

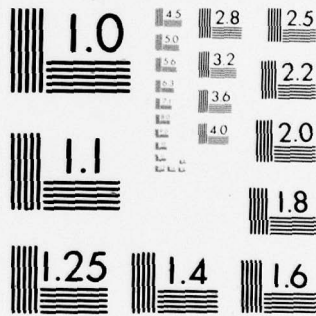
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MEMORANDUM REPORT ARBRL-MR-02990

THE CONVERGENCE OF TWO SERIES
REPRESENTATIONS FOR ASSOCIATED
LEGENDRE FUNCTIONS OF THE FIRST
AND SECOND KIND

James N. Walbert
Kathleen L. Zimmerman

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February 1980



US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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LIST OF SYMBOLS

θ - Apex angle of a cone, in radians

$\Gamma(x)$ - The Gamma function

$(x)_k = \Gamma(x+k)/\Gamma(x)$, the Pochhammer notation

$k! = \Gamma(k+1)$, factorial notation

a, b, c - arbitrary complex numbers

P_{ν}^{μ} - Associated Legendre function of the first kind, of degree ν and order μ .

Q_{ν}^{μ} - Associated Legendre function of the second kind, of degree ν and order μ .

$\text{Re } \{a\}$ - the real part of the complex number a .

$\text{Im } \{a\}$ - the imaginary part of the complex number a .

$\sinh x, \cosh x$ - the hyperbolic sine and cosine, respectively, of x .

$i^2 = -1$

$O(k)$ - Landau symbol

Other terms are defined as they occur in the text.

I. INTRODUCTION

For stresses in a pressurized cone, the method of Papkovitch-Neuber¹ leads to displacements and strains expressed in terms of Legendre functions. In spherical coordinates, the argument at which these Legendre functions are evaluated is the cosine of the apex angle θ of the cone. Computation of Legendre functions using Gaussian quadratures on certain integral representations² is required in the stress analysis for the minihat gage, a strain-type pressure transducer.³

For the purpose of verification of the quadrature results, use was made of the series representations

$$P_{\nu}^{\mu}(\cos \theta) = \pi^{-\frac{1}{2}} 2^{\mu+1} (\sin \theta)^{\mu} \frac{\Gamma(\nu+\mu+1)}{\Gamma\left(\nu+\frac{3}{2}\right)} \quad (1)$$

$$\times \sum_{k=0}^{\infty} \frac{(\mu+\frac{1}{2})_k (\nu+\mu+1)_k}{k! \left(\nu+\frac{3}{2}\right)_k} \sin[(2k+\mu+\nu+1)\theta]$$

$$Q_{\nu}^{\mu}(\cos \theta) = \pi^{\frac{1}{2}} 2^{\mu} (\sin \theta)^{\mu} \frac{\Gamma(\nu+\mu+1)}{\Gamma\left(\nu+\frac{3}{2}\right)} \quad (2)$$

$$\times \sum_{k=0}^{\infty} \frac{(\mu+\frac{1}{2})_k (\mu+\nu+1)_k}{k! \left(\nu+\frac{3}{2}\right)_k} \cos[(2k+\mu+\nu+1)\theta],$$

where $0 < \theta < \pi$. In at least two references^{4,5} there

¹A. I. Lure', Three Dimensional Problems of the Theory of Elasticity, 1964, paragraph 1.10.

²W. Magnus, and F. Oberhettinger, Formulas and Theorems for the Special Functions of Mathematical Physics, 1949, p67.

³H. Gay, "The Evolution of Gages for Measuring Pressures in Guns and Pockets at the Ballistic Research Laboratories", BRL Memorandum Report No. 1402, 1962. (AD #283309)

⁴Handbook of Mathematical Functions, National Bureau of Standards, U.S. Department of Commerce, 1964, 8.7.1, 8.7.2.

⁵A. Erdelyi, et al, Higher Transcendental Functions, Vol. I, Bateman Manuscript Project, 1953, 3.5(2), 3.5(3).

are no stated restrictions on μ or ν for the convergence of these series. However, calculation of the first 50 partial sums of the series in equation (1) with $\theta = \frac{\pi}{6}$, $\mu = 1$, and $\nu = .05$ indicated that these partial sums were increasing without bound (see Appendix A).

Indeed, since

$$\frac{(\mu + \frac{1}{2})_k (\mu + \nu + 1)_k}{k! (\nu + \frac{3}{2})_k} = \frac{\Gamma(\nu + \frac{3}{2})}{\Gamma(\mu + \frac{1}{2}) \Gamma(\mu + \nu + 1)} \frac{\Gamma(\mu + \frac{1}{2} + k) \Gamma(\mu + \nu + 1 + k)}{\Gamma(1 + k) \Gamma(\nu + \frac{3}{2} + k)}, \quad (3)$$

one sees at a glance that for real μ and ν with $\mu \geq \frac{1}{2}$, the term in brackets is greater than or equal to 1 for every positive integral k , giving scant hope for the convergence of the series in Eqs. (1) and (2) in this case.

It is the purpose of this report to identify the region of convergence of these series, and to indicate by numerical computation the behavior of the partial sums for various values of the parameters μ and ν .

In reference 4, for example, the convergence criteria for series representations of the Legendre functions are not complete, and should be supplemented by the more general convergence criteria for the hypergeometric function.

II. ANALYSIS OF CONVERGENCE

To simplify our notation, we write

$$a = \mu + \frac{3}{2}$$

$$b = \mu + \nu + 1$$

$$c = \nu + \frac{3}{2}. \quad (4)$$

The series in Eqs. (1) and (2) are then

$$S = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{k! (c)_k} \sin [(2k+b)\theta], \quad (5)$$

and

$$C = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{k! (c)_k} \cos[(2k+b)\theta] , \quad (6)$$

respectively.

Since

$$\sin [(2k+b)\theta] = \frac{1}{2i} \left(e^{i(2k+b)\theta} - e^{-i(2k+b)\theta} \right) ,$$

and

$$\cos [(2k+b)\theta] = \frac{1}{2} \left(e^{i(2k+b)\theta} + e^{-i(2k+b)\theta} \right) ,$$

we can write

$$\begin{aligned} 2iS &= \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{k! (c)_k} \left[e^{ib\theta} (e^{i2\theta})^k - e^{-ib\theta} (e^{-i2\theta})^k \right] \\ &= e^{ib\theta} \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{k! (c)_k} (e^{i2\theta})^k - e^{-ib\theta} \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{k! (c)_k} (e^{-i2\theta})^k \\ &= e^{ib\theta} F(a, b; c; e^{i2\theta}) - e^{-ib\theta} F(a, b; c; e^{-i2\theta}); \end{aligned}$$

similarly,

$$2C = e^{ib\theta} F(a, b; c; e^{i2\theta}) + e^{-ib\theta} F(a, b; c; e^{-i2\theta}),$$

where

$$F(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{k! (c)_k} z^k \quad (7)$$

is the hypergeometric function. These representations for S and C as

linear combinations of hypergeometric functions are valid only on the intersections of the regions of convergence of the various forms in which the series in Eq. (7) appears.

It is known (see Appendix B) that on $|z|=1$, the series in Eq. (7) exhibits the following behavior:

- I) Absolute convergence if $\text{Re}\{a+b-c\} < 0$;
- II) Conditional convergence if $z \neq 1$ and $0 \leq \text{Re}\{a+b-c\} < 1$;
- III) Divergence if $\text{Re}\{a+b-c\} \geq 1$.

Since a finite linear combination of convergent series is convergent, we have immediately that the series in Eqs. (5) and (6) are convergent for $\text{Re}\{a+b-c\} < 1$ and $0 < \theta < \pi$. Moreover, use of the triangle inequality shows that these series converge absolutely for $\text{Re}\{a+b-c\} < 0$.

By Eq. (4), one sees that

$$\begin{aligned} \text{Re}\{a+b-c\} &= \text{Re}\left\{\left(\mu + \frac{1}{2}\right) + (\mu + \nu + 1) - \left(\nu + \frac{3}{2}\right)\right\} \\ &= \text{Re}\{2\mu\} \\ &= 2\text{Re}\{\mu\}. \end{aligned}$$

Therefore, the series in Eqs. (5) and (6) converge absolutely for $\text{Re}\{\mu\} < 0$ and at least conditionally for $\text{Re}\{\mu\} < \frac{1}{2}$ if $0 < \theta < \pi$. It should be noted that we have assumed that a, b , and c are not negative integers, so that the expressions involving the gamma function are meaningful; this assumption of course imposes additional discrete restrictions on the values of μ and ν .

Continuing the analysis, we will develop an asymptotic expansion for large k of

$$w(a, b, c, k) = \frac{\Gamma(a+k)\Gamma(b+k)}{\Gamma(1+k)\Gamma(c+k)}. \quad (8)$$

To this end (see Appendix C), we observe that

$$\log \Gamma(k+x) = \log \Gamma(k) + x \log k + O\left(\frac{1}{k}\right), \quad (9)$$

for large k . Using Eq. (9) in Eq. (8), we have

$$\begin{aligned}
\log w(a,b,c,k) &= \log \Gamma(k+a) + \log \Gamma(k+b) - \log \Gamma(k+c) - \log \Gamma(k+1) \\
&= [\log \Gamma(k) + a \log k + O(\frac{1}{k})] \\
&\quad + [\log \Gamma(k) + b \log k + O(\frac{1}{k})] \\
&\quad - [\log \Gamma(k) + c \log k + O(\frac{1}{k})] \\
&\quad - [\log \Gamma(k) + \log k + O(\frac{1}{k})] \\
&= (a+b-c-1) \log k + O(\frac{1}{k}) .
\end{aligned}$$

Thus

$$w(a,b,c,k) = k^{a+b-c-1} [1 + O(\frac{1}{k})] , \quad (10)$$

and so

$$w(a,b,c,k) \sim k^{a+b-c-1} \text{ as } k \rightarrow \infty^6. \quad (11)$$

For real values of x and y ,

$$|\sin(x+iy)| = (\sin^2 x + \sinh^2 y)^{\frac{1}{2}}, \quad (12)$$

and

$$|\cos(x+iy)| = (\cos^2 x + \sinh^2 y)^{\frac{1}{2}}. \quad (13)$$

Since $\sin^2 x \leq$ and $\cos^2 x \leq 1$ and $1 + \sinh^2 y = \cosh^2 y$,

$$(\sin^2 x + \sinh^2 y)^{\frac{1}{2}} \leq \cosh y,$$

and $(\cos^2 x + \sinh^2 y)^{\frac{1}{2}} \leq \cosh y$.

Also, $\sin^2 x \geq 0$ and $\cos^2 x \geq 0$, so that

$$(\sin^2 x + \sinh^2 y)^{\frac{1}{2}} \geq |\sinh y|$$

and $(\cos^2 x + \sinh^2 y)^{\frac{1}{2}} \geq |\sinh y|$

This shows that

$$|\sinh(\theta \cdot \text{Im}\{b\})| \leq |\sin[(2k+b)\theta]| \leq \cosh(\theta \cdot \text{Im}\{b\}) , \quad (14)$$

⁶F. W. J. Olver, *Asymptotics and Special Functions*, 1974, pg. 301.

and

$$|\sinh(\theta \cdot \text{Im}\{b\})| \leq |\cos[(2k+b)\theta]| \leq \cosh(\theta \cdot \text{Im}\{b\}). \quad (15)$$

For fixed θ and b , the upper bound, $\cosh(\theta \cdot \text{Im}\{b\})$, is finite and independent of k . If $0 < \theta < \pi$ and $\text{Im}\{b\} \neq 0$,

$$|\sinh(\theta \cdot \text{Im}\{b\})| = \rho > 0. \quad \text{If } 0 < \theta < \pi \text{ and } \text{Im}\{b\} = 0,$$

$|\sin[(2k+b)\theta]|$ and $|\cos[(2k+b)\theta]|$ can be neither constantly zero nor remain arbitrarily close to zero as k varies. Therefore, the expressions

$$|k|^{\text{Re}\{a+b-c-1\}} |\sin[(2k+b)\theta]|, \quad (16)$$

and

$$|k|^{\text{Re}\{a+b-c-1\}} |\cos[(2k+b)\theta]| \quad (17)$$

will tend to zero as $k \rightarrow \infty$ only if $\text{Re}\{a+b-c-1\} < 0$.

Using Eqs. (8), (11), (16), and (17), we see that the k th term of the series in Eqs. (5) and (6) cannot tend to zero in absolute value as $k \rightarrow \infty$ when $\text{Re}\{a+b-c-1\} \geq 0$. Since

$$a+b-c-1 = 2\mu-1,$$

we conclude that the series in Eqs. (5) and (6) diverge for $\text{Re}\{\mu\} \geq \frac{1}{2}$.

