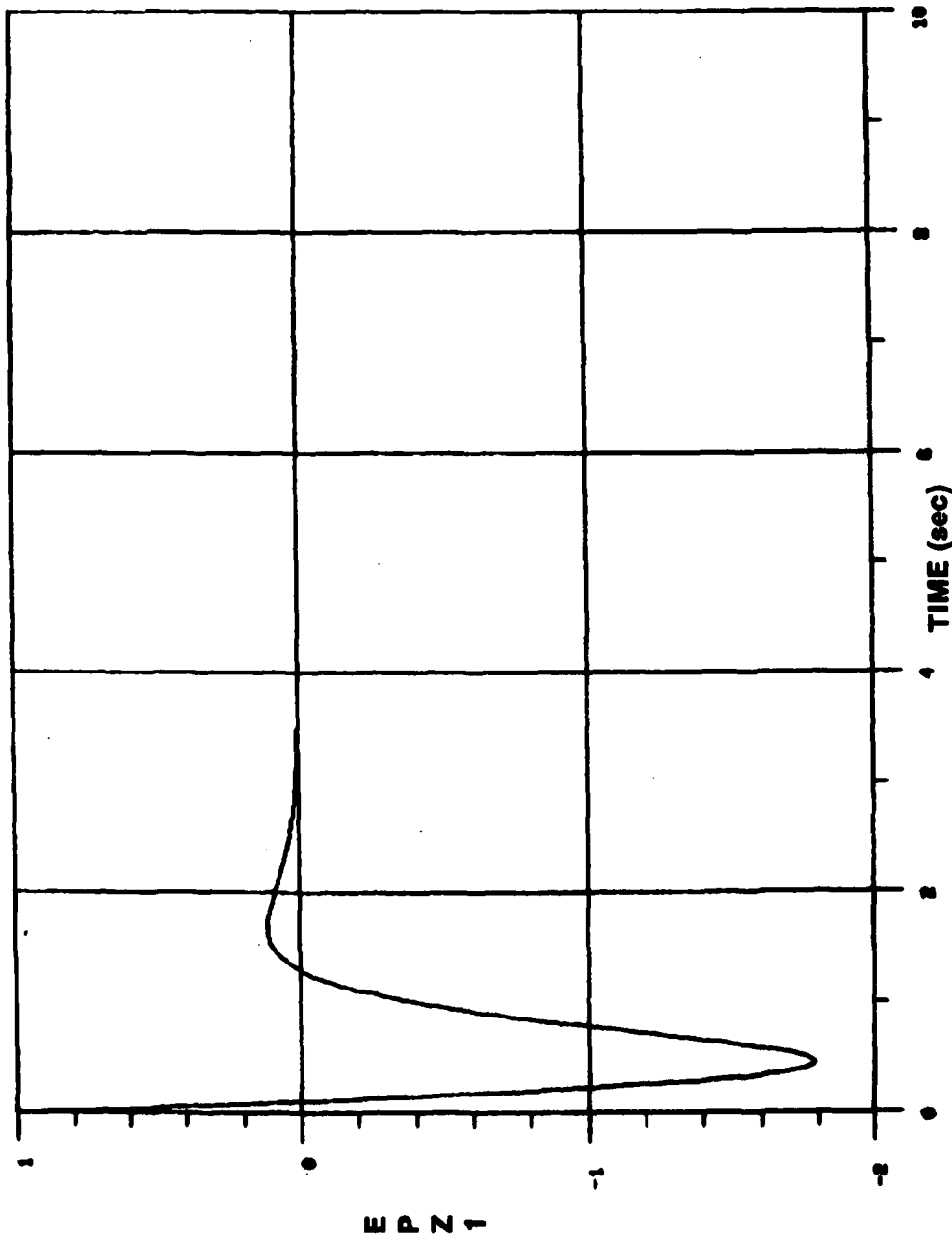


Figure 78. DAC disturbance estimation error, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_3 = 1$, $CRAL = 0$.

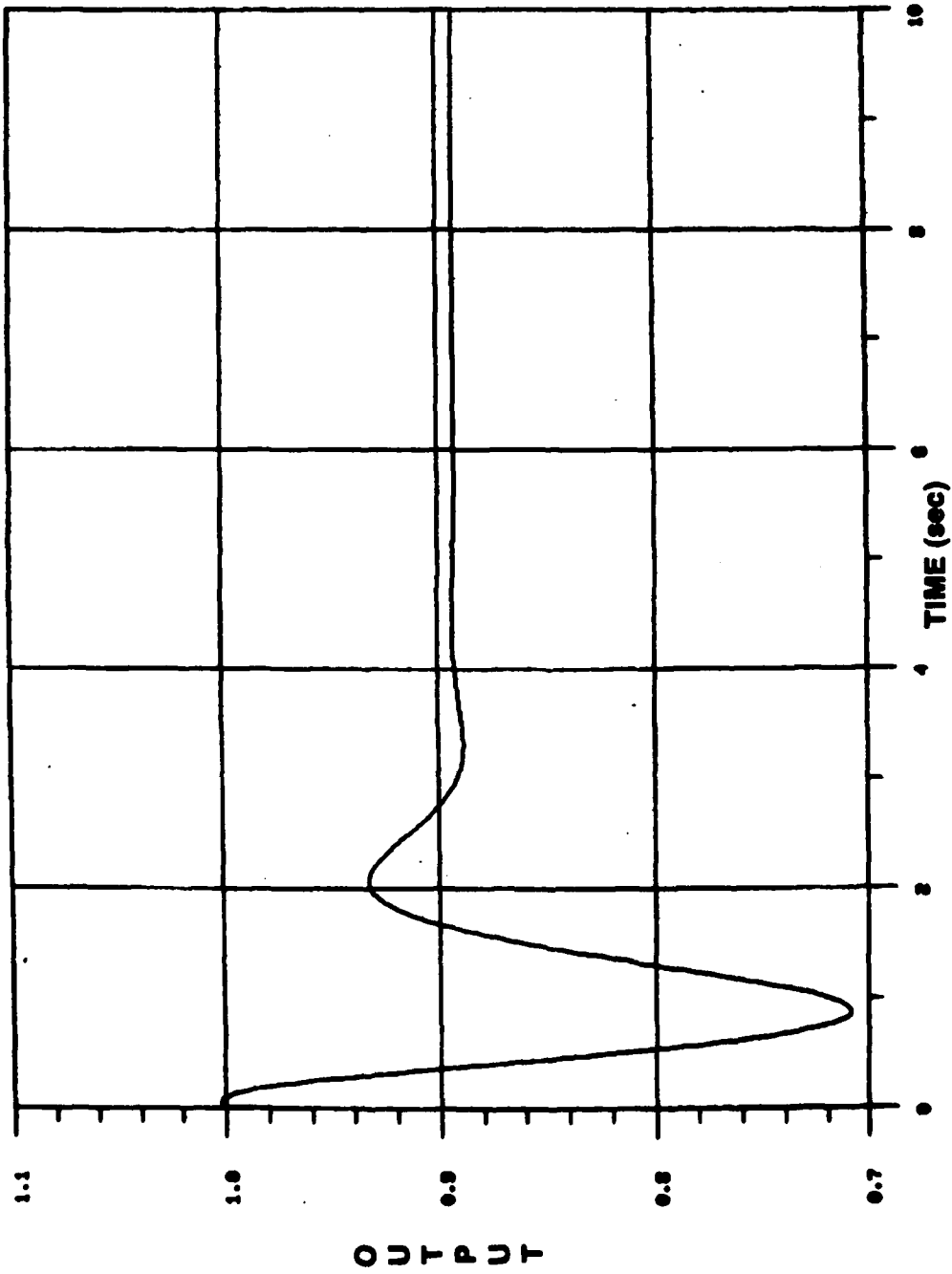


Figure 79. Acceleration loop response, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_3 = 1$, $CRAL = -0.12$.

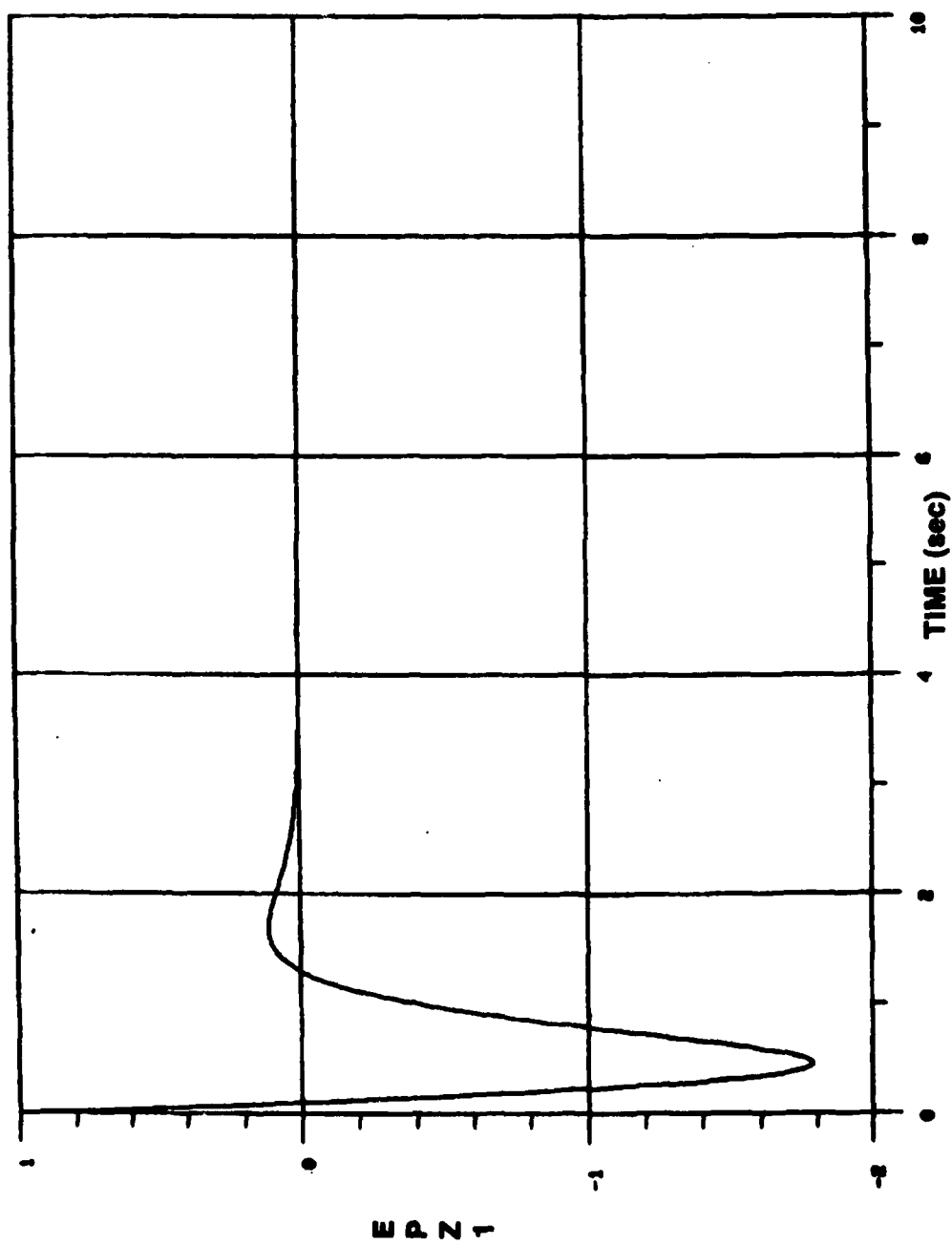


Figure 80. DAC disturbance estimation error, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_3 = 1$, $CRAL = -0.12$.

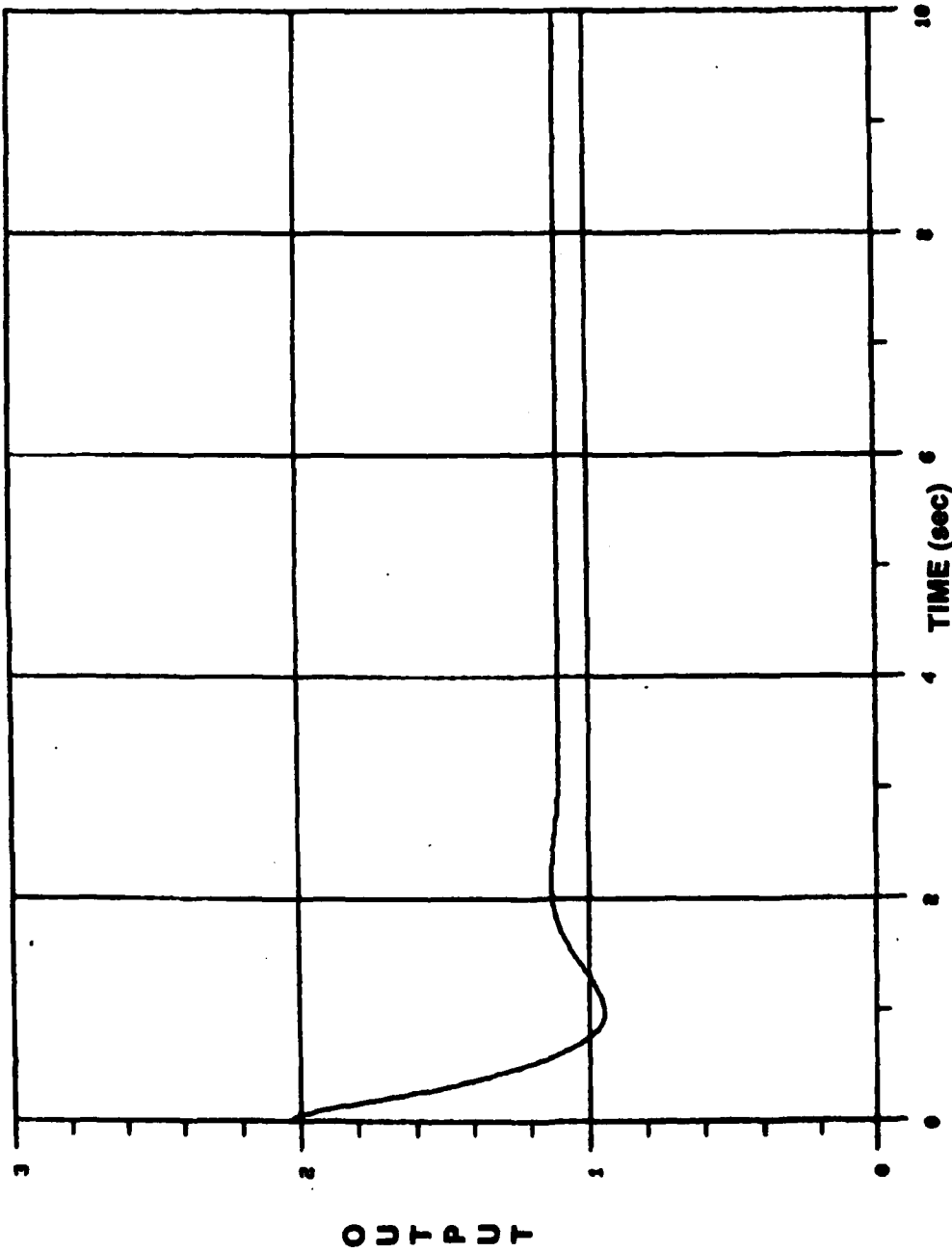


Figure 81. Acceleration loop response, $t_f = 9.85$ sec, $PGO = 1$, $W_3 = 2$,
 CRAL = 0.

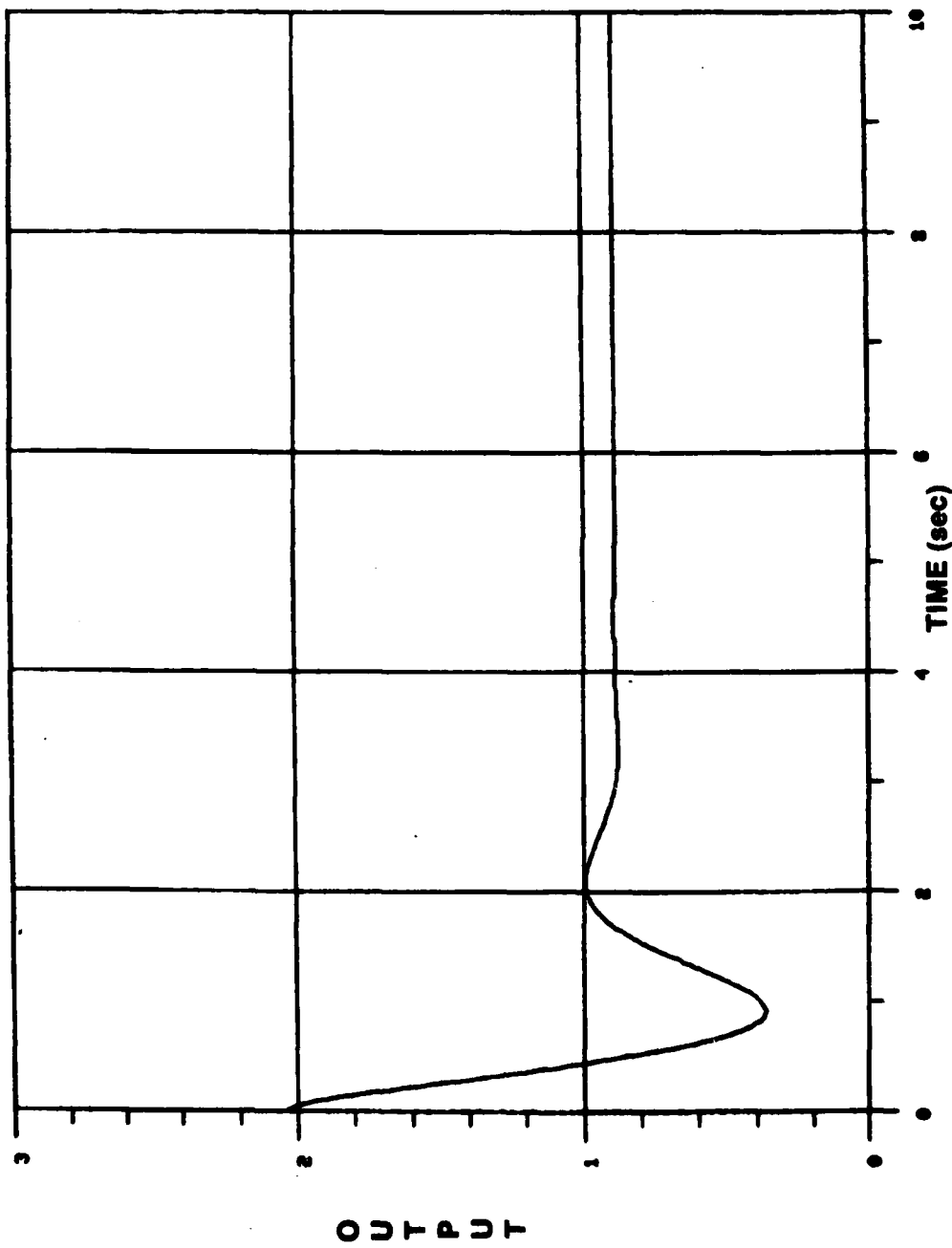


Figure 82. Acceleration loop response, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_3 = 2$,
 $CRAL = -0.12$.

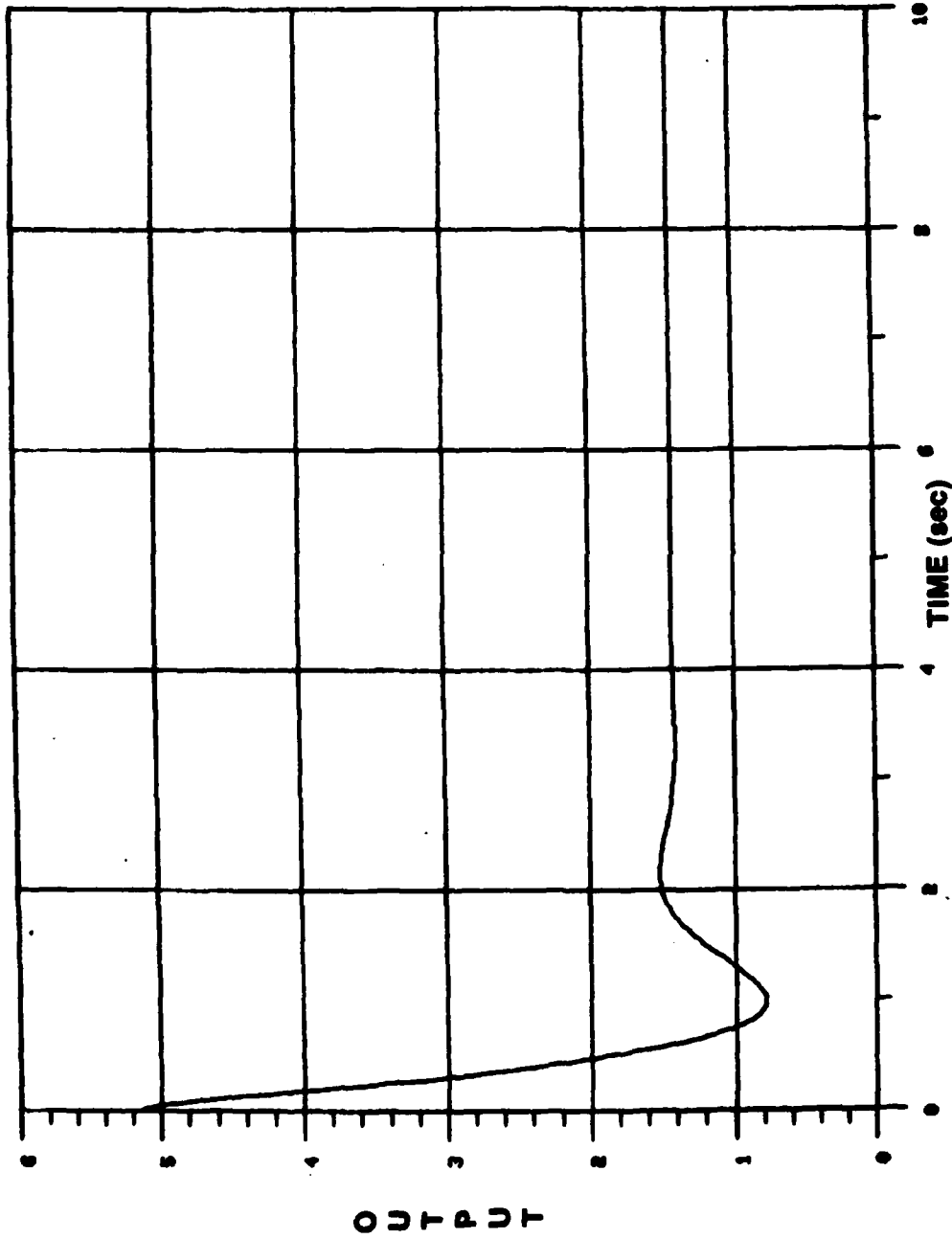


Figure 83. Acceleration loop response, $t_f = 9.85$ sec, $PGO = 1$, $W_3 = 5$,
 CRAL = 0.

