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ACCURACY ESTIMATES OF $1^{\circ} \times 1^{\circ}$ MEAN ANOMALY DETERMINATIONS FROM A HIGH-LOW SST MISSION

Richard H. Rapp and D. P. Hajela

The Ohio State University
Research Foundation
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The method of least squares collocation is used to estimate the accuracy and correlation of 1° x 1° anomalies that could be determined from a high-low satellite to satellite tracking mission. The observed data is taken to be the line of sight acceleration which can be computed from the range rate data. Variables considered in this study were: a) the spherical distance from the center of the 1° x 1° block -

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) within which data is selected for use; b) the accuracy of the "observed" accelerations; c) the height of the low satellite; and d) the data density or interval.

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Typical results indicate that at a low satellite height of 200 km, a data noise corresponding to a range rate accuracy of ±0.015 cm/sec, would yield a $1^\circ \times 1^\circ$ anomaly to an accuracy of about ±8 mgals with an average error correlation coefficient between adjacent blocks of -0.6. Lowering the satellite to 150 km reduces the accuracy to about ±5 mgals but increases the correlation to about -0.9.

This study does not consider the effects of orbit error, nor errors in the degree 12 reference field. In addition, some results could not be obtained when stable matrix inversions could not be obtained. This occured when dense data and/or low data noise was being used.

Foreword

This report was prepared by Richard H. Rapp, Professor, and D. P. Hajela, Post Doctoral Researcher, under Air Force Contract No. F19628-79-C-0027, The Ohio State University Research Foundation No. 711664. (Dr. Hajela is also an Associate Professor, Department of Civil Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.) The contract covering this research is administered by the Air Force Geophysics Laboratory, Hanscom Air Force Base, Massachusetts, with Mr. Bela Szabo, Contract Monitor.

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Introduction

The purpose of this report is to examine the accuracy of the determination of $1° \times 1°$ mean anomalies on the earth using satellite to satellite tracking data. We will not utilize new theories in our analysis. Rather we will use previously applied techniques in real data analysis to postulated satellite configurations and data noise estimates.

A number of previous studies in this general area have been carried out. One of the first simulation studies was carried out by Schwarz (1970). In this study the recovery and accuracies of 5°, 2°, and 1° anomalies from a high low and low low satellite configuration was examined using a data noise of 0.05 mm/sec. For his test orbits with two satellites at 300 km, the accuracy of 5° anomaly recovery was about ±1 mgal with correlation coefficients varying between -0.03 to -0.86. Tests were conducted with 2° blocks with satellites at 200 km. The results here indicated accuracies of about ±6 mgals with correlation coefficients of -0.6 to -0.8 between adjacent blocks. Schwarz also tried to recover $1° \times 1°$ information from 200 km and 100 km satellites. He found that even at the lower altitude, the uncertainties of the anomalies were of the order of ±60 mgals. He concluded $1° \times 1°$ anomaly recovery could not be carried out even from a 100 km altitude.

Hajela (1974) also carried out simulation studies to examine the estimation of 10°, 5°, and 2°.5 anomalies. In this study, Hajela used a high low configuration with data noise of 0.8 mm/sec for a 10 second integration time with satellites at 900 km and 250 km. With the data assumptions made by Hajela he found 2°.5 anomalies could be determined with an accuracy of about ±11 mgals from a satellite at 250 km, and 5° anomalies could be recovered to about ±9 mgals from a 900 km high satellite. The average correlation coefficient between blocks in an east-west direction was -0.2 in both cases.

In the simulations described above, a usual least squares adjustment procedure was carried out. Later work (Rummel, Hajela, Rapp, 1976) suggested that a more rigorous solution to the problem should be sought using least squares collocation techniques. In this report theoretical models and simulation studies were made using as a data type the radial component of the line of sight acceleration between two satellites in a high low configuration. Rummel et al. (ibid) found that 5° anomalies could be found with a predicted accuracy of about ±8 mgals, and to an accuracy of ±4 mgals if the low satellite was at 250 km.

