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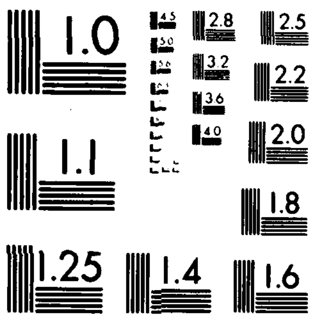


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AIR FORCE HUMAN RESOURCES LABORATORY

# MARKOV RESOURCE UTILIZATION DECISION AID FOR AIR FORCE RECRUITING SERVICE


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Final Report



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AIR FORCE SYSTEMS COMMAND
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This report has been reviewed by the Office of Public Affairs (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

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This technical report presents a recruiting resource and goal allocation decision model developed for the Air Force Recruiting Service. The model uses a Markov probability process to determine the optimum use of recruiting effort and advertising funds to obtain desired recruit input levels. Probability statements are made concerning the likelihood of meeting objectives. The model has considerable variability in user input and output and non-optimum goals can be examined.

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# PREFACE

This report documents work performed for the Air Force Human Resources Laboratory (AFHRL) by ORINCON Corporation under Contract F33615-78-C-0043. A number of people have contributed to this project. Mr. Larry T. Looper was the technical monitor for AFHRL. Major James Wortman of the Air Force Recruiting Service has supported AFHRL and ORINCON during all phases of the project. Dr. Gerald M. Anderson was the ORINCON Program Manager, and Dr. Michael H. Moore and Ms. Nancy D. Anderson were the Principal Investigators. Dr. Moore conceived the general structure of the decision aid and provided technical guidance during its development. Ms. Anderson identified sources of data required by the decision aid. Finally, Mr. Thomas A. Adams developed the computer code for the aid.

## 1.0 INTRODUCTION

In recent years, the military services have faced a supply-limited market for manpower of all types.* It is commonly agreed that this situation is due principally to the elimination, in 1973, of the draft system for obtaining manpower. Because prospects for renewal of the draft are uncertain, continued tightness in military manpower markets is likely.

Headquartered at Randolph Air Force Base, Texas, as a part of Air Training Command, the Air Force Recruiting Service (ATC/RS) is positioning itself so as to be able to continue to meet the Air Force's needs for manpower even under today's difficult recruiting conditions. To do this, ATC/RS is making a number of operational improvements:

- better definition of the manpower markets;
- better recruiting techniques;
- improved efficiency of recruiting managements at several levels.

An example of an improvement of the first of the above types is the ATC/RS's support and use of the "National Recruit Market Network" -- a data base on various manpower markets. An example of an improvement of the second type is the PROMIS system. This is a computer-driven interviewing technique whereby recruiters can deal with potential recruits on an individual basis. An example of an improvement

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* Periods of surplus in manpower markets have occurred from time to time, most often together with economic dips in the civilian sector or with population anomalies (e.g., the baby boom). Such periods have been few and brief.

of the third type is the resource utilization decision tool described in this report.

The Markov Resource Utilization Decision Aid for the Air Force Recruiting Service (hereafter called the Air Force Recruiting Aid (AFRA)) is a tool that recruiting managements at several levels can use to assist them in allocating scarce resources, in assigning recruiting goals, and otherwise in performing their duties. The aid, which is computer-driven, is based on a mathematical model of the recruiting process. The aid should be tested and evaluated before being turned over to the Air Force Recruiting Service for operational use; therefore, the aid must be considered experimental at this point in time. Tests to examine the model's operational capabilities will be conducted at AFHRL.

An overview of the aid is given in Chapter 2 of this report. This overview briefly describes the general structure of the aid and indicates how it works. In particular, the aid's two principal modules -- a computational module and a display module -- are described. In Chapter 3, the aid's computational module is presented. Included here is a description of the recruiting model upon which the module (and indeed the entire aid) is based. In Chapter 4, the aid's display module is described. In Chapter 5, the utility of the aid is illustrated by giving an example. In Chapter 6, a description of appropriate directions for the continued development of the aid is presented.

## 2.0 THE AIR FORCE RECRUITING AID -- AN EXECUTIVE SUMMARY

The Air Force Recruiting Aid (AFRA) is a tool that recruiting managements at several levels can use to assist them in allocating scarce resources to their subordinate recruiting organizations, in assigning recruiting goals to them, and otherwise in performing their duties.

AFRA is computer-driven in either batch or interactive mode. It is tolerant of user errors.

AFRA is runnable on one category of recruits at a time. The definition of this category is open to the user through selection of appropriate inputs. For example, the user could run the aid on unmarried high-school-graduates of ages between 17 and 21.

AFRA is modular in structure. There are two principal modules to the aid: a computational module and a display module. The computational module, the heart of which is a mathematical model of the recruiting process, does most of the computations. The display module post-processes the computational module's output and displays the results to the user.

In this chapter we give an overview of AFRA. In Section 2.1, we describe the aid's modular structure. In Section 2.2, we describe the aid's algorithms -- that is, we indicate how the aid works.

## 2.1 Structure of the Aid

Figure 2.1 shows the overall structure of AFRA. As shown, the aid consists of two modules: a computational module and a display module. These modules are implemented by independent (i.e., separately runnable) computer programs. The computational module:

- accepts inputs from a user;
- computes certain raw outputs;
- writes the raw outputs to a semi-permanent "raw output file" on disk for later access by the display module.

A user can elect to run the computational module in either batch or interactive mode. The display module:

- accesses a raw output file previously created by the computational module;
- post-processes the raw outputs to produce "finished outputs;"
- displays the finished outputs to the user on one or more display terminals.

The display module is runnable only in interactive mode.

The recruiting aid has several kinds of raw and post-processed outputs. Perhaps the single most important kind of output of the aid is an assessment of the probabilities that various numbers of recruits (of some specific category) will be obtained

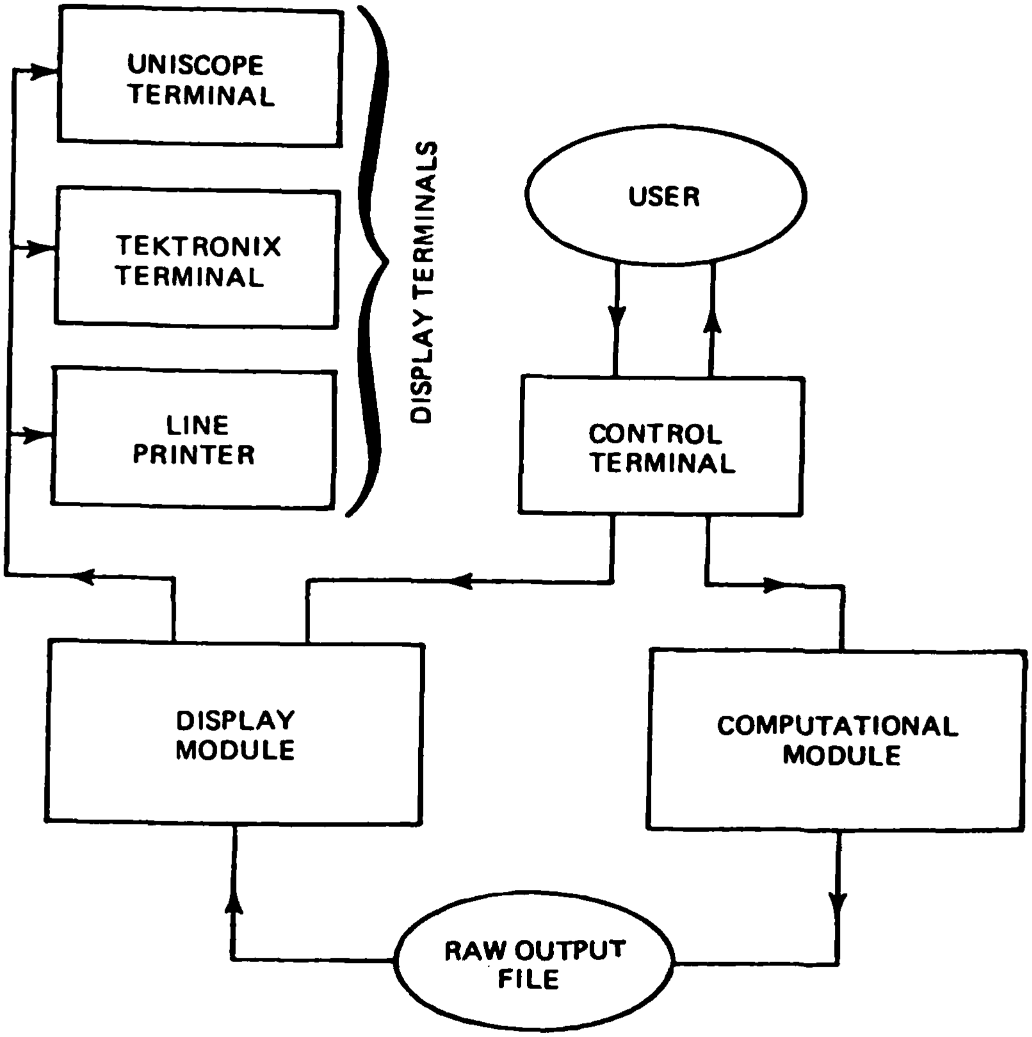



Figure 2.1. Structural design of the recruiting aid.

by all of the subordinate recruiting organizations over a number of months of time that can be as much as a year. This assessment is given as a function of the policy that the recruiting management follows, as specified by the user, with respect to:

- assigning quotas to the subordinate recruiting organizations over time;
- allocating advertising funds to the subordinate recruiting organizations over time;
- allocating other resources (e.g., recruiters) to the subordinate recruiting organizations over time.

The aid can also produce as output, if the user so requests, policies of the above kinds that are jointly "optimal" in a meaningful sense. These outputs can be useful to a recruiting management in determining which particular policies of the above types to adopt.

The inputs to the aid are of two kinds: control inputs and data inputs. The data inputs for the computational module include:

- number of subordinate recruiting organizations;
- number of recruits of the category of interest that the Air Force desires over a certain period of time (typically, a year);
- "difficulty of recruiting" for this category of recruits for each of the subordinate recruiting organizations;
- "recruiting effectiveness" for this category for each of the subordinate recruiting organizations;

- parameters associated with advertising effectiveness for this category;
- parameters associated with market potentials for this category.

The computational module also has a full range of control inputs that offer a user considerable flexibility in controlling the module, e.g., a user can arrange for the module either to do all the computations normally associated with a run, or to redo part of the computations done by an earlier run. All inputs to the display module are control inputs. Using these, a user of the display module can select:

- the kind of information to be displayed;
- the type of information to be displayed;
- the amount of information to be displayed.

These inputs are all described in more detail elsewhere in this report.

## 2.2 Algorithms of the Aid

The aid is based on a mathematical model of the recruiting process. This model is embedded in the computational module. The recruiting model is:

- systems-theoretic;
- analytic;
- stochastic;
- dynamic.

The model is systems-theoretic in that the "most important" elements of the recruiting process are represented in the model. The model is analytic in that it is based on analytic expressions and equations rather than being, say, a Monte-Carlo simulation or a people-driven participation exercise. The model is stochastic in character, as opposed to being deterministic in character. Finally, the model is dynamic in that it represents processes and decisionmaking over a span of time.

The aid's computational module is computationally intensive. "Large" problems require several hours of CPU time on the UNIVAC 1108 even when certain time-reducing options are selected. Large problems, therefore, are usually run in batch mode during non-prime time.

## 3.0 THE COMPUTATIONAL MODULE

The computational module of the Air Force Recruiting Aid:

- accepts inputs from a user;
- computes certain raw outputs;
- writes the outputs to an output file.

A user of the module may elect to run it in either interactive or batch mode.

In this chapter, a macro-description of the recruiting aid's computational module is presented. The description here is "macro" in that it describes the module in terms of its structure, outputs, inputs, and algorithms.

### 3.1 Structure of the Computational Module

Figure 3.1 shows the computational module's major lines of user control and information flow. As shown, a user of the module controls it and provides data to it through a control/input dialogue. If the users choose to run the module in interactive mode, the module prompts them to make all necessary inputs. If the users choose to run in batch mode, they place this information in a disk file which is then accessed by the module. Thus, to run the computational module in batch mode, a user must be familiar enough with it to be able to provide all required information without being prompted.

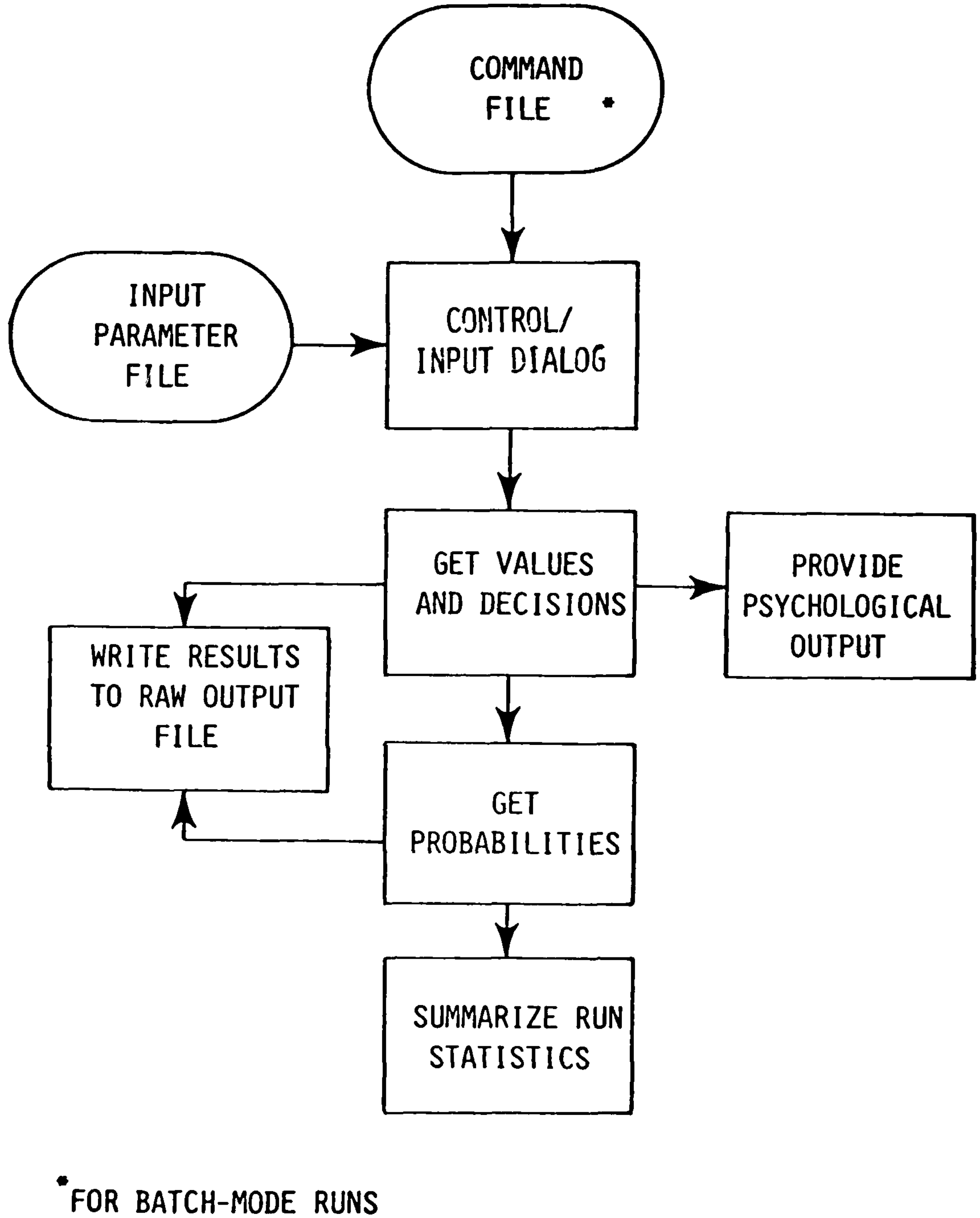



Figure 3.1 Computational module's major lines of user control and information flow.