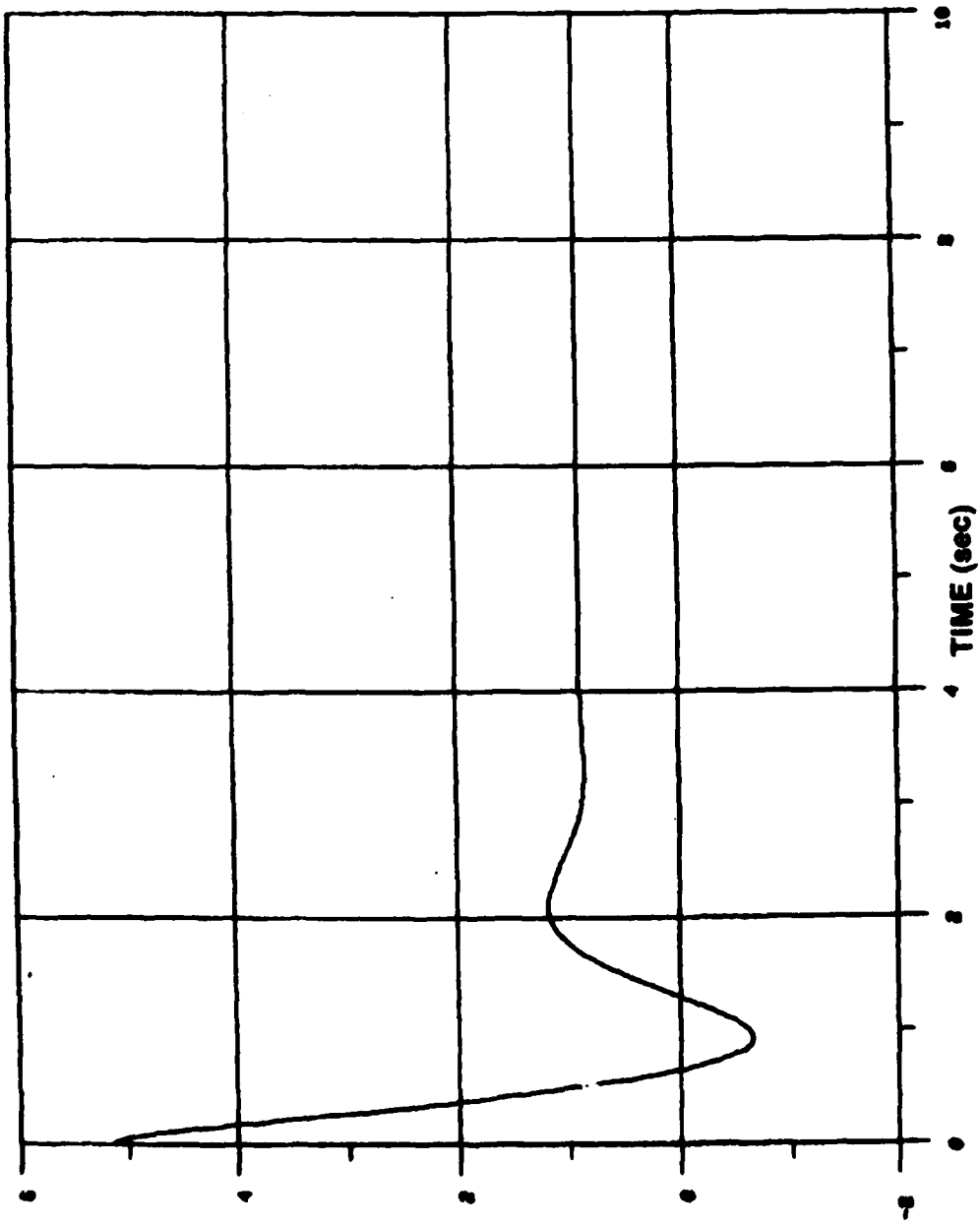


Figure 84. Acceleration loop response, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_3 = 5$,
 $CRAL = -0.12$.

OUTPUT

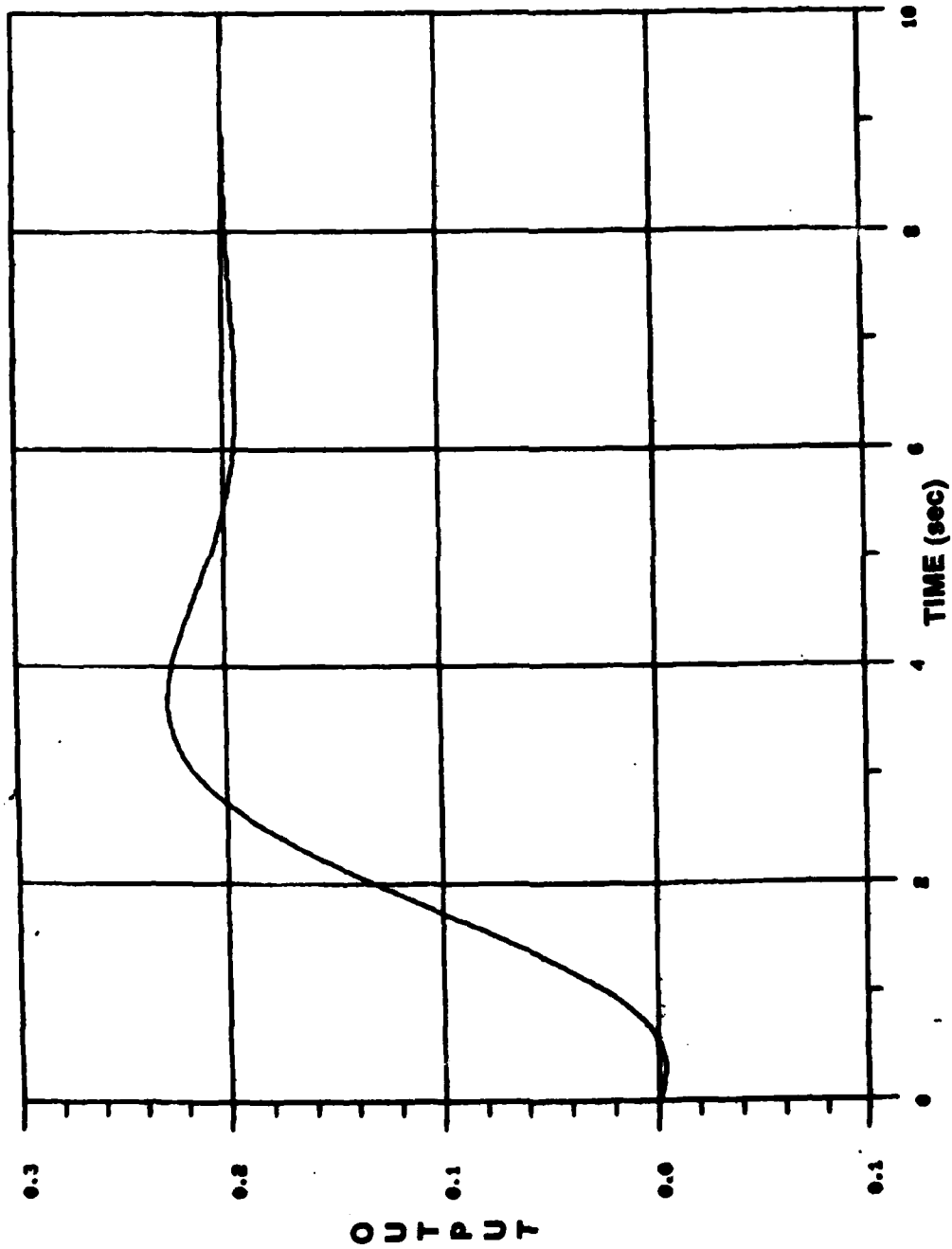


Figure 25. Acceleration loop response, $\eta = 66.7$ sec, $P_{GO} = 0.5$, $W_3 = 0$.

X10⁻⁷

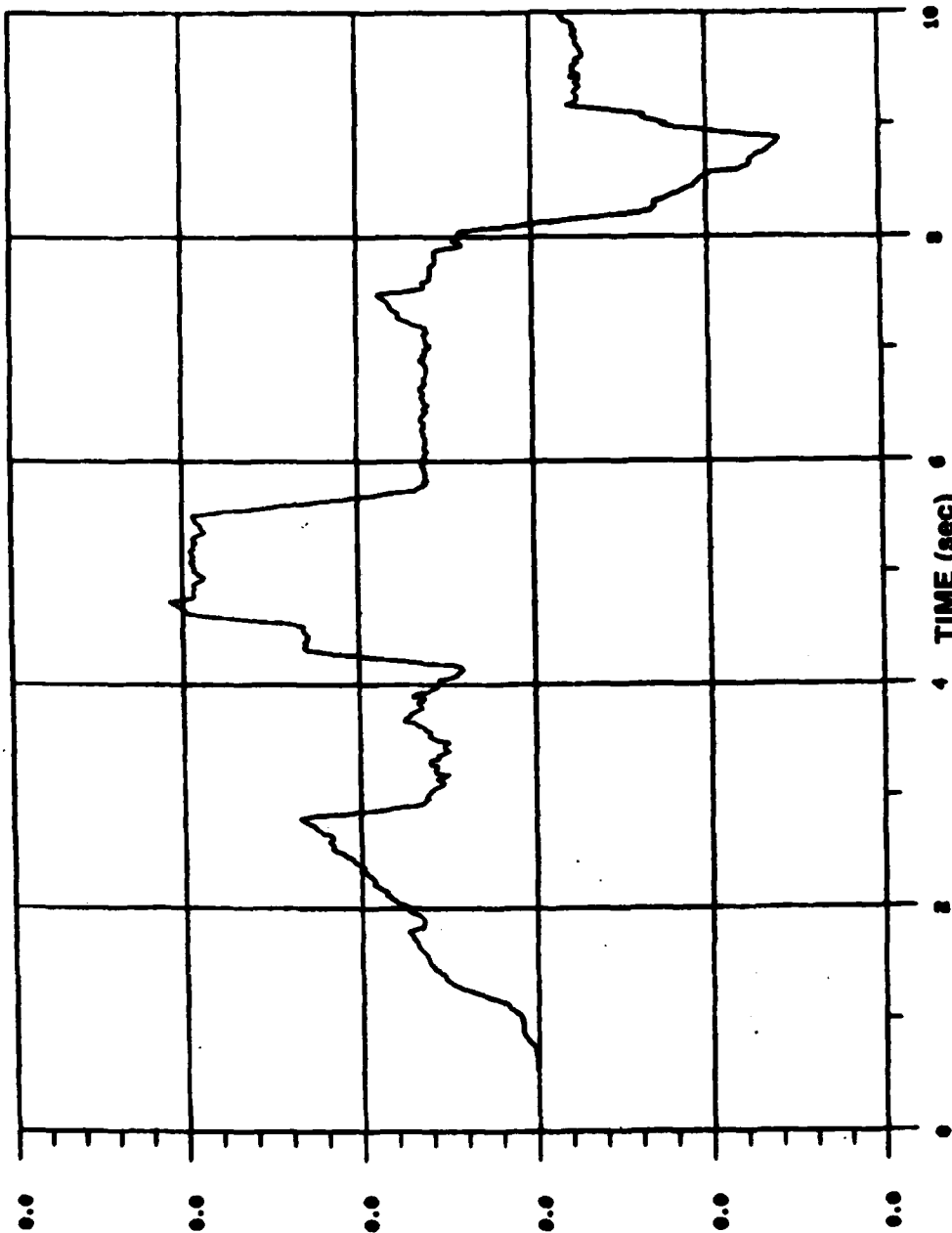


Figure 86. DAC disturbance estimation error, $t_f = 66.7$ sec, $P_{GO} = 0.5$, $W_3 = 0$.

E P N 1

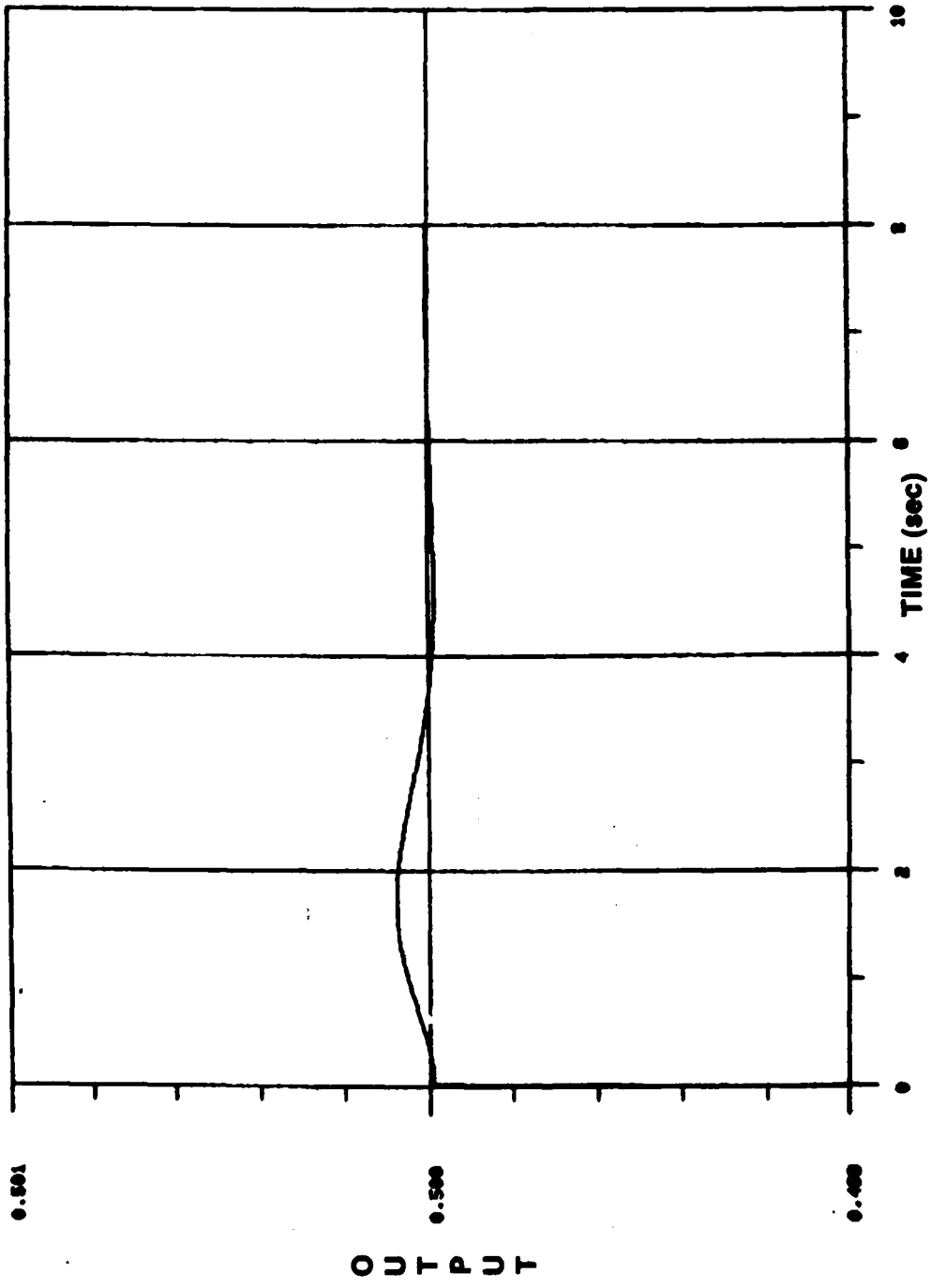


Figure 87. Acceleration loop response, $t_f = 66.7$ sec, $PGO = 0.5$, $W_3 = 0.5$, $CRAL = 0$.

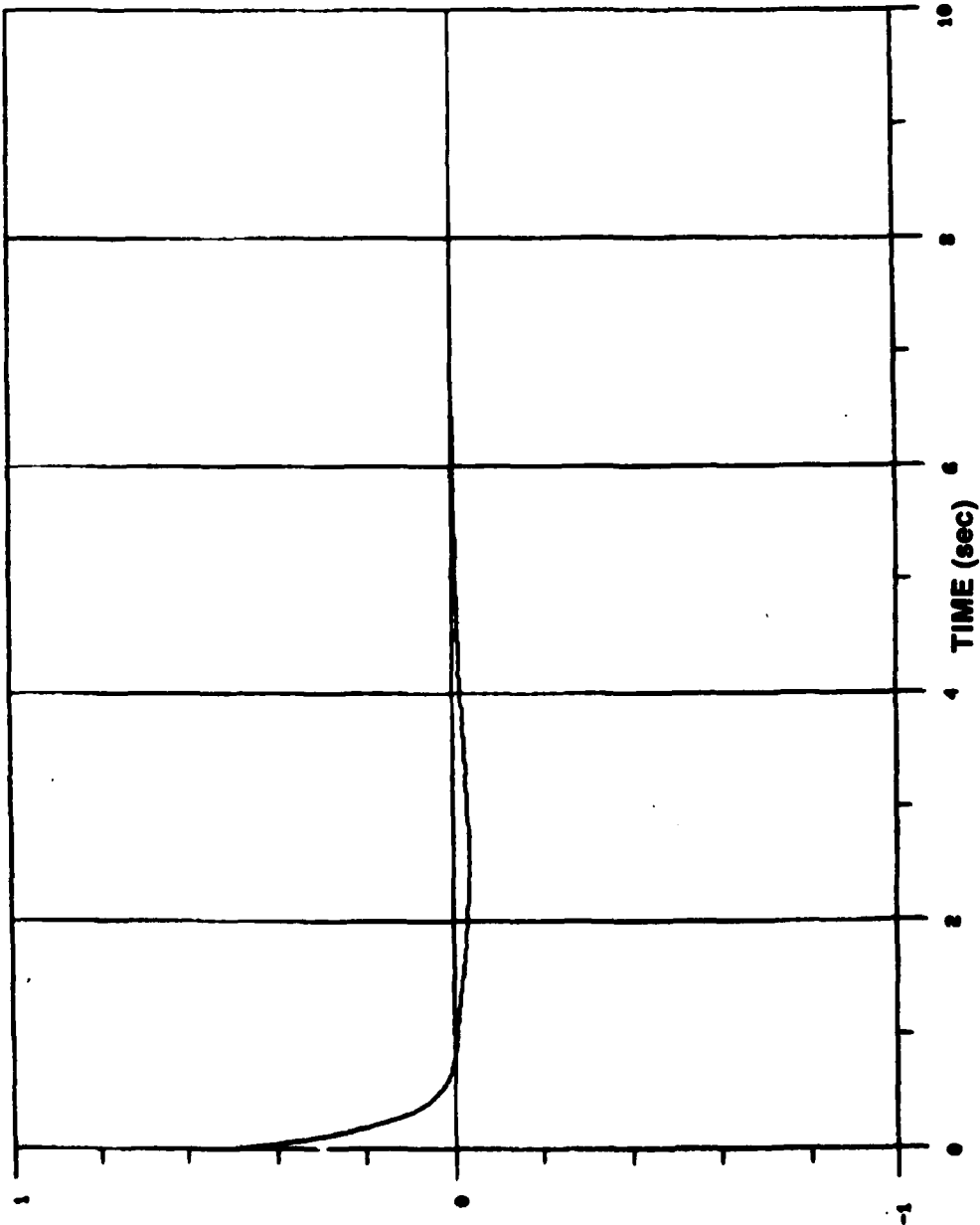


Figure 88. DAC disturbance estimation error, $t_f = 66.7$ sec, $PGO = 0.5$, $\dot{W}_3 = 0.5$, $CRAL = 0$.

E R N T

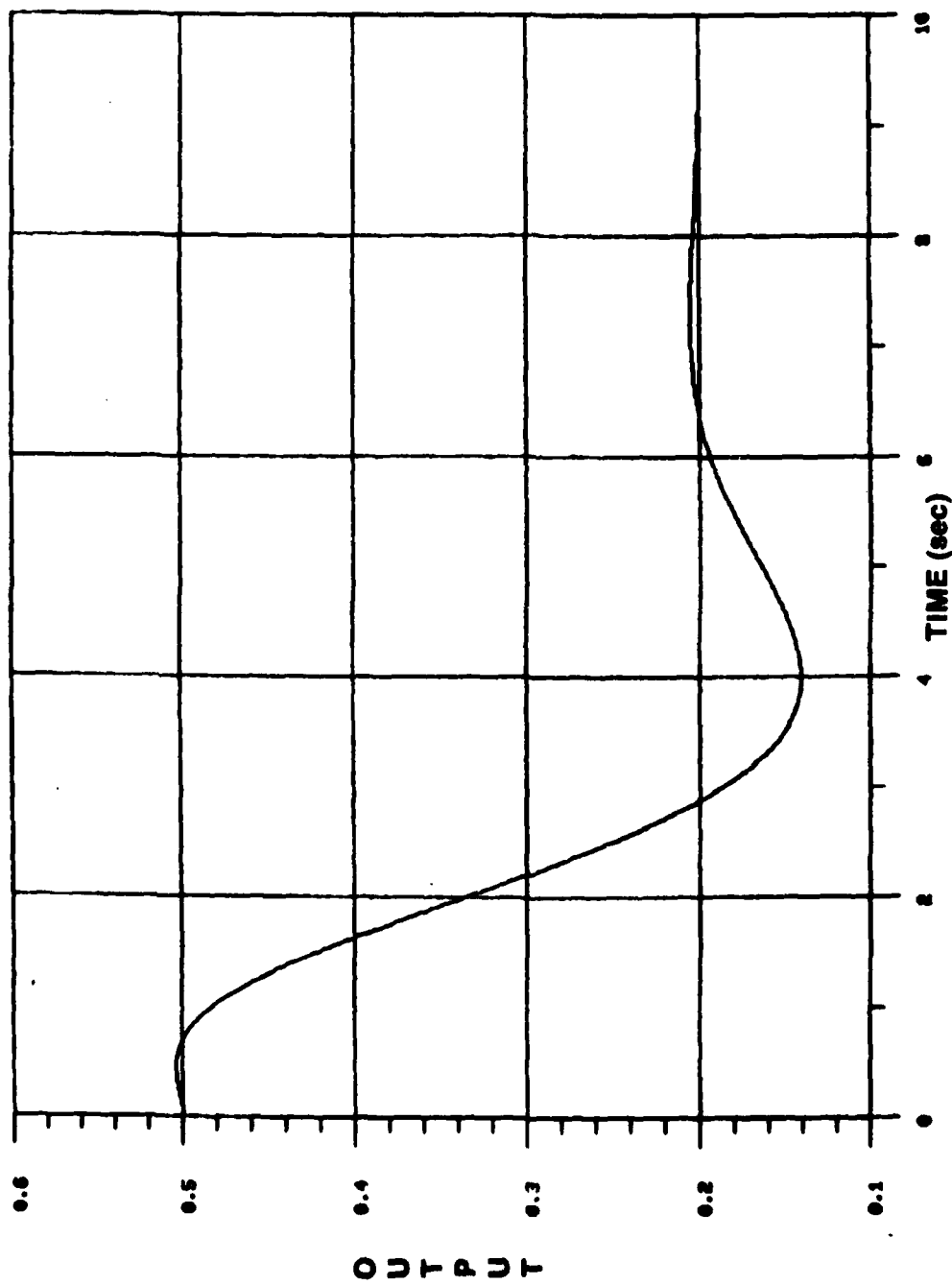


Figure 89. Acceleration loop response, $t_f = 66.7$ sec, $P_{GO} = 0.5$, $W_3 = 0.5$,
 CRAL = -1.50.

For $t_i = 111.4$ sec, an input command and disturbance magnitude of 1.0 were again used. Figures 90 through 94 show the results obtained. Comparing Figures 90, 92 and 94, it can be seen that the disturbance effects are removed. The output on Figure 94 could perhaps be settled out better by more iterations with the state reconstructor roots.

Table 6 gives the components of \underline{K}_1 and \underline{K}_2 and the roots of Equation (29) used in the above three cases along with the value for C_{RAL} in each case.

