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AFGL-TR-79-0276 *AFGL-ERP-*
ENVIRONMENTAL RESEARCH PAPERS, NO. 684

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A Method for Direct Determination of Real Height From Virtual Height Data for the Auroral Region of the Ionosphere

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14 November 1979

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SPACE PHYSICS DIVISION   PROJECT 4643
AIR FORCE GEOPHYSICS LABORATORY
HANSCOM AFB, MASSACHUSETTS 01731
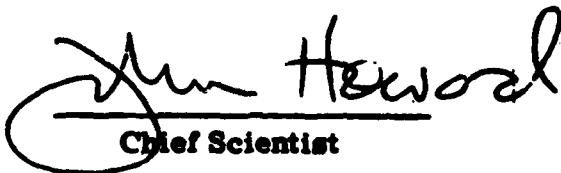
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFGL-TR-79-0276	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A METHOD FOR DIRECT DETERMINATION OF REAL HEIGHT FROM VIRTUAL HEIGHT DATA FOR THE AURORAL REGION OF THE IONOSPHERE		5. TYPE OF REPORT & PERIOD COVERED Scientific. Interim.
		6. PERFORMING ORG. REPORT NUMBER ERP No. 684
7. AUTHOR(s) Milton M. Klein		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Geophysics Laboratory (PHI) Hanscom AFB Massachusetts 01731		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62101F 46430609
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Geophysics Laboratory (PHI) Hanscom AFB Massachusetts 01731		12. REPORT DATE 14 November 1979
		13. NUMBER OF PAGES 19
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Real height
Virtual height data
Auroral region
Electron density
Ionosphere

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
    As part of an effort to determine electron density profiles in the auroral region of the ionosphere, a method has been developed for directly calculating real height from virtual height data obtained at high latitudes. The essential ingredient of the method is the representation of the curved portion of the ordinary ray dispersion curve by an analytic form which reduces the virtual height - real height integral relation to a form of Abel's integral equation. Numerical checks indicate that the present method gives an accurate representation of the dispersion curve. The correctness of the present procedure has

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been verified for the case of a linear electron density profile.

The present method will be applied to the numerical calculation of real height electron density profiles in the auroral region from virtual height data. The accuracy and convenience of the present method will be assessed by comparison of the results with those obtained from standard programs.

Contents

Illustrations

A Method for Direct Determination of Real Height From Virtual Height Data for the Auroral Region of the Ionosphere

1. INTRODUCTION

For the case of no magnetic field, the group index of refraction is of fairly simple form and the virtual height-real height integral relation may be expressed as a special form of Abel's integral equation and solved for real height as a direct function of virtual height. [1]

In the presence of a magnetic field, the group index of refraction becomes quite complicated and, in general, the group height-real height integral relation cannot be reduced to a form of Abel's integral equation. The approach generally used here is the laminar method[2,3] in which the electron density profile is divided into a series of slabs in each of which average values are used to describe the properties of the slab. The integral relation then reduces to a set of simultaneous equations whose coefficients may be calculated, giving the real height as a function of virtual height. Improvements to the laminar method have been provided by Titheridge by use of a single polynomial[4] or a series of overlapping polynomials.[5]

For the particular case of longitudinal propagation, the group index of refraction becomes fairly simple and the group height-real height relation may be reduced to a form of Abel's integral equation. Since, in the auroral region, the angle θ between the magnetic field and the incident ray is less than about 30° for vertical

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propagation, it is natural to inquire whether the group height-real height relation may be expressed, at least approximately, in terms of Abel's integral equation.

