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MEMORANDUM REPORT ARBRL-MR-02993

NUMERICAL CALCULATION OF THREE-DIMENSIONAL DISPLACEMENT EFFECT ON BODIES OF REVOLUTION

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
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Moore's differential equation for the three-dimensional displacement thickness is solved numerically for bodies of revolution. Comparison is made to a known analytic solution for the special case of a cone in supersonic flow. Results are presented for spinning and non-spinning artillery projectile shapes.

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I. INTRODUCTION

The boundary layer formed on an artillery projectile in flight is a narrow region in which viscous effects are important. Outside of this region viscous effects can be ignored. It is, therefore, possible to compute air flows about projectiles in two regions. First an outer region in which viscous effects are ignored and second an inner region where they predominate. The interaction of the inner viscous flow on the outer inviscid flow may be calculated by displacing the body surface outward slightly to account for the air which is pulled along by the viscous forces in the boundary layer. In the three-dimensional case associated with an artillery projectile at angle of attack in a steady air flow, this displacement effect may be computed by solving a differential equation due to Moore[1]. Moore's differential equation is a first order partial differential equation involving certain boundary layer integrals and an inviscid solution for the flow outside of the boundary layer. For the purpose of this report these input quantities will be assumed to be known and the solution of the differential equation alone will be dealt with.

In this report, Moore's equation is first analyzed and a numerical technique developed for its solution. The accuracy of this technique will then be demonstrated by comparison with an exact analytical solution which is available for the special test case of laminar supersonic flow over a cone. Up to the present, displacement effects used in the computation of Magnus forces on artillery projectiles[2] have been obtained from solutions of Moore's equation obtained from a numerical technique developed by Sanders[3]. Comparison is also made with the results of Sanders' technique for a similar case of supersonic flow over a cone with a turbulent boundary layer. It should be noted that the present method is more than 10 times faster than Sanders' technique.

II. THE EQUATION

The usual formula for the displacement thickness in two dimensions is expressed as:

$$\delta^* = \int_0^h (1 - \rho u/\rho_e u_e)\, dy \qquad (1)$$

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1. *F. K. Moore, "Displacement Effect of a Three Dimensional Boundary Layer," NACA Technical Note 2722, June 1952.*

2. *W. B. Sturek, et al, "Computation of Magnus Effects for Yawed, Spinning Body of Revolution,"* <u>*AIAA Journal,*</u> *Vol. 16, pp. 687-692, July 1978.*

3. *R. Sanders, "Three-Dimensional, Steady, Inviscid Flow Field Calculations with Applications to the Magnus Problem," Ph.D. Thesis Department of Mechanical Engineering, University of California-Davis, California, 1974.*

where ρ is the density and u the velocity tangent to the surface. The density ρ_e and velocity u_e are the density and velocity immediately outside of the boundary layer. These edge quantities may be obtained from an inviscid solution. The integral in Equation (1) is taken along a line in the direction normal to the surface. It runs from the surface to a point h which is well outside of the boundary layer. The coordinate system used is shown in Figure 1.

As Moore[1] points out the velocity tangent to the surface may have two components, u and w in three dimensions; see Figure 2. There are consequently two integrals in the form of Equation (1) given by,

$$\delta_x = \int_o^h (1 - \rho u/\rho_e u_e)\ dy \tag{2}$$

$$\delta_z = \int_o^h (1 - \rho w/\rho_e w_e)\ dy \tag{3}$$

The proper displacement thickness, Δ, for steady flow may be obtained from these two integrals by solving the differential equation,

$$\partial\ [\rho_e u_e(\Delta-\delta_x)]/\partial x + \partial\ [\rho_e w_e(\Delta-\delta_z)]/\partial z = 0 \tag{4}$$

which was derived by Moore[1].

This equation has been written in a Cartesian coordinate system shown in Figure 2 with the coordinate x and z directions and the corresponding velocities u and w lying in the plane of the body surface. The y direction is normal to the surface. For projectiles it is more convenient to express Equation (4) in cylindrical coordinate system as shown in Figure 3. In this coordinate system the equation becomes,

$$\partial\ [R\rho_e u_e(\Delta-\delta_x)]/\partial x + \partial\ [\rho_e w_e(\Delta-\delta_\theta)]/\partial \theta = 0 \tag{5}$$

where

$$\delta_x = \int_o^h (1 - \rho u/\rho_e u_e)\ dy \quad \text{and} \quad \delta_\theta = \int_o^h (1 - \rho w/\rho_e w_e)\ dy$$

and where R is the radius from the center line of the projectile to its surface.

Note that the integral δ_θ is undefined where $\rho_e w_e$ is 0. The expression $\rho_e w_e \delta_\theta$ which occurs in Equation (5) does not suffer from the difficulty; however, and in fact, lim δ_θ may be finite as $w_e \rightarrow 0$.