TABLE 6. STATE RECONSTRUCTOR DATA AND C_{RAL} FOR ACCELERATION LOOP WITH DISTURBANCE ON OUTPUT

TIME POINT (SEC)	9.85	66.7	111.4
PARAMETER			
k11	3.653	-0.129	-14.54
k21	195.44	-2.033	-184.34
k31	2159.92	-3.68	-605.05
k41	2668.13	-1.776	-483.25
k12	-12.26	-4.584	7.80
k22	0.0	-2.793	-29.47
λ_1	-3.	-1.	-2.
λ_2	-3.	-1.	-2.
λ_3	-4 + j4	-1.5 + j0.25	-3 + j1
λ_4	-4 - j4	-1.5 - j0.25	-3 - j1
λ_5	-6.	-2.	-5.
λ_6	-6.	-2.	-5.
C_{RAL}	-0.12	-1.50	-0.40

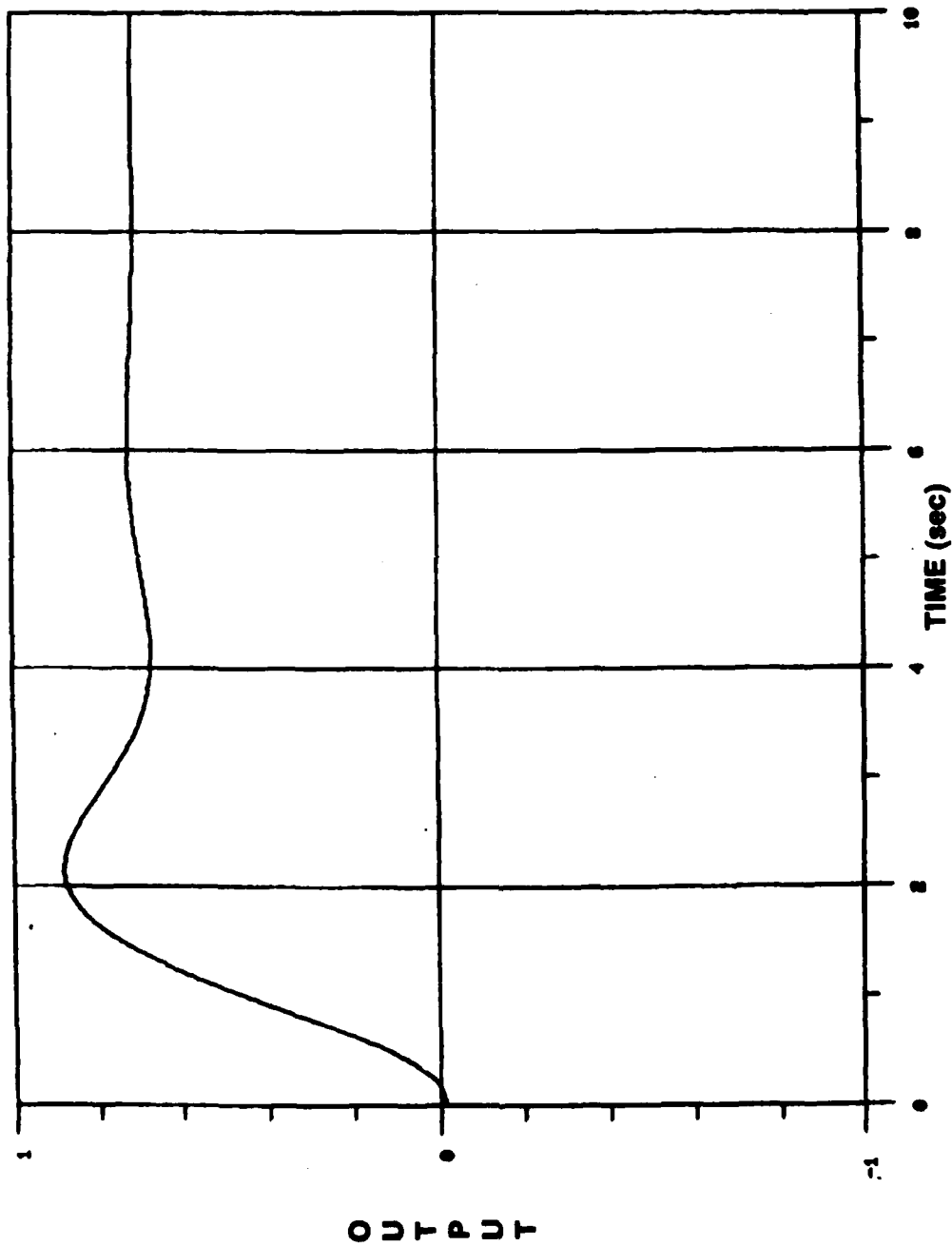


Figure 90. Acceleration loop response, $t_f = 111.4$ sec, $P_{GO} = 1$, $W_3 = 0$.

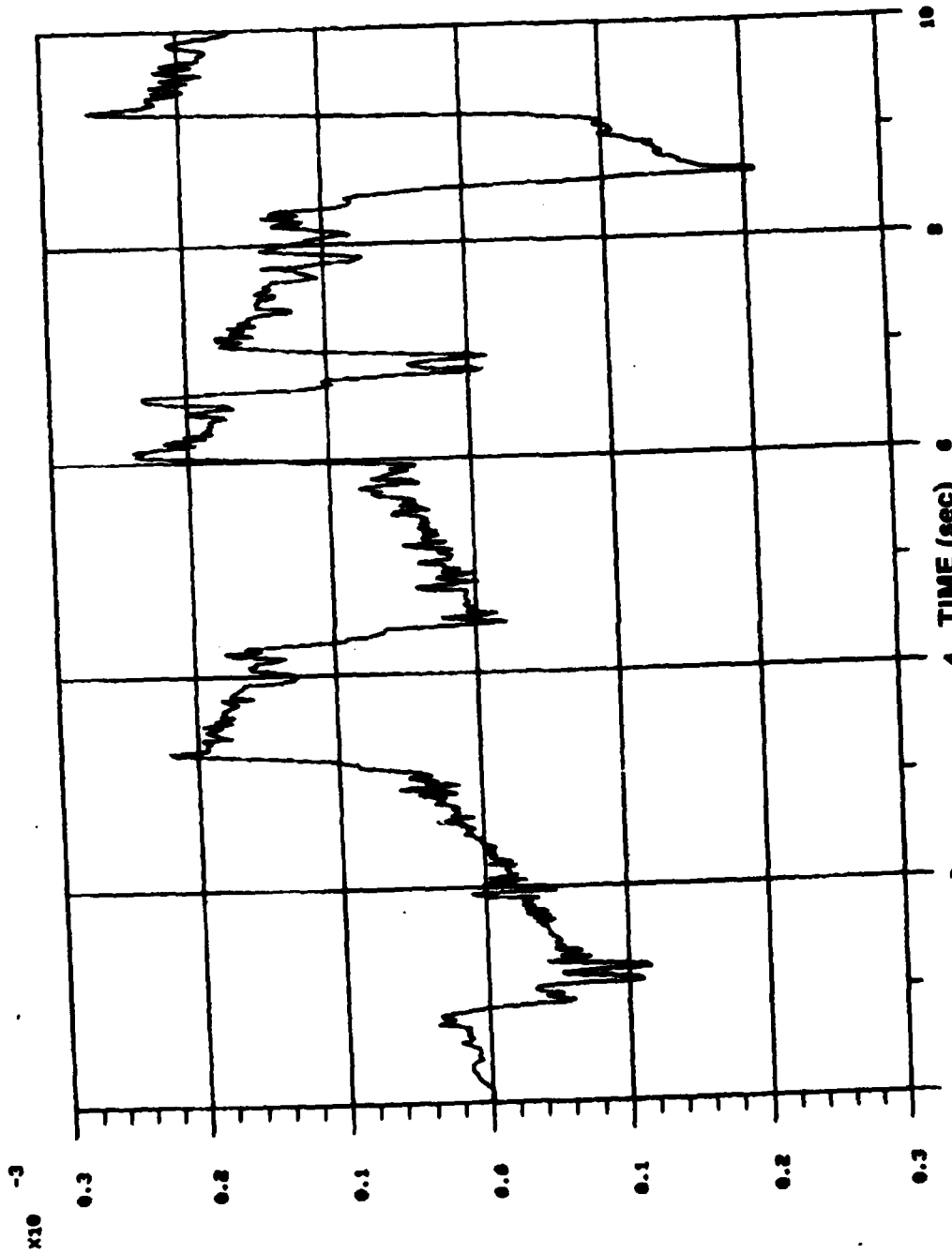


Figure 91. DAC disturbance estimation error, $t_f = 111.4$ sec, $P_{GO} = 1$, $W_3 = 0$.

E P Z 1

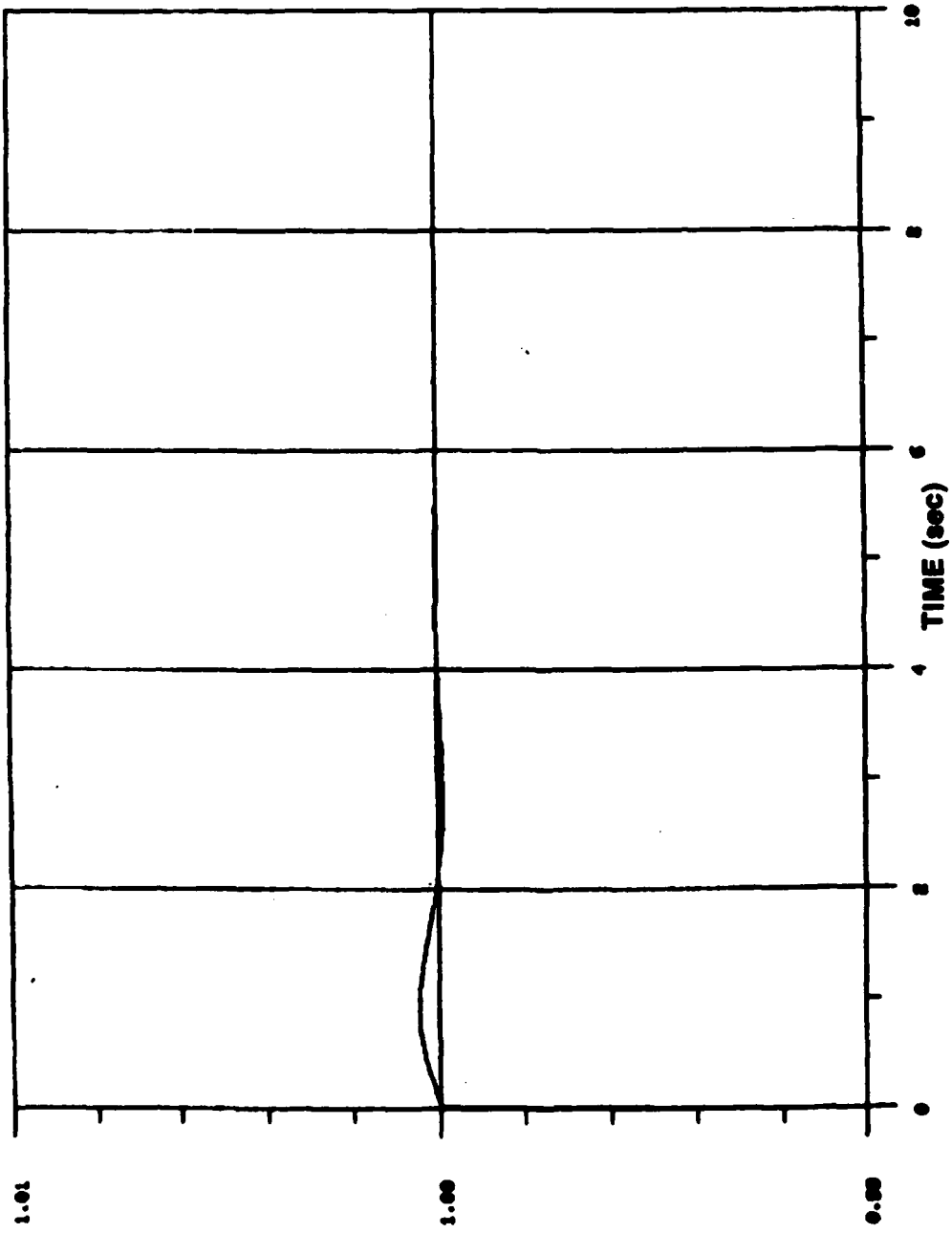


Figure 92. Acceleration loop response, $t_f = 111.4$ sec, $P_{GO} = 1$, $W_3 = 1$, $CRAL = 0$.

OUTPUT

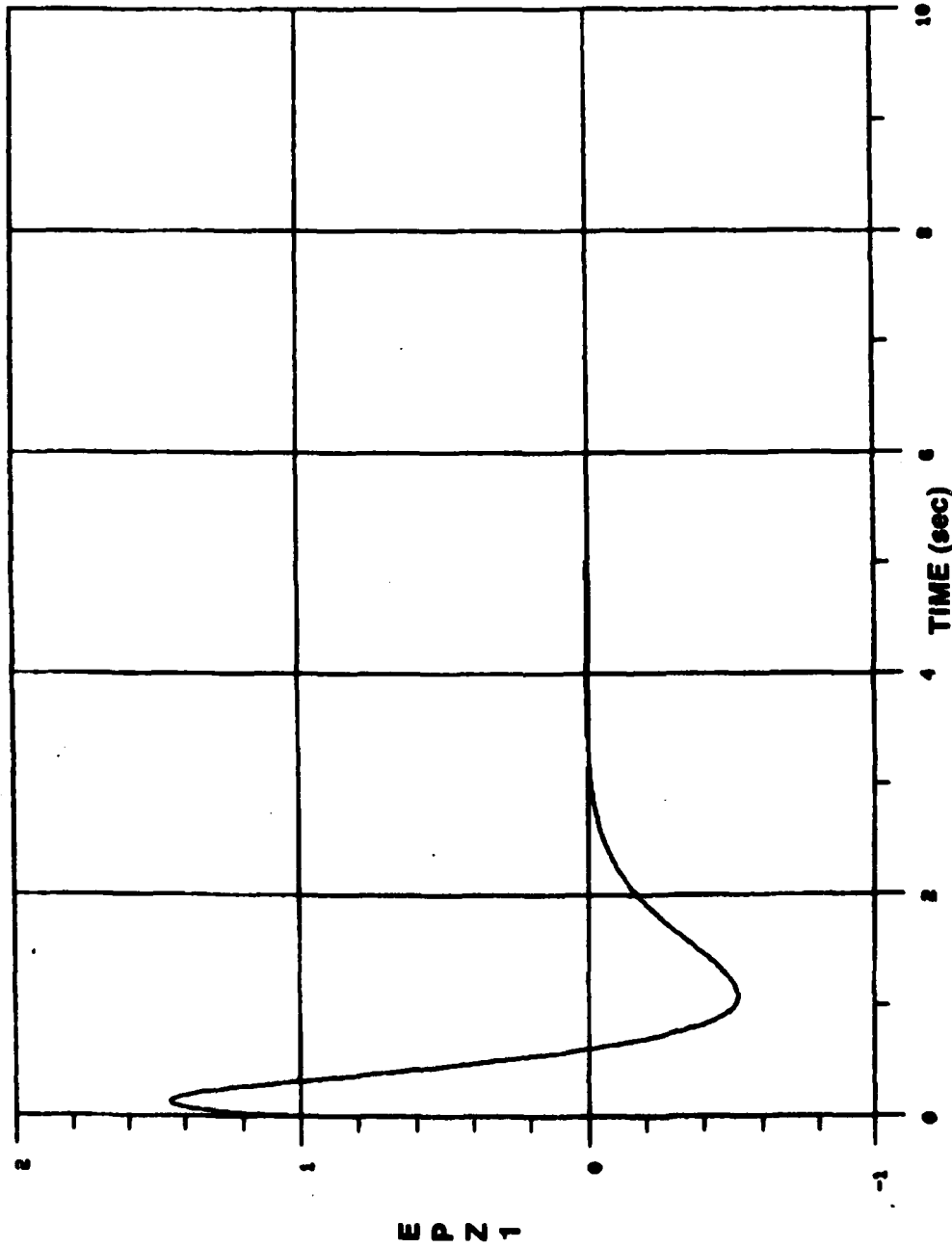


Figure 93. DAC disturbance estimation error, $t_f = 111.4$ sec, $PGO = 1$, $W_3 = 1$, $CRAL = 0$.

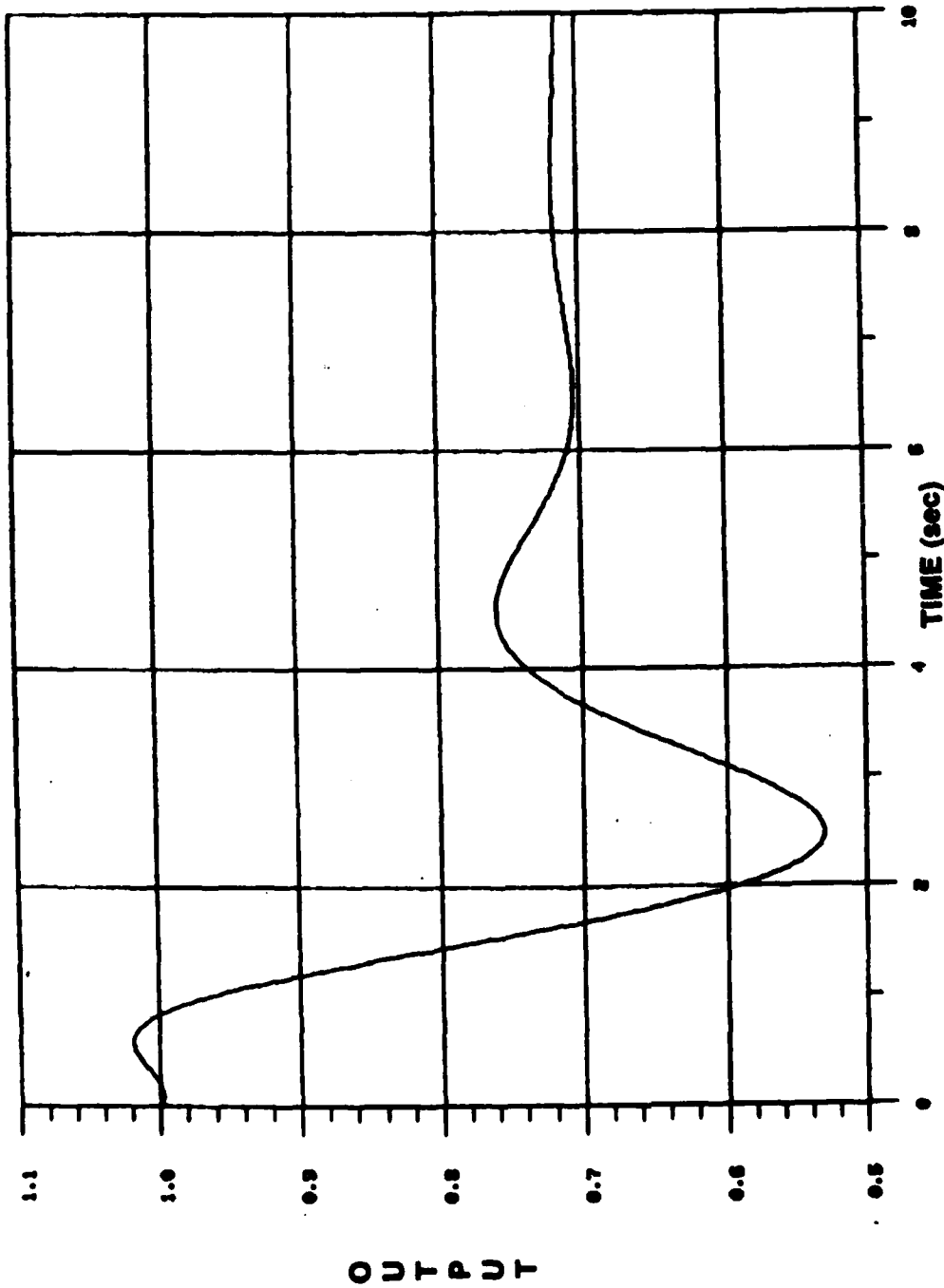


Figure 94. Acceleration loop response, $t_f = 111.4$ sec, $P_{GO} = 1$, $W_3 = 1$, $CRAL = -0.4$.

C. CONCLUSIONS

From the results, in similar fashion to the rate loop, even though no \underline{u}_c existed which would exactly cancel \underline{w}_3 , by using a state reconstructor to estimate the value of the disturbance an implementation was possible whereby the effects of \underline{w}_3 could be minimized. Again, gain switching would be required to implement this in a system.

8. ACCELERATION LOOP WITH INPUT AND OUTPUT DISTURBANCES

A. MODEL

As a last case for this report, the acceleration loop with \underline{w}_1 and \underline{w}_3 both included is considered. If the procedures given in previous sections are followed in attempting to derive a DAC for this case, it becomes necessary to evaluate the determinant of an 8×8 matrix to solve for the components of the gain matrices \underline{K}_1 and \underline{K}_2 . This evaluation is tedious at best with many opportunities for mistake.

Therefore, it is of interest to see if the DACs developed in Sections 5 and 7 can be combined in such a fashion as to continue to function as desired in cancelling the effects of \underline{w}_1 and \underline{w}_3 . A block diagram of the proposed combination is shown in *Figure 95*.

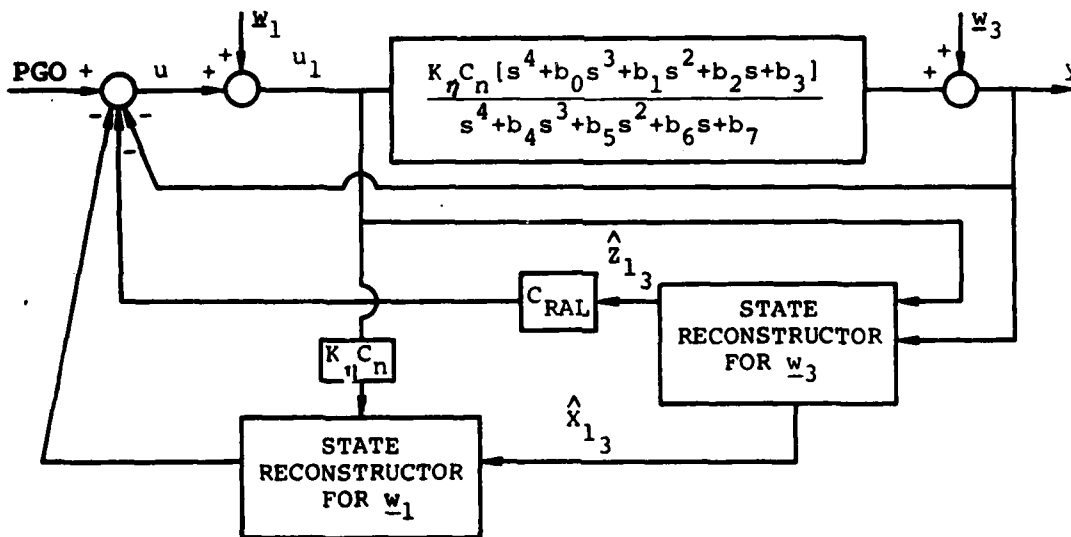


Figure 95. General block diagram of plant/DAC with both acceleration disturbance inputs.

In this development, the same plant state equations as before are used with changes only in nomenclature where necessary. The basic state reconstructor models are as previously developed. In this case, however, the rearrangement of the plant output portion of the data input to the w_1 state reconstructor should be noted. Since this DAC was developed with no disturbance on the plant output, in order for it to function properly it is necessary to use a plant output with w_3 removed. This is possible since the w_3 state reconstructor is also reconstructing the plant states. So, where in Section 5 the plant output is,

$$y = K_n C_n u_1 + x_1 ,$$

here it is formulated as

$$y_{PR} = K_n C_n u_1 + \hat{x}_{1_3} .$$

Thus, it is important in this application for the w_3 reconstructor to settle out as rapidly as possible.

B. SIMULATION AND RESULTS

A listing of the simulation is given in Appendix D. In this particular case, only one time point from *Table 1* (9.85 sec) was used since the purpose here was to see if two DAC's could be operated successfully in a serially connected mode.

Figures 96 through 98 show the loop output, y , and the disturbance estimation errors, ϵ_{w_1} and ϵ_{w_3} , for a nominal run, i.e., input of 1.0, no disturbances. This output compares with similar case results from Section 5 as would be expected. In order to check each reconstructor and see if any undesirable interactions were taking place, two runs were made, one with $w_1 = 1$, $w_3 = 0$, and one with $w_1 = 0$, $w_3 = 1$. The results are shown in *Figures 99 through 104*. In the first case, everything looks okay. In the second case, since the w_3 reconstructor is feeding input to the w_1 reconstructor and has a settling time of several seconds, there are some dynamics induced in the w_1 reconstructor. This in turn causes some dynamics to appear in the output. However, if *Figure 102* is compared to *Figures 96 and 99* on a similar scale, the results do not have such an undesirable appearance. From this it can be seen that some interaction is taking place but not to such an extent that the DAC performance in either case is impaired.

The remainder of the runs were made with disturbance inputs on w_1 and w_3 simultaneously. *Figures 105 through 107* give results for $w_1 = 1$, $w_3 = 2$; *Figures 108 through*

110 for $w_1 = 1. + 0.2t$, $w_2 = 0.5 + 0.1t$ and *Figures 111 through 113* for $w_1 = 1.-0.2t$, $w_2 = 0.5 + 0.5t$. In all these cases, even though the induced dynamics are noted in $\epsilon_{z_{w1}}$, the DAC's performed their function of cancelling the disturbance effects.

C. CONCLUSIONS

From the results in this section it would appear that it is possible to design DAC's for separate disturbances in different parts of a loop, thereby simplifying the size of the matrices involved in the calculations, and combine them in a simple manner to achieve the desired results.

9. CONCLUSIONS

Conclusions have been presented in Sections 5 through 8 regarding the results obtained with the design in each section. Overall, it has been shown that it was possible to cancel out or minimize the effects of the disturbances modeled herein by use of DAC techniques. It was also shown that it was possible to combine two separate DAC's, designed for disturbances at different places in the plant, into a functioning unit which would still perform its overall purpose. This is especially important since the size of the matrices involved in designing a "full-dimensional" observer is directly related to the dimensions of the plant plus disturbance models. Thus, any procedure which can reduce the dimensionality involved is important. In this regard, several of the designs here might be redone utilizing a "reduced-order" observer to see how well such a DAC would perform.

Although it would appear from the results obtained here that a DAC might be very useful in cancelling out unwanted disturbances, the only way to really be sure how one would function in a system application would be to implement one in a 6-DOF simulation and fly it with a severe program of varying disturbance vectors.

