





A NOTE ON THE INCOMPLETE BETA FUNCTION

By

/ Khursheed Alam

Clemson University

Report N107 , \mathcal{T}^{+} Technical Rep**er**t #316



May 25, 1979

Research Supported in part by THE OFFICE OF NAVAL RESEARCH

Task NR 047-202 Contract N00014-75-C-0451

DISTRIBUTION STATEMENT A Approved for public release; Distribution Unlimited

يله

A Note On The Incomplete Beta Function

Khursheed Alam* Clemson University

Abstract

The incomplete beta function arises in various statistical problems. It is known, for example, that the tail probability of the binomial distribution can be expressed as an incomplete beta function. This paper gives some results on a monotonicity property of the incomplete beta function. The given results are shown to have application in a problem of ranking and selection.

ACCESSION fo	и́
NTIS	- Mouse Section 🖌
DDC	G to Section
UNANN YOM , 1	
36811 A.M. 25	
9V	
BA Dicability	TEL AGENTY CODES
BY DICTPIC 124	TEL AGUITY CODES

Key words: Binomial Distribution; Ranking & Selection

AMS Classification: 62E99 *The author's work was supported by the Office of Naval Research under Contract N00014-75-C-0451 1. <u>Main results</u>. There are given below two theorems on the monotonicity property of an incomplete beta function. An application of the given result is shown in the next section. Let

$$I_{p}(a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_{0}^{p} \frac{a-1}{x} (1-x)^{b-1} dx,$$

a > 0, b > 0, 0 < p < 1

denote the incomplete beta function. Let $a = \xi n + \gamma$ and $b = (1-\xi)n + \delta$, where $0 < \xi > 1$, n > 0, $\gamma \ge 0$, $\delta \ge 0$. The following theorem establishes a monotonicity property of $I_p(\xi n + \gamma, (1-\xi)n + \delta)$ in terms of n. Let n' > n and

 $f(p) = I_{p}(n'\xi+\gamma, n'(1-\xi) + \delta) - I_{p}(n\xi+\gamma, n(1-\xi) + \delta).$

<u>Theorem 1.1</u>. Let $0 < p_0 < 1$. If $f(p_0) \le (\ge) 0$ then $f(p) \le (\ge) 0$ for $p \le (\ge) p_0$.

Proof: Let $n\xi + \gamma - 1$ $n(1-\xi) + \delta - 1$ $g_n(x) = x$ (1-x) / $B(n\xi + \gamma, n(1-\xi) + \delta)$

denote the beta density function, and let

 $h(x) = g_{n}'(x) / g_{n}'(x)$

Clearly, h(x) is nondecreasing (nonincreasing) in x for $x \le E(\ge)E$. Consider the function

(1.1)
$$f(p) = \int_{0}^{p} (h(x)-1)g_{n}(x) dx.$$

As x varies from 0 to 1, the integrand on the right hand side of (1.1) is either negative throughout or it changes sign from negative to positive and then to negative. Since f(1) = 0, it follows that f(p) changes sign once from negative to positive as p varies from 0 to 1. The conclusion of the theorem follows immediately. []

Let $\gamma = 0$, $\delta = 1$. If n and n ξ are integer valued then

(1.2)
$$I_p(n\xi, n(1-\xi)+1) = \frac{n}{\xi} (\binom{n}{r} p^r (1-p)^{n-r} \frac{1}{r} = \xi n^{n-r}$$

represents the tail probability of a binomial distribution. Let $y(n) = 3\log T(n)/3n$ denote the digamma function, and let

$$A(n) = \psi(n) - \xi\psi(n\xi) - (1-\xi)\psi(n(1-\xi)) + \xi\log\xi + (1-\xi)\log(1-\xi) - 1/n.$$

Using the integral formula for the digamma function, given by

$$f_{1}(n) = \log n - \frac{1}{2n} - 2 \int_{0}^{\infty} t (e^{2\pi t} - 1)^{-1} (t^{2} + n^{2})^{-1} dt$$

we have

(1.3)
$$A(n) = 2 \int_{0}^{\infty} t \left(e^{2\pi t} - 1\right)^{-1} \left(\frac{z}{t^{2} + n^{2} z^{2}} + \frac{1 - z}{t^{2} + n^{2} (1 - z)^{2}} - \frac{1}{t^{2} + n^{2}}\right) dt - \frac{1}{2n}$$

$$\leq \pi^{-1} \int_{0}^{\infty} \left(\frac{z}{t^{2} + n^{2} z^{2}} + \frac{1 - z}{t^{2} + n^{2} (1 - z)^{2}} - \frac{1}{t^{2} + n^{2}}\right) dt - \frac{1}{2n}$$

= 0 .