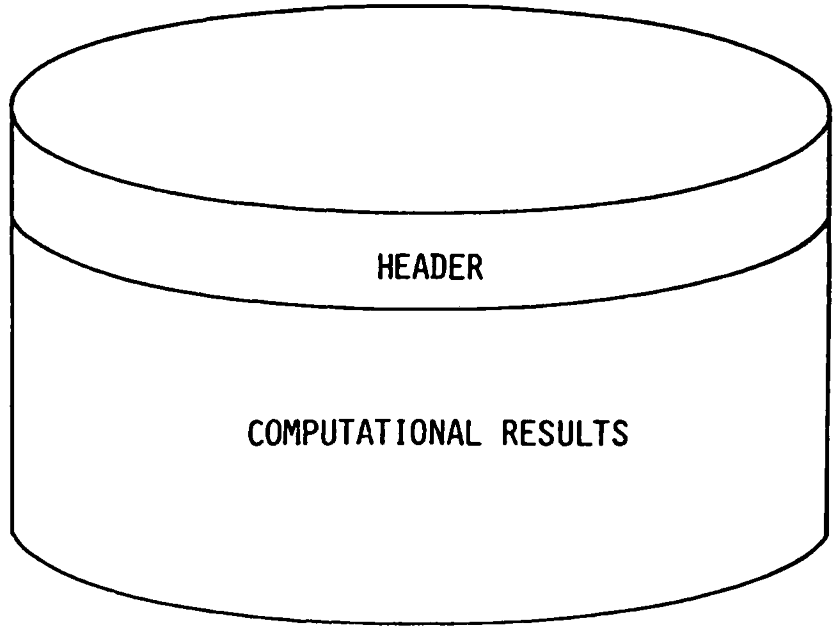
After the module gets all the inputs it needs, it goes on to compute certain raw outputs (these are the "values" and probabilities" in Figure 3.1) and to write them out to a raw output file. The module also provides "psychological outputs" to interactive-mode users in real time during the run to inform them as to the run's status.

Figure 3.2 shows a typical raw output file. This file -- a semi-permanent disk file -- is accessible by the display module. These raw output files all have the same format and information content, as indicated in the figure.

## 3.2 Algorithms of the Computational Module

There are three principal algorithms by means of which the computational module converts inputs into outputs:

- an algorithm for defining and determining the "difficulty of recruiting" for a recruiting organization;
- an algorithm for defining and determining the "recruiting effectiveness" of a recruiting organization;
- an algorithm for estimating the recruiting results of a group of recruiting organizations as a function of:
  - management policy as to assigning quotas for recruits to the subordinate recruiting organizations;




"HEADER" CONTAINS

- CREATOR'S ID
- DATE AND TIME CREATED
- INDICATION OF WHETHER OR NOT DECISIONS ARE OPTIMAL
- RUN PARAMETERS
- INFORMATION ON STATE STRUCTURE

"COMPUTATIONAL RESULTS" CONTAINS

- VALUES
- DECISIONS (OPTIMAL OR NOT)
- PROBABILITIES OF STATE OCCUPANCIES

**Figure 3.2 Format of computational module's raw output disk files.**

- management policy as to allocating advertising funds to the recruiting organizations;
- other factors (e.g., distribution of recruiting personnel among recruiting organizations).

The third of the above listed algorithms, which utilizes the other two, is based on a mathematical model of the recruiting process. This model is the heart of the computational module and, indeed, of the entire Air Force recruiting aid.

### 3.2.1 Difficulty of Recruiting Algorithm

We define the difficulty d(k) of recruiting at time k (for a particular recruiting organization and a certain category of recruits) as

$$d(k) = \frac{\sum_{\ell=k-N}^{k} v_\ell \frac{pop(\ell) - pot(\ell)}{pop(\ell)}}{\sum_{\ell=k-N}^{k} v_\ell} \qquad (1)$$

where

$N$ = a constant;

$pop(\ell)$ = the raw population of eligible age during time period $\ell$;

$pot(\ell)$ = the "market potential" during time period $\ell$;

$v_\ell$ = weight for the $\ell^{th}$ time period.

With this definition, a difficulty of recruiting is a moving estimate of the fraction of an area's eligible population that may be reasonably considered as being available for military service. The constant N determines the width of the time window over which the d(k) will be estimated. The weights $v_\ell$ allow the user to weight different time points differently when calculating the d(k); typically, the more recent time points would be more heavily weighted than the older time points. The raw population figures pop($\ell$) are available as raw data. The market potential figures pot($\ell$) are computable from raw data, as is explained in Section 3.3 on inputs to the computational module.

Each d(k) is a number between 0 and 1, with small values corresponding to low difficulties and large values to high difficulties.

### 3.2.2 Recruiter Effectiveness Algorithm

We define the effectiveness e(k) of a recruiting organization at time k (for a certain category of recruits) as

$$e(k) = \frac{\sum_{\ell=k-M}^{k} w_\ell \bullet d^{\beta}(\ell) \bullet h(\ell)/pot(\ell)}{\sum_{\ell=k-M}^{k} w_\ell} \qquad (2)$$

where:

M = a constant;

$\beta$ = a constant between 0 and 1;

$h(\ell)$ = number of recruits actually obtained by the group during time period $\ell$;

$pot(\ell)$ = the "market potential" during time period $\ell$;

$w_\ell$ = weight for the $\ell^{th}$ time period.

As with the difficulties of recruiting, the recruiter effectivenesses defined above are moving estimates of "success" for a recruiting organization. The constant M determines the width of the time window for the estimates. The weights $w_\ell$ allow the user to weight different time points differently, with recent time points typically being weighted more heavily than old ones. The purpose of the parameter $\beta$ in formula (2) is to keep the value of the recruiter effectiveness e(k) well away from zero for all typical values of the difficulty measures $d(\ell)$. (This $\beta$ can be set to 1 for test purposes, but should later be set to some value $0 < \beta < 1$ on the basis of the observed behavior of the functions $e(\bullet)$). The numbers $h(\ell)$ should be readily available as raw historical data. The computation of the market potentials $pot(\ell)$ is explained in Section 3.3 on inputs to the computational module.

Notice that, as with the difficulty of recruiting d(k), each recruiter effectivenesses e(k) is a number between 0 and 1, with small values corresponding to low effectivenesses and large values to high effectivenesses. Notice also that the effectiveness e(k) of a recruiting organization as defined above depends on the difficulties of $d(\ell)$ of recruiting for the organization for times $\ell \leq k$. The manner of dependence of the e(k) upon these $d(\ell)$ is such that a recruiting organization that does not obtain a large number of recruits over a span of time can still rate as being highly effective as long as the organization's difficulty of recruiting is high, as is reasonable.

The recruiter effectivenesses e(k) characterize not only the effectiveness of each recruiting organization individually, but also that of the recruiting system as a whole. Occasionally, a user of AFRA may wish to determine the effect on the aid's output of making a hypothetical change in the distribution of numbers and talent among the several organizations -- a change which, if actually made, would probably result in new recruiter effectivenesses e(k).

Within limits, it seems reasonable to estimate the new recruiter effectivenesses e(k) that would result from such a redistribution of recruiters by imagining that recruiting effectiveness is a portable and transferable commodity. With this in mind, consider a situation involving two recruiting organizations, and let $n_i$ and $e_i$ denote the number of assigned recruiting personnel, and the effectiveness, for organization i, i = 1, 2. Suppose that we wish to estimate what the effect of transferring m recruiting personnel from organization 1 to organization 2 (where m is "small") would be on the effectivenesses of these organizations. We suggest that these new effectivenesses can be taken as

$$\text{new } e_1 = \text{old } e_1 - \frac{m}{n_1} e_1, \qquad \text{new } e_2 = \text{old } e_2 + \frac{m}{n_1} e_1.$$

### 3.2.3 The Air Force Recruiting Model

In this section the model of the recruiting process that is the heart of the Air Force recruiting aid is presented. In Section 3.2.3.1, the actual recruiting process is described. In Section 3.2.3.2, the model of this process is presented.

#### 3.2.3.1 The Actual Recruiting Process

The actual Air Force recruiting process has, as do many military functions, a hierarchy of managements. The management structure for the recruiting process is shown in Figure 3.3.

As the figure shows, the highest level of management for the recruiting process is the Headquarters, Air Force Recruiting Service (ATC/RS), with offices at Randolph Air Force Base, Texas. The recruiting organizations directly subordinate to the ATC/RS are called "recruiting groups." Currently, there are five recruiting groups, each comprising a number of states and parts thereof. The recruiting organizations directly subordinate to the groups are called "squadrons." The recruiting organizations directly subordinate to the squadrons are called "flights," and those subordinate to the flights are called "offices." The recruiting function per se occurs at office level.

Note that it is not necessarily true that all of the subordinate recruiting organizations under any one management have the same characteristics. Thus, for example, some of the recruiting groups have larger markets for potential enlistees than others, some are larger in physical area than others, and some have higher

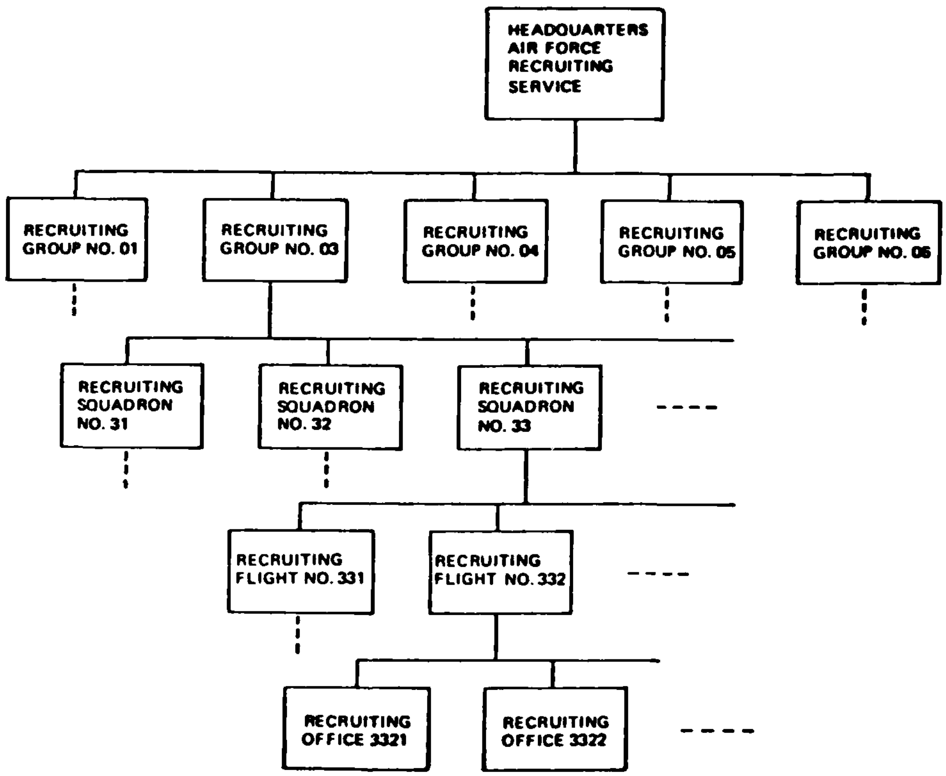



Figure 3. 3 Organization of Air Force Recruiting Service.

"difficulties of recruiting" and "recruiter effectivenesses" than others (see Sections 3.2.1 and 3.2.2 for definitions of these latter quantities).

In the current scheme of things, the Headquarters of the Air Force determines, at the beginning of each year, its need for recruits of various types and communicates these needs for the Headquarters of the Air Force Recruiting Service. This headquarters also provides to the ATC/RS, in addition to recruiting personnel and other fixed resources, certain discretionary resources such as funds for spot advertising purposes. The ATC/RS, upon receiving this information and these discretionary resources, establishes quotas for enlistees for each group on a month-by-month basis, and allocates its discretionary resources to the groups for further disbursement to the subordinate squadrons on a similar schedule. The squadrons, similarly, assign enlistment quotas and allocate resources to the flights, and the flights similarly to the offices.*

#### 3.2.3.2 A Model of the Recruiting Process

In this model of the actual Air Force recruiting process, focus is on the interface between some one recruiting management and all of its directly subordinate recruiting organizations. Thus this model of recruiting can deal with the interface between the Headquarters of the Air Force Recruiting Service and its subordinate

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* Sometimes a recruiting group or squadron retains a portion of the resources that are allocated to it by a higher level of management for its own use, or for the benefit of all of its subordinate recruiting organizations (as in an area-wide advertising campaign).

recruiting groups. Alternatively, the model can represent the interface between a particular recruiting group and its subordinate squadrons, or between a particular squadron and its subordinate flights, or between a particular flight and its subordinate offices.

It is also important to consider in this model two recruiting subprocesses:

- the subprocess that involves a recruiting management assigning quotas for enlistees of each type to its subordinate recruiting organizations;
- the subprocess that involves a recruiting management allocating discretionary resources to its subordinate recruiting organizations.*

The model additionally represents other recruiting subprocesses, such as the distribution of recruiting personnel among recruiting organizations. But these other subprocesses are not represented in as rich a degree of detail as those listed above.

The process by which a recruiting management obtains recruits through the efforts of a collection of subordinate recruiting organizations is modeled as a vector-valued, management-controllable stochastic pure birth process, as illustrated in Figure 3.4. This model can also be described as being a vector-valued Markov decision process with monotone non-decreasing state components. This representation is central to the recruiting model and, indeed, to the

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* The current model is limited to representing only the case where the discretionary resources are funds to be used for advertising purposes.

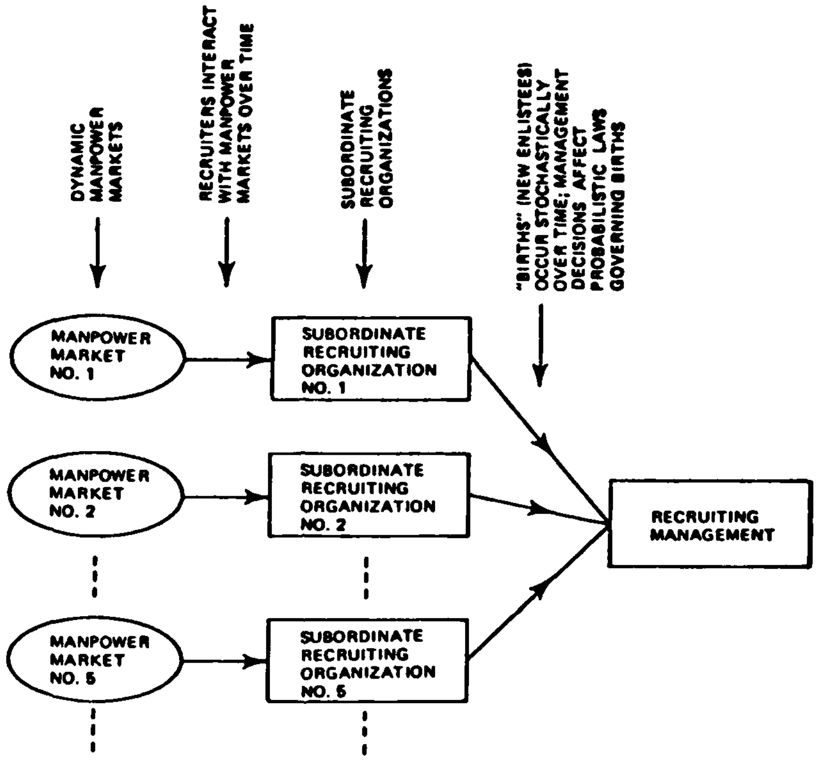


Figure 3.4 Schematic of model by which subordinate recruiting organizations obtain recruits for management.

entire recruiting aid. General discussions of birth processes and Markov decision processes may be found in Feller [1], and Howard [2] or [3].

In this recruiting model, time is taken as being discrete. Typically, time is considered as moving in one-month jumps. It has been convenient to number time periods in inverse order so that, for example, time 12 corresponds to the beginning of time for the model, and time 0 corresponds to a year later.

In this model, "births" correspond, of course, to accessions (i.e., enlistees being gained by the recruiting organizations):* these occur in a stochastic (in fact, Markovian) fashion over time. Accessions are taken as being vector-valued instead of scalar-valued because several recruiting organizations obtain recruits for the recruiting management rather than just one. Accessions are management-controllable in the sense that the management can influence the probabilistic laws that govern the way in which they occur over time by exercising controls on the subordinate recruiting organizations of types that have been mentioned earlier: the assigning of monthly quotas to the recruiting organizations, and the allocating of monthly advertising funds to the recruiting organizations.

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* Note that the recruiting process could be modeled more generally as a stochastic birth/death process, the "deaths" corresponding to losses of enlistees prior to some point in time where they are no longer of interest for recruiting purposes. Such a model, however, seems unnecessarily elaborate at this early stage.