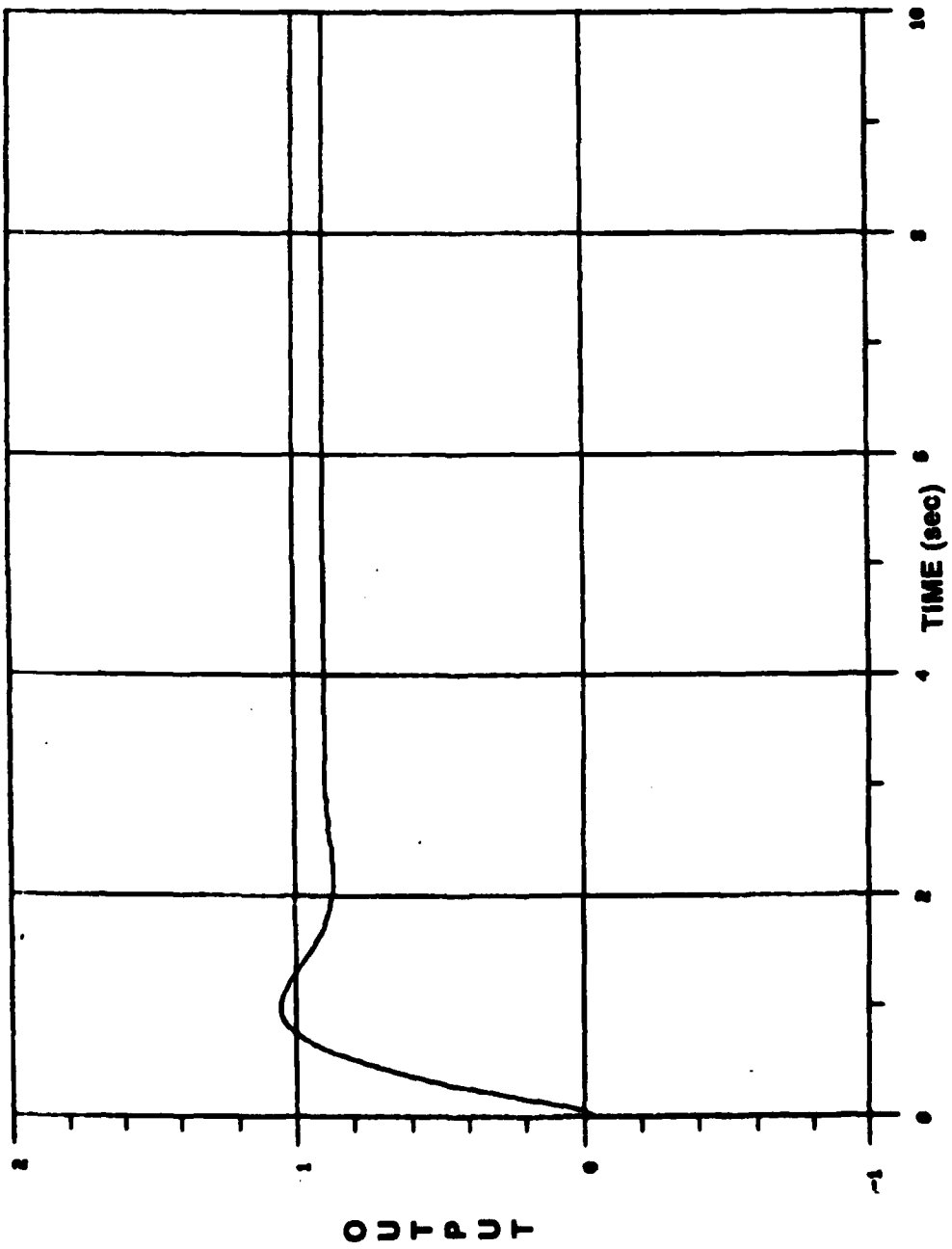


Figure 96. Acceleration loop response, $t_f = 9.85 \text{ sec}$, $PGO = 1$, $W_1 = W_3 = 0$.

x10⁻⁵

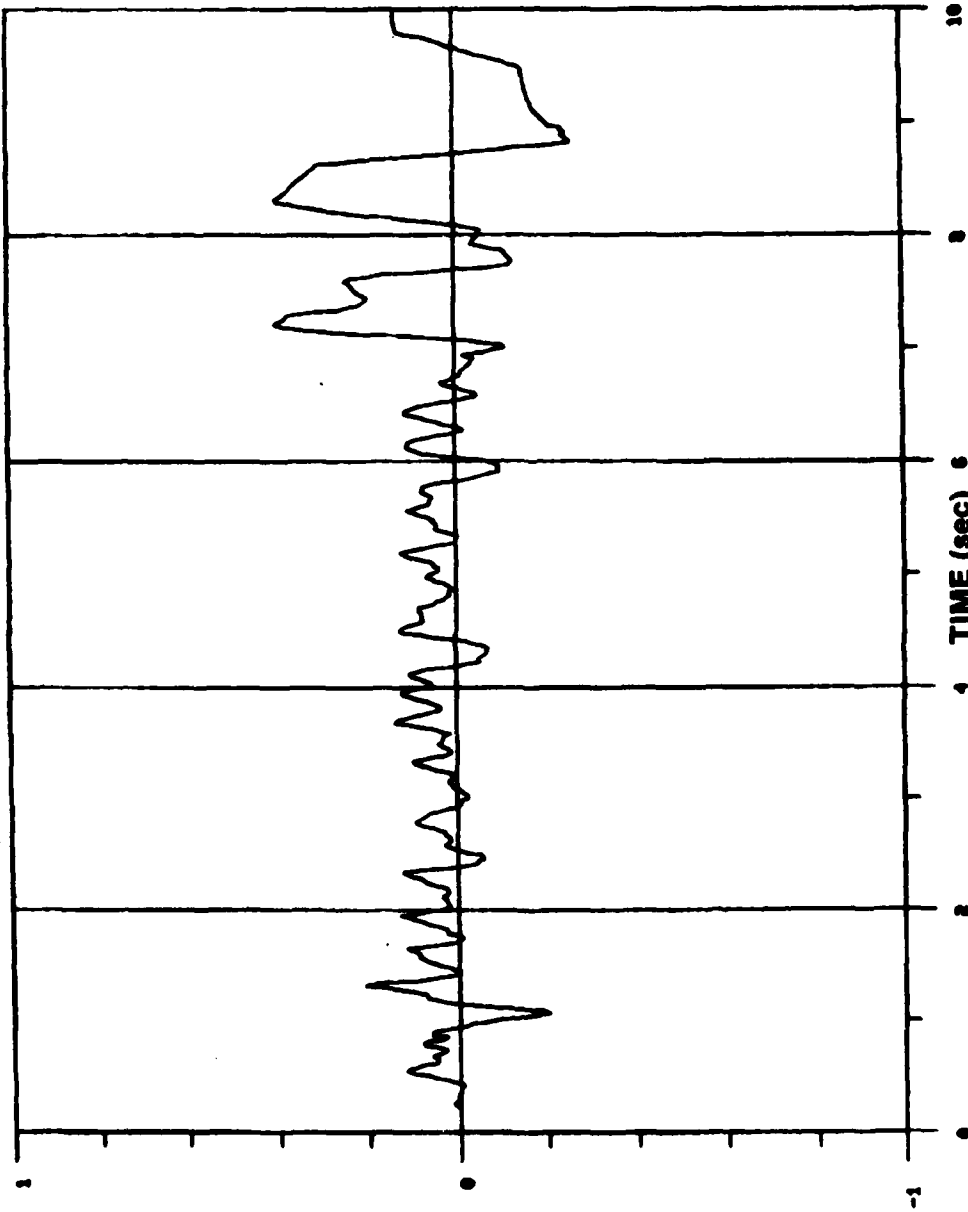
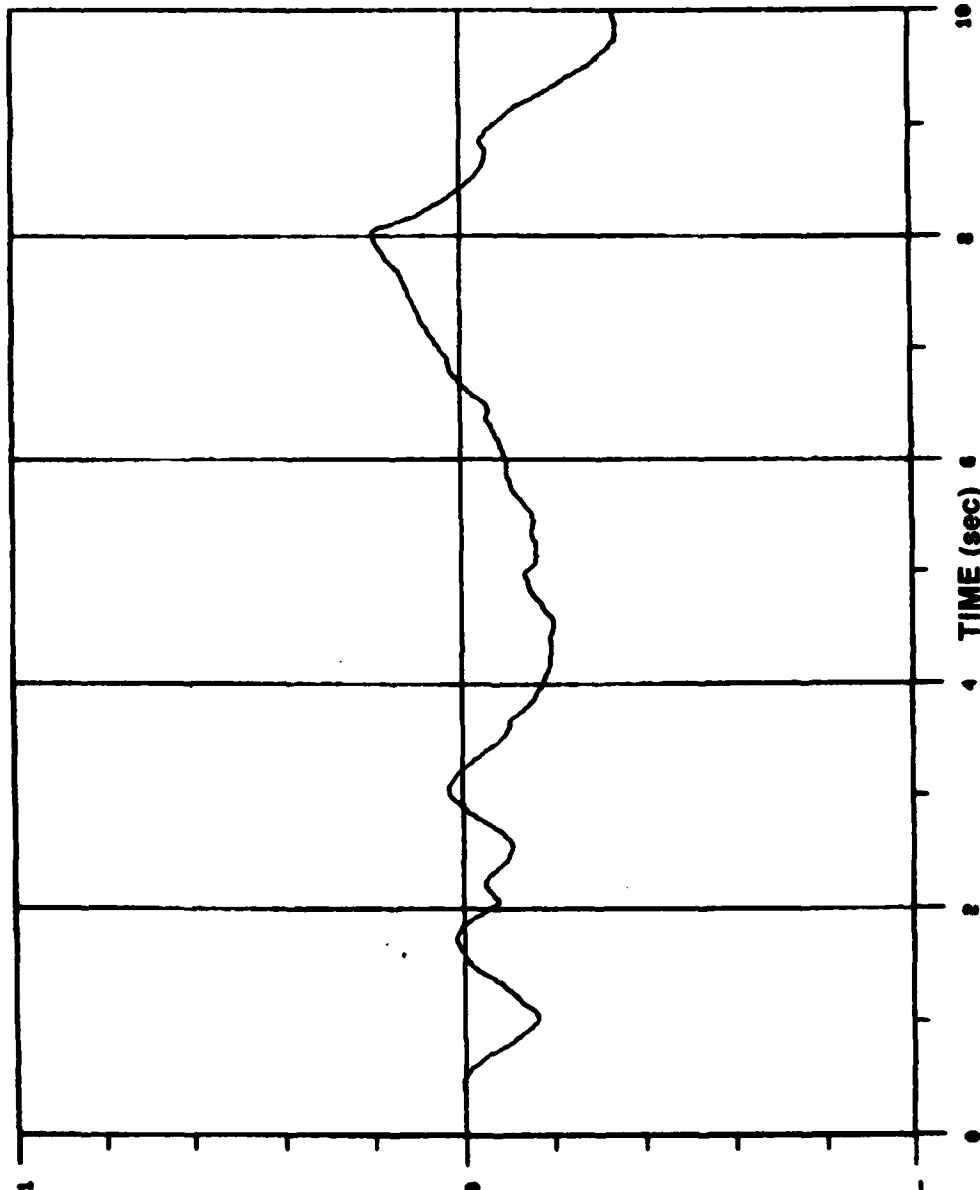


Figure 97. D.C disturbance estimation error for W_1 , $t_f = 9.85$ sec, $PGO = 1$, $W_1 = W_3 = 0$.

E P Z 1

x10⁻⁶



E P Z 3

Figure 98. DAC disturbance estimation error for W_3 , $t_f = 9.85$ sec, $PGO = 1$, $W_1 = W_3 = 0$.

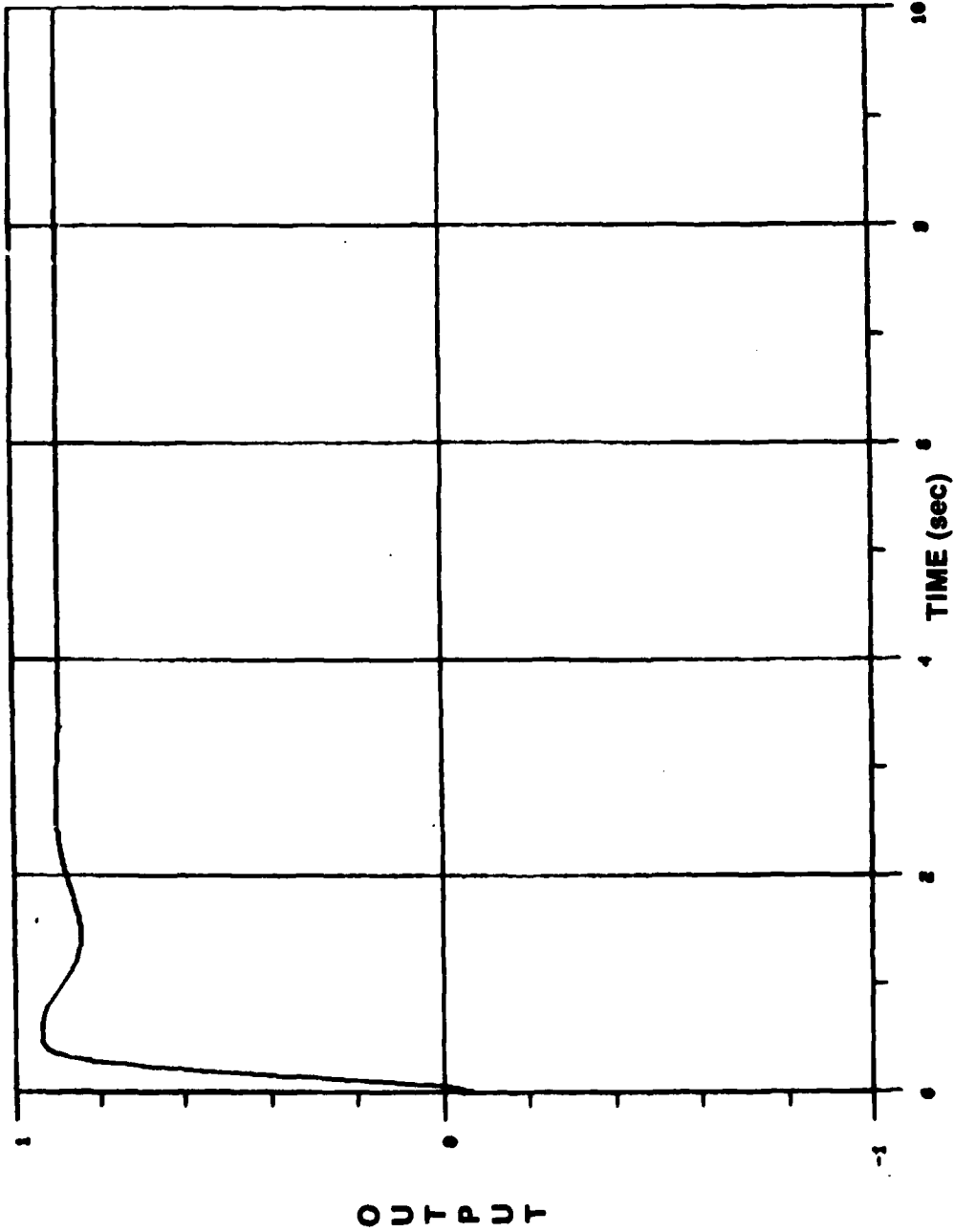


Figure 95. Acceleration loop response $t_f = 9.85$ sec, $P_{GO} = 1$, $W_1 = 1$, $W_3 = 0$.

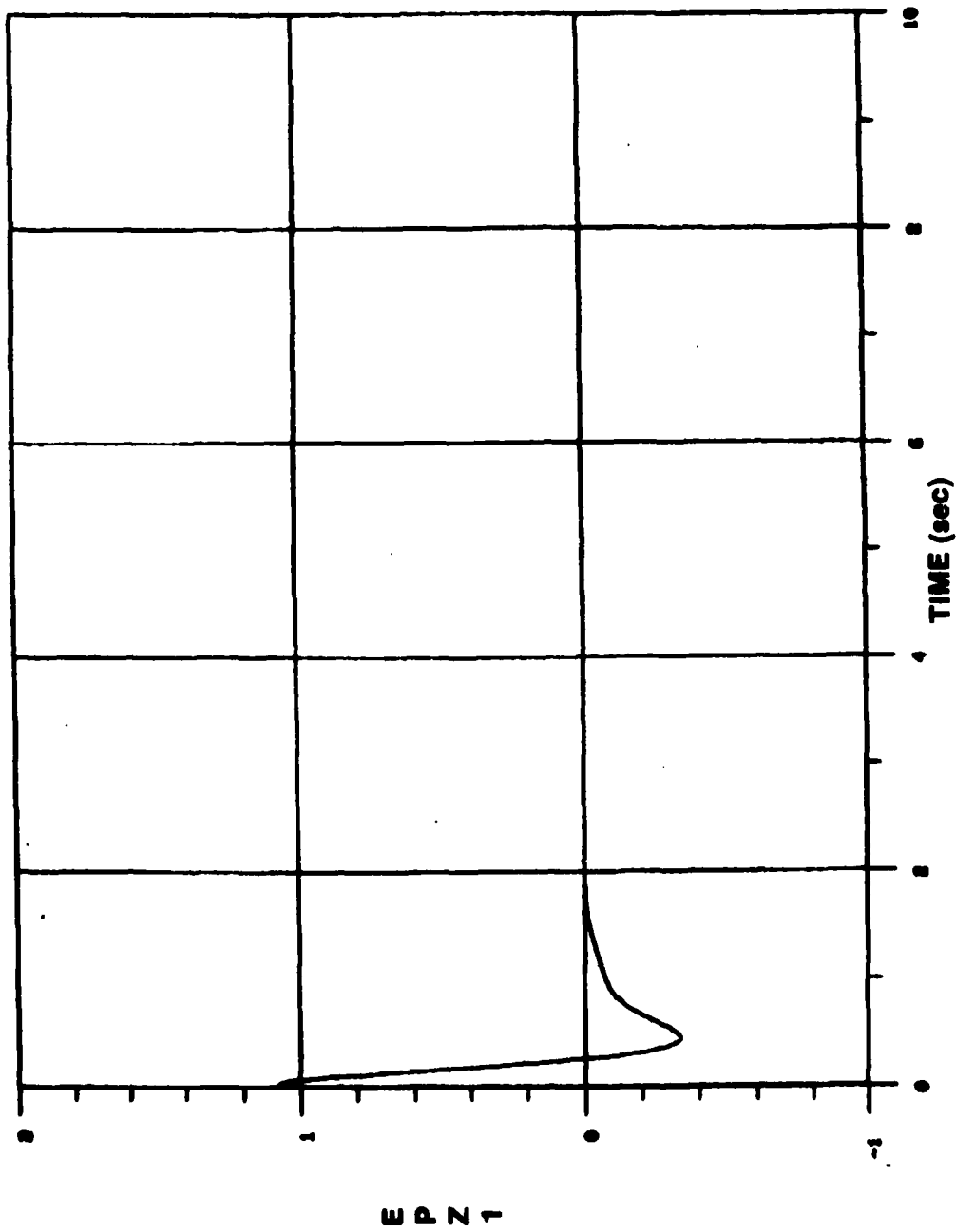


Figure 100. DAC disturbance estimation error for $W_1, t_1 = 9.85 \text{ sec}, PGO = 1, W_1 = 1, W_3 = 0$.

X10⁻⁶

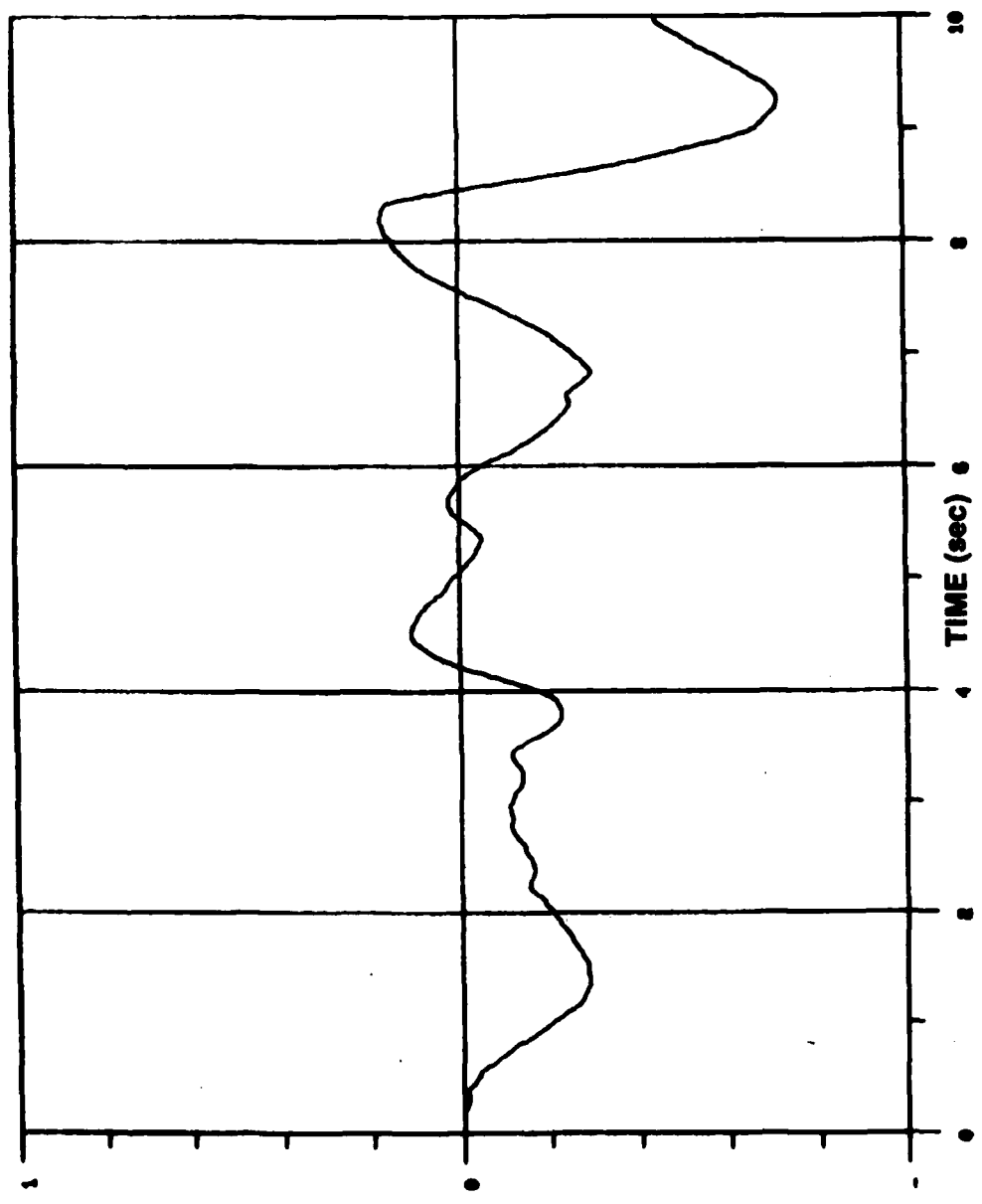


Figure 101. D/C disturbance estimation error for W_3 , $t_f = 9.85$ sec, $P_{GO} = 1.0$, $W_1 = 1.0$, $W_3 = 0$.