For the extraordinary ray, Figure 1, the index of refraction remains virtually unchanged for θ less than about 30°[8] and the real height in the auroral region may be determined from an Abel integral equation in a fairly straightforward manner.
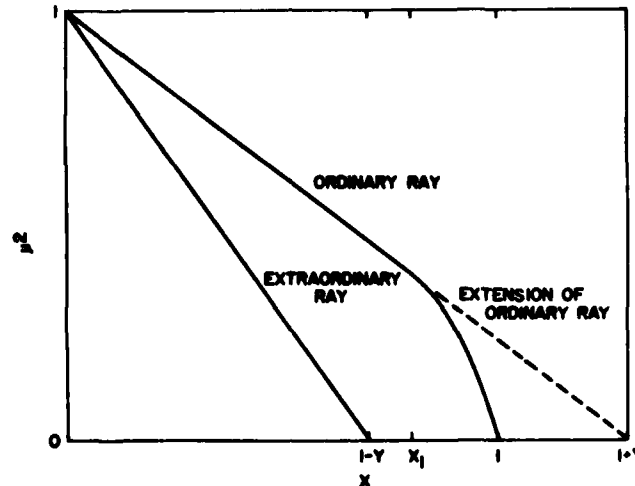
Figure 1. Sketch of Ordinary and Extraordinary Dispersion Curves for θ Less than 30°

For the ordinary ray, however, as seen in Figure 1, the situation is somewhat more complicated. Although μ^2 is almost linear over a portion of $X = f_N^2/f^2$, where μ is phase index of refraction, f_N is plasma frequency and f is probing frequency, the curve bends rapidly at the larger values of X to give a reflection level at $X = 1$. It is found that the nonlinear portion of the curve may be fitted by a simple power law for which an Abel integral equation obtains. Thus, the dispersion relation for the ordinary ray may be regarded as made up of two segments for each of which the real height may be obtained directly.

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6. Davies, Kenneth (1969) Ionospheric Radio Waves, Blaisdell Publishing Company, Waltham, Massachusetts, Chapter 5, pp 130-135.

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2. ANALYSIS

2.1 Extraordinary Ray

For the extraordinary ray the index of refraction in longitudinal propagation is given by

$$\mu^2 = 1 - \frac{X}{1-Y} \tag{1}$$

where

$Y = \frac{f_H}{f}$ is the gyromagnetic ratio

and f_H is the gyromagnetic frequency. As indicated previously, the dispersion relation given by Eq. (1) remains virtually unchanged for θ less than about 30°. We may therefore use Eq. (1) in the analysis of the relation between group height and real height for the auroral region.

The virtual height $h'(f)$, which is a function of input frequency f, is related to the real height h by the integral

$$h'(f) = \int_0^{h_r} \mu' \, dh \tag{2}$$

where h is real height, h_r is the height at reflection, that is, when $\mu = 0$, and μ' is the group index of refraction and given by

$$\mu' = \frac{d}{df} (\mu f) = \mu + f \frac{d\mu}{df} . \tag{3}$$

The group index may now be expressed explicitly as a function of X and Y. For our purposes, however, it will be more convenient to use the first form of Eq. (3). We may then write Eq. (2) as

$$\int_0^{h_r} \frac{d}{df} (\mu f) \, dh = h'(f) . \tag{4}$$

Noting that at the upper limit h_r the index μ vanishes, we can take the derivative outside the integral sign and write

$$\frac{d}{df} \int_0^{h_r} \mu f \, dh = h'(f) . \tag{5}$$

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We may now integrate with respect to f to obtain

$$\int_0^{h_r} f(1 - \frac{X}{1-Y})^{1/2}\, dh = \int_{f_H}^f h'(f)\, df \tag{6}$$

where the lower limit of f is set at f_H since X is negative in the range 0 to f_H when $Y > 1$. Noting that Y is independent of the variable dh, we may transpose the term in $1 - Y$ to the right-hand side and write Eq. (6) in the form

$$\int_0^{h_r} (f^2 - ff_H - f_N^2)^{1/2}\, dh = H'(f) , \tag{7}$$

$$H'(f) = (1 - Y)^{1/2} \int_{f_H}^f h'(f)\, df . \tag{8}$$

To reduce Eq. (7) to the form occurring in Abel's integral equation we introduce new variable F and F_N where

$$F = f^2 - ff_H ,$$

$$F_N = f_N^2 ,$$

which allows us to cast Eq. (7) in the form

$$\int_0^F (F - \xi)^{1/2} u\, d\xi = H'(f) \tag{9}$$

where $\xi = F_N$ is a running variable with maximum value F corresponding to the reflection frequency and $u = \frac{dh}{d\xi}$.