Currently, the model can represent a recruiting (sub)system with a recruiting management and at most five subordinate recruiting organizations. Also, the current model can portray a time span of at most 12 time points. These limits, which arise from computational considerations, are hardwired into the recruiting model. They are discussed later in Section 3.2.3.2.8.

To completely describe a controllable stochastic birth process (alternatively, a Markov decision process), one must specify:

- the "states" of the process;
- a time horizon for the process;
- the rewards/penalties to be associated with changes of state;
- the controls for the process;
- the state transition probabilities as a function of the controls.

We shall take up each of these items below. In the discussion to follow, we shall assume a maximal case: a recruiting (sub)system consisting of a central recruiting management and five subordinate recruiting organizations, and a time horizon of 12 time periods, here interpretable as months.

### 3.2.3.2.1 The State Space

A "state" of the recruiting process is a five-dimensional vector whose $i^{th}$ component is a number of recruits -- expressed in any convenient "person-unit" -- that can have been obtained by the $i^{th}$ recruiting organization at any time. Thus, to say that the recruiting subsystem is in state $s = (s_1, s_2, s_3, s_4, s_5)$ at month k is to say that the $i^{th}$ recruiting organization has obtained $s_i$ person-units of recruits at that time for $i = 1, \ldots, 5$.

### 3.2.3.2.2 The Rewards and Penalties

The rewards and penalties that we associate with the recruiting process are of two kinds, both of the penalty variety:

- a penalty for the allocation (and assumed subsequent expenditure) of advertising funds to a recruiting organization for a month;
- a penalty for setting abnormally high or abnormally low quotas for a recruiting organization for a month.

Note that these penalties depend only on the management controls as to quota and advertising allocations, and not on the state of the recruiting process.

### 3.2.3.2.3 The Control Space

The management controls or decisions that we associate with the recruiting process are of the two types that we have mentioned several times above:

- a management control as to the quota for recruits that it will assign to each of the recruiting organizations for each month;
- a management control as to the advertising funds that it will allocate to each of the recruiting organizations for each month.

When the user has elected to specify controls for the recruiting management, there will be a single known control of each type for each recruiting organization. When the user has elected to determine optimal controls for the recruiting management, these will be obtained from pre-defined sets of controls. In the latter case, the current model is limited to at least three and at most four management controls of each of the two types for each of the five recruiting organizations per month; these are interpretable as "low quota," "medium advertising allocation," etc. The minimum of three controls/organization-month arises from the desire to model the actual controls of a real management with reasonable faithfulness. The maximum of four arises from computational considerations (see Section 3.2.3.2.8).

#### 3.2.3.2.4 The Time Horizon

It is expected that most often the recruiting aid will be run at the beginning of a year for the entire year. This is the case in this example where, as mentioned previously, we have selected a time horizon of 12 months. However, other time horizons (less than 12 months) are also possible. Users of the aid might make such a selection when, for example, they desire at the middle of a recruiting year to update results obtained previously when they ran the aid at the beginning of the year.

### 3.2.3.2.5 The Transition Probabilities

The transition probabilities carry a lion's share of the load of portraying the dynamics of the recruiting process. We therefore pay considerable attention to specifying these probabilities appropriately in what follows below.

First, let $p^{qa}(t, k-1 | s, k)$ denote the probability that a recruiting process in state s at some time k will be in state t at the next time k-1 when the recruiting management makes a decision indexed by:

- q(s, k) as to the monthly quota that it will assign to the recruiting organizations when in state s at time k;
- a(s, k) as to the monthly advertising funds that it will allocate to the recruiting organizations when in state s at time k.

Here, s and t are five-dimensional state vectors: $s = (s_1, s_2, s_3, s_4, s_5)$ and $t = (t_1, t_2, t_3, t_4, t_5)$. Similarly, the control indices q and a are five-dimensional. Here also, the probabilities $p^{qa}(t, k-1 | s, k)$ depend on parameters whose effect is not explicitly exhibited in the notation: market potentials, difficulties of recruiting, recruiter effectivenesses, etc.

We begin by assuming that the subordinate recruiting organizations operate independently.* Thus, we write

$$p^{qa}(t, k-1 \mid s, k) = \prod_{i=1}^{5} p_i^{q_i a_i}(t_i, k-1 \mid s_i, k). \tag{3}$$

Thus, to specify the joint transition probabilities $p^{qa}(t, k-1 \mid s, k)$, we need only specify the marginal probabilities $p_i^{q_i a_i}(t_i, k-1 \mid s_i, k)$ for the individual recruiting organizations.

Consider now one of the recruiting organizations -- say the $i^{th}$. In the discussion below we will fix attention on this $i^{th}$ organization and seldom refer again to the others. For convenience, therefore, we will omit all subscripts "i" whenever no ambiguity is likely to arise from doing so. For example,

$$p_i^{q_i a_i}(t_i, k-1 \mid s_i, k) = p^{qa}(t, k-1 \mid s, k).$$

The matter of selecting an appropriate functional form for $p^{qa}(\bullet, k-1 \mid s_i, k)$ was the subject of much discussion during the early phases of this project. We enquired about this with persons at the Air Force Human Resources Laboratory, the Navy Personnel Research and Development Center, and elsewhere. It appeared then, and still does now, that an appropriate function, while esti-

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* Such dependencies might arise if, for example, it were the case that the market areas of the subordinate recruiting organizations overlapped to some extent.

mable, is not now known. In the absence of information about this function, therefore, we will make a rational assumption. A function that appears to have a reasonable form is shown in Figure 3.5a.

Note that the function $p^{qa}(\bullet, k-1 | s, k)$ shown in Figure 3.5a (called a "tent function") assigns zero probability to states $t \leq s$ and to states $t \geq s + \alpha \cdot q$. This amounts to assuming that existing recruits cannot be lost, and that no more recruits can be obtained than a number $\alpha \cdot q$ that is proportional to the quota assignment index q, as is reasonable.* To other states t -- those between s and $s + \alpha \cdot q$ -- a non-zero probability is assigned.

The maximum likelihood point $s + m(s,k)$ of the tent function shown in Figure 3.5a depends on many factors. For example, for a given state s and decision q as to the quota to be assigned to a recruiting organization, a situation with adverse recruiting conditions (as, e.g., one with a low market potential, high difficulty of recruiting, low recruiter effectiveness, a small amount of advertising funds to be allocated, etc.) might have transition probabilities as shown in Figure 3.5b, whereas a situation with relatively good recruiting conditions might have transition probabilities as shown in Figure 3.5c.

The variable $m(s,k)$ depends on the factors mentioned above.

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* The scaling factor $\alpha$ in Figure 3.5 converts quota assignment indices q into a certain number of persons, expressed in person-units, that is appropriate for a particular recruiting organization.

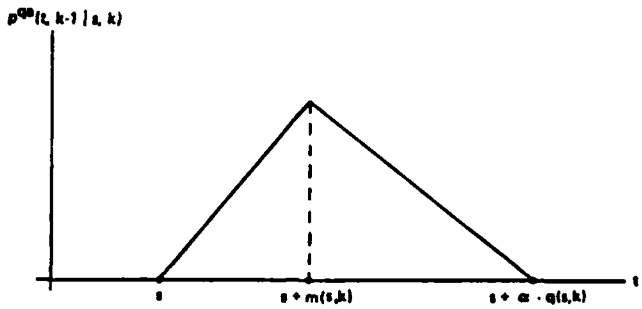


Figure 3.5a The "tent function" -- an assumed functional form for the transition probabilities.

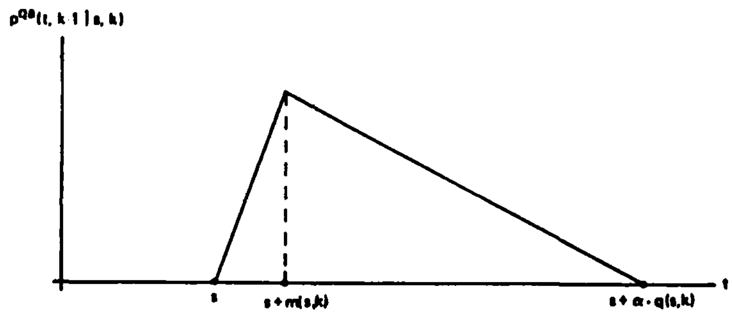


Figure 3.5b Transition probabilities under adverse recruiting conditions.

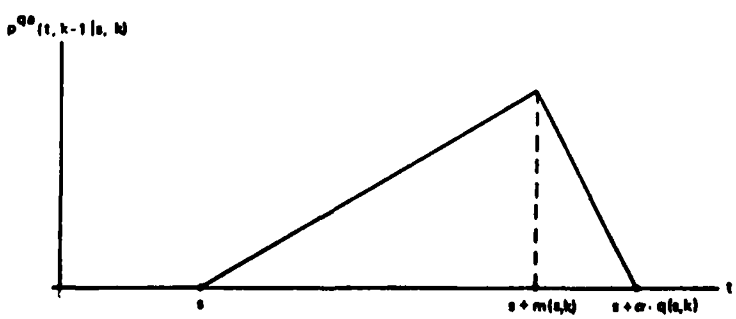


Figure 3.5c Transition probabilities under good recruiting conditions.

For a given index q(s, k) of the quota to be assigned to a recruiting organization, and for a given choice of the index a(s, k) of advertising funds to be allocated, we will estimate m at time k as

$$m(s,k) = \gamma \bullet q(s,k) \bullet (e(k)/d(k)) \bullet r(a,k) \quad (4)$$

where*:

| | | |
|---|---|---|
| $\gamma$ | = | a scaling constant; |
| d(k) | = | "difficulty of recruiting" at time k; |
| e(k) | = | "recruiter effectiveness" at time k; |
| r(a, k) | = | number of accessions expected for time period k when advertising index "a" applies at that time. |

This m has the right qualitative behavior. The users appropriate choice of the scaling factor $\gamma$ should enable them to ensure that m has the right quantitative behavior.

We have already discussed in Section 3.2.1 how we have defined and can compute the difficulty d(k) of recruiting at time k for a recruiting organizations, and in Section 3.2.2 how we have defined and can compute the recruiter effectiveness e(k) at that time for the organization. To complete a specification of the

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* All of these values may be different for different recruiting organizations. All also depend on the category of recruits for which the model is being run.

transition probabilities, therefore, it remains to discuss how to obtain the expected number r(a,k) of recruits that a recruiting organization will obtain at time k if the recruiting management makes allocation "a" of advertising funds to the organization at that time.*

The value of the advertising response function r(a,k) that appears in our formula for the maximum likelihood point "m" of a transition probability function is based on a model due to Vidale and Wolfe [7] that represents the sales-response to advertising over time. After adapting the Vidale-Wolfe advertising model to our use, the basic equations are:

$$\frac{dr(a,k)}{dt} = \rho \bullet A(a) \bullet \frac{pot(k)-r}{pot(k)} - \lambda(r-b) \qquad (5)$$

where:

| | | |
|---|---|---|
| r(a,k) | = | expected number of recruits that will be obtained by a recruiting organization at time k; |
| $\rho(k)$ | = | an advertising response constant; |
| A(a) | = | advertising dollars allocated by the recruiting management to the recruiting organization under management advertising decision index "a"; |

---

* Here, as explained previously, "a" is a decision index that indicates which advertising decision the management makes with respect to the recruiting organization: $1 \leq a \leq 3$ or $1 \leq a \leq 4$ depending on user choice.

pot(k) = market potential at time k;

$\lambda$(k) = a decay constant that measures the rate at which accessions are not achieved at time k because of "forgetting" an advertising message;

b(k) = expected number of recruits that would occur in time period k in the absence of any advertising effort.

Setting the derivative to zero gives the steady state solution of the above equations as

$$r(a,k) = \frac{\lambda b(k) + \rho A(a)}{\lambda + \frac{\rho A(a)}{pot(k)}} . \tag{6}$$

Thus, $r(a,k) \geq b$, and r(a,k) is increasing with A, as is intuitive.

At this point in our discussion of the transition probabilities, we have indicated how to compute or otherwise obtain various functions and parameters that the model needs. Many of these parameters depend on certain other quantities such as market populations pop($\ell$) and potentials pot($\ell$) at times $\ell$. These latter quantities, therefore, must be available to users of the aid in order that they will be able to specify the transition probabilities for the model. However, these latter quantities are well removed from the recruiting model per se. We therefore defer a discussion of where to find these quantities, and how to use them, to Section 3. 3 on inputs to the recruiting aid.

### 3.2.3.2.6 Value Computation

One of the computational module's raw outputs is a set of "values," one such value being associated with each state/time pair. Such a value $v(s,k)$ is a measure of how good it is for a recruiting management to be in state s at time $k \geq 0$. In our scheme of things, values are non-negative and small values are better.

The way that values $v(s,k)$ are computed depends on whether the user of the computational module has chosen to specify control policies for a recruiting management to use in assigning quotas and allocating advertising funds to its subordinate recruiting organizations, or whether it was chosen to have the module determine "optimal" control policies and to imagine that the management employs them. The user makes this choice at running time.

Consider first the case where a user of the module has specified quota and advertising policies on a recruiting management. In this case, the control indices $q = q_i(s,k)$ and $a = a_i(s,k)$ are known to the user (and are available to the module) for all times k. Under these conditions, the $v(s,k)$ are taken as:

$$v(s,0) = \begin{cases} w_1 \quad \sum_{i=1}^{5} s_i - AFN \quad ^{2} & \text{if } \sum_{i=1}^{5} s_i \leq AFN; \\ w_2 \quad \sum_{i=1}^{5} s_i - AFN \quad ^{2} & \text{if } \sum_{i=1}^{5} s_i > AFN \text{ and } LVO = 2; \\ 10^4 & \text{if } \sum_{i=1}^{5} s_i > AFN \text{ and } LVO = 1. \end{cases} \qquad (7a)$$

$$v(s,k+1) = \sum_{t} \prod_{i=1}^{5} p_i^{q_i a_i}(t_i|s_i) \Big[ \underbrace{w_a \cdot \sum_{i=1}^{5} a_i + w_q \cdot \sum_{i=1}^{5} |q_i - 2|}_{\text{penalty term}} \tag{7b}$$

$$+ v(t,k) \Big] \quad (\text{for } k \geq 0).$$

Here, $w_1$, $w_2$, $w_a$ and $w_q$ are weights supplied by the user at running time, and AFN is the Air Force's exogenously specified need for recruits of a certain category over a certain period of time.

Notice that the penalty term in the above formula penalizes the recruiting management for allocating advertising funds and for assigning abnormally high or low quotas to them. The user-assigned weights $w_a$ and $w_q$ make these penalties commensurable with the values $v(s,k)$.

Notice also that the values $v(s,k)$ can be computed recursively from the above formulas for all states s and times $k \geq 0$. For $k \geq 1$, the values $v(s,k)$ are just conditional expected values over what could happen to the recruiting management at later times, given that the management will use the specified control policies as to assigning quotas and allocating advertising funds to its subordinate recruiting organizations.

Consider next the case where a user of the module wishes to examine what happens when the recruiting management employs optimal control policies. In this case, the v(s,k) are taken as:

$$v(s,0) = \begin{cases} w_1 \left( \sum_{i=1}^{5} s_i - AFN \right)^2 & \text{if } \sum_{i=1}^{5} s_i \le AFN; \\ w_2 \left( \sum_{i=1}^{5} s_i - AFN \right)^2 & \text{if } \sum_{i=1}^{5} s_i > AFN; \end{cases} \tag{8a}$$

$$v(s,k+1) = \min_{\substack{q_1 \cdots q_5 \\ a_1 \cdots a_5}} \sum_t \prod_{i=1}^{5} p_i^{q_i a_i} (t_i | s_i) \cdot \underbrace{\left[ w_a \sum_{i=1}^{5} a_i + w_q \sum_{i=1}^{5} |q_i - 2| + v(t,k) \right]}_{\text{penalty term}} \tag{8b}$$

(for $k \ge 0$).

Here, $w_1$, $w_2$, $w_a$, $w_q$ and AFN have the same meanings as described above. As in the previous case, the values v(s,k) can be computed recursively from the above formulas for all states s and times $k \ge 0$, the optimal control policies for quota assignment and advertising allocations being produced (via exhaustive enumeration) as a by-product of this computation. For $k \ge 1$, the v(s,k) are just conditional expected values over what could happen to the recruiting management at later times under the supposition that the management employs optimal control policies.

Besides the values v(s,k) discussed in the preceding section, the computational module also provides as raw output the recruiting management's control policies -- optimal or otherwise -- under which these values were computed.