III. ANALYSIS OF THE EQUATION

Equation (5) is a member of the class of partial differential equations for which characteristic paths exist. Along these paths the partial differential equation may be written as an ordinary differential equation. In order to produce a stable numerical scheme for the solution of Equation (5) it is necessary to understand the nature of these characteristic paths. The characteristic paths of Equation (5) may be obtained if it is rewritten in the form

$$\partial\,(\Delta-\delta_x)/\partial x + (w_e/Ru_e)\partial(\Delta-\delta_x)/\partial\theta = A(\Delta-\delta_x)+B \tag{6}$$

where $A = -(\partial\rho_e u_e R/\partial x + \partial\rho_e w_e/\partial\,\theta)/R\,\rho_e u_e$,

$B = \partial[\rho_e W_e(\delta_\theta - \delta_x)]/\partial\theta$.

Consider paths in the θ, x plane which satisfy the condition

$$d\theta/dx = w_e/Ru_e\ . \tag{7}$$

Along these paths Equation (6) becomes,

$$\partial(\Delta-\delta_x)/\partial x + (d\theta/dx)\partial(\Delta-\delta_x)/\partial\,\theta = A(\Delta-\delta_x)+B$$

or

$$d(\Delta-\delta_x)/dx = A\ (\Delta-\delta_x)+B. \tag{8}$$

The partial differential Equation (5) has been transformed into the ordinary differential Equation (8). The lines along which

$$d\,\theta/dx = w_e/Ru_e$$

are thus characteristic paths of Equation (5). Equation (8) was written in terms of $(\Delta-\delta_x)$ as Δ is approximately equal to δ_x for slender bodies of revolution at small angles of attack and this form emphasizes the difference between Δ and δ_x.

Some facts useful to the development of a solution algorithm are obtained from Equation (8). If $(\Delta-\delta_x)$ is known at any point along a characteristic then Equation (8) can be solved to find its value at all other points along the characteristic. In particular, the knowledge of the solution at one point allows one to determine the solution at a nearby point. This solution may be used to obtain the solution at another point, etc. Further, the solution along any characteristic path is independent of the solution along any other characteristic path.

IV. SOLUTION TECHNIQUE

The existence of the characteristics allows one to produce a marching solution to Equation (6). The solution is known at the tip where the displacement thickness is zero. The solution down stream of the tip may be found by integrating Equation (8). In a numerical marching scheme the solution may be obtained on the surface at a plane cutting the projectile across its centerline in terms of the solution at a nearby plane closer to the tip. The solution at the next plane down the body may then be determined and the process continued until the entire solution has been found. Difference equations can be written using one sided differences to replace the derivatives in Equation (6) giving,

$$[R\rho_e u_e (\Delta-\delta_x)]_{i,j+1}$$

$$= [R\rho_e u_e (\Delta-\delta_x)]_{i,j} + (\Delta x/\Delta\theta) \ \{[\rho_e w_e (\Delta-\delta_\theta)]_{i,j} \tag{9}$$

$$- [\rho_e w_e (\Delta-\delta_\theta)]_{i-1,j}\}$$

where the subscripts refer to grid points given in the computational molecule shown in Figure 4 and where Δx gives the grid separation in the x direction and $\Delta\theta$ gives the grid separation in the θ direction. As $R\rho$, u_e, ρ_e, w_e, δ_θ and δ_x are known functions at all of the grid points and as Δ is known on the plane of grid stations labeled with j it is possible to evaluate the left hand side of Equation (10) and solve for Δ at grid stations j+1. The solution may be marched by this technique from nose to tail.

It should also be noted that if the functions A and B are slowly varying and can be considered to be constant in some neighborhood of a point at which $(\Delta - \delta_x)$ is known, then the solution to Equation (8) in that neighborhood may be written immediately as,

$$(\Delta - \delta_x) = Ce^{Ax} - B/A \tag{10}$$

where

$$C = (\Delta - \delta_x)_o + B/A$$

and where $(\Delta-\delta_x)_o$ is the known value of $(\Delta-\delta_x)$. This local solution implies that the solution will tend to approach the local value of $-B/A$ if one proceeds along the characteristic path in the right direction. The solution will diverge exponentially if one moves in the wrong direction, however. Also, small errors made in a numerical procedure will either grow or damp depending on the direction of march. If A is negative then the poxitive x direction is the direction in which errors will damp exponentially. The characteristic length for this damping is 1/A. This length for the case of a cone in supersonic flow is about equal to the distance from the tip to the point on the body at which one is working. Hence, the damping is not rapid and care must be taken that the solution is started properly. Starting the solution from $\Delta = 0$ at the tip has proved to be sufficient.

Caution must also be employed in setting up the differencing. The solution Δ is dependent only on values of Δ backward along the characteristics and this fact must be born in mind when setting up the computational molecule. The method will be stable only if the domain of dependence of the numerical scheme contains the characteristic path leading to the point at which Δ is to be evaluated. This can be assured if the grid points used in the difference form of the θ derivative term always stradle the characteristic path that contains the point where the new solution is to be obtained. This condition restricts the allowed grid spacing and also forces one to select forward or backward differencing for the θ derivative depending on the direction of w_e.