We may express Eq. (9) in the canonical form of Abel's integral equation by differentiating both sides of Eq. (9) with respect to F. However, the solution can also be obtained by direct use of the Laplace transform, indicating at the same time the simplicity and utility of the Laplace transform technique in this type of integral equation. Applying the Laplace transform to Eq. (9) we obtain

$$\frac{\sqrt{\pi}}{2} \frac{1}{s^{3/2}}\, \bar{u}(s) = \bar{H}(s) \tag{10}$$

where the bar indicates the transformed variable, and s the transform parameter, that is,

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$$\bar{u}(s) = \int_0^\infty u(\xi) \, e^{-s\xi} \, d\xi \tag{11}$$

and we have made use of the relation for the Laplace transform L of a convolution integral for two functions $f(\xi)$ and $g(\xi)$ in terms of the individual transforms

$$\bar{f}(s) \, \bar{g}(s) = L \int_0^\xi f(\xi - \eta) \, g(\eta) \, d\eta . \tag{12}$$

Solving Eq. (10) for $\bar{u}(s)$ we obtain

$$\bar{u}(s) = \frac{2}{\sqrt{\pi}} \, s^{3/2} \, \bar{H}(s) . \tag{13}$$

Since the inverse Laplace transform for s^n does not exist for $n > 0$ except when n is an integer we write Eq. (13) in the form

$$\bar{u}(s) = \frac{2}{\sqrt{\pi}} \, \frac{s^2}{s^{1/2}} \, \bar{H}(s) \tag{14}$$

whose inverse transform is readily calculated as

$$u(\xi) = \frac{2}{\pi} \, \frac{d^2}{d\xi^2} \int_0^\xi \frac{H'(F)}{(\xi - F)^{1/2}} \, dF \tag{15}$$

where F is now the running variable and ξ is the maximum value of F corresponding to the plasma frequency F_N at reflection. Since $u = \frac{dh}{d\xi}$ we can write Eq. (15) as

$$h(\xi) = \frac{2}{\pi} \, \frac{d}{d\xi} \int_0^\xi \frac{H'(F)}{(\xi - F)^{1/2}} \, dF \tag{16}$$

which can be further written in the forms

$$h(F_v) = \frac{2}{\pi} \int_0^{F_v} \frac{1}{(F_v - F)^{1/2}} \, \frac{dH'(F)}{dF} \, dF . \tag{17}$$

$$h(f_v) = \frac{2}{\pi} \int_{f_H}^{f_v} \frac{1}{R} \, \frac{dH'(f)}{df} \, df , \tag{18}$$

where F_v or f_v is the maximum value of F or f corresponding to the reflection height, and

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$$R^2 = f_v{}^2 - f_v f_H - (f^2 - ff_H) .$$

Carrying out the differentiation we obtain

$$h(f_v) = \frac{2}{\pi} \int_{f_H}^{f_v} \frac{(1-Y)^{1/2}}{R} h'(f) \, df + \frac{1}{\pi} \int_{f_H}^{f_v} \frac{1}{fR} \frac{Y}{1-Y} H'(f) \, df . \tag{19}$$

2.2 Ordinary Ray—Linear Segment

As indicated previously, the ordinary ray remains almost linear over some range of X and then begins to bend rapidly toward $\mu = 0$ at X = 1. We shall therefore consider the ordinary ray as made up of a linear and a curved segment joined at the common point $X = X_1$. The point X_1 is chosen at the point at which the μ^2 curve starts to bend away noticeably from the straight line portion of the curve.