3.2.3.2.7 Probabilities of State Occupancy

Another of the computational module's raw outputs is a set of state occupancy probabilities, one such quantity being associated with each state/time pair. A state occupancy probability p(s,k) is simply the probability that the recruiting process will be in state s at time k.*

We use the following formulas to compute the p(s,k):

$$p(s,k_0) = \begin{cases} 1 & \text{if } s = b; \\ 0 & \text{otherwise;} \end{cases}$$

$$p(s,k-1) = \sum_t p^{qa}(s,k-1 \mid t,k) \cdot p(t,k) \quad (\text{for } 1 \le k \le k_0).$$

---

* The state occupancy probabilities are conditional on the process being in some known state b at the beginning of time for the run. The dependence of the p(s,k) on this b; which is a user input, is not explicitly exhibited in the notation.

Here, $k_0$ is the beginning of time for computations (e.g., $k_0 = 12$ for a typical run of the computational module over a 12-month time span), and b is the known initial state. The indices $q = q(s,k)$ and $a = a(s,k)$ are the recruiting management's quota control and advertising control for state s and time k (optimal or not). The $p^{qa}(s, k-1 \mid t, k)$ are the transition probabilities for the recruiting process under these control strategies.

#### 3.2.3.2.8 Computational Considerations

We have already mentioned in our overview of the recruiting aid in Section 2.2 that the aid's computation module can be computationally intense. While several factors contribute to this, the lion's share of this burden is directly traceable to the fact that the sizes of the state and decision spaces that we employ in our model of the recruiting process -- the one that we have embedded in the computational module -- can be "large." Indeed, a reasonable measure of the computational complexity C of the recruiting model is the product $S \cdot D$, where S is the number of states and D is the number of management decisions per state: $C = S \cdot D$.

In this section, we present formulas for the sizes S and D of the state and decision spaces, respectively, under various conditions. We also mention ways that a user of the module can reduce its running time by trading away the recruiting model's built-in flexibility and faithfulness to reality in return for smaller sizes for the state and decision spaces.

In the discussion to follow, let

G = number of subordinate recruiting organizations in the recruiting (sub)system being modeled;

L = number of recruit attainment levels that we distinguish for each recruiting organization;*

Q = number of decisions available to the recruiting management in the (sub)system being modeled with respect to assigning quotas to a particular subordinate recruiting organization;**

A = number of decisions available to the recruiting management in the (sub)system being modeled with respect to allocating advertising funds to a particular subordinate recruiting organization;**

Modeling in a straightforward fashion -- without resorting to "artwork" aimed at reducing the sizes of the state and decision spaces -- a recruiting (sub)system with G recruiting organizations, L distinguishable levels of recruit attainment for each recruiting

---

* This L, therefore, indicates the degree to which we resolve recruits.

** When the user elects to specify the recruiting management's controls, we have Q = A = 1. When the user elects to determine optimal controls, we have Q = 3 or 4 and A = 3 or 4 at the user's choice, as explained previously.

organization, Q management quota-related decisions per recruiting organization and A management advertising-related decisions per recruiting organization requires state and decision spaces having the following sizes:

$$S = L^G;$$

$$D = (QA)^G.$$

When G = 5 and L = 10, and when Q = A = 3 (the usual user choice in the case when they want the module to determine optimal controls), we have $S = 10^5$ and $D = 9^5 = 59049$. A model with this many states and this many decisions per state must be considered "large" even in today's world of large and fast computers.

Clever methods to reduce the sizes of the recruiting model's state and decision spaces are, of course, possible. However, a model that is initially "too large" cannot be made feasible even with the cleverest of artwork. For this reason, we have set limits on the recruiting (sub)system that can be modeled as follows:

- the number of subordinate recruiting organizations can be at most 5;
- the number of quota assignments that the central recruiting management can choose from in assigning quotas to each organization is at most 4;

- the number of funding allocations that the central management can choose from in allocating funds to each organization is at most 4;
- the number of recruiting time periods over which recruiting can occur is at most 12.

These limits are enforced via hardwiring in the computer code that implements the computational module.

Even within these limits, a user of the computational module may still find it appropriate to give up some of the recruiting model's faithfulness and flexibility in order to reduce its computational complexity. Three methods for doing this are discussed below. The first of these methods is directed at reducing the size of the model's state space. The other two are directed at reducing the size of the model's decision space. All three methods are made available to the user of the computational module at running time.

We have noted above that a model of a recruiting subsystem with G subordinate recruiting organizations, each of whose recruiting attainments are distinguished at L levels, has a state space with $L^G$ states. However, some of these states are unlikely ever to be occupied by the real recruiting system. For example, if the recruiting system begins a time of operation in state b = (0, 0, 0, 0, 0) -- that is, with no recruits -- then such states as (0, 0, 0, 0, 10), (1, 3, 2, 10, 2), (8, 8, 7, 1, 9) and others, some of whose components differ by a "large" amount, are unlikely for the recruiting system. More generally, if the recruiting system begins a time of operation in state $b = (b_1, b_2, b_3, b_4, b_5)$, then states $s = (s_1, s_2, s_3, s_4, s_5)$ with

$$|(s_i - b_i)/\alpha_i - (s_j - b_j)/\alpha_j| \geq h$$

for any $i \neq j$ are usually unlikely for it.* Users of the computational module may, therefore, feel that they can, without undue damage to the model's portrayal of the dynamics of the recruiting process, reduce the size of the model's state space by excluding from it these unlikely states.

Limiting the state space to consist of only those states that satisfy the above formula results in a state space size S of

$$S = (h+1)^G + (L-h-1) \cdot [(h+1)^G - h^G].$$

This h is specifiable by a user of the computational module at running time. With G = 5, L = 10 and h = 4, we get S = 13630. This compares favorably with the sizes $S = L^G = 10^5$ that applies when we do not exclude unlikely states in this manner.

We have also noted above that a model of a recruiting (sub) system with G subordinate recruiting organizations and Q and A management decisions per organization concerning, respectively, quota assignments and advertising allocations, has a decision space with $(QA)^G$ decisions. Just as some of the states in a full sized state space seem unlikely as described above, some of the deci-

* These $\alpha_i$ are the same as the scaling factors used to compute the transition probabilities (see Figures 3.5a - 3.5c).

sions in the full decision space may be considered inappropriate in some cases.

Indeed, it appears that a decision may be inappropriate for two reasons. On the one hand, a decision (q, a) -- that is, a decision $q = (q_1, q_2, q_3, q_4, q_5)$ and $a = (a_1, a_2, a_3, a_4, a_5)$ -- might be considered "intra-organization-inappropriate" if there is a mismatch between $q_i$ and $a_i$ for some i, $1 \leq i \leq 5$. For example, a decision (q, a) with $q_4 = 3$ and $a_4 = 1$ might be considered inappropriate in this sense because this decision would assign a larger-than-normal monthly quota to one of the recruiting organizations (namely, recruiting organization #4), while allocating a smaller-than-normal amount of advertising funds to that organization. Similarly, a decision (q, a) with $q_4 = 1$ and $a_4 = 3$ might be considered inappropriate. On the other hand, a decision (q, a) might be considered "extra-organization-inappropriate" if there is a mismatch in the $q_i$ for $1 \leq i \leq 5$, or in the $a_i$ for $1 \leq i \leq 5$. For example, a decision $q = (1, 1, 1, 1, 3)$ might be considered inappropriate in this sense because this decision would assign lower-than-normal monthly quotas to recruiting organizations 1 - 4 while assigning a higher-than-normal monthly quota to organization #5, thus splitting the recruiting load unevenly among the five recruiting organizations.

Consider first the intra-organization decision mismatches. Figure 3.6a shows a matrix of quota and advertising decisions available to any one of the five recruiting organizations. (In the figure, we have assumed that the number Q of quota decisions and the number A of advertising decisions available to the recruiting

Advertising Index

| Quota Index | ① | ② | ③ | ④ |
|---|---|---|---|---|
| ① | (1, 1) | (1, 2) | (1, 3) | (1, 4) |
| ② | (2, 1) | (2, 2) | (2, 3) | (2, 4) |
| ③ | (3, 1) | (3, 2) | (3, 3) | (3, 4) |
| ④ | (4, 1) | (4, 2) | (4, 3) | (4, 4) |

Figure 3.6a Decision matrix for quota assignment of advertising-fund-allocation for a particular recruiting organization.

Advertising Index

| Quota Index | ① | ② | ③ | ④ |
|---|---|---|---|---|
| ① | (1, 1) | (1, 2) | (1, 3) | (1, 4) |
| ② | (2, 1) | (2, 2) | (2, 3) | (2, 4) |
| ③ | ~~(3, 1)~~ | (3, 2) | (3, 3) | (3, 4) |
| ④ | ~~(4, 1)~~ | ~~(4, 2)~~ | (4, 3) | (4, 4) |

Figure 3.6b Decision matrix with two lower diagonals deleted.

Advertising Index

| Quota Index | ① | ② | ③ | ④ |
|---|---|---|---|---|
| ① | (1, 1) | (1, 2) | (1, 3) | ~~(1, 4)~~ |
| ② | (2, 1) | (2, 2) | (2, 3) | (2, 4) |
| ③ | (3, 1) | (3, 2) | (3, 3) | (3, 4) |
| ④ | (4, 1) | (4, 2) | (4, 3) | (4, 4) |

Figure 3.6c Decision matrix with one upper diagonal deleted.

organizations are Q = A = 4.) Because large quotas and low advertising fund allocations for a particular organization may be considered inappropriate as suggested above, we may wish to eliminate decision combinations corresponding to one or more lower diagonals of the decision matrix, as illustrated in Figure 3.6b. Similarly, we may wish to eliminate the decisions corresponding to one or more upper diagonals, as illustrated in Figure 3.6c. We call this decision-eliminating-procedure the "corner cutting procedure."

When the corners of the decision matrices for all of the recruiting organizations are cut in the same way, and if $D_L$ and $D_U$ denote the number of lower and upper diagonals, respectively, that are cut off from each organization's decision matrix, the number of decisions in the decision space is

$$D = \left[ QA - \left( \frac{D_L \cdot (D_L+1)}{2} + \frac{D_U \cdot (D_U+1)}{2} \right) \right]^G .$$

For example, if G = 5 and Q = A = 3, and if $D_L = D_U = 1$, then $D = 7^5 =$ 16807. This compares with the size D = 59049 that applies when we do not exclude inappropriate intra-organization decisions from the decision space.

Consider next the extra-organization decision mismatches. As suggested above, a user of the computational module may consider quota assignments $q = (q_1, q_2, q_3, q_4, q_5)$ for which

$$\max_{1 \leq i \neq j \leq 5} |q_i - q_j| > k_q$$

(where $k_q$ is some user-specified constant) to be so uneven as to be inappropriate. Such users are, in effect, saying that they feel that all indices $q_j$ for admissible quota decision vectors q should come from the same one of a number of subsets of the full set of Q decision indices. Similar comments apply to advertising decisions.

To illustrate these extra-organization mismatches, suppose that Q = 4 (and that the full set of quota decision indices is $\{1, 2, 3, 4\}$). If the user chooses to use the full quota decision space, any of the four quota assignments could be given to each of the five recruiting organizations. Some of these combinations are, of course, uneven. Users could, however, choose to eliminate what they consider to be uneven quota decision vectors q by insisting that all indices $q_j$ come from the subset $\{1, 2\}$ or from the subset $\{2, 3\}$, or from the subset $\{3, 4\}$. Other choices of subsets are also possible.

More generally, suppose that the user divides the Q quota decision indices into $n_Q$ subsets of $m_Q$ indices each, and insists that a quota decision vector $q = (q_1, q_2, q_3, q_4, q_5)$, to be admissible, must draw all of its indices $q_j$ from any one of these subsets. Similarly, suppose that the user divides the A advertising decision indices into $n_A$ subsets of $m_A$ indices each, and defines an admissible advertising decision vector $a = (a_1, a_2, a_3, a_4, a_5)$ as being one for which the $a_j$ all come from any one of these subsets. Under these conditions, the number D of admissible decisions is

$$D = (n_Q n_A) \cdot (m_Q m_A)^G$$

if the subsets do not overlap, and is less than this quantity otherwise.

As an example of the utility of this procedure, suppose that $G = 5$ and that $Q = A = 3$. Suppose also that a decision $(q, a)$ is considered admissible only if its quota indices $q_j$ all come from the subset $\{1, 2\}$ or all from the subset $\{2, 3\}$, and similarly, the $a_j$ must all come from $\{1, 2\}$ or from $\{2, 3\}$. In this case, $n_Q = n_A = 2$ and $m_Q = m_A = 2$, and

$$D \leq (2 \cdot 2) \cdot (2 \cdot 2)^5 = 4096.$$

This compares with the size $D = 59049$ that applies when the user does not exclude inappropriate "extra-organization" decisions in this manner.

Finally, note that a user of the computational module could elect to keep the recruiting model's decision space at a manageable level by excluding from the space both intra-organization-inappropriate decisions and extra-organization-inappropriate decisions. Both options are, in fact, made available to the user at running time.

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* The size of the decision space under these conditions depends on how the user chooses the subsets of admissible quota and advertising indices.

### 3.3 Inputs to the Computational Module

A user of the computational module may elect either to provide the inputs that the module requires at running time, or to have the module obtain all necessary data from a previously prepared disk data file. In the former case, the module prompts the user to make all necessary entries.

Whichever method of data entry is selected by the user, the module:

- displays the inputs to the user;
- checks that the inputs appear to be reasonable;
- warns the user as to any perceived improprieties in the input data;
- makes an input editing capability available to the user.

For the user's convenience all input items carry an identifying number. These ID numbers appear to the user at the time of data entry. They are also known to the module for data editing purposes.

There are 26 types of inputs to the computational module. These are listed in Table 3.1. A user of the module should be able to understand what is needed for most of the items just from the "brief description" given in the table together with the indicated references to other sections of this report where they are dis-

Table 3.1 Inputs of the Computational Module

| Input Number | Name of Input | Brief Description of Input | Relevant Sections |
|---|---|---|---|
| 1 | Air Force Need | Number of recruits (of a given category to be defined parametrically by other inputs) desired by Air Force over time horizon specified in Item 2. Number may be specified in terms of any convenient "person-unit." | 3.2.3.2.6 |
| 2 | Time Horizon | Number of recruiting time periods (typically, months) for which computations are desired. This number is at least 0 and at most 12. | 3.2.3.2.4<br>3.2.3.2.8 |
| 3 | Number of Recruiting Organizations | This number must be between 2 and 5. | 3.2.3.2<br>3.2.3.2.8 |
| 4 | Number of Recruit Attainment Levels for Each Recruiting Organization. | This number determines the degree to which each organization's recruiting attainments are resolved. | 3.2.3.2.1<br>3.2.3.2.8 |
| 5 | Maximum Weighted Difference between State/Starting State Component Values | Maximum weighted difference in state/starting state components for admissible states. This parameter has a significant effect on size of state space. | 3.2.3.2.8 |
| 6 | Type of Computation | Type = 1: Only state occupancy probabilities will be computed.<br>Type = 2: Optimal decisions will be computed.<br>Type = 3: User will specify decisions. | 2.1 |

Table 3.1 (Continued)

| Input Number | Name of Input | Brief Description of Input | Relevant Sections |
|---|---|---|---|
| 7 | Number of Quota Decisions Available to Central Recruiting Management for Each Recruiting Organization | This number must be at least 3 and at most 4. | 2.1<br>3.2.3.2.3 |
| 8 | Number of Funding Decisions Available to Central Recruiting Management for each Recruiting Organization | This number must be at least 3 and at most 4. | 2.1<br>3.2.3.2.3 |
| 9 | Number of Lower Diagonals Eliminated in the Corner Cutting Procedure (Decision Space Reduction Method #1) | This number must be between 0 and 2. Larger numbers result in smaller sizes for the decision space. A choice of 0 means that the method is not employed. | 3.2.3.2.8 |
| 10 | Number of Upper Diagonals Eliminated in the Corner Cutting Procedure (Decision Space Reduction Method #1) | This number must be between 0 and 2. Larger numbers result in smaller sizes for the decision space. A choice of 0 means that the method is not employed. | 3.2.3.2.8 |