The general method alluded to in the above paragraph was implemented by Hajela (1977) to data obtained between the ATS-6 and Geos-3 satellites. With somewhat limited data, Hajela found predicted accuracies of 5° anomalies to be about ±12 mgals although the difference between the predicted and known anomalies was about ±8 mgals.

This method of analysis was extended by Hajela (1978) using a more rigorous theory. With the improved theory and additional data 5° anomalies were recovered to a predicted accuracy of about ± 6 mgals. Using typical data configurations the correlation of the 5° blocks was found to be below 0.1.

Other studies have also been carried out in this general area. For example, Estes and Lancaster (1976) describe the recovery of $5^\circ \times 5^\circ$ blocks from a low-low configuration at 250 km altitude. Other work by Argentiero, and by Rummel (1979) are described in the report on "Applications of a Dedicated Gravitational Satellite Mission" (1979).

It is clear that no final answers on data noise, satellite configuration and theory are available. This report is designed to take a rather specific set of assumptions and to examine what they imply about the earth's gravity field.

The Theory

The theory for the recovery of the gravity anomalies using least squares collocation is taken from Hajela (1978). We deal only with a high-low configuration with a range rate measurement made between the high and low satellites. The actual estimation of a gravity anomaly, Δg, from a line of sight acceleration (T_ℓ) would be:

$$\Delta g = \underline{C}^T_{\Delta g, T_\ell} (\underline{\underline{C}}_{T_\ell, T_\ell} + \underline{\underline{D}})^{-1} \underline{T}_\ell \tag{1}$$

where: $\underline{C}_{\Delta g, T_\ell}$ is a column matrix of the covariances between the anomaly to be predicted and the vector of "observed" line of sight acceleration;

$\underline{\underline{C}}_{T_\ell, T_\ell}$ is the matrix of covariances between the line of sight accelerations;

$\underline{\underline{D}}$ is the noise matrix of the accelerations;

$\underline{T}_\ell$ is the vector of the "observed" line of sight accelerations.

The evaluation of the covariances needed in equation (1) can be carried out using analytical models for potential coefficient variations or anomaly degree variances. In our tests the anomaly degree variance model given by Tscherning and Rapp (1974):

$$c_n = \frac{A\ (n-1)}{(n-2)\ (n+B)} \tag{2}$$

with $A = 425.28$ mgal^2, $B = 24$, with an auxiliary quantity $s = 0.999617$. The details of the evaluation of the covariances are given by Hajela (1978).

The standard deviation, $\sigma_{\Delta g}$, of the predicted anomaly is given, for a single anomaly, as:

$$\sigma^2_{\Delta g} = C_0 - \underline{C}^T_{\Delta g, T_\ell} (\underline{\underline{C}}_{T_\ell, T_\ell} + \underline{\underline{D}})^{-1} \underline{C}_{\Delta g, T_\ell} \tag{3}$$

Here C_0 is the global variance of the anomaly being predicted. Thus, if we are estimating $1^\circ \times 1^\circ$ anomalies, C_0 is the global variance of $1^\circ \times 1^\circ$ anomalies.

The same data vector can be used to predict several anomalies simultaneously. The predicted anomalies will not be a function of the number of anomalies being predicted; however, the predicted anomalies will not be independently estimated. The error covariance matrix, $E_{\epsilon\epsilon}$, of the predicted anomalies will be (Hajela, 1978, p. 19):

$$\underline{\underline{E}}_{\epsilon\epsilon} = \underline{\underline{C}}_{\Delta g,\Delta g} - \underline{\underline{C}}_{\Delta g, r_\ell} (\underline{\underline{C}}_{r_\ell, r_\ell} + \underline{\underline{D}})^{-1} \underline{\underline{C}}_{r_\ell,\Delta g} \tag{4}$$

