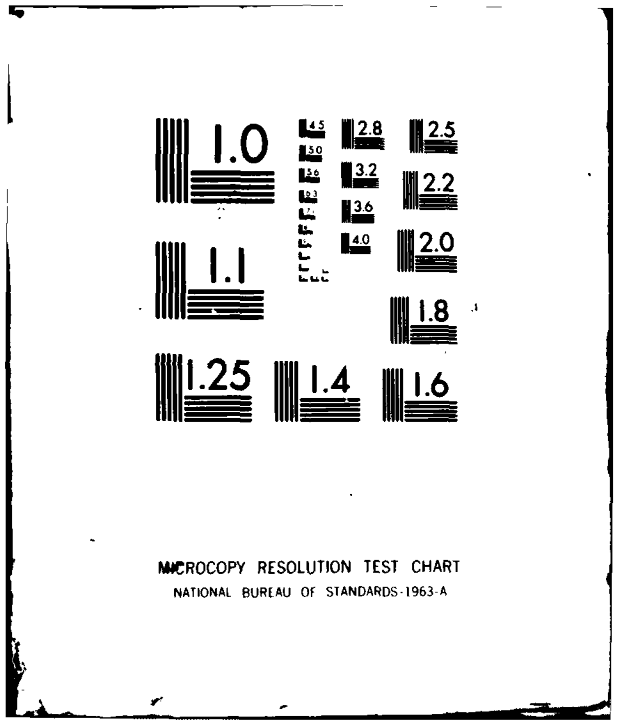
AD-A083 774   MASSACHUSETTS INST OF TECH CAMBRIDGE DEPT OF OCEAN E--ETC   F/G 17/2.1
A RANDOM VIBRATION MODEL FOR CABLE STRUMMING PREDICTION.(U)
1979   M KENNEDY, J K VANDIVER   N00014-75-C-0961

UNCLASSIFIED   NL

END
DATE
FILMED
6 80
DTIC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(6) A RANDOM VIBRATION MODEL FOR CABLE STRUMMING PREDICTION

by

(10) Michael/Kennedy[1]
J. Kim/Vandiver[2]

(12) 20

(15) N00014-75-C-0961

ABSTRACT

A random vibration predictive model for cable strumming response to ocean currents is presented. The model is justified on the basis of an extensive examination of four different field experiments, conducted for the purpose of studying vortex induced cable vibration. The examination of these data reveals that real world cable strumming is predominantly a random process that cannot be predicted on the basis of extending laboratory observations of essentially deterministic phenomena such as lockin. Representative data are presented to support these conclusions.

The proposed predictive model is demonstrated by two examples: the response of a uniform cable in a non-uniform flow and the response of a cable non-uniform in diameter to a uniform flow. The results of the second example are favorably compared to measurements made in a simple field test.

(11) 1979

INTRODUCTION

The prediction of cable vibration response to incident currents, cable strumming, is an as yet unresolved problem in cable systems design.

Observations of real world cable systems reveal the phenomena to be both complex and quite different from published laboratory results. The reasons for this are that field conditions rarely possess the temporal and spatial uniformities of the currents used in laboratory tests, that long field cables have much higher modal densities at the frequencies of interest, and that field cables are rarely uniform in properties over their length, as contrasted to the short uniform sections, commonly chosen for laboratory work.

In particular, the exhaustively studied phenomenon known as resonant lockin, or synchronization of the wake with a natural frequency, is a relatively uncommon aspect of the response of field deployed cables. Instead of the deterministic, periodic response, characteristic of lockin, the motion time history of a typical offshore cable system is usually best described as a random process. The periodic response of a

1. Senior Systems Analyst, Martingale Inc., P.O. Box 217, Mass. Inst. of Technology, Cambridge, Mass. 02139

2. Associate Professor, Massachusetts Institute of Technology, Dept. of Ocean Engineering, Room 5-222, Cambridge, Mass. 02139

single mode is quite rare; more often the response is a superposition of from a few to hundreds of modes responding simultaneously.

The purpose of this paper is to propose a simple stochastic model for the prediction of the non-lockin cable vibration response so frequently observed in the field. The model was formulated after an in depth examination of the data collected on four separate and quite different field experiments. The individual experiments and principal conclusions are reviewed to establish a physical basis for the model. The model is presented and its principal features are demonstrated in two example calculations. In the first example the response of a uniform cable to a spatially non-uniform flow is predicted. The second example is the response prediction for a cable non-uniform in diameter over its length exposed to a uniform flow.

A field experiment is described in which the measured response of a cable non-uniform in diameter is compared to the response predicted in the example.

EXPERIMENTAL EVIDENCE

Laboratory Tests

The literature concerned with fixed and moving cylinders is extensive. The principal results are presented in a recent review paper by King [3]. Laboratory tests of cables are not so numerous and have been generally restricted to investigation of cross flow lockin phenomena on relatively short lengths of cable (0.5 to 5 meters). The flows have been typically low in turbulence and spatially uniform. Only the first few natural response modes have been studied and then only one at a time. Some valuable conclusions are that at lockin the response is self limiting with the antinode responses approaching one diameter single amplitude. Shedding at the antinodes is similar to locked-in moving cylinders and shedding at the nodes is like that of non-moving cylinders [8]. References [1] and [6] are additional sources of experimental cable strumming results.

The laboratory experiments have made valuable contributions to our understanding of the local interaction between the motion of the cylinder and the wake, but cannot simulate the high modal densities and spatial and temporal variations in the flow. The response of actual field deployed cables must be examined to reveal these effects.

Field Tests

Four field experiments are reviewed below. The raw data from three of them were reduced and analyzed by the authors in the past 18 months. The previously reduced data from the fourth experiment known as "fishbite" were also evaluated and reported here.