III. NUMERICAL ASPECTS OF CONVERGENCE

To illustrate numerically the convergence and divergence properties obtained, use was made of the following formulae^{2,4}

$$P_{\nu}^1(\cos\theta) = -\frac{\Gamma(\nu+2)}{\Gamma(\nu)} \sqrt{\frac{2}{\pi}} \frac{(\sin\theta)^{-1}}{\Gamma\frac{3}{2}} \int_0^{\theta} \cos[(\nu+\frac{1}{2})\phi] (\cos\phi - \cos\theta)^{\frac{1}{2}} d\phi; \quad (18)$$

$$P_{\nu}^{\frac{1}{2}}(\cos\theta) = \left(\frac{\pi}{2}\right)^{-\frac{1}{2}} (\sin\theta)^{-\frac{1}{2}} \cos(\nu+\frac{1}{2})\theta; \quad (19)$$

$$Q_{\nu}^{\frac{1}{2}}(\cos\theta) = -\frac{\pi}{2}^{-\frac{1}{2}}(\sin\theta)^{-\frac{1}{2}}\sin(\nu+\frac{1}{2})\theta; \quad (20)$$

$$P_{\nu}^{-\frac{1}{2}}(\cos\theta) = \frac{\pi}{2}^{-\frac{1}{2}}(\nu+\frac{1}{2})^{-1}(\sin\theta)^{-\frac{1}{2}}\sin(\nu+\frac{1}{2})\theta; \quad (21)$$

$$Q_{\nu}^{-\frac{1}{2}}(\cos\theta) = (2\pi)^{\frac{1}{2}}(2\nu+1)^{-1}(\sin\theta)^{-\frac{1}{2}}\cos(\nu+\frac{1}{2})\theta. \quad (22)$$

These associated Legendre functions were computed for

$\nu = -.4, .05, -1.5$, and 1.5 , with $\theta = \frac{\pi}{6}$. From the recurrence formula⁵

$$\begin{aligned} T_{\nu}^{\mu+2}(\cos\theta) + 2(\mu+1)(\cos\theta)(1-\cos^2\theta)^{-\frac{1}{2}}T_{\nu}^{\mu+1}(\cos\theta) \\ + (\nu-\mu)(\nu+\mu+1)T_{\nu}^{\mu}(\cos\theta) = 0, \end{aligned} \quad (23)$$

where T stands for either P or Q , one obtains values for $P_{\nu}^{\frac{3}{2}}(\cos\theta)$ from Eqs. (19) and (21), and values for $Q_{\nu}^{\frac{3}{2}}(\cos\theta)$ from Eqs. (20) and (22).

With ν and θ as above, the sums of the first fifty terms of the series in Eqs. (1) and (2) were computed for $\mu = \pm\frac{1}{2}, \pm 1, 0$, and $\frac{3}{2}$. The values of $P_{\nu}^{-\frac{1}{2}}$ and $Q_{\nu}^{-\frac{1}{2}}$ thus computed agree quite well with those obtained from Eqs. (21) and (22), while the values for $P_{\nu}^{\frac{1}{2}}$, $Q_{\nu}^{\frac{1}{2}}$, and P_{ν}^1 do not agree with the values obtained from Eqs. (13) - (20), as expected, since these values of μ are outside the region of convergence of the series. Indeed, one sees that the terms of the sequence of partial sums increase quite rapidly for $\mu = \frac{1}{2}$ and 1 . This growth behavior is again exhibited in the partial sums when $\mu = \frac{3}{2}$. Using the series approximations for the values $\mu = -1$ and 0 in the recurrence relation (23), the value of P_{ν}^1 obtained agrees well with that computed from Eq. (18). The results are tabulated in Appendix A.

IV. CONCLUSIONS

At $\theta = 0$ or π , the term $(\sin\theta)^{\mu}$ is meaningless, so the series representations in Eqs. (1) and (2) are not valid there. Indeed, ± 1 are singular points of the associated Legendre functions. The series in Eqs. (5) and (6) do converge absolutely for $0 < \theta < \pi$ when $\text{Re}\{\nu\} < 0$, however.

V. SUMMARY

It is shown that the series representations

$$P_{\nu}^{\mu}(\cos\theta) = \pi^{-\frac{1}{2}} 2^{\mu+1} (\sin\theta)^{\mu} \frac{\Gamma(\nu+\mu+1)}{\Gamma(\nu+\frac{3}{2})} \sum_{k=0}^{\infty} \frac{(\mu+\frac{1}{2})_k (\mu+\nu+1)_k}{k! (\nu+\frac{3}{2})_k} \sin[(2k+\mu+\nu+1)\theta]$$

and

$$Q_{\nu}^{\mu}(\cos\theta) = \pi^{\frac{1}{2}} 2^{\mu} (\sin\theta)^{\mu} \frac{\Gamma(\nu+\mu+1)}{\Gamma(\nu+\frac{3}{2})} \sum_{k=0}^{\infty} \frac{(\mu+\frac{1}{2})_k (\mu+\nu+1)_k}{k! (\nu+\frac{3}{2})_k} \cos[(2k+\mu+\nu+1)\theta],$$

for the associated Legendre functions of the first and second kind, respectively, of complex order μ and degree ν with $0 < \theta < \pi$, converge absolutely for $\text{Re}\{\mu\} < 0$, converge at least conditionally for $0 \leq \text{Re}\{\mu\} < \frac{1}{2}$, and diverge for $\text{Re}\{\mu\} \geq \frac{1}{2}$.

Numerical computations for real μ and ν are presented to illustrate the behavior of the partial sums in the regions of convergence and divergence. A method is indicated whereby these series representations may be used to compute the values of $P_{\nu}^{\mu}(\cos\theta)$ for arbitrary complex μ and ν .

VI. ACKNOWLEDGMENTS

Mr. A. S. Elder verified a large portion of the analysis, and provided the authors with a great deal of insight to the problem.

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APPENDIX A

NUMERICAL ASPECTS OF CONVERGENCE AND DIVERGENCE

Tables of the first 50 partial sums (PSP) of $P_{\nu}^{\mu}(\cos\theta)$ and the partial sums (QSP) of $Q_{\nu}^{\mu}(\cos\theta)$ for $\nu = -1.5, -.4, .05, 1.5$; $\mu = \pm 1.0, \pm .5, 0, 1.5$; and $\theta = \pi/6$ are given in this appendix. The functional values $P_{\nu}^{\mu}(\cos\theta)$ and $Q_{\nu}^{\mu}(\cos\theta)$ are listed at the bottom of each table. A discussion of these equations begins on page 12 of the text.

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MU = -1.00 NU = -1.50

THETA = 30.0 DEGREES

K	PSP	WSP
1	-.167108551642066592E-99	.167108551642066592E-99
2	.458744813179650780E+00	.171205895049514944E+01
3	.244737444367757088E+00	.165471584884769308E+01
4	.225154410972202421E+00	.167429888224324775E+01
5	.227394375880306184E+00	.168265854508746235E+01
6	.231051728374650071E+00	.168363852973475775E+01
7	.232457346494350528E+00	.168223291161505729E+01
8	.232154163744093515E+00	.168110141818711965E+01
9	.231431826421794039E+00	.168090786848494665E+01
10	.231073793137333407E+00	.168126590176940728E+01
11	.231166619708905096E+00	.168161233525080525E+01
12	.231420933378204059E+00	.168168047839311810E+01
13	.231561619060528790E+00	.168153979271079337E+01
14	.231521760310901793E+00	.168139103783205925E+01
15	.231404268476736794E+00	.168135955598997748E+01
16	.231335153304527050E+00	.168142867116218723E+01
17	.231355786911404857E+00	.168150567683139861E+01
18	.231419415676681539E+00	.168152272610766988E+01
19	.231458346082933606E+00	.168148379570141782E+01
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40	.231408289525219607E+00	.168146280504010609E+01
41	.231409497400540590E+00	.168146731289217321E+01
42	.231413677471291242E+00	.168146843293875515E+01
43	.231416520255068183E+00	.168146559015497821E+01
44	.231415551887065736E+00	.168146197615639265E+01
45	.231412182864864217E+00	.168146107342961447E+01
46	.231409880054932955E+00	.168146337623954573E+01
47	.231410668276374641E+00	.168146631792201372E+01
48	.231413423212349226E+00	.168146705610488331E+01
49	.231415314588848292E+00	.168146516472838424E+01
50	.231414664461460293E+00	.168146273841994084E+01

PUV (COS THETA) = .261123486338710080E+00

QUV (COS THETA) = .298031510586655852E+01

MU = -1.00 NU = -.40 THETA = 30.0 DEGREES

K	PSP	QSP
1	.813633856749546232E+00	-.382784634055499464E+01
2	.284871085053336490E+00	-.430394647866439731E+01
3	.236535642182184539E+00	-.428824134124941962E+01
4	.233808758707531680E+00	-.427541236314952855E+01
5	.237671811439941889E+00	-.427193405484593058E+01
6	.240114644948309452E+00	-.427272777956741693E+01
7	.240416678864225650E+00	-.427414873742231996E+01
8	.239747650409969615E+00	-.427475113334828242E+01
9	.239180811384473643E+00	-.427456695618430531E+01
10	.239094568498430055E+00	-.427416121530589633E+01
11	.239317675825103320E+00	-.427396032856664725E+01
12	.239530943912509137E+00	-.427402962356882332E+01
13	.239566681646089349E+00	-.427419775638626930E+01
14	.239466621130497154E+00	-.427428785127918304E+01
15	.239364449808307819E+00	-.427425465380421357E+01
16	.239346344493988721E+00	-.427416947499732637E+01
17	.239399527834941158E+00	-.427412158850204352E+01
18	.239456141301466267E+00	-.427413998333238955E+01
19	.239466546090650086E+00	-.427418893401686653E+01
20	.239434982123598783E+00	-.427421735434045360E+01
21	.239400405461301019E+00	-.427420611970184307E+01
22	.239393885506489706E+00	-.427417544572612532E+01
23	.239414127126163610E+00	-.427415722008990780E+01
24	.239436769946629256E+00	-.427416457718825536E+01
25	.239441121715094434E+00	-.427418505064920611E+01
26	.239427374128506652E+00	-.427419742903176908E+01
27	.239411748169980640E+00	-.427419235185007146E+01
28	.239408700511189855E+00	-.427417801374276092E+01
29	.239418459932122391E+00	-.427416922632068326E+01
30	.239429693241146503E+00	-.427417287624403906E+01
31	.239431909923346422E+00	-.427418330491385995E+01
32	.239424733804445244E+00	-.427418976632034093E+01
33	.239416389526758239E+00	-.427418705510016959E+01
34	.239414727197549519E+00	-.427417923445612238E+01
35	.239420156940233186E+00	-.427417434549385051E+01
36	.23942652373773247E+00	-.427417641419177320E+01
37	.239427802174138609E+00	-.427418242876199073E+01
38	.239423595136660236E+00	-.427418621679555077E+01
39	.239418627216291188E+00	-.427418460262037355E+01
40	.239417623005326797E+00	-.427417987417923421E+01
41	.239420948552991952E+00	-.427417688384266700E+01
42	.239424899149243493E+00	-.427417816746920099E+01
43	.239425702277493777E+00	-.427418194589054905E+01
44	.239423028192837342E+00	-.427418435364718850E+01
45	.239419835072523379E+00	-.427418331613950605E+01
46	.239419182731699194E+00	-.427418024711722295E+01
47	.239421364985298508E+00	-.427417828220725644E+01
48	.239423982538757361E+00	-.427417913270193117E+01
49	.239424519596152021E+00	-.427418165935832060E+01
50	.239422715589067396E+00	-.427418328369359554E+01