The contribution of the straight line portion of the curve may be obtained in the same manner as for the extraordinary ray. However, to use the Abel integral relation it is necessary that the upper limit refer to the point X at which μ vanishes. The lower limit may be arbitrary, so long as H'(f) vanishes at the lower limit,[7] a condition which is automatically satisfied. We therefore extend the straight line to $\mu = 0$ at $X = 1 + Y$ and calculate the contribution to the real height of the straight line portion from 0 to X_1 as the difference of the contributions from 0 to $1 + Y$ and from X_1 to $1 + Y$.

For the straight line portion between 0 and $1 + Y$ the real height-virtual height relation is

$$\int_0^{h_r} \frac{d}{df} \, (\mu f) \, dh = \int_0^{f} h'(f) \, df \tag{20}$$

but μ^2 is now given by

$$\mu^2 = 1 - \frac{X}{1+Y} . \tag{21}$$

The lower frequency limit can here be taken as zero since X remains positive for the ordinary ray when Y becomes greater than unity. Following the same procedure as for the extraordinary ray we obtain

$$\int_0^{h_r} (f^2 + ff_H - f_N{}^2)^{1/2} \, dh = H'(f) , \tag{22}$$

$$H'(f) = (1+Y)^{1/2} \int_0^{f} h'(f) \, df , \tag{23}$$

7. Sneddon, I. N. (1972) The Use of Integral Transforms, McGraw-Hill Book Company, New York, Chapter 3, pp 207-211.

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which can be written as

$$\int_0^F (F - \xi)^{1/2} \, u \, d\xi = H'(F) \,, \tag{24}$$

where

$$F = f^2 + ff_H \,,$$

$$\xi = F_N = f_N^2 \,.$$

The real height h_0 is then given by

$$h_0(F_v) = \frac{2}{\pi} \int_0^{F_v} \frac{1}{(F_v - F)^{1/2}} \, \frac{d\,H'(F)}{dF} \, dF \tag{25}$$

or

$$h_0(f_v) = \frac{2}{\pi} \int_0^{f_v} \frac{1}{R} \, \frac{d\,H'(f)}{df} \, df \tag{26}$$

where

$$R^2 = f_v^2 + f_v f_H - (f^2 + ff_H) \,.$$

Carrying out the differentiation we obtain

$$h_0(f_v) = \frac{2}{\pi} \int_0^{f_v} \frac{(1+Y)^{1/2}}{R} \, h'(f) \, df - \frac{1}{\pi} \int_0^{f_v} \frac{1}{fR} \, \frac{Y}{1+Y} \, H'(f) \, df \,. \tag{27}$$

For the interval between X_1 and $1 + Y$ the height relation is

$$\int_{h_1}^{h_r} (f^2 + ff_H - f_N^2)^{1/2} \, dh = H_1'(f) \,, \tag{28}$$

$$H_1'(f) = (1+Y)^{1/2} \int_{f_1}^{f} h'(f) \, df \tag{29}$$

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where the subscript 1 corresponds to X_1 and the frequency f_1 is given by

$$f_1^2 + f_1 f_H = f_{N_1}^2 .$$ (30)

We now write Eq. (28) in the form

$$\int_{\xi_1}^{F} (F - \xi)^{1/2} \, u \, d\xi = H_1'(F)$$ (31)

which may be solved for $h_1(f_v)$ to yield

$$h_1(F_v) = \frac{2}{\pi} \int_{F_1}^{F_v} \frac{1}{(F_v - F)^{1/2}} \, \frac{d H_1'(F)}{dF} \, dF$$ (32)

or

$$h_1(f_v) = \frac{2}{\pi} \int_{f_1}^{f_v} \frac{1}{R} \, \frac{d H_1'(f)}{df} \, df .$$ (33)

The height h_{01} between 0 and X_1 is accordingly given by

$$h_{01} = h_0 - h_1 .$$ (34)

2.3 Ordinary Ray—Determination of Curved Segment

We represent the curved portion of the dispersion curve in the interval X_1 to 1 by

$$\mu^2 = a(1 - X)^\alpha \, \mu_1^2$$ (35)

where a and α are parameters to be determined and μ_1 is the index of refraction given by the straight line portion of the curve at the point X_1, that is,

$$\mu_1^2 = 1 - \frac{X_1}{1 + Y} .$$ (36)

The value of α is between 0 and 1 and approaches zero as θ becomes small.