V. RESULTS

The accuracy of the algorithm may be gauged by a comparison with a formula given by Moore[1] for Δ in terms of δ_x and δ_θ along the surface of a cone at small angle of attack in supersonic flow. This formula states that,

$$\Delta = \delta_x + (2\alpha/3 \sin \theta_c)A_2 (\delta_\theta - \delta_x) \cos \theta$$

where θ_c is the cone half angle, α is the angle of attack, and A_2 is a constant dependent on the cone vertex angle and the Mach number. For a 10^0 half angle cone at Mach 3 the constant A_2 may be taken to be 1.43.

In order to derive this formula Moore assumed that Δ, δ_x, and δ_θ grew along the body as $X^{\frac{1}{2}}$, that the inviscid flow was conical and that δ_θ was constant in θ. A comparison with the results of Moore's formula is seen in Figure 5. This figure shows Δ, δ_x, and δ_θ as calculated for a 10^0 half angle cone at Mach 3 pitched to 2^0 angle of attack. These quantities are shown as a function of azimuthal angle at a station 15 cm. from the tip. The quantities δ_x and δ_θ which are input into Moore's equation for Δ were calculated with a finite difference boundary layer code originally developed by Dwyer and Sanders[4]. These calculations were made for laminar flow as this assumption is necessary to obtain Moore's analytical formula. The accuracy of this code when applied to turbulent flow has been established[2]. Turbulent displacement thicknesses do not, however, exhibit the strong singular behavior seen on the leeward side in this figure and the spike on the leeward side of the cone seen in this figure is probably not well resolved. Further, as seen in Figure 5, the assumption which Moore made in obtaining his analytical solution that δ_θ is constant about the cone is not followed on the leeward side by the δ_θ calculated from this code. One should, therefore, expect to find agreement only on the windward side between the three-dimensional displacement thickness calculated by Moore's formula and that calculated by the technique presented here.

On the windward side of a cone it is possible to solve the displacement thickness equation more accurately. The result is

$$\Delta = \delta_x + (2\alpha/3 \sin\theta_c)A_2(\delta_\theta - \delta_x)/[1+2\ \alpha/3 \sin\theta_c)]$$

Given the input quantities A_2, δ_θ, δ_x, α and θ_c, this formula is exact on the windward side. Comparison of the results of this formula and the present theory are shown in Figure 6. This figure shows the development of the displacement thicknesses Δ, δ_x, and δ_θ along the windward ray for the same case as discussed above. The agreement between the two calculations of Δ is felt to be quite good.

The reason for the development of the technique that is presented here is to improve upon the earlier method of Sanders. A comparison of the results of the two techniques for a turbulent boundary layer on a 10^0 half angle cone at 2^0 angle of attack can be seen in Figure 7. It is evident that there is some discrepancy particularly near the tip. It is felt that the starting conditions used by Sanders are somewhat in error.

Figures 8 and 9 show the displacement thickness for the projectile body shown in Figure 10. These figures show curves made at successive stations along the body. The body shown in this figure has a nose similar in shape to that used on the M549 projectile.

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4. *H. A. Dwyer and B. R. Sanders, "Magnus Forces on Spinning Supersonic Cones. Part I; The Boundary Layer," AIAA Journal, Vol 14, p. 498, April 1976.*

The caliber is, however, enlarged. This body also has a straight cylindrical tail section instead of a boattail as found on the M549. The Mach number is transonic at M = .94 with a well developed shock aft of the corner where the ogive and the tail sections join. The inviscid transonic flow about this body was obtained from a small disturbance potential technique developed by Reklis, Sturek, and Bailey[5]. In both Figures 8 and 9, the angle of attack is 4^0 and the boundary layer is turbulent. The body used for Figure 8 was not spinning while the body used for Figure 9 was spinning at 10,000 rpm. As can be seen in both cases the boundary layer builds up evenly on the nose. It thins over the corner between the nose and the cylindrical portion. It thickens rapidly through the supersonic region aft of this corner and then grows steadily over the afterbody. The boundary layer on the spinning body is skewed slightly to one side in the direction of spin.

VI. CONCLUSIONS

A simple and accurate technique for the solution of the displacement thickness equation has been obtained. The accuracy of the present technique has been established in a comparison with an exact result for a supersonic cone. The present technique has required about 1/10 of the computer time necessary for the earlier method. The present technique has been used to obtain results for spinning projectile shapes in transonic flow and it appears suitable for use in computing displacement effects on realistic projectiles.

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5. *R. P. Reklis, W. B. Sturek, and F. R. Bailey, "Computation of Transonic Flow Past Projectiles at Angle of Attack," Paper 78-1182, presented at AIAA 11th Fluid and Plasma Dynamics Conference, 10 July 1978.*

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1. F. K. Moore, "Displacement Effect of a Three Dimensional Boundary Layer," NACA Technical Note 2722, June 1952.