Table 3.1 (Continued)

| Input Number | Name of Input | Brief Description of Input | Relevant Sections |
|---|---|---|---|
| 11 | Maximum Extra-Organization Difference between Quota Assignments | A control aimed at retaining evenness in quota assignments across recruiting organizations. | 3.2.3.2.8 |
| 12 | Maximum Extra-Organization Difference between Funding Allocations | A control aimed at maintaining evenness in funding allocations across recruiting organizations. | 3.2.3.2.8 |
| 13 | Suboptimal Control "B" Parameters | User-specifiable (suboptimal) control parameters $B_1$-$B_9$, having to do with central management's quota allocations to the recruiting organizations. | 3.3 |
| 14 | Suboptimal Control "C" Parameters | User-specifiable (suboptimal) control parameters $C_1$-$C_9$, having to do with central management's funding allocations to the recruiting organizations. | 3.3 |
| 15 | Gamma | A factor used in scaling the maximum likelihood points M for the recruiting model's transition probabilities. See Equation (4). | 3.2.3.2.5 |
| 16 | Difficulties of Recruiting | One such number, computed from Equation (1), is necessary for each recruiting organization for each time point. | 3.2.1<br>3.2.3.2.5 |

Table 3.1 (Continued)

| Input Number | Name of Input | Brief Description of Input | Relevant Sections |
|---|---|---|---|
| 17 | Recruiter Effectivenesses | One such number, computed from Equation (2), is necessary for each recruiting organization for each time point. | 3.2.2<br>3.2.3.2.5 |
| 18 | Advertising Response Function | One numerical response, computed from Equation (6), is necessary for each advertising decision for each time point. | 3.2.3.2.5<br>3.3 |
| 19 | *Alpha Factors* | Each $\alpha_i$ ($1 \leq i \leq 5$) is a measure of the $i^{th}$ recruiting organization's relative strength (these measures are used in the computation of the recruiting model's transition probabilities). See Figures 3.5a - 3.5c. | 3.2.3.2.5 |
| 20 | Last Value Option | When Option = 1: A fixed penalty of $10^4$ is assigned to all states s whose component sums $\Sigma s_i$ exceed the stated Air Force need.<br>When Option = 2: A variable penalty is assigned to states whose component sums exceed the Air Force need, in accordance with Equations 7a and 8a.<br>Note: For either value of the last value option, a fixed penalty of $10^5$ is assigned to any state one or more of whose components has "slopped over" -- that is, exceeded the number of recruit attainment levels specified in Item #4. | |

Table 3.1 (Continued)

| Input Number | Name of Input | Brief Description of Input | Relevant Sections |
|---|---|---|---|
| 21 | Under-achievement Penalty Coefficient | The weight $w_1$ to be placed on states s at time 0 whose component sums $\Sigma_i s_i$ do not exceed the total specified Air Force need; See Equations 7a and 8a. | 3.2.3.2.6 |
| 22 | Over-achievement Penalty Coefficient | The weight $w_2$ to be placed, when last value option = 2, on states s at time 0 whose component sums exceed the stated Air Force need. See Equations 7a and 8a. | 3.2.3.2.6 |
| 23 | Quota Reward Weight | The weight parameter $w_q$ that makes quota allocations commensurable with recruits gained; see Equations 7b and 8b. | 3.2.3.2.6 |
| 24 | Funding Reward Weight | The weight parameter $w_a$ that makes fund expenditures commensurable with recruits gained; see Equations 7b and 8b. | 3.2.3.2.6 |
| 25 | Starting State | The state b in which the recruiting system will be said to start at the beginning of time for a run of the computational module. | 3.2.3.2.7 |
| 26 | Maximum CPU Time | User's maximum allowable time for all specified time periods for computations. | Additional User Documentation |

cussed in more detail. For other input items, namely input types 13, 14, 16, 17 and 18, amplification seems necessary. These amplifications are given in the remainder of this section.

First, we discuss input types 13 and 14 -- the suboptimal control parameters. As mentioned previously, a user of the computational module can elect to specify the quota controls $q_i(s,k)$ and funding controls $a_i(s,k)$ that the central recruiting management will apply to the $i^{th}$ subordinate recruiting organization for $i = 1, \ldots, 5$ when the recruiting (sub)system is in state s at time k. This is done according to the following scheme:

$$q_i(s,k) = \begin{cases} 1 & \text{if } x_i(s,k) \in (-\infty, 1) \\ 2 & \text{if } x_i(s,k) \in [1, b_9] \\ 3 & \text{if } x_i(s,k) \in (b_9, \infty) \end{cases}$$

$$a_i(s,k) = \begin{cases} 1 & \text{if } y_i(s,k) \in (-\infty, 1) \\ 2 & \text{if } y_i(s,k) \in [1, c_9] \\ 3 & \text{if } y_i(s,k) \in (c_9, \infty) \end{cases}$$

where:

$$x_i(s,k) = b_i \left(\frac{k}{k_{max}}\right)^{-b_6} \cdot \left(1 - \frac{\Sigma}{AFN}\right)^{b_7} \cdot \left(1 + \frac{s_i}{AFN}\right)^{-b_8},$$

$$y_i(s,k) = c_i \left(\frac{k}{k_{max}}\right)^{-c_6} \cdot \left(1 - \frac{\Sigma}{AFN}\right)^{c_7} \cdot \left(1 + \frac{s_i}{AFN}\right)^{c_8},$$

for i = 1,...,5, and where:

$k_{max}$ = time horizon (input item #2);

AFN = Air Force need (input item #1);

$s_i$ = $i^{th}$ component of state vector s;

$\Sigma = \sum_{i=1}^{5} s_i$.

Here, parameters $b_1, \ldots, b_9$ and $c_1, \ldots, c_9$ are input items 12 and 13, respectively. They are provided by the user at running time.

The idea behind the formulas for the $x_i(s,k)$ and $y_i(s,k)$ is to give the user the flexibility to specify many different control policies quickly and efficiently. For example, a user can arrange to have $q_i(s,k) = 1$ for all s and k by choosing $b_1 = b_2 = b_3 = b_4 = b_5 = 0$. Similarly, they can arrange for all $a_i(s,k) = 3$ by taking $c_1 = c_2 = c_3 = c_4 = c_5 = 2$, $c_6 = c_7 = c_8 = 0$ and $c_9 = 1$. More generally, the user can design controls that have the "right" behavior with respect to:

- the time remaining to obtain recruits;
- the number of recruits already obtained by the recruiting (sub)system as a whole;
- the number of recruits already obtained by each of the recruiting organizations.

This is evident from the functional forms for the $x_i(s, k)$ and $y_i(s, k)$ as given above.

Next, we discuss input types 16, 17 and 18 -- the difficulties of recruiting d(k), the recruiter effectivenesses e(k), and the advertising response functions r(a, k). As indicated in the defining equations (1), (2) and (6), respectively, these quantities all depend on market populations pop($\ell$) and market potentials pot($\ell$) at various time points $\ell$.

The market population figures $pop_i(\ell)$ for the $i^{th}$ recruiting organization (i = 1, . . . , 5) are available as raw data. This data can be conveniently obtained from the "non-prior-service military-available" files of the Recruit Market Network data base. This file gives the number of individuals between ages 17 and 21, along with the number that are considered militarily available, for each recruiting area. This data came originally from the 1970 Census of Population and has been updated annually using mobility and morbidity data provided by the Census Bureau. Since the data is organized by recruiting service boundaries, it can be entered directly into the computational module.

Estimates of market potentials $pot_i(\ell)$ for the $i^{th}$ recruiting organization over the $\ell^{th}$ time period can be obtained via regression analysis using data currently collected, or data that could be collected, by the Market Analysis Division of the Air Force Recruiting Service. However, we want these estimates to be fair estimates in the sense that they should <u>not</u> reflect the effectiveness of the recruiters who happen to be in the $i^{th}$ organization at the time to which the data applies.* For this reason, it is necessary to modify the usual methods of regression in a manner now to be explained.

First, do a regression analysis on enlistments:

$$\text{enl}_i(\ell) = \sum_j a_j(\ell)x_{ij} + \sum_j b_j(\ell)y_{ij}$$

where:

$\text{enl}_i(\ell)$ = enlistments obtained by $i^{th}$ recruiting organization during $\ell^{th}$ time period;

$x_{ij}$ = independent variable that is not influenced by recruiter effectiveness;

$y_{ij}$ = independent variable that is influenced by recruiter effectiveness;

$a_j(\ell), b_j(\ell)$ = coefficients applicable to time period $\ell$.

This regression gives us the coefficients $a_j(\ell)$ and $b_j(\ell)$.

Next, we compute the average of the variables $y_{ij}$ that do depend on recruiter effectiveness:

$$\overline{y_j} = \frac{1}{G}\sum_{i=1}^{G} y_{ij},$$

where G is the number of recruiting organizations. Such a $\overline{y_j}$ is a typical value of the $y_{ij}$ from which it was obtained, but the effect

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* This is necessary for several reasons. For example, it is necessary so that the true effectiveness of a recruiting organization can be identified and appropriately recognized.

of differing relative effectivenesses of the recruiting organizations has been washed out. The market potential $pot_i(\ell)$ for enlistments from the $i^{th}$ organization can now be taken as

$$pot_i(\ell) = \sum_j a_j(\ell) x_{ij} + \sum_j b_j(\ell) \overline{y_j}.$$

Note that these estimates of market potential must be obtained for each category of recruit for which one wishes to run the model. Notice also that the choice of the $x_{ij}$'s and the $y_{ij}$'s depends on data availability at a given time and the $R^2$ values that different combinations of these variables produce. The combination that produces the highest $R^2$ should be used.

The important point of the above discussion is that the $x_{ij}$'s must be pure variables like per capita income, unemployment, high school ASVAB leads, advertising response, number of local Air Force Bases, QMA or QHS per square mile, etc. The $y_{ij}$ will be variables like past Air Force accessions, which have tended to receive heavy weights in past regression analyses on enlistments.

As presented, the above equation for the $pot_i(\ell)$ is cross-sectional in form. Whenever new data becomes available, the regressions on enlistments and computations of market potential should be redone. Over time, this will result in a set of values $pot_i(\ell)$ that can be used not only in determining difficulties of recruiting, recruiter effectivenesses, and other quantities, but also in forecasting trends in enlistments.

## 3.4 Outputs of the Computational Module

Each run of the computational module results in the following "raw outputs:"

- identifying information about the run (including input data);
- computational results:
  - values v(s, k);
  - decisions q(s, k) and a(s, k) (optimal or otherwise);
  - state occupancy probabilities p(s, k).

The module writes these outputs on a semi-permanent disk file, as illustrated in Figure 3.2.

As mentioned previously, the raw output disk files produced by the computational module are accessible to the display module. The display module then post-processes the raw outputs to produce "finished outputs." These matters are discussed more fully in Chapter 4 on the display module.

## 4.0 THE DISPLAY MODULE

The display module of the Air Force Recruiting Aid:

- accesses the raw output files produced by the aid's computational module;
- post-processes these outputs as necessary to produce certain "finished" outputs;
- displays the finished outputs in various ways on one or more display terminals, as requested by the user.

The display module is runnable only in interactive mode.

In this chapter, a macro-description of the recruiting aid's display module is given. The description here is "macro" in the same sense that the description of the aid's computational module in Chapter 3 was "macro," namely, in that the display module in terms of its structure, input and outputs is described.*

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* The algorithms used in the display module, being straightforward, are not discussed here.

## 4.1 Structure of the Display Module

Figure 4.1 shows the display module's major lines of user control and information flow. As with the computational module, a user of the display module deals with it through a control dialogue. After the module has all necessary control inputs, it accesses the raw data produced by a run of the computational module, post-processes the raw output as necessary, and displays it to the user in various ways.

## 4.2 Inputs to the Display Module

All inputs to the display module are control inputs. There are no data inputs.

A user of the display module can, via appropriate selection of the control inputs, specify:

- the kind of information to be displayed;
- the type of information to be displayed;
- the amount of information to be displayed.

These output options are all described in the next section.

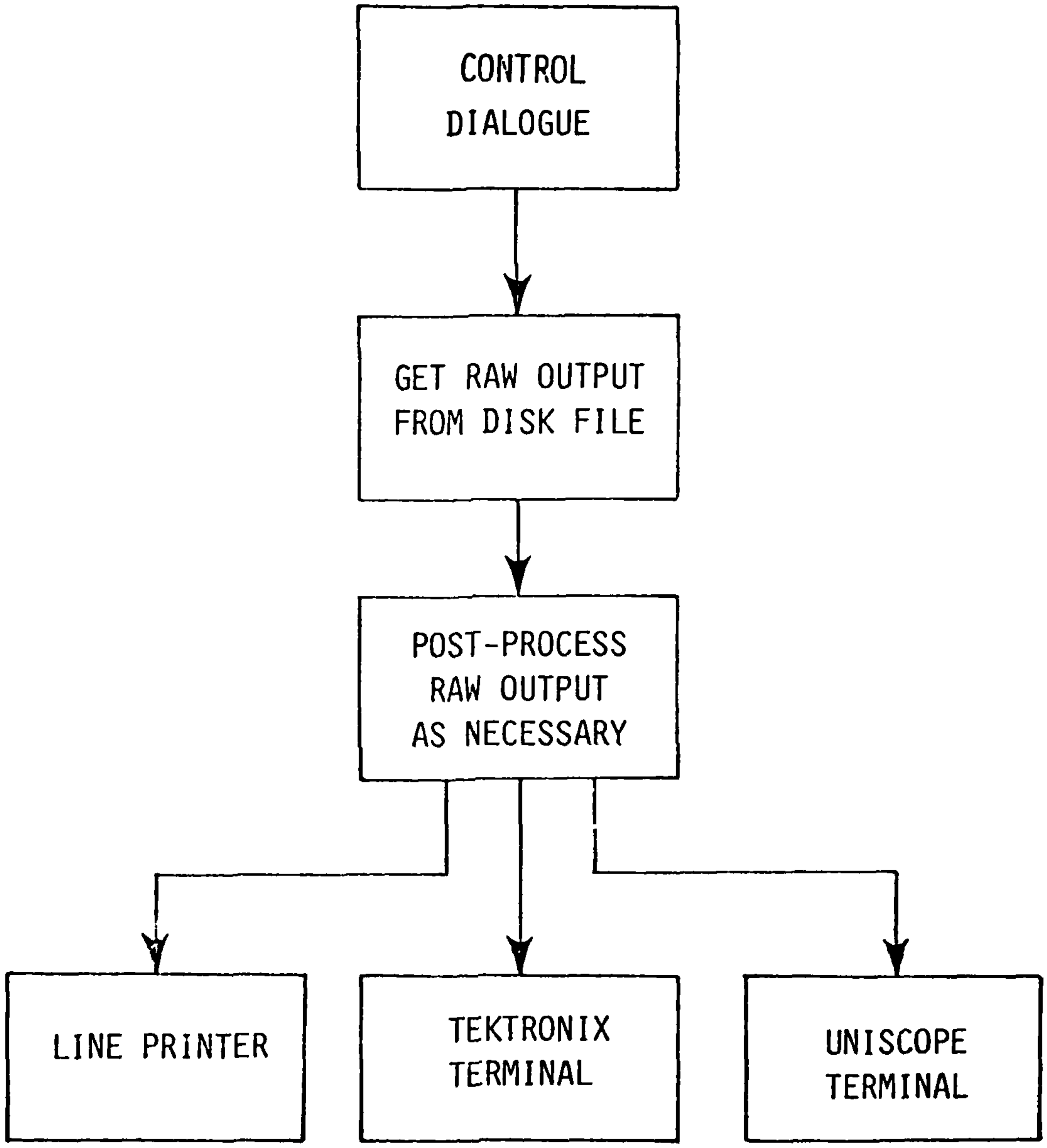


Figure 4.1 Display module's major lines of user control and information flow.

4.3 Outputs of the Display Module

As mentioned in the previous section, a user of the display module can select output options as to:

- the kind of output:
  - raw;
  - post-processed;
- the type of output:
  - values and decisions;
  - state occupancy probabilities;
- the amount of output:
  - all;
  - some.

Not all combinations of options are possible (not all are sensible).

There is one kind of raw output. A user of the computational module who elects to look at raw output will see just exactly that; namely, non-post-processed output from a run of the computational module. Users of the display module who elect to look at raw output cannot select the type of information they wish to see; they will see values, decisions and state occupancy probabilities. They can, however, select the amount of information they wish to see: either all or some. Information in the latter amount is likely to be of use only to persons involved in maintaining the recruiting aid.