PUV (COS THETA) = .270159604400137663E+00

QUV (COS THETA) = -.757579262065869710E+01

MU = -1.00 NU = .05 THETA = 30.0 DEGREES

K	PSP	OSP
1	.573389094124100302E+00	.218968460823165902E+02
2	.262906521267552242E+00	.217282677997457212E+02
3	.231897101684802442E+00	.217472703965057921E+02
4	.232171981201488736E+00	.217577676217403527E+02
5	.236038234720196846E+00	.217598668261245825E+02
6	.237954332598154616E+00	.217586926398348167E+02
7	.237920316675119689E+00	.217573936241425466E+02
8	.237201306854259488E+00	.217570032336622145E+02
9	.236733950151301496E+00	.217572896302181297E+02
10	.236744029047381258E+00	.217576745277308306E+02
11	.236990750834588045E+00	.217578084867314195E+02
12	.237170633693602696E+00	.217576982543736424E+02
13	.237166378987016857E+00	.217575357736843683E+02
14	.237054008737564049E+00	.217574747616169581E+02
15	.236966701927194290E+00	.217575282632991627E+02
16	.236968881592398611E+00	.217576115013550991E+02
17	.237029186936843972E+00	.217576442444855843E+02
18	.237077973899013188E+00	.217576143477967161E+02
19	.237076711929352986E+00	.217575661551197134E+02
20	.237040683668720124E+00	.217575465933702547E+02
21	.237010710306373188E+00	.217575649610703241E+02
22	.237011505313664511E+00	.217575953211739097E+02
23	.237034721310486339E+00	.217576079264317069E+02
24	.237054435681406054E+00	.217575958454496696E+02
25	.237053902927230996E+00	.217575755003885401E+02
26	.237038077392936756E+00	.217575669078244952E+02
27	.237024426482914202E+00	.217575752731129158E+02
28	.237024800746327102E+00	.217575895656560257E+02
29	.237036067683244332E+00	.217575956831036424E+02
30	.237045907414126316E+00	.217575896533088028E+02
31	.237045634519504451E+00	.217575792318836434E+02
32	.237037330661744143E+00	.217575747232567434E+02
33	.237030005661296723E+00	.217575792120227907E+02
34	.237030210726988579E+00	.217575870431656174E+02
35	.237036505793938648E+00	.217575904611080975E+02
36	.237042104941443016E+00	.217575870299460939E+02
37	.237041946964984809E+00	.217575809970685498E+02
38	.237037061635952892E+00	.217575783445513073E+02
39	.237032686070360965E+00	.217575810259013506E+02
40	.237032810339205796E+00	.217575857715370733E+02
41	.237036677325820593E+00	.217575878711394962E+02
42	.237040161378247798E+00	.217575857361094229E+02
43	.237040061870421318E+00	.217575819360588416E+02
44	.237036948762247186E+00	.217575802457790149E+02
45	.237034129579724903E+00	.217575819733762865E+02
46	.237034210491307840E+00	.217575850632649781E+02
47	.237036753614958012E+00	.217575864440684588E+02
48	.237039066890734780E+00	.217575850264912397E+02
49	.237039000215777207E+00	.217575824802773362E+02
50	.237036896003740911E+00	.217575813377834179E+02

PUV (COS THETA) = .267467495283606802E+00
QUV (COS THETA) = .385643088285442081E+02

MU = -1.00 NU = 1.50

THETA = 30.0 DEGREES

K	PSP	QSP
1	.313328534328874664E+00	.313328534328874664E+00
2	.206324849922927818E+00	.342000085152602839E+00
3	.201844920106720291E+00	.358719410841032034E+00
4	.206128708661997875E+00	.363003199396309617E+00
5	.208871723032755789E+00	.362268210910838070E+00
6	.209275966699765140E+00	.360759553006921217E+00
7	.208602964274149631E+00	.360086550581305708E+00
8	.208001016505566735E+00	.360247841999783211E+00
9	.207889624619680709E+00	.360663562177460774E+00
10	.208108648769076134E+00	.360882586326856198E+00
11	.208331173229712727E+00	.360822961077332452E+00
12	.208376726086429295E+00	.360652955501636313E+00
13	.208279497399726506E+00	.360555726814933524E+00
14	.208173754748978007E+00	.360584060472807111E+00
15	.208150828519113776E+00	.360669622327486422E+00
16	.208202226575947327E+00	.360721020384319973E+00
17	.208260552944117619E+00	.360705391881071303E+00
18	.208273677499864374E+00	.360656410372197644E+00
19	.208243273791692187E+00	.360626006664025457E+00
20	.208207752798894691E+00	.360635524485359898E+00
21	.208199549085846767E+00	.360666141159265466E+00
22	.208219003814293076E+00	.360685595887711775E+00
23	.208242219761406788E+00	.360679375193431133E+00
24	.208247685858424691E+00	.360658975441641216E+00
25	.208234493266769126E+00	.360645782849985651E+00
26	.208218494254703280E+00	.360650069772348390E+00
27	.208214671061923048E+00	.360664338122051344E+00
28	.208224025680431426E+00	.360673692740559722E+00
29	.208235515052798240E+00	.360670614172512493E+00
30	.208238293126578344E+00	.360660246260017969E+00
31	.208231420764804896E+00	.360653373898244521E+00
32	.208222893264856416E+00	.360655658834969172E+00
33	.208220811515289954E+00	.360663428030119844E+00
34	.208226007772391891E+00	.360668624287221781E+00
35	.208232510123931555E+00	.360666881987377825E+00
36	.208234110147166632E+00	.360660910619371225E+00
37	.208230086305131882E+00	.360656886777336475E+00
38	.208225015312392014E+00	.360658245545745947E+00
39	.208223759122058191E+00	.360662933711895749E+00
40	.208226938367947760E+00	.360666112957785317E+00
41	.208230969150457317E+00	.360665032912867016E+00
42	.208231973383941336E+00	.360661285062481995E+00
43	.208229417945309924E+00	.360658729623850583E+00
44	.208226161223848456E+00	.360659602254736157E+00
45	.208225345822357057E+00	.360662645379530624E+00
46	.208227430555903794E+00	.360664730113077361E+00
47	.208230099400129173E+00	.360664014998422446E+00
48	.208230770499368398E+00	.360661510421964739E+00
49	.208229047603744655E+00	.360659787526340995E+00
50	.208226833233198145E+00	.360660380865140676E+00

PUV (COS THETA) = .234958820650612321E+00
QUV (COS THETA) = .639253880933468677E+00

	MU = -.50	NU = -1.50	THETA = 30.0 DEGREES
K	PSP		QSP
1	-.4999999999999999360E+00		.866025403784437538E+00
2	0.		.173205080756887508E+01
3	.4999999999999999701-100		.173205080756887508E+01
4	.5833333333333332984-100		.173205080756887508E+01
5	.541666666666666342-100		.173205080756887508E+01
6	.491666666666666372-100		.173205080756887508E+01
7	.474999999999999716-100		.173205080756887508E+01
8	.486904761904761613-100		.173205080756887508E+01
9	.504761904761904460-100		.173205080756887508E+01
10	.511706349206348900-100		.173205080756887508E+01
11	.506150793650793348-100		.173205080756887508E+01
12	.497059884559884262-100		.173205080756887508E+01
13	.493272005772005476-100		.173205080756887508E+01
14	.496477133977133679-100		.173205080756887508E+01
15	.501971639471639171-100		.173205080756887508E+01
16	.504352591852591550-100		.173205080756887508E+01
17	.502269258519258218-100		.173205080756887508E+01
18	.498592787931022926-100		.173205080756887508E+01
19	.496958801002918352-100		.173205080756887508E+01
20	.498420789307011918-100		.173205080756887508E+01
21	.501052368254380338-100		.173205080756887508E+01
22	.502242844444856528-100		.173205080756887508E+01
23	.501160593362605446-100		.173205080756887508E+01
24	.499184308777625210-100		.173205080756887508E+01
25	.498278511676175935-100		.173205080756887508E+01
26	.499111845009509268-100		.173205080756887508E+01
27	.500650306547970806-100		.173205080756887508E+01
28	.501362557260221517-100		.173205080756887508E+01
29	.500701181598845856-100		.173205080756887508E+01
30	.499469654505249798-100		.173205080756887508E+01
31	.498894941861571638-100		.173205080756887508E+01
32	.499432576270173788-100		.173205080756887508E+01
33	.500440640786302819-100		.173205080756887508E+01
34	.500914125634787668-100		.173205080756887508E+01
35	.500468492836213693-100		.173205080756887508E+01
36	.499628156701759912-100		.173205080756887508E+01
37	.499231331304934515-100		.173205080756887508E+01
38	.499606706680309890-100		.173205080756887508E+01
39	.500317944233652706-100		.173205080756887508E+01
40	.500655326149981991-100		.173205080756887508E+01
41	.500334813329469171-100		.173205080756887508E+01
42	.499725057231908195-100		.173205080756887508E+01
43	.499434697185450588-100		.173205080756887508E+01
44	.499711552113468307-100		.173205080756887508E+01
45	.500240093339683951-100		.173205080756887508E+01
46	.500492618592209204-100		.173205080756887508E+01
47	.500251072698489397-100		.173205080756887508E+01
48	.499788538008387640-100		.173205080756887508E+01
49	.499566906802713881-100		.173205080756887508E+01
50	.499779491836727486-100		.173205080756887508E+01

PUV (COS THETA) = .563940766730145087-100
 QUV (COS THETA) = .306998012383946154E+01

MU = -.50 NU = -.40 THETA = 30.0 DEGREES

K	PSP	QSP
1	.523359562429437657E+00	.998629534754572595E+01
2	.523359562429437657E+00	.998629534754572595E+01
3	.523359562429437657E+00	.998629534754572595E+01
4	.523359562429437657E+00	.998629534754572595E+01
5	.523359562429437657E+00	.998629534754572595E+01
6	.523359562429437657E+00	.998629534754572595E+01
7	.523359562429437657E+00	.998629534754572595E+01
8	.523359562429437657E+00	.998629534754572595E+01
9	.523359562429437657E+00	.998629534754572595E+01
10	.523359562429437657E+00	.998629534754572595E+01
11	.523359562429437657E+00	.998629534754572595E+01
12	.523359562429437657E+00	.998629534754572595E+01
13	.523359562429437657E+00	.998629534754572595E+01
14	.523359562429437657E+00	.998629534754572595E+01
15	.523359562429437657E+00	.998629534754572595E+01
16	.523359562429437657E+00	.998629534754572595E+01
17	.523359562429437657E+00	.998629534754572595E+01
18	.523359562429437657E+00	.998629534754572595E+01
19	.523359562429437657E+00	.998629534754572595E+01
20	.523359562429437657E+00	.998629534754572595E+01
21	.523359562429437657E+00	.998629534754572595E+01
22	.523359562429437657E+00	.998629534754572595E+01
23	.523359562429437657E+00	.998629534754572595E+01
24	.523359562429437657E+00	.998629534754572595E+01
25	.523359562429437657E+00	.998629534754572595E+01
26	.523359562429437657E+00	.998629534754572595E+01
27	.523359562429437657E+00	.998629534754572595E+01
28	.523359562429437657E+00	.998629534754572595E+01
29	.523359562429437657E+00	.998629534754572595E+01
30	.523359562429437657E+00	.998629534754572595E+01
31	.523359562429437657E+00	.998629534754572595E+01
32	.523359562429437657E+00	.998629534754572595E+01
33	.523359562429437657E+00	.998629534754572595E+01
34	.523359562429437657E+00	.998629534754572595E+01
35	.523359562429437657E+00	.998629534754572595E+01
36	.523359562429437657E+00	.998629534754572595E+01
37	.523359562429437657E+00	.998629534754572595E+01
38	.523359562429437657E+00	.998629534754572595E+01
39	.523359562429437657E+00	.998629534754572595E+01
40	.523359562429437657E+00	.998629534754572595E+01
41	.523359562429437657E+00	.998629534754572595E+01
42	.523359562429437657E+00	.998629534754572595E+01
43	.523359562429437657E+00	.998629534754572595E+01
44	.523359562429437657E+00	.998629534754572595E+01
45	.523359562429437657E+00	.998629534754572595E+01
46	.523359562429437657E+00	.998629534754572595E+01
47	.523359562429437657E+00	.998629534754572595E+01
48	.523359562429437657E+00	.998629534754572595E+01
49	.523359562429437657E+00	.998629534754572595E+01
50	.523359562429437657E+00	.998629534754572595E+01