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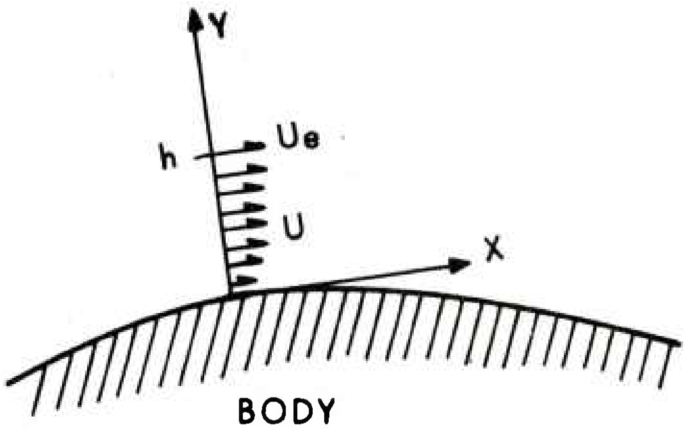

Figure 1. Two-Dimensional Coordinate System

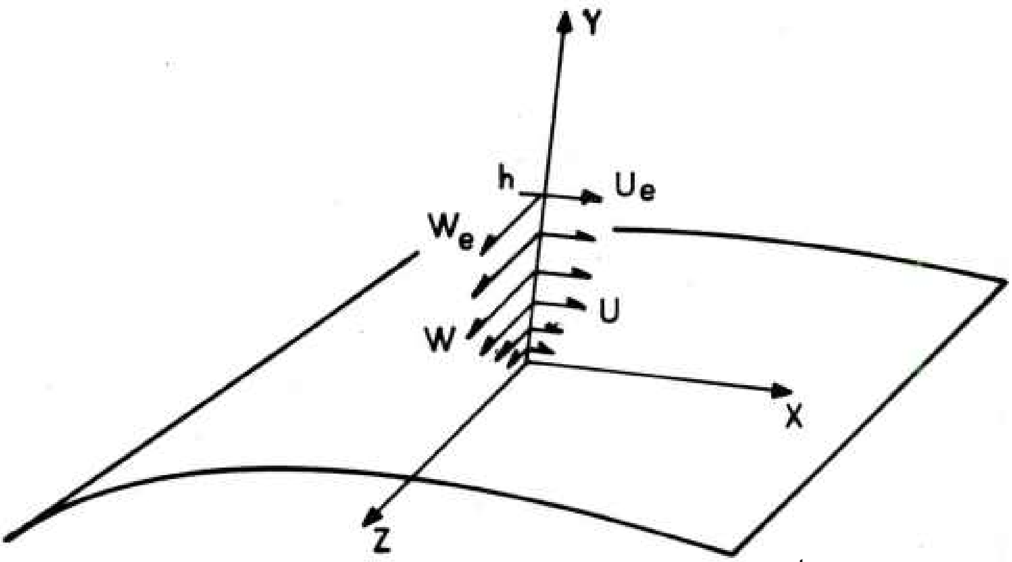

Figure 2. Three-Dimensional Cartesian Coordinate System

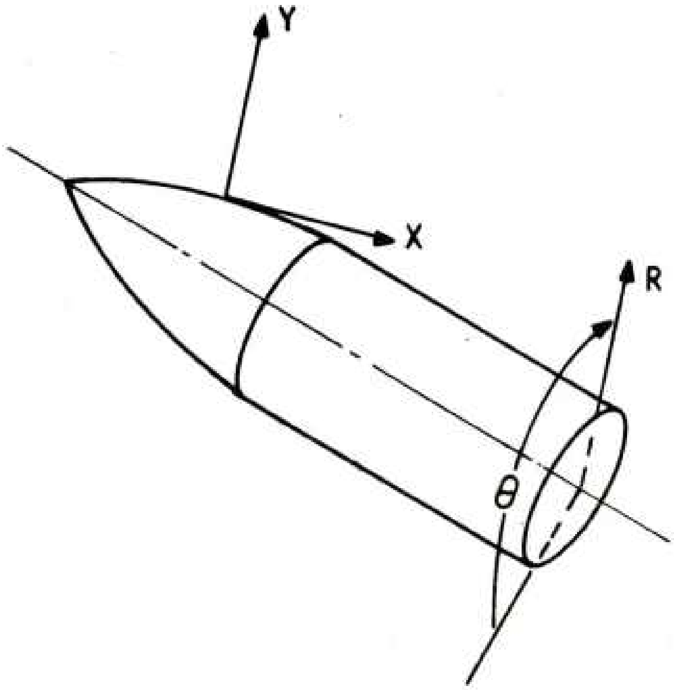

Figure 3. Three-Dimensional Cylindrical Coordinate System

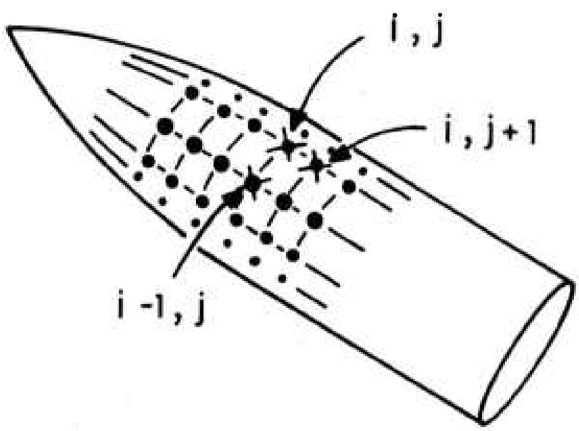

Figure 4. Computational Grid