If n anomalies are being estimated $E_{\epsilon\epsilon}$ will be a n x n matrix. $\underline{\underline{C}}_{\Delta g,\Delta g}$ will be the n x n matrix of the anomaly covariances between the blocks being predicted. $\underline{\underline{C}}_{\Delta g, r_\ell}$ is a matrix expressing the covariances between the n anomalies and the observed line of sight accelerations. Equation (4) can be converted to an error correlation coefficient matrix in order to estimate the error correlation between the predicted anomalies. Note that equation (3) is a special case of equation (4) when $n = 1$. Also note that equation (4) can be evaluated independently from any actual observations; we need only know the locations of the two satellites involved, the accuracy of the data, and an anomaly degree variance model so that we can compute the needed covariances. In this sense we will be performing an error analysis and not a simulation study.

Data Noise

The evaluation of (4) requires we estimate the noise of the observed quantity. In our treatment of this problem the observed quantity is considered to be the line of sight acceleration (with respect to some acceleration implied by a reference gravitational field model). In fact, the observation is a range rate averaged over a certain time interval, say 10 seconds. To obtain accelerations it is convenient to fit spline functions to the data which are then differentiated to yield the accelerations. Usually the accuracy of the range rate data is given or postulated so that we need to propagate this range rate noise to acceleration noise. (Previous work described in Hajela (1978) made certain assumptions on the acceleration noise without propagating range rate noise.)

The error propagation was examined for this study by a Monte Carlo technique. The initial data used were a set of residual (with respect to a 12, 12 potential coefficient field) range rate data for two Geos-3 arcs (439 and 453), Marsh et al. (1977).

121 smoothed $\dot{R}$ values at 10 second intervals were used in each arc for a duration of 20 minutes as the Geos-3 satellite moved in the area (61° N to 3° N, 329° E to 264° E).

An interpolating cubic spline was fitted to the 121 smoothed $\dot{R}$ values separately in each arc using IMSL (1977) subroutine ICSICU and the first derivative of the spline giving $\ddot{R}$ was computed using subroutine DCSEVU. The 121 $\ddot{R}$ values in each arc were treated as known values based on the known 121 smoothed $\dot{R}$ values in each arc. The known values may also be termed as the original or imperturbed values.

The original smoothed $\dot{R}$ values were then impressed by random noise with normal distribution (mean zero, and different standard deviations, e.g. 0.01 cm/sec, etc.). Random numbers with normal distribution were generated using different seed values with IMSL subroutine GGUN. For each combination of cases, described below, both arcs were treated similarly, giving two separate values for each treatment.

The perturbed $\dot{R}$ values simulating $\dot{R}$ with (different) 'observational noise' were approximated in the least squares sense with cubic splines with fixed knots with different (60 to 150 sec., described below) spacings between the knots giving different smoothing to the raw data. This procedure has been described by Hajela (1978, Sec. 2.4, pp. 10-14). The observational 'noise' in the raw (or perturbed) $\dot{R}$ values was thus filtered and a smoothed representation obtained. The first derivative of the 'smoothing' spline gave the 'observed' $\ddot{R}$ data from the raw $\dot{R}$ values.

The RMS value of the differences of 'observed' minus original $\ddot{R}$ gave a measure of the standard deviation (s.d.) of observed $\ddot{R}$ depending on the standard deviation of the observational 'noise' in $\dot{R}$ values. The lowest value of the s.d. of $\ddot{R}$ for different 'smoothing' splines (different spacing of fixed spline knots) for a particular s.d. of $\dot{R}$ noise also indicated the optimum smoothing for that noise 'level'. To avoid any spurious effect due to observations in the end intervals, the differences ($\Delta\ddot{R}$) of observed minus original $\ddot{R}$ were not considered in the last two intervals of the spline at each end. Depending on the spacing of the spline knots (60 to 150 seconds), the RMS values of $\Delta\ddot{R}$ were thus generated based on 60 to 96 differences.