PUV (COS THETA) = .590548027145600779E+00
QUV (COS THETA) = .177002476450372605E+02

MU = -.50 NU = .05 THETA = 30.0 DEGREES

K	PSP	QSP
1	.516391535825313189E+00	.174330860885125786E+01
2	.516391535825313189E+00	.174330860885125786E+01
3	.516391535825313189E+00	.174330860885125786E+01
4	.516391535825313189E+00	.174330860885125786E+01
5	.516391535825313189E+00	.174330860885125786E+01
6	.516391535825313189E+00	.174330860885125786E+01
7	.516391535825313189E+00	.174330860885125786E+01
8	.516391535825313189E+00	.174330860885125786E+01
9	.516391535825313189E+00	.174330860885125786E+01
10	.516391535825313189E+00	.174330860885125786E+01
11	.516391535825313189E+00	.174330860885125786E+01
12	.516391535825313189E+00	.174330860885125786E+01
13	.516391535825313189E+00	.174330860885125786E+01
14	.516391535825313189E+00	.174330860885125786E+01
15	.516391535825313189E+00	.174330860885125786E+01
16	.516391535825313189E+00	.174330860885125786E+01
17	.516391535825313189E+00	.174330860885125786E+01
18	.516391535825313189E+00	.174330860885125786E+01
19	.516391535825313189E+00	.174330860885125786E+01
20	.516391535825313189E+00	.174330860885125786E+01
21	.516391535825313189E+00	.174330860885125786E+01
22	.516391535825313189E+00	.174330860885125786E+01
23	.516391535825313189E+00	.174330860885125786E+01
24	.516391535825313189E+00	.174330860885125786E+01
25	.516391535825313189E+00	.174330860885125786E+01
26	.516391535825313189E+00	.174330860885125786E+01
27	.516391535825313189E+00	.174330860885125786E+01
28	.516391535825313189E+00	.174330860885125786E+01
29	.516391535825313189E+00	.174330860885125786E+01
30	.516391535825313189E+00	.174330860885125786E+01
31	.516391535825313189E+00	.174330860885125786E+01
32	.516391535825313189E+00	.174330860885125786E+01
33	.516391535825313189E+00	.174330860885125786E+01
34	.516391535825313189E+00	.174330860885125786E+01
35	.516391535825313189E+00	.174330860885125786E+01
36	.516391535825313189E+00	.174330860885125786E+01
37	.516391535825313189E+00	.174330860885125786E+01
38	.516391535825313189E+00	.174330860885125786E+01
39	.516391535825313189E+00	.174330860885125786E+01
40	.516391535825313189E+00	.174330860885125786E+01
41	.516391535825313189E+00	.174330860885125786E+01
42	.516391535825313189E+00	.174330860885125786E+01
43	.516391535825313189E+00	.174330860885125786E+01
44	.516391535825313189E+00	.174330860885125786E+01
45	.516391535825313189E+00	.174330860885125786E+01
46	.516391535825313189E+00	.174330860885125786E+01
47	.516391535825313189E+00	.174330860885125786E+01
48	.516391535825313189E+00	.174330860885125786E+01
49	.516391535825313189E+00	.174330860885125786E+01
50	.516391535825313189E+00	.174330860885125786E+01

PUV (COS THETA) = .582685451089739438E+00

QUV (COS THETA) = .308993405707514995E+01

MU = -.50 NU = 1.50

THETA = 30.0 DEGREES

K	PSP	QSP
1	.433012701892218769E+00	.249999999999999680E+00
2	.433012701892218769E+00	.249999999999999680E+00
3	.433012701892218769E+00	.249999999999999680E+00
4	.433012701892218769E+00	.249999999999999680E+00
5	.433012701892218769E+00	.249999999999999680E+00
6	.433012701892218769E+00	.249999999999999680E+00
7	.433012701892218769E+00	.249999999999999680E+00
8	.433012701892218769E+00	.249999999999999680E+00
9	.433012701892218769E+00	.249999999999999680E+00
10	.433012701892218769E+00	.249999999999999680E+00
11	.433012701892218769E+00	.249999999999999680E+00
12	.433012701892218769E+00	.249999999999999680E+00
13	.433012701892218769E+00	.249999999999999680E+00
14	.433012701892218769E+00	.249999999999999680E+00
15	.433012701892218769E+00	.249999999999999680E+00
16	.433012701892218769E+00	.249999999999999680E+00
17	.433012701892218769E+00	.249999999999999680E+00
18	.433012701892218769E+00	.249999999999999680E+00
19	.433012701892218769E+00	.249999999999999680E+00
20	.433012701892218769E+00	.249999999999999680E+00
21	.433012701892218769E+00	.249999999999999680E+00
22	.433012701892218769E+00	.249999999999999680E+00
23	.433012701892218769E+00	.249999999999999680E+00
24	.433012701892218769E+00	.249999999999999680E+00
25	.433012701892218769E+00	.249999999999999680E+00
26	.433012701892218769E+00	.249999999999999680E+00
27	.433012701892218769E+00	.249999999999999680E+00
28	.433012701892218769E+00	.249999999999999680E+00
29	.433012701892218769E+00	.249999999999999680E+00
30	.433012701892218769E+00	.249999999999999680E+00
31	.433012701892218769E+00	.249999999999999680E+00
32	.433012701892218769E+00	.249999999999999680E+00
33	.433012701892218769E+00	.249999999999999680E+00
34	.433012701892218769E+00	.249999999999999680E+00
35	.433012701892218769E+00	.249999999999999680E+00
36	.433012701892218769E+00	.249999999999999680E+00
37	.433012701892218769E+00	.249999999999999680E+00
38	.433012701892218769E+00	.249999999999999680E+00
39	.433012701892218769E+00	.249999999999999680E+00
40	.433012701892218769E+00	.249999999999999680E+00
41	.433012701892218769E+00	.249999999999999680E+00
42	.433012701892218769E+00	.249999999999999680E+00
43	.433012701892218769E+00	.249999999999999680E+00
44	.433012701892218769E+00	.249999999999999680E+00
45	.433012701892218769E+00	.249999999999999680E+00
46	.433012701892218769E+00	.249999999999999680E+00
47	.433012701892218769E+00	.249999999999999680E+00
48	.433012701892218769E+00	.249999999999999680E+00
49	.433012701892218769E+00	.249999999999999680E+00
50	.433012701892218769E+00	.249999999999999680E+00

PUV (COS THETA) = .488602511902919296E+00
QUV (COS THETA) = .443113462726378440E+00

MU = 0.00 NU = -1.50 THETA = 30.0 DEGREES

K	PSP	QSP
1	.917489626359302134-100	-.342411790099030102E-99
2	.626657068657749328E+00	.626657068657749328E+00
3	.947668121875589847E+00	.540642416186564807E+00
4	.100142727967008017E+01	.340010507925414482E+00
5	.894332565788140590E+00	.232915794043474900E+00
6	.779125962216308178E+00	.263785310433279892E+00
7	.753658611194719059E+00	.358830758380041632E+00
8	.812882824648883851E+00	.418054971834206424E+00
9	.883310713573082701E+00	.399183875872338531E+00
10	.900019496455986565E+00	.336825849220704133E+00
11	.859061980519042225E+00	.295868333283759793E+00
12	.808326403493899003E+00	.309462890175173943E+00
13	.795890473610275832E+00	.355874412340219731E+00
14	.827198110728573917E+00	.387182049458517815E+00
15	.866851604759261251E+00	.376556927755922800E+00
16	.876755736060608748E+00	.339594206534460678E+00
17	.851416494041668073E+00	.314254964515520003E+00
18	.818870380602645191E+00	.322975669328277894E+00
19	.810641284149430024E+00	.353687075392061741E+00
20	.831923879869961048E+00	.374969671112592765E+00
21	.859523691273615018E+00	.367574323935732393E+00
22	.866562477068733908E+00	.341305217724754597E+00
23	.848216668143864184E+00	.322959408799884872E+00
24	.824257810722514346E+00	.329379165297507972E+00
25	.818108451577372654E+00	.352328886061165086E+00
26	.834229798580473077E+00	.368450233064265509E+00
27	.855396491543588001E+00	.362778634778361337E+00
28	.860856010833758577E+00	.342403431402542377E+00
29	.846477962186382571E+00	.328025382755166371E+00
30	.827520497781139502E+00	.333105020033093005E+00
31	.822611641411694885E+00	.351425121410918418E+00
32	.835586660439964369E+00	.364400140439187903E+00
33	.852752517836254587E+00	.359800562812464124E+00
34	.857211625407616772E+00	.343158946799725384E+00
35	.845390140500710601E+00	.331337461892819213E+00
36	.829706468587040199E+00	.335539889116441878E+00
37	.825621609267887930E+00	.350784791637289358E+00
38	.836477935077645114E+00	.361641117447046543E+00
39	.850915051408049128E+00	.35772703785280614E+00
40	.854683622409516442E+00	.343708205335873870E+00
41	.844646743136148566E+00	.333671326062505994E+00
42	.831272606769438022E+00	.337254915101429794E+00
43	.827774861544599941E+00	.350308677992456940E+00
44	.837107323426516310E+00	.359641139874373310E+00
45	.849564283016632612E+00	.356303307612054517E+00
46	.852827519785210189E+00	.344124742194596117E+00
47	.844107079359209491E+00	.335404301768595419E+00
48	.832449567782752861E+00	.338527922581263437E+00
49	.829391368549605483E+00	.349941277499037630E+00
50	.837575122762385681E+00	.358125031711817828E+00

PUV (COS THETA) = .945102319402542450E+00
 QUV (COS THETA) = .634760091563271556E+00

MU = 0.00 NU = -.40

THETA = 30.0 DEGREES

K	PSP	OSP
1	.483718232381312815E+00	.148873164043148002E+01
2	.901301469532766985E+00	.157749169753143057E+01
3	.106453580259884789E+01	.139620160437844438E+01
4	.101184759965827758E+01	.123404398958490236E+01
5	.883714269611804624E+00	.120680840951489393E+01
6	.812560358102438076E+00	.128583283411624536E+01
7	.840213206923795422E+00	.137093955168973297E+01
8	.915768137822808203E+00	.138699924801884325E+01
9	.961232297012808759E+00	.133650618384624197E+01
10	.942492101917731445E+00	.127882979391867008E+01
11	.888928521147301482E+00	.126744450335937297E+01
12	.855527818445381587E+00	.130453974178421445E+01
13	.869699333613550190E+00	.134815518071411635E+01
14	.911185587999326050E+00	.135697335630292474E+01
15	.937581474710436404E+00	.132765775418155200E+01
16	.926187890244118656E+00	.129259190684015017E+01
17	.892335226762152656E+00	.128539630108705116E+01
18	.870515648820511814E+00	.130962939741732848E+01
19	.880041799598826729E+00	.133894787484039752E+01
20	.908633315734325235E+00	.134502518920309204E+01
21	.927228835261494243E+00	.132437277249646751E+01
22	.919044204028206846E+00	.129918306769192849E+01
23	.894298580405261981E+00	.129392322301825486E+01
24	.878097049796891709E+00	.131191684567129911E+01
25	.885271342566936333E+00	.133399704842054089E+01
26	.907082995908354304E+00	.133863325845913115E+01
27	.921436613722193855E+00	.132269195088200570E+01
28	.915050637417194301E+00	.130303793673931732E+01
29	.895550990063378119E+00	.129889315874401221E+01
30	.882666917421488119E+00	.131320237106206366E+01
31	.888420662420054386E+00	.133091057732125551E+01
32	.906051454477467708E+00	.133465811786049843E+01
33	.917738936345897421E+00	.132167785423058111E+01
34	.912503517325865415E+00	.130556489130241440E+01
35	.896414694663373998E+00	.130214510647595650E+01
36	.885720433174596434E+00	.131402228712224873E+01
37	.890523193520703898E+00	.132880366357246180E+01
38	.905318067229576274E+00	.133194841105836192E+01
39	.915174693816573956E+00	.132100151821684414E+01
40	.910738541569839300E+00	.130734844547866588E+01
41	.897044980908125618E+00	.130443778930739214E+01
42	.887904305872533706E+00	.131458953739590730E+01
43	.892025848343324674E+00	.132727434080602928E+01
44	.904770694948813094E+00	.132998334157950375E+01
45	.913292382254860403E+00	.132051904900994654E+01
46	.909443781354262153E+00	.130867427337701745E+01
47	.897524710834378241E+00	.130614079672900841E+01
48	.889543496111838489E+00	.131500483368340006E+01
49	.893153059828920310E+00	.132611392851184789E+01
50	.904346850422454578E+00	.132849324215246377E+01