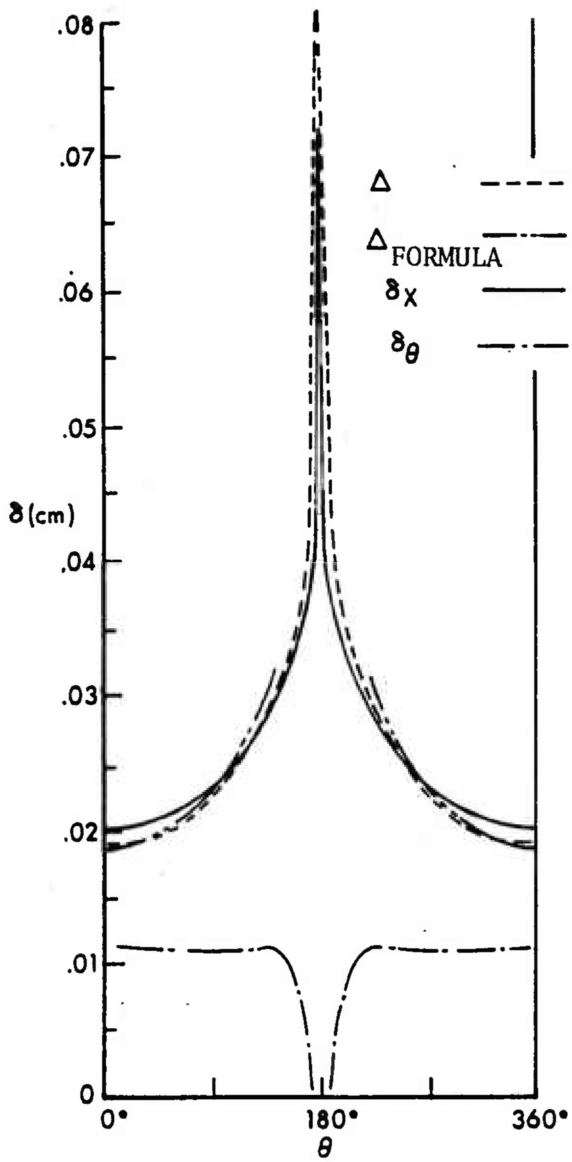

Figure 5. Displacement Thicknesses as a Function of Roll Angle Showing Input Quantities δ_x and δ_θ Together With the Numerical and the Formula Computation of Δ

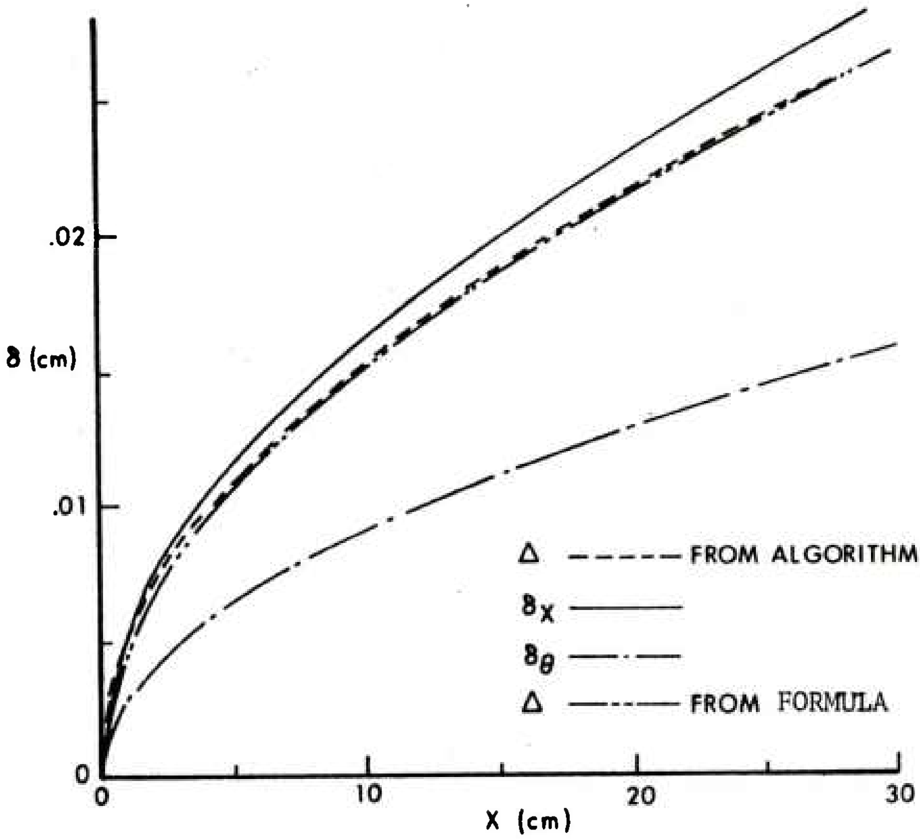

Figure 6. Displacement Thicknesses on Windward Side as a Function of Distance From the Cone Tip Showing Input Quantities δ_x and δ_θ Together With the Numerical and Formula Computation of Δ