E P Z 3

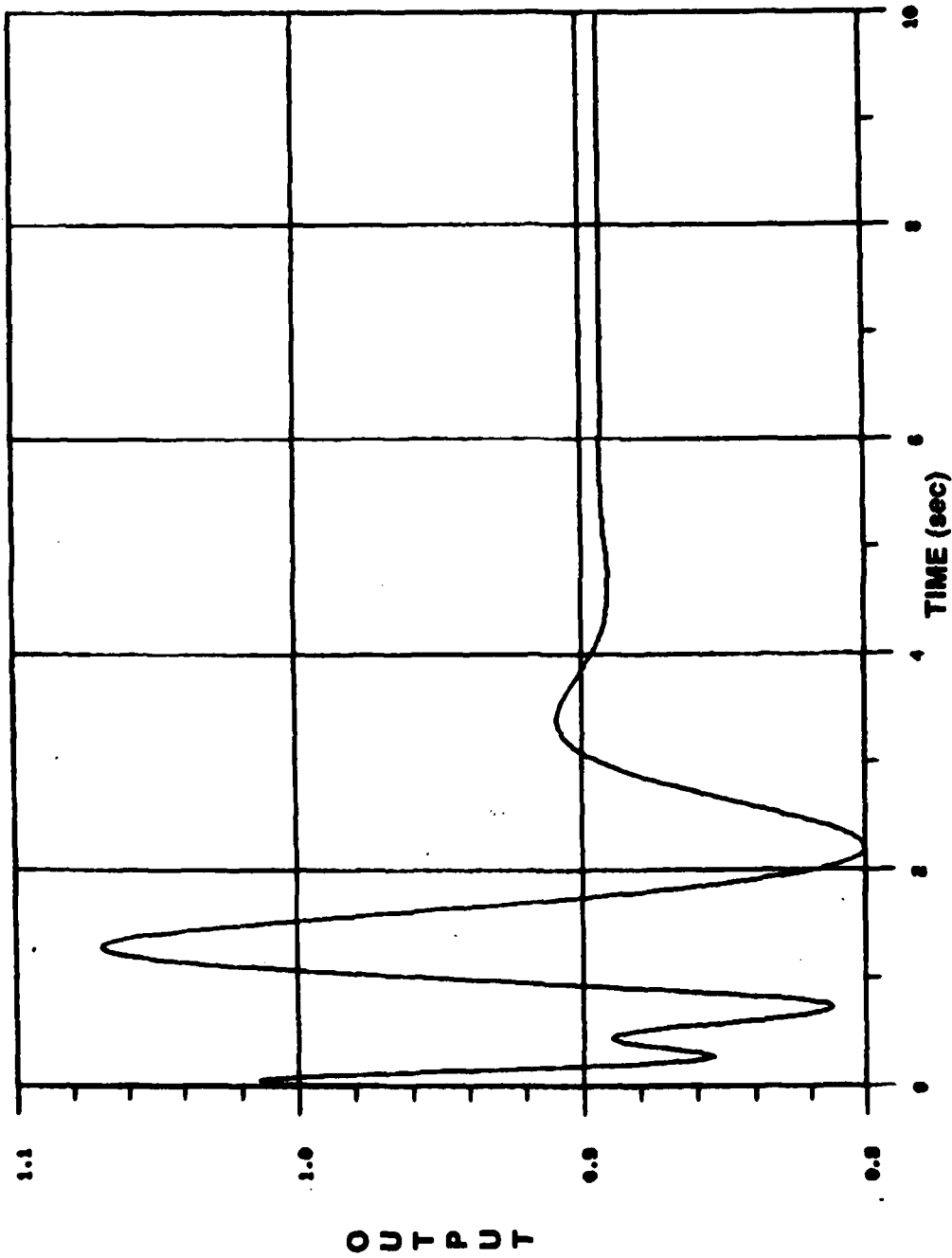


Figure 102. Acceleration loop response, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_1 = 0$, $W_3 = 1$.

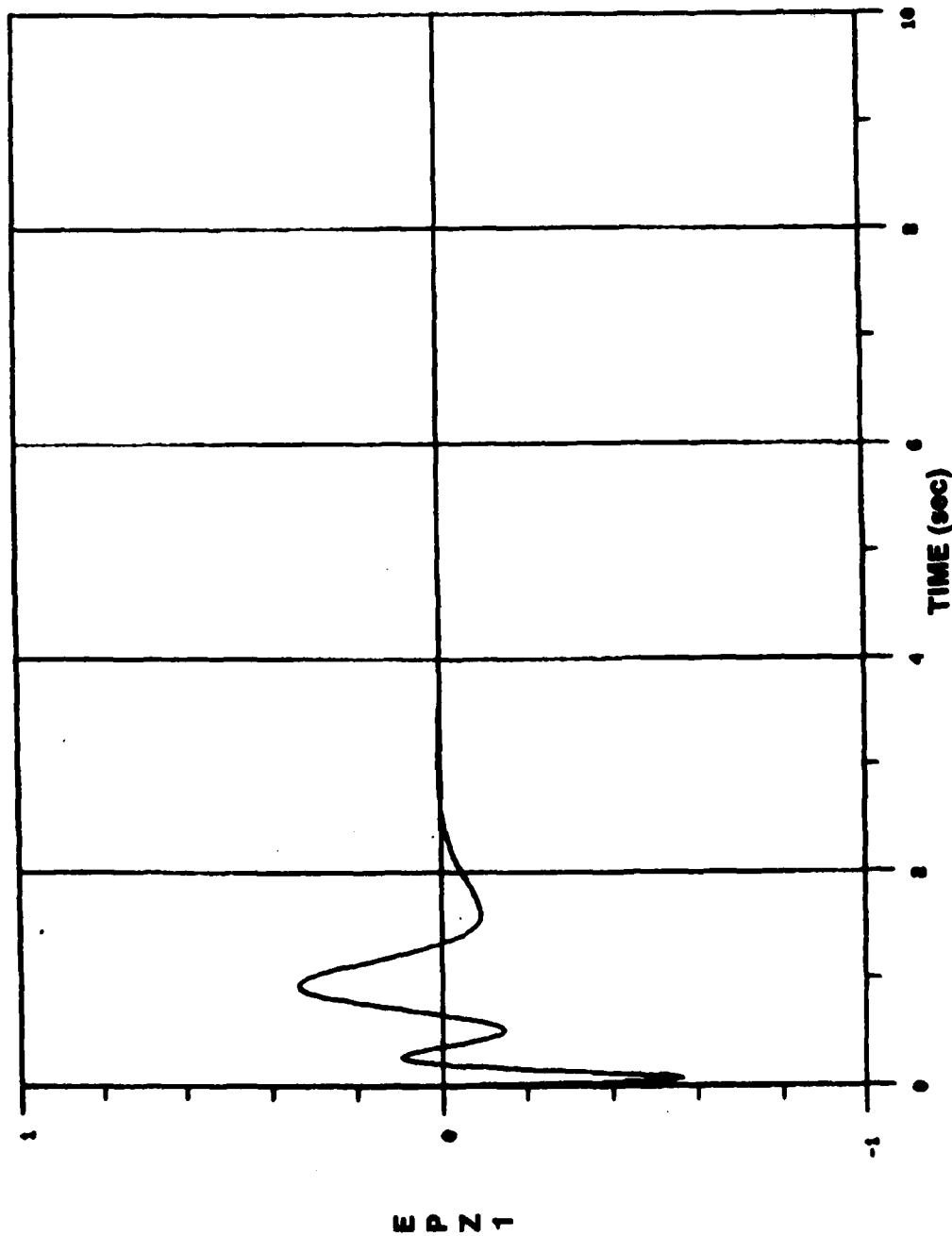


Figure 103. DAC disturbance estimation error for $W_1, t_f = 9.85 \text{ sec}$, $PGO = 1$, $W_1 = 0$, $W_3 = 1$.

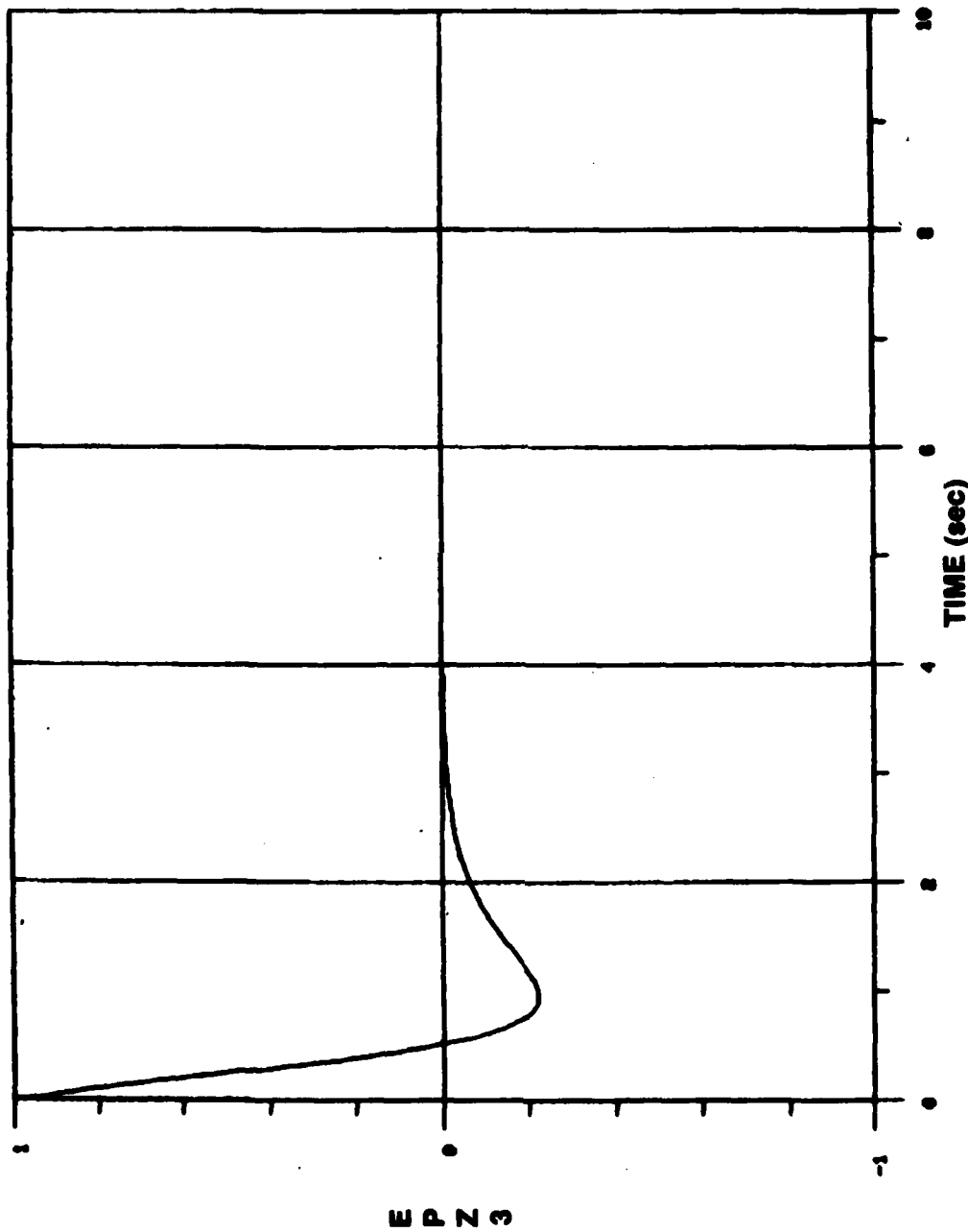


Figure 104. DAC disturbance estimation error for W_3 , $t_f = 9.85$ sec, $PGO = 1$, $W_1 = 0$, $W_3 = 1$.

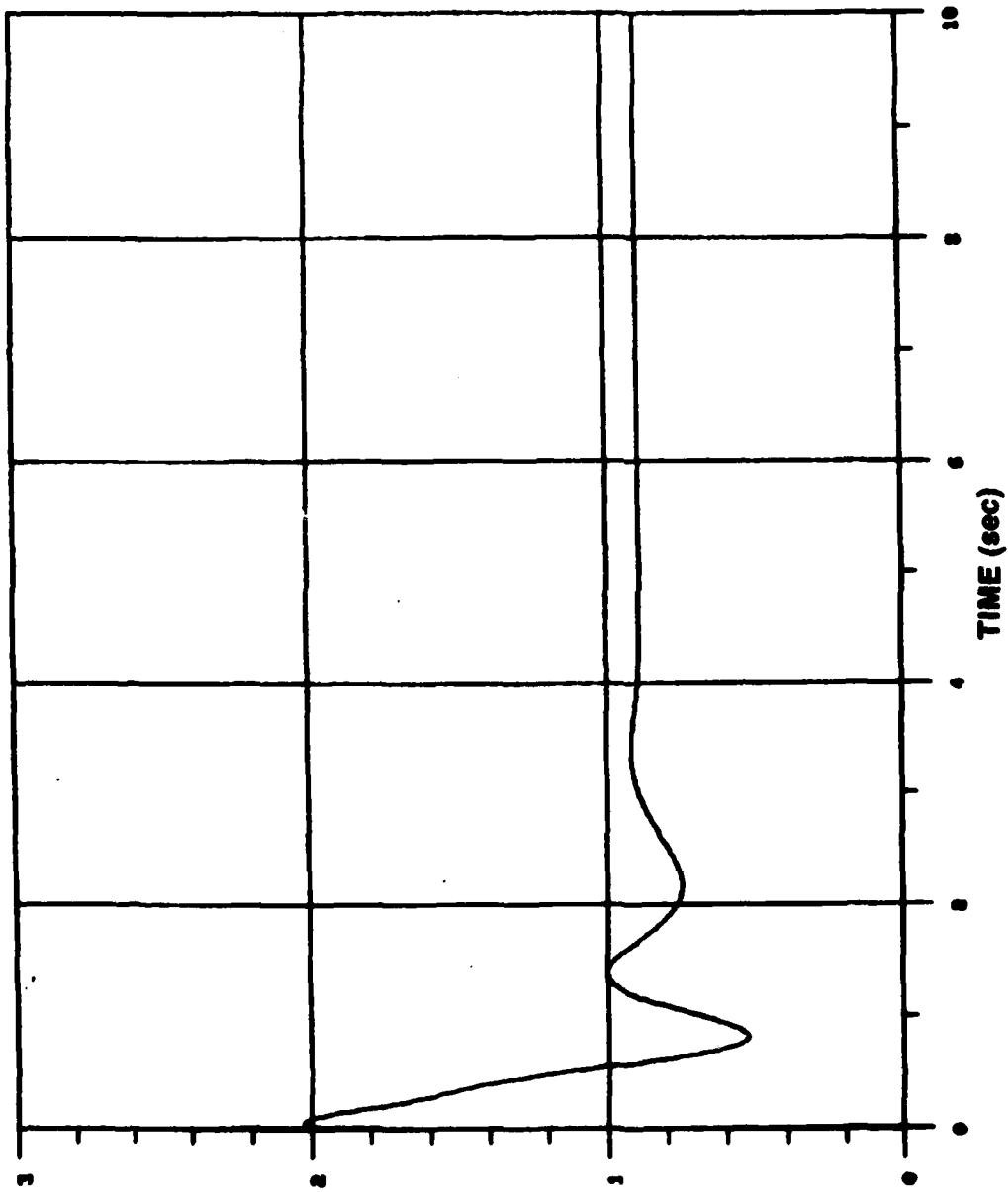


Figure 1.35. Acceleration loop response, $t_f = 9.85$ sec, $P_{GO} = 1$, $W_1 = 1$, $W_3 = 2$.

OUTPUT

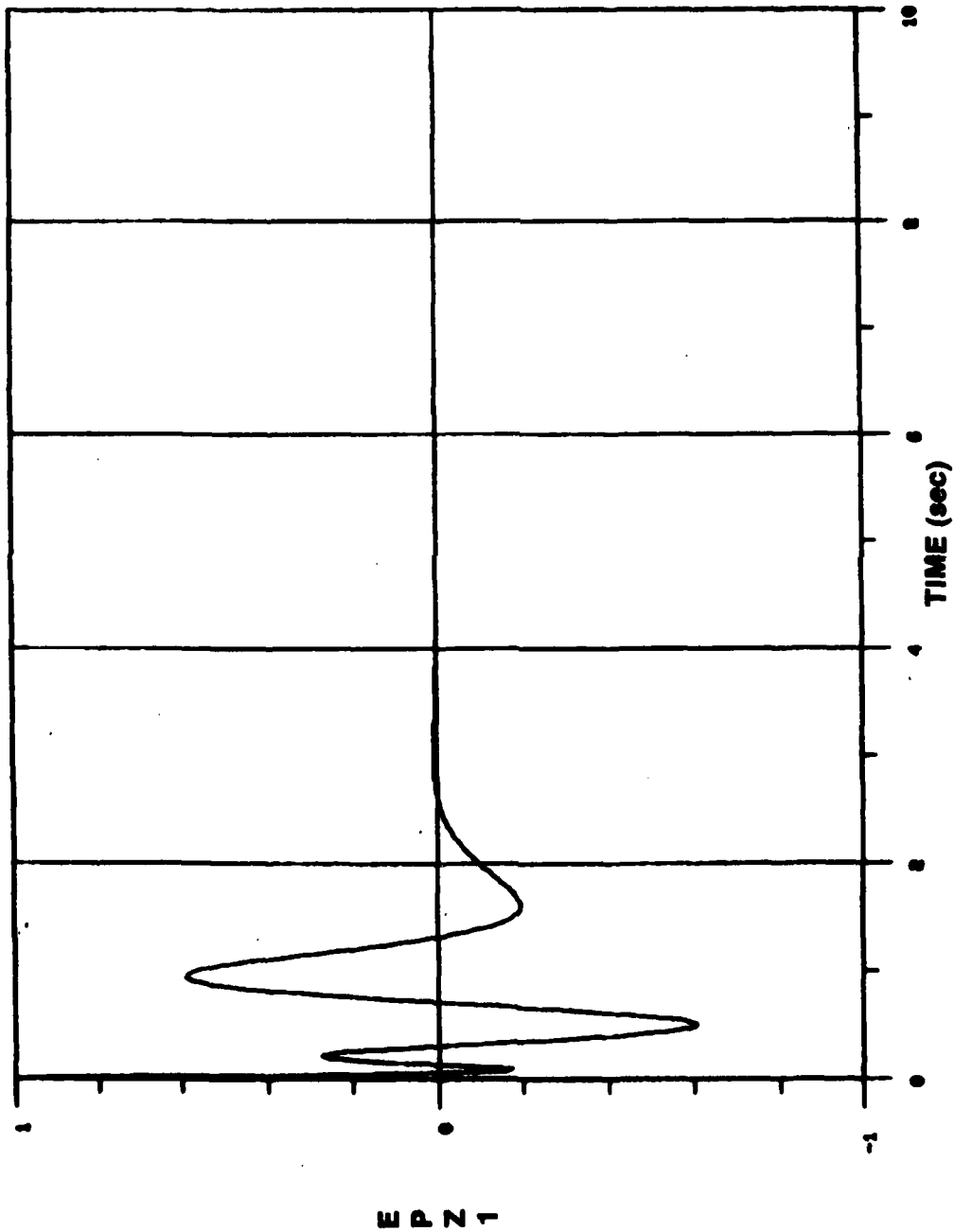


Figure 106. DAC disturbance estimation error for $W_1, t_f = 9.85 \text{ sec}, PGO = 1, W_1 = 1, W_3 = 2.$

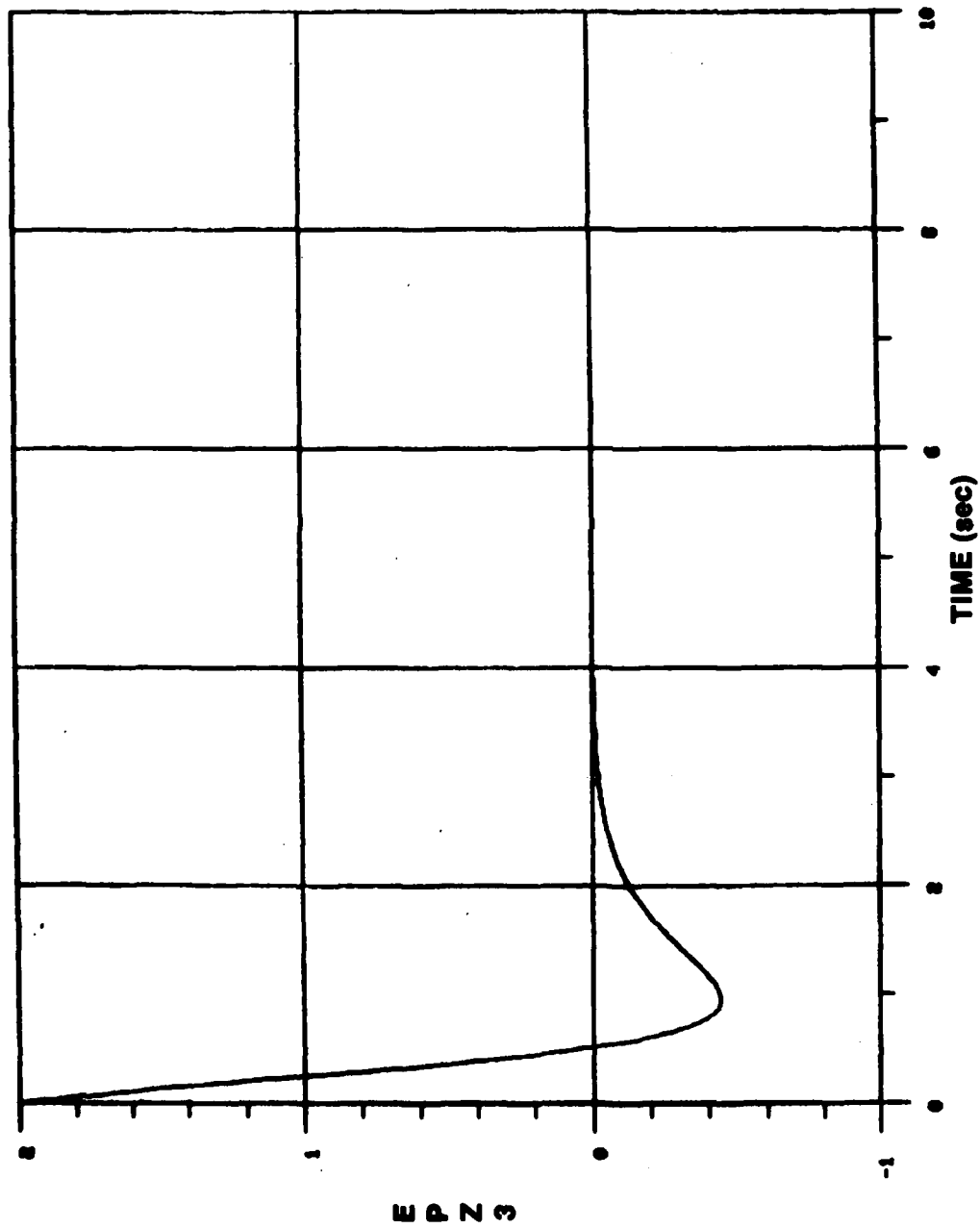


Figure 107. DAC disturbance estimation error for W_3 , $t_f = 9.85$ sec, $PGO = 1$, $W_1 = 1$, $W_3 = 2$.

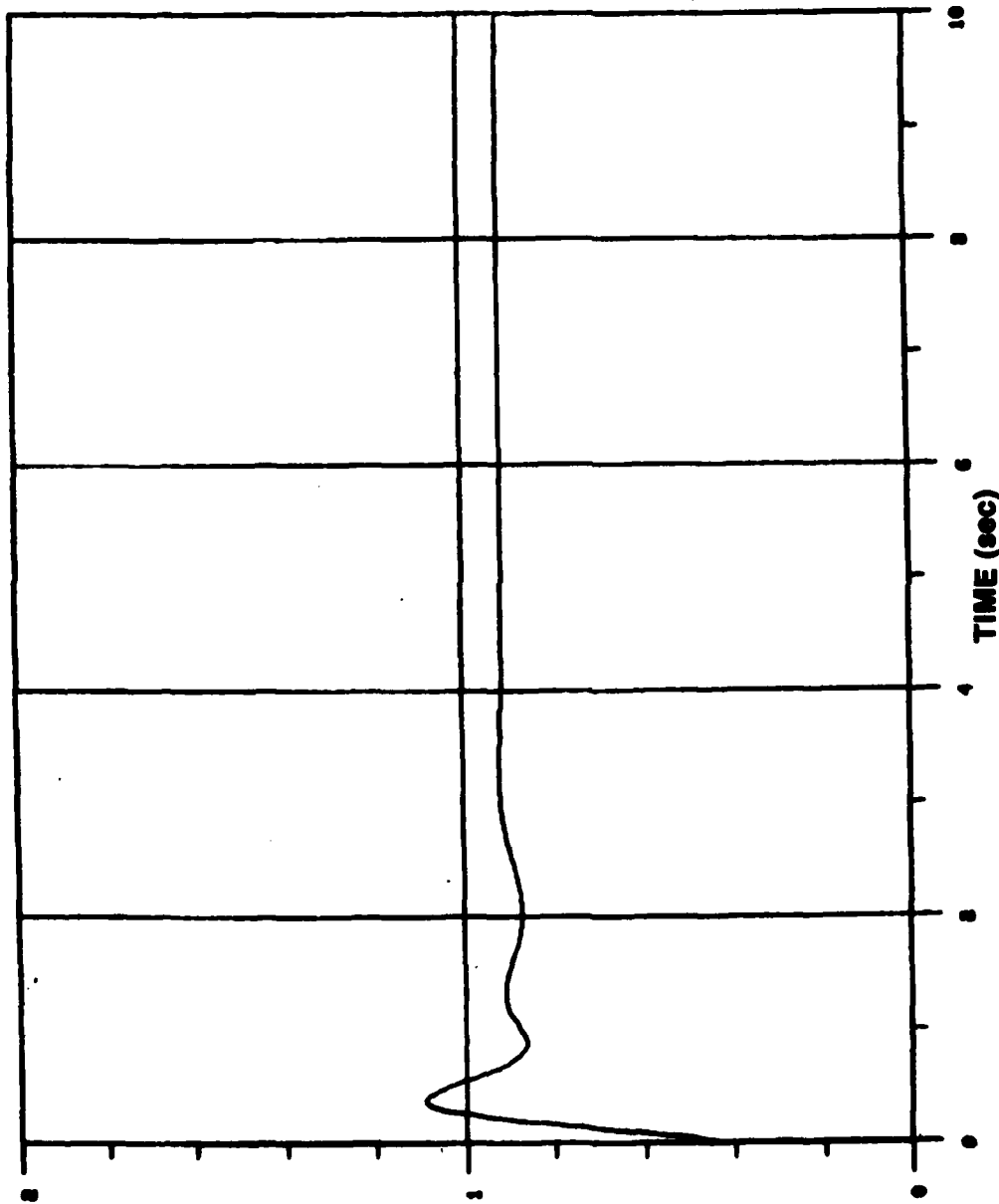


Figure 106. Acceleration loop response, $t_1 = 9.85$ sec, $P_{GO} = 1$, $W_1 = 1 + 0.2t$, $W_3 = 0.5 + 0.1t$.