PUV (COS THETA) = .102044614584513939E+01
QUV (COS THETA) = .235469296295508863E+01

MU = 0.00 NU = .05

THETA = 30.0 DEGREES

K	PSP	QSP
1	.572249628146135628E+00	.933826521149093918E+00
2	.943083311798271430E+00	.924115899393766413E+00
3	.104980823921997599E+01	.727553035254437101E+00
4	.966136593978889083E+00	.591013319802927867E+00
5	.841456810603975845E+00	.594278174831646561E+00
6	.792720944061401262E+00	.684038474935372513E+00
7	.837906164610877192E+00	.757774053322321263E+00
8	.912864500982501054E+00	.755811200205137189E+00
9	.944445999289034612E+00	.697645316086848724E+00
10	.913494866084259005E+00	.647137657148087749E+00
11	.859905352810660753E+00	.648540947939601095E+00
12	.836545587823463039E+00	.691564282997605580E+00
13	.860082559507169495E+00	.729973129949564509E+00
14	.901784331829888204E+00	.728881130610225070E+00
15	.920319021779897419E+00	.694744478278641805E+00
16	.901330534171469489E+00	.663758082738804572E+00
17	.867199835350840669E+00	.664651826541223099E+00
18	.851838137640152650E+00	.692944554144815117E+00
19	.867751437663766077E+00	.718912699636135296E+00
20	.896637786226762521E+00	.718156283978244476E+00
21	.909754092912764059E+00	.693999052191680853E+00
22	.896058742299643486E+00	.671650271187839741E+00
23	.871019739278238919E+00	.672305940557128876E+00
24	.859576117861378387E+00	.693382469313491555E+00
25	.871596157141802618E+00	.712997390692606159E+00
26	.893692217072239281E+00	.712418785000279690E+00
27	.903841529525588778E+00	.693726076810443777E+00
28	.893131600345619522E+00	.676249060809081091E+00
29	.873359442659461530E+00	.676766812979504421E+00
30	.864241406127252448E+00	.693560147202316581E+00
31	.873898746784743626E+00	.709319494847421014E+00
32	.891789308994330719E+00	.708851013988572107E+00
33	.900066316590923118E+00	.693606662373690460E+00
34	.891273178076470877E+00	.679257564453724432E+00
35	.874937208353460890E+00	.679685336875647783E+00
36	.867359181555885449E+00	.693642326005003592E+00
37	.875430083612281714E+00	.706812841142385055E+00
38	.890460033729010288E+00	.706419268047439609E+00
39	.897447942683905395E+00	.693549140780061311E+00
40	.889989638171110189E+00	.681378293977736189E+00
41	.876072340122301150E+00	.681742731252897301E+00
42	.869589281080036237E+00	.693683040649376771E+00
43	.876521422581137450E+00	.704995267453364286E+00
44	.889479451380565674E+00	.704655949527531193E+00
45	.895525692668211642E+00	.693520158354011402E+00
46	.889050367440208341E+00	.682953387955986601E+00
47	.876927896517358808E+00	.683270826028797088E+00
48	.871263324515495587E+00	.693703669823669032E+00
49	.877338318212939158E+00	.703617158538139037E+00
50	.888726460743439680E+00	.703318949531016679E+00

PUV (COS THETA) = .100282042354942522E+01

QUV (COS THETA) = .124660038051107279E+01

MU = 0.00 NU = 1.50 THETA = 30.0 DEGREES

K	PSP	QSP
1	.642022106435681038E+00	.172029304942369042E+00
2	.837852440391227703E+00	-.238010290131776227E-01
3	.790813177321048668E+00	-.199353948741684157E+00
4	.659148487524668768E+00	-.234633396044318433E+00
5	.581839490941143632E+00	-.157324399460793297E+00
6	.605487745461190670E+00	-.690679120816573948E-01
7	.681333164302635586E+00	-.487451933534919712E-01
8	.730025690989281489E+00	-.974377200401378740E-01
9	.714152347250522818E+00	-.156677845359190553E+00
10	.660746476697740480E+00	-.170987905244889657E+00
11	.625152999069583452E+00	-.135394427616732629E+00
12	.637110623957682655E+00	-.907679639964962945E-01
13	.678350257749597482E+00	-.797178374257974779E-01
14	.706411017666753389E+00	-.107778597342953385E+00
15	.696816390468573001E+00	-.143586233526244815E+00
16	.663220402402484865E+00	-.152588251397478767E+00
17	.640056810682505013E+00	-.129424659677498915E+00
18	.648069351965898728E+00	-.995214485101304327E-01
19	.676415104218300102E+00	-.919262270852468070E-01
20	.696138509773027185E+00	-.111649632639973890E+00
21	.689259696382342815E+00	-.137321713709793100E+00
22	.664743656230263911E+00	-.143890766870151621E+00
23	.647569953242448147E+00	-.126717063882335857E+00
24	.653596325204687006E+00	-.104226337533951885E+00
25	.675194991493579786E+00	-.984389923442537508E-01
26	.690403139973064413E+00	-.113647140823738378E+00
27	.685041112098789741E+00	-.133658501282132000E+00
28	.665738966522542253E+00	-.138830495601475482E+00
29	.652092436514014426E+00	-.125183965592947655E+00
30	.656922117780726066E+00	-.107159349721216200E+00
31	.674369382675316123E+00	-.102484369182578917E+00
32	.686745264266555798E+00	-.114860250773818592E+00
33	.682351731806537174E+00	-.131257137139311174E+00
34	.666433975237438855E+00	-.135522287157316267E+00
35	.655112192279197336E+00	-.124200504199074749E+00
36	.659141850796739440E+00	-.109161613874454937E+00
37	.673776735843998303E+00	-.105240208244719612E+00
38	.684209942324576924E+00	-.115673414725298233E+00
39	.680488478460627951E+00	-.129562106944087393E+00
40	.666945049228516008E+00	-.133191057869579848E+00
41	.657271056035583868E+00	-.123517064676647708E+00
42	.660728129804319181E+00	-.110615089726213901E+00
43	.673331641860201558E+00	-.107237988849044299E+00
44	.682349445551023539E+00	-.116255792539866281E+00
45	.679121678747321806E+00	-.128301982246265350E+00
46	.667336062648046971E+00	-.131459928562369280E+00
47	.658891066579714491E+00	-.123014932494036800E+00
48	.661918059698237916E+00	-.111718040381546017E+00
49	.672985483689693731E+00	-.108752533060742599E+00
50	.680926107701536620E+00	-.116693157072585489E+00

PUV (COS THETA) = .768342834261849182E+00
QUV (COS THETA) = -.206833235627626403E+00

MU = .50 NU = -1.50

THETA = 30.0 DEGREES

K	PSP	QSP
1	0.	.9999999999998720E+00
2	.866025403784437538E+00	.1499999999999808E+01
3	.173205080756887508E+01	.99999999999998720E+00
4	.173205080756887508E+01	.252435489670723778E-28
5	.866025403784437538E+00	-.4999999999999360E+00
6	.252435489670723778E-28	-.353409685539013289E-27
7	-.277679038637795768E-27	.99999999999998720E+00
8	.866025403784437538E+00	.1499999999999808E+01
9	.173205080756887508E+01	.99999999999998720E+00
10	.173205080756887508E+01	-.100974195868289511E-27
11	.866025403784437538E+00	-.4999999999999360E+00
12	.113595970351825700E-27	-.833037115913388467E-27
13	-.492249204857910591E-27	.99999999999998720E+00
14	.866025403784437538E+00	.1499999999999808E+01
15	.173205080756887508E+01	.99999999999998720E+00
16	.173205080756887508E+01	.227191940703651400E-27
17	.866025403784437538E+00	-.4999999999999360E+00
18	-.126217744835361849E-27	-.492249204857911367E-27
19	-.934011311781676943E-27	.99999999999998720E+00
20	.866025403784437538E+00	.1499999999999808E+01
21	.173205080756887508E+01	.99999999999998720E+00
22	.173205080756887508E+01	.833037115913388467E-27
23	.866025403784437538E+00	-.4999999999999360E+00
24	.277679038637796156E-27	-.567979851759128500E-27
25	-.934011311781676426E-27	.99999999999998720E+00
26	.866025403784437538E+00	.1499999999999808E+01
27	.173205080756887508E+01	.99999999999998720E+00
28	.173205080756887508E+01	.118644680145240176E-26
29	.866025403784437538E+00	-.4999999999999360E+00
30	-.126217744835361889E-27	-.593223400726200878E-27
31	-.133790809525483447E-26	.99999999999998720E+00
32	.866025403784437538E+00	.1499999999999808E+01
33	.173205080756887508E+01	.99999999999998720E+00
34	.173205080756887508E+01	.156510003595848742E-26
35	.866025403784437538E+00	-.4999999999999360E+00
36	-.169131778079384931E-26	.164083068285970456E-27
37	-.330690491468647942E-26	.99999999999998720E+00
38	.866025403784437538E+00	.1499999999999808E+01
39	.173205080756887508E+01	.99999999999998720E+00
40	.173205080756887508E+01	.292825168018039582E-26
41	.866025403784437538E+00	-.4999999999999360E+00
42	-.171656132976092169E-26	-.130004277180422746E-26
43	-.413994203059986685E-26	.99999999999998720E+00
44	.866025403784437538E+00	.1499999999999808E+01
45	.173205080756887508E+01	.99999999999998720E+00
46	.173205080756887508E+01	.757306469012171333E-28
47	.866025403784437538E+00	-.4999999999999360E+00
48	-.257484199464138253E-26	-.275154683741088918E-26
49	-.499822269548032770E-26	.99999999999998720E+00
50	.866025403784437538E+00	.1499999999999808E+01

PUV (COS THETA) = .977205023805838592E+00

QUV (COS THETA) = .265868077635827064E+01

MU = .50 NU = -.40 THETA = 30.0 DEGREES

K	PSP	QSP
1	.544639035015026385E+00	.838670567945422956E+00
2	.154326856976959898E+01	.786334611702479190E+00
3	.199725906950914519E+01	-.104671912485887531E+00
4	.145262003449411881E+01	-.943342480431310487E+00
5	.453990499739546210E+00	-.891006524188366722E+00
6	-.126217744835361889E-28	-.391275008989621855E-27
7	.544639035015026385E+00	.838670567945422956E+00
8	.154326856976959898E+01	.786334611702479190E+00
9	.199725906950914519E+01	-.104671912485887531E+00
10	.145262003449411881E+01	-.943342480431310487E+00
11	.453990499739546210E+00	-.891006524188366722E+00
12	.252435489670723778E-27	-.164083068285970456E-26
13	.544639035015026385E+00	.838670567945422956E+00
14	.154326856976959898E+01	.786334611702479190E+00
15	.199725906950914519E+01	-.104671912485887531E+00
16	.145262003449411881E+01	-.943342480431310487E+00
17	.453990499739546210E+00	-.891006524188366722E+00
18	.555358077275592311E-27	-.209521456426700736E-26
19	.544639035015026385E+00	.838670567945422956E+00
20	.154326856976959898E+01	.786334611702479190E+00
21	.199725906950914519E+01	-.104671912485887531E+00
22	.145262003449411881E+01	-.943342480431310487E+00
23	.453990499739546210E+00	-.891006524188366722E+00
24	.908767762814605600E-27	-.315544362088404722E-26
25	.544639035015026385E+00	.838670567945422956E+00
26	.154326856976959898E+01	.786334611702479190E+00
27	.199725906950914519E+01	-.104671912485887531E+00
28	.145262003449411881E+01	-.943342480431310487E+00
29	.453990499739546210E+00	-.891006524188366722E+00
30	.164083068285970456E-26	-.283989925879564250E-26
31	.544639035015026385E+00	.838670567945422956E+00
32	.154326856976959898E+01	.786334611702479190E+00
33	.199725906950914519E+01	-.104671912485887531E+00
34	.145262003449411881E+01	-.943342480431310487E+00
35	.453990499739546210E+00	-.891006524188366722E+00
36	.194375327046457309E-26	-.386226299196207380E-26
37	.544639035015026385E+00	.838670567945422956E+00
38	.154326856976959898E+01	.786334611702479190E+00
39	.199725906950914519E+01	-.104671912485887531E+00
40	.145262003449411881E+01	-.943342480431310487E+00
41	.453990499739546210E+00	-.891006524188366722E+00
42	.132528632077129983E-26	-.959254860748750355E-27
43	.544639035015026385E+00	.838670567945422956E+00
44	.154326856976959898E+01	.786334611702479190E+00
45	.199725906950914519E+01	-.104671912485887531E+00
46	.145262003449411881E+01	-.943342480431310487E+00
47	.453990499739546210E+00	-.891006524188366722E+00
48	.116120325248532938E-26	-.260008554360845491E-26
49	.544639035015026385E+00	.838670567945422956E+00
50	.154326856976959898E+01	.786334611702479190E+00