The variations were as follows:

(a) 2 arcs.
(b) Each arc perturbed 10 different times for a particular standard deviation of noise in $\dot{R}$ values.
(c) Different s.d. of noise in $\dot{R}$ tried were:
0.005, 0.01, 0.015, 0.05, 0.07, 0.08, 0.09 cm/sec.
Th last 4 values were representative of 'destruct' mode and the first three of what may be possible in non-destruct doppler data in future missions.
(d) For any given standard deviation of $\dot{R}$ noise, three different smoothing splines were tried to obtain an 'optimum' smoothing. The spline knot spacing of 60, 80, and 100 sec were tried for the first three s.d. of 0.005, 0.01, 0.015 cm/sec. The spline knot spacing of 100, 120, and 150 sec were tried for the last three s.d. of 0.07, 0.08, 0.09 cm/sec. The spline knot spacing of 60, 80, 100, and 120 sec were tried for the s.d. of 0.05 cm/sec.

The 'optimum' smoothing (for the lowest RMS $\Delta\ddot{R}$) was found to be

120 sec spline for 0.07 to 0.09 cm/sec.
80 sec spline for 0.05 cm/sec.
60 sec spline for 0.005 to 0.015 cm/sec.

The mean value of 10 RMS $\Delta\ddot{R}$ in each arc was computed (for the 10 random cases) for a particular standard deviation of $\dot{R}$, and the standard deviation for these 10 RMS $\Delta\ddot{R}$ was also computed. The values for the case of optimum smoothing are given in Table 1 and shown graphed in Figure 1.

Similar Monte Carlo experiments of approximating mathematically smooth functions have been described by Craven and Wahba (1979); and for approximating empirical data by Wold (1974).

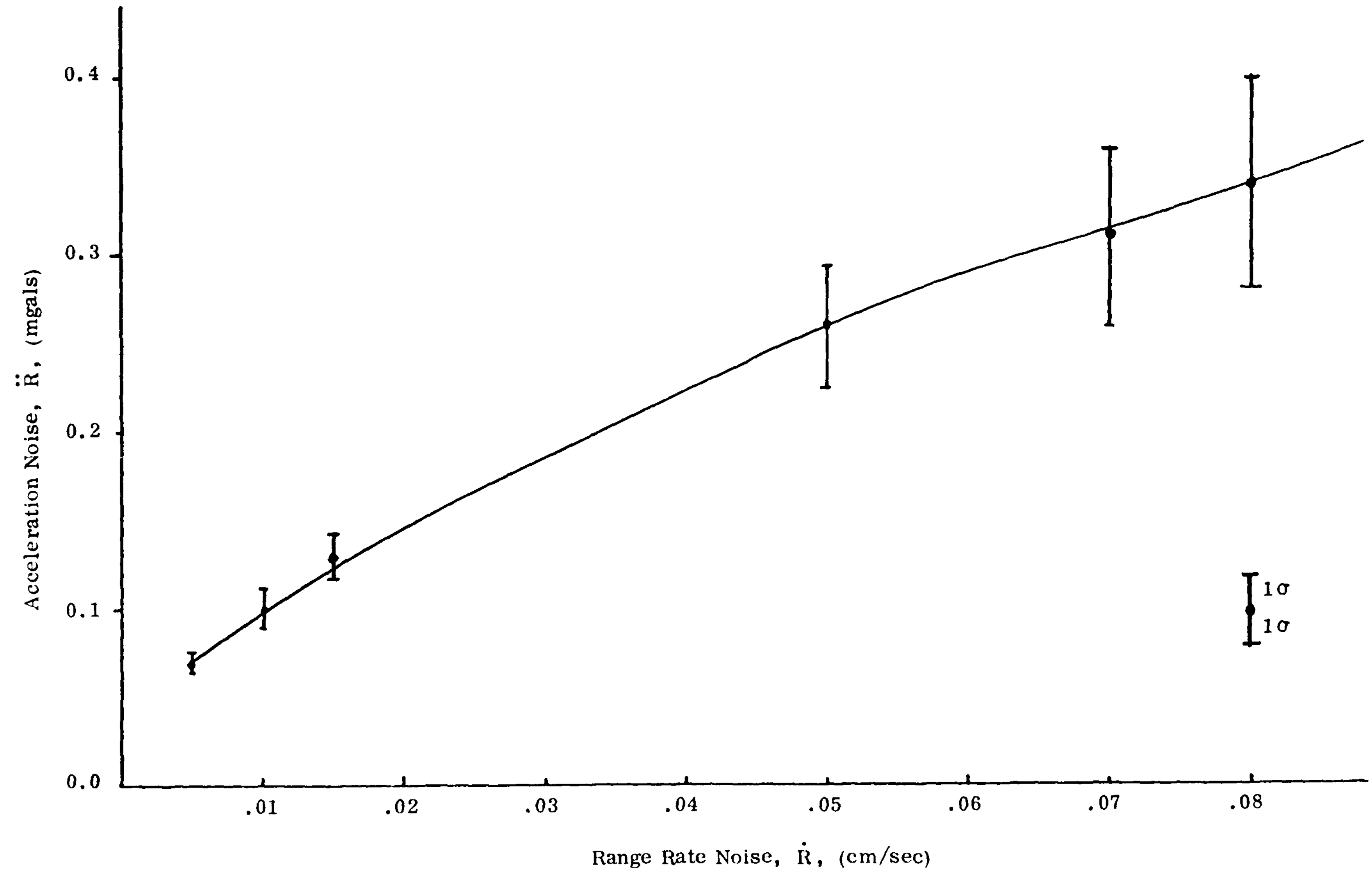
Table 1. Standard Deviation of Line of Sight Accelerations Based on Assumed Data Noise on $\dot{R}$