Bermuda Testspan: From December 1973 to February of 1974 the United States Navy conducted a cable strumming experiment on Plantagent Bank off Argus Island, Bermuda. A 256 meter long, .016 m diameter electromechanical cable was stretched horizontally at a depth of 27.5 meters. The cable without strumming suppression devices was mechanically very complex. Numerous instrument pods, lead weights, and floats were distributed over its length. Two Savonious rotor current meters suspended near mid-span revealed extreme variation in both current speed and direction. Flow reversals occurred with an average period of eight

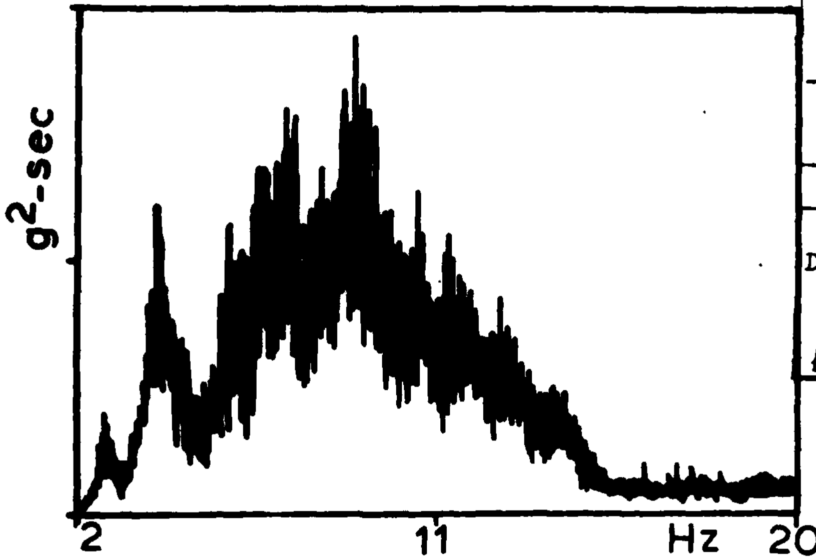


FIGURE 1. BERMUDA TEST: LINEAR ACCELERATION SPECTRUM

seconds, suggesting that long wavelength surface waves were influencing the flow. The flow speed was generally less than 0.5 m/s. Instrumentation failures, signal to noise problems, and inadequate accelerometer calibration data made the analysis of the five year old tapes quite difficult. Nonetheless, the following conclusions were reached. The cable strumming response was typical of a broad band random process. Mode participation varied with flow speed from the 10th to the 150th mode. Figure 1 shows a typical acceleration response spectrum at a point on the cable. This spectrum represents the sum of the two spectral components from a biaxial accelerometer. Due to cable rotation it was not feasible to separate cross flow from in line cable response. This particular spectrum was averaged from 15, 80-second records. It reveals significant (within 10dB of the peak) response over the range of 2 to 4 Hz. More than 10 modes are simultaneously contributing to the response. The conclusions drawn from examination of many hours of data were that a) lockin was rare, b) the response was generally broadband random, and c) over many hours the root mean square (rms) response was self-limiting and constant (to within 2 dB). Though suspect because of poor calibration data, the rms response estimate was less than a cable diameter.

It is the authors' conclusion that the combination of rapid variations in flow velocity, non-uniform cable properties (especially diameter) and high modal density are responsible for the broadband, multimoded response observed in this test. Additional description of this experiment may be found in reference [2].

Seacon II: The experiment was conducted by the Naval Civil Engineering Laboratory in 1975 [4,9]. It consisted of a horizontal delta array buoyed and anchored at three corners, 152 meters below the surface in 880 meters of water in the Santa Monica Basin. One of the horizontal arms of the delta was instrumented with accelerometers to measure the cable strumming response. The unfaired cable, sensors, and electronics were essentially the same as that used in the Bermuda experiment. No lead weights or buoyancy bags were used.

In this experiment the current speeds were on the order of 0.1 m/s, were tidally generated, and were slowly varying. The current meters were more than 200 meters from the test cable and the current and response data were not well synchronized. Data reduction and analysis difficulties encountered in evaluating the four year old raw data tapes were similar to those encountered with the Bermuda data. Due to the lower current speeds the participating natural modes ranged from the 2nd to the 15th with from two to six modes responding simultaneously. The vector averaged (cross plus in line) rms response was self limiting, lockin was rare, and multi-moded random vibration behavior predominated. Even though the current was relatively uniform the response typically spanned a considerable range in frequency. It was concluded that this was a consequence of a large range of cable and instrument housing diameters (from .016 to 0.15 m). For additional information consult reference [2].

Fishbite: Fishbite is the name given to a long cable experiment conducted in 1976 by Softley, Dilley and Rogers [10].

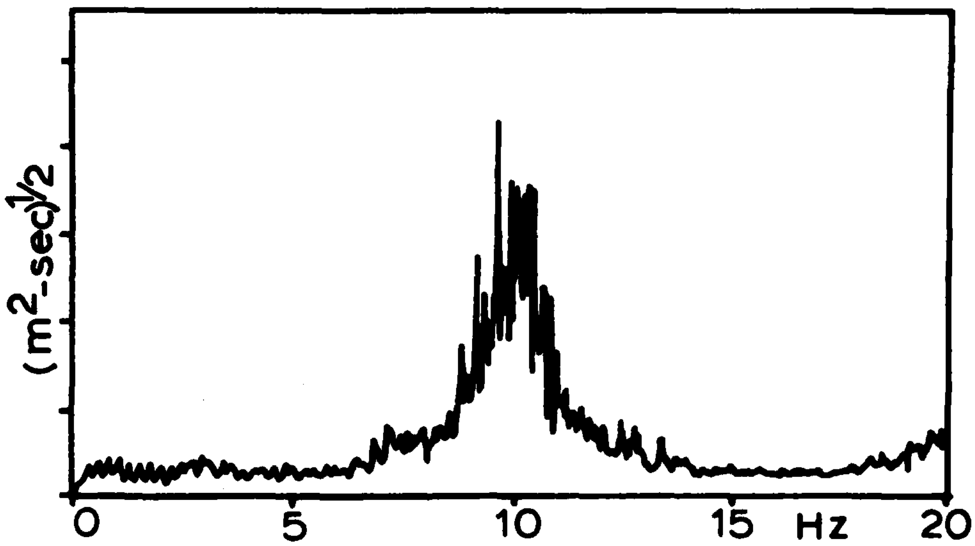


FIGURE 2. FISHBITE: RMS DISPLACEMENT SPECTRUM

A .012 m diameter wire rope 500 meters long was hung over the side of a ship anchored in 1960 meters of water at The Tongue of the Ocean, 77° 52'W and 25° 10'N. The tidal flow was neither spatially nor temporally uniform, varying from 0.1 to 0.4 meters per second. The cable was of uniform diameter with a current meter attached at the halfway point. No other lumped masses were attached to the cable. The response was measured at the top end only. The modal spacing was .025 Hz.