PUV (COS THETA) = .174139210336130303E+01
QUV (COS THETA) = .139374181061235289E+01

MU = .50 NU = .05 THETA = 30.0 DEGREES

K	PSP	QSP
1	.725374371012286709E+00	.688354575693753103E+00
2	.168419410588047853E+01	.404339230989830849E+00
3	.191763946973638364E+01	-.568030689407844508E+00
4	.119226509872409694E+01	-.125638526510159761E+01
5	.233445363855905113E+00	-.972369920397675357E+00
6	-.435451219681998517E-27	-.504870979341447555E-27
7	.725374371012286709E+00	.688354575693753103E+00
8	.168419410588047853E+01	.404339230989830849E+00
9	.191763946973638364E+01	-.568030689407844508E+00
10	.119226509872409694E+01	-.125638526510159761E+01
11	.233445363855905113E+00	-.972369920397675357E+00
12	-.580601626242664689E-27	-.757306469012171333E-27
13	.725374371012286709E+00	.688354575693753103E+00
14	.168419410588047853E+01	.404339230989830849E+00
15	.191763946973638364E+01	-.568030689407844508E+00
16	.119226509872409694E+01	-.125638526510159761E+01
17	.233445363855905113E+00	-.972369920397675357E+00
18	-.176704842769506644E-26	-.143888229112312553E-26
19	.725374371012286709E+00	.688354575693753103E+00
20	.168419410588047853E+01	.404339230989830849E+00
21	.191763946973638364E+01	-.568030689407844508E+00
22	.119226509872409694E+01	-.125638526510159761E+01
23	.233445363855905113E+00	-.972369920397675357E+00
24	-.277047949913619346E-26	-.229716295600358638E-26
25	.725374371012286709E+00	.688354575693753103E+00
26	.168419410588047853E+01	.404339230989830849E+00
27	.191763946973638364E+01	-.568030689407844508E+00
28	.119226509872409694E+01	-.125638526510159761E+01
29	.233445363855905113E+00	-.972369920397675357E+00
30	-.283989925879564250E-26	-.381177589402792904E-26
31	.725374371012286709E+00	.688354575693753103E+00
32	.168419410588047853E+01	.404339230989830849E+00
33	.191763946973638364E+01	-.568030689407844508E+00
34	.119226509872409694E+01	-.125638526510159761E+01
35	.233445363855905113E+00	-.972369920397675357E+00
36	-.381808678126969714E-26	-.482151785271082415E-26
37	.725374371012286709E+00	.688354575693753103E+00
38	.168419410588047853E+01	.404339230989830849E+00
39	.191763946973638364E+01	-.568030689407844508E+00
40	.119226509872409694E+01	-.125638526510159761E+01
41	.233445363855905113E+00	-.972369920397675357E+00
42	-.425353800095169565E-26	-.633613079073516682E-26
43	.725374371012286709E+00	.688354575693753103E+00
44	.168419410588047853E+01	.404339230989830849E+00
45	.191763946973638364E+01	-.568030689407844508E+00
46	.119226509872409694E+01	-.125638526510159761E+01
47	.233445363855905113E+00	-.972369920397675357E+00
48	-.320593071881819198E-26	-.744684694528635144E-26
49	.725374371012286709E+00	.688354575693753103E+00
50	.168419410588047853E+01	.404339230989830849E+00

PUV (COS THETA) = .190040954242058588E+01
QUV (COS THETA) = .716672627040100654E+00

K	MU = .50 PSP	NU = 1.50 QSP	THETA = 30.0 DEGREES
1	.99999999999998720E+00	.757306469012170364E-28	
2	.1499999999999808E+01	-.866025403784437538E+00	
3	.99999999999998720E+00	-.173205080756887508E+01	
4	.201948391736579022E-27	-.173205080756887508E+01	
5	-.49999999999999360E+00	-.866025403784437538E+00	
6	-.315544362088404722E-27	.252435489670723778E-28	
7	.99999999999998720E+00	.328166136571940523E-27	
8	.1499999999999808E+01	-.866025403784437538E+00	
9	.99999999999998720E+00	-.173205080756887508E+01	
10	.403896783473158044E-27	-.173205080756887508E+01	
11	-.49999999999999360E+00	-.866025403784437538E+00	
12	-.643710498660345633E-27	-.252435489670723778E-27	
13	.99999999999998720E+00	.151461293802433750E-27	
14	.1499999999999808E+01	-.866025403784437538E+00	
15	.99999999999998720E+00	-.173205080756887508E+01	
16	.201948391736579022E-27	-.173205080756887508E+01	
17	-.49999999999999360E+00	-.866025403784437538E+00	
18	-.122431212490301032E-26	.328166136571940911E-27	
19	.99999999999998720E+00	.113595970351825597E-26	
20	.1499999999999808E+01	-.866025403784437538E+00	
21	.99999999999998720E+00	-.173205080756887508E+01	
22	.201948391736579022E-27	-.173205080756887508E+01	
23	-.49999999999999360E+00	-.866025403784437538E+00	
24	-.193113149598103690E-26	.681575822110954200E-27	
25	.99999999999998720E+00	.148936938905726925E-26	
26	.1499999999999808E+01	-.866025403784437538E+00	
27	.99999999999998720E+00	-.173205080756887508E+01	
28	.126217744835361889E-27	-.173205080756887508E+01	
29	-.49999999999999360E+00	-.866025403784437538E+00	
30	-.200686214288225403E-26	.111071615455118462E-26	
31	.99999999999998720E+00	.272630328844381473E-26	
32	.1499999999999808E+01	-.866025403784437538E+00	
33	.99999999999998720E+00	-.173205080756887508E+01	
34	-.555358077275592311E-27	-.173205080756887508E+01	
35	-.49999999999999360E+00	-.866025403784437538E+00	
36	-.338263556158769862E-26	.272630328844381680E-26	
37	.99999999999998720E+00	.434189042233644691E-26	
38	.1499999999999808E+01	-.866025403784437538E+00	
39	.99999999999998720E+00	-.173205080756887508E+01	
40	-.555358077275592311E-27	-.173205080756887508E+01	
41	-.49999999999999360E+00	-.866025403784437538E+00	
42	-.478365252926021559E-26	.277679038637796156E-26	
43	.99999999999998720E+00	.358458395332427661E-26	
44	.1499999999999808E+01	-.866025403784437538E+00	
45	.99999999999998720E+00	-.173205080756887508E+01	
46	-.191850972149750071E-26	-.173205080756887508E+01	
47	-.49999999999999360E+00	-.866025403784437538E+00	
48	-.613418239899858780E-26	.206997101529993498E-26	
49	.99999999999998720E+00	.368555814919256509E-26	
50	.1499999999999808E+01	-.866025403784437538E+00	

PIV (COS THETA) = .169256875064326669E+01
 QUV (COS THETA) = -.153499006191973077E+01

	MU = 1.00	NU = -1.50	THETA = 30.0 DEGREES
K	PSP		QSP
1	.458744813179651092-100		.171205895049515060E-99
2	.128404421287136208E+01		.344058609884738085E+00
3	.304651721847128206E+01		-.141841439571518190E+01
4	.210573195706770136E+01		-.492947279028531258E+01
5	-.250253218580559515E+01		-.616425344587751225E+01
6	-.667721800131595250E+01		-.198956763036715490E+01
7	-.485630240327233055E+01		.480618189782630955E+01
8	.303162115623794069E+01		.691974464555551360E+01
9	.960511225893513772E+01		.346253542858316574E+00
10	.690664382334616377E+01		-.972456776138063860E+01
11	-.425518312218534476E+01		-.127153702774917514E+02
12	-.132247394844809157E+02		-.374581391519618043E+01
13	-.964940964293925416E+01		.959749870725448356E+01
14	.478446218423093526E+01		.134650430069990693E+02
15	.161490699506790776E+02		.210043524055092697E+01
16	.116971629307233777E+02		-.145143079484962967E+02
17	-.600792278010507000E+01		-.192583713666365894E+02
18	-.197670962617731023E+02		-.549919788496855717E+01
19	-.144387563083162818E+02		.143864375413314837E+02
20	.653710035846073500E+01		.200069013957453667E+02
21	.226905695077822159E+02		.385343224642388580E+01
22	.164858798293849142E+02		-.193027848785530413E+02
23	-.776048388102112145E+01		-.257995784541476185E+02
24	-.263080831078621465E+02		-.725197922730659346E+01
25	-.192270960522314879E+02		.191746242320445727E+02
26	.828960479981791393E+01		.265477020037199960E+02
27	.292312252560682462E+02		.560608154746966378E+01
28	.212739758906446323E+02		-.240907773727864710E+02
29	-.951294630347011067E+01		-.323401083121393248E+02
30	-.328485126180526948E+02		-.900454199755674062E+01
31	-.240150255812371049E+02		.239624804318400908E+02
32	.100420354930026864E+02		.330880424432600672E+02
33	.357714933211899700E+02		.735858461507278359E+01
34	.260617865845488105E+02		-.288785342526658239E+02
35	-.112653525699867471E+02		-.388803110448877670E+02
36	-.393886614382586794E+02		-.107570021766158347E+02
37	-.288027485126755739E+02		.287501627061604107E+02
38	.117944226084205117E+02		.396281419230462033E+02
39	.423115515641129800E+02		.911101296735373496E+01
40	.308494428631501429E+02		-.336661590665168762E+02
41	-.130177244196604406E+02		-.454203311141869394E+02
42	-.459286492620155805E+02		-.125094062718317995E+02
43	-.335903529813992478E+02		.335377423262664600E+02
44	.135467821076008393E+02		.461680996068807707E+02
45	.488514835571688979E+02		.108633981573127122E+02
46	.356370062628139997E+02		-.384537025006850782E+02
47	-.147700737937844664E+02		-.519602388946615882E+02
48	-.524685362428206577E+02		-.142617764456253969E+02
49	-.383778832760941133E+02		.383252563380191995E+02
50	.152991230824665860E+02		.527079668439157774E+02

PUV (COS THETA) = .172632117610853773E+02
 QUV (COS THETA) = .934224388058987776E+02

MU = 1.00 NU = -.40

THETA = 30.0 DEGREES

K	PSP	QSP
1	.697966A58638996821E+00	.628452182304411485E+00
2	.2646851915203A4340E+01	-.477895826748892819E-02
3	.330621233937490457E+01	-.310682586282114848E+01
4	.113167472873922047E+00	-.598185637424308958E+01
5	-.504463119669334992E+01	-.430598599729680318E+01
6	-.640653836762939488E+01	.210128348540353387E+01
7	-.700715814937718649E+00	.723882918789294411E+01
8	.767399068334997784E+01	.451772209642955912E+01
9	.973929737145194587E+01	-.519878193372218645E+01
10	.151895254164084976E+01	-.126004136640069362E+02
11	-.100740703602049736E+02	-.883361218431806506E+01
12	-.128429842137007813E+02	.419310330139032513E+01
13	-.210759155632982071E+01	.138592942672124732E+02
14	.127042422373985797E+02	.904663773030194484E+01
15	.161768451448643877E+02	-.729067446642411825E+01
16	.292617745455907570E+01	-.192216292444217389E+02
17	-.151046975834117967E+02	-.133630428042707498E+02
18	-.192810301328327556E+02	.628505705493000789E+01
19	-.351496873472032399E+01	.204808825004384981E+02
20	.177350733962494942E+02	.135763252663073233E+02
21	.226151586436168846E+02	-.938267072527891218E+01
22	.433363378184129352E+01	-.258434296467531638E+02
23	-.201356516824096159E+02	-.178928768466724881E+02
24	-.257195039759758281E+02	.837708278051943836E+01
25	-.492247118047054962E+01	.271028152189872032E+02
26	.227661072895875395E+02	.181062507133749387E+02
27	.290537361637662649E+02	-.114747174053124079E+02
28	.574116534857742675E+01	-.324654504502882291E+02
29	-.251667402897762156E+02	-.224228631390790310E+02
30	-.321581524016316397E+02	.104691447901283438E+02
31	-.633002226599230782E+01	.337248976209089156E+02
32	.277972350408346169E+02	.226362795435124783E+02
33	.354924352219128022E+02	-.135667909268520052E+02
34	.714873013361451158E+01	-.390875776187410508E+02
35	-.301978970090111233E+02	-.269529228722355151E+02
36	-.385968888768366575E+02	.125612271584011747E+02
37	-.773759702494758409E+01	.403470583460094752E+02
38	.328284137971896357E+02	.271663624322998591E+02
39	.419312001305265578E+02	-.156588802316660016E+02
40	.855631237385581825E+01	-.457097641457587982E+02
41	-.352290929189087573E+02	-.314830235585990435E+02
42	-.450356758984390048E+02	.146533219979978908E+02
43	-.914518501734778537E+01	.469692651391671698E+02
44	.378596233205332080E+02	.316964770925368949E+02
45	.483700046889919775E+02	-.177509795556155915E+02
46	.996390488212540711E+01	-.523319871474247411E+02
47	-.402603134268898098E+02	-.360131493909243446E+02
48	-.514744945989714737E+02	.167454250043974343E+02
49	-.105527811346181843E+02	.535915013072985069E+02
50	.428908528201712561E+02	.362266119971256837E+02