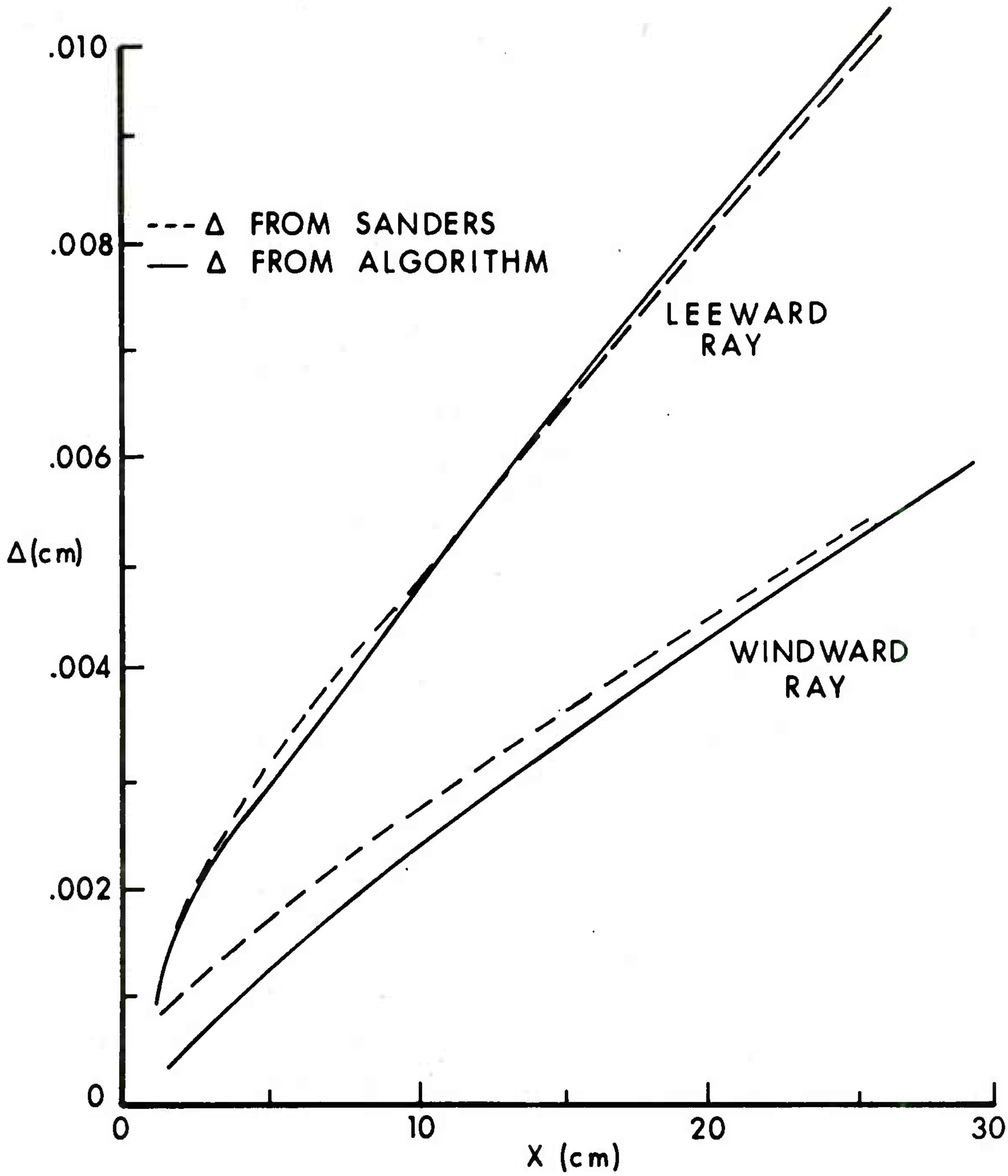

Figure 7. Three-Dimensional Displacement Thickness as a Function of Distance From the Cone Tip Showing Comparison Between the Present Method and the Method of Sanders