Consider the function

$$C(x) = \xi \log x + (1-\xi) \log (1-x) - \xi \log \xi - (1-\xi) \log (1-\xi) + \frac{1}{n}$$

0 < x < 1.

Clearly C(x) is a concave function of x. Let p_n and q_n denote the roots of the equation C(x) = 0, where $p_n < \xi < q_n$. Note that p_n and $q_n \neq \xi$ as $n \neq \infty$. We have

(1.4)
$$\partial^2 I_p(\xi n, (1-\xi)n+1)/\partial p \partial n = \frac{p^n \xi - 1}{B(n\xi, n(1-\xi)+1)} [\xi \log p +$$

 $(1-\xi)\log(1-p) + \psi(n+1) - \xi\psi(n\xi) - (1-\xi)\psi(n(1-\xi)+1)$

$$= \frac{p^{n\xi-1}(1-p)^{n(1-\xi)}}{B(n\xi,n(1-\xi)+1)} [C(p) + A(n)].$$

The second equality in (1.4) follows from the relation $\varphi(n+1) = \varphi(n) + \frac{1}{n}$. In view of (1.3) we have that the right hand side of (1.4) is negative for $p \le p_n$ and $p \ge q_n$. Since $\Im_n(n\xi, n(1-\xi)+1)/\Im n \ne 0$ as $p \ne 0$, it follows that

(1.5) $\partial I_{p}(n\xi, n(1-\xi)+1)/\partial n < 0$

for $p \leq p_n$. Using the relation $I_p(n\xi, n(1-\xi)+1) = 1 - I_{1-p}(n(1-\xi)+1, n\xi)$, we find that the reverse inequality holds in (1.5) for $p \geq q_n$. We have proved the following result.

Theorem 1.2. The incomplete beta function $I_{p}(n\xi, n(1-\xi)+1)$ is

decreasing in n for $p \leq p_n$ and increasing in n for $p \geq q_n$, where p_n and q_n are the roots of the equation C(x) = 0.

<u>Remark 1.1</u>. It can be shown that the monotonicity property given by Theorem 1.2, holds also for the function $I_p(n\xi,n(1-\xi))$, where

$$I_{p}(n\xi, n(1-\xi)) = \sum_{r=\xi n}^{n} {\binom{n-1}{r} p^{r}(1-p)^{n-1-r}}$$

if n and $n\xi$ are positive integers.

Remark 1.2. From Formula (le.6.2) of Rao (1966) we have $C(x) \leq \frac{1}{n} - \frac{(x-\xi)^2}{2} .$

Therefore, $0 < \xi - p_n \leq \sqrt{\frac{2}{n}}$ and $0 < q_n - \xi \leq \sqrt{\frac{2}{n}}$.

2. Application. Consider the following problem of ranking and selection. There are given k populations with cumulative distribution functions (cdf) $F_i(x) = F_i$ (i=1,...,k), and a number x with $0 \le x \le 1$. The distribution functions are unknown but they are assumed to be continuous. Let ξ_i^x denote the x-quantile of F_i . It is assumed for simplicity that ξ_i^x is uniquely determined for each i = 1, ..., k. Given a sample of n observations from each population, it is required to select the population associated with the largest value of ξ_i^x , called the "best" population. We shall assume that nx is integer valued.

Let x_{ij} denote the jth order statistic in the sample from F_i , and let j = nu. Suppose that the population associated with the largest value of x_{ij} is selected as the best population. Let the ith population be the best population. Then the probability of a correct selection (PCS) is given by

(2.1) PCS =
$$n\binom{n-1}{j-1} \int_{0}^{1} u^{j-1} (1-u)^{n-j} \frac{k}{t-1} I_{F} F^{-1}(u) (n\alpha, n(1-\alpha)+1) du. t+i t i$$

For large n, the right hand side of (2.1) is approximately given by

(2.2)

$$\begin{array}{c} k \\ \Pi \quad I \\ t=1 \quad F_t(\xi_i^{\alpha}) \\ t \neq i \end{array}$$