0 2 4 6 8 10

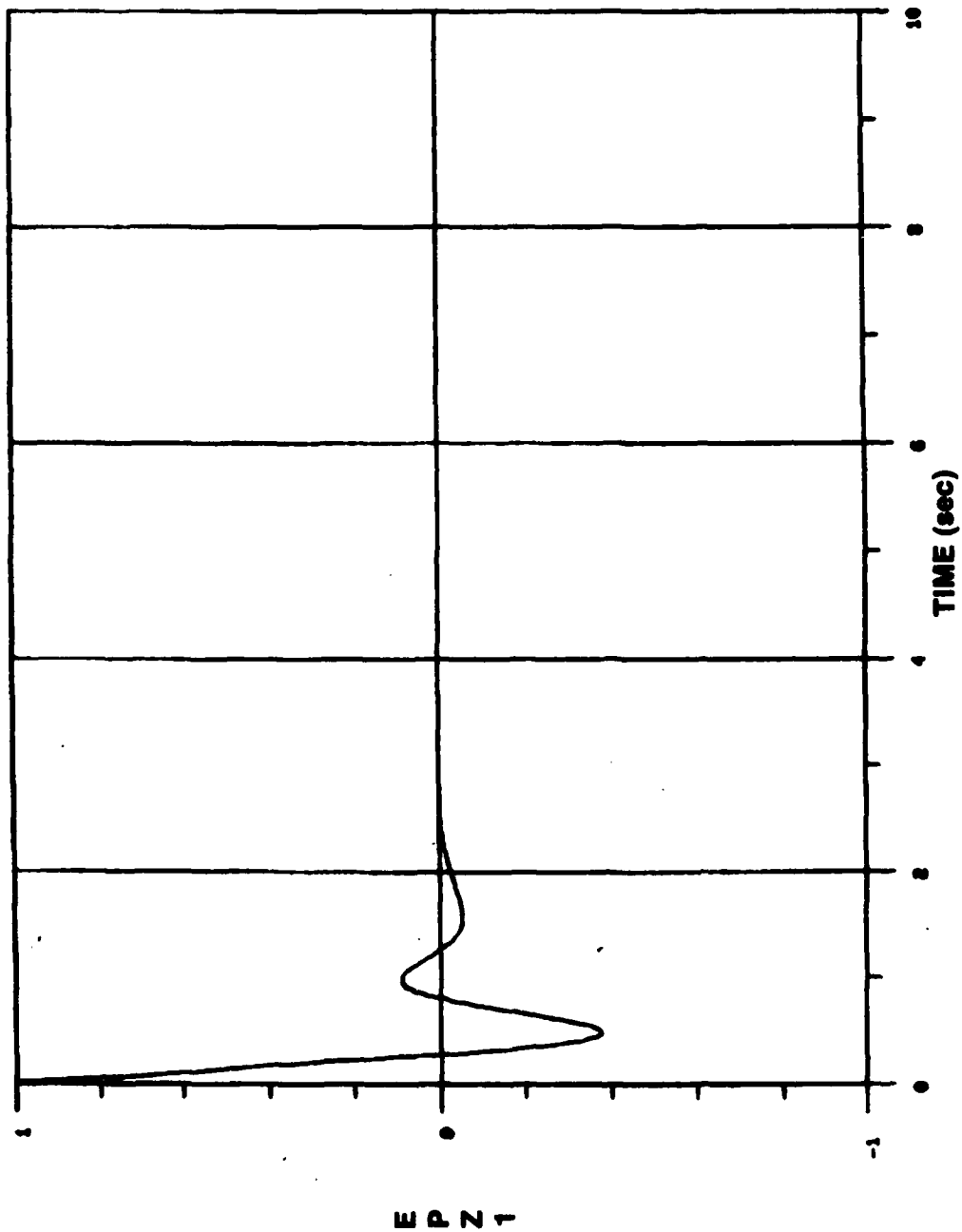


Figure 109. DAC disturbance estimation error for $W_1, t_f = 9.85 \text{ sec}, PGO = 1, W_1 = 1 + 0.2t, W_3 = 0.5 + 0.1t$.

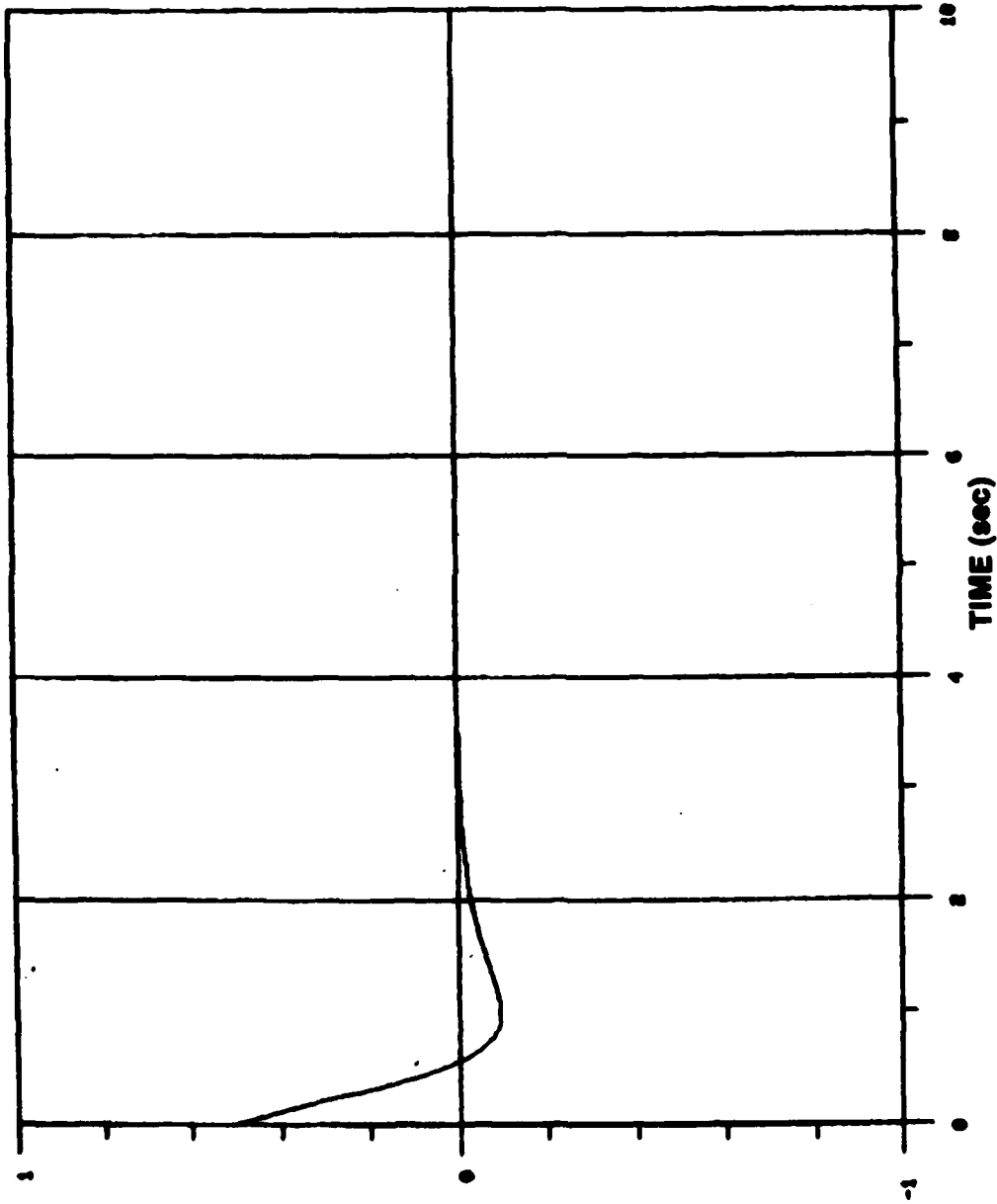


Figure 110. DAC disturbance estimation error in W_3 , $t_f = 9.85$ sec, $PGO = 1$, $W_1 = 1 + 0.2t$, $W_3 = 0.5 + 0.1t$.

E P Z 3

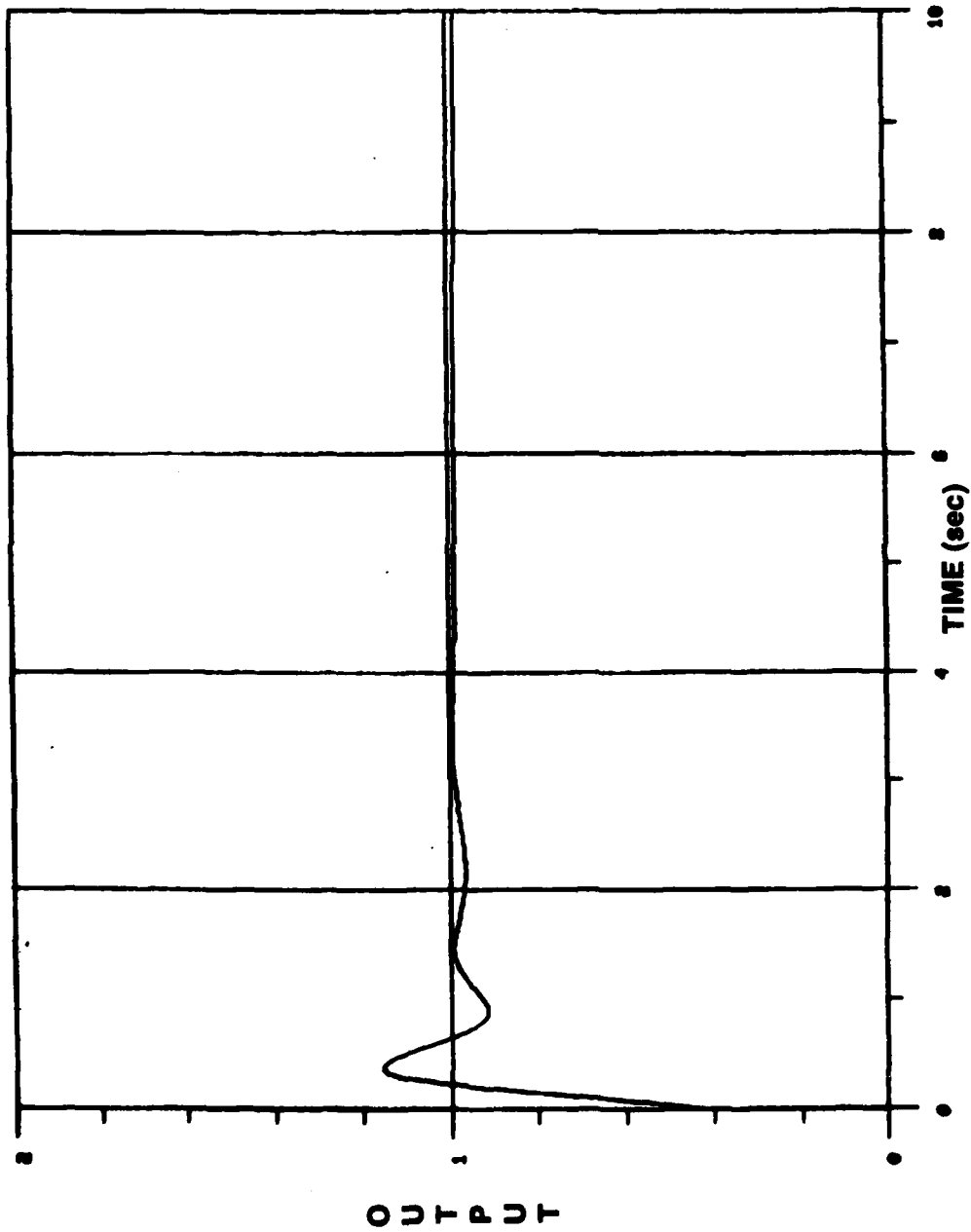


Figure 111. Acceleration loop response, $t_f = 9.85$ sec, $PGO = 1$, $W_1 = 1.0 - 0.2t$, $W_3 = 0.5 + 0.5t$.

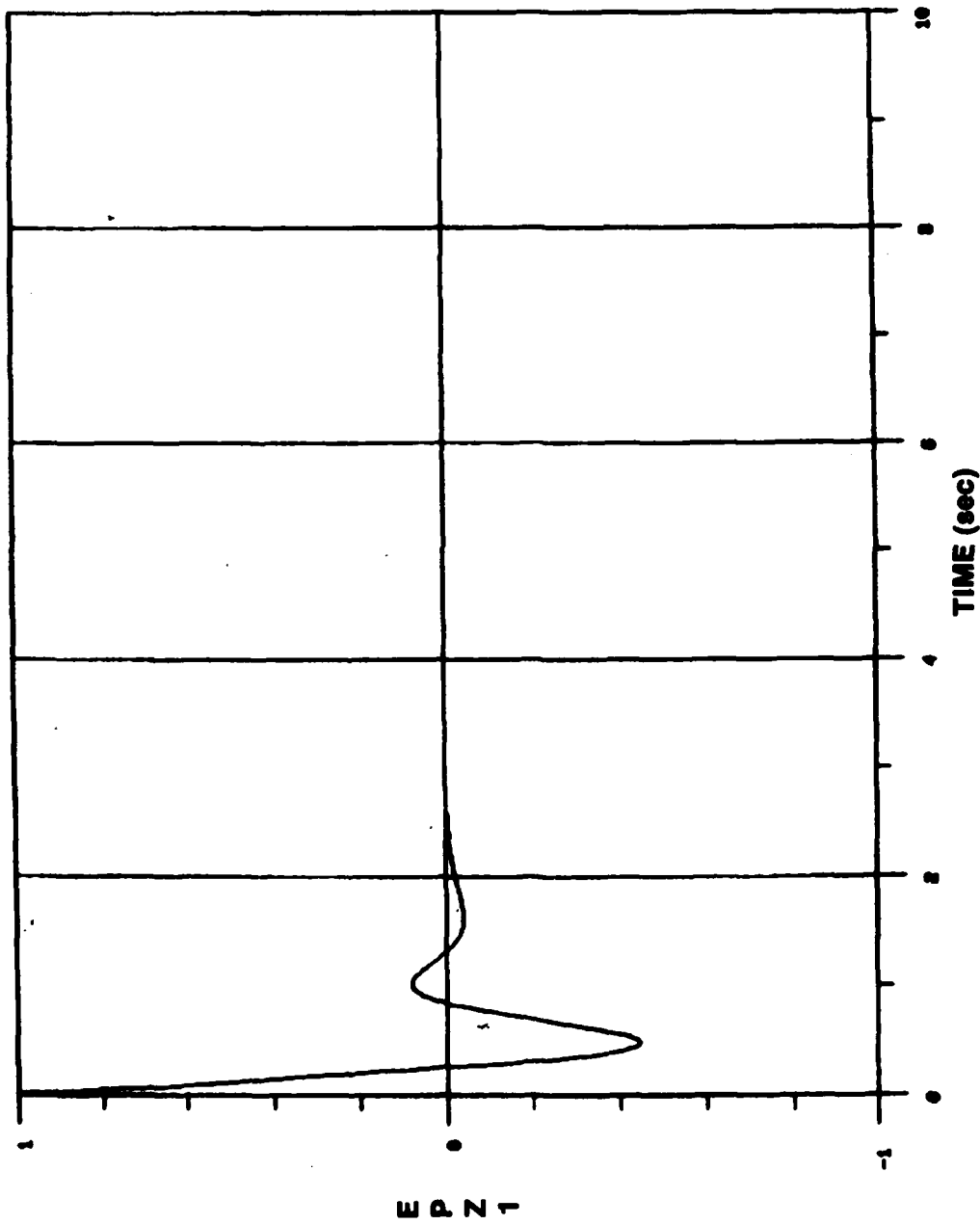


Figure 112. DAC disturbance estimation error for $W_1, t_1 = 9.85 \text{ sec}$, $P G O = 1$, $W_1 = 1.0$
 $- 0.2t, W_3 = 0.5 + 0.5t$.

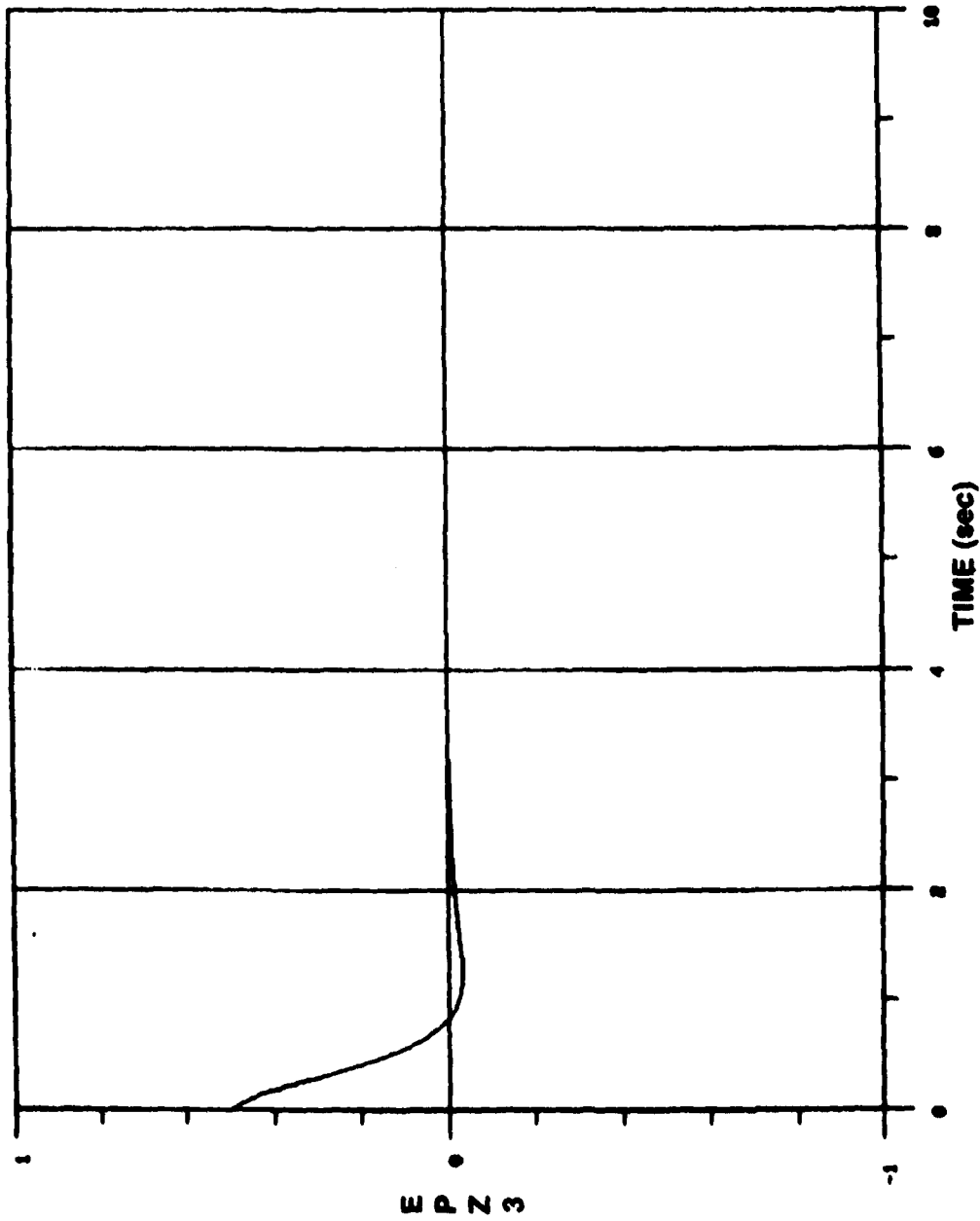


Figure 113. DAC disturbance estimation error for W_3 , $t_f = 9.85$ sec, $PGO = 1$, $W_1 = 1.0$
 $- 0.2t$, $W_3 = 0.5 + 0.5t$.

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1. Johnson, C. D., "Accommodation of Disturbances in Optimal Control Problems," *Int. Journal Control*, Vol. 15, No. 2, 1972, pp. 209-231.
2. Johnson, C. D., "On Observers for Systems with Unknown and Inaccessible Inputs," *Int. Journal Control*, Vol. 21, No. 5, 1975, pp. 825-831.
3. Johnson, C. D., "Accommodation of External Disturbances in Linear Regulator and Servomechanism Problems," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, Vol. AC-16, No. 6, December 1971.
4. Johnson, C. D., "Algebraic Solution of the Servomechanism Problem with External Disturbances," *Transactions of American Society of Mechanical Engineers, Journal of Dynamics Systems, Measurements and Control*, March 1974, pp. 25-35.
5. McCowan, Wayne L., *Investigation of Disturbance Accommodating Controller Design*, U.S. Army Missile Research and Development Command, Redstone Arsenal, Alabama, Report No. T-78-65, July 1978.