Figure 4.2 shows the informational content of raw output: for each state/time pair, a value, a set of decisions concerning quota assignment and advertising fund allocations for a recruiting

| | TIME = 12 | | | | TIME = 11 | | | | TIME = 10 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| STATE | PROB | VALUE | $Q_1Q_2Q_3Q_4Q_5$ | $A_1A_2A_3A_4A_5$ | PROB | VALUE | $Q_1Q_2Q_3Q_4Q_5$ | $A_1A_2A_3A_4A_5$ | PROB | VALUE | $Q_1Q_2Q_3Q_4Q_5$ | $A_1A_2A_3$ | |
| 00000 | 1.000 | 525.4 | 33333 | 33222 | 0.246 | 526.9 | 33333 | 33333 | 0.013 | 527.9 | 33333 | 33333 | |
| 00001 | 0.000 | 525.1 | 33333 | 32222 | 0.108 | 525.7 | 33322 | 33222 | 0.017 | 526.4 | 33222 | 32222 | → |
| 00002 | 0.000 | 524.7 | 33322 | 22221 | 0.008 | 525.0 | 33322 | 33222 | 0.010 | 526.1 | 33332 | 33332 | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | → |
| 96756 | 0.000 | 187.4 | 12232 | 12233 | 0.002 | 189.2 | 12232 | 13233 | 0.001 | 191.3 | 22232 | 23233 | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | → |

Figure 4.2 Raw output of Recruiting Aid.

management with respect to its subordinate recruiting organizations, and a state/time occupancy probability. This information is, of course, interpretable only in the context of the inputs to the computational module from which it was produced.

There are several kinds of post-processed outputs. A user of the display module who elects to look at post-processed outputs must also choose the particular kind of post-processed output desired:

| TYPE | DESCRIPTION |
|---|---|
| 1a. | A graph or table showing the probability that the recruiting (sub)systems will have recruited various numbers of recruits at various times; |
| 1b. | A graph or table showing the probability that a selected subordinate recruiting organization will have recruited various numbers of recruits at various times; |
| 2a. | A graph or table showing the expected number (and standard deviation) of recruits that the recruiting (sub)system will have recruited by various times; |
| 2b. | A graph or table showing the expected number (and standard deviation) of recruits that a selected subordinate recruiting organization will have recruited at various times; |

| TYPE | DESCRIPTION |
|---|---|
| 3a. | The probability that the recruiting (sub)system will be within (Z) % of the Air Force need by time (T); |
| 3b. | The number of recruits that the recruiting (sub) system will obtain by time (T) with probability at least (P). |

Thus, output types 2a and 2b are aggregates of output types 1a and 1b, respectively. We call output types 3a and 3b "user queries." The circled quantities in the above description of these output types are provided by the user in response to prompts from the display module.

Examples of post-processed output types 1a and 1b are shown in Figures 4.3a and 4.3b. Examples of post-processed output types 2a and 2b are shown in Figures 4.4a and 4.4b, respectively.

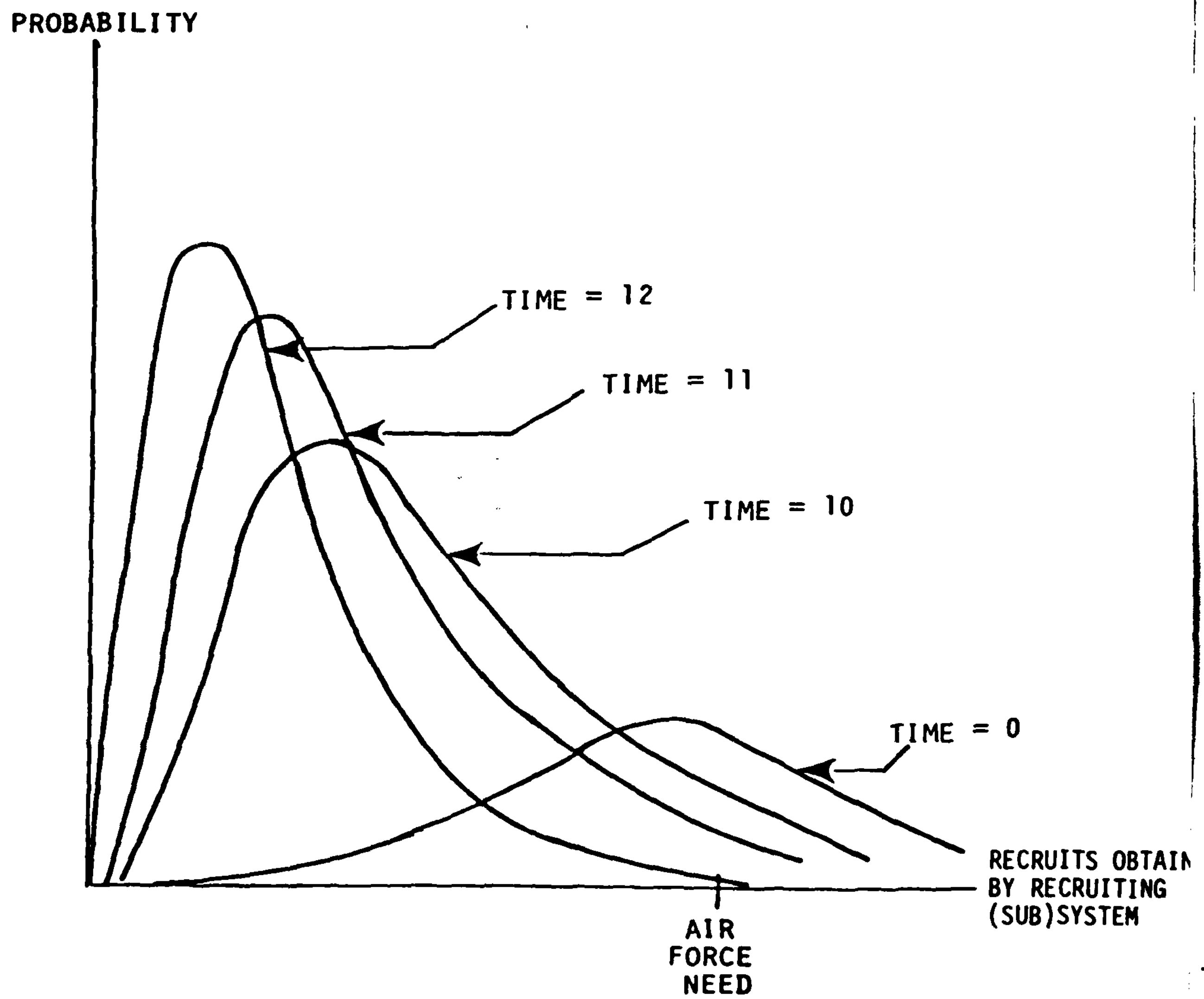


Figure 4.3a Post-processed Output Type 1A -- A graph or table showing probability that the recruiting (sub)system will have recruited various numbers of recruits at various times.

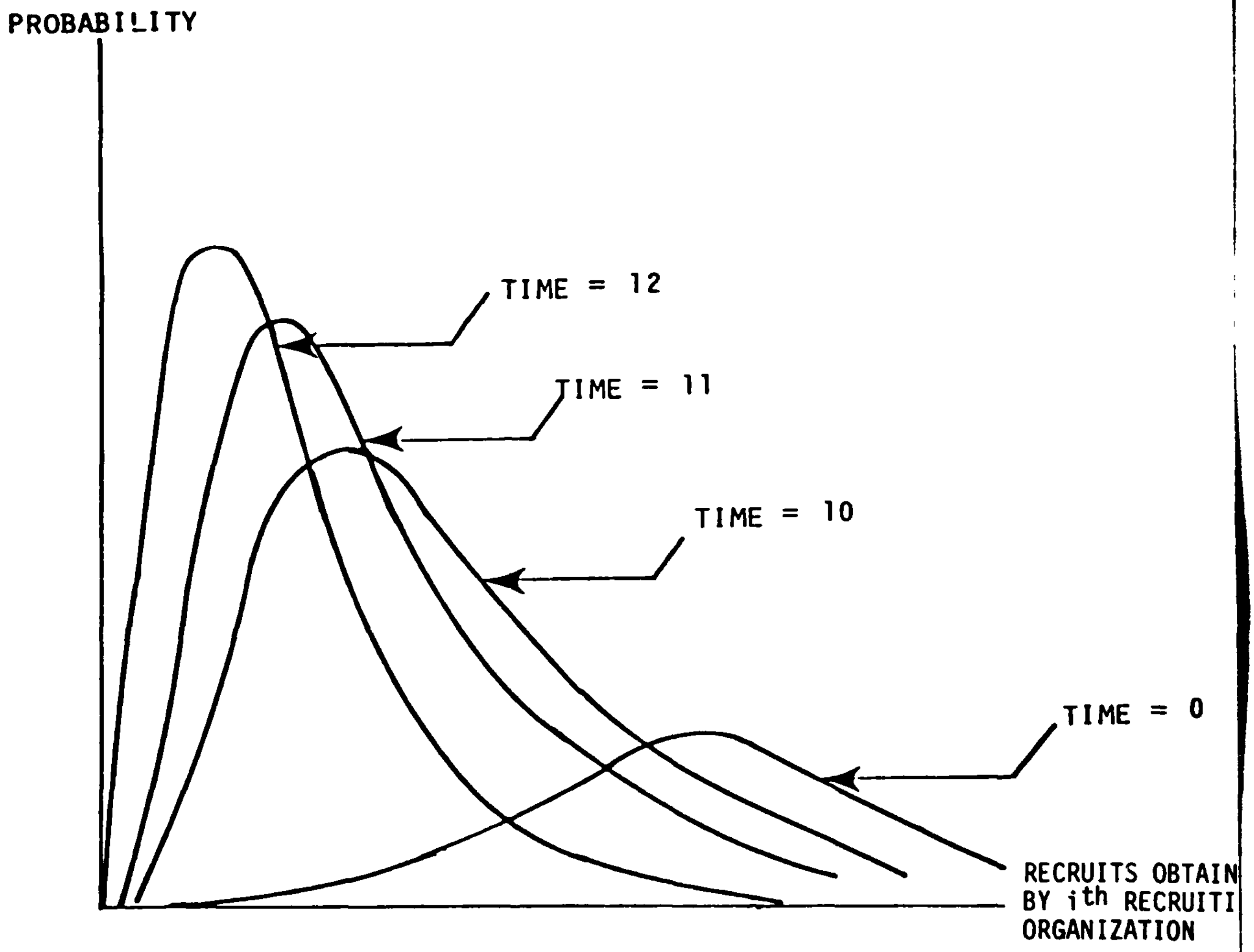


Figure 4.3b Post-processed Output Type 1B -- A graph or table showing the probability that the $i^{th}$ recruiting organization will have recruited various numbers of recruits at various times.

| TIME | MEAN OF SUM | STANDARD DEVIATION OF SUM |
|---|---|---|
| 12 | 0.0 | 4.1 |
| 11 | 10.2 | 9.5 |
| 10 | 19.7 | 27.7 |
| . | . | . |
| . | . | . |
| . | . | . |
| 0 | 120.1 | 411.8 |

NOTE: "SUM" REFERRED TO HERE IS THE SUM OF STATE COMPONENTS (I.E., TOTAL NUMBER RECRUITED).

Figure 4.4a Post-processed Output Type 2A -- A graph or table showing certain aggregates of the probability distribution for the recruiting (sub)system having recruited various numbers of recruits at various times.

| TIME | MEAN OF COMPONENT | STANDARD DEVIATION OF COMPONENT |
|---|---|---|
| 12 | 0.0 | 0.8 |
| 11 | 2.1 | 1.4 |
| 10 | 4.6 | 9.4 |
| . | . | . |
| . | . | . |
| . | . | . |
| 0 | 9.9 | 11.0 |

Figure 4.4b. Post-processed Output Type 2B -- A graph or table showing certain aggregates of the probability distribution for the $i^{th}$ recruiting organization having recruited various numbers of recruits at various times.

## 5.0 AN EXAMPLE

In this section we illustrate AFRA using a test case based on a real-world recruiting situation. Consider a recruiting (sub) system with a recruiting management and two subordinate recruiting organizations. Suppose that the recruiting organizations have market areas that vary significantly in their recruiting potential. Assume that the total number of recruits needed is 28,000. At 2,000 recruits per person-unit, the first input, Air Force Need, is set at 14 person-units. To accommodate the strength differences among the groups, 10 different levels of recruit attainment will be allowed for each organization. We allow three time periods (months) for the goal to be achieved, and three levels of quota and funding allocation for each recruiting organization.

### 5.1 Market Potential Data and Other Inputs

The market characteristics of the two organizations are summarized in Table 5.1. As shown, organization #1 is the weaker of the two because the market potential, pot($\ell$), of its area represents a smaller fraction of the raw age-eligible population than is the case for organization #2. Hence the difficulty of recruiting measure for organization #1 is higher than for organization #2. However, since recruiters in organization #1 have managed to come close to reaching their enlistment goals in the past, the effectiveness rating of this group is also quite high. In contrast, organization #2 can be considered relatively strong. Out of a smaller raw age-eligible population, there are almost twice as many potential recruits as in organization #1's area. Thus the difficulty of recruiting is much

Table 5.1 Market potential for the two recruiting organizations in the test case.

| | GROUP 1 | GROUP 2 |
|---|---|---|
| RAW POPULATION | 110,500 | 90,000 |
| POT($\ell$) | 10,000 | 18,000 |
| DIFFICULTY OF RECRUITING | .91 | .80 |
| h($\ell$) RECRUITS | 9,700 | 18,850 |
| RECRUITER EFFECTIVENESS | .89 | .84 |
| ADVERTISING VARIABLES: | | |
| b(k) | 667 | 1,200 |
| $\rho$ | .04 | .08 |
| $\lambda$ | .1 | .1 |
| EXPECTED ACCESSIONS: | | |
| r(1, k, $\ell$) | 2,150 | 4,047 |
| r(2, k, $\ell$) | 3,072 | 5,781 |
| r(3, k, $\ell$) | 3,686 | 6,937 |

lower and, despite good recruiting success, the effectiveness rating is lower as well.

The advertising variables indicate that the base recruiting rates b(k) that apply in the absence of advertising are set at 20% of the market potential for each time period. Since there are three time periods in this test case, the b(k) and the response figures $r(a, k, \ell)$ are roughly one-third of each group's total market potential. Examination of the response-per-dollar-spent parameter, $\rho$, shows one reason why organization #2 does so well; past figures indicate that the area is twice as responsive to advertising effort as is the area of organization #1. The sales decay parameter $\lambda$ is set at .1 for both groups. The expected accessions figures arise from the steady-state solution of the equation given in Section 3.2.3.2.5. If we take an advertising index of 2 as representing an "average" expenditure level, we can see that dropping the level to 1 reduces the accessions by 30% and increasing the level to 3 raises the accessions by 20%. In terms of numbers of expected enlistees, organization #2 is almost twice as strong as organization #1.

The actual input values for this test case are shown in Table 5.2.

Table 5.2 Inputs for the Test Case.

| Input Number | Name of Input | Input Value |
|---|---|---|
| 1 | Air Force Need | 14 person-units |
| 2 | Time Horizon | 3 time periods |
| 3 | Number of Recruiting Organizations | 2 |
| 4 | Number of Recruit Attainment Levels for Each Recruiting Organization | 10 |
| 5 | Maximum Weighted Difference between State/Starting State Component Values | 4 |
| 6 | Type of Computation | 2 (optimal decisions) |
| 7 | Number of Quota Decisions Available to Central Recruiting Management for Each Recruiting Organization | 3 |
| 8 | Number of Funding Decisions Available to Central Recruiting Management for each Recruiting Organization | 3 |
| 9 | Number of Lower Diagonals Eliminated in the Corner Cutting Procedure (Decision Space Reduction Method #1) | 1 |
| 10 | Number of Upper Diagonals Eliminated in the Corner Cutting Procedure (Decision Space Reduction Method #1) | 1 |

Table 5.2 (Continued)

| Input Number | Name of Input | Input Value |
|---|---|---|
| 11 | Maximum Extra-Organization Difference between Quota Assignments | 1 |
| 12 | Maximum Extra-Organization Difference between Funding Assignments | 2 |
| 13 | Suboptimal Control "B" Parameters | N/A |
| 14 | Suboptimal Control "C" Parameters | N/A |
| 15 | Gamma | .5 |
| 16 | Difficulties of Recruiting | Organization 1 / Organization 2<br>Time: .91 / .80<br>.91 / .80<br>.91 / .80 |
| 17 | Recruiter Effectivenesses | Organization 1 / Organization 2<br>Time: .89 / .84<br>.89 / .84<br>.89 / .84 |
| 18 | Advertising Response Function (Person-Units) | Level 1 / Level 2 / Level 3<br>Group: 1.075 / 1.536 / 1.843<br>2.023 / 2.890 / 3.468 |
| 19 | Alpha Factors | Organization 1 / Organization 2<br>.72 / 1.3 |
| 20 | Last Value Option | 2 (Variable Penalty) |

Table 5.2 (Continued)

| Input Number | Name of Input | Input Value |
|---|---|---|
| 21 | Under-achievement Penalty Coefficient | 1 |
| 22 | Over-achievement Penalty Coefficient | .25 |
| 23 | Quota Reward Weight | .5 |
| 24 | Funding Reward Weight | 1 |
| 25 | Starting State | 000 |
| 26 | Maximum CPU Time | .001 minutes |

## 5.2 Results

Table 5.3 is all of the raw output from the computational module for this example. Thus, Table 5.3 is an example of the output that is illustrated in Figure 4.2. Note that, when the recruiting (sub)system is close to having met its goal of 14 person-units of recruits (this is indicated by the system being in a state whose components sum to a number close to 14) and has a "long" time for recruiting left before the end of the time horizon (i.e., as at times 3 or 2), the quota allocations and funding allocations to the two subordinate recruiting organizations are small. Conversely, when the recruiting (sub)system is far from having met its goal and there is little time for recruiting remaining, these allocations are high. These facts are intuitively plausible.