The repsonse typically included more than a hundred modes between 8 and 12 Hz, centered on 10 Hz. A root mean square displacement spectrum (from Softley et al., Figure 28) is shown in Figure 2. The rms response at the upper end was limited to less than a diameter. The band width of the response can only be attributed to the spatial and temporal variation of the current. The pertinent conclusions are that the response was rms limited and broadband random. Lockin was not observed.

Castine, Maine: Field tests were conducted near Castine, Maine, during the summers of 1975 and 1976 by Vandiver and Mazel [5, 7, 11]. Figure 3 is a schematic illustration of the test site, which was located on a sandbar that was flooded at high tide and exposed at low tide. Tests were conducted on the rising tide. The flow was normal to the 23.3 m long cable and varied from a maximum of 0.7 m/s, shortly after submergence, to 0 m/s at high tide over a period of approximately 2-1/2 hours. The flow was spatially uniform to within 5% over the length of the cable. The mean flow speed variations could be considered as quasi-static when viewed on a time scale of many periods of cable vibration. The data were taken in calm weather and surface waves were not a problem.

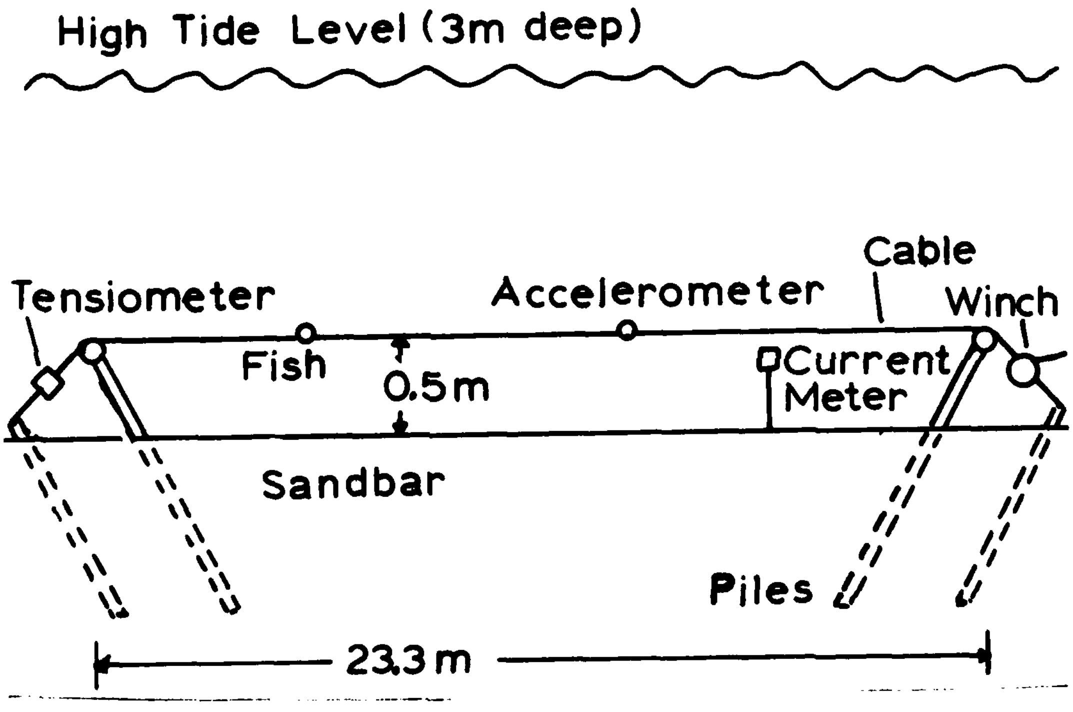


FIGURE 3. CASTINE EXPERIMENT SET-UP

The instrumentation consisted of a tensiometer, an electro-magnetic current meter, very small moveable accelerometers, and a direct displacement measuring device, called the "fish". The fish and accelerometers could measure cross flow vibration without loading the cable. Many different types of cables with and without strumming suppression devices were tested during two months of on-site testing. The cables tested were of various diameters from .006 to .016 m.

The cable strumming was all in the subcritical Reynolds number range, and the vibration response typically included one or more of the first fifteen natural modes. Vibration in line with the flow was negligible when compared to the cross flow vibration.

The experiments were conceived as a stepping stone from idealized laboratory conditions to the almost hopelessly complex conditions typical of the Bermuda test and Seacon II. Deterministic lockin response and non-deterministic non-lockin were observed at Castine.

Single frequency sinusoidal lockin response occurred at discrete steps in flow speed for all cables tested. Responding modes varied from the first to as high as the 15th, depending on the particular cable properties, tension, and flow velocity. The Reynolds number range for observed cross flow vibration was from 300 to 8000. Antinode response was limited to approximately one diameter for all cables tested. In general lockin behavior was exhibited over a smaller range of reduced velocity ($U_R = U/f_p D$) than observed in the laboratory with vibrating rigid cylinders. U is the flow velocity, f_p is the frequency of the largest peak in the response spectrum, and D is the cable diameter. The extremes of the lockin range are evidently reduced by real world spatial and temporal variations in the flow, not revealed by a single point measurement of mean flow speed. As the mean flow speed was observed to slowly move away from a value that insured lockin, the response became progressively more random. This suggests that the portion of the cable near the antinodes experiencing lockin was decreasing.

Even at flow speeds which completely prevented wake synchronization with a natural frequency of the cable, the observed response was quite substantial. The observed response at Castine under such conditions was best described as a narrow band random process. The term "non-lockin" was coined to describe this and all other random vibration cable response. Non-lockin response occurred about half of the time at Castine, and as mentioned before dominated the responses observed in the Bermuda, Seacon II and fishbite experiments.

As a general classification non-lockin occurs when variations in the current or non-uniformities in the cable prevent widespread wake synchronization with a single natural frequency of the cable.

It is conjectured that under such conditions the correlation length of shed vortices is on the order of only a few diameters and that the motion of the cable at any point is poorly correlated to the wake behind it. Under such conditions the lift force spectrum at a point on the cable can be characterized as a band limited random process whose center frequency is adequately characterized by an empirically determined constant such as a reduced velocity or a Strouhal number. In many respects this proposed model of a lift force spectrum is similar to that measured on stationary cylinders.