Standard Deviation of $\dot{R}$ cm/sec	RMS $(\Delta\ddot{R})$ in mgals	
	Mean*	Std. Dev.*
0.005	0.07	0.01
0.010	0.10	0.02
0.015	0.13	0.03
0.05	0.26	0.07
0.07	0.31	0.10
0.08	0.34	0.12
0.09	0.37	0.14

* Mean and standard deviation of RMS $(\Delta\ddot{R})$ are based on 10 cases each in 2 separate arcs.

We note that the range rate noise of ± 0.15 mm/sec is that anticipated to be possible from a high low satellite mission (Committee on Geodesy, 1978, p. 65). This accuracy is about 2.5 to 4 times better than found in the Geos-3/ATS-6 data used by Hajela (1978).

Figure 1. Variation of Acceleration Noise with Range Rate Noise



Results

We can now consider the evaluation of equation (4) using different data noises, different data selection criteria, and different satellite heights above the earth. In all cases, the high satellite was at a height or geocentric distance of ATS-6. The low satellite was considered to be in a nominally polar orbit. The latitude of the blocks being considered was around 25°. All covariances were computed after removing a reference field to degree 12. (Tests were made, in some cases, with reference field to degree 20 and degree 30, but no significant change in the final results was seen.)

We should also note that all covariances, but one, in equation (4) were evaluated by numerical integration of the covariance functions over the block(s) being studied. The $\underline{\underline{C}}_{\Delta g, \Delta g}$ matrix was generated by a series approach using smoothing operators designed for the area of the blocks being considered.

The results we obtained are presented in a series of tables that are described in the following paragraphs.

Variations Caused By Data Selection Distance ψ

The data to be used in any prediction process must be carefully selected because the size of the matrix to be inverted is equal to the number of observations selected for use. To examine this we considered the selection of data only within a specified spherical radius of the center of the $1° \times 1°$ block. The interval of the data was taken as $1° \times 1°$ with three spherical distances and two data noises. The standard deviations and average of the correlation coefficients, ρ, for adjacent blocks are shown in Table 2.