If it is assumed that the α -quantile of the best population is sufficiently larger than the α -quantile of each of the remaining populations, in the sense that

$$F_{\pm}(\xi_{i}^{\alpha}) \geq \alpha + \varepsilon$$
, $t \neq i$

where z is a given positive number, then (2.2) is minimized for

$$F_{\pm}(\xi_{i}^{\alpha}) = \alpha + \varepsilon, \quad t \neq i.$$

Therefore, the minimum probability of a correct selection is approximately given by

(2.3) min PCS =
$$(I_{n+2}(n\alpha, n(1-\alpha)+1))^{K-1}$$

The right hand side of (2.2) is increasing in n for $\varepsilon > \frac{2}{\sqrt{n}}$ by Theorem 1.2 and Remark 1.2. Thus a minimum value of n can be determined for the given selection problem, for which the probability of a correct selection is at least as large as a given number P* $(\frac{1}{k} < P^* < 1)$. The problem of selecting the best population for the largest a-quantile, has been considered by Rizvi and Sobel (1967) and Sobel (1967).

In the application given above, the problem of selecting the population associated with largest median value, that is, when $\alpha = \frac{1}{2}$, is of special interest. For this case the quantity inside the square bracket on the right hand side of (1.4) is given by

(2.4)
$$C(p) + A(n) = \psi(n) - \psi(\frac{n}{2}) + \frac{1}{2} \log (p(1-p))$$
$$= \frac{1}{2} (\psi(\frac{n+1}{2}) - \psi(n) + \log (4p(1-p))).$$

Let p_0 and $1-p_0$ be the values of p obtained by equating the right hand side of (2.4) to zero. Then $I_p(\frac{n}{2}, \frac{n}{2} + 1)$ is decreasing in n for $p < p_0$ and increasing in n for $p > 1-p_0$. To illustrate our result, we give below values of $I_p(\frac{n}{2}, \frac{n}{2} + 1)$ for p = (.5, .55) and

n = 2, 4, 8, (2)14, 20, 100. It appears from the table that $I_{1/2}(\frac{n}{2}, \frac{n}{2} + 1)$ decreases as n varies from 2 to 10 and increases thereafter.

$$I_{p}(\frac{n}{2}, \frac{n}{2} + 1)$$

	n =	2	4	8	10	12	14	20	100
p =	.50	.7500	.6875	.6367	.6230	.6128	.6047	.5881	.5398
p =	.55	.7975	.7585	.7396	.7384	.7393	.7414	.7505	.3654

References

- [1] Rao, C. R. (1966). Linear Statistical Inference. Wiley Publications in Statistics.
- [2] Rizvi, M. H. and Sobel, M. (1967). Nonparametric procedures for selecting a subset containing the population with the largest a-quantile. Ann. Math. Statist. (38) 1788-1303.
- [3] Sobel, M. (1967). Nonparametric procedures for selecting the t populations with the largest a-quantiles. Ann. Math. Statist. (38) 1804-1816.

(7)