Spectra of non-lockin displacement data from Castine typically revealed one to four closely grouped natural frequencies of the cable, and occasionally the shedding frequency which would be expected for a stationary cylinder in that flow. The narrow banded nature of the non-lockin response observed at Castine is consistent with the test conditions: uniform diameter cables in a quite uniform flow. The non-lockin rms cross flow response was approximately one quarter of a diameter for all cables tested and was not measurably Reynolds number dependent. This response level is considerably less than lockin response amplitudes. However, because the displacements are a random process, individual peaks often exceeded two diameters. The displacements could be adequately described by a Gaussian random process. When measured in terms of diameters, the rms response was relatively insensitive to cable material, tension, flow velocity, and the number of participating modes. Furthermore, the reduced velocity remained nearly constant for each type of cable tested for all flow conditions. The observed mean values for reduced velocities varied among the cables from 4.5 to 6.7. Additional data from Castine will be presented later in this paper.

At the end of this extensive review of field data it was concluded that a predictive model for non-lockin cable vibration response was possible and necessary. Such a model would be useful for engineering design purposes if it could provide an estimate of the response spectrum of a deployed cable. From the available experimental data it was felt that such a predictive model could be based upon the concept of a self-limiting response and a band limited local lift force spectrum centered at a frequency established by the local flow velocity and cable diameter.

A STOCHASTIC MODEL

The review of field data briefly described in the previous section indicates that deterministic lockin models are rarely appropriate in the prediction of real world cable strumming response. Non-lockin random vibration generally dominates. In this section a simple stochastic model is proposed for the prediction of random vibration cable response to current flows.

In this analysis the linearizing assumption is made that the cross and in line cable excitations and responses can be decoupled. Since the cross flow response is typically an order of magnitude greater than the in line response it is the only response considered here.

The quantities of interest in a stochastic framework are mean squared responses and power flows. In cable strumming the spatially distributed lift forces arising from the local vortex shedding along the cable, constitute the power input into the cable system. Qualitatively and quantitatively very little is known about the mechanism that governs the input power. It appears to be a non-linear feedback based on mean square amplitude. For the purpose of this model it is assumed that the feedback mechanism is adequately modelled by a uniform spatially distributed feedback level based on the cable's spatially averaged mean square displacement response. The cable length is finite and the principal power outputs are due to internal cable friction and fluid damping. As the system is spatially distributed, power is shared throughout the cable and power output at any point is drawn from the entire cable system.

With these simplifications a form of the spatially distributed excitation may be proposed. f(x,t) is the force per unit length at a location x on the cable at time t and is given by

$$f(x,t) = .5\rho_w D(x) U^2(x) C_L(x,t) \qquad (1)$$

ρ_w: water density
D(x): cable diameter at x
U(x): normal component of the freestream velocity at x
$C_L(x,t)$: local momentary lift coefficient

The local lift coefficient $C_L(x,t)$ is a random variable which is assumed to be zero mean Gaussian, time stationary-ergodic, and locally independent of cable response. The first two assumptions are straightforward and reasonable. The third arises from the previous statement that a uniform feedback level is obtained from a spatially averaged mean square response. As will be demonstrated this assumption is acceptable for cables of uniform diameter in non-uniform flows, but presents obstacles for cables with non-uniform diameters.

Given the above assumptions we can completely characterize $C_L(x,t)$ by either its correlation function or its power spectrum.

$$\text{CORRELATION:} \quad R_c(x,y,\tau) = E[C_L(x,t)\, C_L(y,t+\tau)] \tag{2}$$

$$\text{SPECTRUM:} \quad S_c(x,y,f) = \int_{-\infty}^{\infty} d\tau\, R_c(x,y,\tau) e^{i2\pi f\tau} \tag{3}$$

From experimental results we know that the spatial correlation length for non-lockin is on the order of a few cable diameters. If the assumption is made that the shortest excited spatial wavelength is much larger than this correlation length, then the lift coefficient reduces to:

$$S_c(x,y,f) \approx L\delta(x-y) S_c(x,f) \tag{4}$$

L: cable length
δ(x-y): impulse function at x = y

Because the units of $\delta(x-y)$ are length^{-1}, L, the length of the cable, is introduced in the above equation to maintain consistent units. A visual image of this spatially incoherent process is that of "rain on the roof".

A box car model for the spectrum of the lift coefficient is proposed and illustrated in Figure 4.

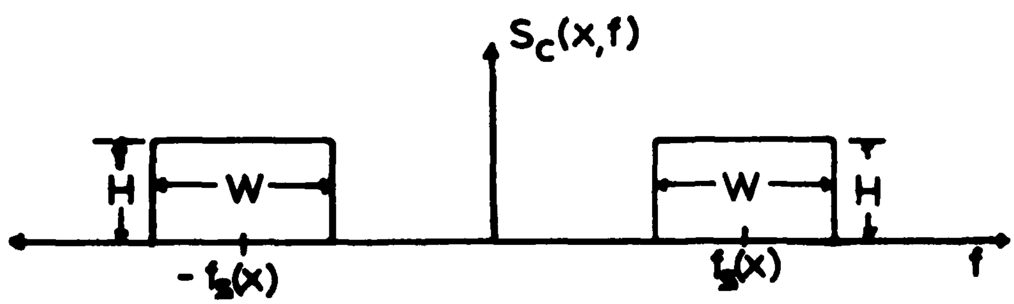


FIGURE 4. BOXCAR LIFT COEFFICIENT SPECTRUM

where:

$$f_s(x) = S_t \frac{U(x)}{D(x)}, \quad \text{local center frequency of vortex shedding} \tag{5}$$

$$W(x) = 2P_t f_s(x), \quad \text{bandwidth of shedding frequencies} \tag{6}$$

$$C = 2WH = \int_{\infty}^{\infty} S_c(x,f)df \text{ , the spatially uniform mean square lift coefficient} \tag{7}$$

This model for the lift coefficient spectrum is dependent upon three dimensionless parameters:

1) S_t: the cable's non-lockin Strouhal number. It determines the center of the spectrum and is inferred from experimental observation of the reduced velocity.

2) P_t: the excitation bandwidth, as a percentage of the S_t dependent shedding frequency.

3) C: mean square lift coefficient appropriate to the cable, flow conditions, and feedback level.

C is spatially distributed and uniform, and incorporates all cable-fluid feedback mechanisms. Its value will be set such that the spatially averaged mean square response is at its appropriate value. A more exact form for $S_c(x,f)$ would include tails at higher and lower frequencies. The boxcar model is felt to be appropriate for the simple response prediction model developed here. Refinements may be warrauted in the future.