APPENDIX A
DIGITAL SIMULATION OF ACCELERATION
LOOP WITH DISTURBANCE ON INPUT


```

0131 EG=XKETASCNS(XK11+80-24)
0132 E7=XKETASCNS(XK21+81-25)
0133 EG=XKETASCNS(XK31+82-26)
0134 EG=XKETASCNS(XK41+83-27)
0135 TIME=0
0136 CONTINUE
0137 IF(TIME.GE.10.) GO TO 1000
0138 J=J+1
0139 DO 100 KUTTA=1,4
0140 U1=C9+C1*TIME
0141 U=POO-ZM1-Y
0142 U1=UM1
0143 X9=XK7ADPSCNPSBZBP2U1-87892V
0144 X03=X4+XK7ADPSCNPSBZBP2U1-88892V
0145 X02=X3+XK7ADPSCNPSBZBP2U1-89892V
0146 X01=X2+XK7ADPSCNPSBZBP2U1-90892V
0147 V=X1+XK7ADPSCNPSBZBP2U1
0148 X0H1=EP20H1+X0B+EKZM1-XK11SV+XKETASCNS(XK11+80-24)2U
0149 X0G=E120H1+X0K+EKZM1-XK21SV+XKETASCNS(XK21+81-25)2U
0150 X0G=E220H1+X0M+EKZM1-XK31SV+XKETASCNS(XK31+82-26)2U
0151 X0M=E320H1+EKZM1-XK41SV+XKETASCNS(XK41+83-27)2U
0152 ZM1=XK120H1+XK12B+XKETASCNS(XK12B+84-28)2U
0153 Z0B=XK220H1+XK22B+XKETASCNS(XK22B+85-29)2U

```

FORTRAN IV-PLUS V02-51 /TR:BLOCKS/UR 12:40:50 00-MAY-70

```

0154 GO TO (30,60,30,40),KUTTA
0155 CONTINUE
0156 NP=NP-1
0157 IF(KUTTA.GT.1)GO TO 31
0158 IF(NP.GT.0)GO TO 31
0159 NP=10
0160 BUFFER(1)=TIME
0161 BUFFER(2)=Y
0162 EP21=X1-ZM1
0163 BUFFER(3)=EP21
0164 BUFFER(4)=DELTC
0165 WRITE(2)BUFFER
0166 CONTINUE
0167 TIME=TIME+.5207
0168 40 CONTINUE
0169 60 CALL NUMOK
0170 100 CONTINUE
0171 IF(J.LT.20) GO TO 1010
0172 EP21=X1-ZM1
0173 WRITE (5,5010),Z21,Y,DELTC,TIME
0174 FORMAT(2X,4I62.6,2X1)
0175 5010 IJJ=IJJ+1
0176

```

```

0177 J=0
0178 GO TO 1010
0179 CONTINUE
0180 GO TO 3333
0181 END
  
```

```

FORTRAN IV-PLUS 1000-S1 12:40:50 00-NOV-70
  
```

PROGRAM SECTIONS		ATTRIBUTES	
NUMBER	NAME	SIZE	
1	SCORE1	000132	RU, I, COM, LCL
2	SIBATA	000328	RU, B, COM, LCL
3	SIBATA	000314	RU, B, COM, LCL
4	SIBATA	000328	RU, B, COM, LCL
5	STEPS	000324	RU, B, COM, LCL
6	.00005	000130	RU, B, COM, GBL

VARIABLES											
NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
A	R24	4-000170	AD	R24	4-000174	AD	R24	4-000320	ADP	R24	4-000304
ADP	R24	4-000316	ADP	R24	4-000324	ADP	R24	4-000314	ADP	R24	4-000374
B1	R24	4-000328	B2	R24	4-000340	B3	R24	4-000344	B3P	R24	4-000374
B3P	R24	4-000370	B4	R24	4-000314	B4P	R24	4-000324	B4P	R24	4-000304
B6	R24	4-000424	B6P	R24	4-000436	B7	R24	4-000438	B7P	R24	4-000470
CH1P	R24	4-000504	CR	R24	4-000144	CRP	R24	4-000144	CB	R24	4-000504
MILLIC	R24	4-000504	E1	R24	4-000504	EPX1	R24	4-000504	EPZ1	R24	4-000504
E1	R24	4-000504	E2	R24	4-000504	E3	R24	4-000504	EC	R24	4-000516
E2	R24	4-000504	E3	R24	4-000504	EPX1	R24	4-000504	EPZ1	R24	4-000504
E3	R24	4-000504	ED	R24	4-000528	EPX1	R24	4-000504	ET	R24	4-000504
J	102	4-000544	KUTTA	102	4-000528	GAN	R24	4-000118	LJ1	102	4-000528
CH1P	R24	4-000504	KUTTA	102	4-000528	MP	R24	4-000120	CH1P	R24	4-000504
CH2P	R24	4-000504	CH1P	R24	4-000120	CH2P	R24	4-000120	CH1P	R24	4-000504
CH3P	R24	4-000504	CH2P	R24	4-000120	CH3P	R24	4-000120	CH2P	R24	4-000504
U1	R24	4-000504	CH3P	R24	4-000120	CH4P	R24	4-000120	CH3P	R24	4-000504
U2	R24	4-000504	U1	R24	4-000548	CH4P	R24	4-000120	CH4P	R24	4-000504
X01	R24	4-000104	U2	R24	4-000548	CH5P	R24	4-000120	CH5P	R24	4-000504
X02	R24	4-000104	X01	R24	4-000548	CH6P	R24	4-000120	CH6P	R24	4-000504
X03	R24	4-000104	X02	R24	4-000548	CH7P	R24	4-000120	CH7P	R24	4-000504
X04	R24	4-000104	X03	R24	4-000548	CH8P	R24	4-000120	CH8P	R24	4-000504
X05	R24	4-000104	X04	R24	4-000548	CH9P	R24	4-000120	CH9P	R24	4-000504
X06	R24	4-000104	X05	R24	4-000548	CH10P	R24	4-000120	CH10P	R24	4-000504
X07	R24	4-000104	X06	R24	4-000548	CH11P	R24	4-000120	CH11P	R24	4-000504
X08	R24	4-000104	X07	R24	4-000548	CH12P	R24	4-000120	CH12P	R24	4-000504
X09	R24	4-000104	X08	R24	4-000548	CH13P	R24	4-000120	CH13P	R24	4-000504
X10	R24	4-000104	X09	R24	4-000548	CH14P	R24	4-000120	CH14P	R24	4-000504
X11	R24	4-000104	X10	R24	4-000548	CH15P	R24	4-000120	CH15P	R24	4-000504
X12	R24	4-000104	X11	R24	4-000548	CH16P	R24	4-000120	CH16P	R24	4-000504
X13	R24	4-000104	X12	R24	4-000548	CH17P	R24	4-000120	CH17P	R24	4-000504
X14	R24	4-000104	X13	R24	4-000548	CH18P	R24	4-000120	CH18P	R24	4-000504
X15	R24	4-000104	X14	R24	4-000548	CH19P	R24	4-000120	CH19P	R24	4-000504
X16	R24	4-000104	X15	R24	4-000548	CH20P	R24	4-000120	CH20P	R24	4-000504
X17	R24	4-000104	X16	R24	4-000548	CH21P	R24	4-000120	CH21P	R24	4-000504
X18	R24	4-000104	X17	R24	4-000548	CH22P	R24	4-000120	CH22P	R24	4-000504
X19	R24	4-000104	X18	R24	4-000548	CH23P	R24	4-000120	CH23P	R24	4-000504
X20	R24	4-000104	X19	R24	4-000548	CH24P	R24	4-000120	CH24P	R24	4-000504
X21	R24	4-000104	X20	R24	4-000548	CH25P	R24	4-000120	CH25P	R24	4-000504
X22	R24	4-000104	X21	R24	4-000548	CH26P	R24	4-000120	CH26P	R24	4-000504
X23	R24	4-000104	X22	R24	4-000548	CH27P	R24	4-000120	CH27P	R24	4-000504
X24	R24	4-000104	X23	R24	4-000548	CH28P	R24	4-000120	CH28P	R24	4-000504
X25	R24	4-000104	X24	R24	4-000548	CH29P	R24	4-000120	CH29P	R24	4-000504
X26	R24	4-000104	X25	R24	4-000548	CH30P	R24	4-000120	CH30P	R24	4-000504
X27	R24	4-000104	X26	R24	4-000548	CH31P	R24	4-000120	CH31P	R24	4-000504
X28	R24	4-000104	X27	R24	4-000548	CH32P	R24	4-000120	CH32P	R24	4-000504
X29	R24	4-000104	X28	R24	4-000548	CH33P	R24	4-000120	CH33P	R24	4-000504
X30	R24	4-000104	X29	R24	4-000548	CH34P	R24	4-000120	CH34P	R24	4-000504
X31	R24	4-000104	X30	R24	4-000548	CH35P	R24	4-000120	CH35P	R24	4-000504
X32	R24	4-000104	X31	R24	4-000548	CH36P	R24	4-000120	CH36P	R24	4-000504
X33	R24	4-000104	X32	R24	4-000548	CH37P	R24	4-000120	CH37P	R24	4-000504
X34	R24	4-000104	X33	R24	4-000548	CH38P	R24	4-000120	CH38P	R24	4-000504
X35	R24	4-000104	X34	R24	4-000548	CH39P	R24	4-000120	CH39P	R24	4-000504
X36	R24	4-000104	X35	R24	4-000548	CH40P	R24	4-000120	CH40P	R24	4-000504
X37	R24	4-000104	X36	R24	4-000548	CH41P	R24	4-000120	CH41P	R24	4-000504
X38	R24	4-000104	X37	R24	4-000548	CH42P	R24	4-000120	CH42P	R24	4-000504
X39	R24	4-000104	X38	R24	4-000548	CH43P	R24	4-000120	CH43P	R24	4-000504
X40	R24	4-000104	X39	R24	4-000548	CH44P	R24	4-000120	CH44P	R24	4-000504
X41	R24	4-000104	X40	R24	4-000548	CH45P	R24	4-000120	CH45P	R24	4-000504
X42	R24	4-000104	X41	R24	4-000548	CH46P	R24	4-000120	CH46P	R24	4-000504
X43	R24	4-000104	X42	R24	4-000548	CH47P	R24	4-000120	CH47P	R24	4-000504
X44	R24	4-000104	X43	R24	4-000548	CH48P	R24	4-000120	CH48P	R24	4-000504
X45	R24	4-000104	X44	R24	4-000548	CH49P	R24	4-000120	CH49P	R24	4-000504
X46	R24	4-000104	X45	R24	4-000548	CH50P	R24	4-000120	CH50P	R24	4-000504

FORTRAN IU-PLUS U02-51 10:04:30 11-MAY-79

```

SUBROUTINE RUNCK
COMMON X,DX,KUTTA,DT,NX
DIMENSION X(10),DX(10),DXA(10),DXA(10)
GO TO (10,30,50,70),KUTTA
10 DO 20 I=1,NX
   X(I)=X(I)
   DXA(I)=DT*DX(I)
20 X(I)=X(I)+5*DXA(I)
   RETURN
30 DT=2.*DT
   MDT=5*DT
   GO 40 I=1,NX
40 X(I)=X(I)+MDT*DX(I)
   RETURN
50 DO 50 I=1,NX
   VBT=DT*DX(I)
   DXA(I)=DXA(I)+2.*VBT
60 X(I)=X(I)+VBT
   RETURN
70 DO 80 I=1,NX
   X(I)=X(I)+(DXA(I)+DT*DX(I))/6.
   RETURN
END

```

FORTRAN IU-PLUS U02-51 10:04:30 11-MAY-79

PROGRAM SECTIONS

NUMBER	NAME	SIZE	ATTRIBUTES
1	SCODE1	000510	164
2	SPDATA	000312	5
4	SUXRS	000135	47
6	.5888.	000130	44

ENTRY POINTS

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
RUNCK		1-000020									

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
DT	R34	6-000123	HDT	R34	4-000126	I	I32	4-000120	KUTTA	I32	6-000126
TDT	R34	4-000122	UDT	R34	4-000132						

ARRAYS

NAME	TYPE	ADDRESS	SIZE	DIMENSIONS
DX	R34	6-000050	000050	20 (10)
DXA	R34	4-000050	000050	20 (10)
X	R34	6-000000	620050	20 (10)
XA	R34	4-000000	600050	20 (10)

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APPENDIX B
DIGITAL SIMULATION OF RATE LOOP
WITH DISTURBANCE ON OUTPUT

```

0001      C
0002      DATE LOOP WITH DMC
0003      COMMON X1,X2,XM1,XM2,ZM1,ZM2
0004      COMMON XH1,XH2,XM3,XM4,ZM3,ZM4,ZM5
0005      COMMON KUT1A,B1,NS
0006      INTEGER*8 LABEL1,LABEL2,LABEL3,LABEL4,LABEL5,LABEL6,LABEL7
0007      DIMENSION LABEL1(5),LABEL2(4),LABEL3(3),LABEL4(5)
0008      DIMENSION BUFFER(3)
0009      DATA LABEL1/79,85,84,90,85,84/
0010      DATA LABEL2/84,73,77,88/
0011      DATA LABEL3/80,72,48/
0012      DATA LABEL4/88,83,48,88,88/
0013      DATA LABEL5/88,83,48,88,88/
0014      CONTINUE
0015      XH1=0.
0016      XH2=0.
0017      XM1=0.
0018      XM2=0.
0019      ZM1=0.
0020      ZM2=0.
0021      X1=0.
0022      X2=0.
0023      XM1=0.
0024      XM2=0.
0025      ZM1=0.
0026      ZM2=0.
0027      TIME=0.
0028      PA=0.
0029      NP=0.
0030      DT=1./856.
0031      XL1=3.
0032      XL2=5.
0033      XL3=-4.
0034      XL4=-4.
0035      XL5=5.
0036      XL6=8.
0037      CASH=0.
0038      I,J=1
0039      ZETA=0.0025
0040      OMEGA=14.54
0041      KSTB=-107.
0042      GAM=0.528
0043      CR=0.1306
0044      Y=0.
0045      J=0.
0046      UC=1.
0047      CO=0.
0048      C1=0.
0049      WRITE(6,2) 'XXXXXXXXXXXXXXXXXXXXXXXXXXXXX'
0050      WRITE(6,3) 'MIT CTRL/2 TO TERMINATE JOB'
0051      WRITE(6,4) 'XXXXXXXXXXXXXXXXXXXXXXXXXXXXX'
0052      WRITE(6,5) 'X1,X2,XL1,XL2,XL3,XL4,XL5,XL6'
0053      READ(5,1) X1,X2,XL1,XL2,XL3,XL4,XL5,XL6
0054      WRITE(6,6) 'CASH,ZETA,OMEGA,KSTB,GAM,CR'
0055      READ(5,8) CASH,ZETA,OMEGA,KSTB,GAM,CR
0056

```

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2063
2064
2065

WRITE(6,2) UC,C9,C1'
READ(5,2)UC,C9,C1
XL50-XL31XKL41-XL30XKL48

FORMAT IU-PLUS U2E-51 13:30:01 00-MAY-79
TRIBLOCKS/UR

2066
2067
2068
2069

TZOWA-2.XZETASOMEGA
OMEGA-ONEGASOMEGA
XK2B--(XL1XKLXLSO)/(CRSOMEGA)
XKL2=(1/(CRSOMEGA))X(-TZOWACRCK2B+(XL1+XL2)XLS0+
1(XL1XKL2)(XL31+XL41))

2099
2061

(XL1+1./CR)(TZOWA-XL1BDCR+XL1+XKL31+XKL41)
XKL1=(1./CR)(TZOWACR(XKL1B-XK2B)+XKL1XKL1BDCR+
1-OMEGA+XLS0+XL1)(XL31+XL41)+XKL2(XL31+XKL41)+XL1XKL2)

2062
2063
2064
2065

B4=XK11-TZOWA-XK1BDCR
B1=XK11-XK1BDCR
B5=XK21-OMEGA-XK1BDCR+OM
B3=XK21-XK1BDCR+OM
B4=XK1B

2067
2068
2069
2070

B5=XK12
B6=XK22
B7=XK2B

2071
2072
2073
2074

WRITE(6,2000)
FORMAT(2M,4XK11,12X,4XK12,12X,4XK21,12X,4XK22/)
WRITE(6,2001)XK11,XK12,XK21,XK22
FORMAT(2X,4(012.6.2X))

2075
2076
2077
2078

WRITE(6,2002)
FORMAT(2M,2M00,12X,2M01,12X,2M02,12X,2M03,12X,2M04,12X,2M06/)
WRITE(6,2004)B0,B1,B2,B3,B4,B6
FORMAT(2X,6(012.6.2X))
WRITE(6,2005)

2079
2080
2081
2082
2083

FORMAT(4M,3XKL1,12X,3XKL2,12X,4XKL31,12X,4XKL32,12X,4XKL41,12X,4X
1XL42,12X,4XCRCH/)
WRITE(6,2032)XKL1,XKL2,XKL31,XKL32,XKL41,XL42,CRCH
FORMAT(4X,7(012.6.2X))
WRITE(6,2012)

2084
2085

FORMAT(2M,4M1ME,12X,2M01,12X,2M02,12X,3M0M1,12X,2M02,12X,2M02,1
12X,3M0M1,12X,1M0/)
WRITE(6,2006)
FORMAT(2M,3M2M0,12X,1M0,12X,4M0M1,12X,4M0M2,12X,4M2M1,12X,4M2

2086
2087
2088
2089
2090
2091
2092
2093

CONTINUE
IF(TIME.GE.10.) GO TO 1000
J=J+1
DO 100 KUTTA=1,4
UR=COGISTINE
UR=UC-Y-CRCHZ /1
DELTA=UR
XDE=XK1BDCR+DELTA-OMEGA5X1

```

0004      X01=X0+XKTBDDEL70-TZ000X01
0005      V=CRI(X1+UB)
0006      X0M1=(XK11SCR-TZ000)Z0M1+X0M0+XK11SCRZ0M1-XK11BY+XKT00UR
0007      X0M2=(XK21SCR-0REGAS)Z0M1+X0M1+XK21SCRZ0M1-XK21BY+XKT000UR
0008      Z0M1=XK12SCR(X01+Z01)+Z0M1+Z0M0-XK12BY
0009      Z0M2=XK22SCR(X01+Z01)+Z0M1+Z0M0-XK22BY
0010      GO TO (30,60,30,40),KUTTA
0011      30 CONTINUE
0012      NP=NP-1
0013      IF(KUTTA.GT.1) GO TO 31
0014      IF(NP.GT.0) GO TO 31
0015      NP=10
0016      EPSV=UC-Y
0017

```

```

FORTRAN IU-PLUS V08-51      13130101      00-MAY-79
TRIBLOCKS/UR

```

```

0107      EPZ1=0-ZM1
0108      BUFFER(1)=TIME
0109      BUFFER(2)=Y
0110      BUFFER(3)=ZM1
0111      WRITE(2)BUFFER
0112      31 CONTINUE
0113      TIME=TIME+.500T
0114      40 CONTINUE
0115      60 CALL RUMOK
0116      100 CONTINUE
0117      IF(J.LY.32) GO TO 1010
0118      WRITE (6,9015)TIME,X1,M01,M2,M02,UB,ZM1,Y
0119      FORMAT(2X,5(O12.5,2X))
0120      WRITE (6,9017)Z0M,U,X0M1,X0M2,Z0M1,Z0M2
0121      FORMAT(2X,5(O12.5,2X))
0122      J=0
0123      J=0
0124      GO TO 1010
0125      1000 CONTINUE
0126      4000 CONTINUE
0127      GO TO 3333
0128      STOP
0129      3
0130      26
0131      U NO PATH TO THIS STATEMENT
0132      END

```


PROGRAM SECTIONS

NUMBER	NAME	SIZE	ATTRIBUTES
1	SCORE1	00336	RU, I, COM, LCL
2	SPDATA	00234	RU, D, COM, LCL
3	SIDATA	00044	RU, D, COM, LCL
4	SUMMS	00322	RU, D, COM, LCL
5	STEPS	00010	RU, D, COM, LCL
6	.9886.	00070	RU, D, OVR, GBL

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
B0	R24	4-00026	B1	R24	4-00032	B2	R24	4-00038	B3	R24	4-00044
B5	R24	4-00052	B6	R24	4-00058	B7	R24	4-00064	CR	R24	4-00070
C0	R24	4-00062	C1	R24	4-00068	DELTA	R24	4-00074	DT	R24	4-00080
EPZ1	R24	4-00070	GAM	R24	4-00076	IJI	R24	4-00082	KUTTA	R24	4-00088
HP	R24	4-00076	HA	R24	4-00082	OMEGA	R24	4-00088	J	R24	4-00094
YZURA	R24	4-00082	U	R24	4-00088	UC	R24	4-00094	OR	R24	4-00100
U3	R24	4-00088	XDH1	R24	4-00094	XDME	R24	4-00100	UR	R24	4-00106
MH1	R24	4-00094	XDE	R24	4-00100	XDTD	R24	4-00106	XD1	R24	4-00112
KXZ1	R24	4-00100	XKZ2	R24	4-00106	XL50	R24	4-00112	XK11	R24	4-00118
XL31	R24	4-00106	XL32	R24	4-00112	XL41	R24	4-00118	XL1	R24	4-00124
X0	R24	4-00112	Y	R24	4-00118	ZDH1	R24	4-00124	XL2	R24	4-00130
ZH1	R24	4-00118	ZME	R24	4-00124	ZDH1	R24	4-00130	XL3	R24	4-00136
									ZDME	R24	4-00136

ARRAYS

NAME	TYPE	ADDRESS	SIZE	DIMENSIONS
BUFFER	R24	4-00004	00014	6 (3)
B004'	3-000130	2006'	2006'	3-000348
B008'	3-000236	1333	1-000000	2012' 4000

FUNCTIONS AND SUBROUTINES REFERENCED

RUNEX

TOTAL SPACE ALLOCATED - 00-0000 1102

FORTRAN IU-PLUS V02-S1 08:54:31 10-PMY-79
 /TR:BLOCKS/UR

```

2001 SUBROUTINE BUNOK
2002 COMMON X,DX,KUTTA,DT,MK
2003 DIMENSION X(6),DX(6),MA(6),DMA(6)
2004 DO 10 (I=1,6),KUTTA
2005 DO 20 I=1,MK
2006 MA(I)=X(I)
2007 DMA(I)=DTDX(I)
2008 X(I)=X(I)+SDMA(I)
2009 RETURN
2010 DT=2.SDT
2011 DT=DT*.SDOT
2012 DO 40 I=1,MK
2013 MA(I)=DMA(I)+DTDX(I)
2014 X(I)=X(I)+MOTDX(I)
2015 RETURN
2016 DO 60 I=1,MK
2017 VOT=DTDX(I)
2018 MA(I)=DMA(I)+2.SDOT
2019 X(I)=X(I)+VOT
2020 RETURN
2021 DO 80 I=1,MK
2022 X(I)=X(I)+DMA(I)+DTDX(I)/6.
2023 RETURN
2024 END
  
```

FORTRAN IU-PLUS V02-S1 08:54:31 10-PMY-79
 /TR:BLOCKS/UR

PROGRAM SECTIONS

NUMBER	NAME	SIZE	ATTRIBUTES	
1	SCORER1	000510	164	RU, I, COM, LCL
2	SPRINTA	000012	5	RU, B, COM, LCL
4	SLUDES	000076	31	RU, B, COM, LCL
6	.S0000.	000070	28	RU, B, COM, LCL

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APPENDIX C
DIGITAL SIMULATION OF ACCELERATION
LOOP WITH DISTURBANCE ON OUTPUT


```

0000 2000 FORMAT(2H,2000,12X,2H01,12X,2H02,12X,2H03,12X,2H04,12X,2H05,12X,2
      12X,12X,2H07)
0001 WRITE (6,2000)
0002 WRITE (6,2000)
0003 2000 FORMAT(2H,4H01,12X,4H02,12X,4H03,12X,4H04,12X,4H05,12X,12X,
      12X,4H07,12X,4H08)
0004 WRITE (6,2001)
0005 2001 FORMAT(2H,3H01,12X,3H02,12X,3H03,12X,3H04,12X,3H05,12X,4H
      07,12X,4H08,12X,4H09)
0006 WRITE (6,2002)
0007 2002 FORMAT(2H,2H01,12X,2H02,12X,2H03,12X,2H04,12X,2H05,12X,4H
      07,12X,4H08,12X,4H09,12X,4H10)
0008 WRITE (6,2003)
0009 2003 FORMAT(2H,1H01,12X,1H02,12X,1H03,12X,1H04,12X,1H05,12X,4H
      07,12X,4H08,12X,4H09,12X,4H10,12X,4H11)

```

FORTRAN IV-PLUS USE-61 10:03:55 11-MAY-79

```

0100 WRITE (6,2012)
0101 2012 FORMAT(2H,4H01,12X,4H02,12X,4H03,12X,4H04,12X,4H05,12X,4H
      06,12X,4H07)
0102 WRITE (6,2013)
0103 2013 FORMAT(2H,3H01,12X,3H02,12X,3H03,12X,3H04,12X,3H05,12X,4H
      06,12X,4H07)
0104 1010 CONTINUE
0105 IF (TIME.05.10.) GO TO 1000
0106 J=J+1
0107 DO 100 KUTTA=1,4
      100 CONTINUE
0108 U=U+C*(KUTTA-1)
0109 U=U+C*(KUTTA-1)
0110 X00=-372H1+KUTTA*(23-37)SU
0111 X01=-303H1+KUTTA*(24-36)SU
0112 X02=-263H1+KUTTA*(31-34)SU
0113 X03=-242H1+KUTTA*(30-34)SU
0114 V=V+C*(KUTTA-1)
0115 X04=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0116 X05=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0117 X06=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0118 X07=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0119 X08=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0120 X09=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0121 X10=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0122 X11=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0123 X12=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0124 X13=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0125 X14=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0126 X15=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0127 X16=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0128 X17=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0129 X18=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0130 X19=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0131 X20=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0132 X21=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0133 X22=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0134 X23=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0135 X24=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0136 X25=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0137 X26=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0138 X27=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0139 X28=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0140 X29=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0141 X30=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0142 X31=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0143 X32=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0144 X33=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0145 X34=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0146 X35=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0147 X36=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0148 X37=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0149 X38=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0150 X39=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0151 X40=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0152 X41=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0153 X42=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0154 X43=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0155 X44=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0156 X45=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0157 X46=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0158 X47=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0159 X48=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0160 X49=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0161 X50=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0162 X51=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0163 X52=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0164 X53=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0165 X54=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0166 X55=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0167 X56=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0168 X57=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0169 X58=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0170 X59=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0171 X60=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0172 X61=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0173 X62=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0174 X63=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0175 X64=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0176 X65=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0177 X66=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0178 X67=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0179 X68=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0180 X69=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0181 X70=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0182 X71=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0183 X72=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0184 X73=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0185 X74=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0186 X75=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0187 X76=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0188 X77=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0189 X78=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0190 X79=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0191 X80=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0192 X81=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0193 X82=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0194 X83=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0195 X84=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0196 X85=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0197 X86=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0198 X87=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0199 X88=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0200 X89=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0201 X90=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0202 X91=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0203 X92=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0204 X93=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0205 X94=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0206 X95=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0207 X96=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0208 X97=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0209 X98=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0210 X99=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)
0211 X100=(KUTTA-1)*KUTTA+KUTTA*(KUTTA-1)