Table 5.4 shows the probabilities that various total numbers of person-units of recruits will be obtained by the two subordinate recruiting organizations at various times. Figure 5.1 displays this information graphically. Thus, Table 5.4 and Figure 5.1 are examples of post-processed information Type 1A, as discussed in Section 4.3 and illustrated in Figure 4.3a.

Table 5.5 shows the means and standard deviations of the probability densities tabulated in Table 5.4 and displayed graphically in Figure 5.1. (Table 5.5 also repeats these same probability densities themselves, but to a smaller number of significant figures than in Table 5.4). Thus, Table 5.5 is an example of post-processed output Type 2A, as discussed in Section 4.3 and illustrated in Figure 4.4a. Notice how the mean number of recruits, along with the standard deviation thereof, increase with time in a manner that is evident from the graphs in Figure 5.1. These facts too are intuitively plausible.

Table 5.3 Raw output from the computational module of AFRA.

AIR FORCE RECRUITING MODEL OUTPUT

X----------------TIME = 3----------------X

| STATE | PROB | VALUE | QUOTA ALLOCATIONS | FUNDING ALLOCATIONS |
|---|---|---|---|---|
| 0 0 0 0 0 | 1.0000000 | .378+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 1 0 0 0 0 | .0000000 | .221+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 2 0 0 0 0 | .0000000 | .793+001 | 3 2 0 0 0 | 3 1 0 0 0 |
| 0 1 0 0 0 | .0000000 | .715+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 1 1 0 0 0 | .0000000 | .215+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 2 1 0 0 0 | .0000000 | .116+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 3 1 0 0 0 | .0000000 | .203+001 | 2 1 0 0 0 | 3 1 0 0 0 |
| 0 2 0 0 0 | .0000000 | .239+002 | 3 2 0 0 0 | 2 1 0 0 0 |
| 1 2 0 0 0 | .0000000 | .179+002 | 3 2 0 0 0 | 2 1 0 0 0 |
| 2 2 0 0 0 | .0000000 | .121+002 | 3 2 0 0 0 | 2 1 0 0 0 |
| 3 2 0 0 0 | .0000000 | .674+001 | 3 2 0 0 0 | 2 1 0 0 0 |
| 0 3 0 0 0 | .0000000 | .134+002 | 1 2 0 0 0 | 1 1 0 0 0 |
| 1 3 0 0 0 | .0000000 | .141+002 | 3 2 0 0 0 | 2 1 0 0 0 |
| 2 3 0 0 0 | .0000000 | .107+002 | 2 2 0 0 0 | 1 1 0 0 0 |
| 3 3 0 0 0 | .0000000 | .764+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 3 0 0 0 | .0000000 | .380+001 | 3 2 0 0 0 | 3 1 0 0 0 |
| 0 4 0 0 0 | .0000000 | .539+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 1 4 0 0 0 | .0000000 | .925+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 2 4 0 0 0 | .0000000 | .850+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 3 4 0 0 0 | .0000000 | .709+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 4 0 0 0 | .0000000 | .594+001 | 2 1 0 0 0 | 1 1 0 0 0 |
| 5 4 0 0 0 | .0000000 | .100+001 | 2 1 0 0 0 | 3 1 0 0 0 |
| 0 5 0 0 0 | .0000000 | .235+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 1 5 0 0 0 | .0000000 | .636+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 2 5 0 0 0 | .0000000 | .721+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 3 5 0 0 0 | .0000000 | .721+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 5 0 0 0 | .0000000 | .705+001 | 2 1 0 0 0 | 1 1 0 0 0 |
| 5 5 0 0 0 | .0000000 | .363+001 | 3 2 0 0 0 | 3 1 0 0 0 |
| 1 6 0 0 0 | .0000000 | .311+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 2 6 0 0 0 | .0000000 | .575+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 3 6 0 0 0 | .0000000 | .628+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 6 0 0 0 | .0000000 | .653+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 5 6 0 0 0 | .0000000 | .503+001 | 3 2 0 0 0 | 2 1 0 0 0 |
| 6 6 0 0 0 | .0000000 | .121+001 | 2 1 0 0 0 | 3 1 0 0 0 |
| 1 7 0 0 0 | .0000000 | .126+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 2 7 0 0 0 | .0000000 | .319+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 3 7 0 0 0 | .0000000 | .534+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 7 0 0 0 | .0000000 | .609+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 5 7 0 0 0 | .0000000 | .653+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 6 7 0 0 0 | .0000000 | .405+001 | 2 1 0 0 0 | 1 1 0 0 0 |
| 2 8 0 0 0 | .0000000 | .151+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 3 8 0 0 0 | .0000000 | .334+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 8 0 0 0 | .0000000 | .368+001 | 2 2 0 0 0 | 1 1 0 0 0 |

X----------------TIME = 2----------------X

| STATE | PROB | VALUE | QUOTA ALLOCATIONS | FUNDING ALLOCATIONS |
|---|---|---|---|---|
| 0 0 0 0 0 | .0023782 | .736+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 1 0 0 0 0 | .0190256 | .465+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 2 0 0 0 0 | .0185962 | .173+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 0 1 0 0 0 | .0190256 | .614+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 1 1 0 0 0 | .1522049 | .445+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 2 1 0 0 0 | .1467695 | .245+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 3 1 0 0 0 | .0000000 | .457+001 | 2 1 0 0 0 | 3 1 0 0 0 |
| 0 2 0 0 0 | .0360512 | .476+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 1 2 0 0 0 | .3044097 | .361+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 2 2 0 0 0 | .2975391 | .237+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 3 2 0 0 0 | .0000000 | .115+002 | 3 2 0 0 0 | 3 1 0 0 0 |
| 0 3 0 0 0 | .0000000 | .326+002 | 1 2 0 0 0 | 1 1 0 0 0 |
| 1 3 0 0 0 | .0000000 | .271+002 | 3 2 0 0 0 | 2 1 0 0 0 |
| 2 3 0 0 0 | .0000000 | .187+002 | 3 2 0 0 0 | 2 1 0 0 0 |
| 3 3 0 0 0 | .0000000 | .113+002 | 3 2 0 0 0 | 2 1 0 0 0 |
| 4 3 0 0 0 | .0000000 | .444+001 | 3 2 0 0 0 | 3 1 0 0 0 |
| 0 4 0 0 0 | .0000000 | .127+002 | 1 2 0 0 0 | 1 1 0 0 0 |
| 1 4 0 0 0 | .0000000 | .192+002 | 3 2 0 0 0 | 2 1 0 0 0 |
| 2 4 0 0 0 | .0000000 | .136+002 | 3 2 0 0 0 | 2 1 0 0 0 |
| 3 4 0 0 0 | .0000000 | .922+001 | 3 2 0 0 0 | 2 1 0 0 0 |
| 4 4 0 0 0 | .0000000 | .545+001 | 3 2 0 0 0 | 2 1 0 0 0 |
| 5 4 0 0 0 | .0000000 | .107+001 | 2 1 0 0 0 | 3 1 0 0 0 |
| 0 5 0 0 0 | .0000000 | .415+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 1 5 0 0 0 | .0000000 | .122+002 | 2 2 0 0 0 | 1 1 0 0 0 |
| 2 5 0 0 0 | .0000000 | .972+001 | 3 2 0 0 0 | 2 1 0 0 0 |
| 3 5 0 0 0 | .0000000 | .672+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 5 0 0 0 | .0000000 | .484+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 5 5 0 0 0 | .0000000 | .333+001 | 3 2 0 0 0 | 3 1 0 0 0 |
| 1 6 0 0 0 | .0000000 | .424+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 2 6 0 0 0 | .0000000 | .621+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 3 6 0 0 0 | .0000000 | .519+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 6 0 0 0 | .0000000 | .467+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 5 6 0 0 0 | .0000000 | .472+001 | 2 1 0 0 0 | 1 1 0 0 0 |
| 6 6 0 0 0 | .0000000 | .893+000 | 2 1 0 0 0 | 3 1 0 0 0 |
| 1 7 0 0 0 | .0000000 | .132+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 2 7 0 0 0 | .0000000 | .330+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 3 7 0 0 0 | .0000000 | .485+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 7 0 0 0 | .0000000 | .521+001 | 2 1 0 0 0 | 1 1 0 0 0 |
| 5 7 0 0 0 | .0000000 | .555+001 | 2 1 0 0 0 | 1 1 0 0 0 |
| 6 7 0 0 0 | .0000000 | .339+001 | 2 1 0 0 0 | 1 1 0 0 0 |
| 2 8 0 0 0 | .0000000 | .149+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 3 8 0 0 0 | .0000000 | .326+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 8 0 0 0 | .0000000 | .349+001 | 2 2 0 0 0 | 1 1 0 0 0 |

Table 5.3 (Cont'd.)

AIR FORCE RECRUITING MODEL OUTPUT

| STATE | | | | | X-----TIME = 3-----X PROB | VALUE | QUOTA ALLOCATIONS | FUNDING ALLOCATIONS | X-----TIME = 2-----X PROB | VALUE | QUOTA ALLOCATIONS | FUNDING ALLOCATIONS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 8 | 0 | 0 | 0 | .0000000 | .432+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .408+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 6 | 8 | 0 | 0 | 0 | .0000000 | .492+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .476+001 | 1 2 0 0 0 | 1 1 0 0 0 |
| 7 | 8 | 0 | 0 | 0 | .0000000 | .120+001 | 2 1 0 0 0 | 3 1 0 0 0 | .0000000 | .119+001 | 2 1 0 0 0 | 3 1 0 0 0 |
| 3 | 9 | 0 | 0 | 0 | .0000000 | .217+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .217+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 4 | 9 | 0 | 0 | 0 | .0000000 | .296+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .294+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 5 | 9 | 0 | 0 | 0 | .0000000 | .399+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .397+001 | 2 2 0 0 0 | 1 1 0 0 0 |
| 6 | 9 | 0 | 0 | 0 | .0000000 | .475+001 | 1 2 0 0 0 | 1 1 0 0 0 | .0000000 | .470+001 | 1 1 0 0 0 | 1 1 0 0 0 |
| 7 | 9 | 0 | 0 | 0 | .0000000 | .504+001 | 3 2 0 0 0 | 3 1 0 0 0 | .0000000 | .504+001 | 3 2 0 0 0 | 3 1 0 0 0 |
| 3 | 10 | 0 | 0 | 0 | .0000000 | .100+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .100+001 | 1 1 0 0 0 | 1 1 0 0 0 |
| 4 | 10 | 0 | 0 | 0 | .0000000 | .000 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .000 | 1 1 0 0 0 | 1 1 0 0 0 |
| 5 | 10 | 0 | 0 | 0 | .0000000 | .250+000 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .250+000 | 1 1 0 0 0 | 1 1 0 0 0 |
| 6 | 10 | 0 | 0 | 0 | .0000000 | .100+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .100+001 | 1 1 0 0 0 | 1 1 0 0 0 |
| 7 | 10 | 0 | 0 | 0 | .0000000 | .225+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .225+001 | 1 1 0 0 0 | 1 1 0 0 0 |
| 8 | 10 | 0 | 0 | 0 | .0000000 | .400+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .400+001 | 1 1 0 0 0 | 1 1 0 0 0 |
| 4 | 11 | 0 | 0 | 0 | .0000000 | .250+000 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .250+000 | 1 1 0 0 0 | 1 1 0 0 0 |
| 5 | 11 | 0 | 0 | 0 | .0000000 | .100+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .100+001 | 1 1 0 0 0 | 1 1 0 0 0 |
| 6 | 11 | 0 | 0 | 0 | .0000000 | .225+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .225+001 | 1 1 0 0 0 | 1 1 0 0 0 |
| 7 | 11 | 0 | 0 | 0 | .0000000 | .400+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .400+001 | 1 1 0 0 0 | 1 1 0 0 0 |
| 8 | 11 | 0 | 0 | 0 | .0000000 | .625+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .625+001 | 1 1 0 0 0 | 1 1 0 0 0 |

## Table 5.3 (Cont'd.)

AIR FORCE RECRUITING MODEL OUTPUT

| STATE | PROB (TIME = 1) | VALUE (TIME = 1) | QUOTA ALLOCATIONS (TIME = 1) | FUNDING ALLOCATIONS (TIME = 1) | PROB (TIME = 0) | VALUE (TIME = 0) | QUOTA ALLOCATIONS (TIME = 0) | FUNDING ALLOCATIONS (TIME = 0) |
|---|---|---|---|---|---|---|---|---|
| 0 0 0 0 0 | .0000059 | .130+003 | 3 2 0 0 0 | 3 1 0 0 0 | .0000000 | .196+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 1 0 0 0 0 | .0000942 | .103+003 | 3 2 0 0 0 | 3 1 0 0 0 | .0000004 | .169+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 2 0 0 0 0 | .0004691 | .486+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0000033 | .144+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 0 1 0 0 0 | .0003942 | .108+003 | 3 2 0 0 0 | 3 1 0 0 0 | .0000004 | .169+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 1 1 0 0 0 | .0015979 | .875+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0000089 | .144+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 2 1 0 0 0 | .0075054 | .573+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0000796 | .121+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 1 0 0 0 | .0117908 | .118+002 | 2 1 0 0 0 | 3 1 0 0 0 | .0005363 | .100+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 0 2 0 0 0 | .0055655 | .985+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0000037 | .144+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 1 2 0 0 0 | .0090473 | .708+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0000887 | .121+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 2 2 0 0 0 | .0455324 | .536+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0007960 | .100+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 2 0 0 0 | .0707449 | .328+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0044778 | .810+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 0 3 0 0 0 | .0015079 | .709+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0000197 | .121+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 1 3 0 0 0 | .0241262 | .555+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0004729 | .100+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 2 3 0 0 0 | .1200864 | .420+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0042456 | .810+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 3 0 0 0 | .1686532 | .294+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0169916 | .640+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 4 3 0 0 0 | .0921976 | .128+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0782895 | .490+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 0 4 0 0 0 | .0015079 | .406+002 | 1 2 0 0 0 | 1 1 0 0 0 | .0001064 | .100+003 | 0 0 0 0 0 | 0 0 0 0 0 |
| 1 4 0 0 0 | .0241262 | .421+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0014003 | .810+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 2 4 0 0 0 | .1200864 | .306+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0127058 | .640+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 4 0 0 0 | .1686532 | .213+002 | 3 2 0 0 0 | 2 1 0 0 0 | .0525526 | .490+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 4 4 0 0 0 | .0921976 | .118+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0989361 | .360+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 5 4 0 0 0 | .0000000 | .292+001 | 2 1 0 0 0 | 3 1 0 0 0 | .0742938 | .250+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 0 5 0 0 0 | .0000000 | .146+002 | 1 2 0 0 0 | 1 1 0 0 0 | .0004690 | .810+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 1 5 0 0 0 | .0000000 | .309+002 | 3 2 0 0 0 | 3 1 0 0 0 | .0021221 | .640+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 2 5 0 0 0 | .0000000 | .215+002 | 3 2 0 0 0 | 2 1 0 0 0 | .0201321 | .490+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 5 0 0 0 | .0000000 | .141+002 | 3 2 0 0 0 | 2 1 0 0 0 | .0847149 | .360+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 4 5 0 0 0 | .0000000 | .845+001 | 3 2 0 0 0 | 2 1 0 0 0 | .1656567 | .250+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 5 5 0 0 0 | .0000000 | .760+001 | 3 2 0 0 0 | 3 1 0 0 0 | .1316999 | .160+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 1 6 0 0 0 | .0000000 | .172+002 | 1 2 0 0 0 | 1 1 0 0 0 | .0012176 | .490+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 2 6 0 0 0 | .0000000 | .141+002 | 3 2 0 0 0 | 2 1 0 0 0 | .0130927 | .360+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 6 0 0 0 | .0000000 | .879+001 | 3 2 0 0 0 | 2 1 0 0 0 | .0575278 | .250+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 4 6 0 0 0 | .0000000 | .521+001 | 2 2 0 0 0 | 1 1 0 0 0 | .1188471 | .160+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 5 6 0 0 0 | .0000000 | .289+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0763394 | .900+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 6 6 0 0 0 | .0000000 | .794+000 | 2 1 0 0 0 | 2 1 0 0 0 | .0301472 | .400+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 1 7 0 0 0 | .0000000 | .333+001 | 1 2 0 0 0 | 1 1 0 0 0 | .0000000 | .360+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 2 7 0 0 0 | .0000000 | .777+001 | 1 2 0 0 0 | 1 1 0 0 0 | .0000000 | .250+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 7 0 0 0 | .0000000 | .524+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .160+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 4 7 0 0 0 | .0000000 | .300+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .900+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 5 7 0 0 0 | .0000000 | .226+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .400+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 6 7 0 0 0 | .0000000 | .241+001 | 2 1 0 0 0 | 1 1 0 0 0 | .0000000 | .100+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 2 8 0 0 0 | .0000000 | .194+001 | 1 2 0 0 0 | 1 1 0 0 0 | .0000000 | .160+002 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 8 0 0 0 | .0000000 | .300+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .900+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 4 8 0 0 0 | .0000000 | .242+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .400+001 | 0 0 0 0 0 | 0 0 0 0 0 |

Table 5.3 (Cont'd.)