This simple model for the mean square lift coefficient may be used with equation (1) to derive an expression for the distributed lift force spectrum.

$$S_F(x,y,f) = \int_{\infty}^{\infty} d\tau E[f(x,y)f(x,t+\tau)]e^{i2\pi f\tau} \tag{8}$$

$$= \frac{(.5\rho_w)^2 \ (D(x)U(x))^3 \ CL\delta(x-y)B(f,x)}{4P_t S_t} \tag{9}$$

$$B(f,x) = 1 \qquad (1-P_t)S_t \frac{U(x)}{D(x)} \leq |f| < (1+P_t)S_t \frac{U(x)}{D(x)} \tag{10}$$

$$= 0 \quad \text{elsewhere}$$

The cable model must be obtained next. The key assumption is that it is modelled as a linear time invariant system. A modal decomposition approach is used here. This requires knowledge of the individual natural frequencies f_j, mode shapes $\phi_j(x)$, and modal damping ratios ξ_j. The modal transfer functions $H_j(f)$ for a cable with fixed ends, and virtual mass per unit length $m(x)$ are completely specified by:

$$M = \int_0^L dx \ m(x) \tag{11}$$

$$M\delta_{jk} = \int_0^L dx \ \phi_j(x)\phi_k(x) \ m(x) \tag{12}$$

$$H_j(x) = \frac{\frac{1}{(2\pi f_j)^2 M}}{1 - \frac{f^2}{f_j^2} + 2i\xi_j \frac{f}{f_j}} \qquad (13)$$

The modal force spectrum is obtained from the mode shapes and distributed force spectrum as shown in the next equation.

$$S_{F_{jk}}(f) = \int_0^L dx \int_0^L dy\ \phi_j(x)\ \phi_k(y)\ S_F(x,y,f) \qquad (14)$$

In this analysis the contributions to the response resulting from the modal overlap cross terms ($j \neq k$) are assumed small compared to the terms for which $j = k$. Damping generated modal coupling is also neglected. With these assumptions the response spectrum may be obtained as follows:

$$S_D(x,y,f) \simeq \sum_j \phi_j(x)\ \phi_j(y) |H_j(f)|^2\ S_{F_j}(f) \qquad (15)$$

where $S_{F_j}(f)$ is reduced from equation (14), and takes the form:

$$S_{F_j}(f) = \int_0^L dx \int_0^L dy\ \phi_j(x)\phi_j(y) S_F(x,y,f) \qquad (16)$$

$$= \frac{(.5\rho_w)^2}{4P_t S_t}\ CL \int_0^L dx\ \phi_j^2(x)\Big(D(x)U(x)\Big)^3 B(f,x) \qquad (17)$$

The total displacement response spectrum is therefore the sum of the individual modal response spectra, summed over all modes included in the excitation bandwidth defined by the function B(f,x) from equation (10).

It remains to establish values for the three parameters C, S_t and P_t. C results from an empirically imposed limit on the spatially averaged mean square response A^2, expressed in diameters squared, where

$$A^2 = \frac{1}{L}\int_0^L dx\ \frac{1}{D^2}\int_{-\infty}^{\infty} df\ S_D(x,x,f) \qquad (18)$$

In other words, C is adjusted so that the empirically observed limits on A, the rms response, are satisfied. Given experimentally determined values for A, P_t and S_t, the response is now completely characterized.

Modal Parameter Values

The data from six cables tested at Castine were used to develop a preliminary data base for A, P_t and S_t. The non-lockin cross flow displacement time histories were divided into five second records from which individual raw spectra were computed. The root mean square displacement obtained from each spectrum when divided by the appropriate cable diameter, provided an estimate of A. S_t was estimated as the reciprocal of the observed reduced velocity

$$S_t = \frac{1}{U_R} = \frac{f_p D}{U} \tag{19}$$

where f_p: frequency of the highest spectral peak. P_t was estimated by dividing the 5 dB down bandwidth of the response spectrum by twice the center frequency of the band.

$$P_t = \frac{\Delta f_{5dB}}{2f_p} \tag{20}$$

Five dB down was an arbitrary definition of significant response level. For some cases the response spectrum contained only one sharp modal response peak. In such cases only an upper bound estimate on P_t could be estimated by dividing the computed modal separation frequency by twice the frequency of the observed peak. Table 1 briefly describes the six cables. Material, diameter, virtual mass per unit length and the tension range are given. The flow speed varied with the tidal cycle. The virtual mass per unit length m, was estimated by adding the displaced mass per unit length of water (the added mass) to the wet (saturated) mass per unit length of the cable. Table II presents the results of the parameter estimation for A, S_t and P_t. The number of records used and the standard deviation for each value are given. Upper bound estimates are denoted by the note U.B. in the standard deviation column.

EXAMPLE RESPONSE PREDICTIONS

A Uniform Cable in a Non-Uniform Flow

A long cable of length L and diameter D with fixed ends and constant tension is exposed to a normal flow field, which is constant but different for the two portions of the cable.

$$U(x) = U_1 \qquad 0 < x < L_1 \tag{21}$$

$$= U_2 \qquad L_1 < x < L \tag{22}$$

The natural frequencies and mode shapes are specified by:

$$f_j = jC_p/2L \tag{23}$$

$$\phi_j(x) = \sqrt{2} \sin \frac{j\pi x}{L} \tag{24}$$

where C_p: phase velocity of transverse waves.

By assuming that the cable is long compared to the longest excited spatial wavelength, a piecewise approximation to the integral contained

Table I. CASTINE TEST CABLES

Description	Diameter (m)	Virtual Mass per unit length (kg/m)	Range of Tension (nt)
Sampson, "Blue Streak", 12 strand single braid, polyester and polypropylene	.010	.177	310-1025
Philadelphia Resins "Phillystran", 7 strand Kevlar core rope with a polyurethane jacket	.0123	.233	490-2000
Wire rope, 3 x 9 torque balanced galvanized plow steel with a polyethylene jacket	.0071	.148	270-2600
Wall Rope Works "Uniline" polyester cable, woven jacket	.0131	.290	1150-1500
Philadelphia Resins, PS 29 EM 1 Four conductors inside single braided Kevlar, protected by a woven Dacron jacket	.0071	.110	850-1110
Cortland Line Co., parallel fiber Kevlar with a braided jacket	.0061	.081	1700-2200

****

Table II. PARAMETER ESTIMATES (MEAN AND STANDARD DEVIATION)