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Since $(1-X)$ goes to zero at $X=1$, the derivative of μ^2 with respect to Y vanishes at $X=1$. The exact dispersion curve is also known to be independent of Y at $X=1$, so that the form for μ^2 in Eq. (35) has the proper qualitative dependence upon Y. In fact, a computational check of μ^2 from Eq. (35) as a function of Y was in good agreement with the exact dispersion equation.

The slope of the μ^2-X curve given by Eq. (35) is infinite at $X=1$, whereas the exact curve has the form, near $X=1$,[8]

$$\mu^2 = \frac{1}{\sin^2 \theta} \ (1-X) \tag{37}$$

with a slope of magnitude $\dfrac{1}{\sin^2 \theta}$. However, since θ is less than 30°, the slope from Eq. (37) is generally large. Thus, the μ^2 curve given by Eq. (35) should generally be in good agreement with the exact dispersion curve.

The values of a and α were determined by fitting the value of μ^2 in Eq. (35) to those of the exact dispersion curve at X_1 and a point X_2 between X_1 and 1. The results obtained for $\theta = 10°$, 20° and 30°, and for $Y = 0.3$ are shown in Figure 2 where it is seen that the fitted curves are in good agreement with the exact curves. The accuracy of the fit was found to be fairly insensitive to a moderate change in the point X_2, say from 0.9 to 0.8. In addition, although the slope at X_1 was not determined, the value from Eq. (35) was in fairly good agreement with that obtained from the exact dispersion curve. It thus appears that the form chosen for μ^2 in Eq. (35) is a useful and accurate representation of the curved portion of the exact dispersion curve.

2.4 Calculation of Real Height for Curved Segment of Dispersion Curve

The apparent height $h'(f)$ for the interval X_1 to 1, with μ^2 now given by Eq. (35) is then

$$\int_{h_1}^{h_r} f a^{\frac{1}{2}} \mu_1 (1-X)^{\frac{\alpha}{2}} \ dh = \int_{f_1}^{f} h'(f) \, df \tag{38}$$

which can be expressed as

$$a^{\frac{1}{2}} \int_{h_1}^{h_r} (f^2 - f_N^2)^{\frac{\alpha}{2}} \frac{\mu_1}{f^{\alpha-1}} \ dh = \int_{f_1}^{f} h'(f) \, df \ . \tag{39}$$