AIR FORCE RECRUITING MODEL OUTPUT

| STATE | X----TIME = 1---- PROB | VALUE | QUOTA ALLOCATIONS | FUNDING ALLOCATIONS----X | X----TIME = 0---- PROB | VALUE | QUOTA ALLOCATIONS | FUNDING ALLOCATIONS----X |
|---|---|---|---|---|---|---|---|---|
| 5 8 0 0 0 | .0000000 | .270+001 | 2 1 0 0 0 | 1 1 0 0 0 | .0000000 | .100+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 6 8 0 0 0 | .0000000 | .323+001 | 2 1 0 0 0 | 1 1 0 0 0 | .0000000 | .000 | 0 0 0 0 0 | 0 0 0 0 0 |
| 7 8 0 0 0 | .0000000 | .704+000 | 2 1 0 0 0 | 2 1 0 0 0 | .0000000 | .250+000 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 9 0 0 0 | .0000000 | .212+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .400+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 4 9 0 0 0 | .0000000 | .261+001 | 2 2 0 0 0 | 1 1 0 0 0 | .0000000 | .100+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 5 9 0 0 0 | .0000000 | .346+001 | 2 1 0 0 0 | 1 1 0 0 0 | .0000000 | .000 | 0 0 0 0 0 | 0 0 0 0 0 |
| 6 9 0 0 0 | .0000000 | .412+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .250+000 | 0 0 0 0 0 | 0 0 0 0 0 |
| 7 9 0 0 0 | .0000000 | .503+001 | 3 2 0 0 0 | 3 1 0 0 0 | .0000000 | .100+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 3 10 0 0 0 | .0000000 | .100+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .100+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 4 10 0 0 0 | .0000000 | .000 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .000 | 0 0 0 0 0 | 0 0 0 0 0 |
| 5 10 0 0 0 | .0000000 | .250+000 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .250+000 | 0 0 0 0 0 | 0 0 0 0 0 |
| 6 10 0 0 0 | .0000000 | .100+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .100+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 7 10 0 0 0 | .0000000 | .225+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .225+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 8 10 0 0 0 | .0000000 | .400+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .400+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 4 11 0 0 0 | .0000000 | .250+000 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .250+000 | 0 0 0 0 0 | 0 0 0 0 0 |
| 5 11 0 0 0 | .0000000 | .100+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .100+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 6 11 0 0 0 | .0000000 | .225+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .225+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 7 11 0 0 0 | .0000000 | .400+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .400+001 | 0 0 0 0 0 | 0 0 0 0 0 |
| 8 11 0 0 0 | .0000000 | .625+001 | 1 1 0 0 0 | 1 1 0 0 0 | .0000000 | .625+001 | 0 0 0 0 0 | 0 0 0 0 0 |

Table 5.4 Probabilities that various total numbers of person-units of recruits will be obtained by the two subordinate recruiting organizations at various times.

AIR FORCE RECRUITING MODEL OUTPUT

PROBABILITY DENSITY ASSOCIATED WITH PERSON-UNITS RECRUITED BY ALL RECRUITING ORGANIZATIONS

| PERSON-UNITS RECRUITED | X-----PROBABILITIES----- X---TIME PERIOD---> X-- 3--X | X-- 1--X | X-- 2--X | |
|---|---|---|---|---|
| 0. | .000000 | .000006 | .002376 | 1.000000 |
| 1. | .000001 | .000188 | .035051 | .000000 |
| 2. | .000016 | .002542 | .208952 | .000000 |
| 3. | .000188 | .018061 | .453173 | .000000 |
| 4. | .001916 | .362457 | .297539 | .000000 |
| 5. | .010613 | .214957 | .000000 | .000000 |
| 6. | .031820 | .308743 | .000000 | .000000 |
| 7. | .102192 | .280851 | .000000 | .000000 |
| 8. | .196744 | .292198 | .000000 | .000000 |
| 9. | .297478 | .000000 | .000000 | .000000 |
| 10. | .250547 | .000000 | .000000 | .000000 |
| 11. | .078339 | .000000 | .000000 | .000000 |
| 12. | .030147 | .000000 | .000000 | .000000 |
| 13. | .000000 | .000000 | .000000 | .000000 |
| 14. | .000000 | .000000 | .000000 | .000000 |
| 15. | .000000 | .000000 | .000000 | .000000 |
| 16. | .000000 | .000000 | .000000 | .000000 |
| 17. | .000000 | .000000 | .000000 | .000000 |
| 18. | .000000 | .000000 | .000000 | .000000 |
| 19. | .000000 | .000000 | .000000 | .000000 |
| 20. | .000000 | .000000 | .000000 | .000000 |
| 21. | .000000 | .000000 | .000000 | .000000 |
| 22. | .000000 | .000000 | .000000 | .000000 |

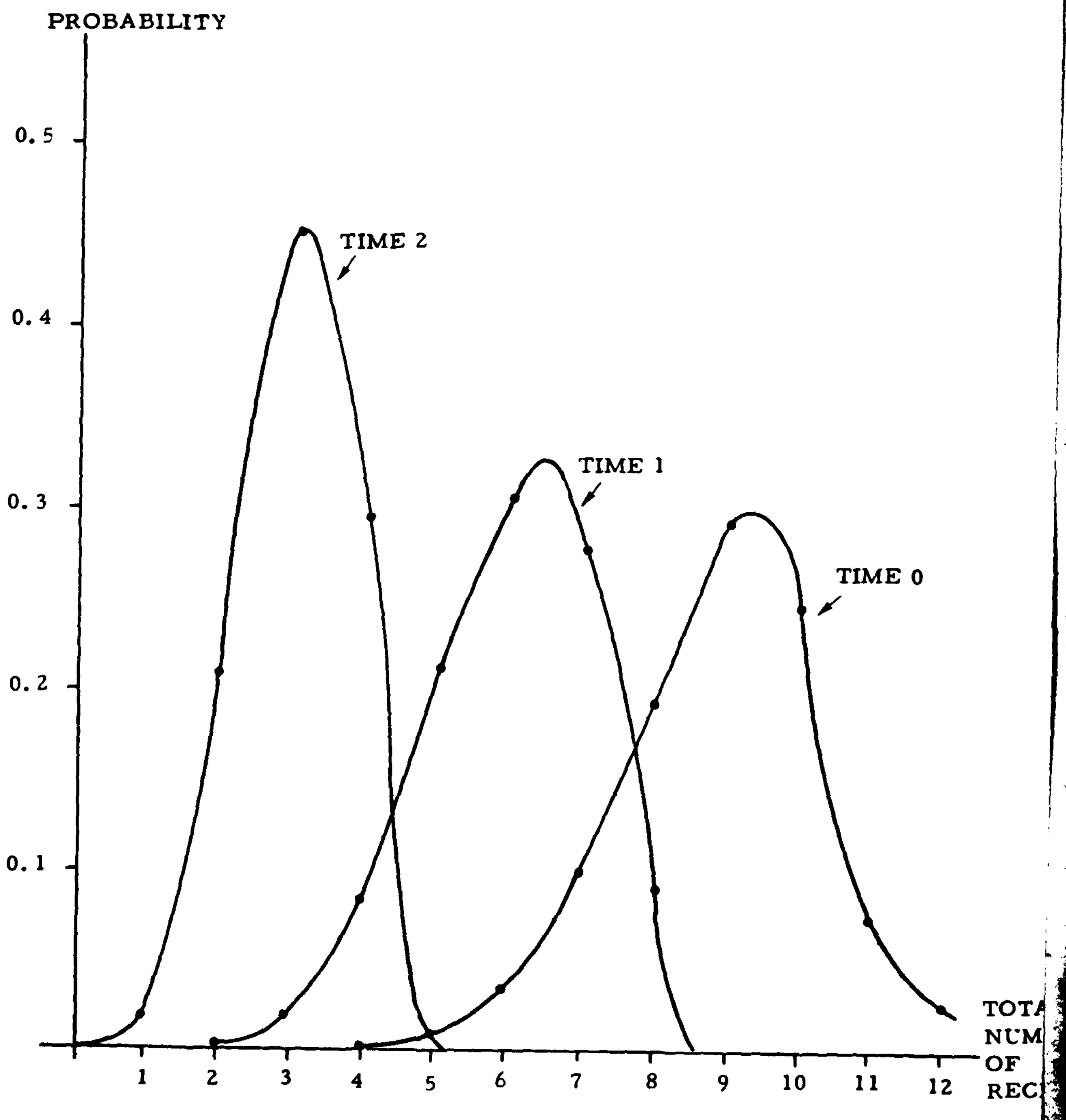


Figure 5.1 Probabilities of having various numbers of recruits at various times.

Table 5.5 Means and standard deviations of probability densities.

AIR FORCE RECRUITING MODEL OUTPUT

PROBABILITIES OF NUMBERS:
ALL RECRUITING ORGANIZATIONS

| TIME | EXP SUM | ST DEV OF SUM | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 8.948 | 1.396 | .000 | .000 | .000 | .000 | .002 | .011 | .032 | .102 | .197 | .297 | .251 | .078 | .030 | .000 | .000 | .000 |
| 1 | 6.020 | 1.184 | .000 | .000 | .003 | .018 | .062 | .215 | .309 | .281 | .092 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 2 | 3.005 | .825 | .002 | .038 | .209 | .453 | .298 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 3 | .000 | .000 | 1.000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |

PROBABILITIES OF NUMBERS:
ALL RECRUITING ORGANIZATIONS

| TIME | EXP SUM | ST DEV OF SUM | 16 | 17 | 18 | 19 | 20 | 21 | 22 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 8.948 | 1.396 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 1 | 6.020 | 1.184 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 2 | 3.005 | .825 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |
| 3 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 |

@GRAFT PRINTS AFFA32C9

## 6.0 DIRECTIONS FOR ADDITIONAL WORK

The Air Force Recruiting Aid (AFRA) is "working" on the UNIVAC 1108 at the Air Force Human Resources Laboratory (AFHRL) in the sense that it runs in what is now considered to be an appropriate manner. However, there are several directions in which additional work on the aid seems necessary before it will be ready to turn over to operational users:

- testing and evaluation;
- improved availability and reliability of inputs;
- refinements and extensions.

We discuss each of these areas very briefly in turn below.

### 6.1 Testing and Evaluation

It is recommended that AFRA be tested and evaluated in the following ways before being turned over to operational users:

- evaluate the appropriateness and utility of the aid's outputs;
- evaluate the feasibility of the aid's inputs;
- determine the sensitivity of the aid to various parameters, functional forms and submodels;
- evaluate the aid's ease of use.

The need for these tests and evaluations scarcely requires comment. AFRA's outputs and inputs need to be evaluated for suitability and feasibility, respectively, in order to be sure that the aid is providing information that is in fact useful to users without being too costly in terms of the effort required to get it. Determining AFRA's sensitivities to the items mentioned above is vital because these sensitivities dictate the degree to which input data must be accurate and the degree to which refinements and extensions to the aid's functional forms and submodels are necessary. Finally, AFRA should be used, not just sit "on the shelf;" therefore, an assessment of the aid's ease of use should be made.

## 6.2 Improved Availability and Reliability of Inputs

Currently, AFRA's utility is limited by two factors having to do with the aid's input data, namely:

- availability of the input data;
- reliability of the input data.

In particular, many of the aid's input parameters are not readily available to users because they must assemble these data before running the aid on the basis of other data at a lower level of aggregation. Similarly, there may be questions about the reliability of this data.

The matter of data availability may be handled by creating and using a data base. The data base would be maintained and updated on a periodic basis using modern data base management techniques. The matter of data reliability can be addressed first by determining the degree to which the data needs to be accurate (via sensitivity analysis -- see Section 6.1) and employing modern estimation techniques. In fact, it appears that the estimation process can take place as part and parcel of the data base management process.

## 6.3 Refinements and Extensions

There are several directions in which AFRA can be:

- refined -- improved in the details of the recruiting model's portrayal of some aspect of the recruiting process;
- extended -- improved in the fundamentals of the recruiting model's portrayal of some aspect of the recruiting process.

These now appear to have mainly to do, respectively, with:

- a refinement in the portrayal of the effect of advertising on recruiting;
- an extension improving the way that the recruiting model accounts for recruits being gained by the subordinate recruiting organizations for the recruiting management.

As explained in Section 3.2.3.2.5, AFRA's recruiting model portrays advertising as affecting manpower markets in the manner first described by Vidale and Wolfe [7]. While the Vidale-Wolfe model is a cornerstone of the mathematical theory of advertising, it is undeniably old. More importantly, it is now recognized as being not particularly faithful to the dynamics of advertising.

Much good work in the mathematical theory of advertising has recently been done. In particular, researchers have recently shown how to:

- embed the Vidale-Wolfe model of the dynamics of advertising in a larger structure involving decision making and control (Sasieni [5]);
- represent the times by which the responses to advertising lag the advertising expenditures which produce them (Mann [4]);
- postulate the existence of a stock of "goodwill" that is buildable and maintainable through advertising but which can deteriorate through forgetting (Tapiero [6]);
- model advertising as affecting manpower markets in a stochastic (rather than deterministic) fashion (Tapiero [6]).

These concerns seem legitimate, and the models proposed for dealing with them convincing. The applicability to AFRA of the contributions of these authors should be considered.

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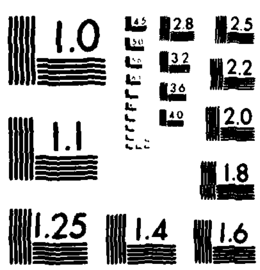


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# REFERENCES


1. William Feller, "An Introduction to Probability Theory and its Applications, Vol. 1," John Wiley & Sons, New York, 1950, 461 pp.

2. Ronald A. Howard, "Dynamic Programming and Markov Processes," MIT Press, 1960, 136 pp.

3. Ronald A. Howard, "Dynamic Probabilistic Systems, Volume I: Markov Models," John Wiley & Sons, New York, 1971, 576 pp.

4. Don H. Mann, "Optimal Theoretic Advertising Stock Models: A Generalization Incorporating the Effects of Delayed Response from Promotional Expenditure," Management Science 21 (1975), 823-832.

5. Maurice W. Sasieni, "Optimal Advertising Expenditure," Management Science 18 (1971), P64-P72.

6. Charles S. Tapiero, "Optimum Advertising and Goodwill under Uncertainty," Operations Research 26 (1978), 450-463.

7. M. L. Vidale and H. B. Wolfe, "An Operations Research Study of Sales Response to Advertising," Operations Research 5 (1957), pp. 370-381.