Cable	Number of Samples	$\overline{A}$	σ_A	$\overline{S}_t$	σ_S	$\overline{P}_t$	σ_P
Blue Streak	140	.26	.04	.15	.020	.08	.03
Phillystran	43	.20	.04	.22	.015	.10	.03
Wire Rope	6	.28	.07	current meter failed			
Uniline	12	.20	.02	.19	.004	.10	U.B.*
PS 29	40	.28	.05	.16	.007	.12	U.B.*
Cortland	43	.29	.04	.17	.010	.13	U.B.*

*Denotes upper bound.

in equation (17) may be obtained.

$$\int_0^L dx\ \phi_j^2(x)(D(x)U(x))^3 B(f,x) \simeq D^3[U_1^3 L_1 B_1(f) + U_2^3 L_2 B_2(f)] \tag{25}$$

where
$$B_1(f) = 1 \quad (1-P_t)S_t \frac{U_1}{D} \le |f| < (1+P_t)S_t \frac{U_1}{D}$$
$$= 0 \quad \text{elsewhere} \tag{26}$$
$$B_2(f) = 1 \quad (1-P_t)S_t \frac{U_2}{D} \le |f| < (1+P_t)S_t \frac{U_2}{D}$$
$$= 0 \quad \text{elsewhere} \tag{27}$$

This leads directly to an expression for the modal exciting force $S_{F_j}(f)$ and hence to the response spectrum.

$$S_D(x,y,f) \simeq \frac{\rho_w^2 CLD^3}{16\ P_t S_t} \sum_j \phi_j(x)\phi_j(y)|H_j(f)|^2\ [U_1^3 L_1 B_1(f) + U_2^3 L_2 B_2(f)] \tag{28}$$

The relative impact of the flows U_1 and U_2 on the response may now be evaluated. Let $P_1(x)$ and $P_2(x)$ be the local mean square responses at a point x due to the two flow fields. $P_1(x)$ and $P_2(x)$ may be obtained by requiring that x = y and integrating the expression for $S_D(x,x,f)$ over the frequency bands defined by $B_1(f)$ and $B_2(f)$ respectively. The result for $P_1(x)$ is shown in equation (29). The result for $P_2(x)$ may be obtained by replacing the subscript 1 by a 2 in equation (29).

$$P_1(x) \simeq \int_{-\infty}^{\infty} df\ \frac{\rho_w^2 CLD^3}{16\ P_t S_t} \sum_j \phi_j^2(x)|H_j(f)|^2\ U_1^3 L_1 B_1(f) \tag{29}$$

$P_1(x)$ and $P_2(x)$ depend on the mode shapes $\phi_j(x)$. To eliminate this x dependence, spatially averaged values may be computed, denoting the result as P_1 and P_2. For example, P_1 is given by

$$P_1 = \frac{1}{L}\int_0^L dx\ P_1(x)$$
$$\simeq \int_{\infty}^{\infty} df\ \frac{\rho_w^2 CLD^3}{16\ P_t S_t} \sum_j |H_j(f)|^2\ U_1^3 L_1 B_1(f) \tag{30}$$

The relative spatially averaged mean square response is given by

$$\frac{P_1}{P_2} = \frac{\sum_j U_1^3 L_1 \int_{-\infty}^{\infty} df\ |H_j(f)|^2 B_1(f)}{\sum_j U_2^3 L_2 \int_{-\infty}^{\infty} df\ |H_j(f)|^2 B_2(f)} \tag{31}$$

The integration over frequency can be simplified by the following approximation:

1) The modal damping ratios ξ_j were assumed small (less than 15%). A

mode whose natural frequency was inside the $B_1(f)$ or $B_2(f)$ bands was assumed to have its modal bandwidth completely inside $B_1(f)$ or $B_2(f)$.

2) The spatially averaged mean square response of any mode inside $B_1(f)$ or $B_2(f)$ was assumed equal to any other mode inside $B_1(f)$ or $B_2(f)$ respectively.

Approximation 1 allows a simple expression for the response of a single mode j to a white noise force spectrum of unit amplitude.

$$\int_{-\infty}^{\infty} |H_j(f)|^2 df = \frac{1}{M^2} \frac{1}{2\xi_j (2\pi f_j)^3} \quad (32)$$

Approximation 2 allows the response of each mode included in the band to be represented by the response of an equivalent mode which has a natural frequency equal to the center frequency of the band and a damping ratio typical of all modes in the band. Therefore, within each band the following relations apply.

$$B_1(f): \quad \xi_j = \xi_{j_1} = \text{constant}, \quad f_j = f_{j_1} = S_t \frac{U_1}{D} \quad (33)$$

$$B_2(f): \quad \xi_j = \xi_{j_2} = \text{constant}, \quad f_j = f_{j_2} = S_t \frac{U_2}{D} \quad (34)$$

The number of modes in a band is the product of the bandwidth and the modal density. For a uniform taut string the modal density is simply the reciprocal of the fundamental natural frequency f_1 and has units of modes per Hz. Therefore the number of modes inside $B_1(f)$ and $B_2(f)$ are given by

$$N_1 = \frac{2L}{C_p} \, 2P_t S_t \frac{U_1}{D} \quad (35)$$

$$N_2 = \frac{2L}{C_p} \, 2P_t S_t \frac{U_2}{D} \quad (36)$$

The total response of all modes in the band is given by the product of the number of equally responding modes and the response of the typical mode.

For example, for the band $B_1(f)$

$$\sum_j \int_{-\infty}^{\infty} df |H_j(f)|^2 B_1(f) = \frac{2L}{C_p} \, 2P_t S_t \frac{U_1}{D} \frac{1}{2\xi_{j_1} M^2 (2\pi f_{j_1})^3}$$

A similar result is obtained for the band $B_2(f)$ by replacing the subscripts 1 with a 2. These results may now be substituted in equation (31) to obtain P_1/P_2. The result depends on the choice of damping model assumed.

Two plausible models are:

Constant damping ratio

$$\xi_{j_1} = \xi_{j_2} \tag{38}$$

Frequency dependent damping ratio

$$\xi_{j_1} f_{j_1} = \xi_{j_2} f_{j_2} \tag{39}$$

A frequency dependent damping ratio is equivalent to specifying a constant damping coefficient per unit length of cable. These two models lead to the following predictions for P_1/P_2. The relations given in equations (33) and (34) were used to eliminate frequency from this expression.