```

```

0120 EPZ1=UO-ZH1
0121 BUFFER(3)=EPZ1
0122 BUFFER(4)=U
0123 WRITE(2)BUFFER
0124
0125 31 CONTINUE
0126 TIME=TIME+.5007
0127
0128 40 CONTINUE
0129 GO CALL RUMKX
0130
0131 100 CONTINUE
0132
0133 IF (J.LY.30) GO TO 1010
0134 WRITE (6,2015)TIME,XI,MI,NO,ZH1,U,Y
0135 FORMAT(6,'(G12.6,GX)')
0136 J=J+1
0137
0138 GO TO 1010
0139
0140 1000 CONTINUE
0141
0142 4000 CONTINUE
0143
0144 GO TO 3333
0145
0146 STOP
0147

```

```

CSTOP 3
CEND 25
U NO PATH TO THIS STATEMENT
0148 END

```

FORTRAN IV-PLUS VMS-51 10103:25 11-PMV-79 /TRISLOCKS/UR

PROGRAM SECTIONS

NUMBER	NAME	SIZE	ATTRIBUTES
1	SCORE1	000100	1310
2	SPDATA	000074	04
3	SIBDATA	000042	200
4	SN000	000000	100
5	ST000	000014	0
6	.0000	000130	44

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
A	R24	4-000070	00	R24	4-000000	01	R24	4-000000
B1	R24	4-000000	02	R24	4-000000	02	R24	4-000000
B2	R24	4-000000	03	R24	4-000000	03	R24	4-000000
B3	R24	4-000000	04	R24	4-000000	04	R24	4-000000
B4	R24	4-000000	05	R24	4-000000	05	R24	4-000000
B5	R24	4-000000	06	R24	4-000000	06	R24	4-000000
B6	R24	4-000000	07	R24	4-000000	07	R24	4-000000
B7	R24	4-000000	08	R24	4-000000	08	R24	4-000000
B8	R24	4-000000	09	R24	4-000000	09	R24	4-000000
B9	R24	4-000000	10	R24	4-000000	10	R24	4-000000
B10	R24	4-000000	11	R24	4-000000	11	R24	4-000000
B11	R24	4-000000	12	R24	4-000000	12	R24	4-000000
B12	R24	4-000000	13	R24	4-000000	13	R24	4-000000
B13	R24	4-000000	14	R24	4-000000	14	R24	4-000000
B14	R24	4-000000	15	R24	4-000000	15	R24	4-000000
B15	R24	4-000000	16	R24	4-000000	16	R24	4-000000
B16	R24	4-000000	17	R24	4-000000	17	R24	4-000000
B17	R24	4-000000	18	R24	4-000000	18	R24	4-000000
B18	R24	4-000000	19	R24	4-000000	19	R24	4-000000
B19	R24	4-000000	20	R24	4-000000	20	R24	4-000000
B20	R24	4-000000	21	R24	4-000000	21	R24	4-000000
B21	R24	4-000000	22	R24	4-000000	22	R24	4-000000
B22	R24	4-000000	23	R24	4-000000	23	R24	4-000000
B23	R24	4-000000	24	R24	4-000000	24	R24	4-000000
B24	R24	4-000000	25	R24	4-000000	25	R24	4-000000
B25	R24	4-000000	26	R24	4-000000	26	R24	4-000000
B26	R24	4-000000	27	R24	4-000000	27	R24	4-000000
B27	R24	4-000000	28	R24	4-000000	28	R24	4-000000
B28	R24	4-000000	29	R24	4-000000	29	R24	4-000000
B29	R24	4-000000	30	R24	4-000000	30	R24	4-000000
B30	R24	4-000000	31	R24	4-000000	31	R24	4-000000
B31	R24	4-000000	32	R24	4-000000	32	R24	4-000000
B32	R24	4-000000	33	R24	4-000000	33	R24	4-000000
B33	R24	4-000000	34	R24	4-000000	34	R24	4-000000
B34	R24	4-000000	35	R24	4-000000	35	R24	4-000000
B35	R24	4-000000	36	R24	4-000000	36	R24	4-000000
B36	R24	4-000000	37	R24	4-000000	37	R24	4-000000
B37	R24	4-000000	38	R24	4-000000	38	R24	4-000000
B38	R24	4-000000	39	R24	4-000000	39	R24	4-000000
B39	R24	4-000000	40	R24	4-000000	40	R24	4-000000
B40	R24	4-000000	41	R24	4-000000	41	R24	4-000000
B41	R24	4-000000	42	R24	4-000000	42	R24	4-000000
B42	R24	4-000000	43	R24	4-000000	43	R24	4-000000
B43	R24	4-000000	44	R24	4-000000	44	R24	4-000000
B44	R24	4-000000	45	R24	4-000000	45	R24	4-000000
B45	R24	4-000000	46	R24	4-000000	46	R24	4-000000
B46	R24	4-000000	47	R24	4-000000	47	R24	4-000000
B47	R24	4-000000	48	R24	4-000000	48	R24	4-000000
B48	R24	4-000000	49	R24	4-000000	49	R24	4-000000
B49	R24	4-000000	50	R24	4-000000	50	R24	4-000000
B50	R24	4-000000	51	R24	4-000000	51	R24	4-000000
B51	R24	4-000000	52	R24	4-000000	52	R24	4-000000
B52	R24	4-000000	53	R24	4-000000	53	R24	4-000000
B53	R24	4-000000	54	R24	4-000000	54	R24	4-000000
B54	R24	4-000000	55	R24	4-000000	55	R24	4-000000
B55	R24	4-000000	56	R24	4-000000	56	R24	4-000000
B56	R24	4-000000	57	R24	4-000000	57	R24	4-000000
B57	R24	4-000000	58	R24	4-000000	58	R24	4-000000
B58	R24	4-000000	59	R24	4-000000	59	R24	4-000000
B59	R24	4-000000	60	R24	4-000000	60	R24	4-000000
B60	R24	4-000000	61	R24	4-000000	61	R24	4-000000
B61	R24	4-000000	62	R24	4-000000	62	R24	4-000000
B62	R24	4-000000	63	R24	4-000000	63	R24	4-000000
B63	R24	4-000000	64	R24	4-000000	64	R24	4-000000
B64	R24	4-000000	65	R24	4-000000	65	R24	4-000000
B65	R24	4-000000	66	R24	4-000000	66	R24	4-000000
B66	R24	4-000000	67	R24	4-000000	67	R24	4-000000
B67	R24	4-000000	68	R24	4-000000	68	R24	4-000000
B68	R24	4-000000	69	R24	4-000000	69	R24	4-000000
B69	R24	4-000000	70	R24	4-000000	70	R24	4-000000
B70	R24	4-000000	71	R24	4-000000	71	R24	4-000000
B71	R24	4-000000	72	R24	4-000000	72	R24	4-000000
B72	R24	4-000000	73	R24	4-000000	73	R24	4-000000
B73	R24	4-000000	74	R24	4-000000	74	R24	4-000000
B74	R24	4-000000	75	R24	4-000000	75	R24	4-000000
B75	R24	4-000000	76	R24	4-000000	76	R24	4-000000
B76	R24	4-000000	77	R24	4-000000	77	R24	4-000000
B77	R24	4-000000	78	R24	4-000000	78	R24	4-000000
B78	R24	4-000000	79	R24	4-000000	79	R24	4-000000
B79	R24	4-000000	80	R24	4-000000	80	R24	4-000000
B80	R24	4-000000	81	R24	4-000000	81	R24	4-000000
B81	R24	4-000000	82	R24	4-000000	82	R24	4-000000
B82	R24	4-000000	83	R24	4-000000	83	R24	4-000000
B83	R24	4-000000	84	R24	4-000000	84	R24	4-000000
B84	R24	4-000000	85	R24	4-000000	85	R24	4-000000
B85	R24	4-000000	86	R24	4-000000	86	R24	4-000000
B86	R24	4-000000	87	R24	4-000000	87	R24	4-000000
B87	R24	4-000000	88	R24	4-000000	88	R24	4-000000
B88	R24	4-000000	89	R24	4-000000	89	R24	4-000000
B89	R24	4-000000	90	R24	4-000000	90	R24	4-000000
B90	R24	4-000000	91	R24	4-000000	91	R24	4-000000
B91	R24	4-000000	92	R24	4-000000	92	R24	4-000000
B92	R24	4-000000	93	R24	4-000000	93	R24	4-000000
B93	R24	4-000000	94	R24	4-000000	94	R24	4-000000
B94	R24	4-000000	95	R24	4-000000	95	R24	4-000000
B95	R24	4-000000	96	R24	4-000000	96	R24	4-000000
B96	R24	4-000000	97	R24	4-000000	97	R24	4-000000
B97	R24	4-000000	98	R24	4-000000	98	R24	4-000000
B98	R24	4-000000	99	R24	4-000000	99	R24	4-000000
B99	R24	4-000000	100	R24	4-000000	100	R24	4-000000
B100	R24	4-000000	101	R24	4-000000	101	R24	4-000000
B101	R24	4-000000	102	R24	4-000000	102	R24	4-000000
B102	R24	4-000000	103	R24	4-000000	103	R24	4-000000
B103	R24	4-000000	104	R24	4-000000	104	R24	4-000000
B104	R24	4-000000	105	R24	4-000000	105	R24	4-000000
B105	R24	4-000000	106	R24	4-000000	106	R24	4-000000
B106	R24	4-000000	107	R24	4-000000	107	R24	4-000000
B107	R24	4-000000	108	R24	4-000000	108	R24	4-000000
B108	R24	4-000000	109	R24	4-000000	109	R24	4-000000
B109	R24	4-000000	110	R24	4-000000	110	R24	4-000000
B110	R24	4-000000	111	R24	4-000000	111	R24	4-000000
B111	R24	4-000000	112	R24	4-000000	112	R24	4-000000
B112	R24	4-000000	113	R24	4-000000	113	R24	4-000000
B113	R24	4-000000	114	R24	4-000000	114	R24	4-000000
B114	R24	4-000000	115	R24	4-000000	115	R24	4-000000
B115	R24	4-000000	116	R24	4-000000	116	R24	4-000000
B116	R24	4-000000	117	R24	4-000000	117	R24	4-

NAME	TYPE	ADDRESS	SIZE	DIMENSIONS
ARRAYS				
U	R24	4-000328		
XORG	R24	4-000329		
YORG	R24	4-000330		
ZORG	R24	4-000331		
ONEGMS	R24	4-000178		
UC	R24	4-000179		
XORG	R24	4-000180		
YORG	R24	4-000181		
ZORG	R24	4-000182		
ONEGMS	R24	4-000183		
UC	R24	4-000184		
XORG	R24	4-000185		
YORG	R24	4-000186		
ZORG	R24	4-000187		
ONEGMS	R24	4-000188		
UC	R24	4-000189		
XORG	R24	4-000190		
YORG	R24	4-000191		
ZORG	R24	4-000192		

ARRAYS

NAME	TYPE	ADDRESS	SIZE	DIMENSIONS
BUFFER	R24	4-000000	800000	8 (4)

LABELS

LABEL	ADDRESS	LABEL	ADDRESS	LABEL	ADDRESS
30	1-004004	31	1-004000	40	1-004070
1000	1-000774	1010	1-003376	2000	3-000000
2000	3-000300	2010	3-000400	3000	3-000114
3000	3-000324	3004	3-000000	3007	3-000106

FUNCTIONS AND SUBROUTINES REFERENCED

RUNEX

TOTAL SPACE ALLOCATED - 000748 1777

FORTRAN 70-PLUS USE-51 10:04:30 11-000-70

```

SUBROUTINE RUNEX
COMMON X,DM,KUTTA,DT,IM
DIMENSION X(10),DM(10),DM1(10)
GO TO (10,20,30,40,50),KUTTA
10 DM=1;IM
X(I)=X(I)
DM(I)=DM(I)
20 X(I)=X(I)+DM(I)
30 DM=DM+DT
40 DM1=DM
5011

```

TIME	ADDRESS	LABEL	ADDRESS
DM1	1-004070	100	1-004070
DM2	3-000316	2000	3-000316
DM3	3-000328	3000	3-000328
DM4	1-000000	4000	1-000000

ONEGMS	ADDRESS	LABEL	ADDRESS
UC	4-000314	40	4-000314
XORG	4-000315	2000	4-000315
YORG	4-000316	3000	4-000316
ZORG	4-000317	4000	4-000317

ONEGMS	ADDRESS	LABEL	ADDRESS
UC	4-000178	40	4-000178
XORG	4-000179	2000	4-000179
YORG	4-000180	3000	4-000180
ZORG	4-000181	4000	4-000181

ONEGMS	ADDRESS	LABEL	ADDRESS
UC	4-000182	40	4-000182
XORG	4-000183	2000	4-000183
YORG	4-000184	3000	4-000184
ZORG	4-000185	4000	4-000185

ONEGMS	ADDRESS	LABEL	ADDRESS
UC	4-000186	40	4-000186
XORG	4-000187	2000	4-000187
YORG	4-000188	3000	4-000188
ZORG	4-000189	4000	4-000189


```

0018 DO 40 I=1,NX
0019 DDA(I)=DDA(I)+DTDX(I)
0020 X(I)=X(I)+DTDX(I)
0021 RETURN
0022 DO 50 I=1,NX
0023 DDT(I)=DDA(I)+D.DTDT
0024 DDT(I)=DDA(I)+D.DTDT
0025 RETURN
0026 DO 70 I=1,NX
0027 DDT(I)=DDA(I)+D.DTDX(I)/5.
0028 RETURN
0029 END

```

FORTRAN IV-PLUS UAB-51 10:04:30 11-MAY-79 /TR:BLOCKS/UR

PROGRAM SECTIONS

NUMBER	NAME	SIZE	ATTRIBUTES
1	SCORE1	104	RU,I,CON,LCL
2	SPRINTA	5	RU,D,CON,LCL
4	SUACC	47	RU,D,CON,LCL
6	.S000.	44	RU,D,CON,LCL

ENTRY POINTS

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
MAIN	1	000000						

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
DT	R24	0-000120	HDT	R24	4-000120	I	I32	4-000120
DT	R24	4-000120	UDT	R24	4-000120	KUTTA	I32	6-000120

ARRAYS

NAME	TYPE	ADDRESS	SIZE	DIMENSIONS
DX	R24	0-000000	00000	20 (10)
DYA	R24	4-000000	00000	20 (10)
X	R24	0-000000	00000	20 (10)
YA	R24	4-000000	00000	20 (10)

APPENDIX D

**DIGITAL SIMULATION OF COMPOSITE ACCELERATION
LOOP WITH DISTURBANCE ON INPUT AND OUTPUT**

FORTRAN IU-PLUS VMS-51 00:10:30 23-MAY-70
/TRIBLOCKS/UR

C ACCEL LOOP WITH BOTH U1 AND U2
COMMON X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,
X15,X16,X17,X18,X19,X20,X21,X22,X23,X24,X25,X26,
X27,X28,X29,X30,X31,X32,X33,X34,X35,X36,X37,X38,X39,X40,
X41,X42,X43,X44,X45,X46,X47,X48,X49,X50,X51,X52,X53,X54,
X55,X56,X57,X58,X59,X60,X61,X62,X63,X64,X65,X66,X67,X68,
X69,X70,X71,X72,X73,X74,X75,X76,X77,X78,X79,X80,X81,X82,
X83,X84,X85,X86,X87,X88,X89,X90,X91,X92,X93,X94,X95,X96,
X97,X98,X99,X100,X101,X102,X103,X104,X105,X106,X107,X108,
X109,X110,X111,X112,X113,X114,X115,X116,X117,X118,X119,
X120,X121,X122,X123,X124,X125,X126,X127,X128,X129,X130,
X131,X132,X133,X134,X135,X136,X137,X138,X139,X140,X141,
X142,X143,X144,X145,X146,X147,X148,X149,X150,X151,X152,
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X989,X990,X991,X992,X993,X994,X995,X996,X997,X998,X999,
X1000

3333 CONTINUE

ZETA-0.005
ONE-14.54
MKT-187
CAR-0.525
ONE-5-49
XETA-317.8
CR-0.1388
A-8.

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0134 XKZ1-85-848XK12-XKZ2-(X3L1(X3L2+X3L3)+X3L4)+X3L5+X3L6)+X3L2X(
1X3L3+X3L4)+X3L5+X3L6)+X3L9+X3L31(X3L5+X3L6)+X3L41(X3L5+X3L6)+
2X3L5X3L6)
0135 XK11-84-XK12+X3L1+X3L2+X3L31+X3L41+X3L5+X3L6
0136 XK22--X11X22X150X15X16/(XKCN8ONEGNSKZZ12)
0137 XK12-(1/(XKCN83))11(-822XKCNKZ2)
1+X11X22X150X15(X3L5+X3L6)+X11X22X15X16(X3L31+X3L41)+X1508X15X16X
2(X11+X12))
0138 XK11--XKCN8K12-84+X11+X12+X131+X141+X15+X16
0139 XK21--XKCN8K22-2-222XKCNK12+85-X11(X12+X131+X141+X15+X16)-
1X13(X131+X141+X15+X16)-X131(X15+X16)-X150-X141(X15+X16)-X15X16
0140 XK31--XKCN81-KK12-2-222XKCNK22+86-X11X12(X131+X141+X15+X16)+
1X11X131(X15+X16)+X11X150+X11X141(X15+X16)+X11X15X16
2X12X131(X15+X16)+X11X15X16+X11X15X16
3(X15+X16)+X11X15X16+X11X15X16
0141 XK41--XKCN8K12-XKCN81XK22+87-X11X12X131(X15+X16)-X11X12X
1X150-X11X12X141(X15+X16)-X11X12X15X16-X11X1508(X15+X16)-
2X11X131(X15X16)-X11X141(X15X16)-X12X1508(X15+X16)-(X12X131+
3X12X141+X150)X15X16
0142 WRITE (6,2067)ZETA,CR,KKTD,ONEGA,KKETA,GAR,ONEGNS,CH
0143
0144 2057 FORMAT(2X,6(G12.6,2X))
0145 WRITE (6,2067)ZETA,OPGAD,KKTD,CRDP,CMRDP,ORGN8D,KKTD8P,CHDP
0146 2067 FORMAT(2X,8(G12.6,2X))
0147 WRITE(6,2070)K11,KK21,KK31,KK41,KK12,KK22
2070 FORMAT(2X,6(G12.6,2X))

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FORTMAN IU-PLUS U02-51 00:10:30 23-MAY-79
/TRIBLOCKS/UR
0148 WRITE (6,2071)K311,K321,K331,K341,K312,K322
0149 FORMAT(2X,6(G12.6,2X))
0150 WRITE (6,2085)
0151 2085 FORMAT(2X,3(K11,12X,3(K12,12X,4(K13,12X,4(K13,12X,4(K14,1,12X,4H
1X14,12X,3X15,12X,3X16))
0152 WRITE (6,2087)X11,X12,X131,X132,X141,X142,X15,X16
0153 WRITE (6,4085)
0154 4085 FORMAT(2X,4(K311,12X,4(K312,12X,5(K313,12X,5(K313,12X,5(K314,1,
12X,5(K314,12X,4(K315,12X,4(K315,12X,4(K316))
0155 WRITE (6,2097)X311,X312,X313,X314,X315,X316)
0156 WRITE(6,9070)
0157 FORMAT(2X,4(K311,12X,4(K312,12X,4(K313,12X,4(K314,1,
0158 WRITE (6,9074)K311
0159 9074 FORMAT(2X,6(G12.6)
0160 E0-KK11-24
0161 E1-KK21-36
0162 E2-KK31-36
0163 E3-KK41-37
0164 E4-KCN8(KK11+80-84)
0165 E5-KCN8(KK21+1-85)
0166 E6-KCN8(KK31+82-86)

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0008 EPZ3-U3-Z3M1
0009 BUFFER(3)-EPZ1
0010 BUFFER(4)-EPZ3
0011 WRITE(2)BUFFER
0012 CONTINUE
0013 TIME=TIME+.5*DT
0014 GO CALL RUNKX
0015 CONTINUE
0016 IF(.J.LT.30) GO TO 1010
0017 WRITE (6,2015)TIME,X1,XM1,U1,ZM1,U3,Z3M1,Y
0018 FORMAT(2X,1612.6,2X)
0019 WRITE (6,5025)U1,U1
0020 FORMAT(2X,31612.6,2X)
0021 J=0
0022 GO TO 1010
0023 CONTINUE
0024 GO TO 3333
0025 END
0026

```

FORTRAN IU-PLUS U02-51 09:10:30 23-MAY-78 /TR:BLACKS/AR

PROGRAM SECTIONS

NUMBER	NAME	SIZE	ATTRIBUTES
1	SCORE1	010744	2000
2	SPDATA	000432	141
3	SIMDATA	000470	196
4	SUMMS	000054	218
5	STEPS	000040	16
6	.0000.	000210	68

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
A	R24	4-000005	AS	R24	4-000072	A9	R24	4-000302	ADP	R24	4-000302
A1BP	R24	4-000302	AS	R24	4-000378	AZBP	R24	4-000378	B9	R24	4-000432
B1	R24	4-000432	B1BP	R24	4-000498	B2	R24	4-000498	BZBP	R24	4-000498
BZBP	R24	4-000498	B4	R24	4-000456	B4BP	R24	4-000456	B5	R24	4-000482
B5	R24	4-000482	B4BP	R24	4-000456	B7	R24	4-000472	BZBP	R24	4-000482
BP	R24	4-000456	B7	R24	4-000472	B7BP	R24	4-000472	C9	R24	4-000482
BPBP	R24	4-000456	CRCH	R24	4-000072	CRBP	R24	4-000072	C9	R24	4-000482
C1	R24	4-000156	C4	R24	4-000102	C4BP	R24	4-000102	E3	R24	4-000078
C1BP	R24	4-000156	E1	R24	4-000002	E1BP	R24	4-000002	E3	R24	4-000078
E3	R24	4-000002	E3	R24	4-000002	E3BP	R24	4-000002	GMF	R24	4-000048
E3BP	R24	4-000002	E7	R24	4-000018	E7BP	R24	4-000018	GMF	R24	4-000048
GMF	R24	4-000126	J	I22	4-000032	KUTTA	I22	4-000032	MX	I22	4-000048
GMFBP	R24	4-000126	ONEGAS	R24	4-000042	ONEGAS	R24	4-000042	MX	I22	4-000048
ONEGAS	R24	4-000042	ONEGAS	R24	4-000042	ONEGAS	R24	4-000042	ONEGAS	R24	4-000048
									ONEGAS	R24	4-000048


```

0001 SUBROUTINE BLANK
0002 DIMENSION X(16),BX(16),XA(16),DHA(16)
0003 GO TO (10,30,50,70),KUTTA
0004 DO 80 I=1,NX
0005 XA(I)=X(I)
0006 DHA(I)=DTDX(I)
0007 RETURN
0008 20 X(I)=X(I)+.5SDHA(I)
0009 RETURN
0010 30 X(I)=X(I)+.5SDHA(I)
0011 DHA(I)=DHA(I)+DTDX(I)
0012 RETURN
0013 40 X(I)=XA(I)+HOTSDX(I)
0014 RETURN
0015 50 DO 60 I=1,NX
0016 DHA(I)=DHA(I)+2.*SUBT
0017 RETURN
0018 60 X(I)=XA(I)+UBT
0019 RETURN
0020 70 DO 80 I=1,NX
0021 X(I)=XA(I)+(DHA(I)+DTDX(I))/6.
0022 RETURN
0023 END

```

FORTMAN IV-PLUS V02-51 00:12:15 23-MAY-70
/TR:BLOCKS/UR

PROGRAM SECTIONS

NUMBER	NAME	SIZE	ATTRIBUTES
1	SCORE1	00010	164
2	SPDATA	00012	5
4	SUMES	00025	130

ENTRY POINTS

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
BLANK	1	00000									

VARIABLES

NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS	NAME	TYPE	ADDRESS
BT	R24	4-000408	HOT	R24	4-000416	I	IS2	4-000402	KUTTA	IS2	4-000404
TST	R24	4-000412	UBT	R24	4-000420						

ARRAYS

NAME	TYPE	ADDRESS	SIZE	DIMENSIONS
BK	R24	4-000100	000100	20 (16)

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