MERORT DOCUMENTATION PAGE MERORT DOCUMENTATION PAGE MERORT DOCUMENTATION PAGE MERORT NUMBER MERORT NUMBER N=107 MERORT NUMBER N=107 MERORT NUMBER N=107 Addressing colspan="2">N=107 N=107 N=107 <	UNCLASSIFICATION OF THIS PAGE (Mon Data Entered)	
AFFORT HUMBER 1 down Activity of 1 Article 1 down activity of Activity Activity of Activity of Activity Activity of	REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
N=107 AD-AC8.3.529 * TVFE or AFFORT & FENDO COLERED * A Note On The Incomplete Beta Function" * A Note On Waraley * Boondard State * Software Converted Co	REPORT NUMBER 2 GOVT ACCESSION NO	3. RECIPIENT'S CATALOG NUMBER
Thild (and denome) 1 type of REPORT & FEMOR Concerns "A Note On The Incomplete Beta Function" 1 FERTYREND Concerns Afford & FEMOR Concerns Authod::::::::::::::::::::::::::::::::::::	N-107 AD-AU83529	
"A Note On The Incomplete Beta Function" * FERTENENDOUS AFFORT NUMBER Aut-03:0 Khursheed Alam NO0014-75-C-0451 * FERTENENDOUS STATEMENT IN AND AND ADDRESS Clemeon University Dept. of Mathematical Sciences Clemeon University Dept. of Mathematical Sciences Clemeon South Carolina 29551 * FERTENENDOUT AND AND ADDRESS Clemeon South Carolina 29551 * State Science Scie	TITLE (and Subility)	S. TYPE OF REPORT & PERIOD COVERED
A Note on the incomplete beta Function Personne of Areor Number Khursheed Alam Personne of Anthematical Sciences Personne of Mathematical Sciences P		
Pressing of August Technics of August Technics of August Technics of August Technics of August Noool4-75-C-0451 Personal Constraint Howered Investig Cleason Durversity Dept. of Mathematical Sciences If a domain of the August South august Sout	"A Note On The Incomplete Beta Function"	
Autorotion 1		· PERFORMING ORS REPORT NUMBER
Khursheed Alam N00014-75-C-0451 PERFORMANCE SECTION AND ADDRESS PERFORMANCE SECTION AND ADDRESS PERFORMANCE SECTION AND ADDRESS Clemson University Dept. of Mathematical Sciences PR 047-202 Clemson, South Carolina 20631 PERFORMANCE Section 2063 PR 047-202 Contractuation of Naval Research May 25, 1979 Code 446 y 54 May 25, 1979 Code 446 y 54 Performed and the Aboress Atlandton, Va. 20217 Performed and the Aboress Image: State of Naval Research May 25, 1979 Code 446 y 54 Performed and the Aboress Image: State of Naval Research May 25, 1979 Code 446 y 54 Performed and the Aboress Image: State of Naval Research May 25, 1979 Code 446 y 54 Performed and the Aboress Image: State of Naval Research May 25, 1979 Code 446 y 54 Performed and the Aboress Image: State of Naval Research May 25, 1979 Image: State of Naval Research May 25, 1979 Image: State of Naval Research May 25, 1979 Image: State of Naval Research May 26, 1979 <	- AUTHOR(s)	S CONTRACT OF GRANT HUMBER(+)
Middle Side of Allin N00014-75-6-0451 Person University Person University Dept. of Mathematical Sciences NR 047-202 Cleason, South Carolina 29531 Areowy Oxy Controlling Office On Xaval Research May 25, 1979 Controlling Office On Xaval Research May 25, 1979 Controlling Office Of Xaval Research May 25, 1979 Controlling Office Of	Khurshood llam	
PERFORMATION CAME AND ADDRESS 0 PROTABLE CENTRY PROFESS Clemson University Dept. of Mathematical Sciences 0 PROTABLE AND ADDRESS Clemson South Carolina 29511 0 PROTABLE AND ADDRESS 0 Control and office and And Address 0 PROTABLE AND ADDRESS 0 Control and office and And Address 0 PROTABLE AND ADDRESS 0 Control and office and And Address 0 1 REPORT CATE Control and office and And Address 0 0 10 10 Control and office and And Address 0 0 1 10 10 Control and office and Address 0 0 10 <td< td=""><td></td><td>N00014-75-C-0451</td></td<>		N00014-75-C-0451
Clement University Dept. of Mathematical Sciences Clement, South Carlina 29631 Controllina 2001 Carlina 29631 Controllina 2001 Carlina 29631 Controllina 2001 Carlina 29631 Controllina 2001 Carlina 2003 Control Controlling Office Carlington, Va. 22217 Controlling Carling Control Statement New Connecting Office Controlling Carling Carling Control Statement New Connecting Office Control Statement of New Control Statement New Connecting Control Cont	PERFORMING ORGANIZATION NAME AND ADDRESS	10 PROGRAM ELEMENT PROJECT, TASK
Super Content of Mathematical Sciences Cleason, South Carolina 29631 Source of Naval Research Office of Naval Research Code 436 % 3% Arlington, Va. 22217 John Toking Alency NAME & ADDRESSUP different New Convoluting Differ Super Caster of Mark and ADDRESSUP different New Convoluting Differ John Toking Alency NAME & ADDRESSUP different New Convoluting Differ John Toking Alency NAME & ADDRESSUP different New Convoluting Differ John Toking Alency NAME & ADDRESSUP different New Convoluting Differ John Toking Alency NAME & ADDRESSUP different New Convoluting Differ John Toking Alency NAME & ADDRESSUP different New Convoluting Differ John Toking Alency NAME & ADDRESSUP different New Convoluting Differ John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency NAME & ADDRESSUP different New Convolution John Toking Alency Name New Convolution John Toking Alency Name Research John Toking Alency Name Research Research Resea	Clemson University	AREA & WORK UNIT NUMBERS