Constant damping coefficient

$$\xi_{j_1} f_{j_1} = \xi_{j_2} f_{j_2}$$

$$\frac{P_1}{P_2} = \frac{L_1}{L_2} \frac{U_1^2}{U_2^2} \tag{40}$$

Constant damping ratio

$$\xi_{j_1} = \xi_{j_2}$$

$$\frac{P_1}{P_2} = \frac{L_1}{L_2} \frac{U_1}{U_2} \tag{41}$$

These results suggest that in general the response spectrum of field cables exposed to shear flows will be biased toward those modes excited by regions of higher flow velocity. To be more conclusive would require a field experiment with a known spatially varying flow. Furthermore, our present knowledge of cable damping mechanisms prevents more definitive conclusions.

A Non-Uniform Cable in a Uniform Flow

Consider a two segment cable with lengths, diameters, and virtual masses per unit length L_ℓ, D_ℓ, m_ℓ and L_s, D_s, m_s where the subscripts ℓ and s refer to segments with the larger and smaller diameters, respectively. A uniform mean square lift coefficient, C, is assumed to be set to an appropriate value, and P_t and S_t are assumed to be the same for both segments. The quantity to be predicted is P_ℓ/P_s, the relative spatially averaged mean square displacement response, where P_ℓ is the spatially averaged mean square displacement response of the modes excited by flow incident on D_ℓ and P_s is the response to flow incident on D_s. If the longest excited spatial wavelength is assumed to be much shorter then both L_ℓ and L_s, then by piecewise integration techniques similar to that in the previous example the relative mean square response weighting P_1/P_2 may be obtained, as shown in equations (42) and (43).

Regardless of the chosen form of damping the response is strongly weighted in favor of the large diameter segment. The major and unjustified assumption in the above example is that there is a uniform mean square lift coefficient, C, that is the same over both cable segments. At the present time we do not have a sufficient understanding of the fluid cable interaction to assign appropriate values for C on each segment of a cable with varying diameter. Nonetheless, the result of the above analysis suggests that the cable response will be highly dominated by the modes excited by the shedding from the larger diameter segments of a cable. This is an important qualitative result in understanding and predicting cable strumming.

Constant damping coefficient

$$f_{j_\ell} \xi_{j_\ell} = f_{j_s} \xi_{j_s}$$

$$\frac{P_\ell}{P_s} = \frac{L_\ell}{L_s} \frac{D_\ell^4}{D_s^4} \frac{m_s}{m_\ell} \qquad (42)$$

Constant damping ratio

$$\xi_{j_\ell} = \xi_{j_s}$$

$$\frac{P_\ell}{P_s} = \frac{L_\ell}{L_s} \frac{D_\ell^5}{D_s^5} \frac{m_s}{m_\ell} \qquad (43)$$

To test this qualitative prediction a simple experiment was conducted by slowly towing a nearly vertical, heavily weighted, composite cable over the stern of a ship. Due to length constraints on this paper only a brief account will be presented here. A more complete description may be found in reference [2].

COMPOSITE CABLE EXPERIMENT

A two diameter composite cable was fabricated from a single 39 meter length of the .0071 m diameter wire rope tested at Castine. The rope had three lays with an exterior sheath of polypropylene. The sheath and two of the lays were removed from half of the cable length. This resulted in a diameter ratio $D_\ell/D_s = 3.0$, where the subscripts ℓ and s denoted ℓarge and small. A large weight was attached to the end with the larger diameter D_ℓ. The cable was then lowered to its full length over the stern. As the vessel moved slowly forward at speeds less than 0.5 m/s, the cross flow and in line acceleration response at several points near the upper end was recorded. After recording sufficient data the cable was reeled in 3.05 m, resulting in an increase in the ratio of immersed lengths, L_ℓ/L_s from 1.16 to 1.49. Again the response of the small diameter section was recorded, followed by pulling in another 3.05 m section. This process was repeated until only the large diameter cable remained in the water and $L_\ell/L_s = \infty$. At this point the cable was reversed and the experiment was repeated. This time the response on the larger cable was measured for discrete values of L_ℓ/L_s from .86 to 0.0. After computing corrections for mode shape and mass loading of the accelerometers the average mean square displacement response ratios P_ℓ/P_s were computed and compared to predicted values. The measured values of P_ℓ and P_s reflect the sum of both in line and cross flow spectral components. However, the cross flow response was by far the dominant component. The response typically included approximately ten modes. Table III presents predicted and measured values for the experiment with the large cable on top. The reverse experiment with the small cable at the top confirmed these results.

Regardless of the damping model chosen the results are remarkably close. Even for the composite cable that had the larger diameter cable spanning only 12% of its total submerged length, over 90% of the mean square displacement response was generated by flow over the larger diameter cable segment. The predictions for this example come from a model which was claimed to be at best a qualitative indicator of expected response. The authors still hold this to be true. The predictions would likely diverge substantially from measured values at very low values of $L_\ell/(L_\ell + L_s)$.

The response of a composite cable when dominated by modes excited in the larger diameter segment has several important but not necessarily obvious features. First, each excited mode has a mode shape which spans the entire cable length. However, because of the change in the mass per unit length, the wave length and mode shape amplitude are larger in the smaller diameter segment. In the experiment just described whenever the larger diameter cable spanned more than a third of the total length, the modes excited by flow over the larger diameter contributed greater than 99% of the mean square response, independent of whether the response was measured on the large or small cable. When measured on the large diameter cable the response was limited to approximately 1/3 of its own diameter, rms. When measured on the small cable the response typically exceeded 2 diameters (of the small cable) rms. This was a necessary consequence of the relative amplitudes of the mode shapes in the two segments. From laboratory experiments with driven cylinders the lift coefficient has been observed to decrease dramatically when the amplitude approaches one diameter. This suggests that under such circumstances the mean square lift coefficient on the smaller segment drops to near zero, which would explain the greater than predicted dominance of the excitation on the larger diameter segment. This also suggests that the small diameter segment acts as a damper or a sink of vibrational energy, thus increasing the modal damping of the excited modes.

TABLE III. Prediction vs measured response for a composite cable.

	Predicted for $\xi_{j_\ell} f_{j_\ell} = \xi_{j_s} f_{j_s}$	Predicted for $\xi_{j_\ell} = \xi_{j_s}$	Measured
$\frac{L_\ell}{L_\ell + L_s}$	$\frac{P_\ell}{P_\ell + P_s}$	$\frac{P_\ell}{P_\ell + P_s}$	$\frac{P_\ell}{P_\ell + P_s}$
.12	.71	.88	.92
.24	.84	.94	.98
.33	.89	.96	.99
.40	.92	.97	.999

CONCLUSIONS

Cable strumming response for real world cable systems in the oceans is not adequately described by resonant lockin models. The phenomenon is extremely complex and yet well described using a stochastic framework. The key response characteristics are:

1) The mean square displacement response (normalized by cable diameter) is limited. This limit is relatively independent of cable length, virtual mass, damping, number of excited modes, and flow speed for subcritical Reynolds numbers.