PUV (COS THETA) = .483971447812410585E+02

QUV (COS THETA) = .642099979395653848E+02

MU = 1.00 NU = .05 THETA = 30.0 DEGREES

K	PSP	OSP
1	.101062077464806388E+01	.548722309768178576E+00
2	.295584166507395871E+01	-.643310584991070075E+00
3	.286655392606633338E+01	-.405307177180721581E+01
4	-.112318773002931388E+01	-.621932474406924379E+01
5	-.595649672473083955E+01	-.325746918279232646E+01
6	-.577856521404204922E+01	.353746107080482009E+01
7	.118677411960816040E+01	.731933176192599338E+01
8	.890685255309004541E+01	.258846161338620268E+01
9	.864029785306437685E+01	-.759085169897491206E+01
10	-.130023397830134701E+01	-.129881201142522409E+02
11	-.119068297883696148E+02	-.648838984236070596E+01
12	-.115516571789737009E+02	.707510489497352517E+01
13	.136394050635195950E+01	.140877022724580950E+02
14	.148569627168610664E+02	.581916762746850694E+01
15	.144131743260753414E+02	-.111284272705446441E+02
16	-.147743349253152553E+01	-.197563233563763508E+02
17	-.178568391031640979E+02	-.971901068891708632E+01
18	-.173244359986984570E+02	.106126435955791169E+02
19	.154115254310566210E+01	.208558224213852506E+02
20	.208069178479732331E+02	.904974625844344581E+01
21	.201859006411560886E+02	-.146659440668456133E+02
22	-.165465121584454078E+01	-.265243961808529518E+02
23	-.238067617643170768E+02	-.129495653777860805E+02
24	-.230971308377111706E+02	.141501463788141362E+02
25	.171837318103011282E+01	.276238657251902518E+02
26	.267568195153772446E+02	.122802860777029381E+02
27	.259585751242462555E+02	-.182034373620818315E+02
28	-.183187348192884735E+01	-.332924198283136574E+02
29	-.297566490750111378E+02	-.161800953362420929E+02
30	-.288697913981527450E+02	.176876329587193282E+02
31	.189559641705165414E+01	.343918756245714737E+02
32	.327066965750283302E+02	.155108091643049834E+02
33	.317312257425396089E+02	-.217409190176262368E+02
34	-.200909732524161302E+01	-.400604197349202811E+02
35	-.357065185598430135E+02	-.194106134440732626E+02
36	-.346424346693334104E+02	.212251108972733698E+02
37	.207282065591027520E+01	.411598680397949005E+02
38	.386565603041746437E+02	.187413235502324169E+02
39	.375038634307509794E+02	-.252783940823158676E+02
40	-.218632183008823777E+01	-.468284063893940001E+02
41	-.416563778130589862E+02	-.226411249751022872E+02
42	-.404150680157483369E+02	.247625836944945531E+02
43	.225004534436565807E+01	.479278501690884677E+02
44	.446064160079661919E+02	.219718328436869662E+02
45	.432764933339999403E+02	-.288158650592646937E+02
46	-.236354664806108261E+01	-.535963848992508685E+02
47	-.476062306547070046E+02	-.258716324824123512E+02
48	-.461876951424673313E+02	.283000531881377852E+02
49	.242727025535972589E+01	.546958257386133749E+02
50	.505562665080598222E+02	.252023389025749222E+02

PUV (COS THETA) = .570466378938233001E+02
QUV (COS THETA) = .446699826396948176E+02

	MU = 1.00	NU = 1.50	THETA = 30.0 DEGREES
K	PSP		QSP
1	.160505526608920260E+01		-.430073262355922606E+00
2	.235768347521206716E+01		-.323891997801202715E+01
3	-.533873799600301569E+00		-.613047725282439587E+01
4	-.560296435676092774E+01		-.477221853167297624E+01
5	-.725834217316422042E+01		.140573558486653690E+01
6	-.192816296228938916E+01		.673591479574136816E+01
7	.645275581969027403E+01		.449025437627908886E+01
8	.899249081789166128E+01		-.498816567476933954E+01
9	.125152030580918091E+01		-.127291361868518199E+02
10	-.104176624099737599E+02		-.960238810182656566E+01
11	-.138375431279701317E+02		.316078049356102596E+01
12	-.369386870164155226E+01		.133044549198896054E+02
13	.112554985479275725E+02		.929878403801128444E+01
14	.155538915327123862E+02		-.674303697210327637E+01
15	.301096673333675227E+01		-.192859617714789103E+02
16	-.152148567925160616E+02		-.144023670763344913E+02
17	-.203909584615884015E+02		.491510733778554835E+01
18	-.545057626577562264E+01		.198554895335983272E+02
19	.160496768176708196E+02		.140945140828240971E+02
20	.221030326014730651E+02		-.849691725861680738E+01
21	.476624316747117537E+01		-.258337066926186971E+02
22	-.200072154062045568E+02		-.191956784740764117E+02
23	-.269375431627792439E+02		.666865682656515525E+01
24	-.720501443098432621E+01		.264011855583600730E+02
25	.208408537451429487E+02		.188863178295370454E+02
26	.286479663300868718E+02		-.102502229978840680E+02
27	.652014946177519881E+01		-.323780398661957410E+02
28	-.247975817356863500E+02		-.239864790830609415E+02
29	-.334813486532338791E+02		.842178025431221480E+01
30	-.895856268980455312E+01		.329445662177415408E+02
31	.256306399637202348E+02		.236764172998931273E+02
32	.351909665967207611E+02		-.120032074314187324E+02
33	.827344546369540946E+01		-.389207285644440841E+02
34	-.295869385359049418E+02		-.287760692466189704E+02
35	-.400237540963389895E+02		.101746566941463420E+02
36	-.107116744099945910E+02		.394867363804907405E+02
37	.304196700153264430E+02		.284656258581196086E+02
38	.417329201617313212E+02		-.137559984869994362E+02
39	.100264185371621999E+02		-.454625001115685575E+02
40	-.343757142003168053E+02		-.335649845023415423E+02
41	-.465653563088775195E+02		.119273791728880635E+02
42	-.124645405923551052E+02		.460281948894104779E+02
43	.352082437590199829E+02		.332543108215164591E+02
44	.482742437804046013E+02		-.155086651099869740E+02
45	.117792005975916777E+02		-.520037082927998976E+02
46	-.391641249915237934E+02		-.383534853414406587E+02
47	-.531064552954426899E+02		.136799997286918894E+02
48	-.142172552763111882E+02		.525691997478233910E+02
49	.399965211458351181E+02		.380426621368678473E+02
50	.548151590028692997E+02		-.172612472445473074E+02

PUV (COS THETA) = .618522834598657480E+02
 QUV (COS THETA) = -.305947641500301026E+02

MU = 1.50 NU = -1.50 THETA = 30.0 DEGREES

K	PSP	QSP
1	.499999999999999701-100	.866025403784438128-100
2	.199999999999999744E+01	.151461293802434073E-27
3	.499999999999999360E+01	-.519615242270662523E+01
4	-.999999999999998720E+00	-.155884572681198757E+02
5	-.209999999999999731E+02	-.155884572681198757E+02
6	-.359999999999999539E+02	.103923048454132505E+02
7	-.149999999999999808E+02	.467653718043596271E+02
8	.409999999999999475E+02	.467653718043596271E+02
9	.769999999999999022E+02	-.155884572681198771E+02
10	.319999999999999598E+02	-.935307436087192555E+02
11	-.779999999999998962E+02	-.935307436087192555E+02
12	-.143999999999999811E+03	.207846096908264986E+02
13	-.659999999999999113E+02	.155884572681198754E+03
14	.115999999999999855E+03	.155884572681198754E+03
15	.220999999999999721E+03	-.259807621135331287E+02
16	.100999999999999875E+03	-.233826859021798138E+03
17	-.170999999999999777E+03	-.233826859021798138E+03
18	-.323999999999999581E+03	.311769145362397488E+02
19	-.152999999999999800E+03	.327357602630517387E+03
20	.226999999999999713E+03	.327357602630517387E+03
21	.436999999999999444E+03	-.363730669589463791E+02
22	.205999999999999740E+03	-.436476803507356521E+03
23	-.299999999999999612E+03	-.436476803507356521E+03
24	-.575999999999999258E+03	.415692193816529993E+02
25	-.275999999999999643E+03	.561184461652315522E+03
26	.373999999999999525E+03	.561184461652315522E+03
27	.724999999999999076E+03	-.467653718043596296E+02
28	.346999999999999560E+03	-.701480577065394408E+03
29	-.464999999999999401E+03	-.701480577065394408E+03
30	-.899999999999998844E+03	.519615242270662498E+02
31	-.434999999999999439E+03	.857365149746593160E+03
32	.556999999999999291E+03	.857365149746593160E+03
33	.108499999999999861E+04	-.571576766497728800E+02
34	.523999999999999333E+03	-.102883817969591180E+04
35	-.665999999999999143E+03	-.102883817969591180E+04
36	-.129599999999999834E+04	.623538290724795002E+02
37	-.629999999999999189E+03	.121589966691335030E+04
38	.775999999999999010E+03	.121589966691335030E+04
39	.151699999999999806E+04	-.675499814951861305E+02
40	.736999999999999060E+03	-.141854961139890869E+04
41	-.902999999999998840E+03	-.141854961139890869E+04
42	-.176399999999999774E+04	.727461339178927507E+02
43	-.860999999999998894E+03	.163678801315258694E+04
44	.103099999999999868E+04	.163678801315258694E+04
45	.202099999999999742E+04	-.779422863405993810E+02
46	.985999999999998742E+03	-.187061487217438508E+04
47	-.117599999999999849E+04	-.187061487217438508E+04
48	-.230399999999999705E+04	.831384387633060011E+02
49	-.112799999999999855E+04	.212003018846430309E+04
50	.132199999999999831E+04	.212003018846430309E+04

PUV (COS THETA) = .149171725890026572E+04
QUV (COS THETA) = .375765567157950091E+04

MU = 1.50 NU = -.40 THETA = 30.0 DEGREES

K	PSP	QSP
1	.980107176607203394E+00	.499389549713500831E+00
2	.450252356197797981E+01	-.178809439734960999E+01
3	.401579916891860279E+01	-.110753490705671351E+02
4	-.105967078277706114E+02	-.185207932662956930E+02
5	-.319828073103788968E+02	-.463249787341252016E+01
6	-.300673113118871550E+02	.319173430986048368E+02
7	.142157129402746711E+02	.544806709356602835E+02
8	.685615657431380786E+02	.191880614666865737E+02
9	.642752509268409841E+02	-.625996974297129237E+02
10	-.257164080161840510E+02	-.108452737903407089E+03
11	-.128118084362320193E+03	-.419523117280723681E+02
12	-.120518903515844758E+03	.103048696718291572E+03
13	.312195075534340943E+02	.180363278823936292E+03
14	.196773077665860586E+03	.728515333119700833E+02
15	.184918983576833823E+03	-.153338056309940609E+03
16	-.446042970540894446E+02	-.270286009042847713E+03
17	-.288405831155823898E+03	-.111959441563979543E+03
18	-.271354776611872819E+03	.213394060859060208E+03
19	.519914910160854642E+02	.378147213214541528E+03
20	.389137059330145492E+03	.159202321138500921E+03
21	.365946997118897110E+03	-.283290425711250195E+03
22	-.672603749414867901E+02	-.504020606684617563E+03
23	-.512846047690890006E+03	-.214653887381134045E+03
24	-.482574930599971332E+03	.362953435520910744E+03
25	.765316633282287853E+02	.647832474107475991E+03
26	.645653510735992803E+03	.778240424946279087E+03
27	.607359291553030850E+03	-.452456805633641681E+03
28	-.936846416783760867E+02	-.809656530828716639E+03
29	-.801438733967518519E+03	-.350035649179535873E+03
30	-.754179365480140299E+03	.551726820703843180E+03
31	.104840024489864058E+03	.989419061502739681E+03
32	.966322431883402518E+03	.429965844735304579E+03
33	.909155866879235042E+03	-.660837196077115067E+03
34	-.123877097264757335E+03	-.118719378147514494E+04
35	-.115418388998570944E+04	-.518104726959185029E+03
36	-.108616808125237972E+04	.779714216407857515E+03
37	.136916574500991281E+03	.140290697540033260E+04
38	.135114382277237464E+04	.614378580505577398E+03
39	.127133672309750969E+04	-.908431597041670352E+03
40	-.157837741700630533E+03	-.163663235862390247E+04
41	-.157108151574546276E+04	-.718861120720081512E+03
42	-.147854107791668959E+04	.104691562263295375E+04
43	.172761313361610455E+03	.188829621580025474E+04
44	.180011768340290916E+04	.831478632257097545E+03
45	.169390186020785479E+04	-.119524000852730754E+04
46	-.195566574985995683E+03	-.215797226227498923E+04
47	-.205213161124677848E+04	-.952304830462225322E+03
48	-.193129835547306991E+04	.135333103937913189E+04
49	.212374241071721581E+03	.244558678270250611E+04
50	.231324401377500609E+04	.108126599998986502E+04