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8. Davies, Kenneth (1969) <u>Ionospheric Radio Waves</u>, Blaisdell Publishing Company, Waltham, Massachusetts, Chapter 5, p 125.
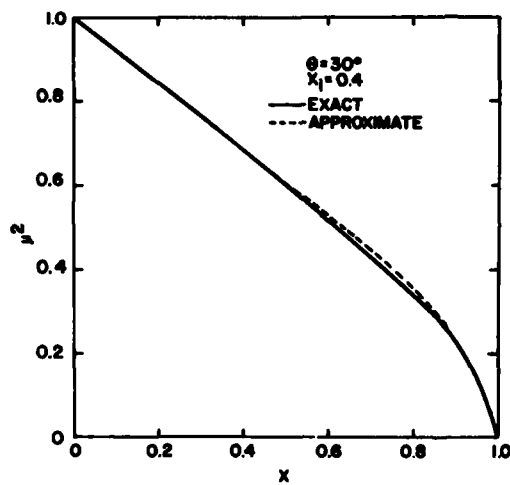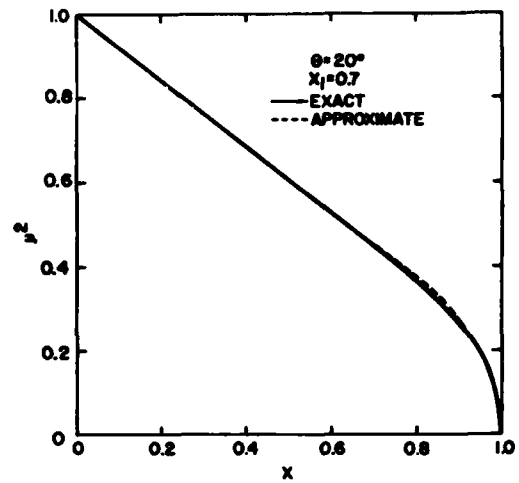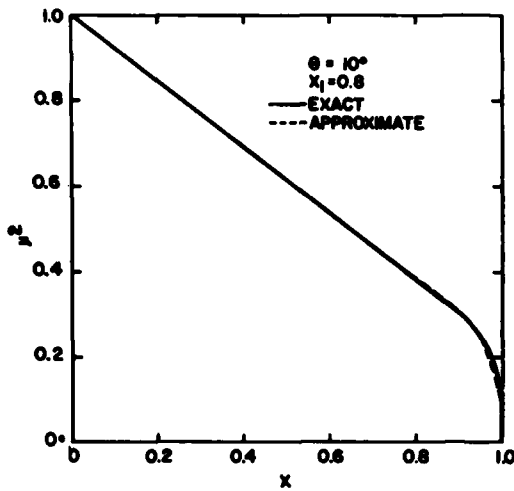
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Figure 2. Comparison of
Exact and Approximate
Dispersion Curves for
Several Values of θ ;
$Y = 0.3$

The terms in μ_1 and f are independent of real height h and we can accordingly
write Eq. (39) in the form

$$\int_{h_1}^{h_r} (f^2 - f_N^2)^{\frac{\alpha}{2}} \, dh = H'(f) \tag{40}$$

where

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$$H'(f) = \frac{f^{\alpha-1}}{a^{1/2}\mu_1} \int_{f_1}^{f} h'(f)\,df .$$
(41)

Changing to the variables F and ξ we write Eq. (40) in the canonical form

$$\int_{\xi_1}^{F} (F-\xi)^{\frac{\alpha}{2}} \, u\,d\xi \; = H'(F)$$
(42)

which yields under Laplace transformation

$$\frac{\bar{u}(s)\,\Gamma(\frac{\alpha}{2}+1)}{s^{\frac{\alpha}{2}+1}} = \bar{H}(s) .$$
(43)

Solving Eq. (43) for $\bar{u}(s)$ we have

$$\bar{u}(s) = \frac{1}{\Gamma(\frac{\alpha}{2}+1)} \; s^{\frac{\alpha}{2}+1}\,\bar{H}(s)$$
(44)

which may be written as

$$\bar{u}(s) = \frac{1}{\Gamma(\frac{\alpha}{2}+1)} \; \frac{s^2}{s^{1-\frac{\alpha}{2}}} \; \bar{H}(s).$$
(45)

The inverse transform of Eq. (45) is then

$$\frac{dh}{d\xi} = \frac{1}{\Gamma(1+\frac{\alpha}{2})} \; \frac{d^2}{d\xi^2} \int_{F_1}^{\xi} \frac{1}{\Gamma(1-\frac{\alpha}{2})} \; H'(F)\,dF$$
(46)

which may be written as

$$h_1 = \frac{\sin\frac{\pi\alpha}{2}}{\frac{\pi\alpha}{2}} \int_{F_1}^{F_v} \frac{1}{(F_v-F)^{\alpha/2}} \; \frac{dH'(F)}{dF} \; dF$$
(47)

or

$$h_1 = \frac{\sin\frac{\pi\alpha}{2}}{\frac{\pi\alpha}{2}} \int_{f_1}^{f_v} \frac{1}{(f_v^2-f^2)^{\alpha/2}} \; \frac{dH'(f)}{df} \; df$$
(48)

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where we have used

$$\Gamma(1+\frac{\alpha}{2})\ \Gamma(1-\frac{\alpha}{2}) = \frac{\frac{\pi\alpha}{2}}{\sin\frac{\pi\alpha}{2}} \tag{49}$$