Table 2. $1° \times 1°$ Anomaly Standard Deviations and Correlation Coefficients as a Function of Spherical Distance (ψ) from the Center of the Block; Low Satellite at 200 km, Data Spacing $1° \times 1°$.

	Data Noise (.13 mgals, .015 cm/sec)		Data Noise (.20 mgals, .035 cm/sec)	
$\psi°$	Std. Dev.	Average ρ	Std. Dev.	Average ρ
2°	8.7 mgals	-.43	9.5 mgals	-.34
3°	7.5	-.60	8.7	-.45
4°	7.1	-.68	8.4	-.49

We see from this table that the increase of data reduces the standard deviation from 8.7 mgals to 7.1 mgals as ψ goes from 2° to 4° (noise = ±.13 mgals). However, at the same time the error correlation between adjacent blocks increases by 58%. Somewhat decreased accuracies and correlations are found with the poorer noise case.

Variations Using Different Data Noise Values and Different Low Satellite Heights

We next examined how the predicted accuracies would depend on the data noise when we put the close satellite at three different elevations, and selected data at $1^\circ \times 1^\circ$ intervals out to the spherical distance of 3°. These results are shown in Table 3.

Table 3. $1^\circ \times 1^\circ$ Anomaly Accuracies as a Function of Data Noise and Low Satellite Height with Data Taken out to a Spherical Distance of 3° at a $1^\circ \times 1^\circ$ Interval.

Data Noise		H = 250 km		H = 200 km		H = 150 km	
$\dot{R}$ (cm/sec)	$\ddot{R}$ (mgals)	Std. Dev.	Avg. ρ	Std. Dev.	Avg. ρ	Std. Dev.	Avg. ρ
.015	0.13	±10.2 mgals	-.31	7.5	-.60	4.5*	-.85*
.035	0.20	11.2 mgals	-.22	8.6	-.45	5.5*	-.65*
.10	0.40	12.8 mgals	-.08	10.6	-.25	7.3	-.56

* Extrapolated values and not results from actual estimation.

A number of observations can be made from this table. First, we see that the standard deviations of the recovered anomalies are not linear with the accuracy of the data. This is because the accuracy estimate is made up of two components; one the representation part (caused by lack of data) and two the noise part. Thus, as the acceleration accuracy decreases by a factor of 3, the resultant standard deviations of the $1^\circ \times 1^\circ$ anomalies only changes by 12%. However, we do see a significant decrease in the error correlation between the blocks as the noise increases.

Second, we see that there is the expected improvement of the accuracies as the height of the low satellite decreases. However, the penalty in this is the increased error correlation of the adjacent blocks. Note that two noise cases for H = 150 km did not yield meaningful results because of matrix stability problem and that the values given in the table have been extrapolated from the other data.

Variations Using Different Data Selection Intervals

In the previous tests we have taken data at a $1^\circ \times 1^\circ$ spacing out to a defined spherical distance. We now look at the variation in accuracy caused by varying the data interval. These results are shown in Table 4 for a low satellite height of 200 km and an acceleration noise of ±0.20 mgals (±.035 cm/sec in range rate).

Table 4. $1^\circ \times 1^\circ$ Anomaly Accuracies as a Function of Data Spacing Variation; Low Satellite Height is 200 km and Data Noise is ± 0.20 mgals.

	$\psi = 2^\circ$		$\psi = 4^\circ$		$\psi = 6^\circ$	
Data Spacing	Std. Dev.	Avg. ρ	Std. Dev.	Avg. ρ	Std. Dev.	Avg. ρ
$\frac{1}{2}^\circ \times \frac{1}{2}^\circ$	7.7 mgals	-.56				
$1^\circ \times 1^\circ$	9.5 mgals	-.34	8.4 mgals	-.49		
$1\frac{1}{2}^\circ \times 1\frac{1}{2}^\circ$					10.6 mgals	-.33
$2^\circ \times 2^\circ$			13.0 mgals	-.22	12.9 mgals	-.23

We see that from this table that doubling the data information from $1^\circ \times 1^\circ$ to $\frac{1}{2}^\circ \times \frac{1}{2}^\circ$ yields only a 10% decrease in the predicted standard deviation ($\psi = 2^\circ$). The increased data also causes the error correlation to increase.