2) The excitation, arising from vortex shedding, is a narrow band random process centered about a local "Strouhal" shedding frequency.

A simple model incorporating the above characteristics explains much of what is experimentally observed in large ocean cable systems. The effect of a spatially non-uniform current was predicted and remains to be verified by experiment. The effect of a non-uniform cable diameter was examined and qualtatively borne out by experiment.

In summary this initial attempt at a stochastic model has provided considerable insight to vortex excited cable vibration. The model may be improved. Additional work on the mean square lift coefficient model is warranted as well as further research on modal damping and its role in vortex excited vibration response.

ACKNOWLEDGEMENTS

The work was funded by the Office of Naval Research and the Naval Civil Engineering Laboratory. The contract monitor Dallas J. Meggett is thanked for his patience and enthusiasm.

We would also like to thank Robert Welsh (Bermuda Testspan) and Eric Softley for their time and access to their logbooks and experimental notes.

NOMENCLATURE

A^2:	mean square amplitude limit in cable diameters
$B(f,x)$:	local excitation bandwidth function
$B_1(f)$:	excitation bandwidth function for cable segment 1
$B_2(f)$:	excitation bandwidth function for cable segment 2
C:	spatially uniform mean square lift coefficient
$C_L(x,t)$:	local instantaneous mean square lift coefficient
C_p:	speed of wave propagation in a cable
$D(x)$:	local cable diameter
D_ℓ:	diameter of large diameter cable segment
D_s:	diameter of small diameter cable segment
$f(x,t)$:	local instantaneous force per unit length
f_j:	natural frequency of mode j
f_p:	frequency of highest spectral peak
$f_s(x)$:	local shedding frequency

$H(x)$:	local height of spectral box
$H_j(f)$:	transfer function for mode j
L:	cable length
L_1:	length of cable segment 1
L_2:	length of cable segment 2
L_ℓ:	length of large diameter cable segment
L_s:	length of small diameter cable segment
M:	virtual mass of cable; also modal mass
m_ℓ:	virtual mass per unit length of large diameter cable
m_s:	virtual mass per unit length of small diameter cable
P_1:	spatial average mean square displacement response in bandwidth 1
P_2:	spatial average mean square displacement response in bandwidth 2
P_ℓ:	average mean square displacement response due to large diameter cable excitation
P_s:	average mean square displacement response due to small diameter cable excitation.
P_t:	excitation bandwidth fraction
$R_c(x,y,\tau)$:	space-time correlation function for the lift coefficient
$S_c(x,y,f)$:	space-time spectrum for the lift coefficient
$S_c(x,f)$:	space-time spectrum approximation for the lift coefficient
$S_D(x,y,f)$:	space-time spectrum for cable displacement response
$S_f(x,y,f)$:	space-time spectrum for excitation force per unit length
$S_{F_{jk}}(f)$:	modal force cross-spectrum (j,k)
$S_{F_j}(f)$:	modal force auto-spectrum approximation
S_t:	"Strouhal" number for a "non-lockin" cable
$U(x)$:	local normally incident flow velocity
U_1:	normally incident flow velocity in region 1
U_2:	normally incident flow velocity in region 2
U_R:	reduced velocity
$W(x)$:	local width of spectral box
Δf_{5dB}:	bandwidth of 5dB spectral band
$\delta(x-y)$:	derac delta or impulse function
δ_{jk}:	Kronecher delta
ξ_j:	modal damping ratio for mode j

ρ_w: density of the fluid

$\phi_j(x)$: eigenfunction for mode j

REFERENCES

1. Griffin, O.M., "Vortex-Excited Strumming Vibration of Marine Cables", ASCE Preprint 3095, 1977 ASCE Fall Convention, San Francisco, CA, October 1977.

2. Kennedy, M.B., "A Linear Random Vibrations Model For Cable Strumming", Massachusetts Institute of Technology, Ph.D. Thesis, Department of Ocean Engineering, May 1979.

3. King, R., "A Review of Vortex Shedding Research and Its Application", Ocean Engineering, Vol. 4, pp. 141-147, Pergamon Press, Great Britain, 1977.

4. Kretschmer, T.R., Edgerton, G.A., Black, S.A., Albertsen, N.D., "Seafloor Construction Experiment, SEACON II: An Instrumented Tri-Moor for Evaluating Undersea Cable Structure Technology", Technical Report R-848, Civil Engineering Laboratory, Naval Construction Battalion Center, Port Hueneme, CA, December 1976.

5. Mazel, C.H., "Vortex-Excited Vibrations of Marine Cables", Massachusetts Institute of Technology, Master of Science Thesis, Department of Ocean Engineering, May 1976.

6. Pattison, J.H., "Measurement Technique to Obtain Strumming Characteristics of Model Mooring Cables in Uniform Currents", Report SPD-766-01, David W. Taylor Naval Ship Research and Development Center, April 1977.

7. Pham, T.Q., "Evaluation of the Performance of Various Strumming Suppression Devices on Marine Cables", Massachusetts Institute of Technology, Master of Science Thesis, Department of Ocean Engineering, February 1977.

8. Ramberg, S.E., Griffin, O.M., "The Effects of Vortex Coherence, Spacing, and Circulation on the Flow-Induced Forces on Vibrating Cables and Bluff Structures", NRL Report 7945, Naval Research Laboratory, January 1976.

9. Skop, R.A., Griffin, O.M., Ramberg, S.E., "SEACON II Strumming Predictions", NRL Memorandum Report 3383, Naval Research Laboratory, Washington, D.C., October 1976.

10. Softley, E.J., Dilley, J.F., Rogers, D.A., "An Experiment to Correlate Strumming and Fishbite Events on Deep Ocean Moorings", GE Document No. 77SDR2181, General Electric: Re-Entry and Environmental Systems Division, 1977.

11. Vandiver, J.K., Mazel, C.H., "A Field Study of Vortex-Excited Vibrations of Marine Cables", Proceedings: 1976 Offshore Technology Conference, Paper Number OTC 2491, Houston, May 1976.