Cleanson, South Carolina 29631 047202 Control Line Generation 00077202 Control Line Generation 0007720 Code 436 % 54 Arlington, Va. 20217 9 The Distribution Statewent of the Record Approved for public release: distribution unlimited. Control Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation Block 20, 11 different free Record Control Line Statewent of the decreation different free Record Control Line Statewent of th	Dept. of Mathematical Sciences	NB 047 200
<pre>1 controlling of the wate and Address Office of Naval Research Cdd 446 % 34 Arlington, Va. 20217 * DONTONING ADENCY NALE & ADDRESS(II different the Controlling Differ Unclassified * Distribution Statement of the Address * Controlling of the A</pre>	Cleason, South Carolina 29631	047-202
Office of Naval Research May 25, 1979 Code 436 % 30' 9 Arlington, Va. 20217 9 * USNYTONNG ADENCY NAME & ADDRESS(If different line Controlling Differ, 11 SEC.RTY CLASS.forther report) Unclassified * USNYTONNG ADENCY NAME & ADDRESS(If different line Controlling Differ, 11 SEC.RTY CLASS.forther report) Unclassified * Distribution STATEMENT of Nil Record) Approved for public release; distribution unlimited. * Distribution STATEMENT (of the second in Block 20, 11 different line Report) * SUPPCEMENTARY MOTES * SUPPCEMENTARY MOTES <td>- CONTROLLING OFFICE NAME AND ADDRESS</td> <td>12 REPORT DATE</td>	- CONTROLLING OFFICE NAME AND ADDRESS	12 REPORT DATE
Code 446 9.54 Arlington, Va. 20217 4 UDETORNING ASENCY NAME & ADDRESS(() different from Controlling Differ) 5 SECLASSIFICATION DOWNGRADING 5 DISTRIBUTION STATEMENT of Net Reserve Approved for public release: distribution unlimited. 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 6 DUPPLEMENTANY MOTES 5 SUPPLEMENTANY MOTES 5 SUPPLEMENTANY MOTES 5 SUPPLEMENTANY MOTES 5 STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerced minered in Block 20, () different (me Reserve) 5 DISTRIBUTION STATEMENT (of the exerc	Office of Naval Research	May 25, 1979
Artington, va. 2211 Tublitoning Adence values a Addressifi different from Controlling Office, Tudlitoning Adence values a Addressifi different from Controlling Office, Tudlitoning Adence values a Addressifi different from Controlling Office, Tudlitoning Adence values and for the Addressifi Approved for public release; distribution unlimited. Approved for public release; distribution unlimited. Constant Button Statement (of the addressifi different from Recent) Constant Button Constant addressifi different from Recent) Constant Button Constant addressifi different from Recent) Constant Button Constant addressifi different from Recent) Constant addressifi different from Recent Constant addressifi different addressifi different from Recent) Constant addressifi different addressifi different from Recent Constant addressifi different addressifi different from Recent Cons	Code 436 4 54	1 3 NUMBER OF PAGES
Unclassified The <u>SECTION STATEMENT of Not Report</u> Approved for public release: distribution unlimited. DISTRIBUTION STATEMENT of the abstract microd in Block 20, if different from Newrit DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the abstract microd in Block 20, if different from Newrit) DISTRIBUTION STATEMENT (of the incomplete beta function); DISTRIBUTION STATEMENT (of the incomplete beta function). This paper gives some results on a monotonicity property of the incomplete beta function. This paper gives may need to a monotonicity property of the incomplete beta function. This paper gives need results are shown to have application in a problem of ranking and selection. DISTRIBUTION STATEMENT (over site operator in the incomplete beta function).	ATIINGTON, VA. 22217 4 MON: YORING AGENCY NAME & ADDRESS(11 dilloroni from Controlling Office)	13 SECURITY CLASS. (of this report)
Unclassified U		
Distribution Statement of the Accord Distribution Statement of the Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution Statement of the accord in Block 20, 11 different free Accord Distribution can be expressed as an incomplete beta function. This paper gives some results on a monotonicity property of the incomplete beta function. This paper gives some results are shown to have application in a problem of ranking and selection. Distribution can be accord		Unclassified