Carrying out the differentiations we may write Eq. (48) as

$$h_1 = \frac{\sin\frac{\pi\alpha}{2}}{\frac{\pi\alpha}{2}} \int_{f_1}^{f_v} \frac{df}{(f_v{}^2 - f^2)^{\alpha/2}} \left[\frac{f^{\alpha-1}}{a^{1/2}\mu_1}\ h'(f) - \frac{(1-\alpha)}{f}\ H'(f) \right.$$

$$\left. + \frac{1}{2}\ \frac{X_1}{f}\ \frac{Y}{(1+Y)^2}\ \frac{1}{\mu_1{}^2}\ H'(f) \right] . \tag{50}$$

3. VERIFICATION OF REAL HEIGHT FORMULA FOR A LINEAR ELECTRON DENSITY PROFILE

Since the real height formulas derived here are of the same general type, we shall, for convenience, work with the equations for the extraordinary ray. For a linear gradient

$$f_N{}^2 = a\,h \tag{51}$$

where f_N and h are referred to suitable initial values which may be taken as zero. The apparent height $h'(f)$ is then given by

$$h'(f) = \frac{d}{df} \int_0^{h_r} \mu f\, dh$$

$$h'(f) = \frac{d}{df} \int_0^{(1-Y)f_n{}^2} \frac{f}{a} \left(1 - \frac{1}{1-Y}\ \frac{f_N{}^2}{f^2}\right)^{1/2} d(f_N{}^2) \tag{52}$$

where the upper limit corresponds to the value of f_N at which μ vanishes. The integral yield

$$h'(f) = \frac{d}{df} \left[\frac{2}{3a}\ (1-Y)f^3 \right] . \tag{53}$$

The quantities $H'(f)$ and $\frac{dH'(f)}{df}$ are then given by

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$$H'(f) = (1 - Y)^{\frac{1}{2}} \int_{f_H}^{f} h'(f) \, df$$

$$H'(f) = \frac{2}{3a} (1 - Y)^{3/2} f^3 \tag{54}$$

$$\frac{dH'(f)}{df} = \frac{(1 - Y)^{\frac{1}{2}}}{a} f(2f - f_H) \tag{55}$$

which can be expressed in terms of F by

$$\frac{dH'(F)}{dF} = \frac{F^{\frac{1}{2}}}{a} . \tag{56}$$

The real height h_r is then obtained from

$$h_r = \frac{2}{\pi} \int_0^{F_v} \frac{1}{(F_v - F)^{1/2}} \frac{dH'(F)}{dF} \, dF ,$$

$$h_r = \frac{2}{\pi a} \int_0^{F_v} \cdot \frac{F^{\frac{1}{2}}}{(F_v - F)^{1/2}} \, dF . \tag{57}$$

Changing the variable to $G = F^{1/2}$ yields

$$h_r = \frac{4}{\pi a} \int_0^{G_v} \frac{G^2}{(G_v^2 - G^2)^{1/2}} \, dG ,$$

which can easily be evaluated as

$$h_r = \frac{F_v}{a} ,$$

which, from Eq. (51), is identical with h_r. The solutions for the ordinary ray, both linear and curved portions, can easily be verified in a similar manner.

4. SUMMARY AND CONCLUSIONS

A method has been developed for directly calculating real height from virtual height data in the auroral region of the ionosphere. The principle approximation

in the method is the representation of the curved portion of the dispersion curve for the ordinary ray by an analytic form which reduces the virtual height-real height integral relation to a form of Abel's integral equation. Numerical checks show that the present method gives an accurate representation of the exact dispersion curve. It is therefore felt that good results should be obtained when the present procedure is applied to virtual height data. The correctness of the real height formula has been verified for the case of a linear electron density profile.

The present method will be applied to the numerical calculation of real height from virtual height data and its accuracy and convenience will be assessed by comparison of the results with those obtained from standard programs.

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