PUV (COS THETA) = .261021635355212178E+04
QUV (COS THETA) = .191649408553523991E+04

MU = 1.50 NU = .05 THETA = 30.0 DEGREES

K	PSP	QSP
1	.150717337661639680E+01	.361840313976652925E+00
2	.501778171265453763E+01	-.333756897818600929E+01
3	.199301829155776563E+01	-.135489991545322522E+02
4	-.157041142596799259E+02	-.177977047767097253E+02
5	-.348059537351815745E+02	.233143401888123093E+01
6	-.236441506883174299E+02	.400130495992011695E+02
7	.277455996046997127E+02	.523506370789857548E+02
8	.748290525821524250E+02	.273503010174534384E+01
9	.504179337048503067E+02	-.796755261101757451E+02
10	-.521670928971044401E+02	-.104304011996973734E+03
11	-.139622541738995771E+03	-.121451981598627082E+02
12	-.968498308265850797E+02	.132253053911286980E+03
13	.744331306514654340E+02	.173374454754504665E+03
14	.214650957720282941E+03	.256156953793018628E+02
15	.148404378568093075E+03	-.198029007778703879E+03
16	-.109079176353211359E+03	-.259845340127747553E+03
17	-.314449764011442597E+03	-.434298965362318138E+02
18	-.219617040414802957E+03	.276720012936257435E+03
19	.141569766516913554E+03	.363433293340533390E+03
20	.424483497127046080E+03	.653044268544835522E+02
21	.295952352881286064E+03	-.368609444160116657E+03
22	-.186440364628000681E+03	-.484421689169031183E+03
23	-.559287620552522049E+03	-.915226611102260872E+02
24	-.391945779452971057E+03	.473413926674112534E+03
25	.229155507201044077E+03	.622527152837071925E+03
26	.704326670802441844E+03	.121801224527290410E+03
27	.493061856644429276E+03	-.591416835254414077E+03
28	-.284250657721472404E+03	-.778033059120824624E+03
29	-.874136111362234126E+03	-.156423491881845529E+03
30	-.613836047941089380E+03	.722334795124852276E+03
31	.337190352703857002E+03	.950656033244120272E+03
32	.105418047874647023E+04	.195106088397722435E+03
33	.739732889857522711E+03	-.866451181061596140E+03
34	-.402510055633626531E+03	-.114067944998312788E+04
35	-.125899523644057883E+04	-.238132388851090139E+03
36	-.885287845879157927E+03	.102348261828847666E+04
37	.465674303025352329E+03	.134781993456167843E+04
38	.147404492095913125E+04	.285219018465779629E+03
39	.103596545252056637E+04	-.119371248158166285E+04
40	-.541218558364463059E+03	-.157236086175594094E+04
41	-.171386499578755616E+04	-.336649352017959916E+03
42	-.120630117326717670E+04	.137685739616498569E+04
43	.614607358165530059E+03	.181401865678974640E+04
44	.196391999744042489E+04	.392140014731461991E+03
45	.138175954463356025E+04	-.157320073681461420E+04
46	-.700376165913981990E+03	-.207307729443926381E+04
47	-.223874538940316611E+04	-.451974381382454862E+03
48	-.157687603010514569E+04	.178245912875437936E+04
49	.783989518124390191E+03	.234925279992832418E+04
50	.252380570819035116E+04	.515869077194769521E+03

PVV (COS THETA) = .284780978291872870E+04
QUV (COS THETA) = .914354132436944155E+03

	MU = 1.50	NU = 1.50	THETA = 30.0 DEGREES
K	PSP		QSP
1	.259807621135331261E+01		-.14999999999999808E+01
2	.259807621135331261E+01		-.949999999999998784E+01
3	-.103923048454132505E+02		-.16999999999999782E+02
4	-.311769145362397514E+02		-.49999999999999360E+01
5	-.311769145362397514E+02		.29999999999999616E+02
6	.103923048454132505E+02		.53999999999999309E+02
7	.649519052838328166E+02		.22499999999999705E+02
8	.649519052838328166E+02		-.57499999999999253E+02
9	-.207846096908265011E+02		-.10699999999999863E+03
10	-.124707658144959003E+03		-.46999999999999410E+02
11	-.124707658144959003E+03		.959999999999998749E+02
12	.207846096908265029E+02		.17999999999999767E+03
13	.189659563428791822E+03		.824999999999998921E+02
14	.189659563428791822E+03		-.14149999999999821E+03
15	-.311769145362397495E+02		-.26899999999999657E+03
16	-.280592230826157760E+03		-.12499999999999842E+03
17	-.280592230826157760E+03		.19799999999999744E+03
18	.311769145362397533E+02		.377999999999999514E+03
19	.376721050646230331E+03		.17849999999999769E+03
20	.376721050646230331E+03		-.26149999999999667E+03
21	-.415692193816529999E+02		-.50299999999999358E+03
22	-.498830632579836020E+03		-.23899999999999696E+03
23	-.498830632579836020E+03		.335999999999999568E+03
24	.415692193816530037E+02		.647999999999999168E+03
25	.626136366936148342E+03		.31049999999999600E+03
26	.626136366936148342E+03		-.41749999999999468E+03
27	-.519615242270662504E+02		-.808999999999998966E+03
28	-.779422863405993782E+03		-.388999999999999504E+03
29	-.779422863405993782E+03		.509999999999999345E+03
30	.519615242270662542E+02		.989999999999998730E+03
31	.937905512298545855E+03		.47849999999999385E+03
32	.937905512298545855E+03		-.60949999999999222E+03
33	-.623538290724795009E+02		-.11869999999999848E+04
34	-.112236892330463105E+04		-.57499999999999266E+03
35	-.112236892330463105E+04		.719999999999999076E+03
36	.623538290724795046E+02		.14039999999999820E+04
37	.131202848673342287E+04		.68249999999999124E+03
38	.131202848673342287E+04		-.837499999999998930E+03
39	-.727461339178927513E+02		-.16369999999999791E+04
40	-.152766881227574782E+04		-.796999999999998982E+03
41	-.152766881227574782E+04		.965999999999998761E+03
42	.727461339178927551E+02		.18899999999999758E+04
43	.174850529024077939E+04		.922499999999998817E+03
44	.174850529024077939E+04		-.11014999999999859E+04
45	-.831384387633060018E+02		-.21589999999999724E+04
46	-.199532253031934409E+04		-.10549999999999865E+04
47	-.199532253031934409E+04		.12479999999999840E+04
48	.831384387633060055E+02		.24479999999999686E+04
49	.224733592282061541E+04		.11984999999999846E+04
50	.224733592282061541E+04		-.14014999999999821E+04

PUV (COS THETA) = .253584703677615115E+04
 QUV (COS THETA) = -.248409407204407754E+04

APPENDIX B

CONVERGENCE OF THE HYPERGEOMETRIC SERIES ON $|z|=1$

We consider the series

$$\sum_{n=0}^{\infty} d_n z^n,$$

where

$$d_n = \frac{(a)_n (b)_n}{n! (c)_n}$$

and

$$|z|=1.$$

By an extension of a Weierstrass test for convergence (see, e.g., reference 7, page 401), if

$$\frac{d_{n+1}}{d_n} = 1 - \frac{\alpha}{n} - \frac{A_n}{n^\lambda},$$

where α is complex and arbitrary, $\lambda > 1$, and where the A_n are bounded, then the series

$$\sum_{n=0}^{\infty} d_n z^n, \quad |z|=1,$$

- I) converges absolutely for $\text{Re}\{\alpha\} > 1$,
- II) converges conditionally if $z \neq 1$ and $0 < \text{Re}\{\alpha\} \leq 1$,
- III) diverges for $\text{Re}\{\alpha\} \leq 0$.

To use this test in the present case, choose $\alpha = c+1-a-b$, $\lambda=2$, and

$$A_n = -\alpha \left[\frac{c+1+\frac{c}{n} + \frac{ab-c}{\alpha}}{1+\frac{1}{n}(c+1) + \frac{c}{n^2}} \right].$$

The A_n are clearly bounded, and one has

$$n \left[\frac{d_{n+1}}{d_n} - 1 \right] = n \left[\frac{\frac{(a)_{n+1} (b)_{n+1}}{(n+1)! (c)_{n+1}}}{\frac{(a)_n (b)_n}{n! (c)_n}} - 1 \right]$$

$$= \frac{\frac{ab-c}{n} - \alpha}{\frac{c}{n^2} + \frac{(c+1)}{n} + 1}.$$

Thus

$$\frac{d_{n+1}}{d_n} = 1 - \frac{1}{n} \left(\frac{\alpha - \frac{ab-c}{n}}{1 + \frac{1}{n}(c+1) + \frac{c}{n^2}} \right)$$

$$= 1 - \frac{\alpha}{n} + \left[\frac{\frac{\alpha}{n} - \frac{ab-c}{n^2}}{1 + \frac{1}{n}(c+1) + \frac{c}{n^2}} \right]$$

$$= 1 - \frac{\alpha}{n} + \left[\frac{\frac{\alpha(c+1)}{n^2} + \frac{\alpha c}{n^3} + \frac{ab-c}{n^2}}{1 + \frac{1}{n}(c+1) + \frac{c}{n^2}} \right]$$

$$= 1 - \frac{\alpha}{n} - \frac{1}{n^2} (-\alpha) \left[\frac{c+1 + \frac{c}{n} + \frac{ab-c}{\alpha}}{1 + \frac{1}{n} (c+1) + \frac{c}{n^2}} \right]$$

$$= 1 - \frac{\alpha}{n} - \frac{A_n}{n^2},$$

as required.

Then since $\operatorname{Re} \{\alpha\} = \operatorname{Re} \{c+1-a-b\} = -2 \operatorname{Re} \{\mu\} + 1$,
we have the desired results.

APPENDIX C

AN ORDER RELATION FOR $\log \Gamma(z+a)$ FOR LARGE z

From Stirling's formula (see, e.g. references 2, 5, and 8), we have

$$\log \Gamma(z) = (z - \frac{1}{2}) \log z - z + \frac{1}{2} \log(2\pi) + O(\frac{1}{z}).$$

Then

$$\begin{aligned} \log \Gamma(z+a) &= (z+a - \frac{1}{2}) \log(z+a) - (z+a) + \frac{1}{2} \log(2\pi) + O(\frac{1}{z+a}) \\ &= (z+a - \frac{1}{2}) \log(z+a) - (z+a) + \frac{1}{2} \log(2\pi) + O(\frac{1}{z}), \end{aligned}$$

since $\frac{1}{z+a} = \frac{1}{z} - \frac{a^2}{z^2} + \frac{a^3}{z^3} - \dots$

It follows that

$$\begin{aligned} \log \Gamma(z+a) - \log \Gamma(z) &= (z+a - \frac{1}{2}) \log(z+a) - (z+a) \\ &\quad - (z - \frac{1}{2}) \log z + z + O(\frac{1}{z}). \end{aligned}$$

Writing $z+a = z(1+\frac{a}{z})$, we have

$$\begin{aligned} \log \Gamma(z+a) - \log \Gamma(z) &= (z+a - \frac{1}{2}) (\log z + \log(1+\frac{a}{z})) - z - a \\ &\quad - (z - \frac{1}{2}) \log z + z + O(\frac{1}{z}) \\ &= a \log z + (z+a - \frac{1}{2}) \log(1+\frac{a}{z}) - a + O(\frac{1}{z}). \end{aligned}$$

Now if $|z|$ is large enough so that $|\frac{a}{z}| < 1$,

$$\log\left(1+\frac{a}{z}\right) = \frac{a}{z} - \frac{1}{2}\left(\frac{a}{z}\right)^2 + \dots, \text{ and then}$$

$$(z+a-\frac{1}{2})\log\left(1+\frac{a}{z}\right) = a + O\left(\frac{1}{z}\right).$$

It follows that when $|z| > |a|$,

$$\log \Gamma(z+a) = \log \Gamma(z) + a \log z + O\left(\frac{1}{z}\right).$$

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