A DISTRIBUTION STATEMENT of the Recent) Approved for public release: distribution unlimited. 2. DISTRIBUTION STATEMENT (of the abstract missed in Block 20, 11 different from Report) 4. SUPPLIMENTANY NOTES 4. SUPPLIMENTANY NOTES 5. KEY WORDS "Continue on reverse side if necessary and identify by block number: Binomial Distribution; Ranking & Selection 5. Additional distribution; Ranking & Selection 5. Addition of the bin and the state of the state of the bin paper gives some results on a monotonicity property of the incomplete beta function. The given results are shown to have application in a problem of ranking and selection. (D. (AM), 1473 - Distribution of the state of th		1150 DECLASSIFICATION, DOWNGRADING SCHEDULE
<pre>* SUPPCEMENTARY NOTES * XEY *CHOSTCONTINUE on reverse side three and identify by black number: Binomial Distribution; Ranking & Selection * ***********************************</pre>	7. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, If different fre	an Report)
XEY WORDS "Continue on reverse elde if receivery and identify by black number: Binomial Distribution; Ranking & Selection X ABSTRACT Continue on reverse elde if necessary and identify by black number: The incomplete beta function arises in various statistical problems. It is known, for example, that the tail probability of the binomial distribution can be expressed as an incomplete beta function. This paper gives some results on a monotonicity property of the incomplete beta function. This paper gives some results are shown to have application in a problem of ranking and selection. D form 1473 formous of the output of the incomplete beta function.	SUPPLZMENTARY NOTES	
Binomial Distribution; Ranking & Selection CARSTRACT Continue on reverse of a line content of identify by block number: The incomplete beta function arises in various statistical prob- lems. It is known, for example, that the tail probability of the bi- nomial distribution can be expressed as an incomplete beta function. This paper gives some results on a monotonicity property of the incomplete beta function. The given results are shown to have application in a problem of ranking and selection. D firm 1473 correct of the officient CONLASSIFIED CONLASSIFIED	9 KEY WCHOS (Continue on reverse elde if necessary and identify by block number	
The incomplete beta function arises in various statistical prob- lems. It is known, for example, that the tail probability of the bi- nomial distribution can be expressed as an incomplete beta function. This paper gives some results on a monotonicity property of the incomplete beta function. The given results are shown to have application in a problem of ranking and selection.	Rinomial Distribution, Panking & Selact:	
The incomplete beta function arises in various statistical prob- lems. It is known, for example, that the tail probability of the bi- nomial distribution can be expressed as an incomplete beta function. This paper gives some results on a monotonicity property of the incomplete beta function. The given results are shown to have application in a problem of ranking and selection.	Binomial Distribución, Ranking & Select	LOII
The incomplete beta function arises in various statistical prob- lems. It is known, for example, that the tail probability of the bi- nomial distribution can be expressed as an incomplete beta function. This paper gives some results on a monotonicity property of the incomplete beta function. The given results are shown to have application in a problem of ranking and selection.	·	
The incomplete beta function arises in various statistical prob- lems. It is known, for example, that the tail probability of the bi- nomial distribution can be expressed as an incomplete beta function. This paper gives some results on a monotonicity property of the incomplete beta function. The given results are shown to have application in a problem of ranking and selection.		
The incomplete beta function arises in various statistical prob- lems. It is known, for example, that the tail probability of the bi- nomial distribution can be expressed as an incomplete beta function. This paper gives some results on a monotonicity property of the incomplete beta function. The given results are shown to have application in a problem of ranking and selection.	G. ABSTRACT (Cantinue on reverse elde (finecessary and identify by block number)	
D (1473 EDITION OF INOV 65 15 OBSOLETE CONDLAGRIFIED	The incomplete beta function arises in var lems. It is known, for example, that the t nomial distribution can be expressed as an incompl- paper gives some results on a monotonicity propert function. The given results are shown to have app ranking and selection.	rious statistical prob- ail probability of the bi- ete beta function. This y of the incomplete beta lication in a problem of
S N 0102-0.4-6601	DD FORM 1473 FORTION OF SHOVER IS OBSOLETE	
	SN 0102-014-6601	LASSIFIED

ł

j V

and the second

20

San Sana

ure