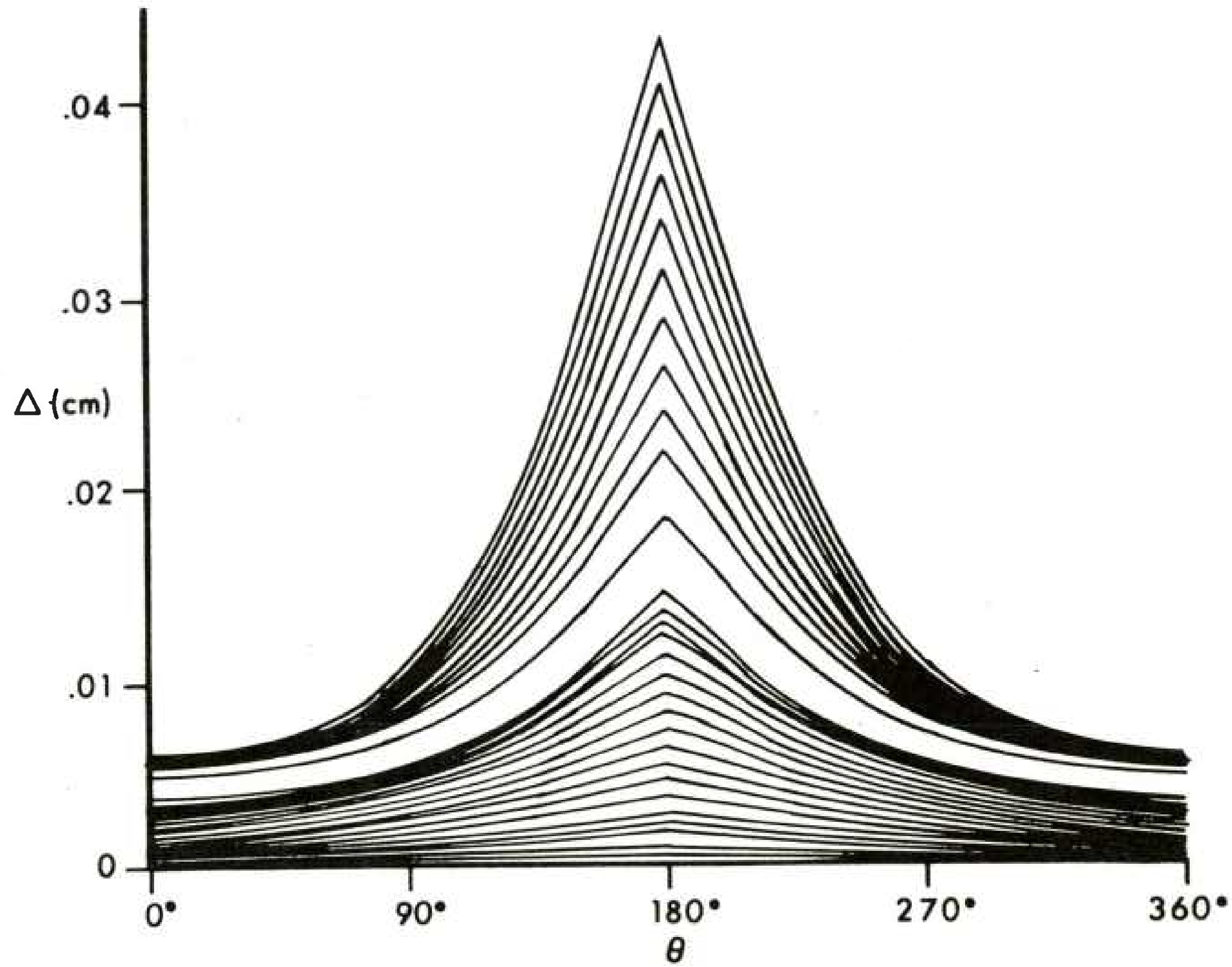

Figure 8. Three-Dimensional Displacement Thickness on Non-Spinning Body as a Function of Roll Position for Successive Stations From Nose to Tail