We note that smaller data noise was tested in runs for Table 4 but that meaningful results were not obtained due to matrix instability problems.

Variations Using Only the Radial Component of the Disturbing Acceleration

The results described so far have been obtained using the line of sight acceleration. The use of such information is more accurate than the use of only the radial component as tested in Rummel et al. (1976) and implemented by Hajela (1977). However, we find that the use of only this component yields more stable matrices to be inverted. Consequently we show in Table 5 some results when we assume we have only the radial component with a data spacing of $0^\circ\!.25 \times 0^\circ\!.25$ within the $1^\circ \times 1^\circ$ block and $0^\circ\!.5 \times 0^\circ\!.5$ from there to 2° from the block center.

Table 5. $1^\circ \times 1^\circ$ Anomaly Standard Deviations Assuming Only the Radial Component of the Line of Sight Acceleration is Used; Data Spacing is $0^\circ\!.25$ and $0^\circ\!.5$.

Noise	H = 250 km	H = 200 km	H = 150 km
± 0.10 mgals	± 10.1 mgals	$\pm$ 7.8 mgals	± 5.4 mgals
0.13	10.5	8.2	5.7
0.40	12.3	10.1	7.5

These accuracies will decrease by about 15% if the line of sight acceleration was used with additional data out to $\psi = 3^\circ$

Summary and Conclusions

We have examined the accuracy of the estimation of $1^\circ \times 1^\circ$ gravity anomalies at the surface of the earth using only postulated range rate data available from a high-low SST mission. The procedure we used was an error analysis through least squares collocation. In this manner a rigorous downward continuation procedure through the covariances of the involved quantities.

Since we used as our observational data line of sight acceleration it was necessary to determine its accuracy based on the accuracy of the expected range rate observations. Through Monte Carlo techniques a range rate noise of 0.015 cm/sec corresponds to an acceleration noise of ± 0.13 mgals.

We examined the variations of the predicted anomaly standard deviations as a function of the spherical cap radius within which data is being used; as a function of data spacing; as a function of the data noise; and as a function of three different low satellite heights.

In brief, at a height of the low satellite equal to 200 km, a data noise corresponding to ± 0.015 cm/sec in the range rate data, we would expect a $1^\circ \times 1^\circ$ anomaly to be determined to about ± 8 mgals with an average correlation coefficient between adjacent blocks of -0.6. If the low satellite height is brought down to 150 km the predicted standard deviation will be about ± 5 mgals with correlations of about -0.9. It appears that additional data or improved accuracy in the range rate observation will not substantially improve these results.

Several words of caution are needed in interpreting the results described here. First, we have considered only two error sources. The first is due to the representation error (only finite data at some specified height) and the data noise error as propagated into the predicted standard deviations. We have completely ignored the error caused by the lack of a sufficiently precise orbit. The effect of orbital element error on the range rate data has been discussed in Rummel et al. (1976). Basically, we expect it to be long wavelength error in contrast to the high frequency information that we are trying to recover. Consequently, we assume that such errors could be removed from the residuals by some filtering process.

We have also assumed our reference field to degree 12 to be perfectly known. If it were not, we should be adding an error component to our final results. We expect this to be small because the resolution in the reference field is much broader than the area ($1^\circ \times 1^\circ$) that we are interested in.

And finally, we emphasize that we had problems in obtaining final results in some cases with very low data noise and/or dense data observations. Such problems seem to be common in least squares collocation solutions when the matrix of the observation covariances ($\underline{\underline{C}}$) has instability in its inversion. Improved analysis techniques are needed to improve the stability problem. Alternately, use of existing ground anomalies could help the situation.

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