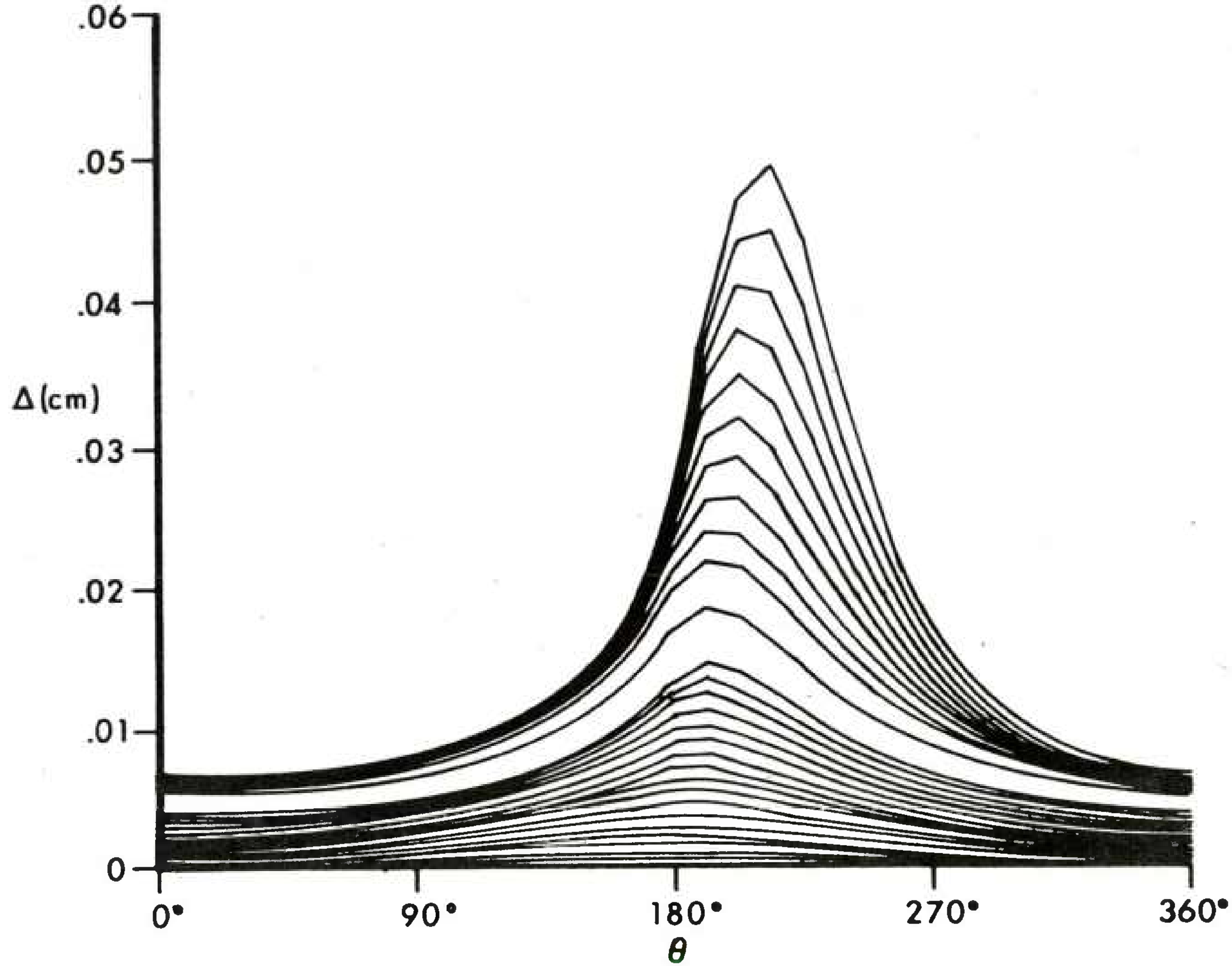

Figure 9. Three-Dimensional Displacement Thickness on a Spinning Body as a Function of Roll Position for Successive Stations From Nose to Tail

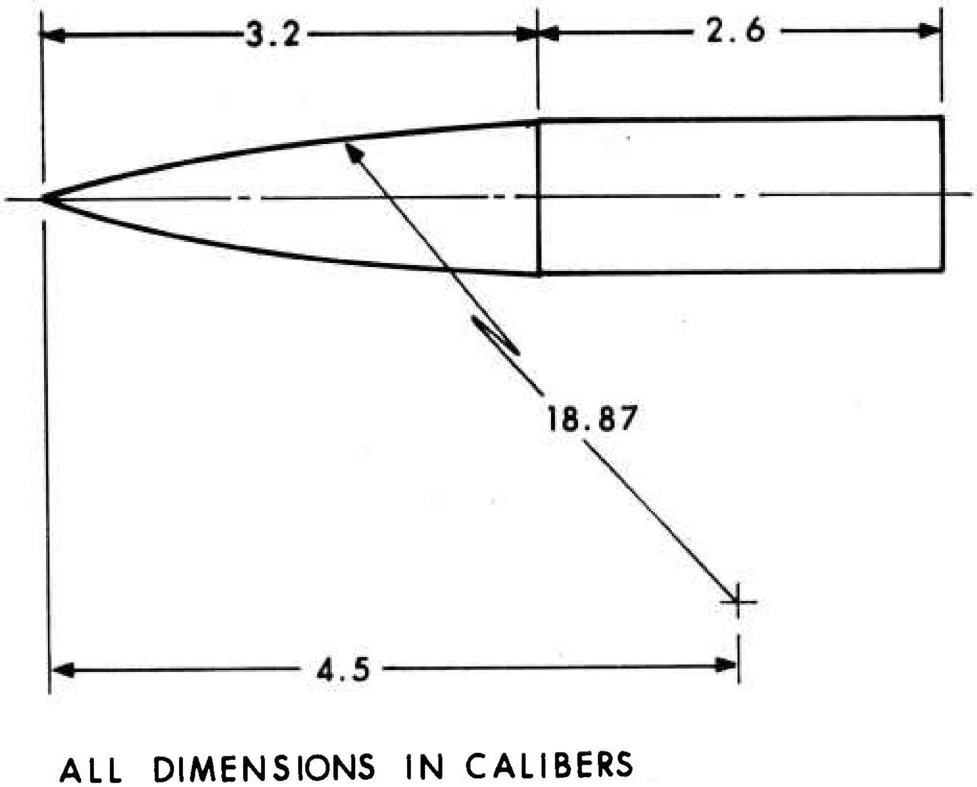

Figure 10. Dimensions of Projectile Shape

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